MEASUREMENT OF MACROSCOPIC PLASMA PARAMETERS WITH A RADIO EXPERIMENT: INTERPRETATION OF THE QUASI-THERMAL NOISE SPECTRUM OBSERVED IN THE SOLAR WIND.

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ABSTRACT

The ISEE-3 SBH radio receiver has provided the first systematic observations of the quasi-thermal (plasma waves) noise in the solar wind plasma. The theoretical interpretation of that noise involves the particle distribution function so that electric noise measurements with long antennas provide a fast and independent method of measuring plasma parameters: densities and temperatures of a two-component (core and halo) electron distribution function have been obtained in that way. The polarization of that noise is frequency dependent and sensitive to the drift velocity of the electron population. Below the plasma frequency, there is evidence of a weak noise spectrum with spectral index -1 which is not yet accounted for by the theory. The theoretical treatment of the noise associated with the low energy (thermal) proton population shows that the moving electrical antenna radiates in the surrounding plasma by Cerenkov emission which becomes predominant at the low frequencies, below about 0.1 f_p.

Introduction

Using the SBH radio experiment on ISEE-3 (Knoll et al., 1978), we have observed systematically the noise spectrum associated with the quasi-thermal fluctuations of the solar wind plasma (Hoang et al., 1980; see also Grigorieva and Slysh, 1970). From the theoretical interpretation of the measured spectra (Andronov, 1966; Meyer-Vernet, 1979), using a bi-maxwellian model, we were able to deduce the density and temperature of the two electron components (Couturier et al., 1981, paper A) The results are in excellent agreement with the parameters measured at the same time on the same spacecraft by the Los Alamos plasma analyzer. Sentman (1982) investigated the electron quasi-thermal noise in a magnetized plasma to account for the noise spectrum observed near plasma resonances in planetary magnetospheres.

The purpose of the present paper is to introduce the physics involved in the plasma and antenna description in order to make clear some fundamental points to solar wind physicists who may not be familiar with the subject developed at length in the quoted literature.

The antenna as a thermometer and densitometer.

Let us first consider a passive receiving antenna in vacuum, in equilibrium with blackbody radiation at temperature T_1. The spectral component e_ω of the voltage measured at the terminals of that antenna is related to the fluctuations of the electromagnetic field inducing that voltage; Nyquist's theorem provides a relation between the output voltage, the antenna radiation resistance in vacuum R_{a0} and its temperature:

\[ e_\omega^2 = 4 K T_1 R_{a0} \]  

(1)
For a thin short dipole antenna of half-length $\lambda$, $R_{a0}$ is given by:

$$
R_{a0} = \left( \frac{\mu_0}{\varepsilon_0} \right)^{\frac{1}{2}} \frac{\omega^2 \lambda^2}{6 \pi c^2}
$$

(2)

Let us now consider the same antenna in equilibrium with blackbody radiation at temperature $T_1$, but immersed in a lossless dielectric. Equation (1) remains valid if we replace $R_{a0}$ by the radiation resistance $R_a$ which takes into account the dielectric permittivity $\varepsilon_r$ which affects the phase velocity of the radiation; namely, if the antenna remains short: $R_a = R_{a0} \varepsilon_r^{\frac{1}{2}}$. The plasma surrounding the antenna may be considered as a pure dielectric when $\omega > \omega_p$ where $\varepsilon_r = 1 - (\omega_p/\omega)^2$, $\omega_p$ is the plasma frequency and we assume there is no static magnetic field. This cold plasma approximation breaks down near the plasma frequency and is of no use for $\omega < \omega_p$ since no transverse mode can propagate and couple the antenna to the blackbody.

Finally, let us consider an antenna immersed in the solar wind, a hot collisionless electron plasma if we neglect the ions in a first approximation. Now, longitudinal modes do propagate and contribute predominantly to the voltage measured at the antenna terminals. In fact, there is a detailed energy balance which allows a separate calculation of the transverse and longitudinal modes contributions with appropriate Nyquist's formulas. When $\omega > \omega_p$, the transverse modes couple the antenna to radio-sources; since the solar wind is optically thin for this mode, it contributes an output voltage given by: $e_T = 4 K T_{\text{eff}} R_T$ where $T_{\text{eff}}$ is an effective temperature deduced from the integration of the electromagnetic flux incident on the antenna taking into account its directivity pattern. $R_T$ is the transverse mode radiation resistance which is the classical radiation resistance $R_a$ previously mentioned.

The longitudinal modes couple the antenna to the natural electrostatic oscillations of the plasma. Landau damping provides the thermalisation process between electric field fluctuations associated with longitudinal plasma waves and the electrons. The electrons with velocity larger than the plasma wave velocity are slowed down and transfer energy to the plasma waves; electrons which are slower than the plasma waves are accelerated and damp the waves. At thermal equilibrium, electric field fluctuations and the electron temperature are related by the fluctuation-dissipation theorem (Sitenko, 1967). Thus, the output voltage associated with these fluctuations is again given by: $e_L = 4 K T_L R_L$, where $R_L$ is the radiation resistance of the antenna for the longitudinal modes.

The method used to evaluate that radiation resistance will be given later; we now present some analytical results (paper A) which describe the resonance peak of the radiation resistance near the plasma frequency for a dipole antenna of half length $\lambda > \lambda_D = v_T/(2 \omega_p)$ the Debye length, $v_T = (2K T/m)^{\frac{1}{2}}$ is the electron thermal velocity.

For low frequencies $\omega < \omega_p$, $R_L$ can be analytically approximated by:

$$
R_L = \frac{(\pi \mu_0)^{\frac{1}{2}} c}{4 \varepsilon_0^{\frac{1}{2}}} \frac{\lambda_D}{v_T} = \frac{(\pi \mu_0)^{\frac{1}{2}} c}{4 (2 \varepsilon_0^{\frac{1}{2}} \omega_p \lambda)}
$$

(3)

For high frequencies, $\omega > \omega_p$, $R_L$ decreases with frequency as $\omega^{-3}$.
\[ R_L = \frac{\mu_0 \omega_p^2 c}{2 \varepsilon_0 \omega^3 \lambda} \]  (4)

The resonance peak appears just above the plasma frequency; for \( \omega = \omega_p \), the radiation resistance increases rapidly (cut-off) and reaches a maximum value for:

\[ \frac{\omega^2}{\omega_p^2} \left(1 - \frac{\lambda}{\lambda_D}\right)^{-\frac{3}{2}} \sim 3.5 \]  (5)

which corresponds to the antenna in tune with the plasma waves. The constant 3.5 is valid for cylindrical antennas, it would be slightly different for spherical antennas. So, there is a well defined resonance peak only if \( \lambda/\lambda_D \gg 1 \). The peak value is roughly equal to the low frequency asymptotic value (3) multiplied by \((\lambda/\lambda_D)^2\). In the resonant frequency range the transverse mode contribution can be neglected.

These expressions show that we can deduce the plasma frequency from the cut-off frequency of the spectrum and the electron temperature from the peak noise voltage and the width of the resonance peak.

Basic equations for a multicomponent plasma

Using the preceding model results in a good fit to the observations for the resonance peak position but the predicted values of the noise peak are too low. The noise peak is produced by plasma waves with frequency close to \( \omega_p \) and phase velocity much larger than \( v_T \). The discrepancy between the theoretical predictions based on a one-component model and the observations must therefore be solved by using a multicomponent model with cold core and hot halo electrons. We shall later discuss the limitations imposed by the quasi-thermal assumption on the distribution functions. Now we summarize the results (paper A) derived from a plasma model made of two maxwellian isotropic electron populations.

For a multicomponent plasma Nyquist's theorem is not valid, there is no longer any linear relationship between the output voltage at the antenna terminals and the antenna resistance; these quantities must be evaluated separately. The particle distribution functions set the plasma dielectric properties and are used to evaluate the dispersion tensor \( \Lambda_{ij}(k, \omega) \) and the dielectric permittivity tensor \( \varepsilon_{ij}(k, \omega) \). The antenna geometry sets the current distribution \( j(r) \). The antenna output voltage is related to the fluctuating electric field:

\[ V(t) = \int d^3 r \frac{\mathbf{E}(r, t) \cdot \mathbf{j}(r)}{I_0} \]  (6)

\( I_0 \) is the current flowing at the center of the antenna. The plasma fluctuations give a spectral distribution of the electric field fluctuations \( \mathbb{E}_1 \mathbb{E}_1^* \); this tensor is deduced from the fluctuation-dissipation theorem (Sitenko, 1967):

\[ \mathbb{E}_1 \mathbb{E}_1^* = \frac{2 K k_i_0 k_j}{\varepsilon_0 k^2 \omega q} \frac{T_q}{[\varepsilon_L]} \frac{\text{Im} \varepsilon_L}{q} \]  (7)
Calculating the autocorrelation of the output voltage (6) and using Parseval's theorem, we obtain:

$$V^2 = \frac{2}{8 \pi^3 I_o^2} \int d^3 k J_1(\vec{k}) \mathbf{E}(\vec{k}) \cdot \mathbf{E}(\vec{k})$$

We emphasize that this output voltage at the antenna terminals is not the input voltage at the receiver. The transfer function from the antenna terminals to the receiver involves the antenna impedance and the receiver input impedance (paper A). For ISEE-3, this transfer gain varies with frequency by one order of magnitude due to the variations of the Debye length. The antenna impedance is given by:

$$Z_a = \frac{1}{I_o} \int d^3 r \mathbf{E}(r) \cdot \mathbf{J}(r)$$

Plasma dielectric properties relate the current distribution to the electric field so that, by Fourier transform, we obtain:

$$Z_a = \frac{1}{8 \pi^3 \varepsilon_o \omega I_o^2} \int d^3 k J_1(\vec{k}) \Lambda_{ij}^{-1}(\vec{k},\omega) J_j(\vec{k})$$

For a multicomponent plasma, there is no longer a linear relation between the longitudinal resistance deduced from (10) and the output voltage given in (8). Figure 1 shows the difference between the calculated spectrum of a one-component plasma with electron density $N_c$ and temperature $T_c$ and the calculated spectrum of a two-component plasma with a cold ($N_c, T_c$) and a hot ($N_h, T_h$) electron population. These spectra have been calculated for the receiver input voltage. As shown by Fejer and Kan (1969) and in paper A, the hot population plays a dominant role close to the resonance peak. As compared to the single cold population case, the peak voltage is multiplied by $T_h/T_c$, the asymptotic low frequency value by a factor $1+(N_h T_h^2)/(N_c T_c^2)$ and the high frequency decreasing part of the spectrum by a total pressure factor $(N_h T_h + N_c T_c)/N_c T_c$.

**Observations and discussion**

Figure 1 shows spectra obtained on October 6, 1979 before and after a shock (figure 1 is a corrected version of paper A figure 11). The parameters of the two maxwellian populations used to determine the calculated radio spectrum are given in table I; figure 2 represents the corresponding electron distribution functions (thin line) and the distribution function deduced from the Los Alamos plasma analyzer measurements made at the same time (thick line). The difference between the two curves is essentially due to our use of a two maxwellian model to interpret the radio spectrum while the Los Alamos distribution shows evidence of a hot population with a high energy tail somewhat like a lorentzian (Feldman et al, 1982). Apart from this tail, the shapes of the two distribution functions are essentially the same; note also, from the parameters of table I, that we have not
used the same normalization factor for the two distribution functions; the plasma density given by the radio experiment differs slightly from that given by the plasma analyzer, this difference is less than the roughly ±30% uncertainty in estimating plasma densities from measurements of solar wind electron velocity distributions. We have repeated this fitting exercise many times, and in every case, we obtained the same agreement. We are quite sure that a purposely designed radio experiment tracking the plasma resonance "line" in frequency with a multichannel receiver would be a fast and efficient means to acquire macroscopic plasma parameters with a high time resolution.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(N_h/N_c)</th>
<th>(T_c)</th>
<th>(T_h)</th>
<th>(N_c) (SBH)</th>
<th>(N_c) (LANL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct. 6, 1979, 10:28 UT</td>
<td>0.016</td>
<td>0.92 (10^5) °K</td>
<td>16 (10^5) °K</td>
<td>22 cm(^{-3})</td>
<td>18 cm(^{-3})</td>
</tr>
<tr>
<td>Oct. 6, 1979, 10:58 UT</td>
<td>0.011</td>
<td>1.50 (10^5) °K</td>
<td>16 (10^5) °K</td>
<td>29 cm(^{-3})</td>
<td>31 cm(^{-3})</td>
</tr>
</tbody>
</table>

Figure 1. Thermal noise spectra at the receiver input terminals as measured (closed circles) and computed. The dotted curves were computed with a single (cold) electron population; the solid curves with two populations. In both cases, the plasma parameters are those of table 1 obtained from the distribution functions plotted on figure 2. The horizontal bars show the bandwidth of the receiving channels.

Figure 2. Electron distribution functions on October 6, 1979. The thick line is the function measured with the Los Alamos plasma analyzer. The thin line is the two-maxwellian function used to compute the noise spectra shown on figure 1. The 10 28 UT curves are displaced to the right, for clarity, by the length of the horizontal arrow.
However, some constraints are imposed for such a method to be efficient. We need an antenna length larger than the Debye length. We need electromagnetic cleanliness of the spacecraft to achieve a low instrumental noise level (4 \times 10^{-16} \text{ V}^2\text{Hz}^{-1} has been achieved on ISEE-3). The method cannot be used effectively in the presence of intense radio waves. It is model-dependent and does not describe distribution functions which would differ widely from a multimaxwellian. Finally, the "quasi-thermal" equilibrium between the plasma and the antenna and the use of the fluctuation-dissipation theorem imply (Sitenko, 1967) that the relaxation time of the observed distribution function towards a monomaxwellian distribution function (exact thermal equilibrium) is long compared to the time needed to acquire a complete spectrum. The noise spectrum arising from an unstable distribution function cannot be interpreted assuming quasi-thermal equilibrium.

The effect of the antenna velocity relative to the solar wind

Further developments of the theoretical interpretation have been motivated by the ISEE-3 observations. For \omega_p \omega_m, the observed noise spectrum is not flat as predicted by our models. An electric noise component with a spectral index -1 has been identified, a spin modulation of the plasma noise is observed on the spinning antennas and the anisotropy is clearly correlated with the solar wind velocity (Hoang et al., 1982). These observations led us to take into account the thermal ion distribution function and the velocity of the antenna relative to the solar wind plasma. Longitudinal electron and ion plasma waves are Doppler-shifted in the antenna frame; this effect is particularly important for low frequency waves (\omega_p \omega_m) which are the slowest. The ion contribution to the noise calculated in that case yields a bump in the low frequency part of the spectrum but it is far from being sufficient to explain the observed \omega^{-1} noise spectrum. The anisotropy introduced by the Doppler shift of the waves results in a spin modulated noise (Couturier et al., 1983) which is frequency-dependent; the direction of the minimum electric field changes by 90° in the plasma resonance peak region in agreement with the observations.

A property of this model was predicted earlier by Andronov (1966): for typical solar wind parameters, the ISEE-3 antenna resistance may become negative below 0.1 \text{ f}_{\text{p}} when the dipole antenna is parallel to the solar wind velocity. In such a case, we get an unstable situation similar to the beam-plasma instability; Cerenkov effect due to the antenna velocity explains this behaviour. When such a process is predominant, the low frequency electric field measurements are quantitatively and qualitatively affected since the antenna gain has nothing in common with the gain evaluated in vacuum. We cannot encounter such a situation on ISEE-3 because our lowest receiver frequency is 30 kHz.

In conclusion, we wish to emphasize the fact that passive electric field measurements with antennas long as compared to the Debye length have been demonstrated to be useful tools for plasma diagnosis.

REFERENCES