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Determining the Alpha Dynamo Parameter in Incompressible Homogeneous Magnetohydrodynamic Turbulence

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DETERMINING THE ALPHA DYNAMO PARAMETER IN INCOMPRESSIBLE HOMOGENEOUS MAGNETOHYDRODYNAMIC TURBULENCE

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Abstract

We show that alpha, an important parameter in dynamo theory, can be proportional to either the kinetic, current, magnetic, or velocity helicity of the fluctuating magnetic field and fluctuating velocity field. The particular helicity to which alpha is proportional depends on the assumptions used in deriving the first order smoothed equations that describe the alpha effect. In two cases, viz. when alpha is proportional to either the magnetic helicity or velocity helicity, alpha can be determined experimentally from two-point measurements of the fluctuating fields in incompressible, homogeneous turbulence having arbitrary symmetry. For the other two possibilities, alpha can be determined if the turbulence is isotropic.

1. Introduction

A fundamental problem in dynamo theory is the study of how nature transforms toroidal magnetic fields into poloidal ones. The inverse transformation, changing poloidal fields into toroidal ones, can be easily accomplished by differential rotation. This problem has been extensively studied for the kinematic turbulent dynamo where one does not impose self-consistency on the velocity and magnetic fields, but assumes the statistics of one is known, usually the velocity field, and solves for the other in terms of the first. Parker (1955) and Steenbeck, Krause and Rädler (1966) developed a dynamo theory in this context that is now known as the "alpha effect". The essence of the theory is to show that under certain conditions involving a lack of mirror symmetry in the turbulence, there is an electromotive force that is proportional to the mean magnetic field $\underline{B}_o$. From Ohm's law there will then be a similar term in the current density that produces a poloidal component to the magnetic field. The alpha effect is the lowest order approximation in a more general framework, mean field electrodynamics, in which the electromotive force due to fluctuations is represented as a series expansion in powers of the mean field.

For α, the constant of proportionality between the electromotive force $\underline{E}$ and $\underline{B}_o$, to be nonzero, some feature of the turbulence must lack mirror symmetry. In the usual derivation of the alpha effect (Steenbeck, Krause, and Rädler, 1966; Moffatt, 1978), α is related to a weighted integral of the kinetic helicity spectrum and hence appears to be dependent on a lack of mirror symmetry of the fluctuating velocity field $\underline{v}$. Recently, Keinigs (1983) has rederived α and has argued that instead of being a function of the kinetic helicity spectrum, α is actually proportional to the total current helicity.

Thus, it would appear that nonzero values of α are possible even if the weighted integral of the kinetic helicity spectrum is zero but the current helicity is finite. In this paper we first show that α can be proportional to any of four helicities of the turbulence depending on the assumptions used in deriving the alpha effect. We then address the outstanding question raised by Moffatt (1981) of how one can determine α experimentally.

The paper is organized as follows. In section 2 we review the standard derivation as presented by Moffatt (1978). We find that the physical nature of the asymmetries in the turbulence can be understood in terms of a pseudoscalar H_v which characterizes the velocity field and is mathematically analogous to the magnetic helicity H_m. We show that α is directly proportional to this function, which we call the "velocity helicity". We then review the derivation of the alpha effect as presented by Keinigs (1983) and discuss the conditions under which α is proportional to the current helicity H_J. We conclude with two new derivations of α made under slightly different assumptions about the statistics of the turbulence and find that α can be proportional to either the kinetic helicity H_k or the magnetic helicity. In section 3 we address the question of the measurability of α in homogeneous turbulence in circumstances in which an experiment provides two-point field covariances with collinear separations. The work of Matthaeus et al. (1982) and Matthaeus and Goldstein (1982) demonstrating the measurability of H_m in homogeneous turbulence of arbitrary symmetry is generalized. In the two cases in which α is proportional to either H_m or H_v, we show that α can be determined independent of the symmetry of the turbulence if the dissipation coefficients are known. Where α is proportional to either H_J or H_k, we show that the assumption of isotropy is sufficient to permit determination of α, again subject to knowing the appropriate dissipation coefficient. The results are summarized in section 4.

2. Helicity and the Alpha Dynamo Problem

The steps leading to the alpha effect equation are well-documented (Moffatt, 1978; Krause and Rädler, 1980). We present a review here to demonstrate explicitly the relationship between the assumptions contained in derivations of the alpha effect equation and the nature of the asymmetries that must be present in the turbulent fields (either $\underline{v}$ or $\underline{b}$) to ensure nonzero values of the alpha effect parameter.

We begin by considering a turbulent, electrically conducting fluid, which is described by the equations of incompressible magnetohydrodynamics (MHD):

$$\partial \underline{B}/\partial t = \nabla \times (\underline{V} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1a}$$

$$\nabla \cdot \underline{B} = 0 \tag{1b}$$

$$\partial \underline{V}/\partial t + (\underline{V} \cdot \nabla)\underline{V} = -\nabla P + (\underline{J} \times \underline{B}) + \nu \nabla^2 \underline{V} \tag{2a}$$

$$\nabla \cdot \underline{V} = 0 \tag{2b}$$

where $\underline{B}$ is now normalized to Alfvén speed units $[\underline{B} \rightarrow \underline{B}/\sqrt{(4\pi\rho_o)}]$, ρ_o is the mean density, P is the pressure which in incompressible MHD is the solution of a Poisson equation derivable from (2), and $\underline{J} = \nabla \times \underline{B}$ is the electric current density. Both the resistivity η and the viscosity ν have dimensions of (length)^2/time as is typical of transport coefficients. The MHD equations (1-2) can be related in a straightforward way to a system of dimensionless units in which η and ν pla, the role of inverse magnetic Reynolds number and

inverse mechanical Reynolds number, respectively. It is useful to rewrite (2) in terms of the vorticity $\underline{\omega} = \nabla \times \underline{V}$ by taking the curl of (2a) yielding

$$\partial\underline{\omega}/\partial t = \nabla \times (\underline{J} \times \underline{B}) - \nabla \times (\underline{\omega} \times \underline{V}) + \nu\nabla^2 \underline{\omega} \tag{3}$$

which completely specifies the time evolution of $\underline{V}$ when (2b) is valid and there is no contribution due to potential flow.

To continue, we separate the fields $\underline{B}$ and $\underline{V}$ and the induction equation into mean and fluctuating parts. We follow Krause and Rädler (1980) and treat the mean as the expectation of an ensemble of identical systems. There are occasions when it may be more convenient to define the mean values in terms of integrations over space and/or time, but the results will not always be indentical. We first separate $\underline{B}$ and $\underline{V}$ into mean and fluctuating parts: $\underline{B} = \underline{B}_o + \underline{b}$ and $\underline{V} = \underline{V}_o + \underline{v}$. We assume that $\underline{V}_o \equiv \langle\underline{V}\rangle = 0$ and that $\underline{B}_o \equiv \langle\underline{B}\rangle$ is in some sense "slowly varying" in time and space, i.e., that its spatial and time derivatives are negligible compared to the derivatives of $\underline{b}(\underline{x},t)$. The mean and fluctuating parts of the induction equation (1a) become

$$\partial\underline{B}_o/\partial t = \nabla \times \langle\underline{v} \times \underline{b}\rangle + \eta\nabla^2 \underline{B}_o \tag{4}$$

$$\partial\underline{b}/\partial t = \nabla \times (\underline{v} \times \underline{B}_o) + \Delta[\nabla \times (\underline{v} \times \underline{b})] + \eta\nabla^2 \underline{b} \tag{5}$$

where Δ denotes the difference between the quantity in brackets and its mean. For future reference we note that the vorticity equation can be similarly rewritten in the form

$$0 = \nabla \times \langle\underline{J} \times \underline{b}\rangle - \nabla \times \langle\underline{\omega} \times \underline{v}\rangle \tag{6}$$

$$\partial\underline{\omega}/\partial t = \nabla \times (\underline{J} \times \underline{B}_o) + \Delta[\nabla \times (\underline{J} \times \underline{b})] - \Delta[\nabla \times (\underline{\omega} \times \underline{v})] + \nu\nabla^2 \underline{\omega} \tag{7}$$

Return now to equation (5) and note that $\underline{b}$ is generated by the source term $\underline{v} \times \underline{B}_o$. Because (5) is a linear equation for $\underline{b}$, $\underline{b}$ and $\underline{B}_o$ will be linearly related if, for example, we assume that $\underline{b} = 0$ at some initial time. Thus, the mean turbulent electromotive force $\underline{E} = \langle \underline{v} \times \underline{b} \rangle$ is also a linear functional of $\underline{B}_o$. In turbulent media, the fluctuating quantities at a certain place and time will have a finite correlation with fluctuating quantities at some other place and time only if the separation with respect to both space and time is not too large. Thus, to determine $\underline{E}$ at a given point, we anticipate that $\underline{v}$, $\underline{b}$, and $\underline{B}_o$ need only be known in a neighborhood of that point. The linear relationship between $\underline{E}$ and $\underline{B}_o$ should then be given approximately by a Taylor series expansion which converges rapidly because $\underline{B}_o$ weakly varies on scales over which $\underline{B}_o$ and $\underline{b}$ are correlated. The lowest order term in this expansion is

$$E_i = \alpha_{ij} B_{oj} \tag{8}$$

The reader is referred to Moffatt (1978) and Krause and Rädler (1980) and references therein for a more complete discussion of these issues. In isotropic turbulence $\alpha_{ij} = \alpha\delta_{ij}$, and equation (4) becomes the alpha effect equation

$$\partial\underline{B}_o/\partial t = \alpha(\nabla \times \underline{B}_o) + \eta\nabla^2 \underline{B}_o \tag{9}$$

The Velocity Helicity, H_v

In the usual approach, the fluctuating part (5) of the induction equation is used to relate $\underline{v}$ and $\underline{b}$. But instead of arriving at an expression for α as a weighted integral of the kinetic helicity spectrum (Moffatt, 1978), we will show that a new helicity can be defined, the velocity helicity H_v, which is proportional to α. Further advantages of introducing H_v are detailed in section 3. The derivation proceeds in the usual way by making the assumption of "first-order smoothing" (equivalent to a quasilinear approximation), which states that the difficult nonlinear term $\Delta[\nabla\times(\underline{v}\times\underline{b})]$ may be dropped. A detailed discussion of the physical regimes in which the neglect of these nonlinear terms is justified can be found in Moffatt (1978) and Krause and Rädler (1980). Circumstances that appear to be sufficient for first order smoothing are the low magnetic Reynolds number and the "weak turbulence" limits. In this paper, we will assume that first-order smoothing is always appropriate.

Upon expanding the remaining double cross product, equation (5) becomes

$$\frac{\partial b}{\partial t} - \eta\nabla^2\underline{b} = (\underline{B}_o\cdot\nabla)\underline{v} \tag{10}$$

The relationship between the fluctuating magnetic field $\underline{b}$ and the fluctuating velocity field $\underline{v}$ is easily expressed by taking the Fourier transform of (10)

$$\underline{b}(\underline{k},\omega) = i(\underline{B}_o\cdot\underline{k})\underline{v}(\underline{k},\omega)/(-i\omega + \eta k^2) \tag{11}$$

Note that at this stage we could equally well have written of $\underline{v}(\underline{k},\omega)$ in terms of $\underline{b}(\underline{k},\omega)$. Substituting (11) into the Fourier integral expansion of $\underline{E}$ gives

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$$\underline{E} = \int \frac{i\underline{B}_o \cdot \underline{k}'}{-i\omega + \eta k'^2} \langle \underline{v}^*(\underline{k},\omega) \times \underline{v}(\underline{k}',\omega') \rangle \, e^{[-i(\underline{k}-\underline{k}')\cdot\underline{x} + i(\omega-\omega')t]} d^3k d^3k' d\omega d\omega' \quad (12)$$

We restrict our attention to homogeneous turbulence and take the usual low frequency limit (cf. Moffatt, 1978) which can be justified at least in the small magnetic Reynolds number regime where $|\omega| \ll \eta k^2$. In this limit the integrals over ω and ω' can be done (assuming that the fluctuations are time stationary). We now introduce the energy spectrum tensor $\Phi_{ij}(\underline{k})$, defined as the Fourier transform of the homogeneous two-point velocity correlation matrix $R_{ij}(\underline{x}) = \langle v_i(\underline{x}) v_j(\underline{x}+\underline{r}) \rangle$ (Batchelor, 1970). Because $\Phi_{ij}(\underline{k})\delta(\underline{k}-\underline{k}') = \langle v_i^*(\underline{k}) v_j(\underline{k}') \rangle$, equation (12) becomes

$$E_i = \int \frac{i\underline{B}_o \cdot \underline{k}}{\eta k^2} \epsilon_{ilm} \Phi_{lm}(\underline{k}) \, d^3k \quad (13)$$

The matrix Φ_{lm} can always be decomposed into its symmetric part Φ^s_{lm} and antisymmetric part Φ^a_{lm}. Because $\epsilon_{ilm}\Phi^s_{lm} = 0$, E_i depends only on Φ^a_{lm}. Matthaeus et al. (1982) and Matthaeus and Goldstein (1982) have shown that for a solenoidal field, the antisymmetric part of the energy spectrum tensor can have only one independent pseudotensor form which depends on a single pseudoscalar function even in $\underline{k}$. In the case of the magnetic power spectrum, this function is the magnetic helicity spectrum $H_m(\underline{k})$ defined by

$$\int H_m(\underline{k}) d^3k = H_m \equiv \langle \underline{A} \cdot \underline{b} \rangle \quad (14)$$

where $\underline{A}$ is the vector potential of the fluctuating magnetic field. In incompressible flow, the velocity field is solenoidal and it is convenient to define a similar function for the velocity field. We will refer to this

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function as the velocity helicity H_v and denote its spectral decomposition $H_v(\underline{k})$. The velocity helicity as used here is essentially the dot product of $\underline{v}$ with a vector whose curl is $\underline{v}$ in the same way that the magnetic helicity is the dot product of the $\underline{b}$ with the vector potential $\underline{A}$, where $\nabla \times \underline{A} = \underline{b}$.

The exact relationship between $\phi^a{}_{ln}(\underline{k})$ and $H_v(\underline{k})$ can be found as follows. First, one can "uncurl" in Fourier space so that

$$H_v(\underline{k}) = \text{trace}\ [i\epsilon_{ijl}k_j\phi^a{}_{lm}(\underline{k})/k^2] \tag{15}$$

The unique form of $\phi^a{}_{lm}$ is (cf. Matthaeus and Goldstein, 1982)

$$\phi^a{}_{lm} = (i/2)\epsilon_{lmn}k_nH_v(\underline{k}) \tag{16}$$

This can in turn be inserted into (13), so that

$$E_i = -B_{oj}\int \frac{k_ik_j}{\eta k^2} H_v(\underline{k})d^3k \tag{17}$$

We define $\alpha = 1/3\ \alpha_{ii}$ to be consistent with the isotropic condition $\alpha_{ij} = \alpha\delta_{ij}$. Then, from (8) and (17), we arrive at a simple expression for α_1 (our first expression for α) that is equivalent to the one found in Krause and Rädler (1980) and Moffatt (1978).

$$\alpha_1 = -\frac{1}{3\eta}\int H_v(\underline{k})\ d^3k = -H_v/(3\eta) \tag{18}$$

If $H_v(\underline{k})$ in (18) is replaced by (15), Moffatt's expression for α is recovered. Written in this way, it is clear that α is related to the linkage of stream lines in the same way that the kinetic helicity reflects the linkage of

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vorticity tubes and the magnetic helicity reflects linkage of magnetic flux tubes (Moffatt, 1978; Turner and Christiansen, 1981).

The Current Helicity, H_J

The approach used above in evaluating alpha is not the only one that can be taken. Recall that we could have solved for $\underline{v}(\underline{k},\omega)$ in terms of $\underline{b}(\underline{k},\omega)$ in (11). This is the approach taken by Keinigs (1983). Proceeding in that way leads to a new form for α which is related to the current helicity, $H_J = \langle \underline{J} \cdot \underline{b} \rangle$. We briefly review the derivation for completeness and to clarify some aspects of the derivation given by Keinigs. In the low frequency limit, the equation for E_i becomes

$$E_i = \int \frac{i\eta k^2}{B_{oj}k_j} \epsilon_{ilm} S_{lm}(\underline{k}) d^3k \tag{19}$$

Here, the relevant spectrum tensor is $S_{lm}(\underline{k})$, which is the Fourier transform of the two-point magnetic correlation $\langle b_l(\underline{x}) b_m(\underline{x}+\underline{r}) \rangle$. The antisymmetric part of this tensor has the unique form $S^a_{ij} = i\epsilon_{ijl}k_l H_m(\underline{k})/2$ (Matthaeus et al., 1982; Matthaeus and Goldstein, 1982), so that $\underline{E}$ becomes

$$E_i = -\eta \int \frac{k^2 k_i}{B_{oj}k_j} H_m(\underline{k})\, d^3k \tag{20}$$

To extract from (20) a scalar expression for α we will assume that in this case the turbulence is isotropic so that $\alpha_{ij} = \alpha\delta_{ij}$. Therefore, $\alpha = E_i B_{oi}/B_o^2$ and we find

$$\alpha_2 = -\frac{\eta}{B_o^2} \int k^2 H_m(\underline{k})\, d^3k \tag{21}$$

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Using the definition of $H_J(\underline{k})$, (21) can be immediately rewritten as

$$\alpha_s = -\frac{\eta}{B_o^2} H_J \tag{22}$$

This is the result found by Keinigs. We emphasize that the two expressions (18) and (22) for α are not in general equal. In the first case, it was assumed that the statistics of the velocity field were known. In the second case, the statistics of the magnetic field were assumed given and the velocity fluctuations were found. It is essential to keep in mind that because these derivations are done within the spirit of the (non-self-consistent) kinematic dynamo, there is no a priori reason for the two expressions for α to be equal. We return to this issue in the next section, but first we show that there are two additional sets of assumptions that can lead to expressions for the alpha effect parameter.

The Magnetic and Kinetic Helicities, H_v and H_k

In deriving both (18) and (22) we used the fluctuating part of the induction equation (5) to obtain the appropriate relationship between the fluctuating magnetic and velocity fields. There is an alternative possibility, namely to use the fluctuating part of the vorticity equation (7). Then, by solving for either $\underline{v}$ in terms of $\underline{b}$, or vice versa, two new expressions for α can be found. As we shall see, these new expressions are proportional to either the magnetic helicity, or the kinetic helicity. One proceeds exactly as before. Note that in equation (7) there are two nonlinear "Δ" terms. In the spirit of this parallel derivation, we assume that these terms can be

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neglected, i. e. we extend the concept of first-order smoothing to include neglect of these nonlinearities. It is beyond the scope of this paper to explore the conditions under which this approximation might be valid, but it is expected that they are substantially the same as those necessary to arrive at (18) or (22). Equation (7) then becomes

$$\frac{\partial \underline{\omega}}{\partial t} = (\underline{B}_o \cdot \nabla)\underline{J} + \nu \nabla^2 \underline{\omega} \tag{23}$$

and the Fourier transform is

$$-i\omega = i(\underline{B}_o \cdot \underline{k})\underline{J}(\underline{k}) - \nu k^2 \; \underline{\omega}(\underline{k}) \tag{24}$$

Provided that contributions from potential flow can be neglected, (24) implies that

$$\underline{v}(\underline{k},\omega) = \frac{i\underline{B}_o \cdot \underline{k}}{(-i\omega + \nu k^2)} \; \underline{b}(\underline{k},\omega) \tag{25}$$

In general (25) and (11) cannot be simultaneously true which is expected because the first order smoothing approximations are different in the two cases. Proceeding in an exactly analogous way, by substituting first for $\underline{v}(\underline{k},\omega)$ and then for $\underline{b}(\underline{k},\omega)$ in the Fourier integral expansion of $\underline{E}$, and taking the low frequency limit, two more expressions for α emerge: one involving the magnetic helicity H_v, and the other involving the kinetic helicity $H_k = \langle \underline{\omega} \cdot \underline{v} \rangle$

$$\alpha_3 = \frac{H_m}{3\nu} \tag{26}$$

and

$$\alpha_4 = \frac{\nu}{B_o^2} \int k^2 H_v(\underline{k})\, d^3k = \frac{\nu H_k}{B_o^2} \tag{27}$$

Note that the derivation of (26) directly parallels that of (18) for α_1 and no assumptions need be made about the symmetries of the turbulence. In contrast, the derivation of (27) parallels that of (21) for α_2, in which it is explicitly assumed that the fluid is isotropic.

We have shown that the alpha effect parameter can be proportional to any of the four helicities derivable from statistics of the magnetic and velocity fields. Which helicity appears is a direct consequence of the assumptions and approximations used in the derivation. The four different expressions for α, equations (18), (22), (26) and (27), indicate that the lack of mirror symmetry required for a nonzero alpha parameter can take the form of linkages of stream lines, tubes of current, magnetic flux tubes, or vorticity tubes, respectively.

The basic question now arises how one might evaluate any or all of these expressions for α in an experiment. This is addressed in the next section.

3. The Evaluation of Helicity and α

It is clear that the first obstacle to determining α in any particular experimental situation is the necessity of measuring the appropriate helicity. This problem has been treated in some detail for the magnetic helicity by Matthaeus et al. (1982) and Matthaeus and Goldstein (1982) who demonstrated that magnetic helicity can be determined whenever the two-point correlation function of the magnetic field is experimentally available for an appropriately wide range of spatial separations in a single direction. This is not an unusual experimental situation and is found, for example, in wind tunnels or in the interplanetary solar wind, where some version of the G. I. Taylor

"frozen-in-flow" hypothesis (Taylor, 1938) allows interpretation of two-time single point covariances as two-point single time covariances. However, the discussion here will also be relevant to cases in which multiple experimental probes are constrained to have separations in one cartesian direction. The theoretical basis for making this determination is that the antisymmetric part of the energy spectrum tensor depends directly on either H_m or H_v. We apply the previous results to the new helicities, and conclude that the velocity helicity can be evaluated in homogeneous turbulence without further specification of spatial symmetry, while the kinetic and current helicities are determinable in isotropic turbulence. The techniques developed here will be expressed in terms of velocity field quantities, but all results apply as well to the analogous magnetic field quantities.

Assume that the correlation function is known for collinear separations in the 1-direction. Following Batchelor (1970) we define the reduced energy spectrum tensor as

$$\Phi^r_{ij}(k_1) = (1/2\pi) \int R_{ij}(r_1,0,0)e^{-ik_1r_1}\, dr_1$$

$$= \int \Phi_{ij}(\underline{k})\, dk_2dk_3 \tag{28}$$

Now consider the imaginary part of (28). Because Φ_{ij} is Hermitian (by the combination of the reality condition $\Phi_{ij}(\underline{k}) = \Phi_{ij}^*(-\underline{k})$, and the homogeneity property $\Phi_{ij}(\underline{k}) = \Phi_{ji}(-\underline{k})$), the only imaginary part of Φ_{ij} is an antisymmetric pseudotensor, which has the unique form (16). After integrating over the 2- and 3-directions, (28) becomes

$$\mathrm{Im}\, \Phi^r_{23}(k_1) = \iint \mathrm{Im}\Phi_{23}(\underline{k})\, dk_2dk_3 = (k_1/2) \iint H_v(\underline{k})\, dk_2dk_3$$

or

$$H_v^r(k_1) = 2 \text{ Im } \phi^r_{23}(k_1)/k_1 \tag{29}$$

Finally, the total velocity helicity is obtained by integrating over k_1 with the result

$$H_v = \int H_v(\underline{k})\ d^3k = \int H_v^r(k_1)\ dk_1 \tag{30}$$

Therefore, the velocity helicity is measurable in that its spectrum is contained in the spectral matrix of velocity correlations in one direction. This result is completely equivalent to the results on the magnetic helicity discussed in Matthaeus et al. (1982) and Matthaeus and Goldstein (1982). If the resistivity η is also known, α_1 can be determined via this analysis. Equivalently, knowing ν and H_m, one can determine α_3.

The kinetic helicity H_k (and the current helicity H_J) can also be evaluated in similar fashion. However, in this case it is necessary to assume that the turbulence is isotropic in addition to being homogeneous. Recall that

$$H_k = \int k^2\ H_v(\underline{k})\ d^3k$$

$$= \int dk_1 k_1^2 \int\int dk_2 dk_3 H_v(\underline{k}) + \int dk_2 k_2^2 \int\int dk_3 dk_1 H_v(\underline{k}) + \int dk_3 k_3^2 \int\int dk_1 dk_2 H_v(\underline{k})$$

Since isotropy is assumed, all three integrals must be exactly equal, and

$$H_k = 6\int k_1 \text{ Im}\phi^r_{23}(k_1) dk_1 \tag{31}$$

An exactly analogous result obtains for H_J. Again, if ν and/or η are known independently, one can evaluate α_4 and/or α_2. Thus, at least for isotropic turbulence, all four helicities can be determined from the reduced power spectrum. In turbulence of arbitrary symmetry, H_v and H_m can still be found. These results are summarized in Figure 1 where the relationships between various expressions and the assumptions made in deriving them are indicated.

Numerical methods for constructing the requisite correlation spectra are well established. Two current techniques for constructing these functions are the Blackman-Tukey "mean-lagged-product" (Blackman and Tukey, 1958) and fast Fourier transform (FFT). A comparison of these two methods can be found in Matthaeus and Goldstein (1982) along with several examples illustrating the determination of H_m from interplanetary magnetic field data. Application of the FFT technique to the determination of the other three helicities using both interplanetary magnetic field and plasma data from the Voyager spacecraft will be reported elsewhere.

4. Conclusions and Summary

Methods for determining the alpha dynamo parameter in homogeneous, incompressible MHD turbulence have been presented. Four distinct helicities of the turbulence have been indentified as being related to the alpha dynamo problem. Which helicity is most closely related to α is completely dependent on the applicability of the approximations made in deriving the alpha effect equation. The recent results of Keinigs (1983) relating α to the current helicity are therefore not in disagreement with the original derivation by Steenbeck, Krause, and Rädler (1966), reviewed by Moffatt (1978 and 1981), in

which α was related to a weighted integral of the kinetic helicity spectrum $H_v(\underline{k})$. That integral of the helicity spectrum is the quantity we have called the velocity helicity in direct analogy to the magnetic helicity. Unlike the kinetic helicity, which is only measurable in isotropic turbulence, the velocity helicity can be measured in MHD turbulence of arbitrary symmetry.

We have presented two new approximations for α; one proportional to the kinetic helicity and the second proportional to the magnetic helicity. The particular form for α most appropriate in a given physical context depends on the validity of the approximations made in each derivation. The two new expressions for α were derived using a form of the first order smoothing approximation whose general range of validity has not been investigated. In addition, numerical techniques that can be utilized to evaluate α in MHD turbulent media have been described. These techniques may be useful in experimental situations where two-point collinear covariances are measured and η and ν are known. An application of these techniques using magnetic field and fluid velocity data obtained in the solar wind will be presented in a separate publication.

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Figure Caption

A summary of the relationships possible between α and the four helicities of the turbulent fluid. α_1 is the traditional result, α_2 is the result found by Keinigs (1983). In both cases the first order smoothing approximation is made on the induction equation. α_3 and α_4 are obtained by making the first order smoothing approximation on the vorticity equation. If the dissipation coefficients η and ν are known, α_1 and α_3, can be determined in homogeneous (and stationary) turbulence having arbitrary symmetry so long as the two-point correlation function can be measured for collinear separations. α_2 and α_4 can be determined in isotropic turbulence.

	VELOCITY FIELD STATISTICS	MAGNETIC FIELD STATISTICS
ARBITRARY SYMMETRY	$\alpha_1 = -\frac{H_V}{3\eta} = -\frac{2}{3\eta}\sum_{k_1}\frac{\mathrm{Im}\,\Phi^r_{23}(k_1)}{k_1}$	$\alpha_3 = \frac{H_m}{3\nu} = \frac{2}{3\nu}\sum_{k_1}\frac{\mathrm{Im}\,S^r_{23}(k_1)}{k_1}$
ISOTROPIC	$\alpha_4 = \frac{\nu H_k}{B_0^2} = \frac{6\nu}{B_0^2}\sum_{k_1} k_1 \cdot \mathrm{Im}\,\Phi^r_{23}(k_1)$	$\alpha_2 = -\frac{\eta H_J}{B_0^2} = -\frac{6\eta}{B_0^2}\sum_{k_1} k_1 \cdot \mathrm{Im}\,S^r_{23}(k_1)$

INDUCTION EQUATION

VORTICITY EQUATION

ORIGINAL PAGE IS
OF POOR QUALITY