FINAL REPORT
HINGE SPECIFICATION FOR A SQUARE-FACETED TETRAHEDRAL TRUSS

Louis R. Adams
Astro Research Corporation (Subcontractor)
Carpinteria, CA 93013

Contract NAS1-16923
20 January 1984
# TABLE OF CONTENTS

**SECTION 1:**  
INTRODUCTION .................................................. 1

**SECTION 2:**  
GEOMETRIC ANALYSIS ............................................. 2  
2.1 Even Right-Handed Cluster ............................ 3  
2.2 Odd Left-Handed Cluster ............................... 5  
2.3 Surface Diagonal Members SA1 and SA2 ............... 6  
2.4 Symmetries ................................................. 10  
2.5 Application .................................................. 11

**REFERENCES** ...................................................... 12
LIST OF TABLES AND FIGURES

Table 1. Relative Member Lengths, Even Right Cluster .......... 13
Table 2. Relative Member Lengths, Odd Left Cluster .......... 14
Table 3. Hinge Specification .................................. 15

Figure 1. Square-faceted tetrahedral truss ...................... 16
Figure 2. Transition: SA members fold inward as ribs shear ...... 17
Figure 3. Rib packaging: Adjacent ribs lie next to each other ...... 18
Figure 4. Complete packaging ..................................... 19
Figure 5. Even right cluster, surface space ..................... 20
Figure 6. Even right cluster, rib space .......................... 21
Figure 7. Even right cluster, king space .......................... 22
Figure 8. Hinges, even right cluster, deployed, king space ....... 23
Figure 9. Hinges, even right cluster, rib packaged .............. 24
Figure 10. Hinges, even right cluster, fully packaged ............ 25
Figure 11. Even right cluster, queen space ...................... 26
Figure 12. Odd left cluster, surface space ...................... 27
Figure 13. Odd left cluster, rib space ........................... 28
Figure 14. Odd left cluster, king space ........................... 29
Figure 15. Odd left cluster, queen space ....................... 30
Figure 16. Hinges, odd left cluster, deployed, king space ....... 31
Figure 17. Hinges, odd left cluster, rib packaged ............... 32
Figure 18. Hinges, odd left cluster, fully packaged ............. 33
Figure 19. Determination of SA hinges: deployed configuration .... 34
Figure 20. Determination of SA hinges: packaged configuration ...... 35
Figure 21. Sequentially deployable tetratruss .................... 36
Figure 22. Tetrahedral truss relative size ....................... 37
Figure 23. Even right cluster .................................... 38
Figure 24. Odd left cluster ...................................... 39
LIST OF SYMBOLS

\( a_1, b_1, c_1 = \hat{e}_1 \) components, member \( S_{A1} \)
\( a_2, b_2, c_2 = \hat{e}_2 \) components, member \( S_{A2} \)
\( \alpha = \) angle, rib plane to surface normal
\( \beta_1 = S_{A1} \) midhinge orientation angle, deployed
\( \beta_2 = S_{A2} \) midhinge orientation angle, deployed
\( d = \) member outside diameter
\( d_1, e_1, f_1 = \hat{e}_2 \) components, member \( S_{A1} \)
\( d_2, e_2, f_2 = \hat{e}_2 \) components, member \( S_{A2} \)
\( \hat{e}_1, \hat{e}_2 = \) reference unit vectors, deployed
\( \hat{e}_1', \hat{e}_2' = \) reference unit vectors, packaged
\( \gamma = \) half angle, core rib members
\( \hat{h} = \) hinge vector
\( H = \) truss depth
\( i, j, k = \) unit orthogonal vectors
\( I = \) interrib member
\( P = \) structural node identifying term
\( P' = \) structural node, mirror image of node \( P \)
\( Q = \) structural node identifying term
\( Q' = \) structural node, mirror image of node \( Q \)
\( R_1 = \) core rib member 1
\( R_2 = \) core rib member 2
\( S_{A1} = \) surface diagonal member 1
\( S_{A2} = \) surface diagonal member 2
LIST OF SYMBOLS (concluded).

SBl = surface chord member 1
SB2 = surface chord member 2
SR1 = surface rib member 1
SR2 = surface rib member 2
t = member wall thickness
X, Y, Z = surface coordinates
X_R', Y_R', Z_R' = rib coordinates
X_K', Y_K', Z_K' = king coordinates
X_Q', Y_Q', Z_Q' = queen coordinates

Subscripts
SAI, etc = (see symbol list)
ER = even right
EL = even left
OR = odd right
OL = odd left
SECTION 1
INTRODUCTION

The tetrahedral truss is a class of structure which is based on the tetrahedron unit; that is, members are connected in an array of nodal points which display a repeating pattern whereby nodes mark the vertices of a triangular pyramid. The pyramid may be equilateral in all respects or may be of arbitrary depth, as is the case for the structure in Reference 1. The tetrahedron may be further generalized insofar as member lengths may be changed so that the surface members are arrayed in nonequilateral triangles. In this study the surface members are arrayed in isosceles right triangles; a square pattern results with diagonal members stiffening each square. Further generalization has been demonstrated possible whereby member lengths are sized to create a doubly curved surface. Reference 2 documents this structure, which has a 50-m surface radius of curvature and packages in the manner described herein.

The purpose of this study was to determine an efficient packaging methodology for this structure and to specify single-degree-of-freedom hinges which allow this packaging. The method presented is for a nominally flat surface but also applies for curved-surface arrays. Attention is limited to hinge position and orientation requirements where members intersect at nodal points. Hinge requirements at the knee joints, as well as cluster modifications for curved surfaces, are specified in Reference 2.
SECTION 2
GEOMETRIC ANALYSIS

The truss configuration to be packaged is shown in deployed form in Figure 1. The upper and lower surfaces are flat and consist of members arranged in squares each having a diagonal stiffening member. Connecting the upper and lower surfaces are rib members and interrib members. The nodal points thus consist of nine member intersections, except at the edge of the truss. The rib members and interrib members are all of the same length. The orientations of the folding diagonals and of the interrib members alternate across each rib plane.

Packaging of the truss is accomplished by allowing the members labeled SA to fold inward (into the truss) as shown in Figure 2. This causes adjacent ribs to shear and package next to each other as shown in Figure 3. This motion makes all remaining surface members lie along the rib direction. The next step is to fold these members outward (out of the truss) so that all members ultimately are packaged against each other as shown in Figure 4. Note that the fully packaged members lie parallel to the member marked "king" in Figure 1.

The immediate effect of introducing king members is the elimination of a hinge at each node since each king member is unhinged, while the other eight members at a node are hinged to it. There are two node types marked as P and Q in Figure 1. Each node in the structure is of the type P or Q or is a mirror image of either.

In this tetrahedral truss configuration, there are two basic types of member clusters, noted here as even and odd, and each is represented in two mirror-image forms.

1. Even, right-handed. Surface diagonals point toward right (x+), interrib member down toward left
2. Even, left-handed. Surface diagonals point toward left (x-), interrib member down toward right (mirror image of Item 1)
3. Odd, right-handed. Surface diagonals point toward right (x+), interrib member down toward right
4. Odd, left-handed. Surface diagonals point toward left (x-), interrib member down toward left (mirror image of Item 3)
Hinge positions and directions at nodal points are specified according to the following criteria:

- The assembly is unstressed in the fully deployed, the rib packaged, or the fully packaged condition. Some stress is acceptable in the transitions between these states.

- Where possible, deployed member centerlines pass through the nodal point. Deviations from this to meet packaging requirements are acceptable to a limit of 1/10 member diameter.

- Hinge pins pass through member centerlines. If this is not possible, a lightly preloaded hard stop is provided to make member end bending load constant. Preloading is accomplished by setting the hinge geometry so that the hard stop makes contact slightly before full deployment.

### 2.1 EVEN RIGHT-HANDED CLUSTER

The cluster at node P is termed the even right-handed cluster. It is shown in Figure 5 with relative member lengths given in Table 1 for the case of a flat truss in a coordinate space consistent with surface geometry. The nine members emanating from the joint are labeled as follows: SA1 and SA2 are surface diagonal members, SB1 and SB2 are surface chord members, SR1 and SR2 are surface rib members, R1 and R2 are core rib members, and I is the interrib member. Note that the members are numbered according to proximity to the I member. The truss depth is arbitrarily chosen as 5/4 times the surface rib member length in order that the SA members have sufficient package room.

In order to determine packaging requirements, it is necessary to perform two rotations into king space. The first rotation is about the X axis, placing the Z axis in the rib plane, using the transformation

\[
\begin{align*}
X_R &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} X \\
Y_R &= \begin{bmatrix} 0 & \cos \alpha & \sin \alpha \end{bmatrix} Y \\
Z_R &= \begin{bmatrix} 0 & -\sin \alpha & \cos \alpha \end{bmatrix} Z
\end{align*}
\]

where
\[ \alpha = \tan^{-1}\left(\frac{Y}{Z}\right)_{R2} = \tan^{-1}\left(\frac{1/2}{5/4}\right) = 21.801 \text{ degrees} \]

Figure 6 shows this view in rib space. A second rotation, about \( Y_R \), aligns \( Z_R \) with the king member. This transformation is

\[
\begin{align*}
\begin{bmatrix}
X_K \\
Y_K \\
Z_K
\end{bmatrix} &= \begin{bmatrix}
\cos \gamma & 0 & \sin \gamma \\
0 & 1 & 0 \\
-\sin \gamma & 0 & \cos \gamma
\end{bmatrix} \begin{bmatrix}
X_R \\
Y_R \\
Z_R
\end{bmatrix} \\
\gamma &= \tan^{-1}\left(\frac{X_R}{-Z_R/R2}\right) = \tan^{-1}\left(\frac{0.5}{1.346}\right) = 20.375 \text{ degrees}
\end{align*}
\]

See Figure 7 for this view, in king space, of the even right-handed cluster.

Packaging of these members requires hinges with precise placement and direction. Figures 8, 9, and 10 show various views of the hinge in the deployed and packaged configurations of this cluster; several of the hinges can be specified by inspection. Member R2 is the king member and is not hinged. Members SB1 and SB2 necessarily hinge to SR2, and SR1, SR2, and R1 hinge on their respective lines of action with hinge pins parallel to \( Y_K \). Specification of hinges for SA1, SA2, and SB2 must be made with consideration of node Q, the odd left-handed cluster. Hinges for members I and SB1 are determined as follows.

Rotation of coordinates from rib space (Figure 6) through the angle \( -\gamma \) results in a view down member R1 which may be termed the queen member. Figure 11 shows this queen space view. It is required that the centerlines of members SB1 and I pass through the nodal point when deployed, and for efficient packaging, they may lie against members SR2 and R2, respectively, with centerlines separated by member diameter \( d \) along the \( Y_Q \) axis. These requirements are satisfied by directing the hinges parallel to member R1 and placing them on the intersection of the member edges. Such hinges are shown in Figure 8.
2.2 ODD LEFT-HANDED CLUSTER

The cluster at node Q shown in Figure 1 is termed the odd left-handed cluster. It is shown in Figure 12 and differs from node P not only in handedness but in placement of the interrib member. Figure 12 is to be compared to Figure 5, being in surface space and configured for a flat truss. The nine members are named using the same system as with node P. The coordinates are rotated through angle $\alpha$ to rib space, through angle $\gamma$ to king space, and $-\gamma$ to queen space, as before. These views are shown in Figures 13, 14, and 15. Member lengths are given in Table 2.

Direction and position of hinges are determined by packaging requirements. Figures 16, 17, and 18 show various views of the hinge in the deployed and packaged configurations of this cluster. A major difference between this odd cluster and the even cluster is in the motion of the I member. The even I member rotates to, attaches to, and packages adjacent to the king member. In contrast, the odd I member rotates to and attaches to the queen member but rotates a second time to similarly package adjacent to the king member.

As with the even cluster, members SBI and SB2 hinge to SR2, and SRI, SR2, and R2 have hinge pins parallel to YK at positions shown in Figure 16. Member R1 is the king member and is not hinged. Member SB2, odd, is hinged in the same manner as SBI, even. Hinges for I and SBI are determined as follows.

In the deployed configuration, the centerlines of the I and SBI members pass through the nodal point. The I member is hinged to and packages against member R2 in a manner such that it packages, after the rotation of R2, adjacent to R1. The SBI member packages near SR2. These requirements are satisfied, with the qualification that the deployed I centerline passes within 1/10 member diameter of the nodal point, by directing the hinges parallel to R1 and placing them on the I and SBI centerlines at a distance of a full member diameter off the rib plane. Such hinges are shown in Figure 16.

Hinges for members SA1 and SA2 are specified by parametric study with the corresponding hinges in the even cluster.
2.3 SURFACE DIAGONAL MEMBERS SAI AND SA2

Four separate hinge determinations are necessary to specify the end hinges for the diagonal surface members. In Figure 19, members SAI and SA2 hinge about the even right-handed cluster P. It is seen that the motion of SAI about that node affects not only the required hinge at P but also at Q; similarly, the motion of SA2 about node P affects the hinges required for that member at nodes P and Q.

In Reference 1, a general equation is given for hinge determination:

\[ \hat{h} = (\hat{e}_1 - \hat{e}_1') \times \hat{e}_2 \times \end{equation} \]

where \( \hat{h} \) is the direction vector of the desired hinge, \( \hat{e}_1 \) and \( \hat{e}_2 \) are unit vector directions after deployment in a body mounted on the hinge, and \( \hat{e}_1' \) and \( \hat{e}_2' \) are packaged orientations of \( \hat{e}_1 \) and \( \hat{e}_2 \).

In most cases, it is advantageous to choose the member centerline as \( \hat{e}_1 \) and another hinge in the member as \( \hat{e}_2 \). In king space, as in Figure 20, all packaged members are aligned with \( z_K \) so that

\[ \hat{e}_1' = -k \]

and all packaged midhinges are oriented as

\[ \hat{e}_2' = -i \]

The deployed centerline unit vectors \( \hat{e}_1' \) for SAI and SA2 have two distinct forms for each of the two cluster types. For SAI on the even right-handed cluster,

\[ \hat{e}_{1SIAR} = a_1 \hat{i} + b_1 \hat{j} + c_1 \hat{k} \]

and for SA2 on the even right-handed cluster,
\( \hat{e}_{1_{SA2_{ER}}} = a_2 \hat{i} + b_2 \hat{j} + c_2 \hat{k} \)

For the truss configuration shown in Figure 1, the above vectors are obtained from Table 1 by normalization of vectors \( SA_1 \) and \( SA_2 \) in king space.

\[
\begin{align*}
  a_1 &= -0.571438 \\
  b_1 &= -0.656532 \\
  c_1 &= 0.492366 \\
  a_2 &= -0.754298 \\
  b_2 &= 0.656532 \\
  c_2 &= 0
\end{align*}
\]

The deployed midhinge on \( SA_1 \) is normal to the member centerline, but can be at any angle of rotation. Its unit vector is given by

\( \hat{e}_{2_{SA1_{ER}}} = d_1 \hat{i} + e_1 \hat{j} + f_1 \hat{k} \)

The deployed midhinge on \( SA_2 \) is similarly defined.

\( \hat{e}_{2_{SA2_{ER}}} = d_2 \hat{i} + e_2 \hat{j} + f_2 \hat{k} \)

Thus, the surface diagonal hinges about the even right-handed cluster \( P \) are given by

\[
\mathbf{h}_{SA1_{ER}} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_{1+1} \\ d_{1+1} & e_1 & f_1 \end{bmatrix}
\]

\( (1) \)
On the other end of member SA1 is node Q, an odd left-handed cluster, and an identical cluster is at the end of SA2. The hinges required here are different from those at node P because the member centerlines are exchanged in notation and reversed in direction; that is,

\[ e_{1_{SA1OL}} = -e_{1_{SA2ER}} = -a_2 \hat{i} - b_2 \hat{j} - c_2 \hat{k} \]

and

\[ e_{1_{SA2OL}} = -e_{1_{SA1ER}} = -a_1 \hat{i} - b_1 \hat{j} - c_1 \hat{k} \]

This change in notation is because of configuration (compare Figures 5 and 12).

The deployed midhinges change in notation only:

\[ e_{2_{SA1OL}} = e_{2_{SA2ER}} = d_2 \hat{i} + e_2 \hat{j} + f_2 \hat{k} \]

and

\[ e_{2_{SA2OL}} = e_{2_{SA1ER}} = d_1 \hat{i} + e_1 \hat{j} + f_1 \hat{k} \]
Thus, the surface diagonal hinges about the odd left-handed cluster $Q$ are given by

$$\hat{h}_{SA1OL} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -a_2 & -b_2 & -c_2 + 1 \\ d_2 + 1 & e_2 & f_2 \end{vmatrix}$$ (2)

and

$$\hat{h}_{SA2OL} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -a_1 & -b_1 & -c_1 + 1 \\ d_1 + 1 & e_1 & f_1 \end{vmatrix}$$

Parametric analysis whereby the angles $\beta_1$ and $\beta_2$ of the deployed midhinges (see Figure 19) are allowed to vary, thus setting various values to $d_1', e_1', f_1'$ and $d_2', e_2', f_2'$ indicates the following solution which has certain manufacturing advantages and on subsequent modeling tests has shown to have minimal deployment strain. At $\beta_1 = 41.04$ degrees and $\beta_2 = 19.47$ degrees, the midhinge vectors are given in king space by

$$d_1 = -0.356686$$
$$e_1 = 0.739040$$
$$f_1 = 0.571485$$
$$d_2 = -0.431016$$
$$e_2 = -0.495200$$
$$f_2 = 0.754322$$
From Eqs. (1) and (2), directions are

\[ \hat{h}_{SA1} = 0.7543 \hat{i} - 0.6565 \hat{j} \]

\[ \hat{h}_{SA2} = 0.6565 \hat{i} + 0.7543 \hat{j} \]

\[ \hat{h}_{SA1} = -0.7986 \hat{i} + 0.6019 \hat{k} \]

\[ \hat{h}_{SA2} = -0.6640 \hat{i} + 0.7477 \hat{k} \]

Note that the hinges on the even hinge body are horizontal (no \( Z \) component) and on the odd hinge body are in the rib plane (no \( Y \) component).

The locations of the SA1 and SA2 hinges satisfy three criteria:

o The hinge pins pass through the member centerlines
o The packaged members are parallel to the king member, displaced in \( X \) and \( Y \) by one member diameter
o The extension of the deployed member centerline passes within 1/10 member diameter of the nodal point

2.4 SYMMETRIES

The hinge geometries thus obtained for nodes P and Q are valid for all nodes in the upper surface. The configuration about node P exists for each node in the same rib, and the Q configuration exists at all nodes on the two adjacent ribs. This pattern repeats such that alternating ribs are of type P or Q in the upper surface.

The P configuration has been termed the even right-handed cluster and Q the odd left-handed cluster. This notation has been established in order to specify the hinge geometry for the lower, as well as the upper, surface. The lower surface configuration is such that it is composed of even left-handed clusters and odd right-handed clusters which are shown as nodes P' and Q', respectively, in Figure 19.
2.5 APPLICATION

The hinges specified here are summarized in Table 3. Verification of these concepts requires assembly of a deployable structure made from parts manufactured in accordance to these specifications. Such a model was constructed in a configuration identical to that shown in Figure 1 and is shown in Figures 21 through 24. It is documented in Reference 2.

Figures 23 and 24 show two hinge assemblies as they appear in the finished model. The photographs of the cluster shown in Figure 23 correspond in all respects to the drawings in Figure 8 of the even right cluster. The cluster shown in Figure 24 corresponds to Figure 16 of the odd left cluster. These hinges were manufactured by numerically controlled milling methods using 6061-T6 aluminum alloy. The sketches shown in Figures 8, 9, and 10 serve as assembly drawings of the even right cluster, as well as indicating packaging rotations. Figures 16, 17, and 18 serve similarly for the odd left cluster.

The hinges were bonded to the graphite/epoxy tubes using rapid-setting epoxy with node-to-node distances controlled by accurate fixturing. The member length (side of square) is nominally 0.85 meter (33.5 in.), and the truss depth is 1.1 meter (43.3 in.). Graphite/epoxy tubing with an outside diameter of 14.0 mm (0.550 inch) and a wall thickness of 0.64 mm (0.025 inch) was used. Hinge material was aluminum alloy 6061-T6. In the model shown, the fixtured member lengths were actually varied slightly using analytical methods described in Reference 2 to produce a doubly curved surface having a 50-m (164-ft.) radius of curvature.
REFERENCES


<table>
<thead>
<tr>
<th>SURFACE</th>
<th>RIB</th>
<th>KING</th>
<th>QUEEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA1</td>
<td>-1.000000</td>
<td>-1.000000</td>
<td>-0.808135</td>
</tr>
<tr>
<td></td>
<td>-1.000000</td>
<td>-0.928477</td>
<td>-0.928477</td>
</tr>
<tr>
<td></td>
<td>0.000000</td>
<td>0.371391</td>
<td>0.696311</td>
</tr>
<tr>
<td>SA2</td>
<td>-1.000000</td>
<td>-1.000000</td>
<td>-1.066738</td>
</tr>
<tr>
<td></td>
<td>1.000000</td>
<td>0.928477</td>
<td>0.928477</td>
</tr>
<tr>
<td></td>
<td>0.000000</td>
<td>-0.371391</td>
<td>0.000000</td>
</tr>
<tr>
<td>SB1</td>
<td>0.000000</td>
<td>-0.928477</td>
<td>-0.928477</td>
</tr>
<tr>
<td></td>
<td>-1.000000</td>
<td>0.371391</td>
<td>0.348155</td>
</tr>
<tr>
<td>SB2</td>
<td>0.000000</td>
<td>0.000000</td>
<td>-0.129302</td>
</tr>
<tr>
<td></td>
<td>1.000000</td>
<td>0.928477</td>
<td>0.928477</td>
</tr>
<tr>
<td></td>
<td>0.000000</td>
<td>-0.371391</td>
<td>-0.348155</td>
</tr>
<tr>
<td>SR1</td>
<td>-1.000000</td>
<td>-1.000000</td>
<td>-0.937437</td>
</tr>
<tr>
<td></td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td></td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.348155</td>
</tr>
<tr>
<td>SR2</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.937437</td>
</tr>
<tr>
<td></td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td></td>
<td>0.000000</td>
<td>0.000000</td>
<td>-0.348155</td>
</tr>
<tr>
<td>R1</td>
<td>-0.500000</td>
<td>-0.500000</td>
<td>-0.937437</td>
</tr>
<tr>
<td></td>
<td>0.500000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td></td>
<td>-1.250000</td>
<td>-1.346291</td>
<td>-1.087986</td>
</tr>
<tr>
<td>R2</td>
<td>0.500000</td>
<td>0.500000</td>
<td>0.000000</td>
</tr>
<tr>
<td></td>
<td>0.500000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td></td>
<td>-1.250000</td>
<td>-1.346291</td>
<td>-1.436141</td>
</tr>
<tr>
<td>I</td>
<td>-0.500000</td>
<td>-0.500000</td>
<td>-0.808135</td>
</tr>
<tr>
<td></td>
<td>-0.500000</td>
<td>-0.928477</td>
<td>-0.928477</td>
</tr>
<tr>
<td></td>
<td>-1.250000</td>
<td>-0.974901</td>
<td>-0.739830</td>
</tr>
<tr>
<td>SURFACE</td>
<td>RIB</td>
<td>KING</td>
<td>QUEEN</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>SA1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.000000</td>
<td>1.000000</td>
<td>1.066738</td>
<td>0.808135</td>
</tr>
<tr>
<td>-1.000000</td>
<td>-0.928477</td>
<td>-0.928477</td>
<td>-0.928477</td>
</tr>
<tr>
<td>0.000000</td>
<td>0.371391</td>
<td>0.000000</td>
<td>0.696311</td>
</tr>
<tr>
<td>SA2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.000000</td>
<td>1.000000</td>
<td>0.808135</td>
<td>1.066738</td>
</tr>
<tr>
<td>1.000000</td>
<td>0.928477</td>
<td>0.928477</td>
<td>0.928477</td>
</tr>
<tr>
<td>0.000000</td>
<td>-0.371391</td>
<td>-0.696311</td>
<td>0.000000</td>
</tr>
<tr>
<td>SB1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000000</td>
<td>0.000000</td>
<td>0.129302</td>
<td>-0.129302</td>
</tr>
<tr>
<td>-1.000000</td>
<td>-0.928477</td>
<td>-0.928477</td>
<td>-0.928477</td>
</tr>
<tr>
<td>0.000000</td>
<td>0.371391</td>
<td>0.348155</td>
<td>0.348155</td>
</tr>
<tr>
<td>SB2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000000</td>
<td>0.000000</td>
<td>-0.129301</td>
<td>0.129302</td>
</tr>
<tr>
<td>1.000000</td>
<td>0.928477</td>
<td>0.928477</td>
<td>0.928477</td>
</tr>
<tr>
<td>0.000000</td>
<td>-0.371391</td>
<td>-0.348155</td>
<td>-0.348155</td>
</tr>
<tr>
<td>SR1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.000000</td>
<td>1.000000</td>
<td>0.937437</td>
<td>0.937437</td>
</tr>
<tr>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>0.000000</td>
<td>0.000000</td>
<td>-0.348155</td>
<td>0.348155</td>
</tr>
<tr>
<td>SR2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1.000000</td>
<td>-1.000000</td>
<td>-0.937437</td>
<td>-0.937437</td>
</tr>
<tr>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>0.000000</td>
<td>0.000000</td>
<td>0.348155</td>
<td>-0.348155</td>
</tr>
<tr>
<td>R1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.500000</td>
<td>0.500000</td>
<td>0.000000</td>
<td>0.937437</td>
</tr>
<tr>
<td>0.500000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>-1.250000</td>
<td>-1.346291</td>
<td>-1.436141</td>
<td>-1.087986</td>
</tr>
<tr>
<td>R2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.500000</td>
<td>-0.500000</td>
<td>-0.937437</td>
<td>0.000000</td>
</tr>
<tr>
<td>0.500000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>-1.250000</td>
<td>-0.346291</td>
<td>-1.087986</td>
<td>-1.436141</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.500000</td>
<td>0.500000</td>
<td>0.129302</td>
<td>0.808135</td>
</tr>
<tr>
<td>-0.500000</td>
<td>-0.928477</td>
<td>-0.928477</td>
<td>-0.928477</td>
</tr>
<tr>
<td>-1.250000</td>
<td>-0.974901</td>
<td>-1.087985</td>
<td>-0.739830</td>
</tr>
</tbody>
</table>
### TABLE 3. HINGE SPECIFICATION (King Space, d = member diameter)

<table>
<thead>
<tr>
<th>POSITION</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>VECTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>EVEN RIGHT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SA1</td>
<td>-d</td>
<td>-d</td>
<td>*</td>
<td>Parametric study</td>
</tr>
<tr>
<td>SA2</td>
<td>-d</td>
<td>d</td>
<td>0</td>
<td>Parametric study</td>
</tr>
<tr>
<td>SB1</td>
<td>*</td>
<td>-d/2</td>
<td>*</td>
<td>Parallel to queen</td>
</tr>
<tr>
<td>SB2</td>
<td>*</td>
<td>d</td>
<td>*</td>
<td>Parallel to king</td>
</tr>
<tr>
<td>SR1</td>
<td>-3/2d</td>
<td>0</td>
<td>*</td>
<td>Perpendicular to rib plane</td>
</tr>
<tr>
<td>SR2</td>
<td>-1/2d</td>
<td>0</td>
<td>*</td>
<td>Perpendicular to rib plane</td>
</tr>
<tr>
<td>R1</td>
<td>-d</td>
<td>0</td>
<td>*</td>
<td>Perpendicular to rib plane</td>
</tr>
<tr>
<td>R2**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>*</td>
<td>-d/2</td>
<td>*</td>
<td>Parallel to queen</td>
</tr>
</tbody>
</table>

| ODD LEFT  |     |     |     |                             |
| SA1       | d  | -d | *   | Parametric study            |
| SA2       | d  | d  | *   | Parametric study            |
| SB1       | *  | -d | *   | Parallel to king            |
| SB2       | *  | d/2| *   | Parallel to queen           |
| SR1       | 3/2d| 0  | *  | Perpendicular to rib plane  |
| SR2       | 1/2d| 0  | *  | Perpendicular to rib plane  |
| R1**      |    |    |     |                             |
| R2        | -d | 0  | *   | Perpendicular to rib plane  |
| I         | *  | -d | *   | Perpendicular to king       |

*Position set to direct deployed member centerline toward center of action (nodal point)

**Unhinged king member
Figure 1. Square-faceted tetrahedral truss.
Figure 2. Transition: SA members fold inward as ribs shear.
Figure 3. Rib packaging: Adjacent ribs lie next to each other.

054A
Figure 4. Complete packaging.
Figure 5. Even right cluster, surface space.
Figure 6. Even right cluster, rib space.
Figure 7. Even right cluster, king space.
Figure 8. Hinges, even right cluster, deployed, king space. 059A
Figure 9. Hinges, even right cluster, rib packaged.
Figure 10. Hinges, even right cluster, fully packaged.
Figure 11. Even right cluster, queen space.
Figure 12. Odd left cluster, surface space.
Figure 13. Odd left cluster, rib space.
Figure 14. Odd left-cluster, king space.
Figure 15. Odd left cluster, queen space.
Figure 16. Hinges, odd left cluster, deployed, king space.
Figure 17. Hinges, odd left cluster, rib packaged.
Figure 18. Hinges, odd left cluster, fully packaged.
Figure 19. Determination of SA hinges: deployed configuration. 070A
Figure 20. Determination of SA hinges: packaged configuration.
Figure 21. Sequentially deployable tetratruss.
Figure 22. Tetrahedral truss relative size. 

P033(a-b)
Figure 23. Photographs of even right cluster.
Figure 24. Photographs of odd left cluster.
Title and Subtitle
Final Report, Hinge Specification for a Square-Faceted Tetrahedral Truss

Abstract
A square-faceted tetrahedral truss is geometrically analyzed. Expressions are developed for single-degree-of-freedom hinges which allow packaging of the structure into a configuration in which all members are parallel and closely packed in a square pattern. Deployment is sequential, thus providing control over the structure during deployment.
DO NOT DETACH FROM REPORT
PLEASE LINE THROUGH YOUR NAME
BEFORE returning to the library

<table>
<thead>
<tr>
<th>Name</th>
<th>Mail Stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theresa Walker</td>
<td>219</td>
</tr>
</tbody>
</table>

NASA Langley (May 1979)