

N84 15599 Dg

A NASTRAN DMAP PROCEDURE FOR CALCULATION OF BASE  
EXCITATION MODAL PARTICIPATION FACTORS

by

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SUMMARY

A common analysis procedure at the Goddard Space Flight Center is to analyze a structure to predict responses that will occur during a sinusoidal or random vibration test in which a predetermined motion is input by the shaker at the shaker/structure interface. Often the loads produced in these tests are a controlling factor in the design of the structure. The response analyses are generally performed in two stages: a modal analysis to determine the undamped eigenvalues and eigenvectors of the structure "cantilevered" at the interface where the structure will attach to the shaker, followed by a subsequent steady state frequency response analysis using the modal information. The output of the modal analysis, while giving information that is useful in comparing with the eigenvalues and eigenvectors obtained in the test, does not give a sufficiently clear indication of which eigenvectors will be important contributors in the subsequent frequency response analysis. The reason for this is that the vibration test loading is really a distributed inertial loading on the whole structure known in terms of the input acceleration as opposed to some known force at a given location in other forced response analyses.

By using the eigenvector data obtained in the normal mode analyses, a modal participation factor can be calculated which gives a clear indication of the relative importance of each calculated mode in terms of its response to any base acceleration input. This factor, like the generalized mass or generalized stiffness already calculated in NASTRAN, is a property of the structure. In addition, however, it is also a property of the base excitation.

This paper presents a technique for calculating the modal participation factors for base excitation problems using a DMAP alter to the NASTRAN real eigenvalue analysis Rigid Format. The DMAP program automates the generation of the "seismic" mass to add to the degrees of freedom representing the shaker input directions and calculates the modal participation factors. These are shown in the paper to be a good measure of the maximum acceleration expected at any point on the structure when the subsequent frequency response analysis is run.

Structures that are analyzed to predict what their responses will be in a vibration test quite often have one feature in common; namely that they are rigidly attached to a vibration shaker at a number of interface degrees of freedom. Such situation, therefore, can be analyzed by multi-point constraining all of the interface degrees of freedom to any convenient single interface grid point with specification of the shaker motion at this grid point. The shaker motion, or base excitation, as it were, is then specified for a nonredundant set of degrees of freedom (DOF) that can easily be identified on a NASTRAN SUPORT bulk data card. In the following derivation of the base excitation modal participation factors, it is assumed that the structure to be analyzed is to have a base motion at a nonredundant set of DOF and that the interface DOF have been constrained to this set of nonredundant DOF. In addition, the derivation is restricted to systems with real eigenvectors.

## DERIVATION OF MODAL PARTICIPATION FACTORS

To derive an equation for the modal participation factors due to base excitation consider a general structural model, as shown in Figure 1. Presumably this structure would be rigidly connected to the shaker at several points, denoted in this example as 1 through 4. This situation can be simulated by creating some central point,  $r$ , and connecting 1 through 4 to  $r$  via rigid elements. Then  $r$  is a set of nonredundant DOF at which the shaker motion can be specified. The equations of motion of this structure, using the displacement set notation of NASTRAN, and ignoring damping for the moment, are \*

$$\mathbf{M}_{qq} \ddot{\mathbf{u}}_q + \mathbf{K}_{qq} \mathbf{u}_q = \mathbf{P}_q \quad (1)$$

Where  $\mathbf{P}_q$  contains the loads necessary to drive the  $r$  DOF by the specified base motions imparted by the shaker.

Following elimination of multi-point constraints (via MPC or rigid element cards), single point constraints (SPC's) and DOF omitted via Guyan reduction (OMIT's),  $\mathbf{u}_q$  in equation (1) is reduced to  $\mathbf{u}_a$  and the equations are

$$\mathbf{M}_{aa} \ddot{\mathbf{u}}_a + \mathbf{K}_{aa} \mathbf{u}_a = \mathbf{P}_a \quad (2)$$

where

$$\mathbf{u}_a = \begin{Bmatrix} \mathbf{u}_1 \\ \mathbf{u}_r \end{Bmatrix} \quad (3)$$

\*Bold face letters will be used to designate matrices, including row and column vectors.

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The  $u_r$  DOF are identified on SUPORT bulk data cards and  $u_l$  are all remaining DOF in  $u_a$ . Partitioning equation (2):

$$\begin{bmatrix} M_{ll} & M_{lr} \\ M_{lr}^T & M_{rr} \end{bmatrix} \begin{bmatrix} \ddot{u}_l \\ \ddot{u}_r \end{bmatrix} + \begin{bmatrix} K_{ll} & K_{lr} \\ K_{lr}^T & K_{rr} \end{bmatrix} \begin{bmatrix} u_l \\ u_r \end{bmatrix} = \begin{bmatrix} 0 \\ P_r \end{bmatrix} \quad (4)$$

where  $P_r$  are the forces exerted by the shaker on the structure and result in the base motions  $u_r$ . The forces  $P_r$  are not known, a priori; rather the motions (accelerations, velocities or displacements) at the  $r$  DOF are known. In this context the forces in  $P_r$  are really forces of constraint which are necessary to produce the desired base motion. Notice in equation (4) that there are no applied loads for the  $l$  DOF. The situation being simulated here is one in which the only excitation comes about due to the shaker driving the structure at the  $r$  DOF.

The DOF  $u_l$  can be considered to be made up to two terms:

$$u_l = u_l^c + u_l^r \quad (5)$$

where

$u_l^c$  = displacements of the  $l$  DOF when the  $r$  DOF are fixed

$u_l^r$  = rigid body displacements of the  $l$  DOF due to the  $r$  DOF

Thus  $u_l^c$  would be the displacement DOF for a structure that was "cantilevered" or fixed at the  $r$  DOF. The displacements,  $u_l^r$  at the  $l$  DOF due to the  $r$  DOF not being fixed can be determined simply from geometric considerations and is

$$u_l^r = D_{lr} u_r \quad (6)$$

where matrix  $D_{lr}$  is data block DM generated by module RBMG3 in NASTRAN whenever SUPORT bulk data cards defining the existence of  $r$  DOF is present. The columns of  $D_{lr}$  are rigid body displacements at the  $l$  DOF due to a unit motion at one of the  $r$  DOF. They are rigid body displacements due to the fact that the  $r$  DOF are a nonredundant set. Substituting (5) and (6) into (4) the two matrix equations for the unknowns  $u_l^c$  and  $P_r$  (since  $u_r$  is known) are

$$M_{ll} \ddot{u}_l^c + K_{ll} u_l^c = -(M_{ll} D_{lr} + M_{lr}) \ddot{u}_r \quad (7)$$

$$P_r = M_{lr}^T \ddot{u}_l^c + K_{lr}^T u_l^c + (M_{lr}^T D_{lr} + M_{rr}) \ddot{u}_r \quad (8)$$

where use of the fact that

$$K_{rr} = K_{lr}^T K_{ll}^{-1} K_{lr}$$

$$D_{lr} = -K_{ll}^{-1} K_{lr}$$

and thus

$$K_{ll} D_{lr} + K_{lr} = 0 \quad (9)$$

has been made. This is discussed in the NASTRAN Theoretical Manual, Reference 1. Equation (7) is the well known form for base acceleration input where the problem is solved for the motions  $u_l^c$  as if the base were fixed but with an equivalent inertia loading on each of the DOF that is the rigid body inertia force due to the base moving. Equation (8) merely solves for the forces required to drive the base at the specified motion. A better form for  $P_r$  results if equations (9) and (7) are used to eliminate the stiffness term in (8). The result is

$$P_r = (M_{ll} D_{lr} + M_{lr}) \ddot{u}_l^c + \bar{M}_{rr} \ddot{u}_r \quad (10)$$

where

$$\bar{M}_{rr} = D_{lr}^T M_{ll} D_{lr} + D_{lr}^T M_{lr} + M_{lr}^T D_{lr} + M_{rr} \quad (11)$$

is the rigid body mass matrix of the structure relative to the SUPORT DOF and is data block MR generated by NASTRAN in module RBMG4 when SUPORT bulk data cards are used. Equation (7) can be diagonalized with a modal transformation using the "cantilevered" modes of the structure (that is, the modes of the structure when constrained at the  $r$  DOF). Define

$\phi_x^j = j^{\text{th}}$  eigenvector of structure when constrained at  $r$  DOF.

$\Phi_{ln} = [\phi_x^1 \ \phi_x^2 \ \dots \ \phi_x^n]$  = matrix whose columns are eigenvectors,  $\eta$

Then the modal transformation is

$$u_x^c = \Phi_{ln} \xi_n = [\phi_x^1 \ \phi_x^2 \ \dots \ \phi_x^n] \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{bmatrix} = \sum_{j=1}^n \phi_x^j \xi_j \quad (12)$$

where  $\xi_n$  is a vector of modal DOF  $\xi_i$  ( $i=1, 2, \dots, n$ ) with  $n$  being the number of modes used in the dynamic analysis. Substituting (12) into (7) and premultiplying by  $\Phi_{\lambda n}^T$  the result is

$$m_{nn} \ddot{\xi}_n + k_{nn} \xi_n = - \Phi_{\lambda n}^T (M_{\lambda\lambda} D_{\lambda r} + M_{\lambda r}) \ddot{u}_r \quad (13)$$

where

$$m_{nn} = \Phi_{\lambda n}^T M_{\lambda\lambda} \Phi_{\lambda n} \quad (14)$$

and

$$k_{nn} = \Phi_{\lambda n}^T K_{\lambda\lambda} \Phi_{\lambda n} \quad (15)$$

are both diagonal matrices with  $m_{nn}$  containing the generalized masses (denoted as  $M_j$ ) and  $k_{nn}$  the generalized stiffnesses (denoted as  $k_j$ ) of the cantilevered modes.

With the addition of damping, each individual modal equation in equation (13) is, after dividing through by the generalized mass:

$$\ddot{\xi}_i + 2\gamma_i \omega_j \dot{\xi}_i + \omega_j^2 \xi_i = - \frac{1}{m_j} \phi_c^i (M_{\lambda\lambda} D_{\lambda r} + M_{\lambda r}) \ddot{u}_r \quad (16)$$

The damping factor,  $\gamma_i$ , is specified through use of the TABDMP bulk data card.

Define

$$f_r^i = \frac{1}{m_j} \phi_{\lambda}^{i,T} (M_{\lambda\lambda} D_{\lambda r} + M_{\lambda r}) \quad (17)$$

The row matrix  $f_r^i$  contains as many terms as there are DOF identified on the SUPPORT card, that is, those that will be given a known motion from the shaker. Each term in  $f_r^i$  is identified as being a modal participation factor since the solution to equation (16) for some specified base motion at one DOF will be proportional to the corresponding term in  $f_r^i$ .

As an example of this consider the situation when the base motion is a steady state sinusoidal input at some resonant frequency  $\omega_j$  with an acceleration (complex) magnitude  $\ddot{u}$ . In this case:

$$\ddot{u}_r = \ddot{u}_r e^{i\omega_j t}$$

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The solution to equation (16) for the modal acceleration  $\ddot{\xi}_j$  is

$$\ddot{\xi}_j = \ddot{\xi}_j^0 e^{i\omega t}$$

where

$$\ddot{\xi}_j = -i \left( f_r^i \phi_j \ddot{u}_r \right) \quad (18)$$

and

$$\phi_j = \gamma_j$$

is the modal amplification in mode  $j$ . Equation (18) indicates the modal acceleration is, of course, lagging the base acceleration input by 90 degrees.

The significance of the modal participation factors in  $f_r^i$  is evident from equation (18). At resonance, the modal acceleration magnitude is equal to the product of the modal amplification, the participation factor and the magnitude of the input shaker acceleration. For systems with relatively small damping, the magnitude of the accelerations at grid points throughout the structure can be approximated by a single mode response (the resonating mode) so that, for example, using only one mode in equation (12)

$$\begin{aligned} \ddot{u}_x(\omega) &\approx \phi_x^i \ddot{\xi}_j \\ &= -i \phi_x^i f_r^i \phi_j \ddot{u}_r \end{aligned} \quad (19)$$

The modal participation factors, together with the eigenvectors calculated in a real eigenvalue analysis can therefore be used to obtain estimates of what the magnitude of the grid point accelerations will be in a subsequent forced base response analysis. The modal participation factors can be easily calculated during the real eigenvalue analysis run (in Rigid Format 3, for example). A matrix of all participation factors for all modes is:

$$F_{nr} = \left\{ f_r^i \right\} = m_{nn}^{-1} \Phi_{kn}^T (M_{kk} D_{kr} + M_{kr}) \quad (20)$$

That is, each row of  $F_{nr}$  is an  $f_r^i$  (the participation factors for one mode). Each column of  $F_{nr}$  gives the participation factors for all modes for one DOF of base acceleration. For example, column 1 of  $F_{nr}$  contains the participation factors for base acceleration in the first DOF entered on the SUPORT card.

The modal participation factors can be used to easily calculate what is commonly referred to as modal mass. Consider equation (10) for the base loads (the shaker forces required to produce  $\ddot{u}_r$ ). Using the modal transformation in equation (12) the loads  $P_r$  in equation (10) can be written as

$$P_r = \sum_{j=1}^n P_r^j + \bar{M}_{rr} \ddot{u}_r \quad (21)$$

where

$$P_r^j = (M_{\lambda\lambda} D_{\lambda r} + M_{\lambda r})^T \phi_{\lambda}^j \ddot{\xi}_j$$

Using equation (18), the complex amplitude of  $P_r^j$  is  $\bar{P}_r^j$  where

$$P_r^j = \bar{P}_r^j e^{i\omega t}$$

$$P_r^j(\omega_j) = -i (M_{\lambda\lambda} D_{\lambda r} + M_{\lambda r})^T \phi_{\lambda}^j f_r^j Q_j \ddot{u}_r$$

Using equation (17):

$$\begin{aligned} P_r^j(\omega_j) &= -i (M_{\lambda\lambda} D_{\lambda r} + M_{\lambda r})^T \phi_{\lambda}^j \frac{1}{m_j} \phi_{\lambda}^{jT} (M_{\lambda\lambda} D_{\lambda r} + M_{\lambda r}) Q_j \ddot{u}_r \\ &= -i m_{rr}^j Q_j \ddot{u}_r \end{aligned} \quad (22)$$

where

$$m_{rr}^j = (M_{\lambda\lambda} D_{\lambda r} + M_{\lambda r})^T \phi_{\lambda}^j \frac{1}{m_j} \phi_{\lambda}^{jT} (M_{\lambda\lambda} D_{\lambda r} + M_{\lambda r}) \quad (23)$$

is the  $j^{\text{th}}$  mode modal mass matrix. It is a square matrix of size  $r$  and from equations (21) and (22) it is seen that its use is in determining the  $j^{\text{th}}$  mode contribution to the base forces (required shaker forces). For systems with small damping the forces  $P_r$  at a resonance  $\omega_j$  would be approximately  $P_r^j$  so that equation (22) would give an approximation to the total forces. It is interesting to note that if the sum of all modal mass matrices is denoted as

$$\sum_{j=1}^n m_{rr}^j = m_{rr} \quad (24)$$

and if all of the eigenvectors are used ( $n=l$ ) then:

$$m_{rr} = (M_{il} D_{lr} + M_{lr})^T M_{ll}^{-1} (M_{il} D_{lr} + M_{lr}) \quad (25)$$

If there are no masses at any of the  $r$  set of D.O.F then

$$m_{rr} = \bar{M}_{rr} \quad (26)$$

That is, the sum of the modal masses is the rigid body mass matrix (data block MR from module RBMG4) when all masses are at the  $r$  set of DOF. Thus, modal masses can be calculated in a real eigenvalue analysis and can be used to estimate how many modes will have to be used in the subsequent base excitation dynamic analysis to make sure a high percentage of the complete system mass is included (since modal truncation, in effect, truncates mass). This would be useful in transient analysis where the single mode response at resonance is not a valid approximation. The modal mass matrix  $m_{rr}^i$  can easily be determined from the modal participation factors  $f_r^i$ . From equations (17) and (23) it is seen that

$$m_{rr}^i = m_j f_r^j f_r^j \quad (27)$$

#### DMAP ALTER DESCRIPTION

The DMAP Alter package for calculating the modal participation factors and modal masses is described in detail in Appendix A. The calculations are done, of course, in the real eigenvalue analysis NASTRAN Rigid Format and in particular in either COSMIC Rigid Format 3 or MSC Rigid Formats 3 or 25. The description in Appendix A is an alter package for MSC Rigid Format 25 or COSMIC Rigid Format 3. However, calculation of the modal masses in the COSMIC version is not convenient at this time due to a limitation in the COSMIC version of module DIAGONAL.

As recommended in the MSC documentation, Reference 2, base motion problems are solved by putting all DOF which are to have a specified motion on SUPORT cards and also connecting fictitious large masses to these DOF. If the large masses are very large in comparison to the complete structure mass then, when a force is applied to these DOF, their acceleration will be determined to be very nearly the force divided by the large mass. Making the force equal to the large mass times the desired acceleration at these DOF will then result in the NASTRAN solution for these accelerations to be the desired base accelerations.

Thus, the first portion of the DMAP Alter automates the inclusion of the large (seismic) masses at the DOF identified on SUPORT bulk data cards. The large mass is taken as  $10^8$  times the diagonal of the rigid body mass matrix (data

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block MR calculated in module RBMG4). The bulk data deck for the real eigenvalue analysis with the DMAP Alter program should include SUPORT cards for the nonredundant DOF at which base motion is desired. As mentioned previously, the actual base DOF that are connected to the shaker are to be rigidly constrained to some nonredundant set of DOF that are then included on SUPORT cards. (and not single point constrained) to identify them as the base motion DOF.

The second part of the DMAP Alter calculates the matrix  $F_{nr}$  which contains the modal participation factors. These are printed out as matrix data block FNR.

The third part of the DMAP alter calculates the diagonal terms of the modal mass matrix  $m_{rr}$ . These can only be calculated in the MSC version since they require a feature in the DIAGONAL module (namely raising all elements of a matrix to a power) not available in the COSMIC DIAGONAL module. The matrix which contains the diagonal terms of  $m_{rr}^j$  is data block MJRR and row  $j$  of this matrix contains the diagonal terms for mode  $j$ . This matrix is also printed out in terms of percent of the corresponding mass in the rigid body mass matrix as matrix data block MJRRP. Addition of each column should give a value of 100 if all modes are used and if there are no masses (other than the fictitious large mass) at the  $r$  set of DOF.

SAMPLE PROBLEM

Figure 2 shows a sample problem using the DMAP Alter for a beam which is to be given a base motion at grid point 11. The aluminum beam is 2.54m (100 inches) long with a mass of 9.1kg (beam weight is 20lb). Section properties for the beam are:  $.0013m^2$  ( $2.0in^2$ ) area and  $8.33 \times 10^{-7}m^4$  ( $2.0in^4$ ) moment of inertia. The DOF which are to be given base motion should be the higher numbered DOF in the problem to avoid numerical problems with the large fictitious masses added to these DOF. The fact that the sample problem initially has a nonredundant interface with the shaker (at grid point 11) is of no consequence. As mentioned earlier, if the interface where the structure is to be connected to the shaker is redundant, the interface points can be rigidly connected to some one point that is then called the base motion point.

Appendix B contains selected pages from the NASTRAN run for this sample problem including the bulk data deck listing. The input data is seen to resemble any real eigenvalue run for the beam cantilevered modes except that some of the DOF at the cantilever point (grid point 11) are identified on a SUPORT bulk data card instead of constrained. For the sample problem these were taken to be DOF 1, 3 and 5 signifying that these will be the DOF that would be given a base motion in subsequent dynamic analyses. The remaining DOF at grid point 11 are constrained.

As mentioned previously, the DMAP Alter automates the inclusion of the large fictitious masses at the DOF identified in the  $r$  set. The values of these masses are contained in matrix data block MAAB output in the DMAP Alter and included in Appendix B. As seen from MAAB, the large masses are  $10^8$  times the rigid body masses output from the grid point weight generator (taking into account the latter is in weight units and MAAB is in mass units).

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The eigenvalues for the sample problem are included in Appendix B. The first three are rigid body modes due to the three DOF on the SUPORT card. The remaining modes are those that would have been found if the SUPORT DOF had been constrained instead. This is due to the fact that large masses are connected to the SUPORT DOF in order that the elastic modes will be the same as if these DOF were constrained. Several of the modes are shown on Figure 3.

The modal participation factors are shown in Appendix B as the matrix data block FNR. These are also shown in Table 1. Each column in the table is a column of  $F_{nr}$  and there are 3 columns; one each for the SUPORT DOF. Each row in the table shows the terms in  $F_{nr}$  (which are the rows  $f_v^i$ ) which are the participation factors for the mode corresponding to that row of the table.

The second column of Table 1 gives the participation factors for base motion in DOF 3 at grid point 11; namely lateral shaker motion. From Table 1 it is seen that mode 1, with a frequency of 10.9hz, has a participation factor of 1.56 while mode 5 does not contribute at all. This is due to the fact that mode 5, as seen from Figure 3 is an uncoupled longitudinal mode of the beam and could have been so identified by visual inspection of the mode shapes for this simple beam problem. For more complicated structures the participation factors are more helpful in identifying significant modes for various base excitations than the visual inspection of the eigenvector data.

The use of the participation factors in estimating grid point accelerations that would be found in a steady state sinusoidal shaker simulation is demonstrated on Figure 4. The figure shows comparison of data from an actual frequency response run using Rigid Format 8 along with the estimated accelerations based on the modal participation factors in table 1.

The solid line is the acceleration at the beam tip as a function of the shaker frequency,  $\omega$ , with an input acceleration in the Z lateral direction of 1.5g's at grid point 11. The simple calculation using the participation factors and a single mode response with equation (19) gives acceleration magnitudes indicated by the data points on Figure 4. The good agreement shows the value of knowing the participation factors when the modal analysis is run rather than waiting to determine the frequency response in a later run. The data point for the 490.6hz mode on Figure 4 is not a mode in resonance since the 490.6 mode is a longitudinal mode and the base response in the example is lateral.

The output in Appendix B also shows the diagonals of the modal mass matrices  $m_{rr}^i$  as output matrix MJRR. Expressed as a percent of the corresponding mass term in the complete structure rigid body mass matrix,  $M_{rr}$ , the result is output matrix MJRRP. As mentioned previously, if there are no masses at the DOF (other than the fictitious large mass put there for use calculating the participation factors) and if all modes of the structure are used, the columns of MJRRP should add to 100. Table 2 shows the terms from MJRRP. As with the modal participation factors these also clearly indicate which modes are significant in terms of base motion in certain directions. For general base excitation problems, the data from MJRRP is useful in identifying significant modes to be included in the analysis. The columns of Table 2 do not add up to 100 percent for base motion DOF 1 or 3 since the set did have mass. The beams' mass was distributed to all eleven grid points so that the end grid points (1 and 11) each have 5 percent of the total.

## CONCLUSIONS

Knowledge of the modal participation factors for base motion can be useful in determining the significant modes to be used in forced base motion problems. Obtaining the participation factors when the modal analysis is run, allows estimates to be made of the subsequent structure responses to the base motion prior to running the base motion dynamic analysis. The participation factors use for steady state base motion frequency response analyses was demonstrated; however they are equally as valid for transient analysis.

## REFERENCES

1. The NASTRAN Theoretical Manual (Level 17.5), NASA SF-221(05), December 1978
2. MSC NASTRAN Applications Manual, MSR-32 (VAX), The MacNeal/Schwendler Corporation, May/June 1981

Table 1  
 Modal Participation Factors  
 for Bead Problem

Modal Participation Factors From  
 for Base Acceleration at G.P.11 In:

Mode No.	Frequency (Hz)	DOF-1	DOF-3	DOF-5
4	10.94	0.	1.5569	-113.590
5	67.82	0.	-.8446	17.800
6	188.0	0.	.4736	-6.124
7	364.5	0.	-.3137	2.923
8	490.6	1.2706	0.	0.
9	595.9	0.	.2161	-1.590
10	878.2	0.	-.1593	.9801
11	1202.	0.	.1371	-.7370
12	1460.	-.4165	0.	0.
13	1545.	0.	.1154	-.5618
14	1861.	0.	-.0806	.3669
15	2086.	0.	-.0453	.1987
16	2393.	-.2414	0.	0.
17	3267.	-.1632	0.	0.
18	4061.	.1171	0.	0.
19	4755.	-.0854	0.	0.
20	5332.	.0613	0.	0.
21	5777.	-.0414	0.	0.
22	6081.	.0240	0.	0.
23	6234.	-.0079	0.	0.

Table 2

Modal Masses  
for Beam Problem

Diagonals of Modal Mass Matrices\*  
for Base Acceleration at G.P.11 In:

Mode No.	DOF-1	DOF-3	DOF-5
4	0.	61.073	97.030
5	0.	18.854	2.4995
6	0.	6.4685	.3228
7	0.	3.3013	.0856
8	80.724	0.	0.
9	0.	1.9882	.0321
10	0	1.3149	.0149
11	0.	.9087	.0078
12	8.6749	0.	0.
13	0.	.6166	.0044
14	0.	.3585	.0022
15	0.	.1171	.0007
16	2.9142	0.	0.
17	1.3315	0.	0.
18	.6854	0.	0.
19	.3647	0.	0.
20	.1878	0.	0.
21	.0858	0.	0.
22	.0288	0.	0.
23	<u>.0031</u>	<u>0.</u>	<u>0.</u>
Totals	95.0002	95.0008	100.000

\*Expressed as percent of total rigid body mass of structure.

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Note: Points 1-4 Are Actual  
Structure/Shaker Rigid  
Interface Points,  
Simulated by Single Point  $r$

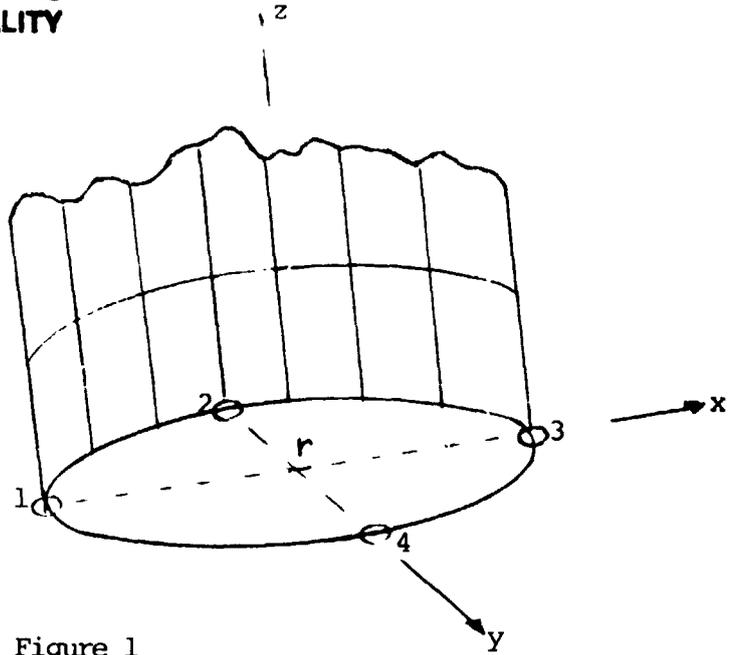


Figure 1

General Base Excitation Problem

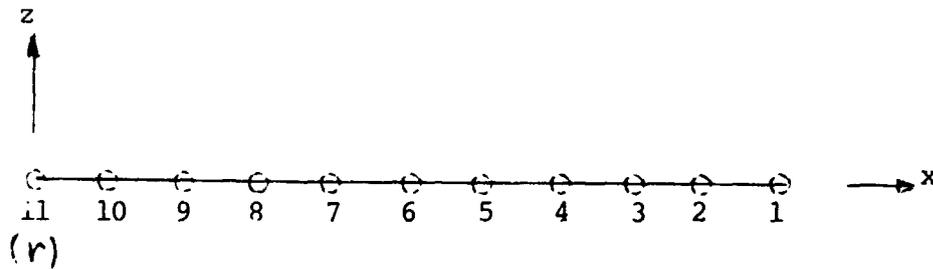


Figure 2

Sample Beam Problem  
With Base Excitation At Grid Point 11

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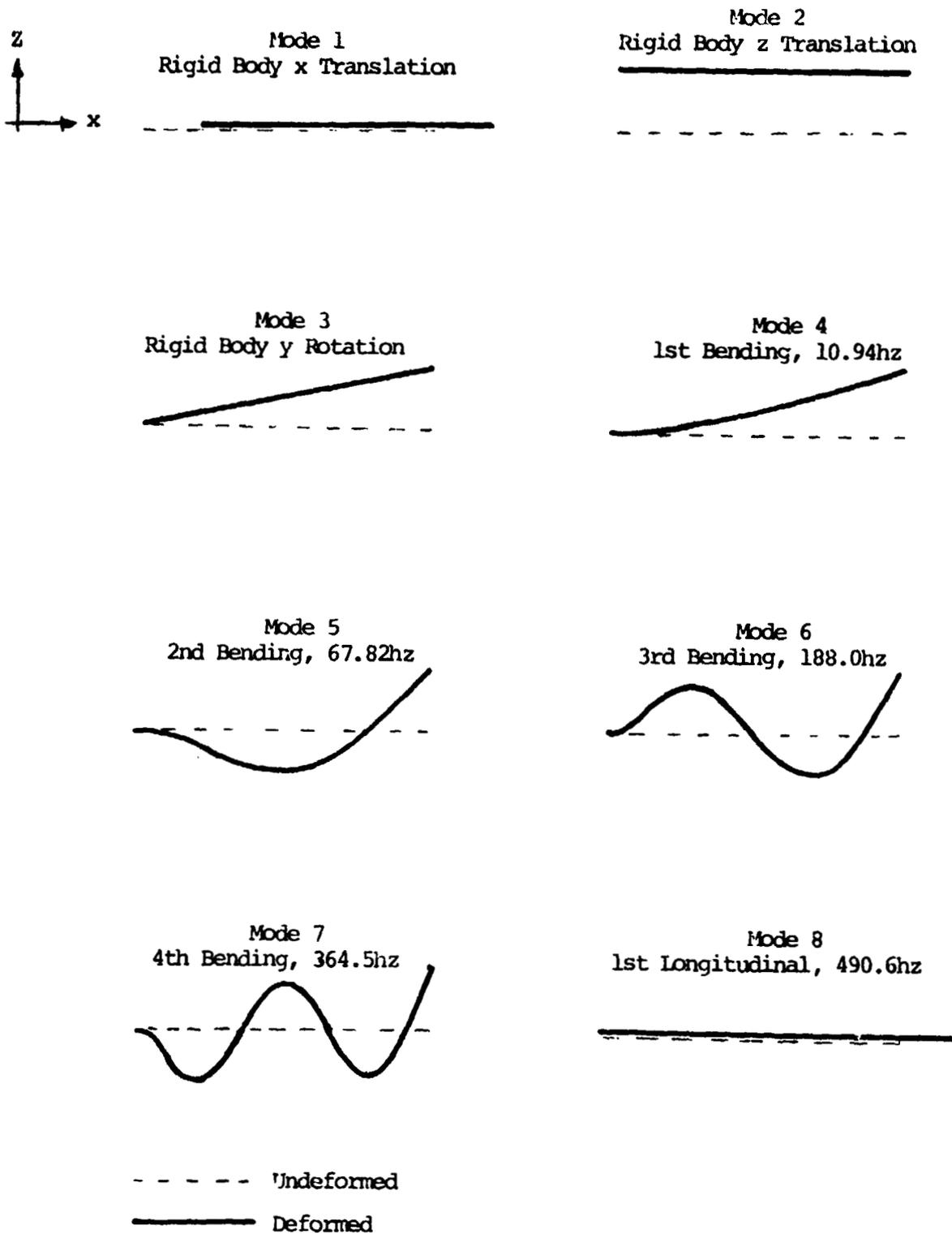


Figure 3  
Beam Mode Shapes

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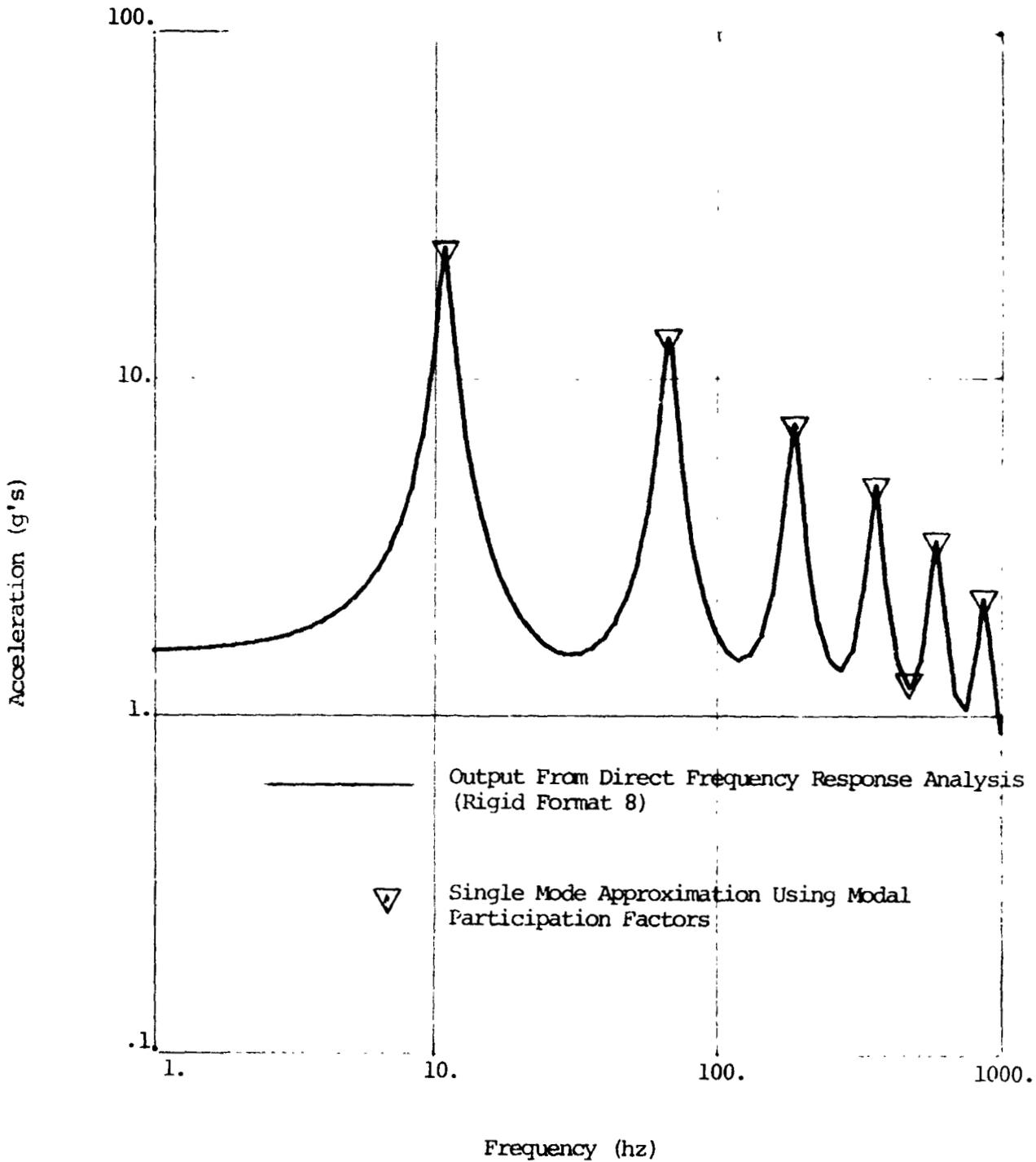


Figure 4

Acceleration at G.P. 1 of Beam due to  
1.5g Sinusoidal Base Acceleration at G.P. 11

DMAP Alter for Determining Base Excitation Modal Participation Factors  
(Real Eigenvalue Analysis Rigid Formats)

```

1  ALTER a1
2  PARAM //C,N,NOP/V,N,T=-1 $
3  PARAMR //C,N,DIV/V,N,GRAVO/C,N,1.0/V,Y,WTMASS=-1.0 $
4  PARAMR //C,N,COMPLEX/V,N,OUTR/V,N,GRAVO/C,N,0.0/V,N,GRAV $
5  DIAGONAL MR/MRD/C,N,SQUARE $
6  ADD MRD,/WRD/V,N,GRAV $
7  ADD MRD,/MBASE/C,N,(1.E8,0.) $
8  VEC USET/CP/C,N,A/C,N,L/C,N,R $
9  MERGE, , ,MBASE,CP,/MAAB/C,N,-1 $
10 MATGPR GPL,USET,SIL,MAAB//C,N,A/C,N,A $
11 ADD MAAB,MAA/MAAT $
12 EQUIV MAAT,MAA/T $
13 ADD MR,MBASE/NEWMR $
14 EQUIV NEWMR,MR/T $
15 ALTER a2
16 PARTN PHIA, ,CP/PHIL,PHIR, ,/C,N,1/C,N,2/C,N,2/C,N,2 $
17 GKAM, ,PHIL,MI,LAMA, , , ,CASECC/MGNN, ,KGNN,PHILN/C,N,-1/C,N,-0/
C,Y,LFREQ=.000001/C,Y,HFREQ=1.E20/C,N,-1/C,N,-1/C,N,-1/
V,N,NOCUP/V,N,FMODE $
18 ADD MGNN,/WGNN/V,N,GRAV $
19 MPYAD MLL,DM,MLR/ML3/C,N,0/C,N,1/C,N,0/C,N,2 $
20 ADD ML3,/WL3/V,N,GRAV $
21 MPYAD PHILN,WL3,/FNRO/C,N,1/C,N,1/C,N,0/C,N,2 $
22 SOLVE WGNN,FNRO/FNR/C,N,1 $
23 MATPRN FNR, , , // $
24 DIAGONAL FNR/MJRRO/C,N,WHOLE/C,N,2.0 $
25 MPYAD WGNN,MJRRO,/MJRR $
26 MATPRN MJRR, , , $
27 TRNSP MJRR/MJRRT $
28 SOLVE WRD,MJRRT/MJRRDT/C,N,1 $
29 ADD MJRRDT,/MJRRPT/C,N,(100.,0.) $
30 TRNSP MJRRPT/MJRRP $
31 MATPRN MJRRP, , , // $
32 ENDALTER

```

I. Generation of Large Masses for SUPORT Points for Base Motion Analyses

1. DMAP statements 2 through 14 are inserted after module RBMG4 has run. For COSMIC Rigid Format 3 (April 1982 version) the Alter is at  $a_1 = 69$  and for MSC Rigid Format 25 (Dec. 1981, version 61B)  $a_1$  is 140.
- 2-4. PARAM modules set the values of several parameters to be used. Parameter GRAV is the inverse of the regular WTMASS parameter.
5. DIAGONAL extracts the diagonal of the rigid body mass matrix, MR ( $\bar{M}_r$  in equation (11)) and calls the result matrix data block MRD.
6. Matrix data block WRD is MRD multiplied by parameter  $GRAV=1/WTMASS$  so that WRD is generally in weight units if a PARAM WTMASS card is used in the bulk data.
7. Matrix data block MBASE is  $10^8$  times MRD. These give the large fictitious masses which are put on the r DOF for base motion problems.
- 8-9. Module MERGE, using partitioning vector CP generated in module VEC, expands the large fictitious mass matrix, MBASE, up to full "a" set size. Output matrix MAAB has the large fictitious masses for the r DOF but in a matrix of "a" size.
10. The large masses are printed out and can be seen to only exist at the r DOF.
- 11-12. The large masses are added to the normal mass matrix MAA and the result equivalenced back to MAA so that when MAA is called for in later modules the sum of the normal MAA and the large masses MAAB will be used.
- 13-14. The real eigenvalue analysis module, READ, uses the rigid body mass matrix, MR, when there are SUPORT DOF. Since the large masses were added at the r DOF, the sum of the MR (from the finite element model) and the large masses (in MBASE) is to be used for MR in module READ.

II. Generation of Modal Participation Factors

15. Following module READ and the output of the eigenvalues and eigenvalue analysis summary, the statements 16 through 23 are inserted for calculation of the modal participation factors. For COSMIC Rigid Format 3 (April 1982) the alter is at  $a_2 = 77$  and for MSC Rigid Format 25 (Dec. 1981 version 613)  $a_2$  is 146.

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- 16-17. Module PARTN extracts the eigenvectors for the  $k$  DOF and module GKAM extracts from this the elastic ( $\omega > 0$ ) eigenvectors. The result is called PHILN and is matrix  $\Phi_{kn}$  used in equation (12). Matrix data block MGNN is the matrix of generalized masses,  $M_{nn}$ , in equation (14).
18. Convert MGNN to weight units if a PARAM WTMASS bulk data card is present.
- 19-20. Calculate  $(M_{ii}D_{ir} + M_{ir})$  and call the result matrix data block ML3. Note that matrix data block DM is matrix  $D_{ir}$ . Matrix data block WL3 is ML3 in weight units if PARAM WTMASS is present in bulk data.
- 21-23. Calculate  $F_{nr} = m_{nn}^{-1} \Phi_{kn}^T (M_{ii}D_{ir} + M_{ir})$  and print out  $F_{nr}$ , the matrix of modal participation factors.

III. Statements 24 through 31 calculate and print the modal masses. These cannot currently be calculated in COSMIC NASTRAN since the COSMIC version of module DIAGONAL does not have a feature which allows calculation of a matrix which contains the terms of some input matrix raised to some specified power.

24. Module DIAGONAL calculates a matrix, MJRRO, whose terms are the squares of the terms in the modal participation factor matrix, FNR.
- 25-26. By multiplying MJRRO by the generalized mass (or weight) matrix the diagonal terms of the modal mass matrices are obtained. Matrix MJRR is so arranged that its  $j^{\text{th}}$  row is the diagonal of the  $j^{\text{th}}$  modal mass matrix,  $m_{rr}^i$ , in equation (27). This matrix is printed out.
- 27-31. The modal mass terms are divided by the corresponding rigid body mass terms for the complete structure and multiplied by 100. The resulting matrix, MJRRP, is printed out and gives the diagonals of the modal masses (in the rows of MJRRP) in percent of the rigid body mass. If there are no masses (other than the large fictitious masses) at the  $r$  DOF then the columns of MJRRP should add to 100 if all modes are used.

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Appendix B

Sample Problem Real Eigenvalue Analysis  
and Modal Participation Factor Calculation

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N A S T R A N   E X E C U T I V E   C O N T R O L   D E C K   E C H O

```
ID TEST,DMAPBASE
APP DIS,
SOL 25.0
TIME 2
DIAG 8,14
ALTER 140
PARAM //C.N.NOP/V.N.T=-1 $
PARAMR //C.N.DIV/V.N.GRAVD/C.N.1.O/V.Y.WTMAS=-1.0 $
PARAMR //C.N.COMPLEX/V.N.OTR/V.N.GRAVD/C.N.O.O/V.N.GRAV $
DIAGONAL MR/MRD/C.N.SQUARE $
ADD MRD./WRD/V.N.GRAV $
ADD MRD./MBASE/C.N.(1.E8.O.) $
VEC USET/CP/C.N.A/C.N.L/C.N.R $
MERGE. ...MBASE.CP./MAAB/C.N.-1 $
MATGPR GPL,USET.SIL,MAAB//C.N.R/C.N.R $
ADD MAAB,MAA/MAAT $
EQUIV MAAT,MAA/T $
ADD MR.MBASE/NEWMR $
EQUIV NEWMR,MR/T $
ALTER 146
PARTN PHIA.,CP/PHIL,PHIR.,/C.N.1/C.N.2/C.N.2/C.N.2 $
GKAM. ,PHIL,MI,LAMA,....CASECC/MGNN.,KGNN,PHILN/C.N.-1/C.N.O/
C.Y.,LFREQ=.00001/C.Y.,HFREQ=1.E20/C.N.-1/C.N.-1,C.N.-1/
V.N.NOCUP/V.N.FMODE $
MGNN./WGNN/V.N.GRAV $
MLL,DM,MLR/ML3/C.N.O/C.N.1/C.N.O/C.N.2 $
ML3./WL3/V.N.GRAV $
MPYAD PHILN,WL3./FNRO/C.N.1/C.N.1/C.N.O/C.N.2 $
SOLVE WGNN, FNRO/FNR/C.N.1 $
MATPRN FNR,....// $
DIAGONAL FNR/MURRO/C.N.WHOLE/C.N.2.0 $
MPYAD WGNN,MURRO./MURR $
MATPRN MURR,.... $
TRNSP MURR/MURRT $
SOLVE WRD./MURRT/MURRDT/C.N.1 $
ADD MURRDT./MURRPT/C.N.(100.O.) $
TRNSP MURRPT/MURRP $
MATPRN MURRP,....// $
ENDALTER
CEND
```

LONGITUDINAL AND LATERAL MODES

C A S E C O N T R O L D E C K E C H O

CARD  
COUNT

1 TITLE = TEST OF DMAP PROGRAM TO GEN. MODAL PART FACTORS FOR BASE ACCEL  
2 SUBTITLE = CANTILEVERED BEAM  
3 LABEL = LONGITUDINAL AND LATERAL MODES  
4 ECHO = UNSORT  
5 METHOD = 1  
6 OUTPUT  
7 VECTOR = ALL  
8 SPCF = ALL  
9 BEGIN BULK

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LONGITUDINAL AND LATERAL MODES

	1	2	3	4	5	6	7	8	9	10
GRDSET										
GRID 1				100.	0.					
GRID 2				90.	0.					
GRID 3				80.	0.					
GRID 4				70.	0.					
GRID 5				60.	0.					
GRID 6				50.	0.					
GRID 7				40.	0.					
GRID 8				30.	0.					
GRID 9				20.	0.					
GRID 10				10.	0.					
GRID 11				0.	0.					
BAROR							0.	1.		1
CBAR 1			1	1	2					
CBAR 2			1	2	3					
CBAR 3			1	3	4					
CBAR 4			1	4	5					
CBAR 5			1	5	6					
CBAR 6			1	6	7					
CBAR 7			1	7	8					
CBAR 8			1	8	9					
CBAR 9			1	9	10					
CBAR 10			1	10	11					
PBAR 1			1	2.0	2.0					
MAT1			10.+6	0.3	0.1					
SUPPORT		11								
ASET1	13		1	THRU	11					
EIGR	1		GIV							
+E2	MAX					23			1.-4	+E2
PARAM	GRDPNT	11								
PARAM	WTMASS	.002591								
ENDDATA										

INPUT BULK DATA CARD COUNT = 32

TOTAL COUNT = 32

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LONGITUDINAL AND LATERAL MODES  
 OUTPUT FROM GRID POINT WEIGHT GENERATOR

REFERENCE POINT = 11  
 M D  
 \* 2.000000E+01 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 \*  
 \* 0.000000E+00 2.000000E+01 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+03 \*  
 \* 0.000000E+00 0.000000E+00 2.000000E+01 0.000000E+00 -1.000000E+03 0.000000E+00 \*  
 \* 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 \*  
 \* 0.000000E+00 0.000000E+00 -1.000000E+03 0.000000E+00 6.700001E+04 0.000000E+00 \*  
 \* 0.000000E+00 1.000000E+03 0.000000E+00 0.000000E+00 0.000000E+00 6.700001E+04 \*

DIRECTION  
 MASS AXIS SYSTEM (S) X-C.G. Y-C.G. Z-C.G.  
 X 2.000000E+01 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 \*  
 Y 2.000000E+01 5.000000E+01 0.000000E+01 0.000000E+00 0.000000E+00 \*  
 Z 2.000000E+01 5.000000E+01 0.000000E+01 0.000000E+00 0.000000E+00 \*  
 I(S)  
 \* 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 \*  
 \* 0.000000E+00 1.700000E+04 0.000000E+04 0.000000E+00 \*  
 \* 0.000000E+00 0.000000E+00 0.000000E+00 1.700000E+04 \*  
 I(Q)  
 \* 0.000000E+00 1.700000E+04 \*  
 \* 1.000000E+00 0.000000E+04 \*  
 \* 0.000000E+00 1.000000E+00 0.000000E+00 \*  
 \* 0.000000E+00 0.000000E+00 1.000000E+00 1.000000E+00 \*

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TEST OF DMAP PROGRAM TO GEN MODAL PART FACTORS FOR BASE ACCEL.  
 CANTILEVERED BEAM

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LONGITUDINAL AND LATERAL MODES

	POINT	MAAB	VALUE	POINT	VALUE	POINT	VALUE	POINT	VALUE
COLUMNS	1 (		1-T1)	THRU	20 (		10-T3)	ARE NULL.	
COLUMN	21 (		11-T1).						
	11 T:		5.18200E+06						
COLUMN	22 (		11-T3).						
	11 T3		5.18200E+06						
COLUMN	23 (		11-R2).						
	11 R2		1.73597E+10						

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MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERALIZED STIFFNESS
1	11	0.0	0.0	0.0	5.182000E+06	0.0
2	22	0.0	0.0	0.0	5.182000E+06	0.0
3	23	0.0	0.0	0.0	1.735970E+06	0.0
4	21	4.727787E+03	6.875890E+01	1.034332E+01	1.305986E-02	6.172530E+01
5	20	1.815695E+05	4.261097E+02	6.781746E+01	1.369489E-02	2.486573E+03
6	19	1.395168E+06	1.181172E+03	1.879894E+02	1.494427E-02	2.084977E+04
7	18	5.246433E+06	2.290509E+03	3.645459E+02	1.738694E-02	9.121943E+04
8	10	9.503404E+06	2.082759E+03	4.906363E+02	2.591000E-02	2.462332E+05
9	17	1.401962E+07	3.744278E+03	5.959203E+02	2.205616E-02	3.092189E+05
10	16	3.044624E+07	5.517811E+03	8.781869E+02	2.686599E-02	8.179684E+05
11	15	5.704086E+07	7.552540E+03	1.202024E+03	2.504681E-02	1.428692E+06
12	9	8.413236E+07	9.172369E+03	1.459828E+03	2.591000E-02	2.179870E+06
13	14	9.420654E+07	9.706006E+03	1.544753E+03	2.398174E-02	2.259237E+06
14	13	1.367145E+08	1.163250E+04	1.860919E+03	2.858839E-02	3.908448E+06
15	12	1.717434E+08	1.310509E+04	2.085740E+03	2.950574E-02	5.067415E+06
16	8	2.260851E+08	1.503613E+04	2.393074E+03	2.591000E-02	5.857865E+06
17	7	4.214662E+08	2.052964E+04	3.267394E+03	2.591000E-02	1.092019E+07
18	6	6.511505E+08	2.551765E+04	4.061260E+03	2.591000E-02	1.687131E+07
19	5	8.926549E+08	2.987737E+04	4.755125E+03	2.591000E-02	2.312869E+07
20	4	1.122339E+09	3.350133E+04	5.331902E+03	2.591000E-02	2.907981E+07
21	3	1.317720E+09	3.630042E+04	5.777391E+03	2.591000E-02	3.414214E+07
22	2	1.459673E+09	3.820567E+04	6.080621E+03	2.591000E-02	3.782013E+07
23	1	1.534302E+09	3.917017E+04	6.234125E+03	2.591000E-02	3.975376E+07

LONGITUDINAL AND LATERAL MODES

MATRIX FNR (GIND NAME 101 ) IS A DB PREC 3 COLUMN X 20 ROW RECTANG MATRIX.

COLUMN ROW	1	ROWS 5 THRU 20	20	3	COLUMN X	20	ROW	RECTANG	MATRIX.
5)	1.2706D+00	0.0000D+00	0.0000D+00	0.0000D+00	-4.1653D-01	0.0000D+00	0.0000D+00	-2.4142D-01	-1.6319D-01
15)	1.1708D-01	-8.5408D-02	6.1280D-02	-4.1421D-02	2.4008D-02	-7.8702D-03			
COLUMN ROW	2	ROWS 1 THRU 12	12						
1)	1.5569D+00	-8.4463D-01	4.7360D-01	-3.1367D-01	0.0000D+00	2.1613D-01	-1.5926D-01	1.3712D-01	0.0000D+00
11)	-8.0610D-02	-4.5339D-02							1.1542D-01
COLUMN ROW	3	ROWS 1 THRU 12	12						
1)	-1.1359D+02	1.7800D+01	-5.1238D+00	2.9231D+00	0.0000D+00	-1.5900D+00	9.8005D-01	-7.3703D-01	0.0000D+00
11)	3.6689D-01	1.9869D-01							-5.6180D-01

THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN = 10  
 THE DENSITY OF THIS MATRIX IS 50.00 PERCENT.

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LONGITUDINAL AND LATERAL MODES

MATRIX MURR (GIND NAME 101 ) IS A DB PREC 3 COLUMN X 20 ROW RECTANG MATRIX.

COLUMN ROW	1	ROWS 5 THRU 20	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
5)	1.6145D+01	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
15)	1.3708D-01	7.2945D-02	3.7552D-02	1.7157D-02	5.7638D-03	6.1940D-04																
COLUMN ROW	2	ROWS 1 THRU 12	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
1)	1.2215D+01	3.7707D+00	1.2937D+00	6.5026D-01	0.0000D+00	3.9765D-01	2.6298D-01	1.8174D-01	0.0000D+00													
11)	7.1697D-02	2.3409D-02																				
COLUMN ROW	3	ROWS 1 THRU 12	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20			
1)	6.5010D+04	1.6746D+03	2.1630D+02	5.7338D+01	0.0000D+00	2.1521D+01	9.9593D+00	5.2511D+00	0.0000D+00													
11)	1.4852D+00	4.4957D-01																				

THE NUMBER OF NON-ZERO TERMS IN THE DENSEST COLUMN = 10  
 THE DENSITY OF THIS MATRIX IS 50.00 PERCENT.

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