Forced Convection Heat Transfer to Air/Water Vapor Mixtures

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**NOMENCLATURE**

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<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>A</td>
<td>heat transfer surface area of individual test plate segment</td>
</tr>
<tr>
<td>$A^*$</td>
<td>dimensionless function of collision integrals</td>
</tr>
<tr>
<td>$c_p$</td>
<td>molal isobaric specific heat</td>
</tr>
<tr>
<td>d</td>
<td>jet hole diameter</td>
</tr>
<tr>
<td>D</td>
<td>mass diffusion coefficient</td>
</tr>
<tr>
<td>$G_j$</td>
<td>mass flux based on jet hole area</td>
</tr>
<tr>
<td>$\bar{G}_j$</td>
<td>mean value of $G_j$ over jet array</td>
</tr>
<tr>
<td>$H$</td>
<td>function found in rigorous expression for mixture viscosity</td>
</tr>
<tr>
<td>$h$</td>
<td>convective heat transfer coefficient resolved in streamwise direction, averaged across span</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td>$k^0$</td>
<td>thermal conductivity with frozen internal degrees of freedom</td>
</tr>
<tr>
<td>$L$</td>
<td>streamwise length of heat transfer surface</td>
</tr>
<tr>
<td>M</td>
<td>molecular weight</td>
</tr>
<tr>
<td>$M^*$</td>
<td>ratio of molecular weights</td>
</tr>
<tr>
<td>$m_i$</td>
<td>mass fraction of component i</td>
</tr>
<tr>
<td>Nu</td>
<td>Nusselt number resolved in streamwise direction, averaged across span $\frac{hd}{k}$</td>
</tr>
<tr>
<td>$P$</td>
<td>mixture pressure</td>
</tr>
<tr>
<td>$P_i$</td>
<td>partial pressure of component i</td>
</tr>
<tr>
<td>$P_0$</td>
<td>plenum pressure</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number, $\mu c_p / k$</td>
</tr>
<tr>
<td>$P_{sat}$</td>
<td>saturation pressure</td>
</tr>
<tr>
<td>Q</td>
<td>heat rate from individual test plate segment</td>
</tr>
<tr>
<td>R</td>
<td>ideal gas constant</td>
</tr>
</tbody>
</table>
Re_j  = jet Reynolds number G_jd/μ
Re_j  = mean jet Reynolds number G_jd/μ
S    = Sutherland constant
T    = mixture temperature
T_B  = boiling temperature at one atmosphere
T_o  = plenum temperature
T_r  = fluid reference temperature for defining heat transfer coefficient, equivalent to adiabatic wall temperature
T_s  = heat transfer surface temperature
W    = humidity ratio (mass ratio of water vapor to air)
X_i  = mole fraction of component i
x    = streamwise location along heat transfer surface measured from upstream end of channel
x_n  = streamwise jet hole spacing
y_n  = spanwise jet hole spacing
z    = channel height of jet plate-to-impingement surface spacing

Greek
μ    = dynamic viscosity
Ψ_ij = Sutherland coefficient for viscosity
Φ_ij = Sutherland coefficient for thermal conductivity

Subscripts
1,2 refer to the individual components of an arbitrary binary system of gases or specifically to air (1) and steam (2), depending on the context
m subscript refers to arbitrary binary mixture or specifically to air/steam mixture depending on context
SUMMARY

The applicability of forced convection heat transfer data and empirical correlations based on experiments with dry or nearly dry air to situations involving air/water vapor mixtures (humid air) is considered. The particular application which motivated the present study is the contingency cooling of helicopter gas turbine engine components using evaporatively cooled (and therefore humid) air. Heat transfer coefficients were measured using both dry and humid air in the same forced convection cooling scheme and were compared using appropriate nondimensional parameters (Nusselt, Prandtl and Reynolds's numbers). A forced convection scheme with a complex flow field, two-dimensional arrays of circular jets with crossflow, was utilized with humidity ratios (mass ratio of water vapor to air) up to 0.23. Results of a survey of the literature regarding the dynamic viscosity, thermal conductivity and specific heat of air, steam and air/steam mixtures are reported. Methods for the determination of gaseous mixture properties from the properties of their pure components are reviewed in detail. Convenient methods for the determination of these properties with good confidence are described and the need for more experimentally determined property data for humid air is discussed. It is concluded that dimensionless forms of forced convection heat transfer data and empirical correlations based on measurements with dry air may be applied to conditions involving humid air with the same confidence as for the dry air case itself, provided that the thermophysical properties of the humid air mixtures are known with the same confidence as their dry air counterparts.
1. INTRODUCTION

Forced convection heat transfer is an active area of experimental investigation motivated by a virtually unlimited number of applications that are of interest to designers of thermal systems. One important application for forced convection heat transfer is the cooling of gas turbine engine components utilizing air drawn from the compressor section of the engine. Metzger and Mayle (1983) have discussed the continual improvement of gas turbine engine performance as made possible in part by improvements in gas turbine component cooling technology.

Gas turbine engine component cooling fulfills two very important requirements that relate to the service life of component materials; namely: the maintenance of low overall component temperatures and the reduction in magnitude of local temperature gradients. The failure to meet either of these requirements, even for a relatively short period of time, can significantly shorten the service life of turbine engine components. Nevertheless, special situations do exist where a temporary need arises for an increased engine power output which can only be realized by a corresponding increase in the turbine inlet temperature (and hence an exposure of the engine components to greater than desirable temperatures). Van Fossen (1983) provides two examples where such a need can arise in the operation of gas turbine engines for helicopter service. One example is an emergency situation where a twin engine helicopter loses one engine and an increase in power beyond the maximum power rating is required from the remaining engine for a brief
time in order to make a safe landing. A second example is on a hot day at high altitude where increased power is required for safe helicopter take-off.

A provision for the temporary extra cooling capability needed for gas turbine engines in special situations such as those just described has been investigated. Van Fossen (1983) has studied the feasibility of water injection (and evaporation) into the turbine coolant air of helicopter engines. This process is intended to lower the turbine coolant temperature during special situations by an amount necessary to maintain the turbine blades at their normal operating temperatures and thus prevent a reduction of blade stress rupture life. Van Fossen used a computer model in his feasibility studies and concluded that water injection shows promising potential for actual engine use. Various humidity ratios (mass ratio of water vapor to air) for the humid air coolant were considered in Van Fossen’s analysis with the maximum value being approximately 0.10. Other investigators have considered humidity ratios as high as 0.16 (Hirschkrön et al. 1981).

The design analysis of turbine cooling depends extensively on the use of nondimensional heat transfer data and empirical correlations based on experiments with dry or nearly dry air. The water injection and evaporation technique discussed above requires heat transfer design analysis for situations where the coolant air stream may contain significant quantities of water vapor. It might be expected that existing dry air correlations and/or data could be applied to these humid air cases provided the relevant nondimensional parameters (such as Nusselt,
Reynolds and Prandtl numbers) are evaluated using the appropriate humid air thermophysical properties. The primary objective of this thesis is to verify this expectation by direct comparison of dimensionless heat transfer coefficients obtained from experiments using dry air and humid air for the same test model geometry. Humidity ratios as high as 0.23 are investigated.

Because of the importance of accurate property data for use in making the comparisons of the dimensionless heat transfer data in this study, a literature survey was conducted covering experimental data for the dynamic viscosity, thermal conductivity and specific heat of air, steam and air/steam (humid air) mixtures as well as methods for determination of gaseous mixture properties from the properties of their pure components. The results of this survey are discussed in Section 2.

The basic experimental facility used in this study is the same as that used by Florschuetz et al. (1980, 1981) for earlier heat transfer studies motivated by the investigation of the air cooling of gas turbine engines. The facility was appropriately modified for humid, in addition to dry air experimentation. A description is given in Section 3. The particular cooling scheme modeled by this facility is that of jet impingement by two-dimensional arrays of circular jets. An example of the application of this scheme in a gas turbine engine is the internal cooling of the midchord section of an airfoil as illustrated in Fig. 1.1. Note that the jet flow from each row of jets, after impingement, is constrained to exit in such a manner as to create a crossflow for the remaining downstream jets. This example scheme is rather complicated,
Fig. 1.1 Example of Internally Cooled Gas Turbine Airfoil Utilizing Jet Array Impingement
involving many possible variations in the geometry and flow distributions as well as complex flow interactions between the crossflow (which can be likened to a channel-type flow) and the jet flow. The jet array impingement scheme thus serves as a non-trivial example of a forced convection heat transfer configuration for which to carry out the desired comparisons.

Section 4 outlines the experimental procedures and data reduction. Section 5 presents the comparisons of the heat transfer data for the dry and humid air tests as well as a discussion of these results. Conclusions based on the results of the study are given in Section 6.

The authors are not aware of any previous investigations into the applicability of nondimensional dry air forced convection heat transfer data to situations involving humid air. Serksnis et al. (1978), using hydrogen/carbon dioxide mixtures and Pickett et al. (1979), using helium/argon mixtures, found that the well known Dittus-Boelter and Colburn analogy correlations both significantly overpredicted their circular tube heat transfer data in the fully developed region. This was attributed to the low Prandtl numbers of the mixtures used (~ 0.4) which were significantly below the smallest values (~ 0.7) for the data on which the correlations were based. A more recent correlation recommended by Kays (1966) based on a family of numerical solutions including Prandtl numbers down to a value of 0.5 was found to give good agreement with the lower Prandtl number mixture data.
2. THERMOPHYSICAL PROPERTIES

The work reported in this study is concerned with the acquisition and evaluation of heat transfer data that is presented by making use of the following three nondimensional parameters:

\[ \text{Nu} = \frac{h_d}{k}, \quad \text{Re}_j = \frac{G_j d}{\mu}, \quad \text{and} \quad \text{Pr} = \frac{\mu c_p}{k}. \]

It is obvious that in order to evaluate such parameters for given experimental conditions, values of the transport properties of viscosity and thermal conductivity, and the thermodynamic property of specific heat for the gases of interest (namely dry and humid air) must be known. An attempt to obtain the most accurate, up-to-date, and easily used methods for the determination of these properties, with high confidence for engineering calculations, has been made for the data reduction and presentation portions of this work. The results of this effort are described in the following pages. Included are descriptions of several methods encountered in the literature regarding mixture viscosity and thermal conductivity and some original suggestions regarding the evaluation of humid air properties in particular.

The amount of research published regarding the determination of the properties of air, steam and polar-nonpolar gaseous systems (which characterizes humid air) is very great, although the available data for transport properties of the humid air system specifically is surprisingly meager. The authors are not research specialists on thermophysical properties. The following discussion is presented from the viewpoint of one attempting to make intelligent and critical use of the
available information. In that sense, this discussion may be considered quite thorough, and should be useful for those having a similar purpose.

An effort has been made to use terminology that is consistent with that found in the mixture property literature. A specified set of two (or more) component gases is referred to as a 'system.' Air and steam (humid air) is an example of such a system. (Here air is considered as one component). A given system containing specified relative amounts of each component is referred to as a 'mixture.' A humid air system containing a mole fraction of steam of 0.2 is an example of one possible mixture for that system.

2.1 **Steam Properties**

Kestin has recently reported internationally accepted formulations for steam viscosity (1976) and thermal conductivity (1978). The formulation for viscosity, adopted in 1975, and the formulation for thermal conductivity, adopted in 1977, are referred to as the 'Release on Dynamic Viscosity of Water Substance' (RDV75) and the 'Release on Thermal Conductivity of Water Substance' (RTC77). Actually, two formulations are provided for thermal conductivity, one for industrial use, and the other for scientific use. The only significant difference between the two formulations is that the latter accounts for the expected 'singular' behavior of the thermal conductivity in a small region about the critical point and as such is more complicated than the industrial formulation. Since all the calculations for this work were far from the critical point, the industrial formulation was used. The
details regarding the regions of validity for the viscosity and thermal conductivity formulations are given in Appendix C. All of the formulations are lengthy, but the RDV75 and the RTC77 (industrial) formulations are easily coded on the computer. Appendix C lists the FORTRAN IV coding used. The formulations require temperature and density as inputs. The suggested formulation for density for the above relations (Sengers et al. 1982) is the 1967 International Formulation Committee's (IFC67) formulation for industrial use from which the 1967 American Society of Mechanical Engineer's Steam Tables (Meyer et al. 1968) were produced. There exists a newer formulation for density referred to as the Provisional International Association for the Properties of Steam (IAPS) Formulation 1982 that is also suggested and is expected to replace IFC67 upon its formal international acceptance in September, 1984, at the Tenth International Conference on the Properties of Steam, Moscow, USSR (Kestin et al. 1983). The formulation IFC67 was used in this work.

Formulae for the isobaric specific heat for steam are contained in a supplement to the IFC67 formulation for industrial use. Sengers et al. (1982) have indicated that in certain regions these formulae are not acceptable because they do not produce sufficiently smooth results. The IFC67 formulae however, do exhibit smooth behavior in the particular region of interest for this work.

2.2 Air Properties

A large amount of experimental and derived data has been published
for the dynamic viscosity, thermal conductivity and specific heat of air. This data has been critically analyzed in the appropriate Thermo-
physical Properties Research Center (TPRC) data series volumes
(Touloukian et al. 1975, 1970a, 1970b). The recommended values
tabulated therein were interpolated for the evaluation of the properties
of pure air for this work.

2.3 Gas Mixtures

The expressions available for determining the properties of viscosity and thermal conductivity of gas mixtures vary from purely theoretical derivations to direct curve fits of experimental data. Semitheoretical methods exist which use the basic forms of the theoretical expressions and through the use of intuitive assumptions, or the process of backing out the appropriate constants from the available experimental data, modify the expressions in order to obtain better agreement with the experimental data. An extensive and fairly recent survey of the methods for determining viscosity and thermal conductivity of gas mixtures is contained in the applicable volumes of the Thermo-

Theoretical expressions for the viscosity and the thermal conductivity of gas mixtures are based on kinetic theory and can be divided into what are referred to as the 'simple mean-free-path theories' and the 'rigorous theories'.

Nearly ninety years ago, Sutherland (1895), using simple mean-free-
path arguments developed the following form for the viscosity of a mixture of $n$ monatomic non-polar gases:

$$\mu_m = \sum_{i=1}^{n} \frac{\mu_i}{1 + \sum_{j=1, j \neq i}^{n} \Psi_{ij} \frac{x_i}{x_j}}$$

where $\mu_i$ is the viscosity of component $i$.

For a binary mixture, Sutherland's equation simplifies to:

$$\mu_m = \frac{\mu_1}{1 + \Psi_{12}(x_2/x_1)} + \frac{\mu_2}{1 + \Psi_{21}(x_1/x_2)} \quad (2.1)$$

Wassiljewa (1904) derived a similar expression for mixture thermal conductivity:

$$k_m = \frac{k_1}{1 + \Phi_{12}(x_2/x_1)} + \frac{k_2}{1 + \Phi_{21}(x_1/x_2)} \quad (2.2)$$

Neither of the above equations compared very well with experimental data, even for very simple gas mixtures. This was mainly due to an inadequate consideration of the intermolecular forces present in the gas mixtures (Touloukian et al. 1975, 1970a). Both expressions however, are the starting point for a myriad of semitheoretical and semiempirical approaches to the evaluation of mixture viscosity and thermal conductivity, many of which are not limited to monatomic or non-polar gas mixtures. These methods concern themselves with the evaluation of the $\Psi_{ij}$ and $\Phi_{ij}$ which are referred to generally in the literature as the Sutherland coefficients.
The more rigorous derivations of mixture properties, the basis of which is the notable work of Chapman and Enskog (Chapman and Cowling, 1970) account for the effects of intermolecular forces at the expense of producing much more complicated expressions for mixture properties. Kestin (1982) has described the great success of rigorous developments in evaluating the pure and mixture properties of several 'simple' gases and mentions the increased difficulty of evaluating the properties of polyatomic and polar gases with rigorous theory (see also Touloukian et al. 1970a). Thermal conductivity is particularly difficult to evaluate theoretically because it depends strongly upon the transport of internal (rotational and vibrational), in addition to translational molecular energy (Mason and Monchick, 1962, 1965 and Touloukian et al. 1970a). Mason and Monchick (1962) have provided a rigorous expression for gas mixture viscosity that has given very good results for fourteen polar-nonpolar gas systems (humid air not included in comparisons).

The complexity of the rigorous expressions for mixture properties has greatly deterred from their usefulness in typical scientific and engineering applications. The knowledge to be gained from such developments however, has been helpful in the development of useful semitheoretical expressions for viscosity and thermal conductivity which rely on the Sutherland and Wassiljewa forms mentioned earlier. It is noted that rigorous theory can be simplified to these forms (Gambhir and Saxena 1964, Mason and Monchick 1965). One expression for mixture viscosity that takes on the Sutherland form and which is notable because of its many citations and wide use is that of Wilke (1950) where the
Sutherland coefficients are given by

\[
\Psi_{ij} = \frac{[1 + (\mu_i/\mu_j)^{1/2}(M_j/M_i)^{2/4}]^2}{(4/\sqrt{2})[1 + (M_i/M_j)]^{2/4}}
\]

2.4 Air/Steam System (Humid Air)

The semitheoretical and semiempirical methods require experimental data in order to obtain accurate formulations. Experimental measurements for the viscosity and the thermal conductivity of humid air, however, are surprisingly scarce. For thermal conductivity, searches have resulted in only two references for such information. Gruss and Schmick (1928) reported measurements at a single temperature (353 K). Their values for pure steam and pure air are reasonably consistent with the currently recommended values discussed in Sections 2.1 and 2.2. Their mixture values are consistent with predictions based on semitheoretical methods such as those of Lindsay and Bromley (1950) which have been verified for other polar-nonpolar systems. Gruss and Schmick's results are reproduced and discussed in more detail in Section 2.4.2. The second reference is Zakharov (1962). His experimental measurements of dry air thermal conductivity obtained over the range 20 to 60°C with the same apparatus he used for humid air measurements are found to deviate from the recommended air values of Touloukian et al. (1970a) by 10 to 20%. His mixture values are significantly larger than predictions based on the validated method of Lindsay and Bromley referred to above, in some cases by a factor of over two. For these reasons the data of Zakharov is not considered reliable and is therefore not
utilized in this study. For viscosity, the 'disgraceful' lack of experimental data was not corrected until the measurements of Kestin and Whitelaw (1964, 1965), discussed in detail in Section 2.4.1. This lack of experimental data has impeded the progress of the determination of the best method for estimating viscosity and thermal conductivity of humid air mixtures. As a result, one is forced to use a method that reproduces the limited experimental data accurately and assume it is valid at other temperatures, or use a method that has proven accurate for a variety of polar–nonpolar gas systems and assume it is also accurate for humid air mixtures specifically.

2.4.1 Viscosity

As mentioned earlier, Mason and Monchick (1962) have investigated a theoretical expression for mixture viscosity which they have compared with experimental data for fourteen polar–nonpolar systems (approximately 218 mixtures, humid air not included). The expression is:

\[
p_m = \left[ \frac{X_1^3}{H_{11}} + \frac{X_2^3}{H_{22}} - \frac{2X_1X_2H_{12}}{H_{11}H_{22}} \right] \left[ 1 - \frac{H_{12}^3}{H_{11}H_{22}} \right]^{-1},
\]

\[
H_{11} = \frac{X_1^3}{\mu_1} + \frac{2X_1X_2}{\mu_1} \frac{RT}{M_1 + M_2} \frac{PD_{12}}{5M_2} \left( 1 + \frac{3M_2}{5M_2} A_1^p \right),
\]

\[
H_{12} = -\frac{2X_1X_2}{M_1 + M_2} \frac{RT}{PD_{12}} \left( 1 - \frac{3}{5} A_2^p \right).
\]

Here \( H_{12} \) is obtained from \( H_{11} \) by interchanging the subscripts 1 and 2,
\( A_{12} \) is a dimensionless function of tabulated collision integrals and \( D_{12} \) is the mass diffusion coefficient.

Mason and Monchick did not use the above in a purely theoretical way since experimental values were used for the component viscosities. They found that the deviation of mixture viscosity from experiment was always less than 5\%, frequently less than 1\% and averaged 1 to 2\%. In a later paper by the same authors (1965) values of humid air viscosity based on the above expression are tabulated for both a purely theoretical calculation (including the component viscosities) and for a semitheoretical calculation using experimental values for the component viscosities and \( D_{12} \). No experimental values for humid air are presented but the authors do argue qualitatively that the above expression used for humid air should be superior to the method of Wilke (presented earlier). The above equation for mixture viscosity simplifies to the Sutherland form, Eq. (2.1), if the terms involving \( H_{12} \) are neglected.

Kestin and Whitelaw (1964) compared their experimental data for humid air to the curves generated by the theoretical values of Mason and Monchick. The values compare fairly well at low mole fractions of steam but are in poor agreement for mole fractions above about 0.6. This is mainly a result of unexpected inflections which occurred in the data at a mole fraction of steam of about 0.6.

Saxena (1973) has surveyed the methods available for determining Sutherland coefficients for the viscosity of gas mixtures including some discussion of their temperature and composition dependence. From that survey, he highlights two convenient methods for the determination of
the coefficients. The first method uses the following expression:

\[
\frac{\psi_{ij}}{\psi_{ji}} = \frac{\mu_i}{\mu_j} \left( \frac{M_j}{M_i} \right)
\]  (2.3)

This equation, along with one experimental value of mixture viscosity and the Sutherland equation (2.1) determines a pair of coefficients. The second method is due to Saxena and Gambhir (1963) and uses the same expression except the molecular weight ratio is raised to the 0.85 power instead of unity. Mathur and Saxena (1965) compared method two to the experimental data for eleven polar–nonpolar binary gas systems (seventy-nine mixtures, humid air not included) and found an average absolute deviation of 0.4%. Their results confirm the expected composition independence of the coefficients. In addition, they indicate that the coefficients are approximately temperature independent for polar–nonpolar gas mixtures. For five of the ten binary gas systems investigated, data was available at more than one temperature. Mathur and Saxena calculated Sutherland coefficients using data at the lowest temperature and then proceeded to calculate and compare the values of viscosity at the higher temperatures. By doing this, they added ninety-five mixtures (for the same eleven systems) to their comparisons and obtained an average absolute deviation of 1.8%. However, the present authors have noted that the trend in the deviation is for it to increase with temperature. Therefore, it seems appropriate to suggest that the coefficients be calculated using a mixture viscosity value at the temperature of the available data that is closest to the temperature of
interest. Nevertheless, it appears that coefficients evaluated at lower temperatures can be very useful at higher temperatures.

Values for the Sutherland coefficients based on methods one and two described above were calculated by Saxena (1973) for a large number of binary systems (including some polar-nonpolar systems). For each method, values of the two coefficients were calculated at each mixture contained in the experimental data and then each pair of coefficients was used to compute values of mixture viscosity at all other mixtures at the same temperature. Three different deviations were then determined; the deviation between experimental mixture viscosity values and (1) the graphically smoothed values, (2) the calculated values of method one, and (3) the calculated values of method two. Provided that the first deviation, which reflects scatter within the experimental data itself, is not unreasonably large, an inspection of the remaining two deviations indicates the preferred pair of coefficients. If the first deviation and the smallest of the two remaining deviations are approximately equal, the best pair of Sutherland coefficients for reproducing the data within experimental uncertainty at a given temperature is determined. The recommended Sutherland coefficients resulting from this process have been tabulated for each temperature (Saxena 1973). A similar tabulation of Sutherland coefficients for a large number of gas systems utilizing methods one and two has also been included in the work by Touloukian et al. (1975). Here, a different set of error calculations was used to find the recommended coefficients; (1) the mean absolute, (2) the root-mean-square, and (3) the maximum absolute deviation from the
experimental data. The recommended pair of coefficients was typically the one yielding the smallest values for all three deviations. Unfortunately, humid air was not included in either of the two tabulations just described.

The present author has made calculations of the Sutherland coefficients for methods one and two with an experimental value of mixture viscosity from the data of Kestin and Whitelaw at 348 K and a mole fraction of steam of 0.193. This is representative of the typical mixture conditions of interest for this work. Calculated mixture viscosities using the coefficients of both methods reveal that they give essentially identical results. Mixture viscosity calculations using the method one coefficients are compared in Fig. 2.1 with the data of Kestin and Whitelaw (1964) obtained at six different temperature levels ranging from 298 K to 523 K. Calculated values using the Wilke method (1950) have also been included in the comparison. An additional curve at 523 K has been included in Fig. 2.1 using method one Sutherland coefficients evaluated at an experimental data point at that same temperature and a mole fraction of steam of 0.498.

Inspection of Fig. 2.1 makes it immediately apparent that method one is more suitable for humid air viscosity calculations than the Wilke method. Further, it is apparent from the additional method one curve at 523 K (made using the Sutherland coefficients calculated at that temperature) that it is good practice to calculate Sutherland coefficients with a value of experimental mixture viscosity that is at the temperature level which is closest to the temperature level of interest.
Fig. 2.1 Dynamic Viscosity of Humid Air at Several Temperature Levels
The previously mentioned inflections in the experimental data of Kestin and Whitelaw above a mole fraction of steam of about 0.6 become apparent in Fig. 2.1 if one imagines a curve smoothed through the data points and extended to the pure steam values at $X_2 = 1.0$.

In the preceding calculations Kestin and Whitelaw's measured values for the pure component viscosities were used whenever possible. For the cases where the pure component viscosities were not measured by Kestin and Whitelaw, the data described earlier in Sections 2.1 and 2.2 was used (Touloukian et al. 1975 for air and RDV75 for steam). The deviation between measured component values (when supplied) and the more recent data was always one percent or less except for the viscosity of pure air at 523 K where there was a deviation of approximately 2%.

An explanation at this point is appropriate regarding how the pure component viscosity of the steam was determined for those cases where pure steam (gaseous water) does not exist in an equilibrium state at the temperature and pressure of the mixture. In these cases, the viscosity of steam was evaluated at the temperature of the mixture and the corresponding saturation pressure for that temperature.

Studnikov (1970), starting with an empirical formula for the thermal conductivity of mixtures containing a polar component, has developed an analogous empirical formula for humid air viscosity as follows:

$$\mu_m = (X_1\mu_1 + X_2\mu_2) \left(1 + \frac{X_2}{a} - \frac{X_2^2}{a}\right)$$

Studnikov used Kestin and Whitelaw's experimental values at 298 K,
323 K, and 348 K to obtain a value for 'a' of 2.75. This reproduced the experimental data at these temperatures with a maximum divergence of 0.1%. Despite the fact that Studnikov developed his empirical formula using only Kestin and Whitelaw's data at 348 K and below, his formula is found to remain accurate for higher temperature mixtures at low mass fractions.

Tabulated values of Kestin and Whitelaw's humid air viscosity data along with the deviations of this data from the Wilke method, methods one and two (using Sutherland coefficients calculated at 348 K) and the Studnikov formula are contained in Table A.1 of Appendix A. Deviations of the experimental data at 523 K using the method one coefficients evaluated at that temperature are also contained in Appendix A, Table A.2.

It is noted that none of the methods described above (including Mason and Monchick's theoretical expression) predict the inflections contained in the experimental data. Indeed, the form of these equations (assuming Sutherland coefficients independent of mixture composition) does not admit the prediction of such inflections. Therefore, no recommendation can be offered for a suitable method to be used in determining mixture viscosity of humid air at high mole fractions of steam. For low mole fractions of steam (i.e. 0.5 and less) methods one and two (Saxena 1973) and Studnikov's formula appear to work well. The purely empirical nature of Studnikov's formula makes it less appealing, however, and the slightly simpler form of method one over method two suggests that method one is a convenient choice for mixture
viscosity calculations. Method one finds further appeal in that it is exactly the same as the method chosen to calculate the ratio of the Sutherland coefficients for mixture thermal conductivity calculations as will be explained later. Hence, method one (Eq. 2.3) was chosen for the mixture viscosity calculations for this work. However, the accuracy of method two and of Studnikov's formula in the region of interest for this work is such that either of those methods could have been used with the same confidence.

2.4.2 Thermal Conductivity

The added difficulty in calculating theoretical mixture thermal conductivity due to its strong dependence on energy transfers internal to the molecular structure was noted earlier. Because of this, the rigorous theoretical expressions for mixture thermal conductivity are even more unwieldy than those for viscosity. Consequently, the practical appeal of semitheoretical (and semiempirical) methods which are concerned with the determination of the appropriate Sutherland coefficients for use with Eq. (2.2) is again emphasized.

An early semitheoretical expression for mixture thermal conductivity which is analogous to Wilke's viscosity expression in its popularity is that of Lindsay and Bromley (1950) where the Sutherland coefficients are given by

$$\phi_{ij} = \frac{1}{4} \left\{ 1 + \left[ \frac{\mu_i}{\mu_j} \left( \frac{M_j}{M_i} \right)^{3/4} \frac{(1 + S_i/T)^{1/2}}{(1 + S_j/T)} \right] \right\}^{3} \frac{(1 + S_{ij}/T)}{(1 + S_i/T)} .$$

$S$ is the Sutherland constant approximated by $S = 1.5T_B$, $T_B$ being
the boiling temperature of the appropriate component at one atmosphere. 

$S_{ij}$ for a polar–nonpolar mixture is given by $S_{ij} = 0.733 \sqrt{S_i S_j}$. Using this expression, Lindsay and Bromley were able to reproduce the experimental data for twenty gas systems (eighty-five mixtures) with an average deviation of 1.9%. Gruss and Schmick's experimental humid air data was reproduced with an average absolute deviation of 0.9%. Tondon and Saxena (1968) compared Lindsay and Bromley's method and three other methods described as follows: An 'approximate' method where

$$
\Phi_{ij} = \frac{0.85}{2^{1/2}} \left( 1 + \frac{M_i}{M_j} \right)^{-1/2} \left[ 1 + \left( \frac{k_i}{k_j} \right)^{1/2} \left( \frac{M_i}{M_j} \right)^{1/4} \right]^{-2}
$$

and $k_i$, $k_j$ are the component thermal conductivities with frozen internal degrees of freedom; an 'empirical' method where the two Sutherland coefficients were calculated based on two mixture conductivities from the data; and a 'semitheoretical' method that used the expression:

$$
\frac{\Phi_{ij}}{\Phi_{ji}} = \frac{M_i}{M_j} \frac{\mu_i}{\mu_j}
$$

(2.4)

along with one mixture conductivity. After testing these methods against available experimental data for twelve polar–nonpolar gas systems (including humid air), the 'semitheoretical' method was considered most favorable with an absolute average deviation of 1.86% for ninety-seven mixtures. The 'empirical' method was tested for eighty-five mixtures and gave the highest absolute average deviation of 2.70%. Tondon and Saxena suggest that the 'empirical' method may be considered the least attractive method because it requires two mixture conditions.
conductivities, while the 'approximate' and Lindsay–Bromley methods can be advantageous where there are no known values of mixture conductivity. Tondon and Saxena's comparisons for humid air in particular provided a 3.5% absolute average deviation from the experimental data of Gruss and Schmick for the 'empirical' method, while the three remaining methods each gave absolute deviations of approximately 1% or less. Tondon and Saxena's tests for systems where experimental data was available for more than one temperature indicated that, unlike the case for viscosity, no systematic error trend was apparent for higher temperature calculations made with lower temperature Sutherland coefficients. The 'semitheoretical' method, however, was noted to be especially satisfactory for increasing temperature.

The determination of recommended Sutherland coefficients for thermal conductivity based on three methods for a large number of gas mixtures (including humid air) was reported by Touloukian et al. (1970a). Method one uses the following expression:

\[
\frac{\phi_{ij}}{\phi_{ji}} = \frac{k_i}{k_j}
\]

and one mixture thermal conductivity to calculate the coefficients. Similarly, methods two and three require one mixture value and the expressions:

\[
\frac{\phi_{ij}}{\phi_{ji}} = \frac{k_i}{k_j} \frac{59M^* + 88M^* + 150}{150M^* + 88M^* + 59}
\]

where \( M^* = \frac{M_j}{M_i} \)
and

\[
\frac{\phi_{ij}}{\phi_{ji}} = \frac{k_i^0}{k_j^0} \left( \frac{M_i}{M_j} \right)^{0.15}
\]

respectively. The coefficients were calculated at all mixtures for each temperature of the experimental data and the absolute, root-mean-square and maximum deviations were considered in finding the best pair of Sutherland coefficients for each method at each temperature. A best method out of the three was not suggested by Touloukian et al. The authors' own calculations for humid air using the tabulated coefficients indicated that methods one and three fit the experimental data of Gruss and Schmick slightly better than method two. The deviations of the individual experimental data points from the calculated values for these two methods are given in Appendix A, Table A.3.

Deviations are also given in Table A.3 for values of mixture conductivity calculated using Tondon and Saxena's published coefficients for their 'semitheoretical' method, based on Eq. (2.4). Since this method requires the use of pure component viscosities it was decided that a newly calculated pair of coefficients should be found using the more recently recommended pure component viscosity data discussed earlier in Sections 2.1 and 2.2 (Touloukian et al. 1975 for air and RDV75 for steam). Comparisons given in Table A.3 based on these newly calculated coefficients indicate that they result in an average absolute deviation from the experimental data of 0.8%.

In all of the above described mixture conductivity methods, the values used for the conductivity of the pure components were either...
exactly or very nearly the same as those measured by Gruss and Schmick. A benefit of this procedure is that it tends to minimize the effect of any systematic error in the measured values on the determination of the best pair of Sutherland coefficients. Once a recommended pair of coefficients is found, these coefficients can be used along with what may be considered more accurate pure component conductivities in making mixture conductivity calculations.

For the present heat transfer study, mixture conductivity values based on the 'semitheoretical' method of Tondon and Saxena (1968) with the newly calculated Sutherland coefficients were utilized. As already noted, these coefficients accurately reproduced the Gruss and Schmick data with an average absolute deviation of 0.8%, and, in addition, Tondon and Saxena found that this method tested well for increasing temperatures. A graphical comparison with the data of Gruss and Schmick is shown in Figure 2.2. The solid curve in the figure was determined using the newly calculated Sutherland coefficients and the more recent pure component data (Touloukian et al. 1970a for air and RTC77 for steam). This curve lies uniformly above the Gruss and Schmick data by about 4.0%, suggesting that their data may contain a small systematic error.

The procedure used for determining the thermal conductivity of pure steam in cases where pure steam does not exist in an equilibrium state at the temperature and pressure of the mixture, was the same as that used for the determination of steam viscosity under such conditions.
Fig. 2.2 Thermal Conductivity of Humid Air at 353 K
That is, the thermal conductivity was evaluated at the mixture temperature and the corresponding saturation pressure.

The newly calculated coefficients used for the above calculations are an appropriate choice for the evaluation of mixture conductivity until the time that more experimental data is available. In the interim, it remains very difficult, impossible rather, to determine the best Sutherland coefficients for humid air thermal conductivity because of the small amount of experimental data that exists. Any additional experimental data for humid air thermal conductivity at various temperatures and mixture concentrations would be a welcome contribution.

2.4.3 Specific Heat

It can be shown that for a mixture of n ideal gases, the mixture isobaric specific heat is provided by the following expression:

$$c_{pm} = \sum_{i=1}^{n} X_i c_{pi}$$

(2.5)

where the pure component specific heats are evaluated at their individual partial pressures, $P_i$, and the mixture temperature. The concept of partial pressure follows from the Dalton model of gas mixtures where the properties of each component are taken as those that the component would have if it existed separately at the temperature and volume of the mixture. It follows from this that:

$$P = \sum_{i=1}^{n} P_i$$
This expression holds exactly for an ideal gas. Use of the ideal gas law yields:

\[ P_i = X_i P \]

Hsieh (1975) notes that experiment has shown that Dalton's model holds approximately for real gas mixtures in some ranges of temperature and pressure where the ideal gas law itself is quite inaccurate. In such cases, a real gas equation of state should be used in preference over the preceding equation in determining the partial pressures of the components. Van Wylen and Sonntag (1978) have mentioned, however, that even for a saturated gas-vapor mixture (such as humid air at saturation) the treatment of the gaseous phase as a mixture of ideal gases often gives good results. This observation seems to be confirmed by the present authors' comparisons of mole fractions determined utilizing two different methods; (1) the above ideal gas relations plus observations of humid air mixture dewpoint temperatures and (2) mole fraction calculations determined directly from measured air and steam flow rates (Sections 4.3 and 5.1). In light of this, Eq. (2.5) was used to determine humid air specific heat in the present study.
3. EXPERIMENTAL FACILITY

As mentioned earlier, the test-unit assembly used in this study was the same as that used by Florschuetz et al. (1980, 1981) in earlier heat transfer studies with dry air. Fig. 3.1 shows the basic test model geometry and nomenclature for the jet array impingement scheme investigated. Spanwise and streamwise views of the test-unit assembly which incorporates the basic test model geometry are shown in Fig. 3.2. The assembly basically consists of a plenum in which either dry or humid air is introduced, a jet orifice plate through which the heat transfer fluid exits forming jets, and a segmented copper heat transfer test plate upon which the jets of fluid emerging from the plenum impinge.

The incorporation of the test unit assembly into the overall experimental test facility is shown in Fig. 3.3. Separate air and steam sources are individually regulated and are combined to form the desired humid air mixture. The air supply consists of a compressor with an aftercooler. The aftercooler removes essentially all water vapor from the compressed air, so that for the present purpose this air may be considered dry. This was verified by the results of saturation tests made during the experimental runs (Sections 4.3 and 5.1). The air supply passes through filters and regulators before it reaches a flow metering section consisting of a square-edged orifice. Orifice upstream and differential pressures are measured with U-tube or well-type manometers and the air temperature just downstream of the orifice is measured using a copper-constantan thermocouple. (Manometers and
Fig. 3.1 Basic Test Model Geometry and Nomenclature
Fig. 3.2 Test-Unit Assembly
Fig. 3.3 Overall Experimental Facility
thermocouples not shown in Fig. 3.3). An electric heater in the air line allows preheating of the air before it mixes with the steam.

The steam supply comes from the campus physical plant and is typically at 430 K and 0.58 MPa in the main. The steam flow passes through a strainer and regulator before being measured by a variable area flowmeter. A bourdon gauge with 0.1 psi divisions is used to measure the pressure in the steam line just upstream of the flowmeter. The steam temperature is measured with a copper-constantan thermocouple also located just upstream of the flowmeter (not shown in Fig. 3.3).

The 2.66 cm inside diameter air and steam lines are joined together at a common piping tee followed by a pipe flow length of over forty diameters to insure complete mixing of the air and steam flows before reaching the plenum. The entire steam and mixture path including the plenum is wrapped with heating tapes and covered with fiberglass insulation to prevent condensation. Thermocouples mounted at various locations along the path allow the pipe surface temperatures to be monitored.

A brief description of the essential details of the test unit assembly (Fig. 3.2) will now be provided. More extensive details of the assembly can be found in an earlier report by Florschuetz et al. (1980). The assembly consists of a single test plate unit containing the segmented copper test plate and is capable of accommodating several different test configurations by means of interchangeable plenums, spacers and jet plates. The plenum size shown in Fig. 3.2 was used for all of the work reported here. In order to be consistent with the
nomenclature of earlier work it is referred to as the B-size plenum. This plenum, and its matching jet plates, cover just a portion of the entire heat transfer test plate surface available. Packing material in the plenum provides uniform flow characteristics approaching the jet plate. A copper-constantan thermocouple mounted in the plenum and a static pressure tap for a manometer connection (not shown in Fig. 3.2) allow for temperature and pressure measurements to be made at a location inside the plenum just upstream of the jet orifice plate.

All of the jet plates used in this study contain uniform inline patterns of 180 holes (18 spanwise holes by 10 streamwise holes). Hole diameters were 0.254 or 0.127 cm and in all cases the jet holes are counterbored so that the plate thickness at each hole location is the same as the hole diameter.

The various geometries studied are denoted by the plenum size followed by the streamwise and spanwise hole spacings, and the channel height in terms of jet hole diameters in parenthesis. The letters I or S denote inline or staggered arrays respectively. Hence a typical configuration is given as B(5,4,3)I indicating the B-size plenum and an inline array with \( x_n/d \), \( y_n/d \) and \( z/d \) taking on the values 5, 4 and 3 respectively.

The individually heated copper segments which make up the test plate allow streamwise resolved heat transfer coefficients to be determined. A one-for-one matching exists between the centerlines of the first ten segments and the ten immediately opposite spanwise rows of holes in the jet plates. The A.C. power input to the segment heaters
can be individually controlled by the use of variac potentiometers. A total of eleven segment heaters are supplied with power, ten corresponding to the ten spanwise rows of jets and an eleventh acting as a guard element. The dimensions of each copper segment are 12.0 cm in the spanwise direction by 1.27 cm in the streamwise direction by 0.635 cm thick. The heat transfer surface length in the streamwise direction, denoted by L, is taken as the distance from one-half of the streamwise hole spacing \((x_n/2)\) upstream of the first jet hole location to one-half of a streamwise hole spacing downstream of the last jet hole location and is equal to 12.7 cm for all the configurations utilized in this study.

The thermocouple voltages are recorded by a digital data logger with a compensated reference junction. The segment heater power measurements are made after conversion to D.C. by a solid state signal conditioner and are also recorded by the digital data logger.
4. EXPERIMENTAL PROCEDURES AND DATA REDUCTION

4.1 Typical Test Runs

The preparation for a typical test run included the installation of the appropriate channel height spacer and jet plate for the desired geometric configuration onto the test-unit assembly. The various configurations and experimental conditions for which data was obtained are summarized in Table 4.1. The geometries chosen represent a range of jet array flow conditions varying from highly nonuniform (for the B(5,4,1)1 geometry) to essentially uniform (for the B(10,8,3)1 geometry) pressure and row by row jet flow distributions along the streamwise direction of the channel. For each configuration at which a humid air run was made a corresponding dry air run was also performed at roughly the same mean jet Reynolds number. The test run procedures for both the humid and dry air runs are described in the following paragraphs.

At the beginning of a humid air run the air regulators were adjusted for the desired air flow. Air flow measurements were made using standard 1.778 or 2.54 cm diameter square-edged orifice plates according to the methods of the American Society of Mechanical Engineers' (1959) publication Fluid Meters which provides tables of coefficients for various standard square-edged orifice diameters. Power was supplied to the air heater in order to obtain an air temperature which, according to a simple energy balance calculation, would provide the desired mixture temperature upon introduction of the steam flow. Preheating the air in this manner insured that the resultant mixture
Table 4.1 Test Run Summary

<table>
<thead>
<tr>
<th>Geometric configuration</th>
<th>Case</th>
<th>Mean jet Reynolds number Rej</th>
<th>Mixture</th>
<th>Plenum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>W</td>
<td>X₂</td>
</tr>
<tr>
<td>B(5,4,3)I</td>
<td>1</td>
<td>9040</td>
<td>0.139</td>
<td>0.191</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>9570</td>
<td>0.227</td>
<td>0.267</td>
</tr>
<tr>
<td></td>
<td>Dry air</td>
<td>9170</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B(10,8,3)I</td>
<td>3</td>
<td>20200</td>
<td>0.064</td>
<td>0.094</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>19100</td>
<td>0.128</td>
<td>0.170</td>
</tr>
<tr>
<td></td>
<td>Dry air</td>
<td>18500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B(5,4,1)I</td>
<td>5</td>
<td>9280</td>
<td>0.156</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>Dry air</td>
<td>8740</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
temperature would be above that of saturation for the mixture.

The steam and mixture lines, along with the aluminum plenum, were heated to temperatures above the expected saturation temperatures using the installed heating tapes before starting the steam flow. Heat-up times for the pipes and plenum from room temperature were typically thirty to forty minutes. Low power inputs were supplied to the copper test plate segments to bring them above room temperature as well. After the necessary pipe, plenum and air heating was established, the steam regulator was adjusted for the desired steam flow rate. The flow rate was measured using the variable area meter.

After introduction of the steam flow, power to the segment heaters was set to zero and the segment temperatures were monitored while approaching steady state conditions. The time period allowed to reach steady state conditions was typically two hours. Once steady state was attained for this zero segment heater power condition the individual segment temperatures were recorded. In most cases, two non-zero power input levels referred to as the maximum and the one-half power levels followed the zero power condition. These power level settings involved adjusting the individual variac potentiometers for each segment heater in such a way as to achieve an essentially isothermal test surface. Temperature differences between adjacent segments were typically no more than 0.1 K and the temperature difference between any two given segments was typically no more than 0.25 K. Whenever the adjustment to a new power level was made, a time period of typically forty minutes to an hour was allowed to pass in order to achieve steady state conditions.
before recording the necessary data.

The minimum temperature in the plenum for any given humid air run was limited by the saturation temperature for the humid air, which in turn depended on the humidity ratio and the plenum pressure level required to achieve the desired mixture flow rate for that run. The maximum temperatures for the test plate segments were limited by certain materials used in the construction of the test plate, which was not designed for operation at high temperatures. For the tests reported here, the maximum test surface temperatures were 355 K. Surface-to-plenum mixture temperature differences for non-zero power input test conditions ranged from 3 to 12 K.

For mixture Case 2 \((W = 0.23)\) only a zero and a maximum power level condition were run due to the fact that using all three power levels would have required a long run time with a high humidity mixture that was being partially exhausted into the laboratory.

A complete set of data recorded for each power level condition, besides the individual segment temperatures and power inputs, included the plenum pressure and temperature, the air-metering orifice temperature and upstream and differential pressures, the steam temperature and pressure at the flowmeter as well as the flowmeter reading itself, and the barometric pressure.

Dry air test runs followed a simplified procedure to that described above where the additional effort required in preheating the steam lines and adjusting and measuring the steam flow was eliminated. Unlike the cases with humid air, these runs were not constrained by the need for an
elevated plenum fluid temperature in order to avoid saturation conditions. The air for these runs could simply be introduced at the slightly-below-ambient temperatures that resulted from the air compressor's aftercooler as was the practice for prior studies using the same air supply (Florschuetz et al. 1980). However, after the first dry air run had been performed it was decided to perform later runs at similar temperature levels and differences to the humid air runs. This procedure would tend to minimize any bias due to possible differences in heat leaks resulting from two different operating temperatures.

4.2 Segment Heat Transfer Coefficients

The heat transfer coefficient for each active segment of the test plate was evaluated from

\[
h = \frac{\text{Q}}{\text{A}} / (T_s - T_r)
\]

where \( Q \) is the segment heat rate determined from the power measurements after correcting for the segment heat leaks, \( A \) is the heat transfer surface area of the segment, \( T_s \) is the segment surface temperature and \( T_r \) is the adiabatic wall temperature.

Three data sets \( (Q, T_s) \) were available corresponding to the three steady state conditions (zero, half, and maximum power levels described above) recorded for each geometry at a specific flow rate. Values of \( h \) were determined by a linear least squares fit to the three data sets. Values of \( T_r \) also resulted from the fit. It is clear from Eq. (4.1) that only two data sets \( (Q,T_s) \) would be required to determine \( h \) and \( T_r \).
The use of three sets permits a check on the expected linearity of $Q$ with $T_s$. Significant deviation of the three points from a straight line would indicate invalid data. To check the linearity, values of $h$ computed using each of the three possible combinations of two members of each data set (zero/maximum, zero/half and half/maximum power input levels) were compared with values of $h$ from the least squares fit line. For the dry air runs 95% of the individually computed values were within $\pm 3\%$ of the values based on the fit line. For the humid air runs the corresponding result was $\pm 4\%$ for 95% confidence.

A comparison of the data for the dry air runs made for this work with the corresponding (i.e. same geometry and Reynolds number) data from the runs by Florschuetz et al. made approximately four years ago indicated excellent long term reproducibility with deviations of $\pm 3\%$ for 95% confidence and $\pm 4\%$ for 100% confidence.

4.3 Saturation Tests

A saturation test to add confidence to the humidity ratio (or mole fraction) determined from the measured mass flow rates was performed for all of the humid air runs except Case 1 as follows: The inline air heater power was lowered by small increments resulting in a gradual decrease in mixture temperature as measured by the plenum thermocouple. While the mixture temperature decreased the flow exhausting from the jet plate-to-impingement surface channel was carefully observed for any traces of liquid. The mixture temperature at which condensate was first observed was recorded. Then, the air heater power was increased. As
the mixture temperature in the plenum began to rise the temperature
where condensate was no longer observed was recorded. These two
temperatures were always within 0.5 K and their average was used as a
measurement of the mixture saturation (dewpoint) temperature. The
saturation pressure for steam \( (P_{\text{sat}}) \) corresponding to this temperature
was then found from the ASME Steam Tables (Meyer et al. 1968). Using
Dalton's model (Section 2.4.3) and the mixture (plenum) pressure \( (P_o) \),
the mole fraction of steam was then calculated from:

\[
X_a = \frac{P_{\text{sat}}}{P_o}
\]

4.4 Experimental Uncertainties

Florschuetz et al. (1980) examined the experimental uncertainty for
heat transfer coefficients and Nusselt numbers obtained with the same
test-unit assembly used for the experimental work reported in this study
and the same dry air experimental procedures described in Section 4.1.
On the basis of 95% confidence the composite uncertainty for heat
transfer coefficients was about \( \pm 4\% \) for the geometric configurations
that were the same as those considered for this work. Uncertainties for
the 0.127 and 0.254 cm diameter jet hole diameters were \( \pm 2\% \) and \( \pm 1\% \)
respectively. The resulting composite uncertainty for Nusselt numbers
is then conservatively characterized by \( \pm 5\% \).

The air mass flow rate determined using square-edged orifice plates
has an uncertainty of about \( \pm 2\% \). The variable area flow meter used for
steam flow measurement was tested against both the 1.778 and the 2.54 cm
standard square-edged orifices using room temperature air and was found
to have good repeatability and the expected relative accuracy. A calibration curve based upon the 2.54 cm orifice test was made for use in the steam flow measurements. The composite steam mass flow rate uncertainty, taking into consideration the meter calibration, scale reading, and steam temperature and pressure measurements is estimated to be $\pm 4\%$. This uncertainty is primarily dependent upon the uncertainty associated with the fluctuating meter scale reading which ordinarily fluctuated about an apparent mean value. Due to the relatively small contribution to the total flow rate made by the steam, the composite uncertainty for the total humid air mixture flow rate (dry air flow rate plus steam flow rate) is just $\pm 2\%$.

Composite uncertainty for the mean jet Reynolds number for both the dry air and humid air runs based on the jet hole diameter and flow rate uncertainties is $\pm 3\%$. The composite uncertainty in the humidity ratio (and also the mole fraction) based on the air and steam flow rate uncertainties is $\pm 4.5\%$. The uncertainty for mole fraction based on the saturation tests (Section 4.3) is estimated at $\pm 4\%$. This is primarily dependent on the uncertainty in determining the saturation (dewpoint) temperature.
5. RESULTS AND DISCUSSION

5.1 Saturation Tests

The mole fraction determinations based on the saturation tests described in Section 4.3 are compared with those based on the measured flow rates in Table 5.1. As stated in Section 4.4, the uncertainties for the mole fractions based on the saturation tests and the measured flow rates were both about $\pm 4\%$. The two different mole fraction determinations are consistent to well within experimental uncertainty. For reduction of the humid air heat transfer data mole fractions based on the flow rate measurements were used.

5.2 Heat Transfer Coefficients

As mentioned in the introduction, the objective of this study is to verify the expectation that dry air heat transfer data may be applied with confidence to situations involving humid air provided that the correct humid air thermophysical property values are used in evaluating the relevant nondimensional parameters. Heat transfer using jet array impingement has served in this work as an example cooling scheme with which to obtain data for carrying out the comparisons necessary to establish this verification.

Results of prior dry air studies with jet array impingement (Florschuetz et al. 1980) have shown that values of Nusselt numbers resolved to $x_n$ in the streamwise direction (referred to as segment Nusselt numbers) for a given geometry and streamwise location are
Table 5.1  Comparison of Mole Fractions Determined from Measured Flow Rates with Values from Saturation Tests

<table>
<thead>
<tr>
<th>Case</th>
<th>Mole fraction $X_2$ based on:</th>
<th>Percent deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured flow rates</td>
<td>Saturation tests</td>
</tr>
<tr>
<td>2</td>
<td>0.267</td>
<td>0.266</td>
</tr>
<tr>
<td>3</td>
<td>0.094</td>
<td>0.094</td>
</tr>
<tr>
<td>4</td>
<td>0.170</td>
<td>0.169</td>
</tr>
<tr>
<td>5</td>
<td>0.200</td>
<td>0.208</td>
</tr>
</tbody>
</table>
proportional to $Re^m$ where $m$ varies somewhat from segment to segment but is typically about 0.7. Values of $m$ determined from these previous dry air studies were used to interpolate the segment Nusselt numbers of the dry air runs of the present study to match exactly the humid air run Reynolds numbers. The Reynolds numbers for the dry and humid air runs made for any given geometry were very similar (see Table 4.1).

The Prandtl numbers for dry and humid air runs differed by 3 to 7%. It is generally accepted that Nusselt number dependence on Prandtl number (at least for Prandtl number on the order of one which is the case for gases) is proportional to $Pr^n$ where $n$ is either 0.33 or 0.40. A value for $n$ of $1/3$ was chosen here to represent the experimental data. Use of $n = 0.4$, however would not have resulted in a significant difference.

The necessary thermophysical property values for reducing the experimental data in nondimensional form were determined according to the methods described in Section 2.

Figures 5.1, 5.2 and 5.3 show values of $NuPr^{-1/3}$ for humid and dry air runs for five cases (three different geometries). It is apparent that overall the humid and dry air results are quite consistent. Recall that in Section 4.4 the uncertainty in the Nusselt number for 95% confidence was conservatively estimated to be $\pm 5\%$. For the purpose of comparison of dry and humid air data obtained with the same test rig it is appropriate to eliminate any possible contribution made to the overall uncertainty as a result of a systematic error (or bias) in the data. It was noted in Section 4.2 that the long term reproducibility of
Fig. 5.1 $\text{NuPr}^{-2/3}$ for Dry versus Humid Air for B(5,4,3)I Geometry
Fig. 5.2 \( \text{NuPr}^{-1/3} \) for Dry versus Humid Air for B(10,8,3)I Geometry
Fig. 5.3 $\text{NuPr}^{-1/3}$ for Dry versus Humid Air for B(5,4,1)I Geometry
dry air runs for 95% confidence was ± 3%. It therefore seems reasonable to suggest an uncertainty that is slightly less than the overall value, say ± 4% for the purpose of comparison of the dry and humid air data. The height of the data points in Figs. 5.1 through 5.3 represents 8% (± 4%) on the logarithmic scale, reflecting this experimental uncertainty. It can be seen that virtually all of the humid air points are coincident with or overlap the dry air points. These results provide strong verification for the anticipated applicability of the dry air data to the humid air case and also indicate that the suggested methods for the determination of the various thermophysical properties involved may be used with good confidence. The values of $\text{NuPr}^{-1/3}$ for the various cases along with the percentage deviations between the dry and humid air values are presented in tabular form in Appendix B, Table B.1.

The only deviation which clearly exceeds experimental uncertainty is for the first points of Case 5 (Fig. 5.3). Comparison of the dry air value with the data of Florschuetz et al. (1980) for the corresponding geometry and Reynolds number indicates that the air data point is consistent and thus, the humid air value is suspect. Some observations of the jet plate and the first segment of the test plate after the runs of Case 5 suggest a possible explanation for this result. The jet plate and heat transfer test plate were completely cleaned before the dry and humid air tests of Case 5 were run. After Case 5 was run the plenum was removed and the jet and test plates were inspected. Discoloration of both plates observed at the upstream end of the channel (location of segment one), indicated that a thin layer of liquid water (possibly from
inadvertent condensation which occurred while achieving this test condition) may have been trapped against the upstream end wall of the channel and covered a small fraction of the surface of segment one. No cross flow exists at that location to overcome the force of surface tension that may have prevented this condensate from being forced away from the upstream endwall. If some condensate were present at the described upstream location, the evaporation of water from the first segment would have no doubt significantly increased the heat transfer coefficient for that segment as is observed in Fig. 5.3.

It is of interest to examine the effect of the presence of water vapor on dimensional heat transfer coefficients relative to those for dry air. Starting with the relation

$$\frac{1}{Nu} \propto Re^m Pr^{1/3}$$

the ratio of the dry air to humid air heat transfer coefficient for the same geometry and flow rate can be expressed as:

$$\frac{h_a}{h_m} = \frac{k_a}{k_m} \left( \frac{\mu_m}{\mu_a} \right)^m \left( \frac{Pr_a}{Pr_m} \right)^{1/3}$$

Results of calculations based on this expression (using $m = 0.73$) at a temperature of 345 K and humidity ratios varying from 0.0 to 0.25 are plotted in Fig. 5.4. While the effect of using dry air rather than humid air properties is seen to not be extremely large (remaining just under 10% for a humidity ratio of 0.25) it is emphasized that this
Fig. 5.4 Effect of Humidity Ratio on Heat Transfer Coefficient
result has apparently not been previously verified directly by heat transfer measurements with humid air for a range of controlled humidity ratios.
6. CONCLUDING REMARKS

Comparison of nondimensional heat transfer data from convective heat transfer experiments using both dry and humid air of various humidity ratios up to 0.23 has verified the equivalency of the data provided the appropriate thermophysical property values are used in evaluating the relevant nondimensional parameters. This verification is based on experiments made with a complex forced convection cooling scheme (heat transfer to arrays of circular jets with crossflow). Designers who wish to apply nondimensional heat transfer data or correlations based on dry air studies to situations involving humid air heat transfer can do so with essentially the same level of confidence as if they were concerned only with dry air. If nondimensional dry air heat transfer data is evaluated for humid air situations using dry, rather than humid air properties however, the heat transfer coefficients obtained will be in error. The error increases with increasing humidity ratio approaching 10% for a humidity ratio of 0.25.

Several methods for determining the properties of viscosity and thermal conductivity of humid air were described in Section 2. The methods selected for use in the data reduction for this work have proven to be convenient and useful.

The work carried out for this study in reviewing the present state of humid air property determination has resulted in a number of observations which are recounted below. One observation is that any new experimental data for the properties of dynamic viscosity and thermal
conductivity of humid air would be useful for either an addition to the available temperature range, or a verification of the meager existing measurements. For the viscosity of humid air there is a specific need to investigate the inflections that occurred in the experimental data of Kestin and Whitelaw. For thermal conductivity, the need for experimental data is very great. The data of Zakharov, as discussed in Section 2.4, appears to be of questionable value leaving only the very small amount of data of Gruss and Schmick at one temperature and four different mixtures.

The semitheoretical methods used to determine the mixture properties of dynamic viscosity and thermal conductivity for this study require an experimental value of the property at one mixture composition. In the case of viscosity, the data of Kestin and Whitelaw may be sufficient to warrant an extended investigation to determine which mixture value is best for determining the Sutherland coefficients that will reproduce all of the experimental data with the least deviation. Approaches similar to those that have been carried out for other systems (as described in Section 2.4.1) could be used in making this determination. It is unknown whether such an effort would result in Sutherland coefficients that provide better results than those obtained by the suggested approach based on a mixture data point selected at the available temperature level which is closest to the temperature of interest. For thermal conductivity the determination of a 'best' pair of Sutherland coefficients cannot really be accomplished until there is more experimental data available.
The interest in the thermophysical properties of humid air in this study was motivated by the concept of using humid air for gas turbine engine component cooling. Thus, in regard to the need for more experimental humid air thermophysical property data, it should be emphasized that data obtained for high temperatures is of particular interest.
REFERENCES


APPENDIX A

THERMOPHYSICAL PROPERTIES OF HUMID AIR

Percent deviations indicated in all tables of Appendix A are deviations of the experimental data from the calculated values based on the methods indicated.
Table A.1 Dynamic Viscosity of Humid Air at Several Temperature Levels  
Subscripts 1 and 2 refer to air and steam respectively  
Viscosity in Kg/m·s X 10+6

<table>
<thead>
<tr>
<th>Mole fraction of steam</th>
<th>Experimental data (Kestin and Whitelaw 1964)</th>
<th>Wilke method (1950)</th>
<th>Percent Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Method one*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Method two**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Studnikov formula (Saxena 1973)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(Saxena 1973)</td>
</tr>
<tr>
<td>$X_2$</td>
<td>$\mu_m$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=298 K</td>
<td>$\mu_1=18.440$</td>
<td>$\mu_2=9.867$</td>
<td></td>
</tr>
<tr>
<td>0.006</td>
<td>18.451</td>
<td>0.34</td>
<td>0.13</td>
</tr>
<tr>
<td>0.008</td>
<td>18.446</td>
<td>0.40</td>
<td>0.14</td>
</tr>
<tr>
<td>0.011</td>
<td>18.441</td>
<td>0.51</td>
<td>0.15</td>
</tr>
<tr>
<td>0.016</td>
<td>18.419</td>
<td>0.63</td>
<td>0.16</td>
</tr>
<tr>
<td>0.017</td>
<td>18.420</td>
<td>0.68</td>
<td>0.17</td>
</tr>
<tr>
<td>0.032</td>
<td>18.399</td>
<td>1.27</td>
<td>0.18</td>
</tr>
<tr>
<td>0.033</td>
<td>18.374</td>
<td>1.18</td>
<td>0.19</td>
</tr>
<tr>
<td>Avg. Absolute Deviation:</td>
<td>0.72</td>
<td>0.13</td>
<td></td>
</tr>
</tbody>
</table>

| T=323 K                | $\mu_1=19.610$                              | $\mu_2=10.617$      |                     |
| 0.019                  | 19.593                                      | 0.78                | 0.14                |
| 0.024                  | 19.591                                      | 0.99                | 0.19                |
| 0.031                  | 19.575                                      | 1.24                | 0.20                |
| 0.042                  | 19.539                                      | 1.56                | 0.17                |
| 0.062                  | 19.474                                      | 2.16                | 0.14                |
| 0.120                  | 19.247                                      | 3.77                | 0.12                |
| Avg. Absolute Deviation: | 1.75                                       | 0.15                |

(continued next page)
Table A.1 (continued)

<table>
<thead>
<tr>
<th>T=348 K</th>
<th>$\mu_1=20.730$</th>
<th>$\mu_2=11.428$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.055</td>
<td>20.632</td>
<td>1.97</td>
</tr>
<tr>
<td>0.075</td>
<td>20.588</td>
<td>2.67</td>
</tr>
<tr>
<td>0.097</td>
<td>20.497</td>
<td>3.24</td>
</tr>
<tr>
<td>0.132</td>
<td>20.357</td>
<td>4.21</td>
</tr>
<tr>
<td>0.193</td>
<td>20.046</td>
<td>5.62</td>
</tr>
<tr>
<td>0.265</td>
<td>19.586</td>
<td>6.91</td>
</tr>
<tr>
<td>0.317</td>
<td>19.252</td>
<td>7.89</td>
</tr>
<tr>
<td>0.371</td>
<td>18.792</td>
<td>8.34</td>
</tr>
<tr>
<td>0.387</td>
<td>18.781</td>
<td>9.20</td>
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</tbody>
</table>

Avg. Absolute Deviation: 5.56 0.24 0.20 0.30

<table>
<thead>
<tr>
<th>T=423 K</th>
<th>$\mu_1=24.034$</th>
<th>$\mu_2=14.187$</th>
</tr>
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<tbody>
<tr>
<td>0.228</td>
<td>22.287</td>
<td>1.65</td>
</tr>
<tr>
<td>0.332</td>
<td>22.068</td>
<td>5.40</td>
</tr>
<tr>
<td>0.434</td>
<td>20.931</td>
<td>4.90</td>
</tr>
<tr>
<td>0.522</td>
<td>19.654</td>
<td>2.95</td>
</tr>
<tr>
<td>0.549</td>
<td>19.337</td>
<td>2.72</td>
</tr>
<tr>
<td>0.642</td>
<td>17.859</td>
<td>-0.20</td>
</tr>
<tr>
<td>0.647</td>
<td>17.876</td>
<td>0.17</td>
</tr>
<tr>
<td>0.712</td>
<td>16.693</td>
<td>-2.88</td>
</tr>
<tr>
<td>0.743</td>
<td>16.260</td>
<td>-3.62</td>
</tr>
<tr>
<td>0.790</td>
<td>15.666</td>
<td>-4.41</td>
</tr>
</tbody>
</table>

Avg. Absolute Deviation: 2.90 7.46 7.28 5.81

(continued next page)
Table A.1 (continued)

<table>
<thead>
<tr>
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<th>T=473 K</th>
<th>μ₁=26.100</th>
<th>μ₂=16.179</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.311</td>
<td>24.289</td>
<td>4.44</td>
<td>-3.00</td>
</tr>
<tr>
<td>0.402</td>
<td>23.274</td>
<td>3.97</td>
<td>-4.49</td>
</tr>
<tr>
<td>0.492</td>
<td>21.976</td>
<td>2.18</td>
<td>-6.69</td>
</tr>
<tr>
<td>0.605</td>
<td>20.670</td>
<td>1.44</td>
<td>-7.36</td>
</tr>
<tr>
<td>0.632</td>
<td>20.246</td>
<td>0.71</td>
<td>-7.90</td>
</tr>
<tr>
<td>0.697</td>
<td>19.154</td>
<td>-1.44</td>
<td>-9.37</td>
</tr>
<tr>
<td>0.701</td>
<td>19.434</td>
<td>0.21</td>
<td>-7.81</td>
</tr>
<tr>
<td>0.797</td>
<td>17.378</td>
<td>-5.49</td>
<td>-11.70</td>
</tr>
<tr>
<td>Avg. Absolute Deviation:</td>
<td>2.49</td>
<td>7.29</td>
<td>7.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>T=523 K</th>
<th>μ₁=28.178</th>
<th>μ₂=18.221</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.230</td>
<td>27.216</td>
<td>4.12</td>
<td>-1.93</td>
</tr>
<tr>
<td>0.356</td>
<td>26.124</td>
<td>4.64</td>
<td>-3.31</td>
</tr>
<tr>
<td>0.498</td>
<td>24.303</td>
<td>3.03</td>
<td>-5.77</td>
</tr>
<tr>
<td>0.575</td>
<td>23.164</td>
<td>1.52</td>
<td>-7.18</td>
</tr>
<tr>
<td>0.598</td>
<td>22.905</td>
<td>1.42</td>
<td>-7.20</td>
</tr>
<tr>
<td>0.661</td>
<td>21.726</td>
<td>-0.96</td>
<td>-9.02</td>
</tr>
<tr>
<td>0.695</td>
<td>21.240</td>
<td>-1.58</td>
<td>-9.29</td>
</tr>
<tr>
<td>0.764</td>
<td>19.614</td>
<td>-5.92</td>
<td>-12.44</td>
</tr>
<tr>
<td>0.795</td>
<td>19.292</td>
<td>-5.96</td>
<td>-11.96</td>
</tr>
<tr>
<td>0.799</td>
<td>19.219</td>
<td>-6.12</td>
<td>-12.04</td>
</tr>
<tr>
<td>Avg. Absolute Deviation:</td>
<td>3.53</td>
<td>8.01</td>
<td>7.88</td>
</tr>
</tbody>
</table>

Sutherland Coefficients (evaluated at 348 K, x₂=0.193)

*Method one: \( \Psi_{12}=0.8390 \quad \Psi_{21}=0.7438 \)

**Method two: \( \Psi_{12}=0.8646 \quad \Psi_{21}=0.7138 \)
Table A.2 Dynamic Viscosity of Humid Air at 523 K  
Subscripts 1 and 2 refer to air and steam respectively  
Viscosity in Kg/m·s X 10+6

Pure component viscosities: \( \mu_1 = 28.179 \quad \mu_2 = 18.221 \)

<table>
<thead>
<tr>
<th>Mole fraction of steam ( X_2 )</th>
<th>Experimental data (Kestin and Whitelaw, 1964)</th>
<th>Method one* (Saxena 1973)</th>
<th>Percent Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_2 )</td>
<td>( \mu_m )</td>
<td>( \mu_m )</td>
<td>( % )</td>
</tr>
<tr>
<td>0.230</td>
<td>27.216</td>
<td>26.644</td>
<td>2.15</td>
</tr>
<tr>
<td>0.356</td>
<td>26.124</td>
<td>25.619</td>
<td>1.97</td>
</tr>
<tr>
<td>0.498</td>
<td>24.303</td>
<td>24.303</td>
<td>0.00</td>
</tr>
<tr>
<td>0.575</td>
<td>23.164</td>
<td>23.516</td>
<td>-1.50</td>
</tr>
<tr>
<td>0.598</td>
<td>22.905</td>
<td>23.272</td>
<td>-1.57</td>
</tr>
<tr>
<td>0.661</td>
<td>21.726</td>
<td>22.577</td>
<td>-3.77</td>
</tr>
<tr>
<td>0.695</td>
<td>21.240</td>
<td>22.187</td>
<td>-4.27</td>
</tr>
<tr>
<td>0.764</td>
<td>19.614</td>
<td>21.365</td>
<td>-8.19</td>
</tr>
<tr>
<td>0.795</td>
<td>19.292</td>
<td>20.981</td>
<td>-8.05</td>
</tr>
<tr>
<td>0.799</td>
<td>19.219</td>
<td>20.931</td>
<td>-8.18</td>
</tr>
</tbody>
</table>

Average Absolute Deviation: 3.97

* Sutherland Coefficients  
(evaluated at 523 K, \( X_2 = 0.498 \))  
\( \Psi_{12} = 0.8969 \quad \Psi_{21} = 0.9326 \)
Table A.3 Thermal Conductivity of Humid Air at 353 K
Subscripts 1 and 2 refer to air and steam respectively
Thermal conductivity in W/m·K
Pure component conductivities: $k_1=0.02869$  $k_2=0.02190$

<table>
<thead>
<tr>
<th>Mole fraction of steam</th>
<th>Experimental data (Gruss and Schmick 1928)</th>
<th>Methods one and three* (Toulkian et al. 1970a)</th>
<th>'Semitheoretical Method'** (Tondon and Saxena 1968)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_2$</td>
<td>$k_m$</td>
<td>Method one</td>
<td>Method three</td>
</tr>
<tr>
<td>0.197</td>
<td>0.02992</td>
<td>1.3</td>
<td>1.2</td>
</tr>
<tr>
<td>0.306</td>
<td>0.02961</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>0.444</td>
<td>0.02885</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.519</td>
<td>0.02814</td>
<td>-0.6</td>
<td>-0.6</td>
</tr>
<tr>
<td>Avg. Absolute Deviation:</td>
<td>0.6</td>
<td>0.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>

*Sutherland Coefficients

Method one: $\phi_{12}=0.8727$  $\phi_{21}=0.6659$
Published: $\phi_{12}=0.8170$  $\phi_{21}=0.7380$

Method three: $\phi_{12}=0.9003$  $\phi_{21}=0.6398$
Newly calc.: $\phi_{12}=0.8153$  $\phi_{21}=0.7257$

**Sutherland Coefficients
APPENDIX B

TABULAR REDUCED DATA ($\text{NuPr}^{-1/3}$)
### Table B.1 Comparisons of Humid and Dry Air Values of NuPr for All Test Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Segment Number 1</th>
<th>Segment Number 2</th>
<th>Segment Number 3</th>
<th>Segment Number 4</th>
<th>Segment Number 5</th>
<th>Segment Number 6</th>
<th>Segment Number 7</th>
<th>Segment Number 8</th>
<th>Segment Number 9</th>
<th>Segment Number 10</th>
<th>Mean</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>53.6</td>
<td>48.3</td>
<td>48.7</td>
<td>44.5</td>
<td>41.8</td>
<td>39.2</td>
<td>38.1</td>
<td>37.3</td>
<td>38.0</td>
<td>38.8</td>
<td>42.8</td>
</tr>
<tr>
<td>B(5,4,3)I</td>
<td>Dry 54.5</td>
<td>48.6</td>
<td>47.8</td>
<td>43.8</td>
<td>41.5</td>
<td>39.0</td>
<td>37.7</td>
<td>37.6</td>
<td>38.1</td>
<td>39.1</td>
<td>42.8</td>
</tr>
<tr>
<td>W=0.139</td>
<td>Dev.* -1.7</td>
<td>-0.6</td>
<td>1.9</td>
<td>1.6</td>
<td>0.7</td>
<td>0.5</td>
<td>1.1</td>
<td>-0.8</td>
<td>-0.3</td>
<td>-0.8</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>55.1</td>
<td>50.2</td>
<td>49.4</td>
<td>46.8</td>
<td>43.2</td>
<td>40.5</td>
<td>39.5</td>
<td>39.2</td>
<td>40.0</td>
<td>40.6</td>
<td>44.5</td>
</tr>
<tr>
<td>B(5,4,3)I</td>
<td>Dry 56.4</td>
<td>50.5</td>
<td>49.7</td>
<td>45.6</td>
<td>43.2</td>
<td>40.5</td>
<td>39.2</td>
<td>39.1</td>
<td>39.7</td>
<td>40.7</td>
<td>44.5</td>
</tr>
<tr>
<td>W=0.227</td>
<td>Dev.* -2.3</td>
<td>-0.6</td>
<td>-0.6</td>
<td>2.6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.8</td>
<td>0.3</td>
<td>0.8</td>
<td>-0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>57.6</td>
<td>58.1</td>
<td>54.5</td>
<td>56.4</td>
<td>54.6</td>
<td>50.4</td>
<td>46.7</td>
<td>45.6</td>
<td>44.4</td>
<td>43.8</td>
<td>51.2</td>
</tr>
<tr>
<td>B(10,8,3)I</td>
<td>Dry 58.3</td>
<td>56.7</td>
<td>53.8</td>
<td>54.9</td>
<td>53.3</td>
<td>49.2</td>
<td>45.8</td>
<td>44.0</td>
<td>44.1</td>
<td>43.0</td>
<td>50.3</td>
</tr>
<tr>
<td>W=0.064</td>
<td>Dev.* -1.2</td>
<td>2.5</td>
<td>1.3</td>
<td>2.7</td>
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<td>2.4</td>
<td>2.0</td>
<td>3.6</td>
<td>0.7</td>
<td>1.9</td>
<td>1.8</td>
</tr>
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<td>54.4</td>
<td>53.2</td>
<td>50.4</td>
<td>46.1</td>
<td>46.0</td>
<td>44.6</td>
<td>44.0</td>
<td>50.9</td>
</tr>
<tr>
<td>B(10,8,3)I</td>
<td>Dry 56.2</td>
<td>54.6</td>
<td>51.9</td>
<td>52.9</td>
<td>51.3</td>
<td>47.4</td>
<td>44.1</td>
<td>42.4</td>
<td>42.5</td>
<td>41.5</td>
<td>48.5</td>
</tr>
<tr>
<td>W=0.128</td>
<td>Dev.* 3.6</td>
<td>5.9</td>
<td>3.7</td>
<td>2.8</td>
<td>3.7</td>
<td>6.3</td>
<td>4.5</td>
<td>8.5</td>
<td>4.9</td>
<td>6.0</td>
<td>4.9</td>
</tr>
<tr>
<td>5</td>
<td>47.2</td>
<td>31.4</td>
<td>32.3</td>
<td>33.8</td>
<td>37.1</td>
<td>40.8</td>
<td>46.1</td>
<td>53.2</td>
<td>61.6</td>
<td>71.9</td>
<td>45.5</td>
</tr>
<tr>
<td>B(5,4,1)I</td>
<td>Dry 39.9</td>
<td>32.3</td>
<td>31.6</td>
<td>33.3</td>
<td>36.9</td>
<td>39.8</td>
<td>45.1</td>
<td>52.6</td>
<td>61.0</td>
<td>70.1</td>
<td>44.3</td>
</tr>
<tr>
<td>W=0.156</td>
<td>Dev.* 18.3</td>
<td>-2.8</td>
<td>2.2</td>
<td>1.5</td>
<td>0.5</td>
<td>2.5</td>
<td>2.2</td>
<td>1.1</td>
<td>1.0</td>
<td>2.6</td>
<td>2.7</td>
</tr>
</tbody>
</table>

*Percent deviation of humid air value from dry air value
The following pages contain the single precision FORTRAN IV coding for the international formulations RDV75 and RTC77 (Industrial) for the dynamic viscosity and thermal conductivity of water and steam (Kestin 1976, 1978). The coding of RTC77 contains conditional statements for the exponential function arguments. This was done to avoid computer execution error messages. Most computers will handle exponential underflows in such a manner as to not effect the final answer making the need for such conditional statements optional.

The available information regarding the regions of validity for the formulations is contained in the comment statements at the top of each program listing.
C*****************************************************************************
C DYNAMIC VISCOSITY OF WATER SUBSTANCE
C
C FORTRAN IV CODING OF INTERNATIONAL FORMULATION RDV75
C
C (FROM KESTIN, J., "NEW VALUES FOR THE VISCOSITY OF WATER SUBSTANCE", MECHANICAL ENGINEERING, VOL. 98, NO. 7, JULY 1976, P. 79.)
C
VARIABLES:
C
MUSTM=DYNAMIC VISCOSITY (X10+6 KG/M*S)
RHOS=STEAM DENSITY (KG/M**3)  TSK=STEAM TEMP (K)
C
REGION OF VALIDITY:
C
273.2 < TSK < 1073.2 AND
0 < RHOS < 1050 (OR 0 < PRESSURE < 100 MEGAPASCAL).
C
EXTENSIONS TO REGION OF VALIDITY:
C
PRESSURE=1000 MPA FOR 273.2 < TSK < 373.2
PRESSURE=350 MPA FOR 373.2 < TSK < 833.2
C
C*****************************************************************************
C REAL MUZ,MUZERO,MUS,RHOCSTCS,TSK,RHOS
REAL MUSTM,TSK,RHOS
REAL A(4),B(6,5)
DATA A/0.0181583,0.0177624,0.0105287,-0.0036744/
DATA B/0.501938,0.162888,-0.130356,0.907919,-0.551119,
0.146543,0.235622,0.789393,0.673665,1.207552,0.0670665,
-0.0843370,-0.274637,-0.745359,-0.959456,-0.687343,
-0.497089,0.195286,0.145831,0.263129,0.347247,
0.213486,0.100754,-0.032932,-0.0270448,-0.0253093,
-0.0267758,-0.0822904,0.0602253,-0.0202595/
C TCS=647.27
RHOCSTCS=317.763
C SPECIFY RHOS, TSK
RHOS=
TSK=
C CALCULATE MUZERO
MUZ=A(1)
DO 10 K=2,4
KZ=K-1
MUZ=MUZ+A(K)*(TCS/TSK)**KZ
10 CONTINUE
MUZERO = ((TSK/TCS)**0.5)/MUZ

CALULATE MUSTM

MUS = B(1,1)
DO 15 J = 2, 5
   JZ = J - 1
   MUS = MUS + B(1, J)*(RHOS/RHOCS-1.)*JZ
15 CONTINUE

DO 20 I = 2, 6
   IZ = I - 1
   DO 30 J = 1, 5
      JZ = J - 1
      IF (J.EQ.1) GO TO 40
      MUS = MUS + B(I, J)*((TCS/TSK-1.)**IZ)*((RHOS/RHOCS-1.)**JZ)
      GO TO 30
40  MUS = MUS + B(I, J)*((TCS/TSK-1.)**IZ)
30  CONTINUE
20  CONTINUE
MUSTM = MUZERO*EXP(RHOS/RHOCS*MUS)
WRITE(6,*) TSK, RHOS, MUSTM
100 CONTINUE
STOP
END
C******************************
C THERMAL CONDUCTIVITY OF WATER SUBSTANCE
C
C FORTRAN IV CODING OF INTERNATIONAL FORMULATION RTC77(IND)
C
C (FROM KESTIN, J., "THERMAL CONDUCTIVITY OF WATER AND
C STEAM", MECHANICAL ENGINEERING, VOL. 100, NO. 8,
C AUGUST, 1978 P. 46.)
C
C VARIABLES:
C
C THERMAL CONDUCTIVITY=CSTM (W/M**2 K)
C DENSITY=RHOS (KG/M**3) TEMPERATURE=TSK (K)
C
C REGION OF INVALIDITY:
C
C RECTANGLE ABOUT CRITICAL POINT WHERE:
C TSK=CRITICAL TEMPERATURE PLUS OR MINUS 1.5 K AND
C RHOS=CRITICAL DENSITY PLUS OR MINUS 100 KG/M**3
C
C******************************
REAL TSK,RHOS,CSTM
REAL A(4),B(3),BB(2),N4),CW
DATA A/ 1.02811E-2, 2.99621E-2, 1.56146E-2, -4.22464E-3/
DATA B,BB/-0.397070, 0.400302, -0.171587, 2.39219/
DATA D/ 7.01509E-2, 1.18520E-2, 1.69937E-3, -1.02000/
DATA C/ 0.642857, -4.11717, -6.17937, 3.08976E-3, , 8.22994E-2, 10.0932/
TCS=647.3
RHOC S=317.7
C SPECIFY RHOS,TSK
RHOS=
TSK=
C CALCULATE CZERO
CZ=A(1)
DO 10 I=2,4
IZ=I-1
CZ=CZ+A(I)*((TSK/TCS)**IZ)
10 CONTINUE
CZERO=((TSK/TCS)**0.5)*CZ
C CALCULATE CBAR
ARG1=BB(1)*((RHOS/RHOC S+BB(2))**2)
IF(ARG1.LE. -180.2) GO TO 21
IF(ARG1.LE.1.E-7.AND.ARG1.GE.0) GO TO 20
CBAR=B(1)+B(2)*((RHOS/RHOC S)+B(3)*EXP(ARG1))
GO TO 25
20 CBAR=B(1)+B(2)*((RHOS/RHOC S)+B(3))
73
GO TO 25
21 CBAR=B(1)+B(2)*(RHOS/RHOCS)
C CALCULATE DELC
25 DTSTAR=ABS(TSK/TCS-1.0)+C(4)
S=C(6)*(DTSTAR**(-0.6))
Q=2.0+C(5)*(DTSTAR**(-0.6))
R=Q+1.0
ARG2=C(1)*(1.-(RHOS/RHOCS)**2.8)
ARG3=Q/R*(1.-(RHOS/RHOCS)**R)
ARG4=C(2)*((TSK/TCS)**1.5)+C(3)*((RHOCS/RHOS)**5)
IF(ARG2.LE.-180.2) GO TO 31
IF(ARG2.LE.1.E-7.AND.ARG2.GE.0) GO TO 30
DELC1=((D(1)*(TCS/TSK**10))+D(2))*((RHOS/RHOCS)**1.8)*
,EXP(ARG2)
GO TO 35
30 DELC1=((D(1)*(TCS/TSK**10))+D(2))*((RHOS/RHOCS)**1.8)
GO TO 35
31 DELC1=0.
35 IF(ARG3.LE.-180.2) GO TO 41
IF(ARG3.LE.1.E-7.AND.ARG3.GE.0) GO TO 40
DELC2=D(3)*S*((RHOS/RHOCS)**Q)*EXP(ARG3)
GO TO 45
40 DELC2=D(3)*S*((RHOS/RHOCS)**Q)
GO TO 45
41 DELC2=0.
45 IF(ARG4.LE.-180.2) GO TO 51
IF(ARG4.LE.1.E-7.AND.ARG4.GE.0) GO TO 50
DELC3=D(4)*EXP(ARG4)
GO TO 55
50 DELC3=D(4)
GO TO 55
51 DELC3=0.
55 DELC=DELC1+DELC2+DELC3
C CALCULATE CSTM
CSTM=CZERO+CBAR+DELC
WRITE(6,*)TSK,RHOS,CSTM
STOP
END
The applicability of forced convection heat transfer data and empirical correlations based on experiments with dry or nearly dry air to situations involving air/water vapor mixtures (humid air) is considered. The particular application which motivated the present study is the contingency cooling of helicopter gas turbine engine components using evaporatively cooled (and therefore humid) air. Heat transfer coefficients were measured using both dry and humid air in the same forced convection cooling scheme and were compared using appropriate nondimensional parameters (Nusselt, Prandtl and Reynolds numbers). A forced convection scheme with a complex flow field, two-dimensional arrays of circular jets with crossflow, was utilized with humidity ratios (mass ratio of water vapor to air) up to 0.23. Results of a survey of the literature regarding the dynamic viscosity, thermal conductivity and specific heat of air, steam and air/steam mixtures are reported. Methods for the determination of gaseous mixture properties from the properties of their pure components are reviewed in detail. Convenient methods for the determination of these properties with good confidence are described and the need for more experimentally determined property data for humid air is discussed. It is concluded that dimensionless forms of forced convection heat transfer data and empirical correlations based on measurements with dry air may be applied to conditions involving humid air with the same confidence as for the dry air case itself, provided that the thermophysical properties of the humid air mixtures are known with the same confidence as their dry air counterparts.