MINIVER UPGRADE FOR THE AVID SYSTEM

VOLUME I: LANMIN USER'S MANUAL

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FOREWORD

This final report presents work which was conducted for Langley Research Center (LaRC) in response to requirements of Contract NAS1-16983. The work presented was performed by REMTECH Inc., Huntsville, Alabama and is entitled "MINIVER Upgrade for the AVID System." The final report consists of three volumes.

Volume 2: LANMIN Input Guide
Volume 3: EXITS User's and Input Guide

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Section 1.0
INTRODUCTION

The successful design of thermal protection systems for vehicles operating in atmosphere and near-space environments requires accurate analyses of heating rate and temperature histories encountered along a trajectory. For preliminary design calculations, however, the requirement for accuracy must be tempered by the need for speed and versatility in computational tools used to determine such "thermal environments."

The MINIVER program (Ref. 1) over the last decade has been found to provide the proper balance between versatility, speed and accuracy for an aerotherm al prediction tool. The advancement in computer aided design concepts at Langley Research Center (LaRC) in the past few years has made it desirable to incorporate the MINIVER program into the LaRC AVID system (Ref. 2). The purpose of the AVID system is to provide the preliminary design engineer with a useful tool for multi-discipline interactions to perform partial or complete vehicle synthesis.

In order to effectively incorporate MINIVER into the AVID system, several changes to MINIVER were made. The thermal conduction options in MINIVER were removed and a new Explicit Interactive Thermal Structures (EXITS) code was developed. Many upgrades to the MINIVER code were made and a new Langley version of MINIVER called LANMIN was created.

This report is divided into three volumes. Volume I describes the theoretical methods and subroutine functions used in LANMIN. Volume II provides a user
input guide for LANMIN. Volume III describes the EXITS code and provides an input guide.

The documentation presented in Volume I utilized prior documentation where applicable and added new material for upgrade areas. The primary sources of prior documentation are Ref. 1 for the original MINIVER code, Ref. 3 for subroutine descriptions and Ref. 4 for rarefied flow updates.
Section 2.0
GENERAL UTILITY ROUTINES

The program uses several subroutines of general utility such as specification of printing of input and output data, interpolation techniques and the managing of all other subroutines. This section gives a description of these utility routines with a discussion of their capabilities.

2.1 MAIN

MAIN performs the task of managing the rest of LANMIN. This routine is best described by the flow diagram in Figure 2-1. The MAIN routine sets up the input, calls the calculation subroutines in proper sequence, saves and controls the output, and controls the case to case input. Most of the input is accomplished through an array W(700). Many of the input array elements are equivalent to other variable names. A complete description of the input is given in Volume II of this report.

2.2 SUBROUTINE WRINF

Subroutine WRINF is called by MAIN to print out all of the input data for each separate case. This routine prints out the pertinent input information.

2.3 SUBROUTINE VANOUT

Subroutine VANOUT is called by MAIN and is used to print the output in a specified format. The parameters in the CALL statement of VANOUT are as follows:

D - (input) This matrix dimensioned (100, 13) contains 13 variables as a function of time. The 13 variables are: time, altitude, velocity, Mach number, angle of attack, Reynolds number/ft, heat transfer coefficient, recovery enthalpy, equilibrium wall temperature, convective heating rate, heating load, local pressure, and flow type.

IF - (input) Number of time points.
TITLE - (input) This array dimensioned (18) contains an input title for the case.

AMET - (input) This variable corresponding to W(646) determines the output units. AMET = 0.0 corresponds to English units and AMET > 0.0 corresponds to metric units for output.

BPN - (input) Body point number corresponding to W(611).

This subroutine writes to paper and to a file. Each case on the file is ended with a flag having a numerical value of -100. for the variable TIME.

2.4 SUBROUTINE TBLIN

Subroutine TBLIN is a utility linear interpolation routine which is used extensively throughout LANMIN by other routines. The following discussion considers the use of this routine in conjunction with an atmospheric table which does not exist as a routine in the program, but rather comes into the program via input data.

The argument of the routine is TBLIN (X, XX, Y, YY, Z, ZZ, N). The parameter X acts as an independent variable and the parameters Y and Z act as functions of X. When a value X is input to TBLIN, the values Y and Z are returned at the corresponding X. The array XX contains all of the table values of the independent variables X in a one-dimensional array. The one-dimensional arrays YY and ZZ contain all of the table values of the dependent variables. Since each of these arrays (XX, YY, ZZ) contains the same number of elements, the parameter N is used to indicate how many values are contained in each array and thus how large each array is to be dimensioned in TBLIN.

It should be noted that this routine can only be used when the values in array XX are input in nondecreasing order. The results of this linear interpolation routine are not valid if the values in the independent variable array XX
are not listed in increasing order. An example of the use of this routine occurs when a value of altitude (X) is input. The routine will return values of ambient temperature (Y) and pressure (Z) via linear interpolation from the temperature and pressure arrays, YY and ZZ.

Upon entering the routine, a check is made to determine if the independent variable X is less than the first (smallest) value in the array XX. If it is, then the first element of array YY and the first element of array ZZ are returned as Y and Z, respectively, to the calling routine. If the independent variable X is greater than the last (largest) element in array XX, then the last element of array YY and the last element of array ZZ are returned as Y and Z, respectively, to the calling routine. If the value of the independent variable X is not out of the range of the array XX, then linear interpolation is employed to determine the corresponding values of Y and Z.

2.5 SUBROUTINE TINI6

Subroutine TINI6 is a linear interpolation routine. This routine interpolates six or less dependent variables simultaneously corresponding to an input independent variable. The argument of TINI6 contains 16 elements. The first two elements in the argument are values of the independent variable and the array of stored values of the independent variable. The next 12 locations of the subroutine argument occur in six pairs corresponding to the six dependent variables. The first element of the pair is the value of the dependent variable returned by the program, and the second element of the pair is the array of input values corresponding to the values in the independent variable array. The independent and dependent variable arrays may each contain a maximum of 10 values. The last element of the subroutine argument corresponds to the number of values contained in the independent variable input array (10 or less).
The next-to-last element of the subroutine argument contains an integer value. If the value of this integer is greater than 0, then TINT6 performs interpolation assuming the independent variable is nonincreasing. If the value of this integer is less than zero, then TINT6 performs linear interpolation assuming the independent variable to be nondecreasing.

Now consider the case where the independent variable is nondecreasing. Before interpolation is attempted, the input value of the independent variable is checked to determine if it is less than the first element in the independent variable array. If the input value is less than this first element, then the values of the dependent variables returned to the calling routine are set equal to the first element of each array. If the independent variable is larger than the first element, then another check occurs to determine if the independent variable is greater than the last element in the independent variable array. If it is, then the values of the dependent variables returned to the calling routine are the last values in each dependent variable array. If the independent variable passes both tests, then TINT6 inspects to resolve whether or not the independent variable is equal to one of the values in the independent variable input array. If equality occurs, then the value returned for each dependent variable is that value in each dependent array corresponding to the independent variable.

If the value of the independent variable is not found to be less than the first element, greater than the last element, or equal to any of the elements in the input array, then linear interpolation occurs using the two elements of each dependent array which corresponds to the two input independent variables bracketing the input independent variable.
An analogous procedure is performed by TINT6 if the independent variable is nonincreasing instead of nondecreasing.

This routine is written for either a nondecreasing independent variable or a nonincreasing independent variable, such as time increasing during a trajectory.

2.6 FUNCTIONS INTIP1 AND INTIP2

These functions are used to perform linear interpolations in semi $\log_{10}$ space. The arguments of INTIP1 are:

- I - (input) Pressure index
- J2 - (input) Highest temperature index
- J1 - (input) Lowest temperature index
- K - (input) Thermal property index
- F - (input) Delta value of independent variable is $(X-J1)/(X2-J1)$

Values are interpolated from the $G(I, J, K)$ matrix.

The arguments of INTIP2 are:

- X - (input) Lowest value of dependent variable
- Y - (input) Highest value of dependent variable
- F - (input) Delta value of independent variable

Both functions linearly interpolate in $\log_{10}$ space of the dependent variable.
Fig. 2.1 Routine MAIN
Section 3.0

FLUID PROPERTY ROUTINES

In this section, the options of computing the free-stream air properties
are set forth. These include ideal and real gas effects.

3.1 SUBROUTINE AIR62

This is one of the atmosphere routines in MINIVER. The routine describes
the U. S. Standard Atmosphere, 1962 (ref. 2). Given altitude (ft), the routine
yields ambient temperature (degrees R), density (slug/ft^3), pressure (lbf/ft^2),
and speed of sound (ft/sec). The altitude range of the table is from 0 to
2,068,776 feet above sea level. The thermodynamic properties, temperature and
density, are exponential curve-fits to the U. S. Standard Atmosphere, 1962. The
ambient pressure and speed of sound are computed from the ideal gas equations.
This routine does not call any other routines and returns only to the calling
routine.

3.2 SUBROUTINE ATMS4

This is the 1963 Patrick atmosphere routine. The input into the routine is
altitude (ft) and the routine returns ambient temperature (degrees R), density
(slug/ft^3), pressure (lbf/ft^2), and speed of sound (ft/sec). The range of alti-
tudes is from 0 to 2,300,000 feet with errors of less than 3 percent below
400,000 feet. This routine is entire within itself and does not call any other
subroutines. The routine breaks up the altitude span into several ranges. Each
altitude range then has the atmospheric properties calculated via a combination
of polynomials and exponential curve-fits.

3.3 SUBROUTINE VRA71

This is the Vandenberg 1971 reference atmosphere. The input into the
routine is altitude (ft) and the routine returns ambient temperature (degrees

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The range of altitude is from 0 to 2,300,000 ft. This routine does not call any other routines. The subroutine calculates ambient viscosity and molecular weight, but these values are not used.

3.4 ROUTINES MOLIER, BLOCK, BLOCKA AND SLOPE

These are the routines constituting the air Mollier diagram and act as an equation of state to account for real gas effects. The data are contained in the G(I, J, K) matrix.

\[
\begin{align*}
G(I, J, 1) &\quad - \text{Temperature} \\
G(I, J, 2) &\quad - \text{Enthalpy} \\
G(I, J, 3) &\quad - \text{Entropy} \\
G(I, J, 4) &\quad - \text{Viscosity} \\
G(I, J, 5) &\quad - \text{Prandtl Number} \\
G(I, J, 6) &\quad - \text{Speed of Sound} \\
G(I, J, 7) &\quad - \text{Density}
\end{align*}
\]

The block data BLOCK contains the pressure array PT(I4) and the variables G(I, J, K), K = 1,4. The block data BLOCKA contains the variables G(I, J, K), K = 5,7.

Four options may be used to look up properties based on two independent variables. These options are:

\[
\begin{align*}
\text{NOPT} = 0 &\quad \text{P,H (Pressure, Enthalpy)} \\
1 &\quad \text{P,S (Pressure, Entropy)} \\
2 &\quad \text{P,T (Pressure, Temperature)} \\
3 &\quad \text{H,S (Enthalpy, Entropy)}
\end{align*}
\]

Based on the options selected, the limits of the program are checked

\[
\begin{align*}
10^{-20} \leq P \leq 10^9 \text{ Atm.} \\
48 \leq H \leq 75580 \text{ Btu/lbm} \\
200 \leq T \leq 30000 \text{ R} \\
.925 \leq S/R \leq 12.368
\end{align*}
\]

If the independent variables are outside of the preceding range of validity, the routine sets all thermodynamic variables to zero and returns with IDEAL = 1.

Given the independent variables, the dependent variables are obtained by
interpolation in the G matrix. For NOFT = 0, 1, and 2, the interpolation procedure is the same. The pressure index is found corresponding to the table pressures below and above the input pressure level. At the lower pressure level, the dependent variables are determined by two calls. First, the function SLOPE is called to calculate

\[ FT1 = \frac{X - X1}{X2 - X1} \]

where X is the log10 of the other independent variable and the X1 and X2 values are the log10 values of the nearest points in the G matrix for the lower pressure. Given the value of FT1, all of the dependent variables are calculated by linear interpolation in log10 space using FT1 in a call to INTP1. Likewise, the dependent variables are determined at the higher pressure level in the tables. Finally, the dependent variables are determined for the input pressure level by linear interpolation in log10 (pressure) space by calling INTP2.

For NOFT = 3 the interpolation procedure is similar but uses different logic. The G matrix is sorted to obtain the surrounding points of enthalpy and entropy. The nondimensional entropy is used in a call to SLOPE to obtain the parameter

\[ FT1 = \frac{S - S1}{S2 - S1} \]

at the lower pressure level where S is the log10 of the entropy. A call is made to INTP1 to obtain the dependent variables at this pressure level. This process is repeated for the higher pressure level. Finally, the dependent variables are determined at the given enthalpy level by a call to INTP2.
After the calculations and interpolations are made, some variables are converted in units and the compressibility and specific heat ratio are calculated for output.

The arguments of the call for MOLIER are

- H - (input/output) Enthalpy
- P - (input/output) Pressure
- NOPT - (input) Independent Variable Option
- T - (input/output) Temperature
- Z1 - (output) Compressibility Factor
- S - (output) Entropy
- R - (output) Density
- G1 - (output) Specific Heat Ratio

In addition the viscosity is output via the common statement /BANK/.

3.5 SUBROUTINE BINTERP

Subroutine BINTERP can be used to calculate the Prandtl number Pr, Lewis number Le, and dissociation enthalpy Hd for air. The Prandtl number is computed as a function of temperature (degrees R) and as a function of the ratio of the pressure at the stagnation point to standard pressure. The Lewis number and dissociation enthalpy are calculated as a function of temperature (degrees R) and as a function of the ratio of the density at the stagnation point to the density at standard conditions (2.37 \times 10^{-3} \text{ slug/ft}^3). Routine BINTERP is composed of three tables. Table I corresponds to dissociation enthalpy, Table II corresponds to Lewis number, and Table III corresponds to Prandtl number. The first two elements of the routine argument are the temperature (degrees R) and the pressure ratio, or the temperature (degrees R) and the density ratio. The third element of the routine argument is the table number (I, II, III). The last (fourth) element of the routine argument is either Hd, Le, or Pr, depending upon the input. Upon entering this routine, various checks occur to insure that the input values are not less than those values for which a calculation can be performed. If the temperature is negative, the pressure ratio is less than 10^{-6}.
or the density ratio is less than $10^{-3}$, then an error message is printed and 
*ERROR* = 1 is returned to the calling routine. If the tests are passed, then 
BINTRP proceeds to use double interpolation to determine the appropriate output. 
If $Hd < 0$, then $Hd = 0$ is returned. If $Hd > 12450$ Btu/lbm, then $Hd = 12450$ 
Btu/lbm is returned. If $Le < 0.5$, then $Le = 0.5$ is returned to the calling 
routine. If $Pr < 0.678$, then $Pr = 0.678$ is returned. If $Pr > 0.734$ and 
$T > 24000$ degrees R, then $Pr = 0.734$ is returned. Presently this routine is 
used only to determine $Hd$ and $Pr$. Thus, the other one-third of this routine is 
not used.

3.6 SUBROUTINE HANSEN

Subroutine HANSEN is called by routine FLOW to calculate viscosity as a 
function of temperature and pressure. This subroutine calls MOLIER and returns 
viscosity via common HANK. If the viscosity is greater than the switch value of 
$0.29531E-08$ slug/ft sec, the viscosity from MOLIER is returned. If the viscosi-
ty from MOLIER is less than the switch value, the viscosity is calculated based 
on the relations of Hansen and Helms (NACA TN 4359).

$$\mu = \mu_s B/D$$

where

$$FL = \log_{10} \frac{P}{2116}.$$  
$$A = (\frac{T}{1800})(1-0.125FL) - 6.5)/\left(1.5+0.125FL\right)$$

$$B = 1.0 + 0.023\left(\frac{T}{1800}\right)(1 + \tanh(A))$$

$$C = \left(\frac{T}{1800} - 14.5 - 1.5FL\right)/(0.9 + 0.1FL)$$

$$D = e^{C + 1.0}$$

$$\mu_s = 2.27 \times 10^{-4} \frac{T^{0.6}}{(T + 198.6)} \quad \text{(Sutherland)}$$

$$T = \text{Temperature (Degrees R)}$$

$$P = \text{Pressure (lbf/ft}^2)$$

If the value of $B/D < 0.04$ then $B/D$ is set equal to 0.04.
Section 4.0
FLOWFIELD ROUTINES

This section of the report deals with those subroutines that are used to compute the local flowfield and boundary-layer edge conditions around the body under consideration. The methods used to evaluate shock angles, flowfields after shocks, and Prandtl-Meyer expansions are presented.

4.1 SUBROUTINE FLOW

Subroutine FLOW is called by MAIN. The subroutine argument contains two parameters, FF and ALPHA. The element FF is a one-dimensional array containing a possibility of nine consecutive flowfield and pressure flags (always occurring in pairs with the exception of the swept cylinder flowfield FF = 39 and the Prandtl-Meyer expansion flag FF = 29). The element ALPHA is a one-dimensional array containing a possibility of nine consecutive flowfield angles corresponding to the flowfield flags. The transfer of other information, such as upstream conditions, downstream conditions, etc., is performed through common blocks. The free-stream temperature and pressure are first needed to compute free-stream viscosity via subroutine HANSEN. The first check performed by FLOW determines if the Mach number upstream $M_u$ of the first flowfield flag is greater than unity. If $M_u < 1$ and this is the first or second flag in the array FF, then the program ignores the flag and proceeds to calculate stagnation conditions based on the isentropic ideal gas relations. If the flag following the $M_u < 1$ verification calls for a Prandtl-Meyer expansion (FF = 29), then a deceleration of the flow occurs. The flow decelerates to a higher pressure calculated by modified Newtonian theory.

Knowing the local stagnation conditions and a local subsonic modified
Newtonian pressure, all of the other local properties are determined by the
ideal gas isentropic relations. If the deceleration angle is greater than or
equal to 90 degrees, then the local conditions are set equal to stagnation con-
ditions. For subsonic flow, the local conditions determined above are also set
equal to the local conditions along the stagnation line of an infinite swept
cylinder. Having the local flowfield conditions, the Eckert reference proper-
ties are next calculated. If the heat transfer flag is equal to 5 or 7, then
the $\rho_\text{f} u_\text{f}$ properties are also calculated. The routine then returns to MAIN.

If $M_\text{u} < 1$ and this is not the first flowfield or pressure flag, then the
local stagnation conditions are taken to be the same as those corresponding to
the conditions downstream of the previous pressure or Prandtl-Meyer flag. If
the flowfield flag following $M_\text{u} < 1$ calls for a Prandtl-Meyer expansion ($FF =
29$), then a deceleration of the flow occurs. The flow decelerates to a higher
pressure again computed via modified Newtonian theory. Knowing the pressure and
also having the entropy from the (isentropic) upstream conditions, the other
local thermodynamic properties are determined by using the Mollier diagram (su-
broutine MOLIER). If MOLIER returns IDEAL $> 0$, then the program switches to the
ideal gas calculation previously discussed for $M_\text{u} < 1.0$. If MOLIER returns
IDEAL = 0, then FLOW checks to determine if the deceleration angle is greater
than or equal to 90 degrees. If the deceleration angle is greater than 90 de-
grees, then the local conditions are set equal to the local stagnation condi-
tions. For subsonic flow, the final computed local conditions are also set
equal to local conditions along the stagnation line of an infinite swept cyli-
der. Knowing the local flow properties, the Eckert reference properties are
next calculated. If the heat transfer flag is equal to either 5 or 7, then the
$\rho_\text{f} u_\text{f}$ properties are also calculated. The FLOW returns to MAIN. The difference
between $M_u < 1$ and $FF = 1$ or $2$, and $M_u < 1$ and $FF > 2$ is that for $FF = 1$ or $2$ the fluid is treated as an ideal gas, and for $FF > 2$ the fluid is treated as a real gas. Only the equations of state are different.

Now if $M_u > 1$, FLOW checks to determine if $FF = 35$, $FF = 36$, $FF = 38$ (there is no $FF = 37$) or $FF = 39$. Each of these flowfield flags corresponds to a different technique for determining the shock angle $\beta$. For $FF = 35$ or $36$, the value of ALFA is taken as the flow deflection angle for a sharp wedge or sharp cone, respectively. For $FF = 38$, the shock angle is read into the program as ALPHA. And $FF = 39$ corresponds to the swept cylinder flowfield and the shock angle is equal to ALFA. Subroutine FLOW (for $FF = 35$ or $36$) calls routine PCSW to determine the attached shock angle as a function of $M_u$ and the wedge or cone deflection angle. If NERROR $\neq 0$ is returned from PCSW, then the input deflection angle must have been either negative or equal to zero. FLOW then immediately returns to MAIN and transfers to the next data case. The next check is to determine the value of OFT. If OFT $= 1$, then the flow upstream of the wedge must have been subsonic. If OFT $= 2$, then the shock associated with the wedge is detached and the shock angle is, therefore, taken to be 90 degrees. (If the shock is detached, FLOW performs a normal shock expansion (modified Newtonian) to the pressure flag angle associated with the flowfield angle.) If OFT $= 0$, then PCSW has successfully determined the shock angle as a function of upstream Mach number and deflection angle. If OFT $= 1$, then the routine proceeds to make a subsonic calculation as discussed above.

Having obtained a shock angle $\beta$ (either input for $FF = 38$, $\beta = ALFA$ for $FF = 39$, or for a sharp wedge or cone from PCSW) (90 degrees for a detached wedge or cone shock), FLOW next calls DWNSTM to determine the properties downstream of
the shock. Subroutine DWNSTM uses $\beta$ along with an air Mollier diagram (subroutine MOLIER) as an equation of state and the upstream conditions to compute the conditions downstream of the shock. If the properties required from the Mollier diagram cannot be obtained, then IDEAL = 1 or 2 may be returned to FLOW. If IDEAL = 1, then the routine proceeds by employing ideal gas relationships (discussed later). If IDEAL = 0 or 2, then the program proceeds by employing the Mollier diagram as the equation of state.

Once the conditions downstream of the shock are known and assuming IDEAL = 0, FLOW then proceeds to determine the local wall pressure based on the pressure flag (14, 15, 16, 17, 18) following the flowfield flag (35, 36, 38). If FF = 14, then Subroutine TBLIN is called to interpolate the pressure coefficient from an input set of tabular values with coefficient of pressure given as a function of free-stream Mach number. If FF = 15 or 16, then subroutine PCSW is called by FLOW to determine the pressure coefficient for a sharp wedge or cone as a function of $M_a$ and deflection angle. Again if NERROR $\neq$ 0 is returned from PCSW, then FLOW returns to MAIN and skips to the next data case. If OFT = 1 is returned, then the program switches to perform a subsonic calculation. If OFT = 2 is returned (detached shock), then FLOW computes the local modified Newtonian pressure. If OFT = 0 is returned, then the local edge of the boundary-layer pressure is determined based on the pressure coefficient returned from PCSW. If FF = 18, then the local pressure is determined from modified Newtonian theory. The other local thermodynamic properties are obtained from the Mollier diagram routine (MOLIER) as a function of local edge pressure and the downstream entropy. Knowing the local enthalpy and the stagnation enthalpy, the velocity is computed from the energy equation.
If \( FF = 17 \), the pressure angle is the surface angle-of-attack or flow deflection angle. This pressure flag provides the pressure solution for a surface whose shock angle is slightly greater than the surface angle-of-attack or flow deflection angle. The shock angle must be known and input under \( FF = 38 \). The edge of the boundary-layer velocity is computed from the surface angle-of-attack, the shock angle and the upstream velocity. Then the local enthalpy is computed from the energy equation. Knowing the local enthalpy and the entropy behind the shock from DWNSTM (\( FF = 38 \)), the other local thermoodynamic properties are obtained via the air Mollier diagram routine. The wall pressure is an output of the Mollier diagram, rather than being obtained from the pressure coefficient (from PCSW) as was done for \( FF = 15 \) or 16. If the surface angle-of-attack is larger than the shock angle, then the program proceeds to compute the edge conditions via the parallel shock technique (with \( \beta \) as the shock angle). If this occurs, then any following flowfield flags are ignored, and the program proceeds to compute \( \rho x u_x \) and/or Eckert reference properties before returning to MAIN.

If \( FF = 14, 15, 16, 17, 18 \) and no abnormality occurs such as \( \alpha > \beta \) (\( FF = 18 \)), then before proceeding to process the next flowfield flag a check occurs to determine if a Prandtl-Meyer expansion is desired. If a Prandtl-Meyer expansion is desired, then FLOW calls PMEXP to calculate conditions downstream of the expansion. If no Prandtl-Meyer expansion is desired, then FLOW bypasses PMEXP. Next a check occurs to determine if additional flowfield flags (with or without pressure flags) are present. If there are, then the present available local conditions are taken as upstream conditions for the next flowfield. If no more flowfield flags are present, then FLOW proceeds to calculate \( \rho x u_x \) and/or Eckert reference properties.
If FF = 39 is used, then FLOW uses the downstream entropy and the stagnation line enthalpy to compute the other local thermodynamic properties. Having the local thermodynamic properties, the local stagnation and static flow properties are computed. Then the $\rho_x u_x$ and/or Eckert reference properties are computed. The swept cylinder flowfield option may be used alone or as a last flowfield flag in a series. Any flowfield option used after FF = 39 will be ignored by FLOW.

If in the use of the Mollier diagram routine the error message IDEAL = 1 or 2 is returned to FLOW, then FLOW skips the real gas calculation and performs an ideal gas calculation. The Mollier diagram routine (MOLIER) is capable of handling both ideal and real gas calculations. The ideal gas relations used in the program are primarily used when nothing else works. This is to keep the program running rather than stop the program because the Mollier diagram routine cannot extrapolate to a value off the table. Once FLOW switches to the ideal gas calculation, all the flowfield and pressure options are executed using the ideal gas relationships. The ideal gas equation of state is implicitly used instead of MOLIER. The shock angles and pressure coefficients are determined exactly the same way as for a real gas calculation. The downstream conditions are now determined by the ideal gas routine DOWNID instead of DWNSTM and the conditions downstream of the Prandtl–Meyer expansion are computed in the ideal gas routine PMID instead of PMEXPN. All logic and error checks are precisely the same as for the real gas calculation including the ideal gas determination of the $\rho_x u_x$ and/or Eckert reference properties.

4.2 SUBROUTINE DWNSTM

Subroutine DWNSTM is called by FLOW to calculate the conditions downstream of a shockwave (normal, oblique or parallel). This routine is used to calculate
the downstream conditions for both real and ideal gases. However, the calculation procedure is based upon the assumption that a real gas calculation will take place. If subroutine MOLLER determines that an ideal gas calculation is sufficient, then the ideal gas equation of state is used instead of the Mollier diagram values.

The shock angle \( \beta \) is input into DWNSTM through the subroutine argument. For an attached wedge or cone shock, \( \beta \) is determined in subroutine PCSW. If the shock is determined to be detached by PCSW, then \( \beta \) is taken as 90 degrees for input into DWNSTM. If the parallel shock option is chosen, then the shock angle is set equal to the effective angle-of-attack.

Having an input shock angle \( \beta \) DWNSTM next requires the input of conditions upstream of the shock. The upstream conditions are input into DWNSTM via a common block with FLOW. Once the upstream conditions and shockwave angle are available, DWNSTM proceeds to calculate the downstream conditions. First, the normal components of the upstream velocity * and Mach number are calculated. Then an initial guess is made for \( \rho_2/\rho_1 \). If \( V_A \leq 4000 \text{ ft/sec} \), the ideal gas equations are used to make the initial guess. If \( V_A > 4000 \text{ ft/sec} \), the initial guess for \( \rho_2 \) is made using various empirical equations depending on whether \( 4000 \text{ ft/sec} < V_A \leq 8000 \text{ ft/sec} \), \( 8000 \text{ ft/sec} < V_A \leq 14000 \text{ ft/sec} \), \( 14000 \text{ ft/sec} < V_A \leq 26000 \text{ ft/sec} \), or \( 26000 \text{ ft/sec} < V_A \). Then a check occurs to insure that \( \rho_2/\rho_1 > 1.0 \). Subroutine DWNSTM then begins the first of a maximum of 50 iterations to determine the downstream properties.

*This discussion employs the following subscripts: 1, 2, N, T. The subscripts 1 and 2 correspond to conditions upstream and downstream of the shock, respectively. Subscripts N and T correspond to velocity components normal and tangent to the shock, respectively.
Having an initial guess for the downstream density \( \rho_n^{(2)} \), the downstream normal component of velocity, \( V_{2N} \), is calculated from Equation (4-1).

\[
V_{2N} = V_{1N} \left( \frac{\rho_1}{\rho_2} \right) \tag{4-1}
\]

and then the downstream enthalpy and pressure are computed based on \( \rho_n^{(2)} \)

\[
h_2 = h_1 + \left[ \frac{V_{1N}^2 - V_{2N}^2}{2} \right] \tag{4-2}
\]

\[
P_2 = P_1 + \left[ \rho_1 V_{1N}^2 - \rho_2^{(1)} V_{2N}^2 \right] \tag{4-3}
\]

Now \( \rho_n^{(2)} \) is determined from subroutine NOLIER via Equation (4-4).

\[
\rho_2^{(2)} = \rho(h_2, P_2) \tag{4-4}
\]

If \( \frac{\rho_2^{(2)} - \rho_2^{(1)}}{\rho_2^{(1)}} < 0.01 \), then convergence is considered to have been achieved. If convergence is not achieved, then \( \rho_n^{(2)} \) is replaced by \( \rho_n^{(2)} \) and the calculations, beginning with Equation (4-1) are repeated. This process is continued until two consecutive iterations are within 1 percent of one another, or until a maximum of 25 iterations is attained. After 25 iterations alternate damping coefficients are chosen to obtain a guessed density

\[
\rho_n^{(2)} = \rho_2^{(2)} + (1 - \alpha) \rho_2^{(2)} \rho_2^{(2)}
\]

where \( \alpha = 0.7 \) between 26 and 35 iterations and \( \alpha = 0.4 \) between 36 and 50 iterations. Once convergence is obtained, all of the static conditions behind the shock are known via Equations (4-1), (4-2), (4-3) and subroutine NOLIER. The velocity behind the shock is given by

\[
V_2 = \sqrt{\left( \frac{\rho_1}{\rho_2} \right)^2 V_{1N}^2 + (V_1 \cos \beta)^2} \tag{4-5}
\]

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\[ V_{AT} = V_x \cos \beta = V_{ST}. \]

Having the total enthalpy and \( S_\alpha \), all of the total conditions behind the shock are computed. Also this routine calculates the stagnation line enthalpy for the parallel shock. The stagnation line enthalpy is given by

\[ h_{SL} = h_2 + \frac{V_x^2}{2}. \]  

(4-6)

Subroutine DWNSN does not calculate the other stagnation line properties for the parallel shock (swept cylinder stagnation line flowfield). The other properties are calculated after returning to FLOW. Subroutine FLOW uses \( S_\alpha \) and \( h_{SL} \) as inputs to MOLIER in determining the other parallel shock properties.

4.3 SUBROUTINE PCSW

Subroutine PCSW is called by routine FLOW and has the purpose of providing the shock angles and pressure coefficients for sharp wedges and cones. This routine contains four tables: one for the sharp-wedge shock angle as a function of upstream Mach number and deflection angle, one for the sharp-cone shock angle as a function of upstream Mach number and deflection angle, one for the sharp-wedge pressure coefficient as a function of upstream Mach number and deflection angle, and the last one for the sharp-cone pressure coefficient as a function of upstream Mach number and deflection angle. The argument of PCSW contains five parameters. The first parameter corresponds to the upstream Mach number, and the second parameter corresponds to the deflection angle. The third parameter ITABLE, is an integer value:

- ITABLE = 1 indicates shock angle for a sharp wedge is desired
- ITABLE = 2 indicates shock angle for a sharp cone is desired
- ITABLE = 3 indicates pressure coefficient for a sharp wedge is desired
- ITABLE = 4 indicates pressure coefficient for a sharp cone is desired
The fourth parameter, OFFTBL, is calculated in PCSW and may have one of three possible values. If OFFTBL = 1.0 is returned, then the upstream Mach number and the deflection angle correspond to conditions producing a detached shock. If OFFTBL = 0.0, then PCSW has successfully completed its function. The last parameter in the subroutine argument is the value of the shock angle or pressure coefficient for either the wedge or cone, depending upon which is requested from the subroutine.

Having successfully entered the routine, several checks occur. If the upstream Mach number is greater than 26, the \( M_u \) (upstream Mach number) is set equal to 26. If \( M_u < 1 \), then the routine returns OFFTBL = 1. If \( \theta \) (deflection angle) \( < 0 \), then PCSW prints out an error message and returns NERROR = 1. This causes PCSW to return to the MAIN program. The MAIN program then will skip to read input data for the next case if \( W(642) = 0.0 \), or will print the stored output in routine VANNOUT and then read input data for the next case if \( W(642) \neq 0.0 \). If \( \theta > 60 \) degrees, then the shock is taken to be detached for the cone and OFFTBL = 2.0 is returned to the calling program. If \( \theta > 55 \) degrees, then the shock is considered to be detached for the wedge and OFFTBL = 2.0 is returned to the calling routine.

Next, the value of ITABLE (1, 2, 3, 4) is checked to determine which table is to be used. For each value of ITABLE several checks occur to determine if the shock is detached. If these tests are passed, then PCSW proceeds to calculate either a shock angle or a pressure coefficient.

The wedge and cone tables for shock angle and pressure coefficient are represented by curve-fits. The pressure coefficient and shock angle are
curve-fitted as a function of Mach number and deflection angle. Each of the four tables consists of several patches, each corresponding to various regions for each table.

After the value of shock angle or pressure coefficient is calculated, a final check is made to determine if \( \beta \geq 90 \) degrees or \( C_p > 1.8 \). This implies a detached shock and OFFtbl = 2.0 is returned to the calling program.

4.4 SUBROUTINE DOWNID

Subroutine DOWNID is called by subroutine FLOW to calculate the conditions downstream of a shockwave (normal, oblique, and parallel). This routine only performs calculations applicable for an ideal gas. The shock angle \( \beta \) is input into DOWNID through the subroutine argument. The upstream conditions are input into this routine via a common block with FLOW. Since an ideal gas calculation is being undertaken, there is no need for an iteration process as in DWNSTM. The calculation procedure begins with the calculation of the normal component of free-stream Mach number

\[
M_{1N} = M_2 \sin \alpha
\]  

(4-7)

*This discussion employs the following subscripts: 1, 2, N, T. The subscripts 1 and 2 correspond to conditions upstream and downstream of the shock, respectively. Subscripts N and T correspond to velocity components normal and tangent to the shock, respectively.*
The downstream pressure, temperature and Mach number are given by

\[
\begin{align*}
\frac{P_2}{P_1} &= \frac{2\gamma M_{1N}^2 - (\gamma - 1)}{(\gamma + 1)} \\
T_2 &= \frac{P_2}{P_1} \frac{(\gamma - 1) M_{1N}^2 + 2}{(\gamma + 1) M_{1N}^2} \\
M_2 &= \frac{T_1}{T_2} \sqrt{\left[M_1^2 - \frac{2}{(\gamma + 1)} \left(\frac{P_2}{P_1} - \frac{T_2 P_1}{T_1 P_2}\right)\right]} \tag{4-10}
\end{align*}
\]

The total pressure behind the shock, the static density, local speed of sound, downstream velocity and static enthalpy and given by

\[
\begin{align*}
P_{s2} &= P_1 \left[1 + \frac{(\gamma - 1)}{2} M_2^2 \right] \tag{4-11} \\
\rho_2 &= \frac{P_2}{RT_2} \tag{4-12} \\
a_2 &= \sqrt{\gamma RT_2} \tag{4-13} \\
V_2 &= M_2 a_2 \tag{4-14} \\
h_2 &= 0.24 T_2 \tag{4-15}
\end{align*}
\]

and then DOWND returns to FLOW.

4.5 SUBROUTINE PMEXPN

Subroutine PMEXPN is the real gas Prandtl-Meyer expansion routine for air. The input conditions to the routine are upstream static properties and stagnation enthalpy. If the Prandtl-Meyer expansion angle \( \theta_{pm} \) is greater than 103.2 degrees, then the program conveys an error message to the user.
The differential equation describing the real gas expansion is

\[ \frac{dF}{d\theta_{pm}} = \frac{2F}{\sqrt{\frac{2F}{a^2} - 1}} \]  \hspace{1cm} (4-16)

with \( F = h_o - h = \frac{u^3}{2} \)

and \( a = a(h,s), \ s = s_a = \text{const.} \)

where \( a = a(h,s) \) is a functional representation of the Mollier chart (Subroutine MOLIER).

Knowing the value of \( \theta_{pm} \), one desires to determine enthalpy, \( h \), at the end of the expansion fan. Once \( h \) is determined, all other properties are immediately available. To determine \( h \), the above equation must be solved for \( F \). This is achieved by employing the fourth-order Runge-Kutta method.

If \( 86 \text{ degrees} < \theta_{pm} < 103.2 \text{ degrees} \), then \( \theta_{pm} \) is divided into twelve equal segments for numerical integration. If \( 68.8 \text{ degrees} < \theta_{pm} < 86 \text{ degrees} \), then \( \theta_{pm} \) is separated into ten equal divisions. If \( 51.6 \text{ degrees} < \theta_{pm} < 68.8 \text{ degrees} \), then \( \theta_{pm} \) is split into eight equal intervals. If \( 34.4 \text{ degrees} < \theta_{pm} < 51.6 \text{ degrees} \), then \( \theta_{pm} \) is separated into six equal segments. If \( 17.2 \text{ degrees} < \theta_{pm} < 34.4 \text{ degrees} \), then \( \theta_{pm} \) is divided into four equal intervals. And, if \( \theta_{pm} < 17.2 \text{ degrees} \), then \( \theta_{pm} \) is divided into two equal divisions. Having successfully determined \( F \) at the end of the expansion fan, all of the properties downstream of the fan are returned to the main program.

4.6 SUBROUTINE PRMID

This is the ideal gas Prandtl-Meyer expansion routine. The input is the Prandtl-Meyer expansion angle, the upstream temperature, pressure and Mach
number. The output consists of local Mach number, temperature, pressure, density, enthalpy, speed of sound and velocity after the expansion. The Mach number after the expansion fan is determined by using the Newton-Raphson iteration method. A maximum of 25 iterations is allowed.
Section 5.0

HEATING ROUTINES

The subroutines used to calculate heat transfer coefficients to various geometries are presented in this section.

5.1 SUBROUTINE FAYRID

Subroutine FAYRID is called by MAIN and is used to compute the heating at the stagnation point of a sphere. The arguments of the CALL statement are RN and ENCL. The nose radius, RN, is input and the laminar value of the heat transfer coefficient, ENCL, is output if a boundary layer calculation is made. The FAYRID subroutine has been modified to compute free molecular flow heating and rarefied flow heating in addition to boundary layer flow heating by the method of Fay and Riddell. The boundary layer flow method has been modified to calculate subsonic flow heating by a modification of the velocity gradient.

The first subroutine called by FAYRID is REGIME. Based on the value of IRE, the output from REGIME, a boundary layer, rarefied or free molecular flow calculation is made.

If IRE = 1 Boundary Layer Flow

A boundary layer flow stagnation point heating calculation is made based on the method of Fay and Riddell as described in Table 5.1. The specific dissociation enthalpy $H_d$ is obtained from BINTRP as a function of the stagnation temperature behind the normal shock and the ratio of the stagnation density behind the normal shock to the density at standard atmospheric conditions. If NERROR $\neq 0$ is returned by BINTRP, then FAYRID returns to MAIN and MAIN reads the next data case. Having obtained the heat transfer coefficient ENCL, FAYRID returns to the calling routine. PCT is set equal to 0.0 for output description purposes.
If IRE = 2 Rarefied Flow

A rarefied flow stagnation point heating calculation is made by calling STHREAT. PCT is set equal to 2.0 for output description purposes. The heat transfer coefficient is output through the labeled common /FREMO/ by the variable HTIL.

If IRE = 3 Free Molecular Flow

A free molecular flow heating calculation is made for a plate perpendicular to the flow direction by calling PMHEAT. PCT is set equal to 10.0 for output description purposes.

One labeled COMMON has been added to FAYRID and it is called /FREMO/. The variables transferred through this statement are:

<table>
<thead>
<tr>
<th>IRE</th>
<th>(output)</th>
<th>1 Boundary Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2 Rarefied Flow</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 Free Molecular Flow</td>
</tr>
<tr>
<td>PCT</td>
<td>(output)</td>
<td>A variable specifying the calculation method used</td>
</tr>
<tr>
<td>HTIL</td>
<td>(output)</td>
<td>Heat transfer film coefficient based on enthalpy</td>
</tr>
<tr>
<td>H</td>
<td>(output)</td>
<td>Heat transfer film coefficient based on temperature</td>
</tr>
<tr>
<td>ATPBA</td>
<td>(output)</td>
<td>Angle of attack plus local body angle</td>
</tr>
<tr>
<td>NONCON</td>
<td>(output)</td>
<td>Integer value of W(646)</td>
</tr>
</tbody>
</table>

If NONCON > 0 a rarefied flow calculation can be made if IRE > 1

If NONCON < 0 only boundary layer calculations can be made (i.e. the program operates identically to the unmodified MINIVER program)

<table>
<thead>
<tr>
<th>AK2</th>
<th>(output)</th>
<th>Rarefaction parameter ZnE^2 from STHREAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHT</td>
<td>(output)</td>
<td>Stanton number from STHREAT</td>
</tr>
<tr>
<td>CHI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZETA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CHMI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AK2CIL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XI</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Not derived from this subroutine or associated calls.

Since SWCYL calls FAYRID, this subroutine should be discussed here. No modifications have been made to SWCYL. However, if W(646) = NONCON is input as greater than zero and SWCYL is called, then a rarefied calculation could be performed if IRE > 1. This method is not recommended.
5.2 SUBROUTINE REGIME

This routine determines which flow regime is appropriate for heating calculations. The Mach number (AM), velocity (UI), density (RI), temperature (T), pressure (P), and characteristic length (X) are input through the argument. The subroutine calls DWNSTM to determine the post-shock compressibility factor. The flow regime is determined by the relations given in Table 5.2 and returns the integer IRE.

IRE = 1 Boundary layer (continuum)
= 2 Rarefied (transitional)
= 3 Free Molecular

Subroutine REGIME is called by FAYRID, SPCHI and SWCYL2.

5.3 SUBROUTINE STHEAT

This subroutine calculates the heat transfer coefficient, HTIL, to the stagnation point of a sphere in the rarefied flow regime. The equational basis for this calculation is given in Table 5.3. This subroutine calls DWNSTM to get the post normal shock temperature, TD, and compressibility factor, Z. Subroutine HANSEN is called to obtain the viscosity, XMU, at post-shock pressure and reference temperature levels.

Subroutine STHEAT is called by FAYRID. The arguments of the CALL statement are:

UI - (input) Free stream velocity
RI - (input) Free stream density
T - (input) Free stream temperature
P - (input) Free stream pressure
AM - (input) Free stream Mach number
GAMMA - (input) Free stream specific heat ratio
HI - (input) Free stream static enthalpy
TW - (input) Wall temperature
HW - (input) Wall enthalpy
R - (input) Body radius
QD - (output) Convective heating rate
H - (output) Heat transfer film coefficient based on temperature
HTIL - (output) Heat transfer film coefficient based on enthalpy
RAW - (input) Laminar adiabatic wall enthalpy
TAW - (output) Laminar adiabatic wall temperature
AK2  - (output) Rarefraction parameter
CHT  - (output) Stanton number

Labeled commons /FRSTW/ and /DNSTW/ are used to transfer information for
the shock calculation.

5.4 SUBROUTINES FMHEAT AND ERF

Subroutine FMHEAT is used to compute free molecular flow heating to any
windward facing surface. The function program ERF is the error function call in
subroutine FMHEAT. The equations programmed in subroutine FMHEAT and function
ERF are given in Table 5.4.

Subroutine FMHEAT is called by FAYRID, SPCHI and SWCL2. The arguments of
the call statement are:

UI  - (input) Free stream velocity
RI  - (input) Free stream density
T  - (input) Free stream temperature
P  - (input) Free stream pressure
MN  - (input) Free stream Mach number
GAMMA  - (input) Free stream specific heat ratio
HI  - (input) Free stream static enthalpy
TW  - (input) Wall temperature
HW  - (input) Wall enthalpy
QC  - (output) Convective heating rate
H  - (output) Heat transfer film coefficient based on temperature
HTIL  - (output) Heat transfer film coefficient based on enthalpy
HAW  - (input) Laminar adiabatic wall enthalpy
TAW  - (output) Laminar adiabatic wall temperature
TH  - (input) Theta, C, surface angle

No common block statements are used to transfer information.

5.5 SUBROUTINE SWCL2

Subroutine SWCL2 is called by MAIN to compute the heat transfer coefficient
along the stagnation line of a swept cylinder for laminar or turbulent flow.
This is done using empirical relations based on the work of Cato for laminar
flow and Johnson for turbulent flow as given in Table 5.5. Both techniques are
based on empirical adjustments to the stagnation point heat transfer coefficient

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on a sphere obtained from FAYRID. The sphere is taken to have a radius equal to the radius of the cylinder. If the upstream Mach number is less than 1.12 the velocity gradient is adjusted per the relations given in Table 5.1 to yield the appropriate stagnation point heat transfer coefficient, ENF, from FAYRID.

After computing $h_L$ and $h$, the Reynolds analogy factor $S = Pr^{2/3}$ is calculated and SWCYL returns to MAIN. Presently SWCYL contains four parameters: RN the cylinder radius, PHI the sweep angle, ENCL the laminar heat transfer coefficient, and ENCT the turbulent swept cylinder heat transfer coefficient. The parameters RN and PHI are inputs with ENCL and ENCT as outputs.

5.6 SUBROUTINE SWCYL2

Subroutine SWCYL2 is used to compute the heat transfer coefficient along the stagnation line of an infinite swept cylinder. This subroutine has been modified to compute rarefied flow heat transfer coefficients.

An initial check is made on NONCON = W(646) to determine if rarefied flow is to be considered. Subsequently, if NONCON > 0, the subroutine $\text{SWCYL2}$ is called by SWCYL2. Based on the value of IRE, the output from $\text{SWCYL2}$, a boundary layer, rarefied or free molecular flow calculation is made. There are three parameters in the argument of the CALL statement for subroutine SWCYL2:

- RN  - (input) Cylinder radius (ft)
- ENCL - (output) Swept-cylinder stagnation line laminar heat transfer coefficient
- ENCT - (output) Swept-cylinder stagnation line turbulent heat transfer coefficient

If IRE = 1 Boundary Layer Flow

A boundary layer flow calculation is made as described in Table 5.6a and 5.6b both for supersonic and subsonic flow conditions. PCT is set equal to 0.0 for output purposes. Subroutine $\text{MOLIER}$ is called to obtain wall properties.
HANSEN is called to obtain wall viscosity. FAYRID is called to obtain the
sphere stagnation point heat transfer coefficient.

If IRE = 2 Rarefied Flow

A rarefied flow calculation is made by calling CYT. PCT is set equal to
3.0 for output purposes.

If IRE = 3 Free Molecular Flow

A free molecular flow calculation is made for a plate at the local sweep
angle. PCT is set equal to 10.0 for output purposes.

The labeled COMMON has been added to SWCYL2 and it is called /FREMO/. The
variables from this subroutine transferred through the COMMON/FREMO/ are:

AK2CYL - Rarefaction parameter, $\frac{x}{A}$
XI  - Stanton Number parameter, $\xi$

plus the variables defined in section 5.1.

5.7 SUBROUTINE CYT

Subroutine CYT is used to compute rarefied flow stagnation line heating
rates for a right circular cylinder at arbitrary yaw angles. The equations pro-
grammed in this subroutine are given in Table 5.7.

Subroutine CYT is called by SWCYL2. The arguments of the CALL statement
are:

UI  - (input) Free stream velocity
RI  - (input) Free stream density
T   - (input) Free stream temperature
P   - (input) Free stream pressure
AM  - (input) Free stream Mach number
GAMMA - (input) Free stream specific heat ratio
E1  - (input) Free stream static enthalpy
TW  - (input) Wall temperature
HW  - (input) Wall enthalpy
X   - (input) Distance along the surface
QC  - (output) Convective heating rate
H   - (output) Heat transfer film coefficient based on temperature
HTIL - (output) Heat transfer film coefficient based on enthalpy
Subroutine CYT calls DWNSTM to calculate post normal shock temperatures and pressures. Subsequently subroutine HANSEN is called to compute viscosity.

Labeled commons /FRSTM/ and /DNSTM/ are used to transfer information for the shock calculations.

5.8 SUBROUTINE SWCYL3

Subroutine SWCYL3 is also called by MAIN and employs the $\rho x u_x$ technique for computing the heat transfer coefficient along the stagnation line of an infinite swept cylinder as described in Table 5.8. This routine calls subroutines NOLIER and BINTRP. The argument of the CALL statement for SWCYL3 contains the three following parameters:

- RN - (input) Cylinder radius
- ENCL - (output) Laminar heat transfer coefficient along the stagnation line
- ENCT - (output) Turbulent heat transfer coefficient along the stagnation line of an infinite swept cylinder.

All other input information necessary for computation is brought into SWCYL3 through common blocks.

5.9 SUBROUTINE DETRAL

Subroutine DETRAL is called by MAIN and is used in calculating the laminar and turbulent heat transfer coefficients about hemispherical nose shapes. This routine enables the evaluation of circumferential as well as stagnation point heat transfer coefficients. The laminar equations of Lees, given in Table 5.9a, are used to compute the laminar heat transfer coefficient. The turbulent equations of Detra and Hidalgo, given in Table 5.9b, are used to compute the turbulent heat transfer coefficient.
The arguments of the CALL statement for DETRAL contain the following parameters:

- **RN** - (input) Sphere radius
- **RL** - (input) Running length from stagnation point
- **PHI** - (input) Body angle (90 degrees at the stagnation point)
- **ENCL** - (output) Laminar heat transfer coefficient
- **ENCT** - (output) Turbulent heat transfer coefficient

DETRAL calls FATRID to obtain the laminar stagnation point heat transfer coefficient and calls BINTRP to obtain the dissociation enthalpy, \( \text{BD} \).

### 5.10 SUBROUTINE ECKERT

Subroutine ECKERT is called by MAIN and is used to calculate the laminar and turbulent heat transfer coefficients for flow over a flat plate with negligible pressure gradient. The CALL statement for subroutine ECKERT contains 6 parameters:

- **ELL** - (input) Laminar running length (already corrected for crossflow if crossflow option was chosen)
- **ELT** - (input) Turbulent running length (already corrected for crossflow and virtual origin adjustment if these options were chosen)
- **ENL** - (input) Laminar Mangler transformation factor to account for axisymmetric flow correction to flat-plate heating
- **ENT** - (input) Turbulent Mangler transformation factor to account for axisymmetric flow correction to flat-plate heating
- **ENCL** - (output) Laminar heat transfer coefficient based on enthalpy
- **ENCT** - (output) Turbulent heat transfer coefficient based on enthalpy

Subroutine BINTRP is called to determine the Prandtl number \( \text{Pr}_L \) as a function of the Eckert reference temperature for laminar flow (recovery factor is 0.83) and the ratio of the boundary-layer edge pressure to standard atmospheric pressure. The Eckert heat transfer coefficient for laminar flow is calculated using the relations in Table 5.10a. Next subroutine BINTRP is called to determine the Prandtl number \( \text{Pr}_T \) as a function of the Eckert reference temperature for turbulent flow (recovery factor is 0.83) and the ratio of the boundary-layer...
edge pressure to standard atmospheric pressure. The turbulent flow heat transfer coefficient is calculated using the relations given in Table 5.10b.

If the parameter NERROR ≠ 0 is returned from BINTRP, then ECKERT returns to MAIN and proceeds to read in the next data case. The Eckert reference properties (laminar and turbulent) are calculated in subroutine FLOW based on a constant Prandtl number (laminar recovery factor is 0.85 and the turbulent recovery factor is 0.88). Using this constant Prandtl number the Eckert reference temperature is determined. This reference temperature is then used in conjunction with BINTRP to determine the Prandtl number (now considered as a function of temperature and pressure) which was originally taken to be a constant. Strictly speaking, the Prandtl number should be determined by an iterative procedure. However, this probably does not cause any appreciable error.

5.11 SUBROUTINES SPCHI AND FSUBC

Subroutine SPCHI is called by MAIN and is used to calculate the turbulent heat transfer coefficient for turbulent flow, and also the Eckert reference technique is used to compute the laminar heat transfer coefficient. Both correspond to flow over a flat plate with a negligible pressure gradient. The CALL statement for subroutine SPCHI contains seven parameters:

- ELL - (input) Laminar running length (already corrected for crossflow if crossflow option was chosen)
- ELT - (input) Turbulent running length (already corrected for crossflow and virtual origin adjustment if these options were chosen)
- ENL - (input) Laminar Mangler transformation factor to account for axisymmetric flow correction to flat-plate heating
- ENT - (input) Turbulent Mangler transformation factor to account for axisymmetric flow correction to flat-plate heating
- ENCL - (output) Laminar heat transfer coefficient based on enthalpy
- ENCT - (output) Turbulent heat transfer coefficient based on enthalpy
- RANFLG - (input) If a value greater than zero is input into location $19$, then the Von Karman form of the Reynolds analogy factor is used.
An initial check is made on NONCON = W(646) to determine if boundary layer flow is to be considered (yes if NONCON > 0). Subsequently, if NONCON > 0, the subroutine REGIME is called by SPCHI. Based on the value of IRE, the output from REGIME, a boundary layer, rarefied or free molecular calculation is made.

**If IRE = 1 Boundary Layer Flow**

A boundary layer flow calculation is made as described in Table 5.11 for turbulent flow and Table 5.10a for laminar flow. FCT is set equal to 0.0 for output purposes.

SPCHI calls subroutine FSUBC to compute $F_0$ for a real gas using MOLIER.

The argument of FSUBC contains five parameters:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>HE</td>
<td>(input) Enthalpy at the edge of the boundary layer</td>
</tr>
<tr>
<td>HW</td>
<td>(input) Wall enthalpy</td>
</tr>
<tr>
<td>HRECT</td>
<td>(input) Turbulent Eckert reference enthalpy</td>
</tr>
<tr>
<td>PE</td>
<td>(input) Boundary-layer edge pressure</td>
</tr>
<tr>
<td>FCDNV</td>
<td>(output) $1/F_0$</td>
</tr>
</tbody>
</table>

If no problem is encountered with the Mollier diagram, then $F_0$ is returned to SPCHI. If the thermodynamic properties cannot be determined by MOLIER, then IDEAL = 1 is returned to SPCHI and an ideal gas calculation of $F_0$ is undertaken. Having obtained $F_0$, the turbulent heat transfer coefficient is then determined from the Spalding–Chai equations which have been checked and found to be correct. If a value greater than zero is stored in location 319, then the Von Karman Reynolds analogy factor is used in determining the turbulent heat transfer coefficient. However, if zero is stored in location 319, then the Reynolds analogy factor is used as $Pr^{2/3}$.

After the turbulent heat transfer coefficient has been determined, SPCHI next proceeds to compute the laminar heat transfer coefficient by the Eckert reference technique. Both the laminar and turbulent heat transfer coefficients
are corrected for crossflow and for axisymmetric flow effects (via the laminar and turbulent Mangler transformation factors) before being used in the heat transfer coefficient equations.

**If IRE = 2 Rarefied Flow**

NFCS \( \geq 0.0 \), NFCS = W(314)

If NFCS \( \geq 0.0 \), then SPCHI calls SPFP.

The rarefied flow heat transfer coefficient for a sharp flat plate at zero angle of attack is computed. PCT is set equal to 4.0 for output purposes.

NFCS \( \leq 0.0 \)

If NFCS \( \leq 0.0 \) then SPCHI calls CONE. The rarefied flow heat transfer coefficient for a sharp nose cone is computed. PCT is set equal to 5.0 for output purposes.

**If IRE = 3 Free Molecular Flow**

A free molecular flow heating calculation is made for a plate at the local body angle by calling FMHEAT. PCT is set equal to 10.0 for output purposes.

5.12 **SUBROUTINE SPFP**

Subroutine SPFP is used to compute flat plate heating rates in the rarefied flow regime. The equations programmed in this subroutine are given in Table 5.12.

Subroutine SPFP is called by SPCHI. The arguments of the CALL statement are:

- UI - (input) Free stream velocity
- RI - (input) Free stream density
- T - (input) Free stream temperature
- P - (input) Free stream pressure
- AM - (input) Free stream Mach number
- GAMMA - (input) Free stream specific heat ratio
- HI - (input) Free stream static enthalpy
- TW - (input) Wall temperature
- HW - (input) Wall enthalpy
X - (input) Distance along the surface
QC - (output) Convective heating rate
H - (output) Heat transfer film coefficient based on temperature
HTIL - (output) Heat transfer film coefficient based on enthalpy
HAW - (input) Laminar adiabatic wall enthalpy
TAW - (output) Laminar adiabatic wall temperature
CHMI - (output) Rarefaction parameter, $M_\infty \sqrt{C_p/\mu}$

Subroutine SPFP calls HANSEN to compute viscosity for use in calculating the Chapman-Rubesin constant.

No common statements are used.

5.13 SUBROUTINE CONE

Subroutine CONE is used to compute rarefied flow heating rates for a sharp cone. The equations programmed in this subroutine are given in Table 5.13.

Subroutine CONE is called by SPCHI. The arguments of the CALL statement are:

UI - (input) Free stream velocity
RI - (input) Free stream density
T - (input) Free stream temperature
P - (input) Free stream pressure
AM - (input) Free stream Mach number
GAMMA - (input) Free stream specific heat ratio
HI - (input) Free stream static enthalpy
TW - (input) Wall temperature
HW - (input) Wall enthalpy
X - (input) Distance along the surface
QC - (output) Convective heating rate
H - (output) Heat transfer film coefficient based on temperature
HTIL - (output) Heat transfer film coefficient based on enthalpy
HAW - (input) Laminar adiabatic wall enthalpy
TAW - (output) Laminar adiabatic wall temperature
TCD - (input) Semivertex angle of the cone
CHI - (output) Rarefaction parameter, $\chi$ (Equation 5.13.13)
ZETA - (output) Stanton Number parameter, $\xi$
NSB - (input) $NSB = \Theta(\sigma_0)$
    $NSB = 0$ Sharp cone curve fit
    $NSB = 1$ Blunt cone curve fit
Subroutine CONE calls DWNSTM to calculate post normal shock temperature and pressure. Subsequently subroutine HANSEN is called to compute viscosity based on a reference temperature.

Labeled commons /FRSTM/ and /DNSTM/ are used to transfer information for the shock calculation.

5.14 SUBROUTINE RHOMUR

Subroutine RHOMUR is called by MAIN and is used to compute laminar and turbulent heat transfer coefficients over a flat plate using the relations in Table 5.14. Subroutine BINTRP is called by RHOMUR to compute the Prandtl number as a function of reference temperature and as a function of the ratio of the boundary-layer edge pressure to standard atmospheric pressure. Also BINTRP is called to compute the specific dissociation enthalpy as a function of reference temperature and as a function of the ratio of the reference density to standard atmospheric density. The expressions for the laminar and turbulent heat transfer coefficient have been somewhat simplified from the original $\rho_x\mu_x$ expressions. The equivalent running length is determined from CRSFLW and the original $\rho_x\mu_x$ crossflow expressions are not used. Also there is no adjustment to the heat transfer coefficient to account for the increased heating due to axisymmetric flow (Mangler transformation factor). The argument of the CALL statement for RHOMUR contains the four following parameters:

- **ELL** - (input) Laminar running length which may have been corrected for crossflow effects
- **ELT** - (input) Turbulent running length which may have been corrected for crossflow effects and virtual origin adjustment
- **ENCL** - (output) Laminar heat transfer coefficient
- **ENCT** - (output) Turbulent heat transfer coefficient

All other information necessary for computation is brought into RHOMUR through common blocks.
5.15 SUBROUTINE LESIDE

Subroutine LESIDE is called from MAIN after a call to FAYRID, to calculate the average heat transfer coefficient to the leeside of an orbiter. The average heat transfer coefficient is calculated using the relations in Table 5.15. This subroutine is called if NEFLAG = 9. The arguments of the call are:

- **EN** - (input) Radius of scale, full scale EN = 1.0 ft.
- **HWD** - (input) W(21), Windward Wall enthalpy (Btu/lbm)
- **ENCL** - (output) Average turbulent or laminar leeward side heat transfer coefficient
- **ENCT** - (output) = ENCL

The common FLWFLD is used to transfer post normal shock values obtained by the call to FAYRID.

5.16 SUBROUTINES FLAP, DEFL, DSTML, CPIAF AND BTHICK

Subroutine FLAP is called from MAIN when NEFLAG = 10 to calculate the peak reattachment heating to a flap. The equations used for this calculation are given in Table 5.16. The arguments of the call are as follows:

- **EL** - (input) Running length
- **FLENG** - (input) Flap length
- **FANG** - (input) Flap angle
- **TW** - (input) Wall temperature
- **HT** - (input) Total enthalpy
- **ENCLO** - (output) Laminar heat transfer coefficient
- **ENCTO** - (output) Turbulent heat transfer coefficient
- **IQUIT** - (output) Flag IQUIT = 0 Separation occurs
  \[ IQUIT = 1 \text{ No separation occurs} \]
- **PCT** - (output) Transition percentage

A common statement, FLPETO, is used to transfer the following edge properties from MAIN into the subroutine:

- **EDGEP(1,I)** - Pressure
- **EDGEP(2,I)** - Temperature
- **EDGEP(3,I)** - Density
- **EDGEP(4,I)** - Speed of sound
- **EDGEP(5,I)** - Velocity
- **EDGEP(6,I)** - Mach number
- **EDGEP(7,I)** - Viscosity
- **EDGEP(8,I)** - Specific heat ratio

where \( I = 1 \) - Before the flap shock edge conditions
\( I = 2 \) - After the flap shock edge conditions
Subroutine FLAPF calls BTHICK to obtain a laminar and turbulent boundary layer thickness. This calculation is made using Eckert reference conditions in the following equations:

\[ \delta = \frac{5.2}{\sqrt{Re_x}} \] : Laminar boundary layer thickness

\[ \delta = 0.154/(Re_x)^{1/7} \] : Turbulent boundary layer thickness

Next subroutine CPIAF is called to calculate the incipient separation pressure coefficient. This is followed by a call to DEFL to determine if the input flap angle is sufficiently large enough to produce separation. If the input angle is insufficient to cause separation, IQUIT is set equal to 1 and the subroutine returns to the MAIN. If the subroutine returns to the MAIN with IQUIT = 1, then the heating is calculated by a call to SPCH1.

If separation is found to occur, the plateau pressure is calculated and DEFL is called to calculate the dividing streamline angle. Next the separation length is calculated by a call to DSTML. If the separation geometry indicates that impingement will not occur for the input flap length, IQUIT = 1 and the subroutine returns to the MAIN.

If impingement occurs, the shear layer thickness is calculated. Subroutine HANSEN is called to calculate the wall viscosity. Finally, the heat transfer coefficient is calculated for either laminar or turbulent flow with PCT set equal to 0.0 or 1.0. The laminar and turbulent heat transfer coefficients are equated since this routine can calculate only one type for a given flowfield. Note that in MAIN if NEFLAG = 10 and IQUIT = 0 then the call to TRANS is skipped.

5.17 SUBROUTINES FINH, FINPKH AND FINPPK

Subroutine FINH is called by MAIN when NEFLAG = 11 to calculate the peak
interference heating produced by a fin on the adjacent surface. The equations used for this calculation are given in Table 5.17. The arguments of the call are as follows:

- ALP - (input) Effective fin angle of attack
- XP - (input) Length along fin surface
- YP - (output) Normal distance from fin to peak heating
- RL - (input) Running length to fin leading edge
- TW - (input) Wall temperature
- HO - (input) Total enthalpy
- AMPPT - (output) Turbulent pressure amplification
- AMPPL - (output) Laminar pressure amplification
- AMPHL - (output) Laminar heating amplification
- AMPHT - (output) Turbulent heating amplification
- ENCLO - (output/input) Laminar heat transfer coefficient
- ENCTO - (output/input) Turbulent heat transfer coefficient

The common statement, FLPHYS, is used to transfer edge properties upstream of the fin shock from MAIN into the subroutine. The variables transferred are defined in subsection 5.16.

Subroutine FINHE calls BTHICK to obtain the laminar and turbulent boundary layer thickness using the relations given in subsection 5.16. Next the shock angle produced by the fin is computed using a call to PCSW. If the shock is attached, the peak pressure amplification is calculated for laminar and turbulent flow using two calls to FINPFP. The peak heating amplification for laminar and turbulent is then computed using two calls to FINPHE. The input heat transfer coefficients are modified using the heating amplification factors.

Subroutine SPCH1 is always called before the call to FINHE to obtain the undisturbed heat transfer coefficient. Note that in MAIN, if NHFLAG = 11, the heating amplification factor and edge pressure are calculated based on PCT after the call to TRANS.

5.18 SUBROUTINE RADEQT

Subroutine RADEQT is called by MAIN and is used to compute the radiation
equilibrium temperature. The radiation equilibrium temperature is determined via the Newton-Raphson iteration technique. The wall temperature is used as an initial guess and convergence to the radiation equilibrium temperature is considered to be achieved after 50 iterations, or after two successive iterations are within 0.5 percent of one another. The first of these two conditions to occur defines convergence.

The argument of the CALL statement for subroutine RADEQT contains the following five parameters:

ENC - (input) Heat transfer coefficient
HR - (input) Recovery enthalpy
EMIS - (input) Effective emissivity - product of shape factor and emissivity
TW - (input) Conduction or thin-skin wall temperature which is used as an initial guess to the radiation equilibrium temperature in the iteration scheme
TRE - (output) Radiation equilibrium temperature
Stagnation point heating is based on the theory of Fay and Riddell.

\[ q = \frac{N_u}{\sqrt{R_e}} \sqrt{g_c \rho_w u_w \frac{dU_e}{dx}} \frac{(H_t - H_w)}{R_w} \]

where

\[ N_u = 0.76 \frac{P_{r_w}^{a_n} (P_w u_w)}{P_{r_w}^{a_n} (P_w u_w)} \left( \frac{1}{1 + (\kappa - 1) \frac{H_d}{H_t}} \right) \]

\[ \frac{dU_e}{dx} = \frac{1}{R_e} \sqrt{\frac{2g_c}{\rho_t} (P_t - P_\infty)} \] for a sphere and \( M_\infty > 1.22 \)

\[ \frac{dU_e}{dx} = \frac{1}{2R_e} \sqrt{\frac{2g_c}{\rho_t} (P_t - P_\infty)} \] for a cylinder and \( M_\infty > 1.12 \)

\[ \frac{dU_e}{dx} = \frac{U}{R_e} (1.5 - 0.378M_\infty^2 - 0.02625 M_\infty^4) \] for a sphere and \( M_\infty \leq 1.22 \)

\[ \frac{dU_e}{dx} = \frac{U}{R_e} (2.0 - 0.872M_\infty^2 - 0.328M_\infty^4) \] for a cylinder and \( M_\infty \leq 1.12 \)

and the dissociation enthalpy

\[ H_d = C_{0,0} h_0^o + C_{0,1} h_N^o \]

\( h_0^o = 6636.26 \) Btu/Lbm

\( h_N^o = 14456.53 \) Btu/Lbm

**Nomenclature**

- \( q \) = Heating Rate (Btu/ft\(^2\)sec.)
- \( q_c \) = 1 (slug ft./lbp sec.\(^2\))
- \( \rho \) = Density (slug/ft.\(^3\))
- \( \mu \) = Viscosity (Lb sec./ft\(^2\))
- \( \overline{U} \) = Velocity (ft./sec.)
- \( P \) = Pressure (Lb./ft.\(^2\))
- \( H \) = Enthalpy (Btu/lbm)
Table 5.1 (Cont. 1)

- \( L_e \) = Lewis Number
- \( H_d \) = Dissociation enthalpy of air (Btu/Lbm)
- \( C_i \) = Mass fraction of species \( i \) (\( i = O \), oxygen and \( i = N \), nitrogen)
- \( R \) = Nose radius (ft.)
- \( Pr \) = Prandtl Number
- \( \theta \) = Sphere Body Angle

Subscripts:
- \( e \) = Edge
- \( t \) = Stagnation (post shock)
- \( w \) = Wall
- \( \infty \) = Free stream

References

Table 5.2
BOUNDARY LAYER, RAREFIED AND
FREE MOLECULAR FLIGHT REGIME CRITERION

\[ A = \frac{M_\infty}{Z \left( Re_{\infty} \right)^{1/2}} \]

Flight Regime Selection Parameter

If \( A \leq 0.05 \)  
Boundary Layer

If \( 0.05 < A < 3.0 \)  
Rarefied

If \( A > 3.0 \)  
Free Molecular

where

\( M_\infty \) = Free Stream Mach Number

\( Re_{\infty} \) = Free Stream Reynolds Number Based on Running Length or Radius

\( Z \) = Post Normal Shock Compressibility
Table 5.3
RAREFIED FLOW STAGNATION POINT HEAT TRANSFER EQUATIONS

The rarefied flow heating to the sphere stagnation point based on the work of Engel and Praharaj is as follows.

(1) \[ T_r = \frac{(T_\delta + T_W)}{2} \] (Reference temperature)

(2) \[ T_0 = T_\infty \left(1 + \frac{\gamma - 1}{2} M^2 \right) \] (Free stream stagnation temperature)

(3) \[ K^2 = \varepsilon \left( \frac{\rho_\infty U_\infty R}{\mu_r} \right) \left( \frac{T_r}{T_0} \right) \] (Rarefaction parameter)

where \[ \varepsilon = \frac{\gamma - 1}{2\gamma} \]

(4) Heat transfer coefficient

\[ \log_{10}(C_H) = \sum_{i=0}^{2} a_i (\log_{10} \varepsilon K^2)^i \]

- \[ a_0 = -0.235256 \]
- \[ a_1 = -0.303095 \]
- \[ a_2 = -0.0779538 \]

(5) Heat transfer

\[ q = \rho_\infty U_\infty C_H (H_0 - H_W) \]
Table 5.3 (Cont. 1)

Stagnation Point Heat Transfer in Low Reynolds Number Flow

Nomenclature

- \( C_H \): Stanton Number
- \( H \): Enthalpy
- \( M_{\infty} \): Free Stream Mach Number
- \( T \): Temperature
- \( U \): Velocity
- \( Z \): Post Normal Shock Compressibility
- \( \rho \): Density
- \( \gamma \): Specific Heat Ratio
- \( \mu \): Viscosity

Subscripts

- \( \infty \): Free Stream
- \( W \): Wall
- \( \delta = e \): Post Normal Shock
- \( 0 \): Total

Reference

Table 5.4
FREE MOLECULAR FLOW HEATING EQUATIONS

(1) \[ N = \frac{p}{kT_\infty} \] (particles/unit volume)

(2) \[ S = \sqrt{\frac{\gamma}{2}} M_\infty \] (molecular speed ratio)

(3) \[ \psi = \frac{NU_\infty}{2\sqrt{\pi}S} \]

(4) \[ \eta = S \sin \beta \]

(5) \[ n = \psi \left[ e^{-\eta^2} + \sqrt{\pi} \eta (1 + \text{erf} \eta) \right] \]

\[ = \text{number of molecules striking a unit area per unit time} \]

where
\[ \text{erf} \eta = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-x^2} dx \] (error function)

Rational approximation (0 ≤ \eta ≤ \infty) from Abramowitz and Stegun:
\[ \text{erf} \eta = 1 - (a_1 t + a_2 t^2 + a_3 t^3) e^{-\eta^2} + \epsilon(\eta) \]

\[ t = \frac{1}{1 + a_0 \eta} \]
\[ a_0 = 0.47047 \]
\[ a_1 = 0.3480242 \]
\[ a_2 = -0.0958798 \]
\[ a_3 = 0.7478556 \]
\[ |\epsilon(\eta)| \leq 2.5 \times 10^{-5} \]

50
Table 5.4 (Cont. 1)

(6) \[ \varphi = \frac{\psi}{2} e^{-n^2} \]

(7) \[ q = \alpha \left\{ \frac{\gamma+1}{2(\gamma-1)} n k T_w - \left[ \frac{s^2 + \frac{\gamma}{\gamma-1}}{n - \phi} \right] k T_w \right\} \]

These equations are based on the work of Oppenheim.

Nomenclature

- **k** - Boltzmann's Constant
- **M_\infty** - Freestream Mach Number
- **P** - Freestream Pressure
- **q** - Heating Rate
- **T_w** - Wall Temperature
- **T_\infty** - Free Stream Temperature
- **U_\infty** - Free Stream Velocity Heat Ratio
- **\gamma** - Free Stream Specific Heat Ratio
- **\alpha** - Accommodation Coefficient

References


The empirical correlations of Cato for laminar flow and of Johnson for turbulent flow are given below along with a comparison with experimental data.

For laminar flow,

\[ h_L = 0.75 h_0 \left[ 1 - 1.857 \left( \frac{\Lambda}{90^\circ} \right)^2 + 1.097 \left( \frac{\Lambda}{90^\circ} \right)^3 \right] \]

For turbulent flow,

\[ h_t = 0.75 \frac{K_T}{K_L} h_0 \left[ \frac{2\rho_0 U_0 R_N}{\mu_u} \right]^{0.3} \left[ 0.01714 + 0.01235 \sin \left[ 3.53 \left( \Lambda - 10^\circ \right) \right] \right] \]

The laminar multiplication factor, \( K_L \), is included in the definition of \( h_0 \).
Table 5.5 (Cont. 1)

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_o$</td>
<td>Stagnation Point Heat Transfer Coefficient (Sphere)</td>
</tr>
<tr>
<td>$h_L$</td>
<td>Laminar Heat Transfer Coefficient</td>
</tr>
<tr>
<td>$h_t$</td>
<td>Turbulent Heat Transfer Coefficient</td>
</tr>
<tr>
<td>$K$</td>
<td>Multiplier Factor</td>
</tr>
<tr>
<td>$R_N$</td>
<td>Cylinder Radius</td>
</tr>
<tr>
<td>$V_u$</td>
<td>Velocity (Upstream)</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>Density (Upstream)</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Sweep Angle</td>
</tr>
<tr>
<td>$\mu_u$</td>
<td>Viscosity (Upstream)</td>
</tr>
</tbody>
</table>

References


Table 5.6a

**BECKWITH AND GALLAGHER TURBULENT REAL GAS**

**yawed cylinder stagnation line heating**

\[ h_{SL} = \frac{0.0323}{P_r^{0.405}} \left( \frac{u_{\infty} \sin \theta}{\mu_0} \right)^{0.5} \left( g \rho \mu \right)^{0.8} \left( \frac{d\rho}{dx} \right)^{0.2} \]

**with and Gallagher turbulent real gas**

\[ \frac{d\rho}{dx} \bigg|_{x=0} = \frac{1.414}{R} \left( \frac{P_e - P}{\rho_e} \right)^{0.5} \]

**nomenclature**

- \( g \): 32.174 Lbm/slug
- \( h \): Heat Transfer Coefficient Lbm/ft²·sec
- \( P \): Pressure Lbf/ft²
- \( P_r \): Prandtl Number @ Edge Conditions Dimensionless
- \( R \): Radius of Cylinder Feet
- \( u \): Velocity Ft/sec
- \( x \): Distance from Stagnation Line Feet
- \( \Lambda \): Sweep Angle Degrees
- \( \mu \): Viscosity Lbm/ft·sec
- \( \rho \): Density Slugs/ft³

**subscripts:**

- SL: Stagnation Line
- e: Edge Conditions
- o: Total

**superscript:**

- *: Evaluate at Eckert's Reference Enthalpy & Edge Pressure

**reference:**

Beckwith, I. E., and Gallagher, J. J., "local heat transfer and recovery temperatures on a yawed cylinder at a mach number of 4.15 and high reynolds number, NASA TR R-104, 1961."
### Table 5.6b
LAMINAR CYLINDER STAGNATION LINE HEATING

<table>
<thead>
<tr>
<th>$h$</th>
<th>$0.707 h_s \cos^{-1} \Lambda$</th>
</tr>
</thead>
</table>

$h_s$ = Sphere Stagnation Point Heat Transfer Coefficient  
$h$ = Cylinder Stagnation Line Heat Transfer Coefficient  
$\Lambda$ = Sweep Angle

---

**Diagram**

- Laminar Flow  
- Cylinder Data  
- $M_\infty = 3.7$ From  
- Brewer, MSFC

$h / h_{ref}$ vs $\Lambda$, Sweep Angle (Deg.)
The rarefied flow heating to a cylinder based on the work of Engel and Praharaj is as follows:

1. \[ T_r = \frac{1}{2} (T_w + T_0 \cos^2 \Lambda) \]

2. \[ Re_\infty = \frac{\rho_{\infty} U_\infty r}{\mu_\infty} \]

3. \[ C_\pi = \frac{U_\infty T_\infty}{\mu_\infty Tr} \]

4. \[ \kappa_a^2 = \frac{Re_\infty Ze}{\gamma_\infty M_\infty^2 C_\pi (\cos^2 \Lambda + P_\infty \sin^2 \Lambda / \rho_\infty U_\infty^2)^{\frac{1}{2}}} \]

5. \[ \xi = C_H / (\cos^2 \Lambda + P_\infty \sin^2 \Lambda / \rho_\infty U_\infty^2)^{\frac{1}{2}} \]

6. Heat Transfer Correlation

\[ \log_{10} \xi = \sum_{i=0}^{2} a_i (\log_{10} \kappa_a^2)^i \]

\[ a_0 = -0.377656 \]

\[ a_1 = -0.368580 \]

\[ a_2 = -0.0461064 \]

7. Heat Transfer

\[ q = \rho_\infty U_\infty C_H (H_0 - H_W) \]
Table 5.7 (Cont. 1)

CALSPLAN CYLINDER DATA
Vidal and Mittliff (1963)

<table>
<thead>
<tr>
<th>A</th>
<th>T₀</th>
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<tr>
<td>140°</td>
<td>1800° K</td>
<td>0.128</td>
</tr>
<tr>
<td>0°</td>
<td>2700° K</td>
<td>0.085</td>
</tr>
<tr>
<td>20°</td>
<td>2700° K</td>
<td>0.085</td>
</tr>
</tbody>
</table>

AEDC DATA
Run Gauge A Conf. P

<table>
<thead>
<tr>
<th>Run</th>
<th>Gauge</th>
<th>Angle</th>
<th>ET</th>
<th>Conf. P</th>
</tr>
</thead>
<tbody>
<tr>
<td>4883</td>
<td>29</td>
<td>60°</td>
<td>ET</td>
<td>0.177</td>
</tr>
<tr>
<td>4885</td>
<td>29</td>
<td>90°</td>
<td>ET</td>
<td>0.177</td>
</tr>
<tr>
<td>4886</td>
<td>29</td>
<td>0°</td>
<td>ET</td>
<td>0.177</td>
</tr>
<tr>
<td>4887</td>
<td>29</td>
<td>0°</td>
<td>ET</td>
<td>0.177</td>
</tr>
<tr>
<td>4882</td>
<td>-</td>
<td>-</td>
<td>ET</td>
<td>0.177</td>
</tr>
<tr>
<td>4883</td>
<td>-</td>
<td>-</td>
<td>ET</td>
<td>0.177</td>
</tr>
</tbody>
</table>

Cylinder Stagnation Line Heat Transfer Data with Sweep Data Included

Nomenclature

- \( \tilde{C}_H \) Stanton Number
- \( H \) Enthalpy
- \( M_\infty \) Free Stream Mach Number
- \( T \) Temperature
- \( U \) Velocity
- \( Z \) Post Normal Shock Compressibility
- \( \rho \) Density
- \( \gamma \) Specific Heat Ratio
- \( \mu \) Viscosity

Subscripts

- \( \infty \) Free Stream
- \( e \) edge
- \( W \) Wall
- \( \delta \) Post Normal Shock
- \( o \) Total

Reference

The "ρmμμr" method for swept cylinders is defined as follows:

\[ H_{mc,o} = 0.5(H_L + H_W) \quad \rho_{mc,o} = f(H_{mc,o}, \rho_L) \]

\[ \Sigma_o = \frac{\rho_L}{\rho_{mc,o}} \quad \Gamma_o = 0.96(\Sigma_o)^{55} - 0.5 \]

\[ \alpha_r = 0.90 \left( \frac{\rho_{stag} u_{stag}}{\rho_w u_w} \right)^{0.5} \]

\[ P_{r_r} = f(P_L, T_r) \]

\[ E_{L,o} = \left[ 1 + \Gamma_o \right] P_r^{\alpha_r} \quad \bar{E}_{T,o} = \left[ 1 + 0.77 \Gamma_o \right] P_r^{\alpha_r} \]

\[ K_r = \frac{1}{V_L^{\frac{1}{2}} \sqrt{\frac{2}{\rho_L} (P_L - P_u)}} \]

\[ X_{eq_{L,o}} = \frac{1}{2K_r E_{L,o}} \quad X_{eq_{T,o}} = \frac{4}{5K_r E_{T,o}} \]

\[ F_x = \left[ \frac{X_{eq_{T,o}}}{X_{eq_{L,o}}} \right]^{0.333} \]

\[ n_r = \sqrt{\frac{\rho_{stag} u_{stag}}{\rho_L u_L}} \]

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Table 5.8 (Cont. 1)

\[ H_{mc} = H_{mc,0} + 0.206(H_T - H_x)P_r^n \]

\[ \rho_{mc} = f(H_{mc}, P_x) \]

\[ \Sigma_c = \frac{\rho_x}{\rho_{mc}} \]

\[ r_c = \left[ 0.96(\Sigma_c)^{0.55} - 0.5 \right] \]

\[ \Gamma_L = (1 + r_c)P_r^\alpha \]

\[ \mu_T'' = \mu_r \left( \frac{H_T}{H_r} \right)^{1.5} \left( \frac{T_r + 2000}{T_r \left( \frac{H_T}{H_r} \right) + 200} \right) \]

\[ \text{Re}_{r,	ext{cyl}} = \frac{\rho_r \mu_r \nu_x (x_{eq L}, 0)}{F_x \mu_T'' (x_{eq L})} \]

\[ C_{f_r,L} = \frac{0.664}{\sqrt{\text{Re}_{r,	ext{cyl}}}} \]

\[ C_{f_r,t} = \frac{0.370}{[\log_{10}(\text{Re}_{r,	ext{cyl}} + 3000)]^{2.584}} \]

\[ \text{Le} = 1.4 \]

\[ H_{D,r} = f(\rho_r, T_r) \]

\[ L = 1 + (\text{Le}^{0.52} - 1) \frac{H_{D,r}}{H_x} \]

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For laminar flow,
\[ h_L = 0.5g \frac{K_L}{p_r} \left( \frac{\mu_T F_x}{X_{eqL,0}} \right) \text{Re}_{r, cylCF_r,L} \]

For turbulent flow,
\[ h_t = 0.5g \frac{K_T}{p_r} \left( \frac{\mu_T F_x}{X_{eqL,0}} \right) \text{Re}_{r, cylCF_r,t} \]

**Nomenclature**
- **Cf**: Skin Friction Coefficient
- **h**: Heat Transfer Coefficient
- **H**: Enthalpy
- **Le**: Lewis Number
- **P**: Pressure
- **Pr**: Prandtl Number
- **Re**: Reynolds Number
- **R_N**: Cylinder Radius
- **T**: Temperature
- **V**: Velocity
- **ρ**: Density
- **μ**: Viscosity

**Subscripts**
- **L**: Laminar
- **r**: Eckerts Reference
- **stag**: Post Normal Shock Stagnation
- **S**: Stagnation Line
- **T**: Total
- **t**: Turbulent

**References**
Table 5.9a

LEES' HEMISPHERICAL DISTRIBUTION METHOD FOR LAMINAR FLOW

Lees' heating distribution over a sphere for an ideal gas is expressed as:

\[ h = h_{sp} \frac{2\pi \sin \theta \left[ \left( 1 - \frac{1}{\gamma M_\infty} \right) \cos^2 \theta + \frac{1}{\gamma M_\infty} \right]}{\sqrt{D(\theta)}} \]

\[ D(\theta) = \left( 1 - \frac{1}{\gamma M_\infty^2} \right) \left[ \theta^2 - \frac{\theta \sin 2\theta}{2} + \frac{1 - \cos 2\theta}{8} \right] \]

\[ + \frac{4}{\gamma M_\infty^2} \left[ \frac{\theta^2 - \theta \sin 2\theta + 1 - \cos 2\theta}{2} \right] \]

Nomenclature

- \( h \) = Heat Transfer Coefficient
- \( h_{sp} \) = Stagnation Point Heat Transfer Coefficient
- \( M_\infty \) = Free Stream Mach Number
- \( \gamma \) = Free Stream Specific Heat Ratio
- \( \theta \) = Local Body Angle

Reference

Detra and Hidalgo developed a method of calculating the turbulent distribution over a hemisphere.

\[ h_t = 0.029 \frac{g}{\Pr^{0.667}} \left( \frac{\rho_e V_e}{\mu_e} \right)^{0.8} \left[ 1.037 f(\delta)^{0.2} \right] \left[ 1 + 0.58 \frac{H_D e}{H_T} \right] \]

where

\[ f(\delta)^{0.2} = \sum_{i=0}^{N} A_i \delta^i \]

<table>
<thead>
<tr>
<th>( A_i )</th>
<th>( \delta &lt; 25^\circ )</th>
<th>( \delta \geq 25^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.03754</td>
<td>0.96451</td>
</tr>
<tr>
<td>2</td>
<td>0.0043776</td>
<td>0.01107</td>
</tr>
<tr>
<td>3</td>
<td>-0.6187 \times 10^{-4}</td>
<td>0.842558 \times 10^{-4}</td>
</tr>
</tbody>
</table>

Nomenclature

- g: Gravitational Constant
- \( h_t \): Turbulent Heat Transfer Coefficient
- H: Enthalpy
- \( K_T \): Turbulent Multiplier Factor
- L: Running Length
- \( \Pr \): Prandtl Number
- V: Velocity
- \( \rho \): Density
- \( \mu \): Viscosity
- \( \delta \): Sphere Tangency Angle (90 Degrees at the Stagnation Point)

Subscripts

- D: Dissociation
- E: Edge
- T: Total

Reference

Table 5.10a
ECKERT'S LAMINAR FLAT PLATE HEATING METHOD

Eckert's laminar flat plate heating relation can be expressed as:

\[ h = 0.332 \frac{g_c}{(P_e^*)^{1/4}} \sqrt{\frac{\rho^* u_e}{\mu^*}} \]  

(lbm/ft²sec)

where

\[ R_e^* = \frac{\rho^* u_e x}{\mu^*} \]

Reynolds no. evaluated at reference conditions

The reference conditions are obtained from the reference enthalpy

\[ H^* = H_e + 0.5 (H_w - H_e) + 0.22(H_{aw} - H_e) \]

and edge pressure. Other reference properties are evaluated as:

\[ \rho^* = f (H^*, P_e) \]
\[ \mu^* = f (H^*, P_e) \]

For geometries other than a flat plate, the Mangler transformation may be evaluated as:

\[ \tau_m = 1 \quad \text{Flat Plate} \]
\[ = 3 \quad \text{Cone} \]

Nomenclature

- \( g_c = 32.174 \) lbf/slug
- \( h = \) Heat Transfer Coefficient (lbm/ft²sec)
- \( H_e = \) Edge Enthalpy (Btu/lbm)
- \( H_{aw} = \) Adiabatic Wall Enthalpy (Btu/lbm)
- \( H_w = \) Wall Enthalpy (Btu/lbm)
- \( P_e = \) Edge Pressure (atm.)
- \( u_e = \) Edge Velocity (ft/sec)
- \( x = \) Surface Distance From Origin to Point of Interest
- \( \rho^* = \) Reference Density
- \( \mu^* = \) Reference Viscosity
- \( \tau_m = \) Mangler Transformation
Table 5.10b
SCHULTZ-GRUNOW TURBULENT FLAT PLATE METHOD

Turbulent heating relations for a plate using the Schultz-Grunow skin function law can be expressed as:

\[
h = \frac{0.185g}{Pr_* 0.667} \left( \frac{\rho u_e}{\log_{10} \left( \frac{Re_*}{\tau_m} \right)} \right)^{2.584}
\]

where

\[
Re_* = \frac{\rho u_e x}{\mu^*}
\]

Reynolds number evaluated at Eckert reference conditions

For geometries other than a flat plate, the Mangler transformation may be evaluated as

\[
\tau_m = 1 \text{ Flat Plate} \quad 2 \text{ Cone}
\]

Nomenclature

- \( g_c \) = 32.174 lbm/slug
- \( h \) = Heat Transfer Coefficient (lbm/ft²/sec)
- \( Pr_* \) = Prandtl Number at Reference Conditions
- \( u_e \) = Edge Velocity (ft/sec)
- \( x \) = Surface Distance From Origin to Point of Interest
- \( \rho^* \) = Reference Density
- \( \mu^* \) = Reference Viscosity
- \( \tau_m \) = Mangler Transformation

Reference

Table 5.11

SPALDING-CHI METHOD MODIFIED FOR REAL GAS HEAT TRANSFER
FOR TURBULENT BOUNDARY LAYER FLOW

(1) Spalding-Chi define $F_C$, $F_{R_0}$, $F_{R_s}$ (Spalding and Chi) which are functions of Mach Number and Temperature alone such that

$$\frac{1}{2} C_f F_C = \psi_0(F_{R_0} R_e) = \psi_s(F_{R_s} R_s)$$

(2) $F_C = \frac{1}{Z_t e} \left[ \int_0^1 \left( \frac{1}{ZT} \right) \frac{1}{2} d \left( \frac{u}{u_e} \right) \right]^{-2}$

Where $ZT = f(H, P_e)$ & $H = H_w + (H_{aw} - H_w)(\frac{u}{u_e}) - (H_{aw} - H_e)(\frac{u}{u_e})^2$

(3) $F_{R_0} = \left( \frac{H_{aw}}{H_w} \right)^{0.772} \left( \frac{H_e}{H_w} \right)^{0.702}$ (Wallace)

(4) $F_{R_s} = F_{R_0}/F_C$

(5) $C_{f_1} = 1/2 \exp \left\{ \sum_{i=1}^{10} g(i) \left[ \ln \left( \frac{F_{R_s} R_{s e}}{\tau_m} \right) \right]^{1-1} \right\} = 1/2 C_f F_C$

Where

$g(1) = 9.2808635$
$g(2) = -4.7340248$
$g(3) = 6.6858663 \times 10^{-1}$
$g(4) = -4.1876614 \times 10^{-2}$
$g(5) = -5.5054577 \times 10^{-4}$
$g(6) = 2.8367291 \times 10^{-4}$
$g(7) = -2.1249608 \times 10^{-5}$
$g(8) = 8.0162000 \times 10^{-7}$
$g(9) = -1.5900985 \times 10^{-8}$
$g(10) = 1.3236350 \times 10^{-10}$

$$R_{s e} = \frac{\rho_e u_{e s}}{u_e}$$

$$\tau_m = \text{Mangler Transformation (Komar)}$$

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Table 5.11 (Cont. 1)

Nomenclature

\[ \begin{align*}
C_f & \quad \text{Skin Friction Coefficient} \\
h_{\text{TURB}} & \quad \text{Heat Transfer Coefficient For Turbulent Flow} \\
H & \quad \text{Enthalpy} \\
P & \quad \text{Pressure} \\
P_r & \quad \text{Prandtl Number} \\
R_e_{\Omega_s} & \quad \text{Local Reynolds Number Based on Momentum} \\
\sigma & \quad \text{Thickness \( \sigma \) and Characteristic Length} \\
\tau & \quad \text{Respectively} \\
P & \quad \text{Inverse of Von Karman Reynolds Analogy Factor} \\
St & \quad \text{Stanton Number} \\
T & \quad \text{Temperature} \\
u & \quad \text{Velocity} \\
Z & \quad \text{Compressibility} \\
\rho & \quad \text{Density} \\
\end{align*} \]

Subscripts

\[ \begin{align*}
aw & \quad \text{Recovery} \\
e & \quad \text{Local} \\
i & \quad \text{Incompressible} \\
w & \quad \text{Wall} \\
\end{align*} \]

References


Rarefied flat plate heating correlations based on the work of Shorenstein and Probststein are given below.

1. \( T_0 = T_\infty \left(1 + \frac{x}{2} \frac{M_\infty^2}{2}ight) \)

2. \( R \alpha_\infty = \frac{\rho_\infty U_\infty x}{\mu_\infty} \)

3. \( C_v = \frac{H_W}{U_\infty} \frac{T_\infty}{T_W} \)

4. \( \beta = \left(\frac{T_W}{T_0}\right)^{1/2} M_\infty^2 C_v / R \alpha_\infty \)

5. \( C_{h_{SIG}} = (0.368 T_W / T_0 + 0.0684) \left[ M_\infty (C_v / R \alpha_\infty)^{1/2} \right]^{1/2} \)

6. \( \frac{C_H}{C_{h_{SIG}}} = \frac{1}{2} \left[ 1 - \tanh (0.91 \log_{10} \beta + 1.10) \right] \)

for \( \beta < 0.1 \)

7. Heat Transfer

\[ q = \rho_\infty U_\infty C_H (H_0 - H_W) \]

**Nomenclature**

- \( C_H \): Stanton Number
- \( H \): Enthalpy
- \( M \): Mach Number
- \( T \): Temperature
- \( U \): Velocity
- \( X \): Running Length
- \( \rho \): Density
- \( \gamma \): Specific Heat Ratio
- \( \mu \): Viscosity

**Subscripts**

- \( \infty \): Free Stream
- \( w \): Wall
- \( o \): Total

**Reference**

The rarefied flow heating to sharp and blunt cones based on the work of Engel and PraharaJ is as follows:

(1) \( T_R = T_W + \frac{(T_\delta + T_W)}{2} - T_\delta \cos^2 \theta_C/3 \)

(2) \( Re_\infty = \frac{\rho_\infty U_\infty x}{\mu_\infty} \)

(3) \( C^* = \frac{\mu \eta T_\delta}{\mu_\delta T_R} \)

(4) \( \xi = \frac{.9 C_H}{(\sin^2 \theta_C + P_\infty \cos^2 \theta_C/\rho_\infty U_\infty^2)^{.5}} \)

(5) \( \bar{\chi}_C = \frac{Re_\infty Ze}{M^2 v_\infty C_w \cos \theta_C} \)

(6) Correlation Equation

\[
\log_{10}(\xi) = \sum_{i=0}^{2} a_i \left( \log_{10}\bar{\chi}_C \right)^i
\]

\[
\text{Sharp} \quad \{ \begin{array}{l} a_0 = -0.344074 \\ a_1 = -0.349130 \\ a_2 = -0.104455 \\ a_3 = +0.022766463 \end{array} \]

\[
\text{Blunt} \quad \{ \begin{array}{l} a_0 = -0.647813 \\ a_1 = -0.365587 \\ a_2 = -0.0143793 \\ a_3 = +0.003281793 \end{array} \]

(7) Heat Transfer

\[ q = \rho_\infty U_\infty C_H (H_O - H_W) \]
Table 5.13 (Cont. 1)

Experimental Data

\[
\frac{0.9 \text{Gf/} \text{h} \cos \theta \cdot \sin \beta}{(\text{d/} \text{h})^{0}} \cdot K
\]

\[
\chi_{c}
\]

Nomenclature

\begin{align*}
C_H & \quad \text{Stanton Number} \\
H & \quad \text{Enthalpy} \\
M_{\infty} & \quad \text{Free Stream Mach Number} \\
T & \quad \text{Temperature} \\
U & \quad \text{Velocity} \\
Z & \quad \text{Post Normal Shock Compressibility} \\
\rho & \quad \text{Density} \\
Y & \quad \text{Specific Heat Ratio} \\
\mu & \quad \text{Viscosity}
\end{align*}

Subscripts

\begin{align*}
\infty & \quad \text{Free Stream} \\
W & \quad \text{Wall} \\
\delta & \quad \text{Post Normal Shock} \\
o & \quad \text{Total}
\end{align*}

Reference

The "\( \rho_r \mu_r \)" method was developed by Hanks from Boeing and is documented by Nagel and Thomas.

For laminar flow,

\[
h_L = 0.332g \frac{K_L}{\rho_r^{0.645}} L \sqrt{\frac{\rho_r \mu_r V_e}{P_r}}
\]

For turbulent flow,

\[
h_t = 0.185g \frac{K_T}{\rho_r^{0.645}} \frac{L}{\nu_{T''}} \frac{1}{\log_{10}(Re_r + 3000)} \left( \frac{\rho_r V_e}{\rho_r^{0.645}} \right) 2.584
\]

where

\[
L = 1 + (Le^{0.52} - 1) \frac{H_D \mu_r}{H_e}
\]

\[
H_D \mu_r = f(\rho_r, T_r) \quad \text{and} \quad Le = 1.4
\]

\[
\nu_{T''} = \nu_r \left( \frac{H_T}{H_r} \right)^{3/2} \frac{(T_r + 200)}{\left[ T_r \left( \frac{H_T}{H_r} \right) + 200 \right]}
\]

**Nomenclature**

- \( g \): Gravitational Constant
- \( h \): Heat Transfer Coefficient
- \( H \): Enthalpy
- \( L \): Running Length
- \( Le \): Lewis Number
- \( K_L \): Laminar Multiplier Factor
- \( K_T \): Turbulent Multiplier Factor
- \( P_r \): Prandtl Number
Table 5.14 (Cont. 1)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>Re</td>
<td>Reynolds Number</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>V</td>
<td>Velocity</td>
</tr>
<tr>
<td>ρ</td>
<td>Density</td>
</tr>
<tr>
<td>μ</td>
<td>Viscosity</td>
</tr>
</tbody>
</table>

Subscripts

- e: Edge
- D: Dissociation
- L: Laminar
- r: Eckert Reference Condition
- t: Turbulent
- T: Total

References


Table 5.15
AVERAGE SEPARATED LEESIDE ORBITER HEATING

The average separated leeside heating relations are based on correlations of wind tunnel data for the Space Shuttle Orbiter configuration by Bertin and Goodrich. The average leeward heating (turbulent or windward flow) is given by:

$$
S_t = (1.067 \left( \frac{H_{w}}{H_t} \right) + 0.7905)(0.00282(Re_{ns})^{-0.37})
$$

$$
Re_{ns} = \rho_\infty U_\infty R_{ref}/u_{ns}
$$

$$
R_{ref} = 1.0 \text{ foot for full scale}
$$

$$
\bar{q} = \rho_\infty U_\infty (H_t - H_w) S_t
$$

where the windward to total enthalpy ratio replaces the temperature ratio in the original paper.

Nomenclature

- $H_{w}$: Windward Wall Enthalpy
- $H_t$: Total Enthalpy
- $H_w$: Local Wall Enthalpy
- $Re$: Reynolds Number
- $R$: Radius
- $\rho_\infty$: Freestream Density
- $U_\infty$: Freestream Velocity
- $\bar{q}$: Average Leeside Heating Rate
- $\mu_{ns}$: Post Normal Shock Viscosity

Leeward surface area over which the heat-transfer measurements were averaged to obtain $S_{t,lep}$. 
Table 5.15 (Cont. 1)

A comparison of the correlation and data from the original paper are given below.

**Reference**

The procedure for calculating peak flap interference heating follows that of Fivel except for the separation length calculations.

### Table 5.16

**PEAK FLAP INTERFERENCE HEATING**

- **TRANSITION**
  \[
  \frac{Re_0}{Me^3} < 400 \quad \text{Laminar} \\
  \geq 400 \quad \text{Turbulent}
  \]

  where \( Re_0 \) = Undisturbed B.L. edge Reynolds number based on boundary layer thickness
  \( \delta \) = Boundary Layer (B.L.) thickness
  \( Me \) = Edge Mach Number

- **INCIPIENT SEPARATION** (Kessler, Reilly and Mockapetris)
  \[
  C_{p_1} \quad \text{Incipient separation pressure coefficient}
  \]
  - **Laminar**
    \[
    \log_{10} C_{p_1} = -0.361397 \log_{10} \left( \frac{Re_0}{Me^3} \right) - 0.662427
    \]
  - **Turbulent**
    If \( 2.6 \leq \log_{10} \frac{Re_0}{Me^3} \leq 3.8 \)
    \[
    C_{p_1} = 0.2615 \left( |\log_{10} \frac{Re_0}{Me^3} - 3.8| \right)^{3.5} + 0.405
    \]
If \( \log_{10} \frac{\text{Re}}{M_e^3} > 3.8 \)

\[
C_{p_1} = 0.0354 \left( \log_{10} \frac{\text{Re}}{M_e^3} - 3.8 \right)^{1.6} + 0.405
\]

**DEFLECTION ANGLE**

\[
\frac{P_3}{P_1} = 1 + \frac{\gamma}{2} C_p \frac{M_e^2}{P_1}
\]

\[
\tan^2 \theta_D = \left( \frac{\frac{P_3}{P_1} - 1}{\frac{\gamma M_e^2}{P_1} - \frac{P_3}{P_1} + 1} \right)^2 \frac{2 \gamma M_e^2 - (\gamma - 1) - (\gamma + 1) \frac{P_3}{P_1}}{(\gamma + 1) \frac{P_3}{P_1} + \gamma - 1}
\]

\( C_p = \) Separation Region Pressure Coefficient

\( \theta_D = \) Deflection Angle of Separated Streamline

**Note if** \( C_p = C_{p_1} = \) Incipient Separation Coefficient

**then** \( \theta_D = \theta_1 = \) Wedge Angle for Incipient Separation

**PLATEAU PRESSURE (Wuerer and Clayton)**

\[
\left( C_{p_{\text{PLAT}}} \right)_L = 1.60 \left( \text{Re}_x (M_e^2 - 1) \right)^{-1/4}
\]

\[
\left( C_{p_{\text{PLAT}}} \right)_T = 1.70 (\text{Re}_x)^{-1/10} (M_e^2 - 1)^{-1/4}
\]

\( C_{p_{\text{PLAT}}} = \) Plateau Pressure Coefficient

\( \text{Re}_x = \) Reynolds Number Based on Distance along a Streamline to the Hinge Line
Laminar (Giles and Thomas)

\[ \frac{L_s}{\delta} = \frac{35}{M_e^3} \left( \frac{P_F - P_{PL}}{Pe} \right) \frac{Re_{ex}^{1/4}}{0.98} \]

Turbulent (Popinski and Ehrlich)

\[ \frac{L_H}{x} = \frac{4}{\left( \frac{P_F}{Pe} \right)^{2/3}} \frac{Re_{ex}^{1/2}}{1.025} \]

\[ L_F = \frac{L_s \sin \theta_D}{\sin(180 - \theta_F)} \]

where

- \( Pe \) = Upstream Edge Pressure
- \( P_{PL} \) = Plateau Pressure
- \( P_F \) = Post Shock Flap Pressure
- \( x \) = B.L. Running Length to Hinge Line
- \( \delta \) = B.L. Thickness Upstream of Separation

Geometry

\[ L_H = \frac{L_s \sin \theta_F}{\sin(180 - \theta_D)/\sin(180 - \theta_F)} \]
PEAK REATTACHMENT HEATING (Bushnell & Weinstein)

\[ Re_{shear} = \frac{\rho_w U_F \delta_s}{\mu_w \sin(\theta_F - \theta_D)} \]

where

- \( U_F \) = Velocity at Reattachment Region
- \( \rho_w \) = Density at Reattachment Region at Wall
- \( \mu_w \) = Viscosity at Reattachment Region at Wall
- \( (\delta_s)_L = \delta + 5 \left( \frac{L_s \mu_s}{\rho_s U_s} \right) \)
- \( (\delta_s)_T = \delta + 1.6 \frac{L_s}{13} \)

where

- \( U_s \) = Shear Layer Edge Velocity
- \( \rho_s \) = Shear Layer Edge Density
- \( \mu_s \) = Shear Layer Edge Viscosity

PEAK STANTON NUMBER AT REATTACHMENT

\[ (St)_{PK} = C_{St} (Re_{shear})^{n_{St}} \]

where

<table>
<thead>
<tr>
<th></th>
<th>Lam</th>
<th>Turb</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{St} )</td>
<td>0.199</td>
<td>0.0204</td>
</tr>
<tr>
<td>( n_{St} )</td>
<td>-0.5</td>
<td>-0.2</td>
</tr>
</tbody>
</table>
Correlation of Peak Heating at Reattachment for Laminar and Turbulent Separated Flows (Bushnell and Weinstein)
Table 5.16 (Cont. 5)

REFERENCES


The peak fin-plate interference heating can be calculated using the method presented by Fivel, which basically came from Hayes. The peak interference heating occurs along a line near the fin on the plate. The peak heating angle is correlated using:

\[ \theta_{pk} = 0.24 (\theta_s - \alpha_F) + \alpha_F \]

The location of the peak heating line is given by:

\[ Y_{FP} = X_F \tan(\theta_{pk} - \alpha_F) \]

The boundary layer thickness at the leading edge of the fin based on the running length to the fin leading edge, \( X_0 \), is required.

The peak pressure ratio is based on correlated test data

\[ \frac{P_{pk}}{P_u} = (Me \sin \theta_s)^{\eta_{pk}} \]
Table 5.17 (Cont. 1)

Likewise, the peak heating ratio is based on a correlation of test data

\[
\frac{h_{pk}}{h_u} = (Me \sin \theta_s - 1.0) n_{st} + 0.75
\]

The exponent \( n_{pk} \) and coefficient \( n_{st} \) are obtained from interpolation of a faired curve through data from Hayes.

<table>
<thead>
<tr>
<th>( \frac{X_a}{\delta} )</th>
<th>( n_{pk} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.25</td>
</tr>
<tr>
<td>1.25</td>
<td>1.50</td>
</tr>
<tr>
<td>2.50</td>
<td>1.71</td>
</tr>
<tr>
<td>3.75</td>
<td>1.87</td>
</tr>
<tr>
<td>5.00</td>
<td>1.98</td>
</tr>
<tr>
<td>6.25</td>
<td>2.03</td>
</tr>
<tr>
<td>7.50</td>
<td>2.09</td>
</tr>
<tr>
<td>8.75</td>
<td>2.14</td>
</tr>
<tr>
<td>10.00</td>
<td>2.18</td>
</tr>
<tr>
<td>11.25</td>
<td>2.20</td>
</tr>
<tr>
<td>12.50</td>
<td>2.23</td>
</tr>
<tr>
<td>13.75</td>
<td>2.25</td>
</tr>
<tr>
<td>15.00</td>
<td>2.26</td>
</tr>
<tr>
<td>17.50</td>
<td>2.29</td>
</tr>
<tr>
<td>20.00</td>
<td>2.32</td>
</tr>
<tr>
<td>22.50</td>
<td>2.34</td>
</tr>
<tr>
<td>25.00</td>
<td>2.36</td>
</tr>
<tr>
<td>27.50</td>
<td>2.38</td>
</tr>
<tr>
<td>30.00</td>
<td>2.40</td>
</tr>
</tbody>
</table>

If \( \frac{X_a}{\delta} > 30 \), \( n_{pk} = 2.4 \)

\( n_{pk} \) is obtained by linear interpolation given \( \frac{X_a}{\delta} \).

<table>
<thead>
<tr>
<th>( \frac{X_a}{\delta} )</th>
<th>( n_{st} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.62</td>
</tr>
<tr>
<td>2.0</td>
<td>2.20</td>
</tr>
<tr>
<td>3.0</td>
<td>2.98</td>
</tr>
<tr>
<td>4.0</td>
<td>3.57</td>
</tr>
<tr>
<td>5.0</td>
<td>3.87</td>
</tr>
<tr>
<td>6.0</td>
<td>4.00</td>
</tr>
<tr>
<td>7.0</td>
<td>4.05</td>
</tr>
</tbody>
</table>

If \( \frac{X_a}{\delta} \leq 0.183 \), \( n_{st} = 1.0 \)

If \( 0.183 < \frac{X_a}{\delta} < 1.0 \),

\[
n_{st} = 0.35612 \ln(\frac{X_a}{\delta}) + 1.62
\]

If \( 1.0 \leq \frac{X_a}{\delta} \leq 7.0 \)

\( n_{st} \) determined by linear interpolation given \( \frac{X_a}{\delta} \)

If \( \frac{X_a}{\delta} > 7.0 \)

\[
n_{st} = 0.35612 \ln(\frac{X_a}{\delta}) + 3.357
\]
Table 5.17 (Cont. 2)

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>Heat Transfer Coefficient</td>
</tr>
<tr>
<td>P</td>
<td>Pressure</td>
</tr>
<tr>
<td>Me</td>
<td>Edge Mach Number Upstream of Fin</td>
</tr>
<tr>
<td>$X_a$</td>
<td>Distance Aft of Fin Leading Edge in Streamwise Direction</td>
</tr>
<tr>
<td>$X_o$</td>
<td>Running Length to Start of Fin in Streamwise Direction</td>
</tr>
<tr>
<td>$X_F$</td>
<td>Running Length Along Fin</td>
</tr>
<tr>
<td>$Y_{FP}$</td>
<td>Peak Heating Location Normal to Fin</td>
</tr>
<tr>
<td>$\alpha_F$</td>
<td>Effective Angle of Attack of Fin</td>
</tr>
<tr>
<td>$\theta_{pk}$</td>
<td>Angular Location of Peak Heating</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>Shock Angle</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Upstream Boundary Layer Thickness</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subscripts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>pk</td>
<td>Peak</td>
</tr>
<tr>
<td>u</td>
<td>Undisturbed</td>
</tr>
</tbody>
</table>

References


Section 6.0

LENGTH AND TRANSITION ROUTINES

This section describes the subroutines which determine boundary layer transition, running length corrections and cross flow adjustments.

6.1 SUBROUTINE TRANS AND EDPRAM

The purpose of routine TRANS is to determine if the flow is laminar, transitional or turbulent. The equations for each of the transition criteria are presented in Table 6.1. If a transition option greater than 8.0 is used, then an error message is printed from TRANS and the routine returns NERROR = 1 to MAIN. This causes MAIN to void the remainder of this problem and read in data for the next case. If the flow is determined to be laminar, then the heat transfer coefficient returned from TRANS is equal to the laminar heat transfer coefficient. If the flow is fully turbulent, then the heat transfer coefficient returned from TRANS is the turbulent heat transfer coefficient. If the flow is determined to be transitional, then the heat transfer coefficient returned from TRANS is determined by linear interpolation between the laminar and turbulent values.

The interpolation parameter for TRANS = 1 and 2 is time (TIME). The interpolation parameter for TRANS = 3 is the Reynolds number based on running length (corrected for crossflow) or body diameter and free-stream properties. However, no correction is made to account for axisymmetric flow. For TRANS = 4, the interpolation parameter is compressible momentum thickness Reynolds number. Transition option 5 uses

\[
\text{PARA} = \psi
\]

as the interpolation parameter. For transition option 6, the interpolation
parameter is the laminar running length corrected for crossflow and corrected for axisymmetric flow (Mangler transformation factor). Transition option 7 uses the input running length as the interpolation parameter. Transition option 8 uses Re₉/Me as the interpolation parameter. Whenever the flow is laminar, then PCT = 0.0, turbulent PCT = 1.0, and transitional 0.0 < PCT < 1.0. The parameter PCT represents how close the flow is to being fully turbulent based on the above interpolation parameters. The parameter PCT is also used in the output routine (VANOUT) to identify the flow type. It should also be noted that the recovery enthalpy for transitional flow is also determined by linear interpolation between the laminar and turbulent recovery enthalpies.

The following is a list of the parameters used in the CALL TRANS statement:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENCL</td>
<td>Laminar heat transfer coefficient</td>
</tr>
<tr>
<td>ENCT</td>
<td>Turbulent heat transfer coefficient</td>
</tr>
<tr>
<td>PARA1</td>
<td>Onset of transitional flow parameter</td>
</tr>
<tr>
<td>PARA2</td>
<td>Onset of fully turbulent flow parameter</td>
</tr>
<tr>
<td>PARA</td>
<td>For transition flag 4, this is equal to the compressible momentum thickness Reynolds number. For transition flag 5, this is equal to the KDAC-EAST transition parameter. For transition flag 6, this is equal to the corrected (crossflow and axisymmetric flow) running length to the onset of transition. This is not used for the other transition options.</td>
</tr>
<tr>
<td>TIME</td>
<td>Trajectory time</td>
</tr>
<tr>
<td>ENC</td>
<td>Heat transfer coefficient</td>
</tr>
<tr>
<td>BRE</td>
<td>Recovery enthalpy</td>
</tr>
<tr>
<td>NTR</td>
<td>NTR = 1 corresponds to laminar flow NTR = 2 corresponds to transitional flow NTR = 3 corresponds to turbulent flow</td>
</tr>
<tr>
<td>PCT</td>
<td>Fraction of turbulent flow PCT = 0.0 corresponds to laminar flow PCT = 1.0 corresponds to fully turbulent flow 0.0 &lt; PCT &lt; 1.0 corresponds to transitional flow</td>
</tr>
<tr>
<td>TRFLAG</td>
<td>Transition option</td>
</tr>
<tr>
<td>EL</td>
<td>Geometric running length</td>
</tr>
<tr>
<td>ELFAC</td>
<td>Multiplication factor which, when multiplied times the running length to the onset of transition, gives the running length to the onset of fully turbulent flow</td>
</tr>
<tr>
<td>ENL</td>
<td>Laminar Mangler transformation factor to account for axisymmetric flow corrections</td>
</tr>
<tr>
<td>ELL</td>
<td>Laminar running length corrected for crossflow effects</td>
</tr>
</tbody>
</table>
Another routine which is closely related to TRANS is EDPARM. Routine EDPARM is used to determine the MDAC-E onset of transition parameter solely as a function of angle-of-attack. The angle-of-attack range is from 0 to 70 degrees. There are six data points in the data table (α = 0, 30, 40, 50, 60, and 70 degrees) along with the corresponding values of the transition parameter. Interpolation at angles-of-attack not in the table is performed with the aid of routine TBLIN.

6.2 SUBROUTINE VRUNL

Subroutine VRUNL is called by MAIN and is not called unless heat transfer technique 3, 4, or 5 (flat-plate laminar and turbulent methods) is being used. Subroutine VRUNL insures that the length parameter used in the turbulent heat transfer equations (when using transition options 3, 4, 5, 6, or 7) is corrected for the assumption that the running length should be measured from the onset of transition. The argument of subroutine VRUNL contains the following parameters:

- **TRFLAG** - (input) May have values 3.0, 4.0, 5.0, 6.0, or 7.0 corresponding to transition options 3.0, 4.0, 5.0, 6.0, or 7.0. (It should be noted that VRUNL will not be called unless a positive value is stored in location 29.)
- **ELTRAN** - (output) Running length at onset of transition with or without crossflow correction
- **ELL** - (input) Equivalent running length with or without crossflow correction for laminar flow
- **ELT** - (input) Equivalent running length with or without crossflow for turbulent flow and is corrected in (output) VRUNL to account for virtual origin from the onset of transition
- **ELTP** - (output) Equal to input value of ELT
- **PARA1** - (input) This parameter denotes the onset of fully transitional flow
- **PARA2** - (input) This parameter denotes the onset of fully turbulent flow
- **ENL** - (input) Laminar multiplication factor

If PARA2 < PARA1, then PARA = PARA2. If PARA2 > PARA1, then PARA = PARA1.

If TRFLAG < 3.0, then MAIN does not call VRUNL. Therefore, VRUNL is really only
meaningful for TRFLAG = 3.0, 4.0, 5.0, 6.0, or 7.0. The values TRFLAG = 1.0 or 2.0 correspond to time dependent transition and are handled through the main routine. If TRFLAG = 6.0, then VRUNL calls EDPARM as a function of angle-of-attack to determine PARA. The value of ELTRAN is given by

\[ ELTRAN = L_{TR} \]

If TRFLAG = 7.0, then the logarithm (base 10) of the transition Reynolds number RELG, based on edge conditions, is determined as a function of edge Mach number \( M_e \) from a built-in table (North American Rockwell technique). The value of ELTRAN is given by

\[ ELTRAN = (10^{\frac{Re_{TR} \mu_e}{\rho_e u_e}}) \]

The interpolation to determine \( \log_{10} Re_{TR} \) is performed with the aid of routine TBLIN. If TRFLAG = 5, then ELTRAN is the same as for option 5. However, the value of PARA = PARA1 is input through the routine argument to define transition onset instead of being determined by EDPARM. If TRFLAG = 4, then the running length at the onset of transition is determined based on the compressible momentum thickness Reynolds number:

\[ ELTRAN = \left[ \frac{Re_{\theta} \mu_e}{\frac{.664}{\rho_e u_e}} \right]^2 \frac{1}{\rho^* \mu^* u_e} \]

where PARA = Re\( \theta \) is input to define the onset of transition. If TRFLAG = 3, then the running length at the onset of transition is determined based on the Reynolds number computed using boundary-layer edge conditions:

\[ ELTRAN = \frac{Re_L \mu_e}{\rho_e u_e} \]

where \( Re_L \) = PARA.
After the running length at the onset of transition is calculated, the turbulent running length is stored (ELTP = ELT) and a check is made to determine if ELL < ELTRAN. If ELL < ELTRAN, then ELT = ELTP. If ELL > ELTRAN, then ELT = ELT - ELTRAN. Thus when VRUNL returns to the calling routine, ELTP contains the input value of ELT and ELT may or may not contain the input value of ELT depending on whether ELL < ELTRAN or ELL > ELTRAN. If ELL > ELTRAN, then ELT contains the turbulent running length measured from the onset of transition.

6.3 SUBROUTINE CRSFLW

Subroutine CRSFLW is called by MAIN and the purpose of this routine is to account for the effects of streamline divergence on basic geometries at angle-of-attack. Two types of geometry are considered in this routine: a constant width shape (rectangle) and a sharp-edge triangle (delta wing). For each of the two geometries it is possible to get an ideal or real gas calculation for both laminar and turbulent flow. In each circumstance, a correction factor is applied to the input geometric length, and this corrected running length is used in the flat-plate heat transfer equations in place of the input geometric length. The parameters in the argument of the CALL CRSFLW statement are listed and defined below:

CFFLG - (input) Type of crossflow correction selected
- Constant width rectangle assuming ideal gas chordwise velocity gradient
- Constant width rectangle using a real gas chordwise velocity gradient
- Sharp-edged delta configuration assuming ideal gas chordwise velocity gradient
- Sharp-edge delta configuration using a real gas chordwise velocity gradient

ELL - (output) Laminar running length corrected for crossflow
ELT - (output) Turbulent running length corrected for crossflow
EL - (input) Pr real geometric running length input into MINIVER
ELRDA - (input) Delta sweep angle
DUSBO - (input) Rectangle width
CORH - (input) Rectangle corner radius
UE - (input) Velocity at the edge of the boundary layer
PE - (input) Pressure at the edge of the boundary layer
RH0E - (input)  Density at the edge of the boundary layer
PU - (input)  Pressure upstream of the previous shock (if no shock, then freestream pressure)
ALPHA - (input) Surface effective angle-of-attack
UDOT - (output) Designates the nondimensional crossflow, stagnation-point, velocity gradient at a point on the centerline (of the wing) as a ratio to the velocity gradient at the stagnation point of a sphere with a diameter equal to the planform width
VU - (input) Velocity upstream of the previous shock (if no shock then freestream Mach number)
XMACh - (input) Mach number upstream of the previous shock (if no shock then freestream Mach number)

Having entered CRSFLW, the first check that occurs is to determine if ALPHA < 0, if so then ELL = EL. If ALPHA > 0, then the decision is made as to which crossflow option should be used. If CFFLG = 1, the necessary program input is CORNR and DSUBO. The routine then proceeds to compute the parameter \( \frac{xV}{D_0} \). If \( \frac{xV}{D_0} < 10^{-5} \), then no crossflow is used. This indicates the velocity gradient is so small that crossflow is insignificant. If \( \frac{xV}{D_0} > 6 \), then the routine shifts to a simplified calculation of ELL and ELT (this situation corresponding to the case of a very large velocity gradient). If \( 10^{-5} < \frac{xV}{D_0} < 6 \) then the standard crossflow correction expressions are used to compute ELL and ELT and then CRSFLW returns to MAIN. If CFFLG = 3, then the ideal gas crossflow correction for a sharp-edged delta is used to compute ELL and ELT. When CFFLG = 3, the only necessary input is the delta sweep angle, ELMBDA.

If CFFLG = 2, then the crossflow correction for a constant width rectangle using a real gas chordwise velocity gradient is used to compute ELL and ELT. The required input are DSUBO and UDOT. If CFFLG = 4, then the crossflow correction for a sharp delta configuration using a real gas chordwise velocity gradient is employed to compute ELL and ELT the required inputs are the delta sweep.
normal component of upstream Mach number XMACHU is less than one. If XMACHU < 1, then a subsonic technique is used to compute the crossflow correction. If XMACHU ≥ 1, then a supersonic technique is used for crossflow correction. However for CFFLG = 1 or 3, no distinction is made for XMACHU < 1 and XMACHU ≥ 1.
Table 6.1
TRANSITION OPTIONS

There are currently eight transition options in the LANMIN code consisting of the following:

1. **Time Dependence: Laminar to Turbulent**

   \[ t \leq t_I \quad \text{Laminar} \]
   \[ t \geq t_{II} \quad \text{Turbulent} \]
   \[ t_I < t < t_{II} \quad \text{Transitional} \]
   \[ n = \frac{t-t_I}{t_{II}-t_I} \quad \text{Percentage of Fully Turbulent Flow} \]

2. **Time Dependence: Turbulent to Laminar**

   \[ t \leq t_I \quad \text{Turbulent} \]
   \[ t \geq t_{II} \quad \text{Laminar} \]
   \[ t_I < t < t_{II} \quad \text{Transitional} \]
   \[ n = \frac{t_{II}-t}{t_{II}-t_I} \quad \text{Percentage of Fully Turbulent Flow} \]

3. **Reynolds Number Dependence**

   \[ Re_\infty = 2\rho_\infty U_\infty R/\mu_\infty \quad \text{for Swept Cylinders} \]
   \[ Re_X = \rho_e U_e X/\mu_e \quad \text{for Plate Options} \]
   \[ Re \leq Re_I \quad \text{Laminar} \]
   \[ Re > Re_{II} \quad \text{Turbulent} \]
   \[ Re_I < Re < Re_{II} \quad \text{Transitional} \]
   \[ n = \frac{Re-Re_I}{Re_{II}-Re_I} \quad \text{Percentage of Fully Turbulent Flow} \]

4. **Compressible Momentum Reynolds Number Dependency**

   \[ Re_\theta^* = 0.664 \frac{\sqrt{\rho^* \mu^* Re_X}}{\sqrt{\rho_e U_e \tau_L}} \quad \text{Compressible Momentum Thickness Reynolds Number} \]

---

on
Table 6.1 (Cont. 1)

\[
\begin{align*}
Re_\theta^* & \leq Re_\theta^I \quad \text{Laminar} \\
Re_\theta^* & \geq Re_\theta^II \quad \text{Turbulent} \\
Re_\theta^I & < Re_\theta^* < Re_\theta^II \quad \text{Transitional} \\
\eta & = \frac{Re_\theta^* - Re_\theta^I}{Re_\theta^II - Re_\theta^I} \quad \text{Percentage of Fully Turbulent Flow}
\end{align*}
\]

5. MDAC-E Transition Parameter

This transition option was developed by Masek and Kipp

\[
\psi = \frac{Re_\theta^*}{[Me(\rho e u_e/u_e)^{\alpha^2}]}
\]

\[
\begin{align*}
\psi & \leq \psi_I \quad \text{Laminar} \\
\psi & \geq \psi_{II} \quad \text{Turbulent} \\
\psi_I & < \psi < \psi_{II} \quad \text{Transitional} \\
\eta & = \frac{\psi - \psi_I}{\psi_{II} - \psi_I} \quad \text{Percentage of Fully Turbulent Flow}
\end{align*}
\]

6. MDAC-E Transition Table Look-up Dependency

The built in tabular function of angle of attack from Masek and Kipp was developed for conical flows. Transition is based on the parameter \(\psi\) from option 5.

\[
\begin{array}{c|c}
\alpha & \log_{10} \psi_{TR} \\
\hline
0 & 1.0 \\
30 & 1.0 \\
40 & 1.0412 \\
50 & 1.1383 \\
60 & 1.30103 \\
70 & 1.69897 \\
\end{array}
\]

\{ \text{Linear Interpolation is used} \}

Once the transition onset parameter, \(\psi_{TR}\), is calculated, the transition length is determined:

\[
L_{TR} = 2.26 \left( \frac{(\psi_{TR} Me)^2 \rho_e \mu_e}{\rho_e u_e} \right)^{1.6}
\]

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Table 6.1 (Cont. 2)

If a crossflow option has been flagged, the following procedure is used:
Since the transition parameter \( \psi \) was derived for complex shapes using
the laminar crossflow length in the calculation of \( \text{Re}_{\text{BL}} \), it is necessary
to remove this correction factor so that the resulting transition length
can be compared with the geometric length.

For crossflow options for rectangular shapes the transition length is

\[
L_{\text{TR}} = 0.5 \bar{v} \ln \left( \frac{1 - 2L_{\text{TR}}'}{L_{\text{TR}}} \right)
\]

For delta wings options the transition length is

\[
L_{\text{TR}} = B L_{\text{TR}}'
\]

When the Mangler transformation, \( \tau_L \), is used, the transition length is

\[
L_{\text{TR}} = \tau_L L_{\text{TR}}'
\]

Two methods for treating the extent of transition have been provided.
The first is rather simple in that the fully turbulent length, \( L_{\text{FT}} \), is
a factor times the onset length

\[
L_{\text{FT}} = K_{\text{TE}} \cdot L_{\text{TR}}
\]

The second method defines the factor \( K_{\text{TE}} \) in terms of the boundary layer
edge Reynolds number at the onset length, which is defined as

\[
\text{Re}_{\text{TE}} = \frac{\rho e u_L L_{\text{TR}}}{\nu_e}
\]

The table, which is built in, consists of

\[
\begin{array}{|c|c|}
\hline
\log_{10} \text{Re}_{\text{TE}} & K_{\text{TE}} \\
\hline
5 & 5.64 \\
6.518 & 2 \\
6.778 & 1.702 \\
7 & 1.605 \\
7.301 & 1.535 \\
10 & 1.535 \\
\hline
\end{array}
\]

Linear Interpolation is used
Table 6.1 (Cont. 3)

The state of flow is determined by

\[ L_{\text{LCF}} \leq L_{\text{TR}} \quad \text{Laminar} \]
\[ L_{\text{LCF}} \geq L_{\text{FT}} \quad \text{Turbulent} \]
\[ L_{\text{TR}} < L_{\text{LCF}} < L_{\text{FT}} \quad \text{ Transitional} \]
\[ n = \frac{L_{\text{FT}} - L_{\text{LCF}}}{L_{\text{FT}} - L_{\text{TR}}} \quad \text{Percentage of Fully Turbulent Flow} \]

Since the transition criteria was developed for a conical shock flowfield, some judgement must be used in applying it to other situations.

7. NAR Transition Parameter

A criteria developed by North American Rockwell is based on edge Reynolds number as a function of edge Mach number. The built in table is

<table>
<thead>
<tr>
<th>Me</th>
<th>( \log_{10} \text{Re}_{\text{TR}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5.30103</td>
</tr>
<tr>
<td>1.0</td>
<td>5.54407</td>
</tr>
<tr>
<td>1.5</td>
<td>5.81291</td>
</tr>
<tr>
<td>2.0</td>
<td>6.00</td>
</tr>
<tr>
<td>3.5</td>
<td>6.00</td>
</tr>
<tr>
<td>4.5</td>
<td>6.07912</td>
</tr>
<tr>
<td>5.5</td>
<td>6.25527</td>
</tr>
</tbody>
</table>

\( L_{\text{TR}} = \frac{\text{Re}_{\text{TR}} \mu_e}{(\rho_e U_e)} \) Transition length

The transition extent is the same as for option 6. The state of flow determination is based on \( L_{\text{GO}} \) rather than \( L_{\text{LCF}} \).

8. Compressible \( Re_\theta/Me \) Dependency

\[ Re_\theta = 0.554 \sqrt{\frac{\rho^* u^*}{\rho e u_e}} \frac{Re_x}{\tau_L} \]
Table 6.1 (Cont. 4)

\[ R_f = \frac{Re_d}{Me} \]

<table>
<thead>
<tr>
<th>Condition</th>
<th>Turbulence State</th>
<th>Nominal Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_f \leq R_{fI} )</td>
<td>Laminar</td>
<td>( R_{fI} = 150 )</td>
</tr>
<tr>
<td>( R_f \geq R_{fII} )</td>
<td>Turbulent</td>
<td>( R_{fII} = \sqrt{2} R_f )</td>
</tr>
<tr>
<td>( R_{fI} &lt; R_f &lt; R_{fII} )</td>
<td>Transitional</td>
<td></td>
</tr>
</tbody>
</table>

\[ n = \frac{R_f - R_{fI}}{R_{fII} - R_{fI}} \]

Percentage of Fully Turbulent Flow

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Cross Flow Parameter</td>
<td>( \bar{V} )</td>
<td>Chordwise Velocity Gradient</td>
</tr>
<tr>
<td>L</td>
<td>Length</td>
<td>( X )</td>
<td>Cross Flow Corrected</td>
</tr>
<tr>
<td>M</td>
<td>Mach Number</td>
<td>( \rho )</td>
<td>Density</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>( \mu )</td>
<td>Viscosity</td>
</tr>
<tr>
<td>U</td>
<td>Velocity</td>
<td>( \tau )</td>
<td>Mangler Factor</td>
</tr>
</tbody>
</table>

Subscripts

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Diameter</td>
</tr>
<tr>
<td>e</td>
<td>Edge</td>
</tr>
<tr>
<td>Geo</td>
<td>Geometric</td>
</tr>
<tr>
<td>I</td>
<td>Initial</td>
</tr>
<tr>
<td>II</td>
<td>Final</td>
</tr>
<tr>
<td>L</td>
<td>Laminar</td>
</tr>
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<tr>
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<td>Transition Extent</td>
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<tr>
<td>TR</td>
<td>Transition</td>
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<tr>
<td>( \theta )</td>
<td>Momentum Thickness</td>
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References


Section 7.0
APPLICATION TECHNIQUES

The purpose of this section is to provide some guidance to the engineer in applying the LANMIN code in the prediction of heating conditions. In order to help achieve this objective, several sample cases are shown. These cases contain experimental wind tunnel data compared with different options available in LANMIN. Based on these results and past experience with the code, a set of recommended options are given.

7.1 GENERAL APPLICATION

In order to achieve the best prediction method using the LANMIN code several key parameters need to be examined.

1. Determine the shock option which establishes the boundary layer edge entropy level.
2. Determine the geometric Mangler transformation which gives the appropriate running length adjustment for the given geometry.
3. Determine the pressure option which best matches the surface pressure.
4. Determine the heating method and associated Reynolds analogy factor which best fits the experimental heating data.

The shock option is usually selected by determining which option produces the approximate shock angle which processes the boundary layer flow. This is usually established by using wind tunnel schlieren.

The Mangler transformation used in LANMIN is the geometric transformation of the running length for a body into the equivalent length for a flat plate. The mathematical relations which can be used for a compound geometry body which is axisymmetric are as follows:

\[ \tau_m = \frac{r_s}{\int_0^s r ds} \]

Laminar

\[ \tau_m = \frac{r_s^2}{\int_0^s r^2 ds} \]

Turbulent

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where \( r \) is the radius of the body normal to the axis of symmetry and \( \Delta \) is the surface distance from the nose to the point of interest on the body. The preceding equations can be numerically integrated for compound body shapes to obtain the laminar and turbulent Mangler factor as a function of running length.

Selection of the pressure option and Reynolds analogy factor for a problem is based primarily on experience. In order to document some of this experience, several sample cases will be examined. Figure 7.1 shows a comparison of wind tunnel pressure data with the LANMIN tangent cone option and a method of characteristic (MOC) calculation. The data and theories are for the cylinder section of the Space Shuttle external tank at zero angle of attack. For this situation, the tangent cone option does not predict the data well. This option would also correspond to the modified Newtonian pressure option. The MOC results do compare well with data. Thus, results from a more exact theoretical method should be used as input to LANMIN, where available, to establish the edge pressure.

Heating results from LANMIN using both pressure options are shown in Figure 7.2 using the best heating prediction method. The overprediction trend of the tangent cone option, shown in Figure 7.1a, is translated into an over prediction in heating as shown in Figure 7.2. Likewise, when the pressures from the more accurate MOC solution are used, the heating is more accurately predicted.

The Reynolds analogy factor effect is examined in Figure 7.3. The Colburn analogy factor is shown to overpredict the data and a unity Reynolds analogy factor underpredicts the data. The von Karman analogy factor when used with the Spalding-Chi heating method with accurate pressures provides the best comparison with data. This trend is substantiated further by the results given in Figure...
7.4. The pressure at small to moderate angles of attack along the windward streamline is predicted well by both the tangent cone option and NCF solutions. Thus the heating data in Figure 7.4a compares well with calculations using both pressure options. The theory and data comparisons in Figure 7.4b and c demonstrate that the von Karman Reynolds analogy factor gives the best prediction.

Results from the Spalding-Chi option using the von Karman Reynolds analogy are shown in Figure 7.5 along with an independent heating theory for a different geometry. Heating data for a sphere-cone-cylinder (SRB) body are shown. The DIROLIN results are from an integral momentum solution based on the work of Dirlik (Ref. 7). The two theories are in good agreement and compare well with the wind tunnel data.

Another application of the LANMIN code is shown in Figure 7.6. This case is a sharp wedge at three angles of attack. Three-dimensional effects on the wedge produced pressures higher than the two-dimensional wedge pressure option. By using the wedge shock entropy and inputting the measured pressures, the LANMIN results compare well with data and results from an independent theory of Dirlik (Ref. 7).

All preceding results and comparisons have been for turbulent flow. The laminar method of Eckert in LANMIN is compared with wind tunnel data in Figure 7.7. The tangent cone pressure option was used for this calculation. Using more precise pressures would have slightly lowered the predicted values in the range 0.08 < X/L < 0.22 and slightly increased the values in the range 0.22 < X/L < 0.60.
Other comparisons of data and theory have been presented in the tables of Section 5.0. These results along with other experience were used to determine a recommended set of heating options. These options are given in Table 7.1.

7.2 GENERIC ORBITER APPLICATIONS

Heating to a generic orbiter geometry can be calculated by a numerous combination of options for a specific location on the vehicle. The success in accurately predicting the heating to any given location is determined in large part by the proper selection of options and the engineering skill of the user. A general approach is given herein for orbiter application. Specific geometries and flow situations may require modification of the guidelines given.

The approach taken follows the selection of the four key parameters given in Section 7.1 as specifically applied to a generic orbiter shape. Figure 7.8 gives the selected generic orbiter design methods in summary form. The approach taken was to select methods which would best represent measured data. If a design conservatism is to be introduced, it should be introduced as a known constant multiplier. In order to demonstrate and help explain the options selected, a discussion is given of three of the principal areas on an orbiter.

**Nose Region**

The heating to the bottom centerline of an orbiter nose can be calculated using the options in LANMIN by determining an effective sphere value. The effective sphere value is calculated with normal shock entropy. The heating methods of Lees and Detra-Hidalgo are used for laminar and turbulent flow respectively. Consider the laminar case as an example. The heat transfer coefficient is determined by

\[ h = \left( h_0 \sqrt{\frac{R_0}{r}} \right) \left( \frac{h}{h_0} \right) \text{Lees} \]
where

$\theta_o$ is the Fay and Riddell value for radius $R_o$

$r$ is the local radius of the body

$h/\theta_o$ is the Lees distribution ratio at the local body angle plus angle of attack

An example of this method being applied to a sphere-cone geometry is given in Fig. 7.9. The Lees distribution value on the cone is evaluated at $90 - 20 - 15 = 55$ degrees, and the local radius, $r$, is a function of axial location. This rather simple method provides quite good agreement with the data shown.

This method was also applied to the nose region of an orbiter configuration as shown in Fig. 7.10. The area of application was $0 \leq X/L < 0.05$ on the orbiter bottom centerline where theory and data agree for all three angles of attack.

Thus by inputting the body angle and local radius an effective sphere distribution value is calculated which represents the nose region bottom centerline. The effect of angle of attack is accounted for by the program calculating an effective body angle.

**Bottom Centerline Heating**

In order to select a usable method for LANMIN for the bottom centerline of an orbiter, each component of the calculation must be considered. The comparisons given in Fig. 7.10 for $X/L > 0.05$ are sample trials of different options. At one angle of attack, a given set of options may agree with data but not agree with data at other angles of attack. This approach confounds the effects of the different options in a set. Thus, final agreement for one case does not assur
applicability over a wide range of conditions.

Let us consider the shock shape which controls the boundary layer edge conditions. Figure 7.11 presents a correlation of shock shapes first published for delta wing data by Dunavant (Ref. 14). The generic orbiter data from four sources were added in the present study. Virtually all of the data agrees with tangent cone shock theory for angles of attack from 10 to 45 degrees. Most all orbiters fly in this angle of attack range during the highest heating period. Thus the shock option to be used is the tangent cone option.

Determining the correct running length to use is the most complicated and most critical decision to be made. The LANMIN code has cross-flow correction options and Mangler transformations to assist in correcting the actual running length to an equivalent running length. For some geometries these running length corrections can be quite accurate and easy to select. This is not the case for generic orbiter geometries.

Another approach is selected here based on the modification of earlier work by Pond et al (Ref. 15). Minor heat transfer data are used to calculate the effective running length to the body location of interest. This procedure finds the actual flat plate equivalent running length which is required for input into LANMIN. This effective running length is then used for flight prediction calculations. A sample case is described to provide details of the approach.

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* Other analyses have developed complex streamline tracing methods which require an accurate and smooth pressure distribution input. These methods are out beyond the scope of the LANMIN approach.
Pond et al (Ref. 15) tested the geometry shown in Fig. 7.12 to obtain heating data. The heating data were used with Eckerts heating method to calculate the equivalent running lengths shown in Fig. 7.13. For this geometry it is evident that the equivalent running lengths are a strong function of angle of attack and axial location.

Work by Dunavant (Ref. 14) which was later amplified by Newmann and Renfroe (Ref. 16) dealt with obtaining an outflow correction for the running length. For hypersonic flow, this correction can simply be written as

\[ L_e = L (\tan \alpha / \tan \alpha) \]  \hspace{1cm} (7.1)

where

- \( L_e \) = equivalent running length
- \( L \) = actual running length
- \( \alpha \) = angle of attack
- \( \alpha = 90 - \) sweep angle

By looking at the geometry in Fig. 7.12 a single value to use for \( \alpha \) is not evident. However, by taking the data in Fig. 7.13 and using it in the preceding equation, \( \alpha \) can be calculated. The results are shown in Fig. 7.14. All of the 20, 30 and 40 degree angle of attack data collapse to give an effective sweep angle complement as a function of axial location. The \( \alpha = 10 \) data is for an inflow condition and is thus not expected to correlate. The data for \( \alpha = 55 \) exhibit a trend which is not completely understood and may be due to lack of quality paint data for the original heating data. Since the preceding relation does collapse the data in the primary range of interest, it can be used along with limited wind tunnel data to develop equivalent running lengths for input.

To complete the explanation of the procedure, the equivalent running length
data in Fig. 7.13 was used in LANMIN for a completely different orbiter shape. The results from LANMIN are compared in Fig. 7.15 with data from a phase B orbiter geometry. The comparison is quite good and significantly improved over the comparisons made in Fig. 7.10. Note that the tangent wedge pressure option was used. Unfortunately, pressure data are not available for comparison. However, Mach number data on orbiter shapes are usually overpredicted when wedge pressures are used. Thus, wedge pressures are most likely too low. This might explain why the theoretical results are slightly lower than measured in Fig. 7.15.

Thus far only laminar equivalent lengths and results have been discussed. The laminar heating rates are more sensitive to the equivalent length \( h \sim Xe^{3/2} \) than turbulent heating rates \( h \sim Xe^3 \). The laminar and turbulent equivalent lengths are different. For example on a cone the differences are:

\[
Xe_L = X/3 \quad \text{from Mangler relations}
\]
\[
Xe_t = X/2
\]

Thus an adjustment must be made to the laminar equivalent length to obtain a turbulent equivalent length. Since no data were available for turbulent equivalent lengths, the cone ratio was used

\[
Xe_t = 3Xe_L/2
\]

to obtain input to LANMIN for the case shown in Fig. 7.16. The comparison of data and theory is quite good. Note that a 50% increase in the running length when used in turbulent heating equations only reduces the heating by 8 percent.

The procedure to use the equivalent running length approach is as follows:

1. **Obtain laminar wind tunnel data for the generic orbiter of interest.**
2. **Use LANMIN parametrically to generate laminar equivalent running lengths.**

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(3) If data are available, repeat step (1) and (2) to verify the laminar to turbulent transformation.

(4) If limited data are available, use equation 7.1 along with an effective s factor for the limited data to expand the angle of attack range.

(5) If no wind tunnel data are available use Fig. 7.13 or closest geometry equivalent length data (Note a set of plots/tables of equivalent lengths for a set of geometries should be developed as a data base).

(6) Use the equivalent running length in LANMIN for flight applications along with the same shock, pressure and heating rate options used to generate the equivalent running lengths.

This method has been discussed in terms of the bottom centerline. However, it has a much broader application. This method may be applied to any body location which is not influenced by separation or shock interference.

**Wing Leading Edge**

The stagnation line of a wing is approximated well by swept cylinder theory. The shock lies parallel to the wing over a considerable portion of the wing as shown in the schlieren of Fig. 7.17. The laminar flow data agree well with the correlation in LANMIN. A comparison for turbulent flow of Beckwith—Gallagher with data is given in Table 5.6a.
Table 7.1

RECOMMENDED METHODS

<table>
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<tr>
<th>GEOMETRY</th>
<th>HEATING OPTION</th>
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<tr>
<td>SPHERE</td>
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<td>Turbulent: Detra and Hidalgo</td>
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<td>Laminar: Correlation</td>
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<td></td>
<td>Turbulent: Beckwith-Gallagher</td>
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<tr>
<td>CONE, WEDGE, OGIVE &amp; ORBITER</td>
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<td>BOTTOM CENTERLINE</td>
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<td>FLAP</td>
<td>Bushnell and Weinstein</td>
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<td>FIN INTERFERENCE ON PLATE</td>
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Fig. 7.1  Pressure Ratio on the Space Shuttle External Tank Barrel  
(From Ref. 5)
Fig. 22 Heat transfer distribution on the Space Shuttle external tank.

Fig. 23 Heat transfer distribution on the Space Shuttle external tank.

From Ref. 5
Fig. 7.4 Heat-transfer Distribution on the Space Shuttle External Tank for Various Tunnel Conditions (From Ref. 6)
Fig. 7.5: Comparison of Experimental Data and MINIVER and DIRLIN Results for Right SRB Unmated Configuration at $M_\infty = 3$ ($\alpha = 0^\circ$, $\beta = 0^\circ$)
Fig. 7.6: Heating Rate Versus Length for Three Test Conditions at the Ames TPS Calibration Test
Fig. 7.7 Laminar Heat Transfer Distribution on the Space Shuttle External Tank
1. Normal Shock
   Stagnation Point: Fay & Riddell
   Distribution: Laminar - Lees
                  Turbulent - Detra & Hidalgo} With Effective Radius

2. Tangent Cone Shock
   Tangent Wedge Pressures
   Laminar: Eckert With Tunnel Derived Equivalent Running Lengths
   Turbulent: Spalding-Chf With Turbulent $r_m = 2/3$
             Von Karman

3. Swept Cylinder
   Parallel Shock
   Laminar: Correlation
   Turbulent: Beckwith-Gallagher

4. Tangent Cone Shock
   Tangent Wedge Pressures
   Tangent Wedge Shock
   Tangent Wedge Pressures
   Bushnell-Weinstein

5. Normal Shock
   Bertin-Goodrich

Fig. 7.8 LANMIN Generic Orbiter Design Methods
Angle of Attack = 15 Deg.
Mach No. = 7.77
θ - Data, Bushnell et al (Ref. 8)

Fig. 7.9 Windward Centerline Heat Transfer Over a 20-Degree Half Angle Blunt Cone at Angle of Attack
Fig. 7.10 MINIVER Theory Compared With Phase B Orbiter Configuration Data
Fig. 7.11 | Shock-Wave-Angle Correlation
Fig. 7.12 - Geometry of the MSFC 437 Aerodynamic Heating Data Model
Fig. 7.13: Equivalent Running Length For The Bottom Centerline Of The MSFC 437 Orbiter
Fig. 7.14 Effective Sweep Angle Compliment For MSFC 437 Orbiter
Fig. 7.15 LANMIN Theory Compared With Phase B Orbiter Configuration Bottom Centerline Laminar Data
Data From Warmbrod et al (Ref. 9)  

Theory

\[ M = 8.0 \]

\[ Re_{\text{ft}} = 3.75 \times 10^6 / \text{ft} \]

Grit on Nose

\[ \alpha = 40 \]

\[ \tau_m = 0.6667 \]

Equivalent Laminar Running Lengths

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Fig. 7.16 LANMIN Theory Compared With Phase B Orbiter Configuration Bottom Centerline Turbulent Data
Fig. 7.17 Laminar Leading Edge Stagnation Line Heating Compared With the Heat Transfer Option 6 Method
Section VIII
REFERENCES


