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NUMERICAL SIMULATION OF TWO-DIMENSIONAL HEAT TRANSFER IN COMPOSITE BODIES WITH APPLICATION TO DE-ICING OF AIRCRAFT COMPONENTS

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TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>NOMENCLATURE</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. LITERATURE REVIEW</td>
<td>6</td>
</tr>
<tr>
<td>III. FORMULATION OF NUMERICAL METHOD</td>
<td>14</td>
</tr>
<tr>
<td>A. Governing Equations and Boundary Conditions</td>
<td>14</td>
</tr>
<tr>
<td>B. Numerical Technique and Crank-Nicolson Finite Difference Formulation</td>
<td>20</td>
</tr>
<tr>
<td>1. Finite Difference Equations for the Composite Body</td>
<td>22</td>
</tr>
<tr>
<td>2. Finite Difference Equations at Interfacial Points of the Composite Body</td>
<td>24</td>
</tr>
<tr>
<td>3. Finite Difference Equations for the Substrate-Ambient Interface</td>
<td>27</td>
</tr>
<tr>
<td>4. Finite Difference Equations for the Abrasion Shield-Ambient Interface</td>
<td>29</td>
</tr>
<tr>
<td>5. Finite Difference Equations for the Ice Layer</td>
<td>30</td>
</tr>
<tr>
<td>a. Abrasion Shield - Ice Interface</td>
<td>35</td>
</tr>
<tr>
<td>b. Ice-Ambient Interface</td>
<td>41</td>
</tr>
</tbody>
</table>
IV. NUMERICAL SOLUTION BY COMPUTER IMPLEMENTATION ............ 47
   A. Gauss-Seidel Point Iterative Method ...................... 47
   B. Numerical Program Algorithm ............................. 48

V. DISCUSSION OF RESULTS ...................................... 50
   A. Effect of Power Density .................................. 51
   B. Effect of Heater Element Thickness ..................... 52
   C. Effect of Insulation Thickness Ratio and
      Insulation Material .................................... 52
   D. Effect of Substrate Material ............................ 53
   E. Effect of Variable Heater Output ....................... 54
   F. Effect of Initial Ice Layer Thickness .................. 54
   G. Effect of Gap Width .................................... 55
   H. Effect of Heat Transfer Coefficient ..................... 56
   I. Effect of Phase Change .................................. 57
   J. Effect of Abrasion Shield ................................ 59
   K. Effect of Ice Shedding .................................. 60
   L. Effect of Refreezing .................................... 61

VI. CONCLUSIONS AND RECOMMENDATIONS ........................... 62

APPENDIX - COMPLETE PROGRAM LISTING AND SAMPLE INPUT DATA FILE 64
REFERENCES ..................................................... 90
NOMENCLATURE

\( C_p \) specific heat capacity (Btu/lb-°F)

\( E \) dependent variable, \( H \) or \( T \)

\( H \) enthalpy (Btu/lb)

\( h \) heat transfer coefficient (Btu/ft\(^2\)-hr-°F)

\( K \) thermal conductivity (Btu/ft-hr-°F)

\( K_W \) average thermal conductivity (Btu/ft-hr-°F)

\( L \) the total length of the composite slab in the y-direction (ft)

\( L_f \) latent heat of fusion for ice (Btu/lb)

\( q \) rate of heat generation per unit volume (Btu/hr-ft\(^3\))

\( q'' \) rate of heat generation per unit area (Btu/hr-ft\(^2\) or Watts/in\(^2\))

\( T \) temperature (°F)

\( t \) time variable (hr)

\( x \) space coordinate in x-direction (ft)

\( \tilde{x} \) dimensionless space coordinate in x-direction

\( y \) space coordinate in y-direction (ft)

\( \tilde{y} \) dimensionless space coordinate in y-direction

Greek Letters:

\( \alpha \) thermal diffusivity (ft\(^2\)/hr)

\( \Delta t \) size of time step (hr)

\( \Delta \tilde{x} \) dimensionless grid spacing
NOMENCLATURE (CONTINUED)

Greek Letters:

Δy dimensionless grid spacing
θ dimensionless temperature
ρ density (lb/ft³)

Subscripts:

b₁ inner ambient boundary
b₂ outer ambient boundary
g the position in the center line of the gap
h the position in the center line of the heater
i grid point in x-direction
j layer in composite blade
k grid point in y-direction
l liquid (water)
lm liquid at the melting point
m melting point
s solid (ice)
sm solid at the melting point

Superscripts:

Δ evaluated at the previous time level
o evaluated halfway between the previous and present time levels
N value from previous iteration
N+1 value from current iteration
I. INTRODUCTION

Many practical transient heat conduction problems involve phase changes, e.g., the melting of a solid or the solidification of a liquid. These types of phase change problems are known as Stefan problems, after the early work done by Stefan (1889), and they have recently received considerable attention. Two of the oldest examples of this type of problem are the formation of ice on a solid body and the solidification of lava streams. Other applications include ablation of missile skins under aerodynamic heating, casting of metals and glass in molds, freeze drying of foodstuffs, heat exchangers using cryogenics, and the freezing and thawing of soil. The number and diversity of melting/freezing applications has motivated much research, which has been extensively reported in the literature [1].

The formation of ice on aircraft components has a considerable effect on aircraft performance as it increases drag and decreases lift. Thus, an aircraft must be designed with the necessary equipment required for ice removal or prevention. Basically, ice protection systems can be classified as either anti-icing or de-icing.

The anti-icing (or ice prevention) principle involves the prevention of ice formation on the protected area at all times. Typical methods of anti-icing employ the use of chemical systems (freezing point depressants) or the passage of hot, compressed bleed air from the engine
through passages below the surface on which ice formation is to be prevented. Anti-icing methods usually utilize large amounts of energy. On the other hand, the de-icing (or ice removal) principle involves the periodic removal of a layer of ice by mechanical or thermal means. This is accomplished by destroying the adhesion strength between the ice and the aircraft surfaces. Various de-icing techniques that have been investigated and reported upon [2, 3] are pneumatic boots and thermal techniques, the latter comprising cyclic heating of discrete elements by hot fluid or electrothermal means. For de-icing systems, the energy requirements are significantly less than those required for anti-icing systems.

The choice between anti-icing and de-icing techniques depends upon the component geometry and its function, and upon other factors such as the energy source utilized, the sensitivity of the component and the total energy available for ice protection. The advantages and pitfalls of anti-icing and de-icing systems have been reviewed by Baliga [4]. Stallabrass [5] has concluded from his experimental studies that the electrothermal method has the most advantages as a de-icing mechanism, although it does have maintainability/reliability problems. Werner [3] also reported that the electrothermal de-icing technique is the method most commonly used, and it has been applied to both fixed and rotary wing aircraft. For these reasons, the electrothermal means of de-icing will be the major topic of the present work.

The electrothermal de-icing principle was first successfully applied to propeller blades. The purpose of this thermal technique is to raise the composite blade surface-ice interface temperature to above
the melting point of ice, resulting in a very thin liquid interface layer which reduces the ice adhesion to the blade surface. Aerodynamic and/or centrifugal forces can then sweep the unmelted ice from the surface. The basic two-dimensional electrothermal de-icing pad employed in this study is shown in Figure 1. It is essentially a composite body consisting of: (1) a metal substrate (aircraft blade); (2) an inner insulation layer; (3) a heater element; (4) an outer insulation layer; and (5) an abrasion shield. A layer of ice is shown attached to the abrasion shield.

The design of the heating elements is either spanwise on the blade to cause chordwise shedding, or chordwise to cause spanwise shedding. The heater element usually consists either of a woven mat of wires and glass fibers or of multiple strips of resistance ribbon. As pointed out by Werner [3], the key factors in the design of an electrical heating element for a cyclic de-icing system are: (1) the uniformity of the obtained surface temperature at the abrasion shield-ice interface; (2) the amount of insulation required between the heater element and the ice surface; and (3) the amount of insulation required between the heater element and the aircraft blade. Stallabrass [5] has mentioned that heater element configurations may vary considerably. One such configuration consists of an array of parallel wires, woven or knitted with nylon or glass filaments to a surface or plane heater mat; the wires might be 0.02" in diameter, and spaced 24 to the inch, whereas metal ribbons usually have a thickness between 0.001" and 0.005". The gaps which exist between these heating elements can reduce the heater pad de-icing effectiveness, causing non-uniform melting of the ice. The gap width is
approximately 0.040" for metal ribbons and 0.080" for wires.

The two insulation layers, which usually consist of resin impregnated glass cloth, serve to provide electrical insulation for the heating element. In order to direct more heat flow toward the ice layer, it is necessary to use a much greater thickness for the inner insulation than for the outer insulation. A minimum thickness ratio of at least 2:1 has been suggested by Stallabrass [5]. The thickness of the outer insulation typically ranges from 0.010" to 0.020".

The purposes of the abrasion shield, which is generally made of stainless steel, are: (1) to protect the de-icer pad from rain erosion or sand and stone abrasion (especially when flying at high speeds); and (2) to diffuse the heat from the heater element so as to provide more uniform heating, thus minimizing cold spots above the heater gaps. The range of abrasion shield thickness is from 0.010" to 0.020". The material of construction of the substrate may vary widely depending upon the application. An aluminum alloy is considered in this study.

A two-dimensional model of the de-icing pad is the subject of this investigation. The model, which represents a cross-section of the heater pad normal to the run of the heater elements, is shown in Figure 1. It can be seen in the figure that natural boundary lines have been formed by lines of symmetry of the centerlines of the heater strip and the gap between the heater strips. It is assumed that there is perfect adhesion between each layer and therefore no contact resistance to heat flow shall be introduced into the analysis. A point heat source or a finite thickness heater providing either a constant or a time dependent heat supply will be included in this study. A wide range of parametric
studies have been conducted using the model to investigate such phenomena as the effect due to different heating functions on the rate and depth of ice melting, the effect of the abrasion shield toward evening out the temperature field due to non-uniform heating, and the effect of the gap width on the heater pad de-icing performance. A finite difference numerical analysis using the Enthalpy method to simulate the phase change in the ice layer is employed in the model solution.
II. LITERATURE REVIEW

A comprehensive review of the recent literature reveals that the study of electrothermal de-icer pads has either been ignored or that relevant information has not been published in the open literature. Of the investigations found, only the works of Stallabrass [5], Baliga [4] and Marano [6] have considered the effect of the phase change in the ice layer. All of the models have been one-dimensional except for that of Stallabrass, who proposed both one-dimensional and two-dimensional models. A two-dimensional electrothermal de-icer pad accounting for the phase change in the ice layer has been considered in the present study.

Temperature distributions in composite bodies consisting of several layers of material have numerous applications in heat transfer problems, such as in rocket thrust chamber liners, fuel elements for nuclear reactors, and in reentry spacecraft. Several analytical methods may be used to obtain solutions for a limited number of layers with various boundary conditions. The Laplace transform technique, as presented in Carslaw and Jaeger [7], is the most common analytical technique used to solve transient heat conduction problems in composite bodies. In principle, this technique will yield the temperature distribution in any composite slab, but, in practice, if the number of layers exceeds two, the inverse Laplace transform becomes quite difficult. Scott [8] applied this method to determine the temperature distribution in a symmetrical
five-component slab problem, and obtained a complicated solution which is limited to constant physical properties within each slab. Goodman [9] introduced the adjoint-solution technique, which is a method of obtaining the solution to a large class of heat conduction problems in composite bodies from the solution of but one adjoint problem. The adjoint solution arises from consideration of an auxiliary function with the application of Green's theorem to transform the problem from a volume to a surface integral. The primary disadvantages of this method are that the solution provides just the interfacial temperatures and not the temperature profiles within each layer in the composite body, and that only steady state cases were evaluated. The extension of this method to handle the transient case has been done by Bouchillon [10]. Campbell [11] proposed a method analogous to electric transmission line theory to calculate the temperature profile in a composite body. Stallabrass [5] applied Campbell's method to check the accuracy of his numerical method.

Tittle [12, 13] has shown that a one-dimensional multilayer problem can be solved by applying the Orthogonal-Expansion technique. This technique basically involves constructing orthogonal sets from the solutions of each of the layers. An orthogonality factor, called the discontinuous weighting function, is used such that the resulting orthogonal set is applicable to the entire composite media.

The disadvantage of using an analytical technique is that an excessive number of calculations must be done for each temperature desired. Hence, as the number of layers within the body increases, the calculations become prohibitively cumbersome. In addition, analytical methods can not be applied when a phase change is considered.
The most commonly used numerical technique for solving partial differential equations is the finite difference method, or, alternatively, the grid method. From a study of the recent literature, it appears that almost all of the numerical models for de-icer pads have used finite difference methods. The method is considered approximate in that the continuous space domain is replaced by a grid composed of nodal points, and the derivative at a given point is represented by a derivative taken over a finite interval across the point. The differential equation is replaced by a system of difference equations. The difference equations are then solved by algebraic or arithmetic procedures. Using this method, the temperature at any or all of the nodal points within the composite body may be obtained at any or all time steps. The disadvantages of the analytical methods are thus overcome by the use of finite difference numerical methods.

Two types of difference equations have previously been studied [14]: (1) explicit difference equations, which are simple to solve, but which require an uneconomically large number of time steps of limited size; and (2) implicit difference equations, which do not limit the time step, but which require at each time step the solution by iteration of a large set of simultaneous equations. Various finite differencing methods have been presented and are discussed in detail by Rosenberg [15] and Nogotov [16]. Convergence and stability are two important considerations in using a finite difference scheme. The accuracy of any finite difference scheme can be controlled by choosing the space and/or time interval as small as possible at the expense of increased computation time in solving the resulting increased number of algebraic equations. A finite
difference representation of the de-icer pad is shown in Figure 2.

Stallabrass [5] and Gent and Cansdale [17] employed the explicit forward time, central space finite difference technique in their de-icer investigation. In this numerical method, the temperature at each node is calculated directly from the value of the temperature at the previous time step. This method is stable only if certain criteria relating the time step and the grid size are satisfied. The criteria requires that the ratio of $\Delta t$ to $(\Delta x)^2$ must remain less than or equal to $1/2$, i.e., the size of $\Delta t$ must be of the same order of magnitude as $(\Delta x)^2$ for the solution to be stable. A time step of 0.001 sec. or smaller is normally used so that the stability criterion at the most sensitive of the nodal points is satisfied. The excessive number of calculations needed because of this small time step can, however, cause an accumulation of truncation and round-off error. The truncation error for the forward differencing scheme is first order for the time differential and second order for the space differential. Stallabrass [5] used the above method to solve a square grid (i.e., $\Delta x = \Delta y$) two-dimensional model with a point source or a finite thickness heater with constant heat output.

Kaliga [4], Marano [6] and the present study have employed the Crank-Nicolson implicit finite difference scheme in the electrothermal de-icer pad analysis. This numerical scheme is unconditionally stable, and consequently permits an independent choice of the time and space parameters of the grid. The truncation error of this method is second order for both the time and space differential. This allows a time step of 0.1 sec. to be used. In addition, Rosenberg [15] has pointed out that the Crank-Nicolson implicit finite difference scheme is the
preferred method for obtaining numerical solutions to parabolic partial
to solve the resulting set of linear algebraic equations for a one-
dimensional model of the de-icer problem. In order to obtain the
temperature profile at any time step with the effect of the phase change,
the Gauss-Seidel method, which is also known as the Liebmann method, is
recommended. This method is best suited to the solution of systems whose
matrix is sparse but not tridiagonal, i.e., for systems of two- and
three-dimensional problems.

Phase change or moving-boundary problems are important in many
engineering applications, and have been studied by several investigators
over the last two decades. Most studies have been restricted to one-
dimensional cases. The few investigations that do deal with two-
dimensional problems are restricted to specific problems and to a limited
range of parameters. The solution of Stefan problems is inherently
difficult because the interface between the solid and liquid phase is
moving as the latent heat is absorbed or released at the interface; as a
result the location of the solid-liquid interface is not known apriori
and must be determined as a part of the solution. Many solution approaches
such as Integral methods [18], Successive approximation [19], Series
solution [20], Green's function [21] and Perturbation techniques [22]
have been proposed for solving one-dimensional problems with a number
of simplifying assumptions. The exact solution of a more general phase
change problem was discussed by F. Neumann in his lectures in the 1860's,
but his lecture notes containing these solutions were not published
until 1912. Since then, many phase change problems have appeared in the literature, but the exact solutions are limited to a few idealized cases involving semi-infinite or infinite regions, subject to simple boundary and initial conditions [7]. Because of the non-linear nature of such problems, the superposition principle is not applicable and each case must be treated separately. Since exact solutions are not available for these problems, approximate, semi-analytical, and numerical methods are used to solve phase change problems. An extensive review of these techniques has been presented in References [23, 24, 25, 26].

Numerically, the major difficulties are caused by the need to track a continuously moving phase change boundary through a region approximated by a finite number of points. Various methods have been proposed in Reference [23] to overcome this problem, including moving grid points, isotherm migration, reformulation of the problem into integral form, etc. A time dependent grid spacing scheme, also requiring nodal iteration, has been presented by Heitz and Westwater [27] and by Murray and Landis [28]. Crank [29] and Ehrlich [30] employed higher order space differences for the phase change equation. The drawback with this method is the requirement that the media be homogeneous, thus necessitating intricate programming in order to simulate the movement of the phase interface across element boundaries.

Most of these methods use predictor-corrector techniques to locate the position of the solid-liquid interface. In general, such procedures require a large number of trial and error computations. In order to circumvent these difficulties, the Enthalpy method, which is also called
the method of weak solution, has been employed frequently in the recent literature, e.g., see [23, 31, 32, 33]. In this technique, enthalpy is used as a dependent variable along with the temperature, the interface is eliminated from consideration in the calculations, and the problem is made equivalent to one of nonlinear heat conduction without change of phase. Hence, the advantages of this method are: (1) that the problem to be solved is formulated in a fixed region; (2) that no modification of the numerical scheme is necessary in order to satisfy the requirement of accurately tracking the moving phase change interface; (3) that the temporal derivative of the temperature no longer appears in the governing equation; and (4) that it is not necessary to consider the solid and liquid regions separately. In addition, Fox [34] asserts that weak solution techniques are by far the best of the finite difference techniques for any significant problem, and especially those in more than one space dimension. It is shown in Reference [33] that the mathematical representation of the Enthalpy method is equivalent to the conventional formulation of the conservation equations for the solid and liquid regions and the movement of the solid-liquid interface.

Kamenomostskaja [35] and Atthey [32] have applied the Enthalpy method to one-dimensional phase change problems using a finite difference scheme. The latter applied the method to a one-dimensional spot-welding problem. Crowley [36] applied the method to Stefan problems in more than one dimension. Marano [6] used the Enthalpy method in a one-dimensional deicer pad analysis.
Bonacina et al. [37] and Goodrich [38] have proposed a heat capacity formulation of the Stefan problem by associating the latent heat effect with a finite temperature interval about the phase change isotherm. Baliga [4] employed this method in a one-dimensional electrothermal de-icer pad analysis. Stallabrass [5] implemented the phase change process by holding the desired node at the melting point until enough heat had been transferred into the nodal volume to completely melt all of the ice in the volume. Both Baliga's [4] and Stallabrass' [5] techniques are equivalent but less rigorous than the Enthalpy method. Meyer [39] and Shamasunder and Sparrow [33] have used an Enthalpy technique to solve two-dimensional phase change problems.

In the present study, a transient, two-dimensional solution is obtained for the de-icer pad by the use of finite-difference techniques which employ the Enthalpy method to simulate the phase change occurring in the ice layer.
III. FORMULATION OF NUMERICAL METHOD

A. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The following assumptions were made in the development of a two-dimensional, transient, mathematical model for heat conduction in a composite blade with an ice layer:

1. The thermal physical properties of the material composing each layer of the blade are different, but do not depend on temperature;
2. The ambient temperature, blade air temperature and all heat transfer coefficients are constants;
3. There is perfect thermal contact between each layer; and
4. The density change due to melting is negligible, i.e., the effect of the volume contraction of the ice as it melts is neglected.

With the above assumptions, the mathematical formulation for the problem of unsteady heat conduction in a chordwise two-dimensional section with electrothermal heating can be represented as

\[
\rho_j c_{pj} \frac{\partial T_j}{\partial t} = K_j \left( \frac{\partial^2 T_j}{\partial x^2} + \frac{\partial^2 T_j}{\partial y^2} \right) + q_j
\]

(1)

where \( j \) stands for the material in question, as given by
j = 1: blade or substrate \( q_1 = 0 \)

j = 2: lower or inner insulation \( q_2 = 0 \)

j = 3: heater \( q_3 = q_3(t) \)

j = 4: upper or outer insulation \( q_4 = 0 \)

j = 5: abrasion shield \( q_5 = 0 \)

and where

\( \rho_j \) = density of the \( j^{th} \) layer;

\( C_{pj} \) = specific heat capacity of the \( j^{th} \) layer;

\( T_j \) = temperature in \( j^{th} \) layer;

\( K_j \) = thermal conductivity of the \( j^{th} \) layer;

\( q_j \) = rate of heat generation per unit volume in \( j^{th} \) layer;

\( t \) = time variable; and

\( x, y \) = space coordinates.

For the ice layer (\( j = 6 \)), the governing equation for the Enthalpy method is:

\[
\frac{\partial H_6}{\partial t} = \frac{\partial}{\partial x} (K_6 \frac{\partial T_6}{\partial x}) + \frac{\partial}{\partial y} (K_6 \frac{\partial T_6}{\partial y})
\]

where

\( H_6 \) = enthalpy per unit volume within the ice-water layer;

\( T_6 \) = temperature within the ice-water layer; and

\( K_6 \) = thermal conductivity within the ice-water layer.

The enthalpy within both phases (ice and water) can be found from
equation (3). The temperature $T_6$ can then be determined from the following relationship between $H_6$ and $T_6$:

$$H_6 = \begin{cases} 
\rho_s \hat{C}_{ps} T_6 & , T_6 < T_m \\
\rho_1 \hat{C}_{pl} (T_6 - T_m) + \rho_1 (\hat{C}_{ps} T_m + L_f), & , T_6 > T_m 
\end{cases} \tag{4}$$

where

- $\rho_s, \hat{C}_{ps}$ = physical properties of the solid phase;
- $\rho_1, \hat{C}_{pl}$ = physical properties of the liquid phase;
- $T_m$ = melting temperature; and
- $L_f$ = latent heat of fusion per unit mass.

From equation (4),

$$T_6 = \begin{cases} 
\frac{H_6}{\rho_s \hat{C}_{ps}} & , H_6 < H_{sm} \\
T_m & , H_{sm} < H_6 < H_{lm} \\
\frac{(H_6 - H_{lm})}{\rho_1 \hat{C}_{pl}} + T_m & , H_6 > H_{lm} 
\end{cases} \tag{5}$$

with

$$H_{sm} = \rho_s \hat{C}_{ps} T_m$$
$$H_{lm} = \rho_1 (\hat{C}_{ps} T_m + L_f)$$

where $H_{sm}$ and $H_{lm}$ are the enthalpies of the ice and water at the melting point, respectively.
The boundary and initial conditions are as follows:

(i) At interior interfaces, the temperatures and heat fluxes are continuous, i.e.,

\[
T_j|_I = T_{j+1}|_I \quad j = 1, \ldots, 5
\]

\[
-K_j \frac{\partial T_j}{\partial y_j}|_I = -K_{j+1} \frac{\partial T_{j+1}}{\partial y_{j+1}}|_I \quad j = 1, \ldots, 5
\]  

where I denotes the interface.

(ii) At the interior and exterior surfaces of the composite body, Newton's law of cooling can be used to represent the convective heat exchange.

For the lower or inner boundary:

\[
K_j \frac{\partial T_j}{\partial y_j}|_1 = h_{b1} (T_j|_1 - T_{b1}) \quad j = 1
\]  

Where 1 denotes the lower or inner ambient boundary, \( h_{b1} \) is the convective heat transfer coefficient at the boundary and \( T_{b1} \) is the lower ambient (blade air) temperature.

For the upper or outer boundary:

\[
-K_j \frac{\partial T_j}{\partial y_j}|_2 = h_{b2} (T_j|_2 - T_{b2}) \quad j = 6
\]  

where 2 denotes the upper or outer ambient boundary, \( h_{b2} \) is the convective heat transfer coefficient at the boundary and \( T_{b2} \) is the upper ambient temperature.
At the centerline of the heater and the gap, insulated boundary conditions can be used to represent the symmetry, i.e.,

\[
\frac{\partial T}{\partial x} \Big|_{g,j=0} = 0, \quad j = 1, \ldots, 6 \quad (8a)
\]

\[
\frac{\partial T}{\partial x} \Big|_{h,j=0} = 0, \quad j = 1, \ldots, 6 \quad (8b)
\]

where \( g \) denotes the position of the centerline of the gap, and \( h \) denotes the position of the centerline of the heater.

In addition to the above, the initial temperature distribution in the composite blade can be constant or a function of position, and, if desired, constant temperature boundary conditions can be used for the lower and upper surfaces of the composite blade.

To formulate the above equations in terms of dimensionless temperature and distance, the following definitions are made:

\[
\theta = \frac{T - T_{\text{ref}}}{T_{\text{ref}}}, \quad \bar{x} = \frac{x}{L}, \quad \bar{y} = \frac{y}{L}, \quad \alpha_j = \frac{K_j}{\rho_j c_j}
\]

where

\( T_{\text{ref}} = \) the reference temperature (taken to be 32°F in this study);

\( L = \) the total length of the composite slab in the \( y \) direction; and

\( \alpha_j = \) the thermal diffusivity of the \( j^{th} \) layer.

Using these definitions, equations (1) through (8) take the following form:

For each layer in the composite blade:
\[
\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{q_j}{\rho_j C_{pj} T_{ref}}
\]  
(9)

For the ice layer:

\[
\frac{\partial H_6}{\partial t} = \frac{T_{ref}}{L} \left( \frac{\partial}{\partial x} (K_6 \frac{\partial H_6}{\partial x}) + \frac{\partial}{\partial y} (K_6 \frac{\partial H_6}{\partial y}) \right)
\]  
(10)

\[
H_6 = \begin{cases} 
\rho_s C_{ps} \theta_6 T_{ref} & \theta_6 < T_m \\
\rho_l C_{pl} T_{ref} (\theta_6 - \theta_m) + \rho_l (C_{ps} \theta_m T_{ref} + L_f), & \theta_6 > T_m
\end{cases}
\]  
(11)

\[
\frac{H_6}{\rho_s C_{ps} T_{ref}} \theta_6 \leq H_{sm} \quad \text{, } H_{sm} < H_6 < H_{lm}
\]

At the interfacial points:

\[
\theta_j \bigg|_I = \theta_{j+1} \bigg|_I 
\]  
(13a)

\[
-K_j \frac{\partial \theta_j}{\partial y} \bigg|_I = -K_{j+1} \frac{\partial \theta_{j+1}}{\partial y} \bigg|_I 
\]  
(13b)

At the lower and upper boundaries:

\[
K_j \frac{\partial \theta_j}{\partial y} \bigg|_1 = h_{b1} L (\theta_j \big|_1 - \theta_{b1}), \quad j = 1 
\]  
(14a)

\[
-K_j \frac{\partial \theta_j}{\partial y} \bigg|_2 = h_{b2} L (\theta_j \big|_2 - \theta_{b2}), \quad j = 6 
\]  
(14b)
Finally, the insulated boundary conditions at the centerlines of the gap and the heater are:

\[
\frac{\partial \theta}{\partial x} \bigg|_{g,j} = 0, \quad j = 1, \ldots, 6 \tag{15a}
\]

\[
\frac{\partial \theta}{\partial x} \bigg|_{h,j} = 0, \quad j = 1, \ldots, 6 \tag{15b}
\]

Next the above equations will be expressed in finite difference form and arranged for numerical solution.

B. NUMERICAL TECHNIQUE AND CRANK-NICOLSON FINITE DIFFERENCE FORMULATION

Formulation of a difference problem begins with the choice of a difference grid (a discrete set of points), on which a discretized or finite difference version of the original differential equation is to be solved. A significant feature of this technique is associated with the optional choice of grid parameters for characterizing the mesh spacing. For this study, there are three independent variables: two spatial coordinates \( x \) and \( y \), and time \( t \). The respective grid spacings and time increment are \( \Delta x \), \( \Delta y \) and \( \Delta t \). Let \( E_{i,k} \) stand for the value of a dependent variable at the point \( (i, k) \), i.e., either \( T \), \( \theta \) or \( H \). The central difference approximations of \( \frac{\partial^2 E}{\partial x^2} \) and \( \frac{\partial E}{\partial x} \) are:

\[
\frac{\partial^2 E_{i,k}}{\partial x^2} \bigg|_{j} = \frac{E_{i+1,k} - 2E_{i,k} + E_{i-1,k}}{(\Delta x)^2} + \frac{0}{(\Delta x)^2} \tag{16}
\]
These are second-order finite difference analogs. The second order finite difference analog for the time derivative, \( \frac{\partial E}{\partial t} \), is:

\[
\frac{\partial E_{i,k}}{\partial t} \bigg|_j = \frac{E_{i+1,k} - E_{i-1,k}}{\Delta t} + O(\Delta t)^2
\]

(18)

where the superscript \( \Delta \) denotes the value at the previous time level and the superscript \( o \) denotes the value at a half-time step between the previous and present time levels. Equation (18) represents the Crank-Nicolson formulation of the time derivative. The real key to the Crank-Nicolson formulation is the manner of approximating \( \frac{\partial^2 E}{\partial x^2} \) and \( \frac{\partial E}{\partial x} \) without requiring the evaluation of the dependent variable at a time level half-way between the known and the unknown time levels. These derivatives are approximated by the arithmetic average of the finite difference analogs at the previous time level and the present time level. Equations (16) and (17) are approximated as:

\[
\frac{\partial^2 E_{i,k}}{\partial x^2} \bigg|_j = 1/2 \left( \frac{\partial^2 E_{i,k}}{\partial x^2} \bigg|_j + \frac{\partial^2 E_{i,k}}{\partial x^2} \bigg|_j \right)
\]

\[
= \frac{E_{i+1,k} - 2E_{i,k} + E_{i-1,k} + E_{i+1,k} - 2E_{i,k} + E_{i-1,k}}{2 (\Delta x)^2} + 0 (\Delta x)^2
\]

(19)

\[
\frac{\partial E_{i,k}}{\partial x} \bigg|_j = 1/2 \left( \frac{\partial E_{i,k}}{\partial x} \bigg|_j + \frac{\partial E_{i,k}}{\partial x} \bigg|_j \right)
\]

\[
= \frac{E_{i+1,k} - E_{i-1,k} + E_{i+1,k} - E_{i-1,k}}{4 \Delta x} + 0 (\Delta x)^2
\]

(20)
The Crank-Nicolson finite difference equations for the present problem are obtained by substituting the analogs of the above derivatives into the governing equations and boundary conditions.

1. Finite Difference Equations for the Composite Body

Substituting equations (19) and (20) into equation (9) yields:

$$
\frac{\theta_{i,k,j} - \theta_{i,k-1,j}}{\Delta t} = \frac{a_1}{2L^2} \left[ \frac{\theta_{i+1,k,j} - 2\theta_{i,k,j} + \theta_{i-1,k,j} + \theta_{i+1,k,j} - 2\theta_{i,k,j}}{(\Delta x)^2} \right]
$$

$$
+ \frac{q_j}{\theta_j C p_j T_{ref}} \quad j = 1, \ldots, 5 \quad (21a)
$$

Solving for the unknown temperature at node \((i, k)\) in the \(j\)th layer gives:

$$
\theta_{i,k,j} = \theta_{1,k,j} \frac{a_1}{2(L\Delta x)^2} \left[ \theta_{i+1,k,j} + \theta_{i-1,k,j} + \theta_{i+1,k,j} + \theta_{i-1,k,j} \right]
$$

$$
+ \frac{a_1}{2(L\Delta y_j)^2} \left[ \theta_{i,k+1,j} + \theta_{i,k-1,j} + \theta_{i,k+1,j} + \theta_{i,k-1,j} \right] + B_j \theta_1 \Delta_{i,k,j}
$$

$$
+ \frac{q_j''/L\Delta y_j}{\rho_j C p_j T_{ref}} \quad i = 2, \ldots, n-1, \quad j = 1, \ldots, 5 \quad (21b)
$$

where
\[
A_j = \frac{1}{\Delta t} \left( \frac{1}{(L\Delta x)^2} + \frac{a_1}{(L\Delta y_j)^2} \right)
\]

\[
B_j = \frac{1}{\Delta t} - \frac{a_1}{(L\Delta x)^2} - \frac{a_1}{(L\Delta y_j)^2}
\] and

\[
q''_j = \text{the heat source per unit area, and equals } q_j L\Delta y_j.
\]

Equation (21b) is valid for all grid points within each of the layers of the composite aircraft blade, except at the centerlines of the gap and the heater, i.e., at \( i = 1 \) and \( i = n \). Finite differencing equation (15) gives:

\[
\theta_0,k,j = \theta_2,k,j \quad j = 1, \ldots, 6 \tag{22a}
\]

\[
\theta_{n+1,k,j} = \theta_{n-1,k,j} \quad j = 1, \ldots, 6 \tag{22b}
\]

The grid points within each of the layers of the composite body at the centerlines of the gap and the heater are shown in Figures 3a and 3b, respectively. The equations for these points can be determined by substituting equation (22) into equation (21b), producing the following:

At the centerline of the gap:

\[
i = 1, \quad \theta_1,k,j = A_j \left( \frac{a_1}{2(L\Delta x)^2} \left[ 2 \theta_2,k,j + 2 \theta_2,k,j \right] + \frac{a_1}{2(L\Delta y_j)^2} \left[ \theta_1,k+1,j \right.ight.
\]

\[
+ \theta_1,k-1,j + \theta_1,k+1,j + \theta_1,k-1,j \right] + B_j \theta_1,k,j + \frac{q''_j/L\Delta y_j}{\rho_j C_p j T_{ref}}
\]

\[
\left. \right] \right\} \quad j = 1, \ldots, 5 \tag{23a}
\]
At the centerline of the heater:

\[ i = n, \theta_{n,k,j} = A_j \left( \frac{a_i}{2(\Delta x)^2} \right) \left[ \theta_{n-1,k,j} + 2 \theta_{n,k,j} + \frac{a_j}{2(\Delta y_j)^2} \theta_{n,k+1,j} \right] \]

\[ + \theta_{n,k-1,j} + \theta_{n,k+1,j} + \theta_{n,k,j+1} + B_j \theta_{n,k,j} \] \( j = 1, \ldots, 5 \)

(23b)

2. Finite Difference Equations at Interfacial Points of the Composite Body

Let \((i, k)\) denote one of the interfacial nodal points between the slab layers \(j\) and \(j+1\), as shown in Figure 3c. Substitution of equation (20) into equation (13b), with the aid of analog (13a), yields:

\[ \theta_{i,k,j} = \theta_{i,k,j+1} \]

\[ \theta_{i,k,j}^\Delta = \theta_{i,k,j+1}^\Delta \]

\( j = 1, \ldots, 5 \)

(24a)

and

\[ K_j \left( \frac{\theta_{i,k+1,j+1} - \theta_{i,k-1,j+1}}{4 \Delta y_j} \right) = \]

\[ -4 \Delta y_j \]

\[ = -K_{j+1} \left( \frac{\theta_{i,k+1,j+1} - \theta_{i,k-1,j+1}}{4 \Delta y_{j+1}} \right) \]

(24b)

The temperatures \(\theta_{i,k+1,j}, \theta_{i,k+1,j}^\Delta, \theta_{i,k-1,j+1}, \text{ and } \theta_{i,k-1,j+1}^\Delta\)

are fictitious and can be eliminated by equation (21b) when applied to the layers \(j\) and \(j+1\):

\[ \theta_{i,k+1,j} + \theta_{i,k+1,j}^\Delta = \frac{2(\Delta y_j)^2}{A_j} \left( \frac{a_i}{2(\Delta x)^2} \theta_{i,k,j} - A_j \right) \]

\[ \theta_{i+1,k,j} + \theta_{i+1,k,j}^\Delta = \frac{a_i}{2(\Delta x)^2} \theta_{i,k,j} + \]

24
Eliminating the fictitious temperatures by combining equations (24a), (25) and (26) yields:

\[
\begin{aligned}
\theta_{i-1,k,j} &+ \theta^\Delta_{i+1,k,j} + \frac{\alpha_j}{2(\Delta\tilde{y}_j)^2} \left( \theta_{i-1,k,j} + \theta^\Delta_{i,k-1,j} \right) \\
+ Bj \theta^\Delta_{i,k,j} + \frac{q''/L\Delta\tilde{y}_j}{\rho_j c_{pj} T_{ref}} \left( \frac{\Delta\tilde{y}_j}{\alpha_j} \right)^2 = 0 \\
\theta_{i,k-1,j+1} &+ \theta^\Delta_{i,k,j+1} + \frac{2(\Delta\tilde{y}_j+1)}{A_{j+1} \alpha_{j+1}} \left( \theta_{i,k,j+1} - \theta_{i,1,k,j+1} \right) \\
+ \frac{\alpha_{j+1}}{2(\Delta\tilde{y}_{j+1})^2} \left( \theta_{i,k+1,j+1} + \theta^\Delta_{i+1,k,j+1} \right) \\
+ Bj+1 \theta^\Delta_{i,k,j+1} + \frac{q''_{i+1}/L\Delta\tilde{y}_{j+1}}{\rho_{j+1} c_{pj+1} T_{ref}} \left( \frac{\Delta\tilde{y}_{j+1}}{\alpha_{j+1}} \right)^2 = 0 \\
\theta_{i+1,k,j} &+ \theta^\Delta_{i-1,k,j} + \frac{\alpha_j}{2(\Delta\tilde{y}_j)^2} \left( \theta_{i-1,k,j} + \theta^\Delta_{i-1,k,j} \right) \\
+ Bj+1 \theta^\Delta_{i,k,j+1} + \frac{q''_{i+1}/L\Delta\tilde{y}_{j+1}}{\rho_{j+1} c_{pj+1} T_{ref}} \left( \frac{\Delta\tilde{y}_{j+1}}{\alpha_{j+1}} \right)^2 = 0 \\
\end{aligned}
\]
Equation (27) is applicable for the interfacial points within the composite blade, except at the centerlines of the gap and the heater. Applicable equations for the interfacial points at the centerlines of the gap and the heater, as shown in Figure 3c with \( i = 1 \) and \( i = n \), can be determined by substituting equation (22) into equation (27):

At the centerline of the gap:

\[
\begin{align*}
\theta_{1,k,j} & = \frac{1}{2(L_\Delta \Delta y_j)^2 + \frac{K_{j+1} \Delta y_j}{2(L_\Delta \Delta y_j + 1)^2} K_j \Delta y_{j+1} A_{j+1} \alpha_{j+1}} \\
& + \frac{(\Delta y_j)^2 K_{j+1} \Delta y_j}{(\Delta x)^2} \left[ \theta_{2,k,j} + \theta_{2,k,j} + 2 \left[ \frac{(\Delta y_j)^2}{(\Delta x)^2} \right] \right] \\
& + 2 \frac{K_{j+1} \Delta y_j}{K_j \Delta y_{j+1}} \left[ \theta_{1,k+1,j+1} + \theta_{1,k+1,j+1} \right] + \frac{2(L_\Delta \Delta y_j)^2}{\alpha_j} B_j \\
& + \frac{2(L_\Delta \Delta y_j)^2}{\alpha_{j+1} K_j \Delta y_{j+1}} B_{j+1} \left[ \theta_{1,k,j+1} - \theta_{1,k,j+1} \right] + \frac{2(L_\Delta \Delta y_j)^2 q''/L_\Delta \Delta y_j}{\alpha_j} C_{pj+1} T_{ref} \\
& + \frac{2(L_\Delta \Delta y_j)^2 K_{j+1} \Delta y_j q''_{j+1}/L_\Delta \Delta y_{j+1}}{\alpha_{j+1} K_j \Delta y_{j+1}} \left( \alpha_j \rho_{j+1} C_{pj+1} T_{ref} \right)
\end{align*}
\] (27a)

At the centerline of the heater:

\[
\begin{align*}
\theta_{n,k,j} & = \frac{1}{2(L_\Delta \Delta y_j)^2 + \frac{K_{j+1} \Delta y_j}{2(L_\Delta \Delta y_j + 1)^2} K_j \Delta y_{j+1} A_{j+1} \alpha_{j+1}} \\
& + \frac{(\Delta y_j)^2 K_{j+1} \Delta y_j}{(\Delta x)^2} \left[ \theta_{2,k,j} + \theta_{2,k,j} + 2 \left[ \frac{(\Delta y_j)^2}{(\Delta x)^2} \right] \right] \\
& + 2 \frac{K_{j+1} \Delta y_j}{K_j \Delta y_{j+1}} \left[ \theta_{1,k+1,j+1} + \theta_{1,k+1,j+1} \right] + \frac{2(L_\Delta \Delta y_j)^2}{\alpha_j} B_j \\
& + \frac{2(L_\Delta \Delta y_j)^2}{\alpha_{j+1} K_j \Delta y_{j+1}} B_{j+1} \left[ \theta_{1,k,j+1} - \theta_{1,k,j+1} \right] + \frac{2(L_\Delta \Delta y_j)^2 q''/L_\Delta \Delta y_j}{\alpha_j} C_{pj+1} T_{ref} \\
& + \frac{2(L_\Delta \Delta y_j)^2 K_{j+1} \Delta y_j q''_{j+1}/L_\Delta \Delta y_{j+1}}{\alpha_{j+1} K_j \Delta y_{j+1}} \left( \alpha_j \rho_{j+1} C_{pj+1} T_{ref} \right)
\end{align*}
\]
3. Finite Difference Equations for the Substrate-Ambient Interface

The grid under consideration is shown in Figure 4a. At this surface \( j = 1 \), and \((i, k)\) is taken to be the interfacial node. The finite difference representation of equation (14a) is:

\[
\frac{\theta_{i+1,1} - \theta_{i,1} + \theta_{i+1,0} - \theta_{i,0}}{4 \Delta y_1} = K_1 \left( \frac{\theta_{i+1,1} - \theta_{i,1}}{2 (L \Delta y_1)} + \frac{\theta_{i,0}}{2 (L \Delta y_1)} \right)
\]

(28)

Temperatures \( \theta_{i,0,1} \) and \( \theta_{i,1,0} \) are fictitious and must be eliminated. Applying equation (21b) with \( k = 1, j = 1 \) and \( q''_j = 0 \), gives:

\[
\theta_{i,1,1} = \frac{a_1}{2 (L \Delta y_1)^2} \left[ \theta_{i+1,1,1} + \theta_{i-1,1,1} + \theta_{i+1,1,1} + \theta_{i-1,1,1} \right]
\]

+ \[ \frac{a_1}{2 (L \Delta y_1)^2} \left[ \theta_{i,2,1} + \theta_{i,0,1} + \theta_{i,2,1} + \theta_{i,0,1} \right] + B_1 \theta_{i,1,1} \]

(29)

The fictitious temperatures are eliminated by substituting equation (29) into equation (28):
\[ \theta_{1,1,1} = \frac{1}{2(L\Delta y_1)^2} \left\{ \frac{(\Delta y_1)^2}{(\Delta x)^2} \left[ \theta_{i+1,1,1} + \theta_{i-1,1,1} + \theta_{i+1,1,1} + \theta_{i-1,1,1} \right] \right\} + \frac{2(L\Delta y_1)^2}{2h_{b_1}L\Delta y_1} \left\{ \theta_{i,2,1,1} + \theta_{i,2,1,1} \right\} + \frac{2(L\Delta y_1)^2}{2h_{b_1}L\Delta y_1} \left[ \frac{\Delta y_1}{a_1} - \frac{2h_{b_1}L\Delta y_1}{K_1} \right] \theta_{i,1,1,1} + \frac{4h_{b_1}L\Delta y_1}{K_1} \theta_{b_1} \] 

(i = 2, \ldots, n - 1) (30)

Again, the interfacial points for the substrate-ambient boundary at the centerlines of the gap and the heater are shown in Figure 4a with \( i = 1 \) and \( i = n \), respectively. The equations for these points can be determined by substituting equation (22) into equation (30):

At the centerline of the gap:

\[ i=1, \theta_{1,1,1} = \frac{1}{2(L\Delta y_1)^2} \left\{ \frac{(\Delta y_1)^2}{(\Delta x)^2} \left[ \theta_{2,1,1,1} + \theta_{2,1,1,1} \right] \right\} + \frac{2(L\Delta y_1)^2}{2h_{b_1}L\Delta y_1} \left\{ \theta_{1,2,1,1} + \theta_{1,2,1,1} \right\} + \frac{2(L\Delta y_1)^2}{2h_{b_1}L\Delta y_1} \left[ \frac{\Delta y_1}{a_1} - \frac{2h_{b_1}L\Delta y_1}{K_1} \right] \theta_{1,1,1,1} + \frac{4h_{b_1}L\Delta y_1}{K_1} \theta_{b_1} \} \] (31a)

At the centerline of the heater:

\[ i=n, \theta_{n,1,1} = \frac{1}{2(L\Delta y_1)^2} \left\{ \frac{(\Delta y_1)^2}{(\Delta x)^2} \left[ \theta_{n-1,1,1,1} + \theta_{n-1,1,1,1} \right] \right\} + \frac{2(L\Delta y_1)^2}{2h_{b_1}L\Delta y_1} \left\{ \theta_{n,2,1,1} + \theta_{n,2,1,1} \right\} + \frac{2(L\Delta y_1)^2}{2h_{b_1}L\Delta y_1} \left[ \frac{\Delta y_1}{a_1} - \frac{2h_{b_1}L\Delta y_1}{K_1} \right] \theta_{n,1,1,1} + \frac{4h_{b_1}L\Delta y_1}{K_1} \theta_{b_1} \} \] (31b)
4. Finite Difference Equations for the Abrasion Shield-Ambient Interface

When the ice layer is not present or when the ice is shed, the grid under consideration is shown in Figure 4b. At this surface, \( j = 5 \), and \((i, m)\) is taken to be the interfacial node. Finite differencing equation (14b) yields:

\[
-K_5 \left( \frac{\theta_{i,m+1,5} - \theta_{i,m-1,5} + \theta_{i,m+1,5} - \theta_{i,m-1,5}}{4 \Delta y_5} \right)
= h_{b2} L \left( \frac{\theta_{i,m,5} + \theta_{i,m,5}}{2} - \theta_{b2} \right)
\]

(32)

Temperatures \( \theta_{i,m+1,5} \) and \( \theta_{i,m+1,5}^\Delta \) are fictitious and are eliminated by the same procedure as used for the previous substrate-ambient interface boundary:

\[
\theta_{i,m,5} = \frac{1}{2 (L \Delta y_5^2) + \frac{2 h_{b2} L \Delta y_5}{K_5}} \left[ \left( \frac{\Delta y_5}{\Delta x} \right)^2 \left[ \theta_{i+1,m,5} + \theta_{i-1,m,5} \right] + \frac{4 h_{b2} L \Delta y_5}{K_5} \theta_{b2} \right] \in \{2, \ldots, n-1\}
\]

(33)

At the centerline of the gap:

\[
i=1, \quad \theta_{1,m,5} = \frac{1}{2 (L \Delta y_5^2) + \frac{2 h_{b2} L \Delta y_5}{K_5}} \left[ \left( \frac{\Delta y_5}{\Delta x} \right)^2 \left[ \theta_{2,m,5} + \theta_{2,m,5}^\Delta \right] \right]
\]

(34)
At the centerline of the heater:

\[
\begin{align*}
&+ 2[\theta_{1,m-1,5} + \theta_{1,m-1,5}] + \left[\frac{2(\Delta y_5)^2}{a_5} - \frac{2h_{b2}L\Delta y_5}{K_5}\right] \theta_{1,m,5} \\
&+ \frac{4h_{b2}L\Delta y_5}{K_5} \theta_{b2}
\end{align*}
\] (34a)

5. Finite Difference Equations for the Ice Layer

As mentioned before, two equations are needed in the formulation of the Enthalpy method within the ice-water layer \(j = 6\), one to calculate enthalpy and one to calculate temperature at each nodal point in the ice layer. Substitution of analogs (18) and (19) into equation (10) gives:

\[
\begin{align*}
\frac{H_i,k,6 - H_i,k,6}{\Delta t} &= \frac{K_6}{2L^2} \left[ \frac{\theta_{i+1,k,6} - 2\theta_{i,k,6} + \theta_{i-1,k,6}}{(\Delta x)^2} \right] \\
&+ \frac{\theta_{i+1,k,6} - 2\theta_{i,k,6} + \theta_{i-1,k,6}}{(\Delta y_6)^2} \\
&+ \frac{\theta_{i,k+1,6} - 2\theta_{i,k,6} + \theta_{i,k-1,6}}{(\Delta y_6)^2} + \frac{\theta_{i,k+1,6} + \theta_{i,k-1,6}}{\Delta y_6} \\
&- 2\theta_{i,k,6} + \theta_{i,k-1,6}
\end{align*}
\] (35)

Equation (35) can be solved explicitly for \(H_i,k,6\) by using equation (12) to relate \(\theta_{i,k,6}\) to \(H_i,k,6\). Thus, there are three sets of equations.
pertaining to the solid phase, the melting point and the liquid phase. Since the thermal conductivity for each node within the ice-water layer is a function of position, average values of the thermal conductivity have been used. The quantity $KW_1$ is the average value of the thermal conductivity between two adjacent nodes $(i-1, k)$ and $(i, k)$; $KW_2$ is the average value of the thermal conductivity between two adjacent nodes $(i, k)$ and $(i+1, k)$; $KW_3$ is the average value of the thermal conductivity between two adjacent nodes $(i, k)$ and $(i, k+1)$; and $KW_4$ is the average value of the thermal conductivity between two adjacent nodes $(i, k-1)$ and $(i, k)$. Employing this averaging approach and substituting equation (12) into equation (35), we obtain, upon rearrangement, the governing finite difference equation for each phase.

For the solid phase:

$$H_{i,k,6} = \left(1 + \frac{\Delta t}{2L^2\alpha_s C_p b_s \Delta x^2} \left[\frac{KW_1 + KW_2}{(\Delta x)^2} + \frac{KW_3 + KW_4}{(\Delta y_6)^2}\right]\right)^{-1} \times$$

$$\left(H_{i,k,6}^{\Delta} + \Delta t \frac{T_{ref}}{2L^2} \left[\frac{KW_2 \theta_{i+1,k,6} + KW_1 \theta_{i-1,k,6}}{(\Delta x)^2} + \frac{KW_3 \theta_{i,k+1,6}}{(\Delta y_6)^2}\right]\right)$$

$$+ \frac{KW_4 \theta_{i,k-1,6}}{(\Delta x)^2} + \frac{KW_1 \theta_{i,k,6}}{(\Delta x)^2} - \frac{KW_2 \theta_{i,k,6}}{(\Delta x)^2} - \frac{KW_3 \theta_{i,k,6}}{(\Delta y_6)^2}$$

$$+ \frac{KW_4 \theta_{i,k+1,6}}{(\Delta y_6)^2} - \frac{KW_3 \theta_{i,k,6}}{(\Delta y_6)^2}$$

$$i = 2, \ldots, n - 1 \quad H_{i,k,6} \leq H_{sm} \quad (36)$$

Substitution of equation (22) into equation (36) yields:
At the centerline of the gap:

\[
H_{1,k,6} = (1 + \frac{\Delta t}{2L^2C_p\rho_s} \left[ \frac{KW_1 + KW_2}{(\Delta x)^2} + \frac{KW_3 + KW_4}{(\Delta y_6)^2} \right]^{-1}) x
\]

\[
(H_{1,k,6}^\Delta + \frac{\Delta t}{2L^2} \frac{KW_2}{(\Delta x)^2} \left[ \frac{\theta_{2,k,6}}{2} + \frac{KW_3}{(\Delta y_6)^2} \left( \theta_{1,k+1,6} + \theta_{1,k-1,6} \right) \right]
+ 2KW_2 \frac{(\theta_{2,k,6} - \theta_{1,k,6})}{(\Delta x)^2} + \frac{KW_3}{(\Delta y_6)^2} \left( \theta_{1,k+1,6} - \theta_{1,k,6} \right)
- \frac{KW_4}{(\Delta y_6)^2} \left( \theta_{1,k,6} - \theta_{1,k-1,6} \right))}
\]

\[
H_{1,k,6} \leq H_{sm} \quad (37a)
\]

At the centerline of the heater:

\[
H_{n,k,6} = (1 + \frac{\Delta t}{2L^2C_p\rho_s} \left[ \frac{KW_1 + KW_2}{(\Delta x)^2} + \frac{KW_3 + KW_4}{(\Delta y_6)^2} \right]^{-1}) x
\]

\[
(H_{n,k,6}^\Delta + \frac{\Delta t}{2L^2} \frac{KW_2}{(\Delta x)^2} \left[ \frac{\theta_{n-1,k,6}}{2} + \frac{KW_3}{(\Delta y_6)^2} \left( \theta_{n,k+1,6} + \theta_{n,k-1,6} \right) \right]
+ 2KW_2 \frac{(\theta_{n-1,k,6} - \theta_{n,k,6})}{(\Delta x)^2} + \frac{KW_3}{(\Delta y_6)^2} \left( \theta_{n,k+1,6} - \theta_{n,k,6} \right)
- \frac{KW_4}{(\Delta y_6)^2} \left( \theta_{n,k,6} - \theta_{n,k-1,6} \right))}
\]

\[
H_{n,k,6} \leq H_{sm} \quad (37b)
\]

with \( \theta_{i,k,6} = H_{i,k,6}/C_p\rho_s \) \( T_{ref} \) \( i = 1, \ldots, n \)

For the melting point:

\[
H_{1,k,6} = H_{1,k,6}^\Delta + \frac{\Delta t}{2L^2} \left[ \frac{-KW_1 + KW_2}{(\Delta x)^2} + \frac{KW_3 + KW_4}{(\Delta y_6)^2} \right] \theta_{m}
\]

\[
H_{1,k,6} \leq H_{sm}
\]

32
\[ \frac{KW_2 \theta_{i+1,k,6} + KW_1 \theta_{i-1,k,6} + KW_3 \theta_{i,k+1,6} + KW_4 \theta_{i,k-1,6}}{(\Delta x)^2} \]

\[ + \frac{KW_\Delta}{(\Delta x)^2} \left[ \theta_{i-1,k,6} - \theta_{i,k,6} \right] - \frac{KW_2 \Delta}{(\Delta x)^2} \left[ \theta_{i,k,6} - \theta_{i+1,k,6} \right] + \frac{KW_3 \Delta}{(\Delta y_6)^2} \left[ \theta_{i,k,6} - \theta_{i,k-1,6} \right] \]

\[ \theta_{i,k,l+1,6} - \theta_{i,k,l,6} \frac{KW_\Delta}{(\Delta y_6)^2} \left[ \theta_{i,k,l,6} - \theta_{i,k,l-1,6} \right] \]

\[ i = 2, \ldots, n-1, \quad H_{im} < H_{i,k,6} < H_{im} \]  

Substitution of equation (22) into the above equation gives

At the centerline of the gap:

\[ i=1, \quad H_{1,k,6} = H_{1,k,6} + \frac{\Delta t}{2L} \left[ \frac{KW_1}{(\Delta x)^2} + \frac{KW_2}{(\Delta y_6)^2} \right] \theta_{m} \]

\[ + \frac{2KW_2 \theta_{2,k,6} + KW_3 \theta_{1,k+1,6} + KW_4 \theta_{1,k-1,6}}{(\Delta x)^2} \]

\[ + \frac{KW_3 \Delta}{(\Delta y_6)^2} \left[ \theta_{1,k+1,6} - \theta_{1,k,6} \right] - \frac{KW_4 \Delta}{(\Delta y_6)^2} \left[ \theta_{1,k,6} - \theta_{1,k-1,6} \right] \]

\[ H_{sm} < H_{1,k,6} < H_{im} \]

At the centerline of the heater:

\[ i=n, \quad H_{n,k,6} = H_{n,k,6} + \frac{\Delta t}{2L} \left[ \frac{KW_1}{(\Delta x)^2} + \frac{KW_2}{(\Delta y_6)^2} \right] \theta_{m} \]

\[ + \frac{2KW_1 \theta_{n-1,k,6} + KW_3 \theta_{n,k+1,6} + KW_4 \theta_{n,k-1,6}}{(\Delta x)^2} \]

\[ + \frac{KW_3 \Delta}{(\Delta y_6)^2} \left[ \theta_{n,k+1,6} - \theta_{n,k,6} \right] - \frac{KW_4 \Delta}{(\Delta y_6)^2} \left[ \theta_{n,k,6} - \theta_{n,k-1,6} \right] \]
H_{sm} < H_{n,k,6} < H_{lm} \tag{39b}

with \theta_{i,k,6} = \theta_m \quad i = 1, \ldots, n

For the liquid phase:

\begin{align*}
H_{i,k,6} &= (1 + \frac{\Delta t}{2L^2 c_{p1} \rho_1} \left[ \frac{KW_1 + KW_2}{(\Delta \bar{x})^2} + \frac{KW_3 + KW_4}{(\Delta \bar{y}_6)^2} \right]^{-1} \times \\
&\left( H_{i,k,6}^\Delta + \frac{\Delta t}{2L^2} \left[ \frac{KW_1 + KW_2}{(\Delta \bar{x})^2} + \frac{KW_3 + KW_4}{(\Delta \bar{y}_6)^2} \right] \left( \frac{C_{ps} \theta_m}{c_{p1}} + \frac{L'_f}{c_{p1} T_{ref}} - \theta_m \right) + \frac{KW_2}{(\Delta \bar{x})^2} (\theta_{i,k,6} - \theta_{i+1,k,6}) + \frac{KW_1}{(\Delta \bar{x})^2} (\theta_{i-1,k,6} - \theta_{i,k,6}) - \frac{KW_3}{(\Delta \bar{y}_6)^2} (\theta_{i,k,6} - \theta_{i+1,k,6}) - \frac{KW_4}{(\Delta \bar{y}_6)^2} (\theta_{i,k,6} - \theta_{i-1,k,6}) \right) \\
&i = 2, \ldots, n-1 \quad H_{1,k,6} \geq H_{lm} \tag{40}
\end{align*}

By substituting equation (22) into equation (39), the following are obtained:

At the centerline of the gap:

\begin{align*}
&i = 1, \quad H_{i,k,6} = (1 + \frac{\Delta t}{2L^2 c_{p1} \rho_1} \left[ \frac{KW_1 + KW_2}{(\Delta \bar{x})^2} + \frac{KW_3 + KW_4}{(\Delta \bar{y}_6)^2} \right]^{-1} \\
&\times \left( H_{1,k,6}^\Delta + \frac{\Delta t}{2L^2} \left[ \frac{KW_1 + KW_2}{(\Delta \bar{x})^2} + \frac{KW_3 + KW_4}{(\Delta \bar{y}_6)^2} \right] \left( \frac{C_{ps} \theta_m}{c_{p1}} + \frac{L'_f}{c_{p1} T_{ref}} - \theta_m \right) + \frac{KW_2}{(\Delta \bar{x})^2} (\theta_{1,k,6} - \theta_{1+1,k,6}) + \frac{KW_1}{(\Delta \bar{x})^2} (\theta_{1-1,k,6} - \theta_{1,k,6}) - \frac{KW_3}{(\Delta \bar{y}_6)^2} (\theta_{1,k,6} - \theta_{1+1,k,6}) - \frac{KW_4}{(\Delta \bar{y}_6)^2} (\theta_{1,k,6} - \theta_{1-1,k,6}) \right)
\end{align*}
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\[ H_{1,k,6} \geq H_{1m} \quad (41a) \]

At the centerline of the heater:

\[ i=n, \quad H_{n,k,6} = \left\{ 1 + \frac{\Delta t}{2L^2} \left[ \frac{KW_1 + KW_2}{(\Delta x)^2} + \frac{KW_3 + KW_4}{(\Delta y)^2} \right] \right\}^{-1} \]

\[ \times \left\{ H_{n,k,6}^\Delta \frac{\Delta t}{2L^2} \left[ \frac{KW_1 + KW_2}{(\Delta x)^2} + \frac{KW_3 + KW_4}{(\Delta y)^2} \right] \left( C_{ps} \theta_m + \frac{L_f}{C_{pl}} \right) - \theta_m \right\} \]

\[ + \frac{2KW_1}{(\Delta x)^2} \left( \theta_{n-1,k,6}^\Delta - \theta_n^\Delta \right) + \frac{KW_3}{(\Delta y)^2} \left( \theta_{n,k+1,6}^\Delta - \theta_n^\Delta \right) + \frac{KW_4}{(\Delta y)^2} \left( \theta_{n,k-1,6}^\Delta - \theta_n^\Delta \right) \]

\[ + \frac{KW_3}{(\Delta y)^2} \left( \theta_{n,k+1,6}^\Delta - \theta_n^\Delta \right) - \frac{KW_4}{(\Delta y)^2} \left( \theta_{n,k-1,6}^\Delta - \theta_n^\Delta \right) \]} \]

\[ H_{n,k,6} \geq H_{1m} \quad (41b) \]

with

\[ \theta_{i,k,6}^\Delta = \frac{H_{i,k,6}}{T_{ref} \rho_1 C_{pl}} - \frac{1}{C_{ps}} \left( \theta_m + \frac{L_f}{C_{pl}} \right) + \theta_m \quad i=1, \ldots, n \]

\[ a. \quad Abrasion-Shield-Ice Interface \]

Let \((i, k)\) be one of the abrasion shield-ice interface nodes as shown in Figure 3c \(j = 5\). Substitution of analog equation (20) into equation (13b) with the aid of analog equation (13a) and with \(j = 5\) yields:
\[\theta_{i,k,5} = \theta_{i,k,6} \quad \theta_{i,k,5}^\Delta = \theta_{i,k,6}^\Delta \tag{42a}\]

and
\[-K_5 \left( \frac{\theta_{i,k+1,5} - \theta_{i,k-1,5} + \theta_{i,k+1,5}^\Delta + \theta_{i,k-1,5}^\Delta}{4 \Delta \overline{y}_5} \right) \]
\[-K_6 \left( \frac{\theta_{i,k+1,6} - \theta_{i,k-1,6} + \theta_{i,k+1,6}^\Delta + \theta_{i,k-1,6}^\Delta}{4 \Delta \overline{y}_6} \right) \tag{42b}\]

Equation (21b) with \(j = 5\) is used to eliminate the fictitious temperatures \(\theta_{i,k+1,5}^\Delta\) and \(\theta_{i,k+1,5}\), and equation (35) is used to eliminate fictitious temperature \(\theta_{i,k-1,6}^\Delta\) and \(\theta_{i,k-1,6}\). Combining the resulting equation with equations (12) and (42a) yields the following equations for the solid phase, the melting point and the liquid phase.

For the solid phase:

\[H_{i,k,6} = \frac{\Delta t}{C_p s \alpha_a} \frac{\Delta \overline{y}_5}{\alpha_5} + \frac{\Delta t}{2C_p s \alpha_a L^2} \left[ \frac{K_W}{\Delta \overline{y}_6^2} + \frac{2K_W}{(\Delta \overline{y}_6)^2} \right] + 1 \]

\[\left( \theta_{i,k,6}^\Delta - \theta_{i,k,6}^\Delta \right) = \frac{K_W}{\Delta \overline{y}_6^2} \left( \theta_{i,k,6}^\Delta - \theta_{i,k,6}^\Delta \right) + \frac{2K_W}{(\Delta \overline{y}_6)^2} \left( \theta_{i,k+1,6}^\Delta - \theta_{i,k+1,6}^\Delta \right) \]

\[i = 2, \ldots, n - 1 \quad H_{i,k,6} \leq H_{sm} \tag{43}\]
Substituting equation (22) into equation (43) gives

At the centerline of the gap:

\[
1=1, \quad H_{1,k,6} = \left( \frac{\Delta t K_5 \Delta y_5}{C_p \rho_s \Delta y_6 \Delta \alpha_s} + \frac{\Delta t}{2} \left[ \left( \frac{2K_{W_2}}{(\Delta \alpha)^2} + \frac{2K_{W_3}}{(\Delta \alpha)^2} \right) + 1 \right] \right) ^{-1} x + \frac{2K_5}{\Delta y_5 \Delta y_6} \left( \theta_{1,k-1,5}^\Delta + \theta_{1,k-1,5}^\Delta \right) + \frac{2K_{W_3} \theta_{2,k+1,6}^\Delta}{(\Delta \alpha_x)^2} \left( \theta_{2,k+1,6}^\Delta - \theta_{1,k,6}^\Delta \right) + \frac{2K_{W_3}^\Delta}{(\Delta \alpha_x)^2} \left( \theta_{1,k+1,6}^\Delta - \theta_{1,k,6}^\Delta \right)
\]

\[
H_{1,k,6} \leq H_{sm} \quad (44a)
\]

At the centerline of the heater:

\[
1=n, \quad H_{n,k,6} = \left( \frac{\Delta t K_5 \Delta y_5}{C_p \rho_s \Delta y_6 \Delta \alpha_s} + \frac{\Delta t}{2} \left[ \left( \frac{2K_{W_2}}{(\Delta \alpha)^2} + \frac{2K_{W_3}}{(\Delta \alpha)^2} \right) + 1 \right] \right) ^{-1} x + \frac{2K_5}{\Delta y_5 \Delta y_6} \left( \theta_{n-1,k,5}^\Delta + \theta_{n-1,k,5}^\Delta \right) + \frac{2K_{W_3} \theta_{n-1,k,6}^\Delta}{(\Delta \alpha_x)^2} \left( \theta_{n-1,k,6}^\Delta - \theta_{n-1,k,6}^\Delta \right) + \frac{2K_{W_3}^\Delta}{(\Delta \alpha_x)^2} \left( \theta_{1,k+1,6}^\Delta - \theta_{1,k,6}^\Delta \right)
\]

\[
H_{n,k,6} \leq H_{sm} \quad (44b)
\]

with \( \theta_{i,k,6} = H_{i,k,6}/C_p \rho_s T_{ref} \) for \( i = 1, \ldots, n \).
For the melting point:

\[ H_{1, k, 6} = H_{1, k, 6}^\Delta + \frac{\Delta t_{T\text{ref}}}{2L^2} \left( - \frac{2L^2 \bar{\Delta_Y} \bar{K}_5}{a_5 A_5 \bar{\Delta_Y}_6} + \frac{2KW_1}{(\bar{\Delta_X})^2} + \frac{2KW_3}{(\bar{\Delta_Y}_6)^2} \right) \theta_m \]

\[ + (\theta_{i+1, k, 5} + \theta_{i-1, k, 5} + \theta_{i+1, k, 5} + \theta_{i-1, k, 5}) \frac{K_5 \bar{\Delta_Y}_5}{\bar{\Delta_Y}_6 (\bar{\Delta_X})^2} + \frac{KW_2}{(\bar{\Delta_Y}_6)^2} \theta_{i+1, k, 6} \]

\[ + KW_{\Delta} \theta_{i-1, k, 6} + \frac{2KW}{\bar{\Delta_Y}_5 \bar{\Delta_Y}_6} (\theta_{i, k-1, 5} + \theta_{i, k-1, 5}) + \frac{KW_{\Delta}}{(\bar{\Delta_X})^2} \theta_{i, k-1, 6} - \]

\[ \frac{KW_{\Delta}}{(\bar{\Delta_Y}_6)^2} (\theta_{i, k, 6} - \theta_{i+1, k, 6}) + \frac{2KW_3}{\bar{\Delta_Y}_6} \theta_{i, k, 6} + \frac{2KW_{\Delta}}{(\bar{\Delta_Y}_6)^2} \theta_{i, k+1, 6} - \]

\[ \theta_{1, k, 6}^\Delta + \frac{2L^2 \bar{\Delta_Y} B_5 K_5}{a_5 \bar{\Delta_Y}_6} \theta_{1, k, 5} \]

\[ i = 2, \ldots, n - 1 \quad H_{sm} < H_{i, k, 6} < H_{lm} \quad (45) \]

Substitution of equation (22) into the above equation yields

At the centerline of the gap:

\[ i = 1, \quad H_{1, k, 6} = H_{1, k, 6}^\Delta + \frac{\Delta t_{T\text{ref}}}{2L^2} \left( - \frac{2L^2 \bar{\Delta_Y} \bar{K}_5}{a_5 A_5 \bar{\Delta_Y}_6} + \frac{2KW_2}{(\bar{\Delta_X})^2} + \frac{2KW_3}{(\bar{\Delta_Y}_6)^2} \right) \theta_m \]

\[ + \frac{2KW}{\bar{\Delta_Y}_5 \bar{\Delta_Y}_6} (\theta_{2, k, 5} + \theta_{2, k, 5}) + \frac{2KW_2}{(\bar{\Delta_X})^2} \theta_{2, k, 6} + \frac{2K_5}{\bar{\Delta_Y}_5 \bar{\Delta_Y}_6} (\theta_{1, k-1, 5} + \theta_{1, k-1, 5}) \]

\[ \frac{KW_{\Delta}}{(\bar{\Delta_Y}_6)^2} (\theta_{2, k, 6} - \theta_{1, k, 6}) + \frac{2KW_3}{(\bar{\Delta_Y}_6)^2} (\theta_{1, k+1, 6} - \theta_{1, k, 6}) \]

\[ + \frac{2KW_3}{(\bar{\Delta_Y}_6)^2} \theta_{1, k+1, 6} + \frac{2L^2 \bar{\Delta_Y} B_5 K_5}{a_5 \bar{\Delta_Y}_6} \theta_{1, k, 5} \]

\[ H_{sm} < H_{1, k, 6} < H_{lm} \quad (46a) \]
At the centerline of the heater:

\[ n = \Delta_n, k, 6 = H_n, k, 6 + \frac{\Delta t}{2L^2} \left( \frac{2L^2 K_5}{\alpha_5} + \frac{2KW_1}{\Delta x^2} + \frac{2KW_3}{\Delta y_6^2} \right) \theta_m \]

\[ + \frac{2K_5}{\Delta y_6 (\Delta x)^2} \left( \theta_{n-1, k, 5} + \theta_{n-1, k, 5}^\Delta \right) + \frac{2KW_1}{\Delta x^2} \left( \theta_{n-1, k, 6} + \frac{2K_5}{\Delta y_6} (\theta_{n, k-1, 5} - \theta_{n, k-1, 5}) \right) + \frac{2KW_3}{\Delta y_6^2} \left( \theta_{n, k-1, 6} - \theta_{n, k, 6} \right) \]

\[ + \frac{2KW_3}{\Delta y_6^2} \left( \theta_{n, k+1, 6} + \frac{2L^2 K_5}{\alpha_5} \frac{B_5 K_5}{\Delta y_6} \right) \theta_{n, k, 6} \]

\[ H_{on} < H_n, k, 6 < H_{in} \quad (46b) \]

with \( \theta_{i, k, 6} = \theta_m, \quad i = 1, \ldots, n \)

For the liquid phase:

\[ H_{i, k, 6} = \left( \frac{\Delta t}{C_{p1} \rho_1 \Delta y_6} + \frac{\Delta t}{2L^2 C_{p1} \rho_1} \right) \left( \frac{KW_1 + KW_3}{\Delta x^2} \right) + \left( \frac{KW_1}{\Delta x^2} \right) \left( \frac{KW_1}{\Delta y_6^2} \right) + \left( \frac{KW_1}{\Delta y_6^2} \right) \left( \theta_{i, k+1, 6} \right) \]

\[ + \left( \frac{KW_1}{\Delta y_6^2} \right) \left( \theta_{i, k, 6} - \theta_{i-1, k, 6} \right) - \left( \frac{KW_1}{\Delta x^2} \right) \left( \theta_{i, k, 6} - \theta_{i-1, k, 6} \right) + \frac{KW_3}{\Delta y_6^2} \left( \theta_{i, k, 6} - \theta_{i+1, k, 6} \right) \]

\[ + \frac{2KW_3}{\Delta y_6^2} \left( \theta_{i, k+1, 6} - \theta_{i, k, 6} \right) \]
\[
\left( \frac{1}{c_p} \left( \frac{L}{T_{ref}} \right) \right) + \frac{2L^2 \Delta \gamma_5 B_5 K_5}{a_5 \Delta \gamma_6} \theta_{i,k,5}
\]

\( i = 2, \ldots, n - 1 \quad H_{i,k,6} \geq H_{1,m} \quad (47) \)

Substituting equation (22) into equation (47) gives

At the centerline of the gap:

\( i = 1, \quad H_{1,k,6} = \frac{\Delta t}{c_p \rho_1 \Delta \gamma_6 A_5} + \frac{\Delta t}{2L^2 c_p \rho_1} \left[ \frac{2K_2}{\Delta \gamma_6^2} \right] + \frac{2K_3}{(\Delta \gamma_6^2) + 1} \)

\[ x \left( H_{1,k,6} + \frac{2K_5}{\Delta \gamma_5 \Delta \gamma_6} \right) \theta_{2,k,5} + \frac{\Delta t \Delta T_{ref}}{c_p \rho_1 \Delta \gamma_6} \left( \frac{2K_5}{\Delta \gamma_6^2} \right) \theta_{2,k,5} \]

\[ + \frac{2K_5}{\Delta \gamma_5 \Delta \gamma_6} \left( \theta_{1,k-1,5} + \theta_{1,k+1,5} \right) \]

\[ + \frac{2K_5}{\Delta \gamma_5 \Delta \gamma_6} \left( \theta_{1,k,5} \right) - \frac{2K_5}{\Delta \gamma_6^2} \frac{K_5}{A_5} \frac{2K_2}{\Delta \gamma_6^2} + \frac{2K_3}{(\Delta \gamma_6^2) + 1} \]

\[ \left( \frac{1}{c_p} \left( \frac{L}{T_{ref}} \right) \right) + \frac{2L^2 \Delta \gamma_5 B_5 K_5}{a_5 \Delta \gamma_6} \theta_{1,k,5} \quad (48a) \]

At the centerline of the heater:

\( i=n, \quad H_{n,k,5} = \frac{\Delta t}{c_p \rho_1 \Delta \gamma_6 A_5} + \frac{\Delta t}{2L^2 c_p \rho_1} \left[ \frac{2K_2}{\Delta \gamma_6^2} \right] + \frac{2K_3}{(\Delta \gamma_6^2) + 1} \)

\[ x \left( H_{n,k,6} + \frac{2K_5}{\Delta \gamma_5 \Delta \gamma_6} \theta_{n-1,k,5} + \theta_{n-1,k,5} \right) \]

\[ + \frac{\Delta t \Delta T_{ref}}{c_p \rho_1 \Delta \gamma_6} \left( \frac{2K_5}{\Delta \gamma_6^2} \right) \theta_{n-1,k,5} \]

\[ + \frac{2K_5}{\Delta \gamma_5 \Delta \gamma_6} \left( \theta_{n-1,k,5} \right) \]

\[ + \frac{2K_5}{\Delta \gamma_5 \Delta \gamma_6} \left( \theta_{n-1,k,5} \right) - \frac{2K_5}{\Delta \gamma_6^2} \frac{K_5}{A_5} \frac{2K_2}{\Delta \gamma_6^2} + \frac{2K_3}{(\Delta \gamma_6^2) + 1} \]

\[ \left( \frac{1}{c_p} \left( \frac{L}{T_{ref}} \right) \right) + \frac{2L^2 \Delta \gamma_5 B_5 K_5}{a_5 \Delta \gamma_6} \theta_{n-1,k,5} \]
Let \((i, m)\) be one of the ice-ambient interface nodes as shown in Figure 4c. Substitution of analog equation (20) into equation (14b) yields:

\[
\Delta H_{n,k,6} = \frac{\theta_{i,k,6} - \theta_{i,m+1,6}}{\Delta y_6} \Delta y_6
\]

with \(\theta_{i,k,6} = \frac{H_{i,k,6}}{C_p l T_{ref}} - \frac{1}{C_p l} \left( C_p s \Delta T + \frac{L_f}{T_{ref}} \right) + \theta_{i,m+1,6} \), \(i=1, \ldots, n\)

b. Ice-Ambient Interface

Temperatures \(\theta_{i,m+1,6}\) and \(\theta_{i,m+1,6}\) are fictitious and are eliminated using equation (35) and employing the average thermal conductivity approach. Combining the resulting equation with equation (12) gives

For the solid phase:

\[
H_{i,m,6} = \left( 1 + \frac{\Delta T}{2L C_p s p_s^*} \left[ \frac{KW_1}{(\Delta \xi)^2} + \frac{KW_2}{(\Delta y_6)^2} + \frac{2h b_2 L}{\Delta y_6} \right] \right)^{-1} x
\]
\[
\begin{align*}
\{H_{i,m,6}^\Delta + \frac{\Delta t_{\text{ref}}}{2L^2} & \left[ \frac{KW_1^2 \theta_{i+1,m,6} + KW_1^4 \theta_{i-1,m,6}}{(\Delta \mu)^2} + \frac{2KW_4^2 \theta_{i,m-1,6}}{(\Delta Y_6)^2} \right. \\
& \left. + \frac{KW_1^2}{(\Delta \mu)^2} \left( \theta_{i-1,m,6} - \theta_{i,m,6} \right) - \frac{KW_2^2}{(\Delta \mu)^2} \left( \theta_{i,m,6} - \theta_{i+1,m,6} \right) - \frac{2KW_4^2}{(\Delta \mu)^2} \left( \theta_{i,m,6} - \theta_{i,m-1,6} \right) \right] \\
& \left(\theta_{i,m,6} - \theta_{i,m-1,6}^\Delta \right) + \frac{2h_{b2}L}{(\Delta Y_6)} \left(2 \theta_{g2} - \theta_{i,m,6}^\Delta \right) \} \\
& \quad \left(1 = 2, \ldots, n - 1 \right) \quad H_{1,m,6} \leq H_{\text{sm}} \\
\end{align*}
\]

Substitution of equation (22) into the above equation yields

At the centerline of the gap:

\[
\begin{align*}
\{H_{1,m,6}^\Delta + \frac{\Delta t_{\text{ref}}}{2L^2} & \left[ \frac{KW_1^2 \theta_{2,m,6} + 2KW_4^2 \theta_{1,m-1,6}}{(\Delta \mu)^2} + \frac{2KW_4^2}{(\Delta \mu)^2} \left( \theta_{2,m,6} - \theta_{1,m,6} \right) \right. \\
& \left. - \frac{KW_2^2}{(\Delta \mu)^2} \left( \theta_{1,m,6} - \theta_{1,m-1,6} \right) + \frac{2h_{b2}L}{(\Delta Y_6)} \left(2 \theta_{g2} - \theta_{1,m,6}^\Delta \right) \} \\
& \quad \left(1 = 2, \ldots, n - 1 \right) \quad H_{1,m,6} \leq H_{\text{sm}} \\
\end{align*}
\]

At the centerline of the heater:

\[
\begin{align*}
\{H_{n,m,6}^\Delta + \frac{\Delta t_{\text{ref}}}{2L^2} & \left[ \frac{KW_1^2 \theta_{n-1,m,6} + 2KW_4^2 \theta_{n,m-1,6}}{(\Delta \mu)^2} + \frac{2KW_4^2}{(\Delta \mu)^2} \left( \theta_{n,m,6} - \theta_{n,m-1,6} \right) \right. \\
& \left. - \frac{KW_2^2}{(\Delta \mu)^2} \left( \theta_{n,m,6} - \theta_{n,m-1,6} \right) + \frac{2h_{b2}L}{(\Delta Y_6)} \left(2 \theta_{g2} - \theta_{1,m,6}^\Delta \right) \} \\
& \quad \left(1 = 2, \ldots, n - 1 \right) \quad H_{1,m,6} \leq H_{\text{sm}} \\
\end{align*}
\]
ORIGINALLY MADE OF POOR QUALITY.

\[
\frac{2KW_1}{\Delta x} \left( \frac{\Delta}{\Delta x} n-1, m, 6 - \Delta \frac{\Delta}{\Delta x} n, m, 6 \right) - \frac{2KW_4}{\Delta y_6} \left( \frac{\Delta}{\Delta y_6} n, m, 6 - \Delta \frac{\Delta}{\Delta y_6} n, m-1, 6 \right) +
\]

\[
\frac{2h_bL}{\Delta y_6} \left( 2 \Delta \frac{\Delta}{\Delta y_6} n, m, 6 \right)
\]

\[
H_{n, m, 6} \leq H_{sm}
\]

For the melting point:

\[
H_{i, m, 6} = H_{i, m, 6}^\Delta - \frac{\Delta t_{ref}}{2L} \left( \frac{KW_1 + KW_2}{\Delta x^2} + \frac{2KW_4}{\Delta y_6^2} + \frac{2h_bL}{\Delta y_6} \right) \theta_m
\]

\[
= \frac{KW_2}{\Delta x^2} \theta_{i+1, m, 6} + KW_1 \theta_{i-1, m, 6} - \frac{2KW_4}{\Delta y_6^2} \theta_{i, m-1, 6} - \frac{KW_1}{\Delta x^2} \theta_{i, m, 6} + \frac{2h_bL}{\Delta y_6} \theta_{i-1, m, 6}
\]

\[
- \theta_{i, m, 6}^\Delta \right) + \frac{KW_2}{\Delta x^2} \theta_{i, m, 6} - \theta_{i+1, m, 6} + \frac{2KW_4}{\Delta y_6^2} \theta_{i, m, 6} - \theta_{i, m-1, 6}
\]

\[
- \frac{2h_bL}{\Delta y_6} \left( 2 \Delta \frac{\Delta}{\Delta y_6} n, m, 6 \right)
\]

\[
i = 2, \ldots, n - 1 \quad H_{sm} < H_{i, m, 6} \leq H_{lm}
\]

Substituting equation (22) into equation (52) gives

At the centerline of the gap:

\[
i = 1, \quad H_{1, m, 6} = H_{1, m, 6}^\Delta - \frac{\Delta t_{ref}}{2L} \left( \frac{KW_2}{\Delta x^2} + \frac{2KW_4}{\Delta y_6^2} + \frac{2h_bL}{\Delta y_6} \right) \theta_m
\]

\[
- \frac{2KW_2}{\Delta x^2} \theta_{2, m, 6} - \frac{2KW_4}{\Delta y_6^2} \theta_{1, m-1, 6} - \frac{2KW_2}{\Delta x^2} \theta_{2, m, 6} - \theta_{1, m, 6}
\]

43
\[
\frac{2KW_4}{(\Delta y_6)^2} (\theta_{1,m,6} - \theta_{1,m-1,6}) - \frac{2hb_2L}{\Delta y_6} (2 \theta_2 - \theta_{1,m,6})
\]

\[H_{im} < H_{1,m,6} < H_{im}\]  

(53a)

At the centerline of the heater:

\[i = n, \quad H_{n,m,6} = H_n^\Delta \quad \frac{\Delta t_T}{2L} \left( \frac{2KW_1}{(\Delta x)^2} + \frac{2KW_4}{(\Delta y_6)^2} \right) \theta_m\]

\[\frac{2KW_1}{(\Delta x)^2} (\theta_{n-1,m,6} - \theta_{n,m,6} - \theta_{n,m-1,6}) - \frac{2KW_4}{(\Delta y_6)^2} (\theta_{n-1,m,6} - \theta_{n,m,6})\]

\[\frac{2KW_4}{(\Delta y_6)^2} (\theta_{n,m,6} - \theta_{n,m-1,6}) - \frac{2hb_2L}{\Delta y_6} (2 \theta_2 - \theta_{n,m,6})\]

\[H_{im} < H_{n,m,6} < H_{im}\]  

(53b)

For the liquid phase:

\[H_{i,m,6} = 1 + \frac{\Delta t}{2L C_p_1 \rho_1} \left( \frac{KW_1 + KW_2}{(\Delta x)^2} + \frac{2KW_4}{(\Delta y_6)^2} + \frac{4hb_2L}{(\Delta y_6)^2} \right) \theta_m - \frac{1}{C_p_1} \left( \frac{L_f}{T_{ref}} \right)\]

\[\frac{1}{C_p_1} \left( \frac{KW_2 \theta_{i+1,m,6} + KW_1 \theta_{i-1,m,6}}{\Delta x} \right) - \frac{1}{(\Delta x)^2} (KW_2 \theta_{i+1,m,6} + KW_1 \theta_{i-1,m,6}) - \]

\[\frac{2KW_4}{(\Delta y_6)^2} (\theta_{i-1,m,6} - \theta_{i,m,6}) + \frac{KW_1}{(\Delta x)^2} \theta_{i-1,m,6} - \theta_{i,m,6}) + \frac{KW_2}{(\Delta x)^2} (\theta_{i,m,6} - \theta_{i-1,m,6}) - \frac{2hb_2L}{\Delta y_6} (2 \theta_2 - \theta_{i,m,6})\]
\( i = 2, \ldots, n-1, \ H_{i,m,6} \geq H_{1m} \) \hspace{1cm} (54)

Substitution of equation (22) into equation (54) yields,

At the centerline of the gap:

\[
\begin{align*}
  \frac{H_{1,m,6}}{\Delta} &= \frac{\Delta t}{2L^2} \frac{(2KW_4 \Delta)}{(\Delta y_6)^2} \left[ \left( \frac{2KW_2}{(\Delta x)^2} + \frac{2KW_4}{(\Delta y_6)^2} \right) \left[ \theta_m - \frac{1}{C_p} \right] \right] \\
  H_{1,m,6} &= \frac{\Delta t}{2L^2} \left[ \left( \frac{2KW_2}{(\Delta x)^2} + \frac{2KW_4}{(\Delta y_6)^2} \right) \left[ \theta_m - \frac{1}{C_p} \right] \right] \\
  H_{1,m,6} &\geq H_{1m} \hspace{1cm} (55a)
\end{align*}
\]

At the centerline of the heater:

\[
\begin{align*}
  \frac{H_{1,m,6}}{\Delta} &= \frac{\Delta t}{2L^2} \frac{(2KW_4 \Delta)}{(\Delta y_6)^2} \left[ \left( \frac{2KW_2}{(\Delta x)^2} + \frac{2KW_4}{(\Delta y_6)^2} \right) \left[ \theta_m - \frac{1}{C_p} \right] \right] \\
  H_{1,m,6} &= \frac{\Delta t}{2L^2} \left[ \left( \frac{2KW_2}{(\Delta x)^2} + \frac{2KW_4}{(\Delta y_6)^2} \right) \left[ \theta_m - \frac{1}{C_p} \right] \right] \\
  H_{1,m,6} &\geq H_{1m} \hspace{1cm} (55a)
\end{align*}
\]
\[
\begin{align*}
&- \frac{2KW_\Delta}{(\Delta x)^2} (\theta_{n-1,m,6}^\Delta - \theta_{n,m,6}^\Delta) + \frac{2KW_\Delta}{(\Delta y)^2} (\theta_{n,m,6}^\Delta - \theta_{n,m-1,6}^\Delta) \\
&- \frac{2h_{b2}L}{\delta y_6} (2 \theta_{82} - \theta_{n,m,6}^\Delta)
\end{align*}
\]

\[H_{n,m,6} \geq H_{1m}\] (53b)

The above equations are now ready to be solved numerically.
IV. NUMERICAL SOLUTION BY COMPUTER IMPLEMENTATION

A. Gauss-Seidel Point Iterative Method

The Crank-Nicolson finite differencing formulation used in this study results in a set of algebraic equations which have to be solved simultaneously at each time step. The matrix size of the system is usually so large that the inverse by any direct method requires an excessive amount of calculations, quite apart from any considerations of round-off error. Most important, in this study, is the fact that the material phase at any grid point within the ice-water layer must be determined during the calculations. Accordingly, the system of equations has to be solved by an iterative procedure. The Gauss-Seidel method was chosen to perform this iteration because of its desirable convergence properties.

The Gauss-Seidel method is started with an initial approximation which is improved by successive back substitutions. In particular, a first approximation is used to calculate a second approximation, the second approximation is then used to calculate the third, and so on until some convergence criterion is met. For example, equation (21b) can be rewritten in the following form:

\[
\theta_{i,k,j}^{N+1} = \frac{a_j}{2(L\Delta x)^2} \left( \theta_{i+1,k,j}^N + \theta_{i-1,k,j}^N + \theta_{i+1,k,j}^{\Delta} + \theta_{i-1,k,j}^{\Delta} \right)
\]
\[ + \frac{q_j}{2(\Delta y_j)^2} \left[ \theta_{i,k+1,j}^N + \frac{\theta_{i,k+1,j}^{N+1}}{\theta_{i,k-1,j}^N + \theta_{i,k+1,j}^A + \theta_{i,k-1,j}^A} \right] + \]

\[ B_j \frac{\theta_i^A}{\rho_j C_{pj} T_{ref}} \]

If \( \theta_{i,k,j}^N \) denotes the last estimate which is known for all values of \( i \) and \( k \), and \( \theta_{i,k,j}^{N+1} \) denotes the new value, then \( \theta_{i-1,k,j}^{N+1} \) and \( \theta_{i,k-1,j}^{N+1} \) will both have already been calculated before computing \( \theta_{i,k,j}^{N+1} \). All of the other difference equations can be rewritten in the same form as equation (56). These equations are successively solved over the whole grid system during each time step by making a series of passes until the difference between two successive pass values satisfies some specified small value, called the convergence criterion. A convergence criterion of 0.01% was used in this study. A complete discussion of the Gauss-Seidel method is given in References [40] and [41].

B. Numerical Program Algorithm

Figure 5 shows the flow chart of the computer program. The complete program is listed in the Appendix along with a sample input data file. The program can be used for both conventional English units or metric units by modifying the input-output formats. The program can be reduced to describe the one-dimensional thermal de-icer pad by setting the gap width equal to zero. A metric version of the program has been compiled and is available upon request.
Although the following modifications are not presently in the program, they could be easily incorporated: (1) time-varying ambient temperatures; (2) temperature-dependent heat transfer coefficients; (3) initial position-dependent temperature profiles; (4) imperfect interface contact at specified nodes; and (5) temperature-dependent properties, although this would require the use of the Kirchhoff temperature.
V. DISCUSSION OF RESULTS

The computer model formulated in this study has been designed to investigate the effects of a number of parameters on one and two-dimensional de-icer performance. De-icer performance is measured by the de-icing time, which is the time required to raise the ice-abrasion shield interface temperature to 32°F, and, if the phase change is considered, to melt a thin layer of ice. The remaining ice can then be shed as a result of the dynamic forces acting on the outer surface of the aircraft blade.

In order to test the resulting program and algorithm, Figure 6 shows the one-dimensional comparison of results between the present study and those of Campbell [11] (an approximate analytical solution), Stallabrass [5] and Baliga [4]. Layer thickness and layer materials correspond to a standard design de-icer pad and are shown in the inset of Figure 6. The temperature rise plotted on the ordinate is the difference between 32°F and the initial temperature. E.g., if the initial temperature of the de-icer pad is -8°F, then a temperature rise of 40°F is necessary to bring the abrasion shield-ice interface temperature to 32°F, which corresponds to a de-icing time of approximately 7 seconds. It can be seen that excellent agreement is achieved between the present program and the approximate analytical solution. The results of Stallabrass [5] and Baliga [4] show a slightly optimistic temperature rise beyond 5 seconds.
A number of parameters were investigated using the present program, and, wherever possible, the results are compared with the results of Stallabras [5], Baliga [4] and Marano [6] to illustrate de-icer performance. The one-dimensional model is the subject of Sections A through F below. The phase change is not considered in Sections A through H, whereas Sections I through L incorporate the Enthalpy method for the phase change simulation. For all of the following cases, unless otherwise specified, values of \( h_{b1} = 1 \text{ Btu/ft}^2\text{-hr-}^\circ\text{F} \) and \( h_{b2} = 10^6 \text{ Btu/ft}^2\text{-hr-}^\circ\text{F} \) were used. Material property data for this study are presented in Table 1.

A. Effect of Power Density

Figure 7 shows the results of the effect of heater power density on the de-icing time for various ambient temperatures. The power density for the heater ranged from 15 to 40 Watts/\text{in}^2. Good agreement is obtained with the results of Stallabras [5], Baliga [4] and Marano [6]. A slight variation between results occurs because of the more optimistic nature of Stallabras' and Baliga's results as indicated in Figure 6. Examination of Figure 7 reveals that the practical minimum power density for the de-icer is 25 Watts/\text{in}^2, which agrees with the observation made from tests both in the laboratory and on various helicopter blades [5]. For power densities on the order of 15 or 20 Watts/\text{in}^2, particularly at low ambient temperatures, de-icing times are quite long.
B. Effect of Heater Element Thickness

The heater element usually is made up either of a woven mat of wires and glass fibers, or of resistance ribbon. Woven mats are an order of magnitude thicker than resistance ribbon. Figure 8 presents a comparison with the results of Marano [6] for different thickness heaters. Curve 1 represents the response of a point (zero thickness) heater, which is an idealization and yields the best possible results. Curve 2 represents the response of a heater composed of resistance ribbon, and Curve 3 represents the response of a heater composed of a woven mat. It can be seen that good agreement between the two studies is obtained. In addition, Figure 8 shows that heater thickness does not drastically affect de-icer performance. The maximum difference between the de-icing times for the various thickness heaters shown is less than two seconds.

C. Effect of Insulation Thickness Ratio and Insulation Material

In order to obtain optimum de-icer performance, it is desired that a maximum amount of energy released from the heater be directed toward the ice layer. This can be accomplished by increasing the insulation thickness beneath the heater. The effect on energy transfer of increasing this thickness relative to the insulation thickness above the heater is shown in Figure 9. An insulation thickness of 0.010 inches was assumed for the outer epoxy/glass insulation layer for the present investigation. Figure 9 presents a comparison of de-icing times with the results of Baliga [4] for the effect of varying the insulation thickness ratio: inner insulation thickness/outer insulation thickness. Again, good agreement between the
studies is obtained. As shown in Figure 9, when the ratio is increased from 1 through 5 (Curve 1, 2, 3), the de-icing time decreases appreciably. It can be seen that increasing the inner insulation from 0.020" to 0.050" reduces the time required for a 45°F temperature rise from nearly 8.7 seconds to just over 6.9 seconds, a 21% improvement. It is also seen that increasing the insulation thickness ratio from 5 to 10 only has an appreciable effect for very low ambient temperatures (Curves 3 and 4).

Figure 10 shows a comparison with Baliga's results for the effect on the de-icing time when the epoxy/glass inner insulation layer having a thermal conductivity of 0.22 Btu/ft-hr-°F is replaced by a layer of Polytrifluorochloroethylene (KEL-F) of the same thickness but having a thermal conductivity of 0.04 Btu/ft-hr-°F. As shown in Figure 10, for a 45°F temperature rise, the de-icing time is reduced by 40% (Curves 1 and 3). Therefore, the use of an inner insulation material of very low thermal conductivity is recommended if it is also a good electrical insulator.

D. Effect of Substrate Material

An interesting result was obtained when a stainless steel substrate was substituted for the aluminum alloy substrate. It might be expected, because of the lower thermal conductivity and lower thermal diffusivity of the stainless steel, that the de-icing time would be reduced. However, the opposite was in fact found to be the case, as shown in Figure 11, which also contains Stallabrass' [5] results. This result is attributed to the higher thermal capacity per unit volume of the stainless steel.
E. Effect of Variable Heater Output

A number of time-dependent heater outputs have been investigated in this study. In this section, a ramp function heater output is presented and the temperature response at all interfacial nodes is determined. Figures 12a and b depict the temperature response for a point heat source with a ramp function output, 5 seconds on, 1 second off. These results are compared to a constant point heat source. In all cases the ambient temperature was -4°F. Both sources have an average power density of 25 Watts/in². As expected, the temperature at the various interfacial nodes drops as the heater is switched off, and rises after the heater is switched on. The temperature at the outer surface of the ice remains constant at -4°F. This is due to the large heat transfer coefficient at this surface. As indicated in Figures 12a and b, if heat is applied such that the heater is operated for a period of time until the ice-abrasion shield interface reaches 32°F, and then is switched off until another layer of ice forms on the blade surface, the total energy consumption for de-icing can be reduced. The computer program developed in this study has been tested with constant, ramp and sinusoidal heater outputs; however, it can be easily modified to accommodate any other type of variable heater output.

F. Effect of Initial Ice Layer Thickness

In order to determine what effect the ice layer thickness has on de-icer performance, an investigation was performed in which the initial ice layer thickness was varied from 0.10" to 0.50". Figure 13 shows the
results of this investigation, along with the corresponding results obtained by Marano [6]. It can be seen that the de-icing times for the thicker ice layers did not differ appreciably. However, for a thin ice layer (0.10") with a large convection rate at the ice-ambient interface, the de-icing time increased significantly. The reason for this unexpected result is that any ice layer acts somewhat as an "insulator" and therefore slows the passage of heat, thus allowing the supplied thermal energy to raise the abrasion shield-ice interface temperature. Consequently, for thicker initial layers of ice, this rate of temperature rise is relatively high, which results in shorter de-icing times. Conversely, for the case of very thin initial ice layers with high convection rates at the outer surface, the thermal energy supplied to the abrasion shield-ice interface is conducted through the ice layer and is convected to the ambient at high enough rates that the interface temperature rises more slowly, thus increasing the de-icing time.

G. Effect of Gap Width

Earlier attempts at analyzing transient heat flow in de-icer pads with only one-dimensional mathematical models proved to be inadequate for prediction purposes, as a 3 to 5 times greater energy requirement was needed in actual operation as versus that predicted. This difference has been attributed to the effect of two-dimensional heat flow, i.e., the effect of delayed heating over the gap between heater elements. The two-dimensional model of Figure 2 was programmed to investigate the effect of the width of the gap between legs of the heater element. Gap
widths of 0.010", 0.030", 0.050" and 0.070" were investigated. Figure 14 employed an insulation thickness ratio of 2, a heater of zero thickness, but having a width of 0.510", and a 0.010" stainless steel abrasion shield and a 0.100" substrate. The heat transfer coefficients used were $h_{b1} = 10 \text{ Btu/ft}^2\text{-hr-}^\circ\text{F}$ and $h_{b2} = 200 \text{ Btu/ft}^2\text{-hr-}^\circ\text{F}$. Figure 14 shows how much more time is required to achieve a given temperature rise over the gap than over the heater element. The results of Stallabrass [5] are also shown. It can be seen that to raise the temperature over the centerline of a 0.070" gap by 36°F requires 5.2 seconds more time than to raise the temperature over the centerline of the heater element by the same amount. The two-dimensional results of Stallabrass [5] are seen to be more optimistic than those obtained in the present study. Since Stallabrass used an explicit finite difference formulation which required small time and space increments in order to maintain stability and achieve accuracy, it is possible that excessive computer time prevented sufficient testing to find the increment sizes below which repetitive results were obtained. This would account for the difference in the results of the two studies, particularly at large gap widths. A 0.010" gap is seen to have an almost negligible effect on the abrasion shield-ice interface temperature rise.

H. Effect of Heat Transfer Coefficient

The magnitudes of the heat transfer coefficients, $h_{b1}$ and $h_{b2}$ of Figure 1, have a greater effect on de-icer performance than do any of the parameters previously discussed when the blade has a very thin initial ice layer at low ambient temperature. As shown in Table 2, to raise the
temperature over the centerline of a 0.070" gap by 36°F requires 11.3 seconds with heat transfer coefficient values of $h_{b1} = 10 \text{ Btu/ft}^2\cdot\text{hr}\cdot\text{oF}$ and $h_{b2} = 200 \text{ Btu/ft}^2\cdot\text{hr}\cdot\text{oF}$, and 14.2 seconds with $h_{b1} = 1 \text{ Btu/ft}^2\cdot\text{hr}\cdot\text{oF}$ and $h_{b2} = 10^6 \text{ Btu/ft}^2\cdot\text{hr}\cdot\text{oF}$. The initial ice layer thickness is 0.15".

The reason for this phenomenon is already described in the Section F, i.e., the heat is rapidly conducted away from the abrasion shield-ice interface for very thin ice layers, low ambient temperatures, and high heat transfer coefficients. It can be seen from Table 2 that the de-icing time does not change for a temperature rise of 10.8°F. However, for high temperature increases (lower ambient temperatures) and greater gap widths, the heat transfer coefficient effect is very evident.

I. Effect of Phase Change

In the earlier part of this study, the phase change in the ice layer has been neglected, i.e., no account has yet been made for the latent heat needed to melt the ice. The Enthalpy method is applied to simulate the phase change in the ice layer as described in Chapter III. Figure 15 shows the temperature response for the substrate, heater, abrasion shield-ice interface and ice-ambient interface in the two-dimensional de-icer pad. It can be seen that the abrasion shield-ice interface over the center of the gap requires more time (11 sec.) to achieve a temperature of 32°F than does the same interface over the center of the heater element (4.6 sec.).

Figure 16 shows a more detailed comparison with the results of Stallabrass [5] for the effect of the phase change on the abrasion shield-
ice interface temperature. This de-icer pad had a gap width of 0.050", a power density of 25 Watts/in², an initial equilibrium temperature of 23°F, and a stainless steel abrasion shield and substrate 0.010" thick, respectively. Two curves are shown in Figure 16: the interface temperature above the center of the heater element and that above the center of the gap, both plotted as a function of time. Excellent agreement is achieved between the present study and Stallabrass' results. Curves 1, 4 and 6 show the interface temperatures would behave if the latent heat of the ice were not taken into account, i.e., if no phase change took place. These curves show that the interface temperatures increase continuously. Curves 2, 3, 5 and 7 show the temperature response if the latent heat of the ice is taken into account, illustrating that a plateau becomes evident when the abrasion shield-ice interface temperature reaches 32°F. This plateau remains until enough heat has been transferred into the nodal volume to completely melt all of the ice in that volume, after which the interface temperature smoothly increases. However, this plateau is a consequence of the numerical solution technique, and in reality does not occur. As mentioned in Marano [6], the temperature response at the abrasion shield-ice interface and for any point in the ice layer has a very strong nodal dependence. If more than 90 nodes are used in the ice layer, the temperature response curves will not exhibit a plateau. However, such simulations consume a large amount of computer time, and the average temperatures obtained are identical with those resulting when a lesser number of nodes is used [6]. It can be seen from Figure 16 that the phase change does not occur over the center of the
gap until about 4.2 seconds for the present study, as versus 0.9 seconds over the center of the heater.

J. Effect of Abrasion Shield

The abrasion shield has sometimes been called the heat diffuser for it will diffuse or conduct the heat laterally, thus producing greater uniformity of the temperature distribution at the abrasion shield-ice interface. Table 3 employed the same basic de-icer pad as given in Figure 16, with the abrasion shield material being changed each time. Good agreement is shown in Table 3 as compared with the results of Stallabrass [5]. Case 1 is the case presented in Figure 16, i.e., a 0.010" stainless steel abrasion shield. The values tabulated are the de-icing times for the abrasion shield-ice interface to reach 32°F over the center of the heater element and over the center of the gap. Initial equilibrium temperature in all cases was 23°F and the power density was 25 Watts/in². Case 2 shows that replacing the thin stainless steel abrasion shield by the same thickness of a more highly conductive material (nickel) has little effect. This is true also of a 0.010" plating of nickel on 0.010" of stainless steel (Case 3). However, as the thickness of the more conductive material is increased, a great improvement is shown (compare Case 3 and Case 4). Case 5 shows that by locating the better thermal conductor beneath the poorer one, more heat flows parallel to the blade surface from the warmer to the colder regions. In Case 6, an additional insulating layer has been inserted between the two metal layers of Case 5. Although the de-icing time has been increased, the uniformity of the
heat distribution has been greatly improved. However, this process of improving lateral heat flow should not be taken too far because the added material will increase the thermal capacity of the system which in turn will increase the de-icing time.

Table 3 depicts the effect of increasing the thickness of a good conductor (nickel) from zero (Case 1) to 0.020" (Case 4). Further increase in the thickness of the nickel will yield further improvement in de-icing time over the gap before passing through a minimum; then the de-icing time will start to increase. Clearly, narrow heater gaps require a thinner abrasion shield as a heat diffuser to produce greater uniformity of surface temperature, while wider heater gaps require a thicker abrasion shield.

K. Effect of Ice Shedding

As soon as a thin layer of ice is melted, the ice layer plus any water which has formed is shed from the de-icer pad due to the aerodynamic and/or centrifugal forces acting on the outer surface of the composite blade. Once the heater has been turned off, a new ice layer may form. Figure 17 shows the temperature response of the de-icer pad for ice shedding for the first 20 second cycle. The heater was turned on for 10 seconds and then off for 10 seconds, and the ice was replaced every 20 seconds. The ice was shed when the interface over the heater gap reached 32°F and melted. Sharp decreases in the heater element temperatures are shown to occur at 6.8 seconds and at 10.0 seconds. The former decrease is from the ice being shed, and the latter is from the
heater being turned off. The abrasion shield surface temperature drops immediately from 32°F to -4°F when the ice is shed. Only the substrate temperature remains warm after 20 seconds. During ice shedding, the program time step had to be drastically reduced in order to accurately follow the abrupt changes in temperature which occur.

Figure 18 shows the temperature response for the second cycle, from 20 to 40 seconds. The temperature profiles are quite similar to those in Figure 17. The only difference is that, since the substrate temperature is initially hotter because of residual heat, the de-icing time is decreased to 6.2 seconds.

L. Effect of Refreezing

From the results of Section K, it would be interesting to see the composite blade temperature response if the ice layer could not be shed and refreezing occurs. As shown in Figure 19, the heater was turned on for 10 seconds and then was turned off. In this case, the water at the abrasion shield-ice interface begins to refreeze after 12.0 seconds, and has completely solidified after 19.2 seconds.
VI. CONCLUSIONS AND RECOMMENDATIONS

The two-dimensional numerical simulation model developed in this study for the electrothermal de-icer pad has been shown to predict accurate temperature profiles for several boundary conditions and thermal heat sources. Parametric studies using the simulation to investigate such practical composite blade design aspects as the effect due to different heating functions on the rate and depth of ice melting, the effect of the abrasion shield on lateral heat transfer, the effect of the phase change on de-icer performance, etc., have been completed. The results were found to agree comparatively well with previous numerical calculations by Stallabraser [5], Baliga [4] and Marano [6]. The simulation contains the following improvements over Stallabraser's [5] two-dimensional numerical technique for the electrothermal de-icer pad:

(1) The Crank-Nicolson implicit finite difference scheme was used instead of the explicit forward finite difference method. This decreased the total number of calculations and promoted accuracy and stability;

(2) The Enthalpy method was used to simulate the phase change in the ice layer;

(3) The simulation can apply to shedding and refreezing of the ice layer.

(4) In addition, the program can be easily modified to handle time and position-varying ambient temperatures, temperature-dependent heat transfer coefficients, and imperfect contact between layers at specified nodes.
The computer program was run on the University of Toledo IBM 4341 computer. A 20 x 29 mesh was used to represent the 6 layers of the composite body. Typical computing times for a total simulation time of 20 seconds were 11 minutes without the phase change and 18 minutes with the phase change.

Based on the progress thus far, it is recommended that the following additional work be carried out:

(1) A complete experimental study should be conducted on de-icer pads in order to provide an exhaustive examination of the assumptions used in the simulation model and on model accuracy. It was originally anticipated that such experimental data would be available from tests conducted in the NASA Lewis Icing Tunnel. Unfortunately, the experimental study, originally scheduled for Summer, 1982, has been delayed until summer, 1983. At that time, time important analysis should be carried out;

(2) In order to assess the effect of blade geometry on de-icer design, an analysis should be conducted using the true blade shape. This work is already underway at The University of Toledo, where a free-surface coordinate transformation technique is being investigated.

(3) Available model simulations which predict the rate of ice deposition on blade surfaces should be coupled with the internal blade heat transfer programs in order to simulate the total process of ice formation and removal.
APPENDIX

COMPLETE PROGRAM LISTING
AND
SAMPLE INPUT DATA FILE
NUMERICAL SIMULATION OF TWO-DIMENSIONAL HEAT TRANSFER
IN COMPOSITE BODIES WITH PHASE CHANGE.

THIS PROGRAM CAN CALCULATE THE TEMPERATURE PROFILE IN
A COMPOSITE SLAB WHICH HAS CONVECTIVE, CONSTANT TEMP-
ERATURE OR FIXED BOUNDARY CONDITIONS.

THE PROGRAM CAN ALSO BE USED FOR COMPOSITE BODY PROBLEMS
WITH CONSTANT OR VARIABLE HEAT SOURCES.

AK = THERMAL CONDUCTIVITIES OF LAYERS.
ALP = THERMAL DIFFUSIVITIES OF LAYERS.
AKL = THERMAL CONDUCTIVITY OF WATER.
ALPL = THERMAL DIFFUSIVITY OF WATER.
CA = CONSTANT.
CHANGE = FUNCTION PROGRAM TO CORRECTING THERMAL
CONDUCTIVITIES OF EACH NODE IN THE ICE LAYER
FOR PHASE CHANGE
CPS, CPL = SPECIFIC HEAT OF ICE AND WATER PER UNIT VOLUME.
DES, DEL = DENSITY OF ICE AND WATER.
DTAUI = INITIAL TIME STEP.
DTAUM = INTERMEDIATE TIME STEP.
DTAUF = FINAL TIME STEP.
DX = SPACING BETWEEN NODES IN THE X-DIRECTION OF THE
GRID.
DY = SPACING BETWEEN NODES IN THE Y-DIRECTION OF THE
GRID.
EL = LENGTH OF EACH LAYER.
HEAD = HEADINGS.
H1 = HEAT TRANSFER COEFF. AT LOWER BOUNDARY.
H2 = HEAT TRANSFER COEFF. AT UPPER BOUNDARY.
HLAM = LATENT HEAT OF ICE.
IBC1 = 1, IMPLIES TEMPERATURE IS CONSTANT AT X=1.
IBC1 = 2, IMPLIES CONVECTIVE HEAT TRANSFER AT X=1.
IBC2 = 1, IMPLIES TEMPERATURE IS CONSTANT AT X=2.
IBC2 = 2, IMPLIES CONVECTIVE HEAT TRANSFER AT X=2.
ICL = NUMBER OF CYCLE FOR SHEDDING ICE.
ICOUNT = COUNTER ON TIME STEP.
IFREQ = NUMBER OF TIME STEPS BETWEEN SUCCESSIVE
PRINTING OF THE TEMPERATURE PROFILE.
IG = 1, IMPLIES PHASE CHANGE IN ICE LAYER IS NOT
CONSIDERED.
IG = 2, IMPLIES PHASE CHANGE IN ICE LAYER IS
CONSIDERED.
IH = 1, IMPLIES NO HEAT SOURCE.
IH = 2, IMPLIES POINT HEAT SOURCE.
IH = 3, IMPLIES HEAT GENERATION WITHIN SLAB.
IHQ = 1, IMPLIES CONSTANT HEAT SOURCE.
IHQ = 2, IMPLIES STEP FUNCTION HEAT SOURCE.
INQ = 3, IMPLIES RAMP FUNCTION HEAT SOURCE.
IHQ = 4, IMPLIES SINE FUNCTION HEAT SOURCE.
IJ = SLAB WITHIN WHICH HEAT GENERATION OCCURS.
IRF = 1, IMPLIES REFREEZING ICE LAYER IS NOT
       CONSIDERED.
IRF = 2, IMPLIES REFREEZING ICE LAYER IS CONSIDERED.
ISH = 1, IMPLIES SHEDDING ICE LAYER IS NOT CONSIDERED.
ISH = 2, IMPLIES SHEDDING ICE LAYER IS CONSIDERED.
L = NUMBER OF LAYERS IN SLAB
L1 = LOWER SLAB NUMBER FOR POINT HEAT SOURCE.
L2 = UPPER SLAB NUMBER FOR POINT HEAT SOURCE.
M = NUMBER OF NODES IN THE Y-DIRECTION OF THE GRID
MN = INTERFACE NODE NUMBERS.
N = NUMBER OF NODES IN THE X-DIRECTION OF THE GRID
NISP = NUMBER OF TIME STEPS FOR WHICH INITIAL TIME
       STEP IS USED.
NISP = NUMBER OF TIME STEPS FOR WHICH INTERMEDIATE
       TIME STEP IS USED.
NFSP = NUMBER OF TIME STEPS FOR REFREEZING ICE LAYER.
NHSP = NUMBER OF TIME STEPS FOR SHEDDING AT 2ND CYCLE.
NTSP = NUMBER OF TIME STEPS FOR STOPPING THE PROGRAM.
NO = NUMBER OF NODES IN EACH LAYER IN THE Y-DIRECTION
     OF THE GRID
NO1 = LOWER NODE NUMBER FOR FINITE THICKNESS HEATER.
NO2 = UPPER NODE NUMBER FOR FINITE THICKNESS HEATER.
NODE = NODE AT WHICH POINT HEAT SOURCE IS APPLIED.
Q = POINT HEAT SOURCE WATTS/IN\(^2\)IN.
Q2 = VOLUMETRIC HEAT SOURCE WATTS/IN\(^3\)IN\(^3\)IN.
QHEAT2 = FUNCTION PROGRAM FOR STEP FUNCTION HEAT INPUT.
QHEAT3 = FUNCTION PROGRAM FOR RAMP FUNCTION HEAT INPUT.
QHEAT4 = FUNCTION PROGRAM FOR SINE FUNCTION HEAT INPUT.
QV = INPUT FOR VARIABLE HEAT SOURCE.
T = NON-DIMENSIONAL TEMPERATURE.
TE = TEMPERATURE.
TG1 = AMBIENT TEMPERATURE AT LOWER BOUNDARY OF SLAB.
TG2 = AMBIENT TEMPERATURE AT UPPER BOUNDARY OF SLAB.
TIN = INITIAL TEMPERATURE IN SLAB.
TLEN1 = TOTAL LENGTH OF THE SLAB IN THE Y-DIRECTION
        OF THE GRID
TLEN2 = WIDTH OF THE HEATER
TLEN3 = GAP WIDTH
TMP = ICE MELTING TEMPERATURE.
TOFF = OFF TIME OF STEP HEAT INPUT.
TON = ON TIME OF STEP HEAT INPUT.
TR = NON-DIMENSIONAL TEMPERATURE AT PREVIOUS
    TIME STEP
TREF = REFERENCE TEMPERATURE.
TX1 = CONSTANT TEMPERATURE AT LOWER SLAB BOUNDARY.
TX2 = CONSTANT TEMPERATURE AT UPPER SLAB BOUNDARY.

DATA IN/5/,IO/6/
DIMENSION HEAD(42,80),DIF(30,95),THOLD(30,95),CONST(30,95)
DIMENSION PHOLD(30,95),TOLD(30,95),H(30,95),HO(30,95)
DIMENSION T(30,95), TR(30,95), TO(30,95), TEO(30,95), FDIST(30,95)
DIMENSION ALP(7), CPD(7), NO(7), MM(7), DY(7), AK(7), EL(7), TE(30,95)
DOUBLE PRECISION E1, F1, G1, P1, P2
DOUBLE PRECISION A(7), B(7), C(7), D(7), E(7)
COMMON /ScAl/ HSMP, HLMP, AKS, AKL

C
C
C INPUT DATA

C
DO 5 I=1,20
READ(IN,700)(HEAD(I,J), J=1,80)
5 CONTINUE
READ(IN,701)L,N,M,TLEN1,TLEN2,TLEN3
C
DO 10 I=1,N
DO 10 J=1,M
PHOLD(I,J)=0.
TOLD(I,J)=0.
THOLD(I,J)=0.
CONST(I,J)=0.
FDIST(I,J)=0.
H(I,J)=0.
NO(I,J)=0.
T(I,J)=0.
TE(I,J)=0.
TO(I,J)=0.
7E0(I,J)=0.
TR(I,J)=0.
DIF(I,J)=C.
10 CONTINUE
DO 15 K=1,L
A(K)=0.
B(K)=0.
C(K)=0.
D(K)=0.
DY(K)=0.
E(K)=0.
CPD(K)=0.
ALP(K)=0.
AK(K)=0.
EL(K)=0.
DY(K)=0.
NO(K)=0.
MM(K)=0.
15 CONTINUE
C
C INPUT DATA

C
DO 20 I=21,24
READ(IN,700)(HEAD(I,J), J=1,80)
DO 25 K=1,L
READ(IN,702) NO(K), EL(K), AK(K), ALP(K)
25 CONTINUE
DO 30 I=25,27
30 READ(IN,700)(HEAD(I,J), J=1,80)
READ(IN,700) IH, NODE, L1, L2
READ(IN,704) IJ, NO1, NO2, IHQ
READ(IN,705) Q, TON, TOFF, QV
DO 35 I=28,30
35 READ(IN,700) (HEAD(I,J), J=1,80)
READ(IN,706) IBC1, IBC2
READ(IN,716) TX1, TX2
READ(IN,707) TG1, H1, TG2, H2
READ(IN,708) TIN, TREF
DO 40 I=31,33
40 READ(IN,700) (HEAD(I,J), J=1,80)
READ(IN,709) IG, HLAM, ALPL, AKL, DEL, CPL, DES, CPS
READ(IN,710) TMP
DO 45 I=34,36
45 READ(IN,700) (HEAD(I,J), J=1,80)
READ(IN,711) DTAUI, NISP, DTAUM, NMSP, DTAUF
READ(IN,712) IFREQ
READ(IN,713) ISH, IRF, NFSP, ICL, NHSP, NTSP
C
PRINT THE DATA
C
DO 50 I=1,20
50 WRITE(IO,775) (HEAD(I,J), J=1,80)
WRITE(IO,717) L, N, M, TLEN1, TLEN2, TLEN3
DO 55 I=21,24
55 WRITE(IO,775) (HEAD(I,J), J=1,80)
DO 60 K=1,L
WRITE(IO,702) NO(K), EL(K), AK(K), ALP(K)
60 CONTINUE
DO 65 I=25,27
65 WRITE(IO,775) (HEAD(I,J), J=1,80)
IF(IH.EQ.1) GO TO 70
IF(IH.EQ.2) GO TO 75
WRITE(IO,718) IJ, NO1, NO2
GO TO 80
70 WRITE(IO,719)
GO TO 90
75 WRITE(IO,720) NODE, L1, L2
80 IF(IHQ.NE.1) GO TO 85
WRITE(IO,721) Q
85 IF(IHQ.NE.1) WRITE(IO,722) TON, TOFF
IF(IHQ.EQ.1) WRITE(IO,724) QV
IF(IHQ.EQ.2) WRITE(IO,726) QV
IF(IHQ.EQ.3) WRITE(IO,728) QV
IF(IHQ.EQ.4) WRITE(IO,727) QV
90 CONTINUE
DO 95 I=28,30
95 WRITE(IO,775) (HEAD(I,J), J=1,80)
IF(IBC1.EQ.2) GO TO 100
WRITE(IO,729) TX1
GO TO 105
100 WRITE(IO,730) TG1, H1
105 IF(IBC2.EQ.2) GO TO 110
WRITE(IO,732) TX2
GO TO 115

68
110 WRITE(IO,734)TG2,H2
115 WRITE(IO,736)TIN,TREF
WRITE(IO,775)(HEAD(28,J),J=1,80)
  IF(IG.EQ.1)GO TO 120
WRITE(IO,738)
WRITE(IO,775)(HEAD(31,J),J=1,80)
WRITE(IO,775)(HEAD(30,J),J=1,80)
WRITE(IO,775)(HEAD(32,J),J=1,80)
WRITE(IO,775)(HEAD(33,J),J=1,30)
WRITE(IO,750)CPS
WRITE(IO,752)TMP
GO TO 125
120 WRITE(IO,753)
125 WRITE(IO,775)(HEAD(28,J),J=1,80)
DO 130 I=34,36
130 WRITE(IO,775)(HEAD(I,J),J=1,80)
WRITE(IO,754)DTAUI,NISP,DTAUM,NMSP,DTAUF
  IF(ISH.EQ.1)GO TO 135
WRITE(IO,756)
  IF(IRF.EQ.1)GO TO 135
WRITE(IO,758)NFSP,ICL,NHSP
135 WRITE(IO,760)IFREQ,NTSP
WRITE(IO,775)(HEAD(1,J),J=1,80)
WRITE(IO,762)
WRITE(IO,775)(HEAD(18,J),J=1,80)
C
C  INITIAL CONDITIONS
C
MJ=M
IRP=0
INCL=1
NIP=0
JSH=1
G2TIME=TG2/TREF
GITIME=TG1/TREF
TMP=TMP/TREF
X2TIME=TX2/TREF
X1TIME=TX1/TREF
TLEN1=TLEN1/12.
TLEN2=TLEN2/12.
TLEN3=TLEN3/12.
DO 140 J=1,L
140 EL(J)=EL(J)/12.
TIN=TIN/TREF
TON=TON/3600.
TOFF=TOFF/3600.
DTAUI=DTAUI/3600.
DTAUM=DTAUM/3600.
DTAUF=DTAUF/3600.
TAUI=TAUI/3600.
TAUF=TAUF/3600.
TAFI=TAFI/3600.
DO 150 I=1,N
DO 150 J=1,M
FDIST(I,J)=TIN
IF(TIN.LT.TMP) GO TO 145
H(I,J)=CPL*TREF*FDIST(I,J)+DEL*(CPS*TMP*TREF/DES+HLAM)-TMP*TREF*1CPL
145 H(I,J)=CPS*FDIST(I,J)*TREF
150 CONTINUE

C INTERFACE NODE NUMBERING
C
MM(I)=NO(I)
DO 160 I=2,L
MM(I)=MM(I-1)+NO(I)-1
160 CONTINUE

C
AKS=AK(L)
INC=MM(L-1)
DX=(TLEN2+TLEN3)/(N*TLEN1)

C SET UP GAP NODE
C
NODEH=IFIX(TLEN3/(DX*TLEN1))+1
NODEG=NODEH+1
IF(TLEN3.EQ.0.) GO TO 165
WRITE(IO,764)NODEG
GO TO 170
165 WRITE(IO,764)NODEH

C PRINT THE INITIAL TEMPERATURE PROFILE AND ENTHALPYS
C
170 DO 175 I=1,N
DO 175 J=1,M
175 T(I,J)=FDIST(I,J)
DTAU=0.
TAU=0.
WRITE(IO,766) TAU
DO 180 I=1,N
DO 180 J=1,M
180 TEO(I,J)=T(I,J)*TREF
WRITE(IO,768)((TEO(I,J),J=1,M),I=1,N)
HSMP=CPS*TMP*TREF
HLMP=DEL*(CPS*TMP*TREF/DES+HLAM)
HAFP=(HSMP+HLMP)/2.

C CALCULATION OF CONSTANTS
C
LA=0
DTAU=DTAUI
200 ICOUNT=1
MON=MJ-1
NIP=NIP+1

C DETERMINE THE TIME STEP
IF(NIP.EQ.NISP+1) DTAU=DTAUM
IF(JSH.EQ.3) GO TO 210
IF(NIP.EQ.NMSP+1) DTAU=DTAUF

210 IF(NIP.EQ.NHSP+1) DTAU=DTAUI
LA=LA+1
A1=1./DTAU
IF(IRF.EQ.1) GO TO 215
IF(NIP.EQ.NFSP) GO TO 687

C CONVERGENCE CRITERIA

215 EPSI=0.0001

C CALCULATION OF CONSTANTS

NOW=N-1
DO 220 K=1,L
DY(K)=EL(K)/(TLEN1*(NO(K)-1))
C(K)=ALP(K)/(2.*TLEN1*DX*DX)
D(K)=ALP(K)/(2.*TLEN1*DY(K)*DY(K))
A(K)=1./(A1+2.(*(C(K)+D(K)))
B(K)=A1-2.*((C(K)+D(K))
CPD(K)=AK(K)/ALP(K)

220 CONTINUE

NEW=L-1
IF(L.EQ.1) GO TO 230
DO 225 K=1,NEW
E(K)=1./(1./(D(K)*A(K))+AK(K)*DY(K)/AK(K)*DY(K+1)*A(K+1)*

225 CONTINUE

230 DO 240 I=1,N
DO 240 J=1,MJ
TO(I,J)=T(I,J)

240 CONTINUE

DO 250 I=1,N
DO 250 J=INC,MJ
HO(I,J)=H(I,J)

250 CONTINUE

TI=TLEN1*TLEN1
X1=1./(DX*DX)
YL1=DY(L)*DY(L)
YL2=DY(L-1)*DY(L-1)
E1=2.*TLEN1*DY(1)*DY(1)/(A(1)*ALP(1))
F1=4.*H1*TLEN1*DY(1)/AK(1)
G1=1./(E1+F1/2.)
P1=4.*H2*TLEN1*DY(L)/AK(L)
P2=1./(1./(D(L)*A(L))+P1/2.)
VAS=DTAU/(2.*TL*CPS)
VAL=DTAU/(2.*TL*CPL)
VA1=DTAU*(AK(L-1)/(2.*CPS*TL*DY(L)*DY(L-1))
VA2=DTAU*(AK(L-1)/(2.*CPL*TL*DY(L)*DY(L-1))
VA3=DTAU*TREF/(2.*TL)
VA4=AK(L-1)*DY(L-1)*X1/DY(L)
VA5=AK(L-1)/(DY(L-1)*DY(L))
VA9=2.*H2*TLEN1/DY(L)

HEAT TERM INCLUSION

IF(IHQ.EQ.1)Q1=Q
IF(IHQ.EQ.2)Q1=QHEAT2(TAU,DTAU,TON,TOFF,QV)
IF(IHQ.EQ.3)Q1=QHEAT3(TAU,DTAU,TON,TOFF,QV)
IF(IHQ.EQ.4)Q1=QHEAT4(TAU,DTAU,TON,TOFF,QV)

Q2=2.*3.4121*144.*Q1

CALCULATION OF CONSTANTS OF FINITE THICKNESS HEATER

HEAT=Q2*DY(IJ)*TLEN1*DY(IJ)*TLEN1/(TREF*AK(IJ)*EL(IJ))
FACT1=AK(IJ)*DY(IJ-1)/(AK(IJ-1)*DY(IJ))
FACT2=AK(IJ+1)*DY(IJ)/(AK(IJ)*DY(IJ+1))

CALCULATION OF NEW TIME

TAU=TAU+DTAU

IF(L.NE.1) GO TO 295

260 DO 270 I=1,N
    DO 270 J=2,MOW
    THOLD(I,J)=T(I,J)
270 CONTINUE

CONSTANT AMBIENT TEMPERATURE BOUNDARY CONDITIONS FOR ONE LAYER CASE ONLY

DO 275 I=1,N
    T(I,1)=X1TIME
    T(I,M)=X2TIME
275 CONTINUE

CALCULATION OF TEMPERATURES IN THE INTERIOR OF ONE LAYER CASE ONLY

DO 285 J=2,MOW
    T(1,J)=A(1)*(C(1)*2.*(T(2,J)+TO(2,J))+D(1)*(T(1,J+1)+T(1,J-1))
        +ITO(1,J)+TO(1,J-1))+B(1)*TO(I,J))
    DO 280 I=2,NOW
        T(I,J)=A(1)*(C(1)*(T(I+1,J)+T(I-1,J)+TO(I+1,J)+TO(I-1,J))+
            1D(I)*(T(I,J+1)+T(I,J-1)+TO(I,J+1)+TO(I,J-1))+B(I)*TO(I,J))
280 CONTINUE
    T(N,J)=A(1)*(C(1)*2.*(T(N-1,J)+TO(N-1,J))+D(1)*(T(N,J+1)+
        T(N,J-1)+TO(N,J+1)+TO(N,J-1))+B(1)*TO(N,J))
285 CONTINUE

DUMAX=0.

CHECK CONVERGENCE FOR CONSTANT TEMPERATURE BOUNDARY CONDITIONS OF ONE LAYER CASE ONLY

72
DO 290 I=1,N
DO 290 J=2,N
DIF(I,J)=ABS(T(I,J)-THOLD(I,J))
290 DUMAX=AMAX1(DIF(I,J),DUMAX)
GO TO 680
295 IF((IBC1.EQ.1).AND.(IBC2.EQ.1))GO TO 315

GAUSS-SEIDEL REITERATION

300 DO 305 I=1,N
DO 305 J=1,MJ
TOLD(I,J)=T(I,J)
305 CONTINUE

CALCULATION OF TEMPERATURES AT THE INNER-AMBIENT INTERFACE

T(1,1)=G1*(DY(1)*DY(1)*2.*(T(2,1)+TO(2,1))*X1+2.*(T(1,2)+
1TO(1,2))+(B(1)/D(1)-F1/2.)*TO(1,1)+F1*G1TIME)
DO 310 I=2,N
T(I,1)=G1*(DY(1)*DY(1)*(T(I+1,1)+T(I-1,1)+TO(I+1,1)+TO(I-1,1))
+X1+2.*(T(I,2)+TO(I,2))+(B(1)/D(1)-F1/2.)*TO(I,1)+F1*G1TIME)
310 CONTINUE
T(N,1)=G1*(DY(1)*DY(1)*2.*(T(N-1,1)+TO(N-1,1))*X1+2.*(T(N,2)+
1TO(N,2))+(B(1)/D(1)-F1/2.)*TO(N,1)+F1*G1TIME)
GO TO 330

CONVECTION BOUNDARY CONDITIONS FOR ONE-LAYER CASE ONLY

DO 325 I=1,N
T(I,1)=X1TIME
T(I,M)=X2TIME
325 CONTINUE

LIST=2

CALCULATION OF TEMPERATURES IN THE INTERIOR OF THE LAYER
AND AT THE INTERFACE BETWEEN LAYERS

DO 600 K=1,L
NN=MM(K)
NNEW=NN-1
INCI=INC+1
IF(IH.EQ.3) GO TO 335
GO TO 340
335 IF(NN.EQ.NNO2) GO TO 445
340 IF(LIST.GT.NNEW) GO TO 360
DO 355 J=LIST,NNEW
355 CONTINUE
IF(IG.EQ.1) GO TO 345
IF(J.EQ.INC1) GO TO 530
345 T(1,J)=A(K)*C(K)*2.*((T(2,J)+TO(2,J))+D(K)*T(1,J+1)+
 1*T(1,J-1)+TO(1,J+1)+TO(1,J-1))+B(K)*TO(1,J))
DO 350 I=2,NOW
T(I,J)=A(K)*C(K)*D(I-1,J)+D(I,J)+D(I-1,J)+B(K)*D(I,J)
350 CONTINUE
T(N,J)=A(K)*C(K)*2.*((T(N-1,J)+TO(N-1,J))+D(K)*T(N,J+1)+
 1*T(N,J-1)+TO(N,J+1)+TO(N,J-1))+B(K)*TO(N,J))
355 CONTINUE
360 LIST=NN+1
IF(IG.EQ.1) GO TO 375
IF(IH.EQ.1) GO TO 370
IF(IRF.EQ.1) GO TO 370
IF(IRP.EQ.1) GO TO 365
IF(NIP.GE.NFSP) GO TO 370
IF((JSH.EQ.2).AND.(NN.EQ.INC)) GO TO 610
GO TO 370
365 IF((JSH.EQ.3).AND.(NN.EQ.INC)) GO TO 610
370 IF(NN.EQ.INC) GO TO 475
375 IF(NN.EQ.NJ) GO TO 605
IF(IH.EQ.1) GO TO 380
IF(IH.EQ.2) GO TO 385
IF(IH.EQ.1) GO TO 395
380 IF(NN.EQ.NO1) GO TO 430
GO TO 395
385 IF(NN.EQ.NODE) GO TO 390
GO TO 395
390 IF(TLEN.EQ.0.) GO TO 415
395 T(I,NN)=E(K)*((DY(K)*DY(K)/(DX*DX)+DY(K)*AK(K+1)/
 1(DX*DX*AK(K)))**2.*((T(I-1,NN)+TO(I-1,NN))+2.*((T(I,NN-1)+TO(I,NN-1))+
 22.*AK(K+1)*DY(K)*T(I+1,NN)+TO(I,NN+1)))/(AK(K)*DY(K)+AK(K+1)*DY(K+1)))*TO(I,NN))
DO 410 I=2,NOW
IF(IH.EQ.2) GO TO 400
IF(IH.EQ.1) GO TO 405
IF(IH.EQ.3) GO TO 405
400 IF(NN.EQ.NODE).AND.(I.EQ.NODE)) GO TO 420
405 T(I,NN)=E(K)*((DY(K)*DY(K)/(DX*DX)+DY(K)*AK(K+1)/
 1(DX*DX*AK(K)))**2.*((T(I-1,NN)+TO(I-1,NN))+TO(I,NN-1)+TO(I,NN)+2.*
 2(T(I-1,NN)+TO(I-1,NN))+2.*AK(K+1)*DY(K)*T(I+1,NN)+TO(I,NN+1)))/(AK(K)*
 3(DY(K+1))+B(K)*D(K)+AK(K+1)*DY(K)+B(K+1)/(AK(K)*
 4(DY(K+1))+B(K)*D(K)+AK(K+1)*DY(K+1))+TO(I,NN)))
410 CONTINUE
T(N,NN)=E(K)*((DY(K)*DY(K)/(DX*DX)+DY(K)*AK(K+1)/
 1(DX*DX*AK(K)))**2.*((T(N-1,NN)+TO(N-1,NN))+2.*(T(N-1,NN)+
 2TO(N,NN-1)+2.*AK(K+1)*DY(K)*T(N,NN)+TO(N,NN+1)))/(AK(K)*
 3(DY(K+1))+B(K)*D(K)+AK(K+1)*DY(K)+B(K+1)/(AK(K)*
 4(DY(K+1))))*TO(N,NN))
GO TO 600
C
C POINT HEATER
C
74
415 \[ T(1, \text{NODE}) = \text{E(L1)} \ast ((\text{DY}(\text{L1}) \ast \text{DY}(\text{L1})) / (\text{DX} \ast \text{DX}) + \text{DY}(\text{L2}) \ast \text{DY}(\text{L1}) \ast \text{AK}(\text{L2})) / \\
1(\text{DX} \ast \text{DX} \ast \text{AK}(\text{L1})) + 2 \ast ((T(2, \text{NODE}) + \text{TO}(2, \text{NODE})) + 2 \ast (T(1, \text{NODE-1}) + \\
2 \ast \text{AK}(\text{L2}) \ast \text{DY}(\text{L1}) \ast (T(1, \text{NODE+1}) + \text{TO}(1, \text{NODE+1})))) / \\
3(\text{AK}(\text{L1}) \ast \text{DY}(\text{L2}) + \text{B(L1)} / \text{D(L1)} + \text{AK}(\text{L2}) \ast \text{DY}(\text{L1}) \ast \text{B(L2)}) / (\text{AK}(\text{L1}) \ast \\
4 \ast \text{DY}(\text{L2})) \ast \text{TO}(1, \text{NODE}) + 4 \ast \text{TLEN1} \ast \text{DY}(\text{L1}) \ast \text{Q1} \ast 3.4121 \ast 144 / \\
5(\text{TREF} \ast \text{AK}(\text{L1})) \] 
420 \[ \text{DO 425 I=NODEG, NOW} \\
T(I, \text{NODE}) = \text{E(L1)} \ast ((\text{DY}(\text{L1}) \ast \text{DY}(\text{L1})) / (\text{DX} \ast \text{DX}) + \text{DY}(\text{L2}) \ast \text{DY}(\text{L1}) \ast \text{AK}(\text{L2}) / \\
1(\text{DX} \ast \text{DX} \ast \text{AK}(\text{L1})) \ast (T(I+1, \text{NODE}) + \text{TO}(I+1, \text{NODE}) + \\
2 \ast \text{AK}(\text{L2}) \ast \text{DY}(\text{L1}) \ast (T(I, \text{NODE+1}) + \text{TO}(I, \text{NODE+1}))) / \\
3(\text{AK}(\text{L1}) \ast \text{DY}(\text{L2}) + \text{B(L1)} / \text{D(L1)} + \text{AK}(\text{L2}) \ast \text{DY}(\text{L1}) \ast \text{B(L2)}) / (\text{AK}(\text{L1}) \ast \\
4 \ast \text{DY}(\text{L2})) \ast \text{TO}(I, \text{NODE}) + 4 \ast \text{TLEN1} \ast \text{DY}(\text{L1}) \ast \text{Q1} \ast 3.4121 \ast 144 / \\
5(\text{TREF} \ast \text{AK}(\text{L1}))) \\
\text{CONTINUE} \] 
425 \[ \text{DO 430 I=NODEG, NOW} \\
T(N, \text{NODE}) = \text{E(L1)} \ast ((\text{DY}(\text{L1}) \ast \text{DY}(\text{L1})) / (\text{DX} \ast \text{DX}) + \text{DY}(\text{L2}) \ast \text{DY}(\text{L1}) \ast \text{AK}(\text{L2}) / \\
1(\text{DX} \ast \text{DX} \ast \text{AK}(\text{L1})) \ast (T(N-1, \text{NODE}) + \text{TO}(N-1, \text{NODE}) + \\
2 \ast \text{AK}(\text{L2}) \ast \text{DY}(\text{L1}) \ast (T(N, \text{NODE+1}) + \text{TO}(N, \text{NODE+1}))) / \\
3(\text{AK}(\text{L1}) \ast \text{DY}(\text{L2}) + \text{B(L1)} / \text{D(L1)} + \text{AK}(\text{L2}) \ast \text{DY}(\text{L1}) \ast \text{B(L2)}) / (\text{AK}(\text{L1}) \ast \\
4 \ast \text{DY}(\text{L2})) \ast \text{TO}(N, \text{NODE}) + 4 \ast \text{TLEN1} \ast \text{DY}(\text{L1}) \ast \text{Q1} \ast 3.4121 \ast 144 / \\
5(\text{TREF} \ast \text{AK}(\text{L1}))) \\
\text{GO TO 600} \] 

**FINITE THICKNESS HEATER**

430 \[ \text{IF(TLEN3.NE.0.) Q7=0.} \\
\text{IF(TLEN3.EQ.0.) Q7=HEAT} \\
T(1, \text{NO1}) = \text{E(IJ-1)} \ast ((\text{DY}(\text{IJ-1}) \ast \text{DY}(\text{IJ-1})) / (\text{DX} \ast \text{DX}) + \text{DY}(\text{IJ}) \ast \text{DY}(\text{IJ}) \ast \\
1 \ast \text{FACT1} / (\text{DX} \ast \text{DX})) \ast (T(1, \text{NO1}) + \text{TO}(2, \text{NO1})) + 2 \ast (T(1, \text{NO1-1}) + \\
2 \ast \text{FACT1} \ast (T(1, \text{NO1+1}) + \text{TO}(1, \text{NO1+1})) + \text{B(IJ-1)} / \text{D(IJ-1)} + \\
3 \ast \text{FACT1} / \text{D(IJ-1)}) \ast \text{TO}(1, \text{NO1}) + \text{TLEN3} \ast \text{DY}(\text{L1}) \ast \text{Q1} \ast 3.4121 \ast 144 / \\
5(\text{TREF} \ast \text{AK}(\text{L1}))) \\
\text{DO 435 I=2, NOW} \\
\text{IF(I.LT.NODEG) Q8=0.} \\
\text{IF(I.GE.NODEG) Q8=HEAT} \\
T(I, \text{NO1}) = \text{E(IJ-1)} \ast ((\text{DY}(\text{IJ-1}) \ast \text{DY}(\text{IJ-1})) / (\text{DX} \ast \text{DX}) + \text{DY}(\text{IJ}) \ast \text{DY}(\text{IJ}) \ast \\
1 \ast \text{FACT1} / (\text{DX} \ast \text{DX})) \ast (T(I+1, \text{NO1}) + \text{TO}(I+1, \text{NO1})) + 2 \ast (T(I, \text{NO1-1}) + \\
2 \ast \text{FACT1} \ast (T(I, \text{NO1+1}) + \text{TO}(I, \text{NO1+1})) + \text{B(IJ-1)} / \text{D(IJ-1)} + \\
3 \ast \text{FACT1} / \text{D(IJ-1)}) \ast \text{TO}(I, \text{NO1}) + \text{TLEN3} \ast \text{DY}(\text{L1}) \ast \text{Q1} \ast 3.4121 \ast 144 / \\
5(\text{TREF} \ast \text{AK}(\text{L1}))) \\
\text{CONTINUE} \] 
435 \[ \text{GO TO 600} \] 

**MORE THAN TWO NODES IN THE HEATER**

445 \[ \text{IF((NO2-NO1).EQ.1) GO TO 465} \] 

75
DO 455 I=2,N0W
IF(I.LT.NODEG) Q4=0.
IF(I.GE.NODEG) Q4=Q2
T(I,IL)=A(IJ)*(C(IJ)*(T(I+1,IL)+T(I-1,IL)+TO(I+1,IL)+TO(I-1,IL))+
1D(IJ)*(T(I+1,IL)+T(I-1,IL)+TO(I+1,IL)+TO(I-1,IL))+B(IJ)*TO(I,IL)+
2Q4/(EL(IJ)*TREF*2.*CPD(IJ)))
455 CONTINUE

T(N,IL)=A(IJ)*(C(IJ)*2.*(T(N-1,IL)+TO(N-1,IL))+D(IJ)*(T(N,IL+1)+
1T(N,IL-1)+TO(N,IL+1)+TO(N,IL-1))+B(IJ)*TO(N,IL)+Q2/(TREF*EL(IJ)*
22.*CPD(IJ)))
460 CONTINUE

IF(TLEN3.EQ.0.) QS=HEAT
IF(TLEN3.NE.0.) QS=Q5

T(1,NO2)=E(IJ)*((DY(IJ)*DY(IJ)/(DX*DX)+DY(IJ+1)*DY(IJ+1))*FACT2/
1(DX*DX))*2.*((T(1,NO2-1)+TO(1,NO2-1))+2.*FACT2*(T(1,NO2)+TO(1,NO2)))

3D(IJ+1)*TO(1,NO2)+Q5)
DO 470 I=2,N0W
IF(I.LT.NODEG) Q6=0.
IF(I.GE.NODEG) Q6=HEAT
T(I,NO2)=E(IJ)*((DY(IJ)*DY(IJ)/(DX*DX)+DY(IJ+1)*DY(IJ+1))*FACT2/
1(DX*DX))*2.*((T(I+1,NO2)+TO(I+1,NO2))+2.*FACT2*(T(I,NO2)+TO(I,NO2)))
3D(IJ)+B(IJ)*TO(I,NO2)+Q6)
470 CONTINUE

T(N,NO2)=E(IJ)*((DY(IJ)*DY(IJ)/(DX*DX)+DY(IJ+1)*DY(IJ+1))*FACT2/
1(DX*DX))*2.*((T(N-1,NO2)+TO(N-1,NO2))+2.*FACT2*(T(N,NO2)+TO(N,NO2)))
22.*FACT2*(T(N,NO2+1)+TO(N,NO2+1))+(B(IJ)/D(IJ)+B(IJ+1)*FACT2/
3D(IJ+1))*TO(N,NO2)+HEAT)
LIST=NN+1
GO TO 600

CALCULATION OF ENTHALPS AND TEMPERATURES AT THE SHIELD-WATER INTERFACE

IF(ICOUNT.NE.1) GO TO 480
CALL CHANGE(HO(2,INC),HO(1,INC),HO(2,INC),HO(1,INC),HO(1,INC),HO(1,INC),
1PO,AKO,AKO2,AKO3,AKO4)
CONST(1,INC)=AKO2*X1*2.*(TO(2,INC)-TO(1,INC))/YL1
480 CALL CHANGE(H(2,INC),H(1,INC),H(2,INC),H(1,INC),H(1,INC),H(1,INC),IP,
1AK1,AK2,AK3,AK4)
VA6=(TMP*TREF-DEL*(CPS*TMP*TREF/DIEL+HLAM))/TREF
VK1=(AK1+AK2)*X1
VK2=(AK3+AK4)/YL1
VK3=2.*AK3/YL1
VK4=2.*AK4/YL1
GO TO(483,485,487,IP)

ICE

H(1,INC)=1./((VA1/(D(L-1)*A(L-1)))+VAS*(VK1+VK3)+1.)*(HO(1,INC)+
1VA3*(T(2,INC)+TO(2,INC))+2.*VA4*X1*2.*AK2*T(2,INC)+2.*VAS*(
2T(1,INC-1)+TO(1,INC-1))+CONST(1,INC)+B(L-1)*VAS*
3TO(1,INC)/D(L-1)))
T(1,INC)=H(1,INC)/(CPS*TREF)
IF(H(1,INC).LE.HSMP) GO TO 490
IP=1
C MELTING POINT

485 H(1,INC)=HO(1,INC)+VA3*(-(VAS/(D(L-1)*A(L-1)))+VK1+VK3)*TMP+
      1T(1,INC)+TO(1,INC)+2.*VA4+X1*AK1*2.*T(1,INC)+2.*VAS*(T(1,INC-1)+
      2TO(1,INC-1))+VK3*T(1,INC+1)+CONST(1,INC)+B(L-1)*VAS*TO(1,INC)/
      3D(L-1))
      T(1,INC)=TMP
      IF(H(1,INC).GT.HSMP.AND.H(1,INC).LT.HLMP) GO TO 490
      IF(IP.EQ.1.OR.IP.EQ.3) GO TO 490

C LIQUID WATER

487 H(1,INC)=1./(VA2/(D(L-1)*A(L-1)))+VAL*(VK1+VK3)+1.)*(HO(1,INC)+
      1VA3*((T(2,INC)+TO(2,INC))*2.*VA4+X1*AK2*T(2,INC)+2.*VAS*(
      2T(1,INC-1)+TO(1,INC-1))+VK3*T(1,INC+1)+CONST(1,INC)-VAS/(D(L-1)*
      3A(L-1))+VK1+VK3)*VAS*TO(1,INC)/D(L-1)))
      T(1,INC)=H(1,INC)/(CPL*(TREF-CPL)-DEL*(CPS*TMP*TREF/DES+HLAM)/(CPL*TREF
      1)+TMP)
      IF(H(1,INC).GE.HLMP) GO TO 490
      IP=3
      GO TO 483

490 DO 510 I=2,NOW
      IF(ICOUNT.NE.1) GO TO 495
      CALL CHANGE(HO(I-1,INC),HO(I,INC),HO(I+1,INC),HO(I,INC1),
      1HO(I,INC),IPO,AK01,AK02,AK03,AK04)
      CONST(I,INC)=AK01*X1*(TO(I-1,INC)-TO(I,INC))-AK02*X1*TO(I,INC)+
      1TO(I+1,INC)+2.*AK03*(TO(I,INC+1)-TO(I,INC))/YL1
      CALL CHANGE(H(I-1,INC),H(I,INC),H(I+1,INC),H(I,INC1),H(I,INC),IP
      1AK1,AK2,AK3,AK4)
      VK1=(AK1+AK2)*X1
      VK2=(AK3+AK4)/YL1
      VK3=2.*AK3/YL1
      VK4=2.*AK4/YL1
      GO TO(503,505,507),IP

C ICE

503 H(I,INC)=1./(VA1/(D(L-1)*A(L-1)))+VAS*(VK1+VK3)+1.)*(HO(I,INC)+
      1VA3*((T(I+1,INC)+T(I-1,INC)+TO(I+1,INC)+TO(I-1,INC))*VA4+X1*(AK2*
      2T(I-1,INC)+AK1*T(I-1,INC))+2.*VAS*(T(I,INC-1)+TO(I,INC-1))+VK3*
      3T(I,INC+1)+CONST(I,INC)+B(L-1)*VAS*TO(I,INC)/D(L-1)))
      T(I,INC)=H(I,INC)/(CPS*TREF)
      IF(H(I,INC).LE.HSMP) GO TO 510
      IP=1

C MELTING POINT

505 H(I,INC)=HO(I,INC)+VA3*(-(VAS/(D(L-1)*A(L-1)))+VK1+VK3)*TMP+
      1T(I-1,INC)+T(I-1,INC)+TO(I-1,INC)+TO(I-1,INC)+TA4*X1*(AK2*
      2T(I-1,INC)+AK1*T(I-1,INC))+2.*VAS*(T(I,INC-1)+TO(I,INC-1))+VK3*
      3T(I,INC+1)+CONST(I,INC)+B(L-1)*VAS*TO(I,INC)/D(L-1))
      T(I,INC)=TMP
      IF(H(I,INC).GT.HSMP.AND.H(I,INC).LT.HLMP) GO TO 510
      IF(IP.EQ.1.OR.IP.EQ.3) GO TO 510

C LIQUID WATER

507 H(I,INC)=1./(VA2/(D(L-1)*A(L-1)))+VAL*(VK1+VK3)+1.)*(HO(I,INC)+VA3*
      1(T(I+1,INC)-T(I-1,INC)+TO(I+1,INC)+TO(I-1,INC))*VA4+X1*(AK2*
      2T(I+1,INC)+AK1*T(I-1,INC))+2.*VAS*(T(I,INC-1)+TO(I,INC-1))+VK3*
\[
3T(I,INC+1) + \text{CONST}(I,INC)(-VA5/(D(L-1)*A(L-1)) + VK1+VK3)*VA6 + B(L-1)\times\frac{\Partial}(I,INC)/D(L-1))
\]
\[
T(I,INC) = H(I,INC)/\left(\text{TREF}*\text{CPL} - \text{DE}\times(\text{CPS}*\text{TMP} + \text{DES} + \text{HLAM}) / (\text{CPL}^*\text{TREF})\right)\times\text{TMP}
\]
\[
\text{IF}(H(I,INC) < H_{\text{LMP}}) \text{ GO TO 510}
\]
\[
\text{IP} = 3
\]
\[
\text{GO TO 503}
\]
\[
510 \text{ CONTINUE}
\]
\[
\text{IF}(\text{ICOUNT} \neq 1) \text{ GO TO 520}
\]
\[
\text{CALL CHANGE}(\text{HO}(N-1,INC), \text{HO}(N,INC), \text{HO}(N-1,INC), \text{HO}(N,INC), \text{HO}(N,INC), \text{HO}(N,INC), \text{IP}, 1, \text{AKO1}, \text{AKO2}, \text{AKO3}, \text{AKO4})
\]
\[
\text{CONST}(N,INC) = \text{AKO1} \times X1^* \left(\frac{\text{TO}(N-1,INC) - \text{TO}(N-1,INC)}{\text{T(N-1,INC)} + \text{TO}(N-1,INC)}\right) + 2 \times \text{AKO3} \times \left(\frac{\text{TON}(N-1,INC) - \text{TON}(N-1,INC)}{\text{TON}(N-1,INC) + \text{TO}(N-1,INC)}\right)
\]
\[
\text{VK1} = (\text{AK1} + \text{AK2}) \times X1
\]
\[
\text{VK2} = (\text{AK3} + \text{AK4}) / \text{YL1}
\]
\[
\text{VK3} = 2 \times \text{AK3} / \text{YL1}
\]
\[
\text{VK4} = 2 \times \text{AK4} / \text{YL1}
\]
\[
\text{GO TO (523, 525, 527), IP}
\]
\[
\text{ICE}
\]
\[
523 \text{ H(N,INC)} = (\text{VA1}/(\text{D}(L-1) \times \text{B}(L-1)) + \text{VAS} \times (\text{VK1} - \text{VK3}) + 4 \times (\text{VAK4} + \text{X1} \times \text{AK1} + \text{T}(N-1,INC) + \text{TO}(N-1,INC)) / (\text{D}(L-1) \times \text{A}(L-1)) + \text{VAS} \times (\text{VK3} + \text{T}(N,INC+1)) + \text{CONST}(N,INC) + \text{B}(L-1) \times \text{VAS} \times \text{TO}(N,INC) / (\text{D}(L-1))
\]
\[
\text{T(N,INC)} = \text{H(N,INC)}/\left(\text{TREF}*\text{CPL} - \text{DE} \times (\text{CPS} \times \text{TMP} + \text{DES} + \text{HLAM}) / (\text{CPL}^*\text{TREF})\right)
\]
\[
\text{IF}(\text{H(N,INC)}. \text{GE}. \text{HLMP}) \text{ GO TO 510}
\]
\[
\text{IP} = 3
\]
\[
\text{GO TO 503}
\]
\[
525 \text{ MELTING POINT}
\]
\[
525 \text{ H(N,INC) = HO(N,INC) + VA3* (-VA5/(D(L-1)*A(L-1)) + VK1 + VK3)*TM} + (1 \times (\text{TON}(N-1,INC) + \text{TO}(N-1,INC)) \times \text{AK1} + \text{T}(N-1,INC) + 2 \times \text{VAS} \times (2 \times (\text{TON}(N,INC+1) + \text{TO}(N,INC+1)) + \text{CONST}(N,INC) + \text{B}(L-1) \times \text{VAS} \times \text{TO}(N,INC) / (\text{D}(L-1))
\]
\[
\text{T(N,INC) = TMP}
\]
\[
\text{IF}(\text{H(N,INC) \text{GE.HSMP. AND.H(N,INC) \text{LT.HLMP}) GO TO 600}
\]
\[
\text{IF}(\text{IP.EQ.1.OR.IP.EQ.3}) \text{ GO TO 600}
\]
\[
\text{LIQUID WATER}
\]
\[
527 \text{ H(N,INC) = HO(N,INC) + VA3* (-VA5/(D(L-1)*A(L-1)) + VK1 + VK3)*TM} + (1 \times (\text{TON}(N-1,INC) + \text{TO}(N-1,INC)) \times \text{AK1} + \text{T}(N-1,INC) + 2 \times \text{VAS} \times (2 \times (\text{TON}(N,INC+1) + \text{TO}(N,INC+1)) + \text{CONST}(N,INC) + \text{B}(L-1) \times \text{VAS} \times \text{TO}(N,INC) / (\text{D}(L-1))
\]
\[
\text{T(N,INC) = TMP}
\]
\[
\text{IF}(\text{H(N,INC). \text{GE}. \text{HLMP}) \text{ GO TO 600}
\]
\[
\text{IP} = 3
\]
\[
\text{GO TO 523}
\]
\[
\text{CALCULATION OF ENTHALPY AND TEMPERATURES IN THE INTERIOR OF THE ICE-WATER LAYER}
\]
\[
\text{DO 590 J = INC1, MOW}
\]
\[
\text{IF}(\text{ICOUNT} \neq 1) \text{ GO TO 535}
\]
\[
\text{CALL CHANGE} \left(\text{HO}(2,J), \text{HO}(1,J), \text{HO}(2,J), \text{HO}(2,J), \text{HO}(1,J-1), \text{IP}, 1, \text{AKO1}, \text{AKO2}, \text{AKO3}, \text{AKO4}) \right)
\]

78
CONST(1,J)=AKO2*X1*2.*X1*2.*(TO(2,J)-TO(1,J))+AKO3*X1*(TO(1,J+1)-TO(1,J))
1-AKO4*(TO(1,J)-TO(1,J-1))/YL1
535 CALL CHANGE(H(2,J),H(1,J),H(2,J),H(1,J+1),H(1,J-1),IP,AK1,AK2,
1AK3,AK4)
VK1=(AK1+AK2)*X1
VK2=(AK3+AK4)/YL1
VK3=2.*AK3/YL1
VK4=2.*AK4/YL1
GO TO(543,545,547),IP
C ICE
543 H(1,J)=1./(1.+VAS*(VK1+VK2))**(HO(1,J)+VA3*(X1*2.*AK2*T(2,J)+
1AK3*T(1,J+1)+AK4*T(1,J))/YL1+CONST(1,J)))
T(1,J)=H(1,J)/(CPS*TREF)
IF(H(1,J).LE.HSMP) GO TO 550
IP=1
C MELTING POINT
545 H(1,J)=HO(1,J)+VA3*(-(VK1+VK2)*TMP+X1*2.*AK2*T(2,J)+(AK3*T(1,J+1)+
1AK4*T(1,J-1))/YL1+CONST(1,J)))
T(1,J)=TMP
IF(H(1,J).GT.HSMP.AND.H(1,J).LT.HLMP) GO TO 550
IF(IP.EQ.1.OR.IP.EQ.3) GO TO 550
C LIQUID WATER
547 H(1,J)=1./(1.+VAL*(VK1+VK2))**(HO(1,J)+VA3*(-(VK1+VK2)*VA6+X1*12.*AK2*T(2,J)+
1AK3*T(1,J+1)+AK4*T(1,J-1))/YL1+CONST(1,J)))
T(1,J)=H(1,J)/(CPL*TREF)-DEL*(CPS*TREF/DES+HLAM)/(CPL*TREF)+
1TMP
IF(H(1,J).GE.HLMP) GO TO 550
IP=3
GO TO 543
550 DO 570 I=2,NOW
IF(ICOUNT.NE.1) GO TO 555
CALL CHANGE(HO(I-1,J),HO(I,J),HO(I+1,J),HO(I,J+1),HO(I,J-1),
1IPO,AKO1,AKO2,AKO3,AKO4)
CONST(I,J)=AKO1*X1*(TO(I-1,J)-TO(I,J))-AKO2*X1*(TO(I,J)-TO(I+1,J))
1+(AKO3*(TO(I,J+1)-TO(I,J))-AKO4*(TO(I,J)-TO(I,J-1)))/YL1
555 CALL CHANGE(H(I-1,J),H(I,J),H(I+1,J),H(I,J+1),H(I,J-1),IP,
1AK1,AK2,AK3,AK4)
VK1=(AK1+AK2)*X1
VK2=(AK3+AK4)/YL1
VK3=2.*AK3/YL1
VK4=2.*AK4/YL1
GO TO(563,565,567),IP
C ICE
563 H(I,J)=1./(1.+VAS*(VK1+VK2))**(HO(I,J)+VA3*(X1*(AK2*T(I+1,J)+AK1*
1T(I-1,J))+AK3*T(I,J+1)+AK4*T(I,J-1))/YL1+CONST(I,J)))
T(I,J)=H(I,J)/(CPS*TREF)
IF(H(I,J).LE.HSMP) GO TO 570
IP=1
C MELTING POINT
565 H(I,J)=HO(I,J)+VA3*(-(VK1+VK2)*TMP+X1*(AK2*T(I+1,J)+AK1*T(I-1,J)+
1AK3*T(I,J+1)+AK4*T(I,J-1))/YL1+CONST(I,J))
T(I,J)=TMP
IF(H(I,J).GT.HSMP.AND.H(I,J).LT.HLMP) GO TO 570
IF(IP.EQ.1.OR.IP.EQ.3) GO TO 570
LIQUID WATER

\[ H(I,J) = \frac{1}{(1 + VA(I) \cdot (VK1 + VK2)) \cdot (HO(I,J) \cdot VA3 \cdot (-VK1 + 1) + VA6 \cdot X1 \cdot (AK2 \cdot T(I+1,J) + AK1 \cdot T(I-1,J) + AK3 \cdot T(I,J+1) + AK4 \cdot T(I,J-1))/YL1 + CONST(I,J))} \]

\[ T(I,J) = H(I,J)/(TREF*CPL) - DEL*(CPS*TMP*TREF/DES+HLAM)/(CPL*TREF) + TMP \]

IF \( H(I,J) \geq HLMP \) GO TO 570
IP = 5
GO TO 563

570 CONTINUE
IF (ICOUNT .NE. 1) GO TO 575
CALL CHANGE(HO(N-1,J),HO(N,J),HO(N-1,J),HO(N,J+1),HO(N,J-1),
1PO,AK01,AK02,AK03,AK04)
CONST(N,J) = AK01*X1*2.*T(N-1,J) + AK3*T(N,J+1) + AK4*T(N,J-1))/YL1

575 CALL CHANGE(H(N-1,J),H(N,J),H(N-1,J),H(N,J+1),H(N,J-1),IP,AK1,
1AK2,AK3,AK4)
VK1 = (AK1 + AK2)*X1
VK2 = (AK3 + AK4)/YL1
VK3 = 2.*AK3/YL1
VK4 = 2.*AK4/YL1
GO TO(583,585,587),IP

ICE

\[ H(N,J) = \frac{1}{(1 + VAS(VK1 + VK2)) \cdot (HO(N,J) + VA3(X1*AK1*2.*T(N-1,J) +
1(AK3*T(N,J+1) + AK4*T(N,J-1))/YL1 + CONST(N,J))} \]

\[ T(N,J) = H(N,J)/(CPS*TREF) \]

IF \( H(N,J) \leq HSMP \) GO TO 590
IP = 1

MELTING POINT

\[ H(N,J) = HO(N,J) + VA3((-VK1 + VK2)*TMP + X1*2.*AK1*T(N-1,J) +
1AK3*T(N,J+1) + AK4*T(N,J-1))/YL1 + CONST(N,J)) \]

\[ T(N,J) = TMP \]

IF \( H(N,J) > HSMP \) AND \( H(N,J) < HLMP \) GO TO 590
IF (IP.EQ.1 OR IP.EQ.3) GO TO 590

LIQUID WATER

\[ H(N,J) = \frac{1}{(1 + VA(I) \cdot (VK1 + VK2)) \cdot (HO(I,J) \cdot VA3 \cdot (-VK1 + 1) + VA6 \cdot X1 \cdot (AK2 \cdot T(I+1,J) + AK1 \cdot T(I-1,J) + AK3 \cdot T(I,J+1) + AK4 \cdot T(I,J-1))/YL1 + CONST(I,J))} \]

\[ T(N,J) = H(N,J)/(TREF*CPL) - DEL*(CPS*TMP*TREF/DES+HLAM)/(CPL*TREF) + TMP \]

IF \( H(N,J) \geq HLMP \) GO TO 590
IP = 3
GO TO 583

590 CONTINUE

600 CONTINUE
605 IF ((IBC1.EQ.1) AND (IBC2.EQ.1)) GO TO 672
IF (TG.EQ.2) GO TO 615

C

CALCULATION OF TEMPERATURES AT THE OUTER-AMBIENT INTERFACE

C

WITHOUT PHASE CHANGE

C

\[ T(1,MJ) = P2*(YL1*2.*T(2,MJ) + TO(2,MJ))*X1+2.*T(1,MJ-1)+
1TO(1,MJ-1))+(B(L)/D(L)-P1/2.)*TO(1,MJ)+P1*G2TIME) \]
DO 612 I=2,NOW
T(1,MJ) = P2*(YL1*(T(I+1,MJ)+T(I-1,MJ)+TO(I+1,MJ)+TO(I-1,MJ))*X1

80
1+2.*(T(I,MJ-1)+TO(I,MJ-1))+(B(L)/D(L)-P1/2.)*TO(I,MJ)+P1*G2TIME)

612 CONTINUE
T(N,MJ)=P2*(YL1*2.*(T(N-1,MJ)+TO(N-1,MJ))*X1+2.*T(N,MJ-1)+TO(N,MJ-1))+(B(L)/D(L)-P1/2.)*TO(N,MJ)+P1*G2TIME)

612 CONTINUE
T(N,MJ) = P2*(YL1*2.*(T(N-1,MJ)+TO(N-1,MJ))*X1+2.*T(N,MJ-1)+TO(N,MJ-1))+(B(L)/D(L)-P1/2.)*TO(N,MJ)+P1*G2TIME)

GO TO 677

C
C CALCULATION OF TEMPERATURES AND ENTHALPYS AT THE OUTER-
C AMBIENT INTERFACE
C
615 IF(ICOUNT.NE.1) GO TO 617
CALL CHANGE(HO(2,M),HO(1,M),HO(2,M),HO(1,M),HO(1,M-1),IPO,
1AKO1,AKO2,AKO3,AKO4)
CONST(1,M) = AKO2*X1*2.*(TO(2,M)-TO(1,M))+2.*AKO4*(TO(1,M)-
1TO(1,M-1))/YL1
617 CALL CHANGE(H(2,M),H(1,M),H(2,M),H(1,M),H(1,M-1),IP,AK1,
1AK2,AK3,AK4)
VK1=(AK1+AK2)*X1
VK2=(AK3+AK4)/YL1
VK3=2.*AK3/YL1
VK4=2.*AK4/YL1
GO TO(623,625,627),IP

C
ICE
623 H(1,M)=1./(1.+VAS*(VK1+VK4+VA9))*(HO(1,M)-VA3*(-X1*2.*AK2*T(2,M)-
1VK4*T(1,M-1)+CONST(1,M)-VA9*(G2TIME*2.-TO(1,M))))
T(1,M)=H(1,M)/(CPS*TREF)
IP=1
625 H(1,M)=HO(1,M)-VA3*((VK1+VK4+VA9)*TMP-X1*2.*AK2*T(2,M)-VK4*
1T(1,M-1)+CONST(1,M)-VA9*(G2TIME*2.-TO(1,M)))
T(1,M)=TMP
IP=3
627 H(1,M)=1./(1.+VAL*(VK1+VK4+VA9))*(HO(1,M)-VA3*((VK1+VK4+VA9)*
1VA6-X1*AK2+2.)*T(2,M)-VK4*T(1,M-1)+CONST(1,M)-VA9*(G2TIME*2.-
1TO(1,M))))
T(1,M)=H(1,M)/(TREF*CPL)-DEL*(CPS*TMP/TREF+DESLAM)/(CPL*TREF)+
1TMP
GO TO 630
IP=3
630 DO 650 I=2,NOW
IF(ICOUNT.NE.1) GO TO 635
CALL CHANGE(HO(I-1,M),HO(I,M),HO(I+1,M),HO(I,M),HO(1,M-1),IPO,
1AKO1,AKO2,AKO3,AKO4)
CONST(1,M) = -AKO1*X1*(TO(I-1,M)-TO(I,M))+AKO2*X1*(TO(I,M)-
1TO(I+1,M))/YL1
635 CALL CHANGE(H(I-1,M),H(I,M),H(I+1,M),H(I,M),H(I,M-1),IP,AK1,
1AK2,AK3,AK4)
VK1=(AK1+AK2)*X1
VK2=(AK3+AK4)/YL1
VK3=2.*AK3/YL1
VK4=2.*AK4/YL1

81
GO TO (643, 645, 647), IP

C

ICE

643  

\[ H(I,N) = \frac{1}{1 + VAS \times (VK1 + VK4 + VA9)} \times (HO(I,N) - VA3 \times \left( -X1 \times (AK2 \times T(I+1,N) + AK1) \times T(I-1,N) + \text{CONST}(I,N) - VA9 \times (G2TIME \times 2 - TO(I,N)) \right) + \text{VAS} \times (VK4 \times T(I-1,N) + \text{CONST}(I,N) - VA9 \times (G2TIME \times 2 - TO(I,N))) } \]

\[ T(I,N) = H(I,N) / (TREF \times CPS) \]

IF (H(I,N) .LE. HSMP) GO TO 650

IP = 1

C

MELTING POINT

645  

\[ H(I,N) = HO(I,N) - VA3 \times ((VK1 + VK4 + VA9) \times TMP - X1 \times (AK2 \times T(I+1,N) + AK1) \times T(I-1,N) + \text{CONST}(I,N) - VA9 \times (G2TIME \times 2 - TO(I,N)) ) - \text{VAS} \times (VK4 \times T(I-1,N) + \text{CONST}(I,N) - VA9 \times (G2TIME \times 2 - TO(I,N)) ) \]

\[ T(I,N) = H(I,N) / (TREF \times CPS) \]

IF (H(I,N) .GT. HSMP .AND. H(I,N) .LT. HLMP) GO TO 650

IF (IP .EQ. 1 .OR. IP .EQ. 3) GO TO 650

C

LIQUID WATER

647  

\[ H(I,N) = \frac{1}{1 + VAS \times (VK1 + VK4 + VA9)} \times (HO(I,N) - VA3 \times ((VK1 + VK4 + VA9) \times VA6 - X1 \times (AK2 \times T(I+1,N) + AK1) \times T(I-1,N) + \text{CONST}(I,N) - VA9 \times (G2TIME \times 2 - TO(I,N)) ) - \text{VAS} \times (VK4 \times T(I-1,N) + \text{CONST}(I,N) - VA9 \times (G2TIME \times 2 - TO(I,N)) ) \]

\[ T(I,N) = H(I,N) / (TREF \times CPS) \]

IF (H(I,N) .LE. HSMP) GO TO 650

IP = 1

GO TO 643

650  

CONTINUE

IF (ICOUNT .NE. 1) GO TO 655

CALL CHANGE (HO(N-1,M), HO(N,M), HO(N-1,M), HO(N,M), HO(N,M-1), IPO, 1AK01, AK02, AK03, AK04, 1AK01, 1AK02, 1AK03, 1AK04)

\[ \text{CONST}(N,M) = -AK01 \times 2 \times X1 \times (TO(N-1,M) - TO(N,M)) + 2 \times AK04 \times (TO(N,M) - TO(N,M-1)) / YL1 \]

655  

CALL CHANGE (H(N-1,M), H(N,M), H(N-1,M), H(N,M), H(N,M-1), IP, AK1, AK2, 1AK3, AK4, 1AK1, AK2)

\[ VK1 = (AK1 + AK2) \times X1 \]

\[ VK2 = (AK3 + AK4) / YL1 \]

\[ VK3 = 2 \times AK3 / YL1 \]

\[ VK4 = 2 \times AK4 / YL1 \]

GO TO (663, 665, 667), IP

C

ICE

663  

\[ H(N,M) = \frac{1}{1 + VAS \times (VK1 + VK4 + VA9)} \times (HO(N,M) - VA3 \times \left( -X1 \times (AK1 \times 2 \times T(N-1,M) + AK1 \times 2 \times T(N-1,M) + \text{VAS} \times (VK4 \times T(N,M-1) + \text{CONST}(N,M) - VA9 \times (G2TIME \times 2 - TO(N,M)) ) \right) + \text{VAS} \times \left( -X1 \times (AK1 \times 2 \times T(N-1,M) + AK1 \times 2 \times T(N-1,M) + \text{VAS} \times (VK4 \times T(N,M-1) + \text{CONST}(N,M) - VA9 \times (G2TIME \times 2 - TO(N,M)) ) \right) \]

\[ T(N,M) = H(N,M) / (TREF \times CPS) \]

IF (H(N,M) .LE. HSMP) GO TO 670

IP = 1

C

MELTING POINT

665  

\[ H(N,M) = HO(N,M) - VA3 \times ((VK1 + VK4 + VA9) \times TMP - X1 \times AK1 \times 2 \times T(N-1,M) - VK4 \times T(N,M-1) + \text{CONST}(N,M) - VA9 \times (G2TIME \times 2 - TO(N,M)) ) \]

\[ T(N,M) = H(N,M) / (TREF \times CPS) \]

IF (IP .EQ. 1 .OR. IP .EQ. 3) GO TO 670

C

LIQUID WATER

667  

\[ H(N,M) = \frac{1}{1 + VAS \times (VK1 + VK4 + VA9)} \times (HO(N,M) - VA3 \times ((VK1 + VK4 + VA9) \times VA6 - X1 \times 2 \times AK1 \times T(N-1,M) - VK4 \times T(N,M-1) + \text{CONST}(N,M) - VA9 \times (G2TIME \times 2 - TO(N,M)) ) \]

\[ T(N,M) = H(N,M) / (TREF \times CPS) \]

IF (H(N,M) .GE. HLMP) GO TO 670

82
IP=3
GO TO 663
670 GO TO 677
672 DUMAX=0.
C
C CHECK THE CONVERGENCE FOR CONVECTION BOUNDARY CONDITIONS
OF ONE LAYER CASE ONLY
C
DO 675 I=1,N
DO 675 J=2,MO
DIF(I,J)=ABS(T(I,J)-PHOLD(I,J))
DUMAX=AMAX1(DIF(I,J),DUMAX)
675 CONTINUE
GO TO 680
C
C CHECK CONVERGENCE OF THE GAUSS-SEIDEL ITERATION
C
677 DUMAX=0.
DO 678 I=1,N
DO 678 J=1,MJ
DIF(I,J)=ABS(T(I,J)-TOLD(I,J))
DUMAX=AMAX1(DIF(I,J),DUMAX)
678 CONTINUE
C
C CHECK SEE IF NEED ANY MORE ITERATION
C
680 IF(DUMAX.LE.EPSI) GO TO 682
ICOUNT=ICOUNT+1
IF(L.EQ.1) GO TO 260
IF((IBCI.EQ.1).AND.(IBC2.EQ.1))GO TO 315
IF((IBCI.EQ.2).AND.(IBC2.EQ.2))GO TO 300
C
682 TITAU=TAU*3600.
C
Determination of whether ice layer should be shed
C
IF(ISH.EQ.1) GO TO 685
IF(JSH.EQ.3) GO TO 685
IF((JSH.EQ.2).AND.(NIP.LT.NFSP)) GO TO 685
HINC=H(1,INC)
HINC=HAFP+(HINC-HSMP)/2.
IF(HINC.LT.HAFP) GO TO 685
JSH=2
IF(IRP.EQ.1) JSH=3
NISP=NIP
C
Determination of whether the program should be terminated
by physical time
C
685 TCOUNT=0.015
(TAU.GE.TCOUNT) GO TO 999
C
Determination of whether the program should be terminated
83
BY INPUT DATA OF TIME STEPS

IF(NIP.GT.NTSP) GO TO 999

DETERMINATION OF WHETHER TEMPERATURES AND ENTHALPYS
FOR THE ICE LAYER SHOULD BE STORED FOR THE PURPOSE OF
RECREATING ICE AND SHEDDING THE ICE

IF(IRF.EQ.1) GO TO 694
IF(NIP.LT.NFSP) GO TO 694
IF(IRP.EQ.1) GO TO 694
IF((ISH.EQ.1).AND.(JSH.EQ.1)) GO TO 694

687 DO 692 I=1,N
    DO 692 J=INC,M
    T(I,J)=TIN
    IF(TIN.LT.TMP) GO TO 690
    H(I,J)=CPL*TREF*T(I,J)+DEL*(CPS*TMP*TREF/DES+HLAM)-TMP*TREF*
    1CPL
690 H(I,J)=CPS*T(I,J)*TREF
692 CONTINUE
IRP=1

694 IF(ISH.EQ.1) GO TO 695
IF((JSH.EQ.2).AND.(NIP.LT.NFSP))MJ=INC
IF((JSH.EQ.2).AND.(NIP.GE.NFSP))MJ=M
IF((JSH.EQ.3).AND.(NIP.GE.NFSP))MJ=INC
IF((JSH.EQ.1).MJ=M
695 IF(LA.NE.IFREQ) GO TO 200
    TITAU=TAU*3600.

PRINT OUT OF PROGRAM

WRITE(IO,766)TITAU
    DO 697 I=1,N
    DO 697 J=1,MJ
    TR(I,J)=T(I,J)
    TE(I,J)=TR(I,J)*TREF
697 CONTINUE
WRITE(IO,768)((TE(I,J),I=1,N),J=1,MJ)
IF(IRF.EQ.2.AND.NIP.EQ.NFSP) GO TO 699
WRITE(IO,770)ICOUNT
699 LA=0
GO TO 200

700 FORMAT(80A1)
702 FORMAT(7X,I2,7X,F10.5,9X,F13.6,14X,F13.6)
710 FORMAT(47X,F13.6)
Of Poor Quality.
SUBROUTINE FOR CONSTANT STEP FUNCTION HEAT SOURCE

FUNCTION QHEAT2(TAU,DTAU,TON,TOFF,QV)
  TAU=TAU
  TAU=TAU+0.5*DTAU
  IN=IFIX(TAU/(TON+TOFF))
  IP=IN+1
  B =IN*TOFF+IP*TON
  C =IP*(TON+TOFF)
  QHEAT2=QV
  IF((B.LT.TAU).AND.(TAU.LT.C))QHEAT2=0.
  RETURN
END

SUBROUTINE FOR RAMP FUNCTION HEAT SOURCE

FUNCTION QHEAT3(TAU,DTAU,TON,TOFF,QV)
  TAU=TAU
  TAU=TAU+0.5*DTAU
  IN=IFIX(TAU/(TON+TOFF))
  YP=TAU-IN*(TON+TOFF)
  A=QV/TON
  IF(YP.EQ.0) GO TO 20
  IF(YP-TON)10,10,20
10 QHEAT3=YP*A
  GO TO 50
20 QHEAT3=0.
50 RETURN
END

SUBROUTINE FOR SINE FUNCTION HEAT SOURCE

FUNCTION QHEAT4(TAU,DTAU,TON,TOFF,QV)
  TAU=TAU

TAUR=TAUR+0.5*DTAU
IN=IFIX(TAUR/(TON+TOFF))
YP=TAUR-IN*(TON+TOFF)
XP=YP*2.*3.14159/(TON+TOFF)
QHEAT4=5.*SIN(XP)+QV
RETURN
END

SUBROUTINE FOR CORRECTING THERMAL CONDUCTIVITIES FOR PHASE CHANGE

SUBROUTINE CHANGE(HL1,HL2,HL3,HL4,HL5,IPP,AK1,AK2,AK3,AK4)
COMMON /SEAL/HSMP,HLMP,AKS,AKL
DIMENSION HL(5),IP(5),CK(5)
HL(1)=HL1
HL(2)=HL2
HL(3)=HL3
HL(4)=HL4
HL(5)=HL5
DO 50 I=1,5
IF(HL(I).LE.HSMP) GO TO 10
IF(HL(I).GT.HSMP.AND.HL(I).LT.HLMP) GO TO 20
IF(HL(I).GE.HLMP) GO TO 30
10 IP(I)=1
   CK(I)=AKS
   GO TO 50
20 IP(I)=2
   X=(HLMP-HL(I))/(HLMP-HSMP)
   CK(I)=X*AKS+(1.-X)*AKL
   GO TO 50
30 IP(I)=3
   CK(I)=AKL
50 CONTINUE
IPF=IP(2)
AK1=(CK(1)+CK(2))/2.
AK2=(CK(3)+CK(2))/2.
AK3=(CK(4)+CK(2))/2.
AK4=(CK(5)+CK(2))/2.
RETURN
END

//GO.FT06F001 DD SYSOUT=A,OUTLIM=9990
//GO.SYSIN DD *
-------------------------------------------------------------------------------------------------
DATA FOR THE PROBLEM
-------------------------------------------------------------------------------------------------
COMMENTS:
IF PHASE CHANGE IS CONSIDERED IG=2
IF PHASE CHANGE IS NOT CONSIDERED IG=1
FOR CONSTANT TEMPERATURE BOUNDARY CONDITIONS AT X=1 IBC1=1
" " " " " " AT X=2 IBC2=1
" " CONVECTION BOUNDARY CONDITIONS AT X=1 IBC1=2
" " " " " " AT X=2 IBC2=2

87
IF NO HEAT SOURCE IS PRESENT IH = 1
IF POINT HEAT SOURCE IS PRESENT IH = 2
IF INTERNAL HEAT GENERATION IN A SLAB OCCURS IH = 3
IF CONSTANT HEAT INPUT IS USED IHQ = 1
IF STEP FUNCTION HEAT INPUT IS USED IHQ = 2
IF RAMP FUNCTION HEAT INPUT IS USED IHQ = 3
IF SINE FUNCTION HEAT INPUT IS USED IHQ = 4

DATA FOR THE COMPOSITE BODY

TOTAL NUMBER OF SLABS I= 6
TOTAL NUMBER OF NODES IN THE X-DIRECTION OF THE GRID N= 20
TOTAL NUMBER OF NODES IN THE Y-DIRECTION OF THE GRID M= 29
TOTAL LENGTH OF COMPOSITE SLAB IN THE Y-DIRECTION TLEN1= 0.413000 INCHS
TOTAL LENGTH OF THE HEATER OF THE GRID TLEN2= 0.255000 INCHS
GAP WIDTH TLEN3= 0.015000 INCHS

DATA FOR EACH SLAB:

<table>
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<tr>
<th>NUMBER OF NODES</th>
<th>LENGTH</th>
<th>THERMAL CONDUCTIVITY</th>
<th>THERMAL DIFFUSIVITY</th>
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<tr>
<td></td>
<td>IN INCHS</td>
<td>IN B.T.U./HR.*FT.°F</td>
<td>IN FT*FT/HR.</td>
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<tr>
<td>5</td>
<td>0.08700</td>
<td>66.500000</td>
<td>1.650000</td>
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<td>5</td>
<td>0.05000</td>
<td>0.220000</td>
<td>0.008700</td>
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<td>2</td>
<td>0.00400</td>
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<td>0.138000</td>
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<td>0.220000</td>
<td>0.008700</td>
</tr>
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<td>8.700000</td>
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<td>16</td>
<td>0.25000</td>
<td>1.416000</td>
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</tbody>
</table>

DATA FOR THE HEATER:

TYPE OF HEAT SOURCE IH= 3
FOR IH=2 POINT HEAT SOURCE AT NODE= 9
BETWEEN SLAB L1= 2
AND SLAB L2= 3
FOR IH=3 INTERNAL HEAT GENERATION IN SLAB BETWEEN NODE NO1= 9
AND NODE NO2= 10
TYPE OF HEAT SOURCE USED
CONSTANT HEAT INPUT OF Q = 25.000000 WATTS/IN*IN
ON-TIME FOR THE VARIABLE HEAT INPUT TON = 10.000000 SEC.
OFF-TIME FOR THE VARIABLE HEAT INPUT TOFF = 10.000000 SECS.
VARIABLE HEAT INPUT OF QV = 25.000000 WATTS/IN*IN

BOUNDARY AND INITIAL CONDITIONS:

TYPE OF BOUNDARY CONDITION AT X=1 IBC1= 2
AT X=2 IBC2= 2
IF IBC1=1, CONSTANT TEMPERATURE AT X=1 TX1 = 20.000000 DEG.F
IF IBC2=1, " " AT X=2 TX2 = 20.000000 DEG.F
IF IBC1=2, AMBIENT " AT X=1 TG1 = -4.000000 DEG.F
HEAT TRANSFER COEFF AT X=1 H1 = 1.000000
B.T.U./HR.*FT.*FT°F
IF IBC2=2, AMBIENT TEMPERATURE AT X=2 TG2 = -4.000000 DEG.F
HEAT TRANSFER COEFF. AT X=2 H2 = 1000000.000000
B.T.U./HR.*FT.*FT°F
THE INITIAL TEMPERATURE IN THE SLAB TIN = -4.000000 DEG.F
THE REFERENCE TEMPERATURE TREF = 32.000000 DEG.F
PROPERTIES OF WATER:

IS THE PHASE CHANGE CONSIDERED IF YES IG=2 IF NO IG=1    IG= 2

LATENT HEAT OF ICE     HLM = 143.40000 B.T.U./LB
THERMAL DIFFUSIVITY OF WATER    ALPL = 0.0051 FT^2/FT/HR.
" CONDUCTIVITY " "    AKL = 0.32 B.T.U./HR.FT.'F.
DENSITY OF WATER    DEL = 62.40000 LB./CU.FT
SPECIFIC HEAT=DENSITY OF WATER    CPL = 62.21280 B.T.U./CU.FT.'F.
DENSITY OF ICE    DES = 57.40000 LB./CU.FT
SPECIFIC HEAT=DENSITY OF ICE    CPS = 28.81480 B.T.U./CU.FT.'F.
ICE MELTING TEMPERATURE    TMP = 32.00 DEG.'F.

ADDITIONAL DATA:

INITIAL TIME STEP   DTAUI = 0.100000 SECS.
FOR TIME STEPS    NISP = 75
INTERMIDATE TIME STEP   DTAUM = 0.001000 SECS.
FOR TIME STEPS    NMSP =168
FINAL TIME STEP   DTAUF = 0.100000 SECS.
FREQUENCY OF TIME STEP PER PRINT OF OUTPUT IFREQ = 3
IS THE SHEDDING ICE LAYER CONSIDERED IF YES ISH=2 IF NO ISH=1    ISH= 2
DOES THE INITIAL TEMPERATURE CHANGE IF YES IRF=2 IF NO IRF=1    IRF= 2
INITIAL TEMPERATURE IS CHANGED FOR EACH TIME STEPS    NFSP =300
THE CYCLE FOR SHEDDING    ICL = 2
FOR SHEDDING AT 2ND CYCLE FOR TIME STEPS    NHSP =463
FOR TIME STEPS STOP THE PROGRAM    NTSP =600

/*

//
REFERENCES


<table>
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<tr>
<th>Material</th>
<th>Thermal Conductivity</th>
<th>Spec. Heat</th>
<th>Diffusivity</th>
<th>Density</th>
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<td>Btu/ft·hr·°F</td>
<td>W/m·°C</td>
<td>Btu/lb·°F</td>
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<tr>
<td>Aluminum (soft)</td>
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<td>215</td>
<td>0.23</td>
<td>963</td>
</tr>
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<td>62.3</td>
<td>0.105</td>
<td>440</td>
</tr>
<tr>
<td>Nichrome 80/20</td>
<td>7.6</td>
<td>13.2</td>
<td>0.107</td>
<td>448</td>
</tr>
<tr>
<td>Stainless Steel 304</td>
<td>8.7</td>
<td>15.1</td>
<td>0.118</td>
<td>494</td>
</tr>
<tr>
<td>Glass (&quot;Ep&quot;) Glass</td>
<td>0.6</td>
<td>1.0</td>
<td>0.19</td>
<td>796</td>
</tr>
<tr>
<td>Epoxy Resin</td>
<td>0.10</td>
<td>0.173</td>
<td>0.25</td>
<td>1050</td>
</tr>
<tr>
<td>Epoxy/Glass filled 32%</td>
<td>0.22</td>
<td>0.38</td>
<td>0.23</td>
<td>963</td>
</tr>
<tr>
<td>Neoprene</td>
<td>0.108</td>
<td>0.187</td>
<td>0.295</td>
<td>1235</td>
</tr>
<tr>
<td>Nylon</td>
<td>0.14</td>
<td>0.24</td>
<td>0.4</td>
<td>1675</td>
</tr>
<tr>
<td>Polytetrafluoroethylene (Teflon)</td>
<td>0.15</td>
<td>0.26</td>
<td>0.25</td>
<td>1047</td>
</tr>
</tbody>
</table>

95
<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal Conductivity $\frac{\text{Btu}}{\text{ft-hr} \cdot \text{°F}}$</th>
<th>$\frac{\text{W}}{\text{m} \cdot \text{°C}}$</th>
<th>Spec. Heat $\frac{\text{Btu}}{\text{lb} \cdot \text{°F}}$</th>
<th>$\frac{\text{J}}{\text{Kg} \cdot \text{°C}}$</th>
<th>Diffusivity $10^5 \frac{\text{m}^2}{\text{s}}$</th>
<th>Density $\frac{1\text{ lb}}{\text{ft}^3}$</th>
<th>Density $\frac{\text{Kg}}{\text{m}^3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polytrifluoro-chloro-ethylene (KEL-F)</td>
<td>0.04</td>
<td>0.07</td>
<td>0.216</td>
<td>904</td>
<td>0.0014</td>
<td>0.0036</td>
<td>130</td>
</tr>
<tr>
<td>Water $32^\circ \text{F}$</td>
<td>0.320</td>
<td>0.554</td>
<td>0.997</td>
<td>4174</td>
<td>0.0051</td>
<td>0.0132</td>
<td>62.4</td>
</tr>
<tr>
<td>Ice (Pure) $32^\circ \text{F}$</td>
<td>1.293</td>
<td>2.238</td>
<td>0.5057</td>
<td>2117</td>
<td>0.0445</td>
<td>0.115</td>
<td>57.2</td>
</tr>
<tr>
<td>Ice (Pure) $14^\circ \text{F}$</td>
<td>1.356</td>
<td>2.347</td>
<td>0.5038</td>
<td>2109</td>
<td>0.0469</td>
<td>0.121</td>
<td>57.3</td>
</tr>
<tr>
<td>Ice (Pure) $-4^\circ \text{F}$</td>
<td>1.416</td>
<td>2.451</td>
<td>0.5020</td>
<td>2102</td>
<td>0.0492</td>
<td>0.127</td>
<td>57.4</td>
</tr>
</tbody>
</table>

Latent Heat of Fusion = 143.4 Btu/1b (333.6 KJ/Kg)
**TABLE 2. HEATING TIMES REQUIRED TO PRODUCE GIVEN TEMPERATURE RISES OVER THE CENTERLINES OF THE HEATER ELEMENT AND THE GAP BETWEEN ELEMENTS FOR FOUR GAP WIDTHS.**

De-Icer Construction:
- **Substrate**: 0.100" 304 Stainless Steel
- **Inner Insulation**: 0.020" Epoxy/Glass Laminate
- **Heater Element**: Zero Thickness, 0.510" Wide
- **Outer Insulation**: 0.010" Epoxy/Glass Laminate
- **Abrasion Shield**: 0.010" 304 Stainless Steel
- **Ice Layer**: 0.150"

**Power Density**: 25 Watts/in²

<table>
<thead>
<tr>
<th>Temp. Rise</th>
<th>10.8°F</th>
<th>21.6°F</th>
<th>36°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap Width (in.)</td>
<td>Heater</td>
<td>Gap</td>
<td>Heater</td>
</tr>
<tr>
<td>0.010</td>
<td>1.0</td>
<td>1.2</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>(1.0)</td>
<td>(1.2)</td>
<td>(2.5)</td>
</tr>
<tr>
<td>0.030</td>
<td>1.0</td>
<td>1.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>(1.0)</td>
<td>(1.5)</td>
<td>(2.5)</td>
</tr>
<tr>
<td>0.050</td>
<td>1.0</td>
<td>1.9</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>(1.0)</td>
<td>(1.9)</td>
<td>(2.5)</td>
</tr>
<tr>
<td>0.070</td>
<td>1.0</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>(1.0)</td>
<td>(2.5)</td>
<td>(2.5)</td>
</tr>
</tbody>
</table>

\[ h_{b1} = 10, \ h_{b2} = 200 \quad \text{or} \quad h_{b1} = 1, \ h_{b2} = 10^6 \]
### TABLE 3. EFFECT OF VARIOUS ABRASION SHIELD MATERIALS ON ABRASION SHIELD-ICE INTERFACE TEMPERATURE

<table>
<thead>
<tr>
<th>Case</th>
<th>Abrasion Shield</th>
<th>Time to Reach 32°F Over Center of Heater (Sec.)</th>
<th>Time to Reach 32°F Over Center of Gap (Sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.010&quot; S.S.</td>
<td>0.81 0.9</td>
<td>3.73 4.2</td>
</tr>
<tr>
<td>2</td>
<td>0.010&quot; nickel</td>
<td>0.80 0.9</td>
<td>3.37 3.8</td>
</tr>
<tr>
<td>3</td>
<td>0.010&quot; nickel over 0.010&quot; S.S.</td>
<td>1.10 1.2</td>
<td>3.36 3.7</td>
</tr>
<tr>
<td>4</td>
<td>0.020&quot; nickel over 0.010&quot; S.S.</td>
<td>1.35 1.5</td>
<td>1.97 2.2</td>
</tr>
<tr>
<td>5</td>
<td>0.010&quot; S.S. over 0.010&quot; nickel</td>
<td>1.12 1.2</td>
<td>1.59 1.9</td>
</tr>
<tr>
<td>6</td>
<td>0.010&quot; S.S. over 0.010&quot; Epoxy/Glass over 0.010&quot; nickel</td>
<td>2.12 2.2</td>
<td>2.34 2.6</td>
</tr>
</tbody>
</table>

**Other Data as Follows:**

- **Substrate:** 0.100" thick, 304 Stainless Steel
- **Inner Insulation:** 0.020" thick, Epoxy/Glass Laminate
- **Heater Element:** Zero thickness, 0.510" wide
- **Other Insulation:** 0.010" thick, Epoxy/Glass Laminate
- **Gap between Elements:** 0.050" wide
FIGURE 1. TWO-DIMENSIONAL DE-ICER PAD MODEL
FIGURE 2. TWO-DIMENSIONAL FINITE DIFFERENCE REPRESENTATION
FIGURE 3. FINITE DIFFERENCE GRID AT SELECTED POINTS

a. Interior Mesh Point at the Center of the Gap

b. Interior Mesh Point at the Center of the Heater

c. Interfacial Point

FIGURE 3. FINITE DIFFERENCE GRID AT SELECTED POINTS
a. Substrate - Ambient Interface

b. Abrasion Shield - Ambient Interface

c. Ice-Ambient Interface

FIGURE 4. FINITE DIFFERENCE GRID AT SELECTED POINTS
FIGURE 5. FLOW CHART OF COMPUTER PROGRAM
Call change: Determination of the thermal conductivity of the node.

\[ H_{i,k}^N \leq H_{sm} \] IP = 1
\[ H_{sm} < H_{i,k}^N < H_{lm} \] IP = 2
\[ H_{i,k}^N \geq H_{lm} \] IP = 3

IP = 1

Calculate \( H_{i,k}^{N+1} \) and \( T_{i,k}^{N+1} \) using the ice equation (36).

\[ H_{i,k}^{N+1} \leq H_{sm} \] Yes
\[ H_{i,k}^{N+1} > H_{sm} \] No, IP = 1

IP = 2

Calculate \( H_{i,k}^{N+1} \) and \( T_{i,k}^{N+1} \) using the melting point equation (38).

\[ H_{sm} < H_{i,k}^{N+1} < H_{lm} \] Yes
\[ H_{sm} < H_{i,k}^{N+1} \] No
\[ IP = 1,3 \] Yes (not converged)
\[ IP = 3 \] No

IP = 3

Calculate \( H_{i,k}^{N+1} \) and \( T_{i,k}^{N+1} \) using the liquid water equation (40).

\[ H_{i,k}^{N+1} \geq H_{lm} \] Yes
\[ H_{i,k}^{N+1} < H_{lm} \] No, IP = 3

End of pass for node \((i,k)\)

**FIGURE 5. FLOW CHART OF COMPUTER PROGRAM (CONTINUED)---SOLID LIQUID DETERMINATION PROCEDURE FOR ICE-WATER LAYER BY APPLYING ENTHALPY METHOD**
FIGURE 6. COMPARISON OF FINITE DIFFERENCE AND APPROXIMATE ANALYTICAL RESULTS FOR A ONE-DIMENSIONAL DE-ICER PAD
FIGURE 7. EFFECT OF POWER DENSITY ON DE-ICER PERFORMANCE
FIGURE 8. COMPARISON OF RESPONSE OF POINT AND FINITE THICKNESS HEATERS
FIGURE 9. COMPARISON OF DE-ICING TIME FOR VARIOUS RATIOS OF INNER TO OUTER INSULATION THICKNESS
Power Density 25 Watts/in²
Substrate 0.087" 75S-T6 Alum.
Inner Insulation "N" ———
Heater 0.004" Nichrome
Outer Insulation 0.010" Epoxy/Glass
Abrasion Shield 0.012" 304 S.S.
Ice 0.250"

FIGURE 10. EFFECT OF INSULATION THICKNESS RATIO AND THERMAL MATERIAL ON DE-ICING TIME
FIGURE 11. COMPARISON OF DE-ICING TIME FOR VARIOUS SUBSTRATE MATERIALS

- **Power Density**: 25 Watts/in²
- **Substrate**: ---
- **Inner Insulation**: 0.020" Epoxy/Glass
- **Heater**: 0.004" Nichrome
- **Outer Insulation**: 0.010" Epoxy/Glass
- **Abrasion Shield**: 0.012" 304 S.S.
- **Ice**: 0.250"

- Curve 1: 0.087" 7075-T6 Alum.
- Curve 2: 0.087" 304 S.S.

**De-Icing Time (Sec.)**

**Temperature Rise (°F)**

- Present Investigation
- Stallabrass [5]
De-Icer Pad Construction
Is Same as Presented in Figure 7
Heater 0.000" thick
Heater on 5 Sec. and Off 1 Sec.
Power Density 25 Watts/in²

Curve
1. Substrate (Constant Heat Input)
2. Substrate (Ramp Function Heat Input)
3. Abrasion Shield-Ice Interface (Constant Heat Input)
4. Abrasion Shield-Ice Interface (Ramp Function Heat Input)
5. Ice-Ambient

FIGURE 12a. COMPARISON OF INTERFACIAL TEMPERATURE RESPONSE FOR RAMP FUNCTION AND CONSTANT POINT HEAT SOURCE
Power Density: 25 Watts/in²
Substrate: 0.087" 75S-T6 Alum.
Inner Insulation: 0.050" Epoxy/Glass
Heater: 0.004" Nichrome
Outer Insulation: 0.010" Epoxy/Glass
Abrasion Shield: 0.012" 304 S.S.
Ice: ----

Curve | Ice Layer Thickness
--- | ---
1 | 0.50"
2 | 0.25"
3 | 0.10"

Present Investigation
Marano [6]

FIGURE 13. EFFECT OF INITIAL ICE LAYER THICKNESS ON DE-ICER PERFORMANCE
FIGURE 14. TIME DIFFERENCE BETWEEN INTERFACE ABOVE CENTER OF HEATER AND INTERFACE ABOVE CENTER OF GAP TO ACHIEVE A GIVEN TEMPERATURE RISE

- Present Investigation
- Stallabrass [5]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Density</td>
<td>25 Watts/in$^2$</td>
</tr>
<tr>
<td>Substrate</td>
<td>0.100&quot; 304 S.S.</td>
</tr>
<tr>
<td>Inner Insulation</td>
<td>0.020&quot; Epoxy/Glass</td>
</tr>
<tr>
<td>Heater</td>
<td>Zero thickness, 0.510&quot; Wide</td>
</tr>
<tr>
<td>Outer Insulation</td>
<td>0.010&quot; Epoxy/Glass</td>
</tr>
<tr>
<td>Abrasion Shield</td>
<td>0.010&quot; 304 S.S.</td>
</tr>
<tr>
<td>Ice Layer</td>
<td>0.150&quot;</td>
</tr>
</tbody>
</table>

Temperature Rise (°F)

- Gap Width: 0.010", 0.030", 0.050", 0.070"
De-Icer Pad Construction
is Same as Presented in Figure 14
Gap Width 0.050"
De-Icer Pad Construction is Same as Presented in Figure 7
Gap Width 0.030 in.
Heater on 10 Sec. and Off 10 Sec.

Curve
1. Substrate
2. Heater (Over Center of Gap)
3. Heater (Over Center of Heater)
4. Abrasion Shield-Ice Interface
5. Abrasion Center of Gap
6. Ice-Ambient Interface

Time (Sec.)
12
8
4
0

Temperature (°F)
-15
-10
-5
0
5
10
15
20

Figure 17. Temperature Profiles When Ice Layer is Shed, 1st Cycle
De-Icer Pad Construction is  
Same as Presented in Figure 7  
Gap Width 0.030"  
Heater on 10 Sec.

Curve
1. Substrate
2. Heater (Over Center of Heater)
3. Heater (Over Center of Gap)
4. Abrasion Shield-Ice Interface  
   (Over Center of Heater)
5. Abrasion Shield-Ice Interface  
   (Over Center of Gap)
6. Ice-Ambient Interface

**FIGURE 19. TEMPERATURE PROFILES WITH REFREEZING**