INTERPLANETARY ALFVENIC FLUCTUATIONS: A STATISTICAL STUDY OF THE DIRECTIONAL VARIATIONS OF THE MAGNETIC FIELD

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ABSTRACT

Magnetic field data from HELIOS 1 and 2 are used to test a stochastic model for Alfvénic fluctuations recently proposed by A. Barnes. A reasonable matching between observations and predictions is found. A rough estimate of the correlation length of the observed fluctuations is inferred.

Introduction

In a recent paper Barnes (1981) proposes a simple stochastic model for interplanetary Alfvénic turbulence based on the random wandering of a vector of constant length. Let us locate this vector with the usual spherical coordinates \( \theta, \phi \) and assume it initially along the polar axis (\( \theta = 0 \)). As time advances the tip of the vector walks over a sphere, moving at each step of an angle \( \epsilon \) along a great circle in a random direction. In other words if the position of the vector after \( N \) steps is \( (\theta, \phi) \), at the \( (N+1) \)th step the vector will lie at an angle \( \epsilon \) away along a great circle, whose tangent at \( (\theta, \phi) \) makes a random angle \( \psi \) with the unit vector in the \( \theta \) direction. A field direction confinement can be simulated with reflecting boundaries. This model, with certain qualifications, reproduces the well known observational feature of the Alfvénic fluctuations of a well defined direction of minimum variance aligned with the mean magnetic field. The model predicts also the statistical distributions of the field directions in terms of a density of probability \( W(\mu, t) \) and an integrated probability \( P(\mu, t) \), being \( \mu=\cos \theta \). \( W(\mu, t)d\mu \) gives the probability that after a time \( t \) the angle \( \theta \) that the vector makes with its initial position has its cosine between \( \mu \) and \( \mu + d\mu \). \( P(\mu, t) \), integral of \( W \) between \( -1 \) and \( \mu \), gives the probability that, at the time \( t \), \( \cos \theta \) lies between \(-1\) and \( \mu \). Figures 4 and 5 of Barnes (1981) show the evolution with time of these functions.

An observational test of the temporal evolution of the fluctuating field direction as predicted by the model has been performed by using interplanetary magnetic field data (6 s averages) of HELIOS 1 and 2 (Rome/GSFC experiment). We have selected four periods of 2-4 days in the trailing edge of high velocity streams characterized by the presence of fluctuations whose Alfvénic character had been already established by previous analyses (Bavassano et al., 1982a and b). Table 1 of the latter reference gives a list of the periods considered. Three of them refer to HELIOS 2 observations of the same stream during three successive solar rotations. The fourth period refers to an observation of the same stream by HELIOS 1 almost contemporary to the second of the HELIOS 2 stream encounters. To obtain a statistical description of the temporal evolv-
tion of the direction of the measured fluctuating field we have divided the selected periods in intervals of 20 minutes, for each interval we have determined how the magnetic field vector evolves (starting from its initial position at the beginning of the interval) as time advances, and finally for each of the four stream encounters we have grouped the results from the various intervals to have statistical distributions for the parameters characterizing, following the Barnes model, the field evolution with time. The parameters considered are $\epsilon$, the angle between consecutive vectors (6 s averages), $\psi$, the direction of the field variation as defined in the model, and $\theta$, the angle that the magnetic vector has with its initial position at the beginning of the interval. We would note that our results do not change significantly if intervals longer than 20 minutes are taken for the statistics. Finally, all intervals including discontinuities as identified through automatic selection criteria (Mariani et al., 1983) and visual inspection of the data have been rejected.

Experimental Results

In Figure 1 we show the histograms of the angles $\epsilon$ and $\psi$ at the four heliocentric distances considered. The 20 minutes intervals contributing to the statistics are (in order of decreasing R) 111, 110, 104 and 235 respectively. The histograms of $\epsilon$ show that most of the values fall between $0^\circ$ and $15^\circ$. A dependence on heliocentric distance is clearly apparent, the average value of $\epsilon$ being $6.1^\circ$ at 0.87 AU and becoming $11.0^\circ$ at 0.29 AU. These histograms should be compared with the fixed step length assumption in the model. The histograms of $\psi$ do not show on the contrary any dependence on the heliocentric distance. In Figure 1 we give only the curve for 0.87 AU, representative within 1% for all distances. To a good approximation $\psi$ has a uniform distribution, in agreement with the assumption of the model that the direction of each step is random.

![Figure 1. Frequency distributions of $\epsilon$ and $\psi$.](image1)

![Figure 2. Frequency distributions of $\cos \theta$.](image2)

Let us consider now the evolution with time of the magnetic field...
direction in terms of the angle $\theta$ between the current vector and its initial position at the beginning of the 20 minutes interval. In Figure 2 we show the frequency distributions of $\cos \theta$ for different delay times $\tau$ (time elapsed from the beginning of each interval). These histograms correspond to the curves of the function $W$ of the model of Barnes (see his Figures 4 and 5), taking care that $W$ is a probability density. The features of the temporal evolution of $\cos \theta$ are perhaps more clearly seen in Figure 3 in terms of the integrated frequency distributions, giving the number of cases for which $\cos \theta$ is between -1 and a given value. These curves should be compared with those of the function $P$ given by Barnes in his Figures 4 and 5. One is easily convinced that the experimental curves look like those of the model. When the delay time $\tau$ is small (e.g., $\tau=1$ min) most of the values of $\cos \theta$ are close to 1. For increasing $\tau$ the distributions spread out in a quite regular way. The final state is not very different at the various distances but it is reached more rapidly near the Sun. This is clearly seen in Figure 3 where at 0.29 AU the curve for $\tau=10$ min is almost coincident with that for $\tau=20$ min (i.e., final state is already reached after about 10 minutes), whereas at 0.87 AU they differ noticeably (i.e., after 10 minutes some evolution is still necessary before reaching final situation). This different time scale can be related to the radial gradient of the angle $\epsilon$ (see Figure 1), the evolution being more rapid near the Sun where the angle between consecutive vectors is larger. Another experimental evidence is that our curves do not reach a uniform distribution, as those of the model, and this holds also if $\tau$ increases up to 1 hour.

![Figure 3. Integrated frequency distributions of $\cos \theta$.](image)

The evolution of the frequency distributions of $\cos \theta$ has been also seen in terms of variation of their moments. Figures 4 and 5 show the variation with the delay time $\tau$ of the average value of $\cos \theta$, $\langle \cos \theta \rangle$, and of its standard deviation, $\sigma(\cos \theta)$, respectively. The changes in the histograms of $\cos \theta$ shown in Figure 2 are here seen as a regular decrease of $\langle \cos \theta \rangle$ and a corresponding increase of $\sigma(\cos \theta)$. Again it is clearly apparent that the variation is more rapid near the Sun (heavy line). These trends almost completely
disappear after 10-15 minutes, with $\langle \cos \theta \rangle$ remaining around 0.7 and $\sigma(\cos \theta)$ around 0.3 for delay times up to 1 hour. In other words most of the variation of $\langle \cos \theta \rangle$ and $\sigma(\cos \theta)$ takes place in the first minutes ($\sim 10$ near the Sun and $\sim 15$ near the Earth), after that only a very slight dependence from $\tau$ is found. We would notice that the observations made at 0.65 AU by HELIOS 2 and at 0.41 AU by HELIOS 1 are almost contemporary and at the same heliographic latitude. This allows us to exclude that the observed trends are due to slow temporal variations or heliographic latitude dependence.

![Figure 4. $\langle \cos \theta \rangle$ vs. $\tau$.](image)

![Figure 5. $\sigma(\cos \theta)$ vs. $\tau$.](image)

**Concluding Remarks**

As regards the observational test of the Barnes model we can summarize the results as follows: a) the direction of the variation of the magnetic field, as given by the angle $\psi$, has a uniform distribution, in agreement with the assumption of the model that the direction of each step is random; b) the angle $\epsilon$ between consecutive vectors (6 s averages) is of the order of $5^\circ$-$10^\circ$ and increases near the Sun (in the model this angle is taken as a constant); c) the frequency distribution of $\cos \theta$ spreads out when $\tau$ increases but does not reach a uniform distribution (as in the model). In conclusion the model of Barnes, although essentially local, can be considered in reasonable agreement with the experimental results. On the other hand, as already indicated by the author himself, the model can be considerably improved, for example by using a variable step length and by simulating the confinement of the field around the spiral direction. In this way a better matching with the observations could be obtained.

The time scale of the variation of $\cos \theta$ can give some information about the correlation time of the fluctuations. Barnes (1981) shows that, in terms of population statistics, the autocorrelation function of the temporal series of random fluctuating vectors is just given by the average value of $\cos \theta$. His computations show also that, with a reflecting boundary, the average value
of \( \cos \theta \) initially decreases as time advances but beyond the correlation time tends towards a constant value. Figure 4 shows that the decrease of \( \langle \cos \theta \rangle \) lasts 10-15 minutes, the evolution being faster near the Sun, after that this trend almost completely disappears. We can then infer that the correlation time of the observed fluctuations is of the order of 10 minutes near the Sun and 15 minutes near the Earth. With the knowledge of the average solar wind speed in the different periods this gives a correlation length of \( \approx 65 \, R_E \) near the Sun and \( \approx 90 \, R_E \) near the Earth. Our estimate for the correlation length at 0.87 AU agrees with previous determinations near the Earth's orbit (Chang and Nishida, 1973; Fisk and Sari, 1973; Sari and Valley, 1976; Crooker et al., 1982). The decrease of the correlation length approaching the Sun should be essentially related to the greater angular variability of the magnetic field near the Sun.

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REFERENCES


