Theoretical Basis for Design of Thermal-Stress-Free Fasteners

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SUMMARY

A theoretical basis has been developed for the design of fasteners which are free of thermal stress. A fastener can be shaped to eliminate the thermal stress which would otherwise result from differential thermal expansion between dissimilar fastener and sheet materials for many combinations of isotropic and orthotropic materials. The resulting joint remains snug, yet free of thermal stress at any temperature, if the joint is uniform in temperature, if it is frictionless, and if the coefficients of thermal expansion of the materials do not change with temperature. In general, such a fastener has curved sides; however, if both materials have isotropic coefficients of thermal expansion, a conical fastener is free of thermal stress. Equations are presented for thermal-stress-free shapes at both initial and final temperature, and typical fastener shapes are shown.

INTRODUCTION

A recent effort at the NASA Langley Research Center has been aimed at the development of high-temperature structures of carbon-carbon materials. The largest coefficient of thermal expansion (CTE) of carbon-carbon is approximately an order of magnitude lower than the CTE of metals typically used for fasteners. This thermal-expansion mismatch can cause failure of the carbon-carbon material around a standard, snug-fitting, cylindrical fastener. A clearance left around the fastener to accommodate the expansion can make the joint unacceptably loose at low temperatures. Figure 1 shows a cylindrical stainless-steel fastener which was installed in a piece of graphite with a close tolerance fit. When this specimen was heated to 1600°F in a vacuum furnace, the graphite around the fastener failed during the initial thermal cycle. (See fig. 1.)

A biconic fastener with coincident vertices (ref. 1), shown in figure 2(a), was proposed for thermal-stress-free joints in materials with isotropic CTE and is the subject of a patent application. Figure 2(b) shows a specimen which consists of two pieces of graphite joined by steel biconic fasteners, which provide a tight joint at room temperature. This specimen was also heated to 1600°F in a vacuum furnace. The graphite around the conical fastener showed no evidence of damage after four thermal cycles.

Two-dimensional carbon-carbon material, which consists of layers of carbon-fiber cloth in a carbon matrix, was found to have a through-the-thickness CTE which is twice the in-plane CTE. (See ref. 2.) Although the conical fastener, which was designed for isotropic materials, should reduce the thermal stresses, it would not be expected to eliminate them. The cone angle in the carbon-carbon, with its unequal CTE, changes with temperature, but the cone angle in the isotropic metal fastener remains constant. Consequently, the originally snug-fitting conical mating surfaces interfere with one another during heating. This interference causes mechanical constraints to expansion, and thus causes thermal stresses. The use of curved-sided fasteners to eliminate thermal stresses around fasteners in materials whose coefficients of thermal expansion are orthotropic is discussed in reference 3. However, approximations in the analysis resulted in a solution which reduces, but does not eliminate, thermal stresses.
This paper contains the solution for an interference-free interface between two materials with orthotropic coefficients of thermal expansion. The application of this solution to an axisymmetric fastener for a wide range of material combinations is also discussed. Within the framework of the simplifying assumptions, this solution provides a basis for design of thermal-stress-free fasteners in materials with a wide range of CTE. Equations are presented for the fastener shapes at the initial temperature and at elevated temperatures.

SYMBOLS

- $A$: constant of integration
- $b$: axial offset of vertex of cone in conical fastener at initial temperature
- $C$: thermal-expansion term in equation (B4)
- $f$: fraction of total thickness
- $L$: length
- $m$: slope of line
- $p, q$: exponents in equation (4)
- $R$: radius of fastener shank
- $R'$: reference radius of fastener shank
- $R_0, Z_0$: coordinates of specified point on initial boundary
- $r, \theta, z$: cylindrical coordinates
- $T$: temperature
- $t$: thickness
- $t_m$: location of zero-thermal-expansion mismatch (fig. 12)
- $t_w$: washer thickness
- $t'_w$: reference washer thickness
- $X_0, Y_0$: specified point on initial boundary
- $x, y$: rectangular Cartesian coordinates
- $\alpha$: coefficient of thermal expansion (CTE)
- $\bar{\alpha}$: average CTE over a given temperature range
- $\Delta$: difference or change
- $\phi$: cone angle
Subscripts:

- initial or reference conditions
- material 1
- material 2

ANALYSIS

The objective of this two-dimensional analysis is to find an interface between two materials with orthotropic CTE along which the two materials will remain in contact, without interference or separation, as temperature changes. This analysis requires an exact expression for thermal expansion. The basic relation between expansion and temperature change is given by

\[ \frac{dL}{L} = \alpha \, dT \]  \hspace{1cm} (1)

where \( L \) = Length, \( T \) = Temperature, and \( \alpha \) = Coefficient of thermal expansion. Integration of equation (1) produces

\[ L = L_0 \, e^{\bar{\alpha}(T - T_0)} \]  \hspace{1cm} (2)

where \( \bar{\alpha} \) is the average value of \( \alpha \) between temperatures \( T_0 \) and \( T \) and where the subscript \( o \) signifies initial or reference conditions. The Taylor series expansion for equation (2) is

\[ L = L_0 \left[ 1 + \bar{\alpha}(T - T_0) + \frac{2}{2!} \bar{\alpha} (T - T_0)^2 + \frac{3}{3!} \bar{\alpha} (T - T_0)^3 + \ldots \right] \]  \hspace{1cm} (3)

The first two terms of equation (3) give the common engineering approximation for thermal expansion which was used in reference 3. The more precise expression for thermal expansion in equation (2) is used in the present analysis.

Assumptions made in the present analysis are as follows: the CTE of each material is independent of temperature (i.e., \( \alpha = \bar{\alpha} \) at any temperature); there are no thermal gradients in the materials; and there is no friction along the interface between the materials. Based on these assumptions and on equation (2), the following expression for a two-dimensional, thermal-stress-free interface between two materials with different coefficients of thermal expansion (see fig. 3) is derived in appendix A:

\[ y = A \, e^{q(T - T_0)} x^p \]  \hspace{1cm} (4)
where

\[ q = \frac{\alpha_{y1} \alpha_{x2} - \alpha_{y2} \alpha_{x1}}{\alpha_{x1} - \alpha_{x2}} \]

\[ p = \frac{\alpha_{y1} - \alpha_{y2}}{\alpha_{x1} - \alpha_{x2}} \]

and A is an arbitrary constant. The x- and y-axes are parallel to the principal axes of thermal expansion in both materials, and all thermal expansion is relative to the origin of the x,y coordinate system (fig. 3). At the initial temperature, \( T = T_0 \), equation (4) reduces to

\[ y = Ax^D \]  \hspace{1cm} (5)

Two materials in contact along a boundary which, at a given initial temperature, is shaped according to equation (5), will be in continuous, interference-free contact at any temperature if \( p \) and \( q \) are independent of \( T \). Figure 4 shows the family of solutions to equation (5) passing through an arbitrary point \((x_o, y_o)\), which also determines the value of \( A \) \((A = y_o/x_o^D)\). As explained in appendix A, the solution shown in figure 4 can be applied to each of the other three quadrants to produce the shapes shown in figure 5.

Each of the shapes in figure 5 represents a different relationship between the CTE of the two materials. For \( p = 1 \) (fig. 5(a)), the relationship is \( \alpha_{y1} - \alpha_{y2} = \alpha_{x1} - \alpha_{x2} \). In the first quadrant, this condition results in a boundary which is a line passing through the origin and the point \((x_o, y_o)\). A special case of \( p = 1 \) is an interference-free interface between two isotropic materials for which \( \alpha_{y1} = \alpha_{x1} \) and \( \alpha_{y2} = \alpha_{x2} \). For \( 1 < p < \infty \) (fig. 5(b)), the relationship between the coefficients of thermal expansion is \( \alpha_{y1} - \alpha_{y2} > \alpha_{x1} - \alpha_{x2} \). This relationship results in a curve with increasing slope throughout the first quadrant. Similarly, for \( 0 < p < 1 \) (fig. 5(c)), the relationship between coefficients of thermal expansion is \( \alpha_{y1} - \alpha_{y2} < \alpha_{x1} - \alpha_{x2} \), which results in a curve with constantly decreasing slope in the first quadrant. For \( p < 0 \) (fig. 5(d)), \( \alpha_{y1} - \alpha_{y2} \) and \( \alpha_{x1} - \alpha_{x2} \) are of opposite sign. The resulting boundary has decreasing slope throughout the first quadrant and is asymptotic to the x- and y-axes. For the boundary resulting from \( p = \infty \) (fig. 5(e)), \( \alpha_{x1} = \alpha_{x2} \), which implies a thermal expansion mismatch only in the y-direction. Therefore, the interference-free boundary is a vertical line. Similarly, for \( p = 0 \) (fig. 5(f)), \( \alpha_{y1} = \alpha_{y2} \) and the interference-free boundary is a horizontal line. As a result of the assumption that the x- and y-axes are parallel to the principal axes of thermal expansion of both materials, the x- and y-axes are also interference-free interfaces between any two materials.
In general the boundary changes shape with temperature, as indicated in equation (4). However, if the coefficients of thermal expansion of the two materials are such that $\alpha_{y1}/\alpha_{x1} = \alpha_{y2}/\alpha_{x2}$, then $q = 0$, and equation (4) reduces to

$$y = Ax^{\alpha_{y1}/\alpha_{x1}}$$

Therefore, if the two materials have the same ratio of CTE in the y-direction to CTE in the x-direction, then equation (6) defines an interference-free boundary which is independent of temperature.

One other special case is worthy of consideration. If the corresponding coefficients of thermal expansion of one material are much greater than those of the other (i.e., $\alpha_{y1} \gg \alpha_{y2}$ and $\alpha_{x1} \gg \alpha_{x2}$), then $p$ approaches $\alpha_{y1}/\alpha_{x1}$. Thus, for an isotropic metal (material 1) with the CTE roughly an order of magnitude greater than carbon-carbon (material 2), $p$ is close to unity and a portion of the boundary can be closely approximated by a straight line.

APPLICATION TO FASTENERS

Although the solution given by equation (5) was derived for a two-dimensional boundary, the solution can be applied to a three-dimensional fastener. Consider two sheets joined by a fastener of a different material. Figure 6 is a cross-sectional view of such a fastener. If the coefficients of thermal expansion of the sheets and the fastener are isotropic in the plane of the sheets but different in the thickness direction, then the following equation, derived in appendix A, can be used to define the thermal-stress-free shape of an axisymmetric fastener when $T = T_o$:

$$z = Ar^p$$

where

$$A = Z_o/R_o^p$$

In general, as discussed in appendix A, $R_o$ can be a function of $\theta$; however, the simplest shape is an axisymmetric fastener. As in the previous analysis, both materials must expand relative to the origin of the coordinate system. Because a practical fastener cannot come to a point at the origin, a shank of arbitrary radius $R$ must be built into the fastener. A washer of the same material as the sheets is added, as shown in figure 6, to shift the vertex of the conical fastener outside the sheets being joined. The boundary between the shank and the washer is not interference-free; thus, a clearance between shank and washer is required to accommodate the radial thermal expansion mismatch. Another equally important boundary between the two materials is the interface between the fastener head and the washer, which lies in the $z = 0$ plane, and is therefore interference-free for any two materials.
The shape of the load-bearing surface is determined from equation (7), where \( R \) and \( t_w \) (the washer thickness) correspond to \( R_0 \) and \( Z_0 \), respectively. The dimensions \( R \) and \( t_w \) can be varied to produce acceptable shear and bearing areas for the fastener, and, as illustrated in figure 7, these dimensions give a designer considerable control over the proportions of the thermal-stress-free fastener. A general thermal-stress-free fastener is shown on the upper left of figure 7. Doubling the washer thickness and holding the radius constant produces the shape shown on the lower left. Increasing the minimum radius while holding the washer thickness constant produces the shape on the upper right. Doubling the washer thickness while increasing the minimum radius produces the shape shown on the lower right.

The fastener shown in figures 6 and 7 has a shape corresponding to \( 1 < p < \infty \). Figure 8 shows a similar fastener which has a shape corresponding to \( 0 < p < 1 \). As shown in figure 9, for \( p = 1 \) the shape reduces to the conical fastener proposed in reference 1. If the two materials are such that \( p < 0 \), which means that either \( \alpha_{y1} < \alpha_{y2} \) and \( \alpha_{x1} > \alpha_{x2} \) or \( \alpha_{y1} > \alpha_{y2} \) and \( \alpha_{x1} < \alpha_{x2} \), the thermal-stress-free shape does not readily lend itself to a practical thermal-stress-free fastener design, as is evident in figure 5(d). If the two materials have coefficients of thermal expansion which are equal in one direction but not in the other (see figs. 5(e) and 5(f)), it is impossible for a snug-fitting fastener to eliminate the thermal expansion mismatch in the other direction. Also, since the axisymmetric solution for a metallic fastener in carbon-carbon materials closely approximates a cone, the exact axisymmetric thermal-stress-free metallic fastener which joins sheets of carbon-carbon material can be closely approximated by a conical fastener with the vertex slightly offset from the point about which both materials expand. (See fig. 10.) This offset is further discussed in appendix B. Although fasteners of the type discussed herein should not develop thermal stress, further effort is needed to determine whether excessive stress concentrations can develop in mechanically loaded joints which employ these unusually shaped fasteners.

CONCLUDING REMARKS

A theoretical basis for the design of thermal-stress-free fasteners has been developed in this study. The analysis yields the equation of a two-dimensional thermal-stress-free interface between two materials with orthotropic coefficients of thermal expansion. If both materials have coefficients of thermal expansion which are isotropic in one plane, the two-dimensional analysis can be used to design thermal-stress-free fasteners, made from one material, which are used to join pieces of the other material. The two materials remain in contact as the temperature increases, forming a tight joint without interference and providing effective shear transfer. The simplest general shape is an axisymmetric, curved-sided fastener. For two materials with isotropic coefficients of thermal expansion, the shape reduces to the conical fastener proposed by Jackson and Taylor in Astronautics and Aeronautics, June 1983. If the exact shape of the thermal-stress-free fastener is nearly conical it can be approximated by a conical fastener with the vertex slightly offset.

Assumptions made in this analysis are as follows: the coefficients of thermal expansion of both materials are independent of temperature, both materials are uniform in temperature, and the interface between the materials is frictionless. In an actual joint, these requirements are unlikely to be fully satisfied. However, the resulting thermal interferences using a fastener shape defined by the analysis given herein should be significantly smaller than those of a snug-fitting cylindrical fastener. Further research is needed to predict thermal stresses resulting from viola-
tions of the basic assumptions of this analysis. Also, since these fasteners are shaped much differently from conventional fasteners, further work is necessary to determine stress concentration factors in the joint under mechanical loading.

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APPENDIX A

ANALYSIS OF THERMAL-INTERFERENCE-FREE BOUNDARY

Consider a plane which contains two orthotropic materials separated by a boundary along which the materials are in continuous contact. Choose a point in that plane such that all motion of both materials due to thermal expansion is relative to that point. Make that point the origin of an x,y coordinate system which is aligned with the principal axes of thermal expansion of both materials. (See fig. 11.) Consequently, any particle which is initially on the x- or y-axis moves only along that axis during thermal expansion, and any thermal expansion in the first quadrant is independent of the expansion in any other quadrant. Considering each of the other quadrants separately, it can be seen that a solution which is valid for the first quadrant can be individually applied to each of the other quadrants.

Assume that the coefficients of thermal expansion (CTE) for both materials are independent of temperature and location within each material. Also assume that the temperatures are uniform throughout the materials and that there are no frictional forces acting along the boundary.

The objective is to find an initial boundary shape, \( y = f(x) \), such that when the two materials reach a new uniform temperature, the material boundaries will still coincide. Let \((x,y)\) be a point on the boundary at \( T = T_0 \). At each point \((x,y)\), there are adjacent particles of materials 1 and 2. The appropriate coefficients of thermal expansion are defined as follows:

- \( \alpha_{x1} \) CTE of material 1 in the x-direction
- \( \alpha_{y1} \) CTE of material 1 in the y-direction
- \( \alpha_{x2} \) CTE of material 2 in the x-direction
- \( \alpha_{y2} \) CTE of material 2 in the y-direction

Using equation (2), the locations of the particles after a temperature change of \( \Delta T \) can be found as follows:

For material 1,

\[
\begin{align*}
  x_1 &= x \cdot e^{\alpha_{x1} \Delta T} \\
  y_1 &= y \cdot e^{\alpha_{y1} \Delta T}
\end{align*}
\]

(A1)
For material 2,

\[
\begin{align*}
    x_2 &= x e^{x_2^2 \Delta T} \\
    y_2 &= y e^{y_2^2 \Delta T}
\end{align*}
\]  \hspace{1cm} \text{(A2)}

It is required that the original shape of the boundary, \( y = f(x) \), be such that \((x_1, y_1)\) and \((x_2, y_2)\) are on coincident boundaries after a temperature increase of \( \Delta T \). The slope of this boundary at \( T = T_0 + \Delta T \) can be approximated by

\[
\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y e^{y_2^2 \Delta T} - e^{y_1^2 \Delta T}}{x e^{x_2^2 \Delta T} - e^{x_1^2 \Delta T}}
\]  \hspace{1cm} \text{(A3)}

Now as \( \Delta T \to 0 \), the boundary approaches \( y = f(x) \), and the approximate slope of the boundary at \( T = T_0 + \Delta T \) becomes the exact slope of the initial boundary at \( T = T_0 \).

As \( \Delta T \to 0 \), \( \Delta y \to 0 \) and \( \Delta x \to 0 \); therefore,

\[
\lim_{\Delta T \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}
\]  \hspace{1cm} \text{(A4)}

Taking the limit of equation (A3) as \( \Delta T \to 0 \) yields

\[
\frac{dy}{dx} = \frac{(y_2 - y_1)y}{(x_2 - x_1)x}
\]  \hspace{1cm} \text{(A5)}

Integration of equation (A5) gives the initial boundary shape

\[
y = Ax^p
\]  \hspace{1cm} \text{(A6)}

where

\[
p = \frac{\alpha y_1 - \alpha y_2}{\alpha x_1 - \alpha x_2}
\]
APPENDIX A

and \( A \) is the constant of integration. At any other temperature, \( T = T_0 + \Delta T \), and the boundary of each material undergoes thermal expansion governed by equations (A1) and (A2) to the shape given by

\[
y = A e^{\frac{q(T-T_0)}{x}} \tag{A7}
\]

where

\[
q = \frac{\alpha_{x1} - \alpha_{x2}}{\alpha_{y1} - \alpha_{y2}}
\]

and where equation (A7) is obtained by direct substitution of either equation (A1) or (A2) into equation (A6). Thus, any point on the boundary given by equation (A6) at \( T = T_0 \), for material 1 or material 2, will be located on the boundary given by equation (A7) after a temperature change. Consequently, although boundary particles of the two materials move along the interface by generally unequal amounts, the boundaries remain coincident during the thermal expansion. Thus, motion of one material does not constrain motion of the neighboring material or cause separation of the two materials. That is, the expansion is stress-free, yet the two materials are in continuous contact.

The preceding two-dimensional solution can be extended to a special case of a three-dimensional solution. Let \( x \) and \( y \) become \( r \) and \( z \), respectively, in a cylindrical coordinate system, and assume that the coefficients of thermal expansion of both materials are isotropic in all \( r-\theta \) planes. (An example of this type of material is an idealized quasi-isotropic filamentary composite. In the plane of a sheet, the thermal expansion is primarily controlled by the fibers; however, through the thickness, the thermal expansion is controlled by the matrix material.)

As a result of this assumption, the two-dimensional solution is valid at any angle \( \theta \). The thermal-stress-free interface is therefore given as follows for \( T = T_0 \):

\[
z = A r^p \tag{A8}
\]

where

\[
p = \frac{\alpha_{z1} - \alpha_{z2}}{\alpha_{r1} - \alpha_{r2}}
\]

and \( A \) is defined by some specified point \((z_0, r_0)\) on the initial interface as

\[
A = \frac{z_0}{r_0^p}
\]
APPENDIX A

The interface is given as follows for $T = T_0 + \Delta T$:

$$z = A e^{-r \rho}$$

$$q(T-T_0)$$

where

$$q = \frac{\alpha_1 \alpha_2 - \alpha_1 \alpha_2}{\alpha_1 - \alpha_2}$$

In general, $R_\theta$ can be any single-valued function of $\theta$. For example, the shape of a cross section of the fastener parallel to the $r-\theta$ plane could be a square or any other polygon. However, the simplest shape for a fastener is axisymmetric (circular cross section); that is, $R_\theta$ is independent of $\theta$. 
APPENDIX B

CALCULATION OF OFFSET OF VERTEX OF CONICAL FASTENER TO ADJUST THERMAL-EXPANSION MISMATCH

If a conical fastener is used in a joint in which the coefficient of thermal expansion (CTE) of the fastener and the material being joined are such that \( p \) is not equal to 1, the initially mated surfaces will attempt to translate and rotate by different amounts if there is a temperature change. Some adjustment to this thermal-expansion mismatch can be effected by specifying the new location of the hypothetical intersection of the surfaces after unrestrained thermal expansion. The location of the intersection can be adjusted by offsetting the vertex of the cone from the origin along the z-axis. A representative arrangement in the r-z plane before and after free thermal expansion is illustrated in figure 12. Extension to an axisymmetric configuration is effected by revolving the graph about the z-axis.

Before expansion, the two material boundaries have the common equation

\[
z = mr + b
\]  

(B1)

where \( b \) is the offset of the vertex of the cone (see sketch A).
APPENDIX B

After a change in temperature \( \Delta T \), the boundaries of the freely expanding materials have the following equations, which result from combining equation (B1) with equations (A1) and (A2), as follows:

\[
z_1 = m e^{(z_1 - \alpha_1) \Delta T} r_1 + e^{\alpha_1 \Delta T} b
\]

and

\[
z_2 = m e^{(z_2 - \alpha_2) \Delta T} r_2 + e^{\alpha_2 \Delta T} b
\]

Sketch A shows the boundaries before and after a temperature increase.

In figure 12, the dashed line represents the initial interface, and the solid lines represent the boundaries of the two materials at the final temperature. The shaded region between the boundaries represents the thermal-expansion mismatch which results when the coefficients of thermal expansion of the materials are such that \( p = 1 \). The intersection of the two boundaries can be controlled by adjusting the value of \( b \). This has the effect of adjusting the amount of interference present in the joint at the final temperature \( T_0 + \Delta T \). At the point of intersection \( z_1 = z_2 \) and \( r_1 = r_2 \). If the point of intersection in material 2 is initially at \( z = b + R \cot(\phi/2) + tf \), then by simultaneous solution of equations (B2) and (B3),

\[
b = [R \cot(\phi/2) + tf]C
\]

where

\[
C = \frac{e^{(z_1 - \alpha_1) \Delta T} - e^{(z_2 - \alpha_2) \Delta T}}{e^{\alpha_2 \Delta T} - e^{\alpha_1 \Delta T}}
\]

A conical fastener with the vertex offset by the amount given in equation (B4) is essentially a linear approximation of the exact thermal-stress-free shape given by equation (5). The nearer the exact shape is to a cone, the lower the thermal mismatch between the conical fastener and the material being joined. Although equation (B4) provides a means of adjusting the thermal-expansion mismatch between the two materials, the thermal stresses which result from this mismatch depend on the stiffnesses of the materials. Therefore, additional analysis, which accounts for material stiffness, is necessary to calculate the vertex offset, which minimizes thermal stress.
REFERENCES


Figure 1.- Cylindrical stainless-steel fastener in graphite which failed during first thermal cycle.

(a) Schematic of biconic fastener.  (b) Test specimen after four cycles to 1600°F.

Figure 2.- Biconic steel fastener in graphite.
Figure 3. Thermal-stress-free boundary.

Figure 4. Solutions to equation $y = Ax^p$. 
Figure 5.- Thermal-stress-free shapes in x-y plane.

Figure 6.- Thermal-stress-free fastener for $1 < p < \infty$. 
Figure 7.- Illustrations of effect of design parameters $t_w$ and $R$ on fastener proportions.

Figure 8.- Thermal-stress-free fastener for $0 < p < 1$.  
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Figure 9.- Thermal-stress-free fastener for $p = 1$.

Figure 10.- Metallic fastener in carbon-carbon material ($p = 1$).
Figure 11.- Thermal-stress-free boundary between two materials in a plane.
Figure 12. - Thermal expansion of conical fastener with offset vertex.
A theoretical basis has been developed for the design of fasteners which are free of thermal stress. A fastener can be shaped to eliminate the thermal stress which would otherwise result from differential thermal expansion between dissimilar fastener and sheet materials for many combinations of isotropic and orthotropic materials. The resulting joint remains snug, yet free of thermal stress at any temperature, if the joint is uniform in temperature, if it is frictionless, and if the coefficients of thermal expansion of the materials do not change with temperature. In general, such a fastener has curved sides; however, if both materials have isotropic coefficients of thermal expansion, a conical fastener is free of thermal stress. Equations are presented for thermal-stress-free shapes at both initial and final temperature, and typical fastener shapes are shown.