Liquid Management in Low Gravity Using Baffled Rotating Containers

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NOMENCLATURE

$A, B$  constants relating $C$ to $D$

$C$  dimensionless axial curvature at maximum interface distance from rotation axis

$D$  dimensionless baffle spacing

$F = \rho \omega^2 R^3 / \gamma$

$G = \rho g R^2 / \gamma$

$h$  step size along the interface for numerical integration

$i$  index for multilayered system

$K'$  mean curvature of the interface

$L$  half baffle spacing

$n$  number of baffles

$P_G, P_L$  pressure in gas and liquid

$P_{0G}, P_{0L}$  constants in $P_G$ and $P_L$ expressions

$r$  radial coordinate

$R$  maximum distance of an interface from the rotation axis

$r_I$  equation of interface in cylindrical coordinates

$\Delta s$  integration step in the $z$ direction

$V$  void volume

$V_1$  critical volume for interface formation

$V_2$  critical volume for interchange instability

$V_{A, V_B}$  volumes corresponding to configurations in Figure 8

$x$  dimensionless axial coordinate

$y$  dimensionless interface rise

$Y$  first derivative of $y$ with respect to $x$

$y^B$  capillary rise at bottom baffle
NOMENCLATURE (Concluded)

\( y^T \)  capillary rise at top baffle
\( z \)  axial coordinate
\( z^B \)  distance from bottom baffle to that \( z \) where \( y = 0 \)
\( z^T \)  distance from top baffle to that \( z \) where \( y = 0 \)
\( \gamma \)  surface tension
\( \Delta \)  \( = (1 + \gamma^2)^{1/2} \)
\( \epsilon \)  symbolic small parameter
\( \xi \)  capillary rise
\( \theta \)  contact angle
\( \Pi \)  pressure invariant, dimension of inverse length
\( \rho \)  \( \rho_C - \rho_G \)
\( \rho_G, \rho_L \)  gas and liquid densities
\( \phi \)  azimuthal angle
\( \omega \)  rotation rate
LIQUID MANAGEMENT IN LOW-GRAVITY USING BAFFLED ROTATING CONTAINERS

INTRODUCTION

Long duration space missions can require the location and control of large masses of liquids for long times. An extreme example of this problem is a proposed relativity experiment (GP-B\(^1\)) for which over 300 kg of liquid helium will be vented during a period of one year, during which its configuration must be symmetric enough to keep acceleration effects at the design center of mass below 9.8 \( \times 10^{-10} \) m/s\(^2\). It has been proposed that the helium, to be contained in a cylindrical annulus, be controlled by rotating the annulus, and providing baffles to help locate the helium. An ideal result is shown in Figure 1. The purpose of this paper is to examine the realizability of the configuration shown in Figure 1.

Without baffles the helium gas is likely to form a single bubble, which may arrange itself around the axis, or in an off-axis location. (For a full discussion of the unbaffled problem, see Reference 1.) From this it follows that there is likely to be a maximum permissible baffle spacing which will depend on the relative magnitude of the surface tension and centrifugal forces. As will be shown below, that proves to be the case; there is a baffle spacing beyond which the fluid cannot form the interface shown in Figure 1. The two possibilities are shown schematically in Figure 2.

A second question is whether the interfaces will all remain at the same distance from the rotation axis. The layers of liquid are to be interconnected at the outside, so that withdrawal of fluid can be effected evenly from all the layers. Thus if there are pressure differences across the baffles, fluid will be forced from the high pressure layer to the low pressure layer.

The pressure in the liquid is the difference between a centrifugal term and a surface tension term. If the interface moves out, both terms decrease. If the interface moves in, both terms increase. One cannot tell, a priori, whether the difference will increase or decrease. If the difference increases when the interface moves out of the configuration will be unstable because liquid will flow from the “shallow” layer to the “deep” layer. Figure 3 illustrates the situation for two layers. It will be shown below that this instability is possible at low rotation rates, and that it can arise for the presently-designed GP-B experiment.

A third difficulty is the possible fluid dynamic instability of the interface itself. This problem is beyond the scope of this paper.

Experimental analysis of possible fluid dynamical instability, and verification of the theoretical results to be given below, is to be desired. Unfortunately gravity levels of 9.8 \( \times 10^{-10} \) m/s\(^2\) are not easily attainable. Thus the theory will be generalized to include an axial gravity field of arbitrary magnitude. A possible moderate gravity experiment will be discussed, and the results of a simple laboratory experiment will be given. Good agreement is demonstrated between calculation and observation for the latter.
THE EQUATION OF THE INTERFACE

Consider a liquid of density \( \rho_L \) partially filling a cylindrical container of radius \( R_C \) and length \( 2(n+1)L \). Let the cylinder contain \( n \) baffles, evenly spaced at intervals \( 2L \) (and neglect the baffle thickness.) Imagine the container to be rotating at \( \omega \) about its symmetry axis, which is antiparallel to a gravity field of strength \( g \).

Let the remainder of the container be filled with a gas of density \( \rho_G < \rho_L \), and denote the interfacial tension by \( \gamma \). Neglect compressibility.

Construct a cylindrical \( r, \phi, z \) coordinate system with \( r \) measured from the cylinder axis and \( z \) from the bottom of the cylinder. An interface shape

\[
r = r_I(z)
\]

(1)
is to be found, compatible with a state of no relative motion — both gas and liquid are to be in a state of solid corotation with the container. Thus the pressure fields in the gas and liquid must be given by

\[
\begin{align*}
P_G &= P_{0G} + \frac{1}{2} \rho_G \omega^2 r^2 - \rho_G gz \\
P_L &= P_{0L} + \frac{1}{2} \rho_L \omega^2 r^2 - \rho_L gz
\end{align*}
\]

(2)

where \( P_{0G} \) and \( P_{0L} \) are constants.

The interface need not be continuous because of the baffles. Indeed its derivatives are not continuous, and the interface itself can be continuous only in the absence of gravity. There will be \( n+1 \) individual interface equations, the \( i \)th of which is

\[
r_i = R_i - \xi_i(z)
\]

(3)

where \( R_i \) is a constant, representing the maximum distance from the rotation axis to the interface.

Figure 4 shows a single layer, and introduces some additional notation: \( \theta_i \) is the contact angle and \( z_i^T \) and \( z_i^B \) are the distances between the \( \xi_i = 0 \) position and the upper and lower bounding baffles.

The jump in pressure across each interface is equal to \( \gamma K' \), where \( K' \) denotes the mean curvature. Using standard formulas from differential geometry [2] it can be shown that

\[
K' = \frac{1}{r} + \frac{1}{r^2} \frac{d^2 \xi}{dz^2}
\]

(4)
where
\[
\Delta = 1 + \frac{d^2 z}{dz^2} 2^{1/2}
\]  
and the subscript \(i\) has been suppressed.

The interface equation in the \(i\)th layer can be written in terms of local dimensionless coordinates \(x_i\) and \(y_i\), defined by
\[
z = 2(i-1)L + z_i B + R_i x_i
\]
\[
r = R_i = R_i[1 - y_i(x_i)]
\]
The result is
\[
\frac{1}{\Delta} \left[ 1 + \frac{y_i''}{\Delta} \cdot \frac{1}{2} F_i y_i (2-y_i) \right] - G_i x_i = 1 + C_i
\]
where the prime denotes differentiation with respect to argument,
\[
F_i = \rho \omega^2 R_i^3 / \gamma \quad \quad G_i = \rho g R_i^2 / \gamma
\]
\(\rho = \rho_L - \rho_G\) is the density contrast and
\[
C_i = R_i \left( \frac{d^2 z_i}{dz^2} \right)_0 = \left( \frac{d^2 y_i}{dx_i^2} \right)_0
\]
is the axial curvature at the point \(x_i = 0\).

Because the various liquid segments are connected at their outer edges, the liquid pressure must be continuous there. This reduces to the condition that
\[
\Pi = \frac{1}{R_i} \left\{ 1 + C_i + 1/2 F_i - G_i \cdot \frac{L}{R_i} + \frac{z B_i}{R_i} \right\}
\]
be independent of \(i\). (In the absence of gravity, one solution is the intuitive one that \(R_i = R\), a constant.)
Except for the condition (10), the subscript i is not necessary and will be suppressed below where possible.

An interface shape is to be found by integrating equation (9) subject to the boundary condition that

\[ y' = \cot \theta \quad \text{at } x = z^T/R \]

\[ y' = -\cot \theta \quad \text{at } x = -z^B/R \]  

(11)

and the compatibility condition that \( z^T - z^B = 2(L/R) \).

A rescaling of equation (7) assuming \( y \to \epsilon y, z \to \epsilon z, \epsilon^2 G << 1, \epsilon^6 F = O(1) \) leads to the usual equation for a planar interface under a perpendicular gravity field [3], which can be integrated directly. That case is not relevant to the present inquiry, and it is necessary to integrate equation (7) numerically. That is most easily done by rewriting the equation as a pair of first order equations

\[ y' = Y \]

\[ Y' = (1+Y^2) \left\{ (1+Y^2)^{1/2} \left[ 1EC-Gz+Fy(1-1/2y) \right] - \frac{1}{1-y} \right\} \]  

(12)

and then applying a fourth order Runge-Kutta scheme [4] to this system. It is only necessary to note that, for small contact angles, \( Y \) becomes large near the wall. This necessitates a continual adjustment of the interval in \( z \). The easiest way to do this is to fix the interval \( \Delta s \) along the interface and to write the interval in \( z, h \), as

\[ h = \Delta s/(1 + Y^2)^{1/2} \]  

(13)

The actual integration is performed, for a given \( F \), by starting at \( z = 0 \) (where \( y = 0 = y' \)) with a given value of \( C \) and integrating first in the forward direction until \( Y = \cot \theta \) and then, starting again at \( z = 0 \), integrating in the negative direction until \( Y = -\cot \theta \). Finally the compatibility conditions are applied, in effect determining \( L/R \) a posteriori.

RESULTS AND DISCUSSION: STABILITY, AN EXPERIMENT AND GP-B

Interface profiles, and results derived from them, are obtained by choosing \( \omega \) and \( g \), picking values of \( R \) and \( C \), and then integrating equations (12), as described above, until the boundary condition (11) are met. The values of \( z^T, z^B \) and \( 2L \) are recorded, as well as the maximum values of \( y \), the capillary rise at each end, \( y^T \) and \( y^B \). For fixed \( \omega, g \) and \( R \) (hence \( F \) and \( G \) \( z^T, z^B, y^T \) and \( y^B \) all increase as \( C \) decreases. For any \( F \) and \( G \) there appears to exist a minimum value of \( C \), at which the larger \( y^T \) or \( y^B \) reaches unity, which
corresponds to a capillary rise reaching the rotation axis. Decreasing C below this value leads to an apparent change in sign in \( y'' \) before \( y' \) can reach its boundary value. Because one feels that the interface slope should be monotonic, and because the maximum capillary rise cannot exceed the maximum radius \( R \), this critical value of \( C, C_{\text{min}} \) and the value of \( L, L_{\text{max}} \) associated with it are considered to be real limits. Specifically \( 2L_{\text{max}} \) is assumed to be the maximum baffle spacing for which an interface can form.

Figure 5 shows the dimensionless maximum baffle spacing \( D = 2(L/R) \) as a function of \( F \) for \( \theta = 0 = G \). Increasing the contact angle increases \( D \). Figure 6 shows the effect of adding gravity, a decrease in \( D \).

The second type of instability is that outlined in the introduction, for which the dependence of pressure on radius is such that a perturbation of the equilibrium position tends to grow. If the \((i+1)\)st slice is moved outward, so that \( R_{i+1} \rightarrow R_{i+1} + \Delta R_{i+1} \), and the \(i\)th slice is moved inward, so that \( R_i \rightarrow R_i - \Delta R_i \), then the pressure drop from \( i+1 \) to \( i \) is given by

\[
\left( \frac{\partial P_L}{\partial R} \right)_{i+1} \Delta R_{i+1} + \left( \frac{\partial P_L}{\partial R} \right)_{i} \Delta R_{i}
\]

and if that is positive, then the displacement will grow and the pair is unstable.

It is necessary to find the rate of change of the liquid pressure with \( R \), holding \( \omega, g \) and \( L \) fixed. To that end, write the dimensional pressure

\[
P_L = P_{0L} + (P_{0G} - P_{0G}) + \frac{1}{2} \rho_L \omega^2 r^2 - \rho_L gz
\]

\[
= P_{0G} - \gamma \frac{1+C_i}{R_i} - \frac{1}{2} \rho \omega^2 R_i^2 + \rho g 2(i-1) L + z_i^B + \frac{1}{2} \rho \omega^2 R_i^2 - \rho_L g 2(i-1) L + z_i^B + x_i
\]

\[
= \frac{1}{2} \rho L \omega^2 r_i^2 - \rho_L g 2(i-1) L + z_i^B + x_i + P_{0G} - \gamma \Pi_i .
\]

The pressure in question is that at \( x = -z_i^B \) or \( x = z_i^T \). In either case \( x_i + z_i^B \) is independent of \( R_i \) and

\[
\frac{\partial P_L}{\partial R} = -\gamma \frac{\partial \Pi_i}{\partial R} = \frac{\gamma}{R_i^2} \left[ 1 + C_i - F_i - R_i \frac{\partial C_i}{\partial R_i} + G_i \frac{\partial z_i^B}{\partial R_i} \right].
\]

In any such virtual displacement of the interfaces, volume must be conserved. Thus \( \Delta R_i \propto R_i^{-1} \), and the criterion for the interchange instability is

\[
\frac{1}{R_i} \frac{\partial \Pi_i}{\partial R} + \frac{1}{R_{i+1}} \frac{\partial \Pi_{i+1}}{\partial R} < 0 .
\]
If \( G = 0 \) the situation is simplified dramatically because the equilibrium position has \( R_i = R_{i+1}, C_i = C_{i+1}, \) \( F_i = F_{i+1} \) and the neutral curve is given by

\[
R^2 \frac{\partial C}{\partial R} = 1 + F - R \frac{\partial C}{\partial R} = 0.
\]  

(17)

In the case of zero \( g \), where equation (17) applies, \( C \) has been found to be well represented in the form

\[
C = A(F) \frac{R}{L} + B(F)
\]

(18)

so that

\[
\frac{C}{R} = \frac{A}{L} + \frac{F}{R} [A'(R/L) + B']
\]

(19)

where ' denotes derivative with respect to argument, and equation (17) can be rewritten

\[
1 - F + B - 3F[A'(R/L) + B'] = 0.
\]

(20)

that curve is shown in Figure 7. One notes that this is the limiting stability curve for \( F < 1.2 \), and that it gives a stringent limit for \( F < 1.0 \).

The situation for nonnegligible \( G \) is sufficiently complicated that a general analysis seems counterproductive. A possible application to moderate \( g \) (a low \( g \) profile term on an aircraft will be explained as an example.

Consider a two layer model for which \( \gamma = 0.022 \text{ N/m}, \rho = 790 \text{ kg/m}^3, 2L = 20 \text{ mm}, \omega = 1 \text{ s}^{-1} \) and \( g = 0.098 \text{ m/s}^2 \). Figure 8 shows \( \Pi_1 \) and \( \Pi_2 \) as a function of \( R \), where 1 denotes the lower layer and the 2 the upper.

Because all the independent variables other than the void volume have been fixed, \( R \) is a function of gas volume only, and the position of any configuration is determined by that void volume. Denote that by \( V \). For \( V \) sufficiently small, the system cannot form an interface at the given spacing, \( D \). Let this critical volume be \( V_1 \). In this case \( V_1 \sim 9 \text{ ml} \), corresponding to \( R \sim 10 \text{ mm} \). Note that the \( \Pi, R \) curves do not extend to the left of this critical point.

There is a second critical volume, \( V_2 \), below which two interfaces are unstable to the interchange instability. Such a circumstance is shown by the configuration B, which has a total void volume, \( V_B \), less than that of the configuration A, \( V_A \); \( V_A \approx V_2 \). For the state B, \( \partial \Pi / \partial R < 0 \) and \( \partial \Pi_2 / \partial R < 0 \), the calculation shows that
so that the configuration is unstable. The only other configuration with the same volume is one for which the lower layer fills in to the center and the upper layer becomes an apparent first layer. This configuration is that of B', and the theory given in this paper predicts that the B' configuration is that realized.

For volumes greater than that associated with the configuration A, both ∂Π₁/∂R and ∂Π₂/∂R are positive. Such configurations will be stable. So will the alternative configuration A', in which the lower layer is full and upper acts as a lower. The present theory cannot answer the question of the state attained as V goes from V_B to V_A.

One motivation for adding gravity was the hope that useful experiments could be done in the laboratory. Ideally one would want small G and moderate F to attempt to model the proposed space application, for which G << 1 and F = O(1). That proves to be impossible for reasonable values of R and ω. If ρ, g and γ are fixed, then

\[
R = \left[\frac{Gγ}{ρg}\right]^{1/2}
\]

and

\[
ω = \left[\frac{Fγ}{ρR^2}\right]^{1/2} = \left(\frac{ρg^3}{γ}\right)^{1/2} FG^{-3/2}
\]

and, to use ethanol as an exemplary fluid, if G = 0.1 and F = 1, then R = 537 μm and ω = 1.8 \times 10^5 \text{ s}^{-1}.

While a model experiment is impossible, it seemed useful to explore the basic premises of the model in the laboratory. To that end, a set of experiments in a shallow disk were performed. All that could be observed easily was the location of the top interface (when the liquid intersects the upper boundary) and whether the liquid intersects the lower boundary. (If 2L exceeds the maximum value for a given F, the liquid will intersect only the top surface.)

The experiments were carried out in a small petri dish (diameter = 48.06 mm, depth = 8.59 mm) mounted, using double sided tape, to the top of a Genisco model C-181 turntable (maximum rotation rate 22 s⁻¹). Centering accuracy was better than 0.8 mm. Turntable speed was accurate to better than 5 parts per thousand in the range of interest. The working fluid was ethanol (Pharmco 200 proof ethyl alcohol) colored with Higgins India Ink at 2 drops for 25 ml ethanol. Textbook [3] values for density (800 kg/m³) and surface tension (0.022 N/m) were used. A contact angle of zero was assumed.

The measured dependent variable was the apparent intersection radius of the fluid with the upper boundary as a function of rotation rate and air volume. The observation was made visually by watching the position of the intersection against a marked upper surface, marked with concentric circles spaced at 2 mm intervals. Errors arise from nonconcentricity and parallax (because the circles are on the upper surface of the cover, to avoid imposing surface drag on the contact line.) These errors are estimated to be 0.5 mm. The mean position can be estimated somewhat more precisely than 0.5 mm. Positions shown are to 0.1 mm, which may be optimistic.
There is an additional systematic error which may lead one to overestimate the radius. This arises because the visible intersection line is not the true intersection line, but is some point at which the coloring is dark enough to see. The nearer the contact angle is to zero, the more pronounced this effect will be.

The most difficult parameter to control in the experiment is the air volume. Several methods were tried. The final procedure was to fill the container as nearly completely as possible, the last few ml being added through a small hole in the center of the cover through a syringe while the container was spinning at 22 s⁻¹, to center the air bubble; (2) withdraw the desired amount of fluid with the container stationary; (3) plug the hole with paraffin. The last step is necessary because, when the hole is unplugged, the ethanol is sufficiently volatile to evaporate at about 1.2 ml/hr (measured with the container stationary).

Results are shown in Figure 9 for air volumes of 2, 4, and 6 ml. The symbols denote data and the solid lines joining open circles are calculated results. The calculation predicts-bottom exposure for the 6 ml case at rotation rates above 18 s⁻¹, and the observations are consistent with the prediction. The four rightmost symbols on the upper curve showed a clearly exposed bottom. The next two were ambiguous. All the others, on all the curves, showed the bottom covered.

Finally, the consequences of this analysis for the proposed GP-B experiment mentioned in the introduction should be considered. The container for that application is a cylindrical annulus with an inner radius 0.18 m and $R_c = 0.54$ m. The overall length is 2.94 m. Reasonable values for the physical constants of liquid helium are [1]: $\rho = 145$ kg/m³ and $\gamma = 5.3 \times 10^{-1}$ N/m. The design rotation rate is to be 0.01 s⁻¹. The dewar is to be initially 90 percent full of liquid helium. This provides an estimate of $R > 0.242$ m, and gives values of $G$ and $G_F$,

$$F \approx 0.388 \quad G \approx 1.56 \times 10^{-5},$$

the latter based on the necessity of keeping local gravity to $10^{-10} \times$ normal gravity.

For these values of $F$ and $G$ one can find the maximum baffle spacing for which the helium does not reach the surface of the inner annulus. That result is $2L = 155$ mm. This, however, is not the limiting value, because the interchange instability is dominant at this small value of $F$. By repeated evaluation of equation (17) at varying values of $2L$, the critical values is found to be $2L = 18.5$ mm.
REFERENCES


Figure 1. Schematic of an ideal rotating liquid container with baffles.
Figure 2. Sketch of the type I instability, for which the interface cannot form.
Figure 3. Sketch of the type II instability, an unstable "hydrostatic" equilibrium.
Figure 4. Definitional sketch of one layer. All parameters are dimensional and positive as shown. For individual definitions, see text.
Figure 5. Neutral stability curves for zero gravity: D and F are defined in the text. D,F pairs above the curve cannot form an interface.
Figure 6. Neutral stability curves for nonzero gravity: D,F pairs above each curve cannot form interfaces at G the value shown on the curves. Points denote numerical calculations.
Figure 7. Type II neutral stability curves for zero gravity; $D,F$ pairs in the $U_{||}$ field can form interfaces, but these are unstable to the interchange instability. The curve dividing the stable region from the $U_I$ region has been redrawn from Figure 5.
Figure 8. \( \Pi \) versus \( R \) for two layers at 1 percent g. See text for a detailed explanation.
Figure 9. Experimental upper interface versus $\omega$ at fixed void volumes, $\Delta$, 2 ml void volume, $\blacksquare$ 4 ml void volume, $\blacktriangledown$ 6 ml void volume. The curves are theoretical.
Possible static configurations of liquids in rotating cylindrical containers with baffles evenly spaced in the axial direction are found. The force balance is among surface tension, centrifugal force and gravity. Two "instabilities" are found in this parameter space: type I is the inability of the liquid to form an interface attached to the baffles; type II is the inability for multi-baffled configurations to sustain interfaces between each pair of baffles. The type I analysis is confirmed through a laboratory based equipment. Applications to orbiting containers are discussed.