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IMAGES - A Digital Computer Program for Interactive Modal Analysis and Gain Estimation for Eigensystem Synthesis

Ruben L. Jones

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Ruben L. Jones

Langley Research Center
Hampton, Virginia
SUMMARY

An interactive digital computer program for modal analysis and gain estimation for eigensystem synthesis has been written. Both the mathematical and the operational considerations are described; however, the mathematical presentation is limited to those concepts essential to the operational capability of the program. The program is capable of both modal and spectral synthesis of multi-input control systems. It is also user-friendly with scratchpad capability and dynamic memory and can be used to design either state or output feedback systems.

INTRODUCTION

Srinathkumar (refs. 1, 2, and 3) developed several algorithms for the synthesis of control systems by using the concept of eigensystem (or pole) placement. He later programmed one of the algorithms (modal synthesis) for use on the Control Data Corporation (CDC) CYBER computer system and solved several example problems. But that program was written as a subroutine and was intended to operate in a "batch" mode. Moreover, the program was not formally documented for public use and did not include the algorithm for spectral synthesis.

This document describes a digital computer program which is based on the work of Srinathkumar. This program takes advantage of the interactive capability of an on-line terminal within the modern computer complex. It also includes those algorithms for both modal and spectral synthesis as well as numerous other options. The program is called IMAGES, which is an acronym for Interactive Modal Analysis and Gain Estimation System.

This paper describes the interactive program and the mathematical basis for the major algorithms. The reader is referred to references 1 and 2 for those details omitted here. The present paper is intended to introduce the interactive program IMAGES to the scientific community and to serve as a user's guide. Hence, program structure, interactive discourse, and other user-related information are emphasized. An example problem is included to clarify operational procedures and to demonstrate ease of application. It is also intended to clarify certain points not clearly evident within references 1 and 2.

SYMBOLS

A system matrix (numerical subscripts indicate particular partitions of this matrix)

B input matrix (numerical subscripts indicate particular partitions of this matrix)

C matrix defined in equations (23)

C_i, C_k matrices defined by equations (10b) and (15b), respectively
\( D \) matrix defined in equations (23)

\( e_r \) vector with the \( r \)th entry equal to unity and the remaining elements equal to zero

\( F \) matrix defined in equations (9)

\( f^{(k-1)}_r \) vector defined in equation (21)

\( G \) matrix defined in equations (9)

\( g^{(k)}_r \) vector defined in equation (21)

\( h^{(k-1)}_r \) vector defined in equation (21)

\( I \) order identity matrix

\( i,k \) indices

\( \mathcal{K}, \mathcal{K}, \tilde{\mathcal{K}} \) feedback matrices

\( M^{(k)} \) matrix defined in equation (16)

\( m \) number of inputs

\( m^{(k)}_a, m^{(k)}_b \) vectors defined in equation (16)

\( N \) matrix of \((k-1)\) eigenvectors

\( n \) number of states

\( P \) matrix defined below equations (25)

\( Q^{(k-1)} \) matrix defined by equation (18)

\( R^{(k-1)} \) matrix defined by equation (14)

\( r \) number of outputs in output feedback vector

\( r \in \Delta^{(k)} \) implies \( r \) is an element of the set \( \Delta^{(k)} \)

\( S \) matrix defined in equations (9)

\( t \) time

\( u \) input vector defined by equation (2)

\( u \in \mathbb{R}^m \) \( u \) is an \( m \times 1 \) real vector

\( u \) \( p \) external reference input

\( V \) modal matrix (matrix of eigenvectors)

\( v_i, v_k \) eigenvectors

2
w_i, w_k  vectors representing partitions of v_i and v_k, respectively
x  state vector defined by equation (1)
x ∊ R^n  x is an n × 1 real vector
y  output vector defined by equations (23)
z_i, z_k  vectors representing partitions of v_i and v_k, respectively
z_i ∊ R^m  z is an m × 1 real vector
Δ(k)  set of indices defined in equation (17)
δσ_k  perturbed quantities
Λ  eigenvalue matrix
λ_i, λ_k  eigenvalues
σ, σ_k  scalar quantity indicating limiting value

Superscripts:
(1), (2), ..., (k)  indicates an iterative step
T  matrix transpose
-1  matrix inverse

Subscripts:
o  initial value
ref  reference value

Uppercase letters denote matrices; matrix subscripts indicate partitioned quantities. A dot over a symbol denotes a derivative with respect to time; det is an abbreviation for determinant. A tilde (~), caret (^), or bar (¬) over a variable or matrix denotes a transformed quantity.

GENERAL DISCUSSION

Modal Synthesis

Srinathkumar (refs. 1, 2, and 3) both developed and programmed an algorithm for the synthesis of control systems by using the concept of eigensystem (eigenvalue/eigenvector) or pole placement. He derived constraining relations for calculating those feedback gains which will provide the desired modal characteristics of a linear system with multi-inputs. Although this analysis is described in detail in reference 1 and to a lesser degree in reference 2, multivariable synthesis by eigensystem assignment is briefly discussed herein.
The analysis begins with the controllable system

\[ \dot{x} = Ax + Bu \]  

where \( x \in \mathbb{R}^n \) and \( u \in \mathbb{R}^m \). Then, if \( B \) is assumed to be full rank, the state variable feedback law can take the form

\[ u = Kx + u_p \]  

where \( K \) is the feedback matrix, and \( u_p \) is an external reference input. Hence, the problem is to select a \( K \) so that the closed-loop system matrix \( A + BK \) satisfies

\[ [A + BK]v_i = \lambda_i v_i \quad (i = 1, 2, \ldots, n) \]  

where \( \lambda_i \) is the \( i \)-th eigenvalue, and \( v_i \) is the corresponding eigenvector.

Now, since \( m \) represents the number of inputs, equation (3) can be partitioned as

\[ \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{pmatrix} z_{i1} \\ w_i \end{pmatrix} = \lambda_i \begin{pmatrix} z_{i1} \\ w_i \end{pmatrix} \quad (i = 1, 2, \ldots, n) \]  

where \( A_{11}, B_1, \) and \( K_1 \) are \( m \times m \) matrices, and the remaining matrices are compatibly dimensioned. Moreover, \( z_i \in \mathbb{R}^m \), and

\[ v_i = \begin{bmatrix} z_{i1}^T \\ w_i^T \end{bmatrix} \]

where \( z_{i1} \) is the design mode shape (user-specified part of mode shape). Then, completing the multiplication of these partitioned quantities yields

\[ [A_{11} + B_1 K_1]z_i + [A_{12} + B_1 K_2]w_i = \lambda_i z_i \]  

\[ [A_{21} + B_2 K_1]z_i + [A_{22} + B_2 K_2]w_i = \lambda_i w_i \]

Hence, it follows from equation (5) that

\[ K_1 z_i + K_2 w_i = B_1^{-1} [\lambda_i z_i - A_{11} z_i - A_{12} w_i] \]  

\[ (7a) \]
\[ K_1 z_i + K_2 w_i = B_2^{-1} \left[ \lambda_i w_i - A_{22} w_i - A_{21} z_i \right] \quad (7b) \]

provided \( B_1 \) is either nonsingular or can be made nonsingular by reordering the state variables. That is, on combining equations (7a) and (7b) to eliminate \( K_1 \) and \( K_2 \),

\[
\begin{bmatrix} \lambda_i I_{n-m} - F \end{bmatrix} w_i = \begin{bmatrix} G + \lambda_i S \end{bmatrix} z_i \quad (i = 1, 2, \ldots, n) \quad (8)
\]

where \( I_{n-m} \) is an \((n - m)\)th-order identity matrix, and

\[
\begin{align*}
S &= B_2 B_1^{-1} \\
G &= A_{21} - SA_{11} \\
F &= A_{22} - SA_{12}
\end{align*} \quad (9)
\]

Equation (8) is the fundamental eigenvector-constraining relationship used by Srinathkumar in his algorithm for the synthesis of multi-input control systems. It constitutes a set of \( n - m \) linear equations in \( n \) unknowns, each of which represents an element of an eigenvector. Then, if \( \lambda_i \) is not an eigenvalue of \( F \), the \( n - m \) elements of \( w_i \) are uniquely related to the remaining \( m \) eigenvector elements, each of which may be assigned arbitrarily. These assigned elements correspond to the \( z_i \) vector, while the \( n - m \) elements which correspond to the \( w_i \) vector must be computed. That is,

\[
w_i = C_i z_i \quad (i = 1, 2, \ldots, n) \quad (10a)
\]

where

\[
C_i = [\lambda_i I_{n-m} - F]^{-1}[G + \lambda_i S] \quad (10b)
\]

is defined as that modal coupling matrix corresponding to \( \lambda_i \). Hence, after all \( n \) eigenvalues are assigned, up to \( nm \) eigenvector elements can also be arbitrarily assigned through the use of state variable feedback, provided the resulting modal matrix

\[
v = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}
\]

is nonsingular. The reader, however, must be aware that those choices of \( \lambda_i \) and \( z_i \) depend on the desired characteristics of the control system.

Equation (8) can be applied in a variety of ways. Aside from that option of synthesizing those eigenvectors achievable by state feedback, the analysis can be
extended to the problem of output feedback. Moreover, equation (8) can be used to
evaluate the achievability of certain preconceived objectives. For example, there
are occasions where those eigenvalues and eigenvectors chosen to achieve certain
design objectives will not satisfy those constraints imposed by equation (8) on the
choices of $\lambda_i$ and $z_i$. In these instances, it will be necessary to perturb
slightly either $\lambda_i$ or $z_i$ to achieve these design objectives. The option which
provides this capability is the spectral synthesis option. Equation (8) can also be
used to establish the parametric equivalence between the nonunique feedback matrix
$K$ and the arbitrary modal entries. Each of these options is included within IMAGES
and is described briefly in this paper. But, the reader is, once again, referred to
references 1 and 2 for additional information.

The feedback matrix $K$ is easily evaluated by returning to equation (3)
or (4). If a nonsingular modal matrix $V$ and an eigenvalue matrix $A$ should be
chosen to satisfy equation (8), then $\lambda_i$ will not be an eigenvalue of $F$, and the
closed-loop system matrix $A = A + BK$ is uniquely determined by

$$
\hat{A} = \begin{bmatrix}
\hat{A}_{11} & \hat{A}_{12} \\
\hat{A}_{21} & \hat{A}_{22}
\end{bmatrix} = VAV^{-1}
\tag{11}
$$

Moreover, that feedback matrix $K$ required to yield this closed-loop matrix $\hat{A}$ is
computed using the relations

$$
\begin{align*}
K_1 &= B_1^{-1}[\hat{A}_{11} - A_{11}] \\
K_2 &= B_1^{-1}[\hat{A}_{12} - A_{12}]
\end{align*}
\tag{12}
$$

where

$$
K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}
$$

Hence, given a modal matrix $V$ and an eigenvalue matrix $A$, it is possible to
evaluate the feedback matrix $K$, provided the matrices $A$ and $B$ are known.

Spectral Synthesis

The spectral synthesis option is more complicated. This option also assumes
that one may be given those matrices $A$ and $B$ along with the desired eigenvalues
$\lambda_i$ and the eigenvector elements $z_i$. In this instance, however, it is required to
synthesize a nonsingular modal matrix of eigenvectors, each of which not only satis-
fies equation (8) but also is mutually independent of the others. The latter condi-
tion is satisfied provided the $k$th eigenvector $v_k$ is chosen so that

$$
R^{(k-1)} v_k \neq v_k
\tag{13}
$$
where \( R^{(k-1)} \) is the projector of that subspace spanning the previous \((k - 1)\) eigenvectors. That is, if

\[
N = \begin{bmatrix} v_1 & v_2 & \cdots & v_{k-1} \end{bmatrix}
\]

represents the \((k - 1)\) eigenvectors, then

\[
R^{(k-1)} = N[N^TN]^{-1}N^T
\]

(14)

represents the projector of that subspace spanning these eigenvectors, and equation (13) ensures a nonsingular modal matrix.

It is also essential that each eigenvalue \( \lambda_k \) not be an eigenvalue of \( F \) if equation (8) is to be satisfied. If the desired eigenvalue should not combine with \( F \) so that

\[
det[\lambda I_{n-m} - F] \neq 0 \tag{15a}
\]

then

\[
C_k = [\lambda I_{n-m} - F]^{-1}[G + \lambda S] \tag{15b}
\]

is not defined. Spectral synthesis is an iterative procedure which avoids this difficulty by use of equation (8) to guarantee a solution for \( \psi_k \) which will combine with \( z_k \) so that \( \psi_k \) satisfies equation (13) and \( \lambda_k \) satisfies equation (15a). It requires that each eigenvector be evaluated sequentially and allows for perturbations within both \( \lambda_k \) and \( z_k \).

The algorithm depends on constructing an \( n \times n \) matrix \( M^{(k)} \) such that

\[
M^{(k)} = \begin{bmatrix} I_{n-r} & m^{(k)} & 0 \\ - & - & - \\ 0 & -1 & 0 \\ 0 & m^{(k)} & I_{r-1} \end{bmatrix}
\]

(16)

In equation (16), both \( m^{(k)}_a \) and \( m^{(k)}_b \) are vectors which must be evaluated, and the superscript \((k)\) refers to the \( k \)th index chosen from a subset \( \Delta^{(k)} \) of indices \( 1, 2, \ldots, n \). These indices represent the corresponding values of \( \lambda_i \) and \( z_i \), and the superscript represents the \( k \)th permutation of these indices. Hence, a particular index will no longer be resident within the subset \( \Delta^{(k)} \) once it is used in the construction of \( M^{(1)}, M^{(2)}, \ldots, M^{(k)} \).
Now, $M^{(k)}$ is defined so that equation (13) may be satisfied without evaluating $R^{(k-1)}$ explicitly. The construction is such that $v^{(k)}_k$ is transformed into canonical form by

$$M^{(k)}_k v^{(k)}_k = \sigma^*_k e_r$$  \hspace{1cm} (r \in \Delta^{(k)}) \hspace{1cm} (17)$$

where $\sigma^*_k \neq 0$, and $e_r$ is an $n$ vector with the $r$th entry equal to unity and all other entries equal to zero. Then, both the vectors $m^{(k)}_a$ and $m^{(k)}_b$ must be constructed so that

$$\sigma^*_k = v^{(k)}_r$$

Hence, $r$ is both the $r$th element of $v^{(k)}_k$ and the $r$th row of $M^{(k)}$. With these definitions, an $n \times m \ Q$ matrix is defined as

$$Q^{(k-1)} = M^{(k-1)}_1 M^{(k-2)}_2 \cdots M^{(1)}_n \hspace{1cm} (k > 1) \hspace{1cm} (18)$$

where

$$Q^{(0)} = I_n \hspace{1cm} (19)$$

Hence,

$$v^{(k)}_k = Q^{(k-1)} v^{(k)}_k \hspace{1cm} (20)$$

The construction of $M^{(k)}$ and hence $Q^{(k)}$ is such that a nonsingular modal matrix is assured. The definition of $Q^{(k-1)}$ also permits considerable freedom in the choice of $z_k$ and $\lambda_k$ from the list of desired values denoted by $z_i$ and $\lambda_i$. These values may be permuted at will to find the best combination of independent eigenvectors to achieve the desired design objectives. The procedure also permits perturbations within both $z_k$ and $\lambda_k$ to ensure that both equation (13) and equation (15a) are satisfied. These and other points should become clearer as the remaining steps are outlined.

The $v^{(k)}_k$th eigenvector is synthesized by using the $r$th row of equation (18). It is first required to compute

$$[g^{(k)}_r]^T = f^{(k-1)}_r + h^{(k-1)}_r C_k$$  \hspace{1cm} (21)$$

where

$$[f^{(k-1)}_r \quad h^{(k-1)}_r]$$
is the rth row of the transformation \( q^{(k-1)} \), and \( h_r^{(k-1)} \) is an \((n - m)\) row vector, while \( C_k \) is evaluated using equation (15b). Then,

\[
\sigma_k = [g_r^{(k)}]^T z_k^{(k)}
\]  

(22)

which must not be equal to zero. (See eq. (17).) In the event \( \sigma_k = 0 \), the user must either select another \( r \), perturb \( z_k \) to \( z_k + \delta z_k \), or return to perturb \( \lambda_k \) once again. (See eqs. (15).) Normally, the order of application is as listed (i.e., select another \( r \), etc.), with all remaining values of \( r \) being tried before perturbing either \( z_k \) or \( \lambda_k \). Hence, on finding those values of \( z_k \) and \( \lambda_k \) which will satisfy those conditions specified, \( w_k \) can be evaluated by using equation (8). The results will be a \( v_k \) which is independent of those eigenvectors computed earlier. Moreover, subsequent eigenvectors computed in the same manner will also be independent. But, \( M^{(k)} \) must be constructed from equation (17) before these steps are repeated for subsequent eigenvectors and eigenvalues.

Output Feedback Control System

The synthesis can be extended to an output feedback system. For example, consider systems of the form

\[
\dot{x} = Ax + Bu
\]
\[
y = Cx + Du
\]

(23)

where \( y \) is an \( r \times 1 \) output vector with \( r < n \). Then, an approximation to the state feedback law (eq. (2)) can be found using the control law

\[
u = \tilde{K}y + u_p
\]

(24)

That is, on combining equations (23) and (24), the closed-loop system takes the form

\[
\dot{x} = [A + \tilde{B}K]x + \hat{B}u_p
\]
\[
y = [C + D\tilde{K}]x + \hat{D}u_p
\]

(25)

where \( \tilde{K} = \tilde{P}C \), \( \hat{B} = BP \), \( \hat{D} = DP \), and \( P = [I_m - \tilde{K}D]^{-1} \). Hence, \( P \) is the equivalent feedforward matrix, and \( K \) is the equivalent state feedback matrix. Then, for every state feedback matrix \( K \) derived by using the control law \( u = Kx + u_p \), there exists an output feedback approximation

\[
\tilde{K} = \tilde{K}[C + D\tilde{K}]^\dagger
\]

(26)

where the superscript dagger (\( \dagger \)) indicates the generalized inverse.
These algorithms are more general than is apparent from the preceding descriptions. Throughout the preceding discussion, the emphasis has been on real variables. It is, however, possible to extend the theory to complex variables. In fact, Srinathkumar has shown in references 1 and 2 that complex pairs can be treated using the same equations with very little modification. Those modifications have been included within the program and will not be discussed herein. Both real and complex eigenvalues and eigenvectors may be entered into the program without regard for order. The user, however, must be aware that the expected number of eigenvalues is diminished by one for each complex number entered. This information will be especially useful when running IMAGES.

PROGRAM DESCRIPTION

The digital computer program IMAGES takes advantage of the interactive capability of the modern computer complex in the modal analysis and gain estimation for control system synthesis. The emphasis is on simplicity with a minimum of demand on the user. "Control" variables and other decision-making logic have been reduced to a series of numbered lists from which the user must select one. Moreover, dynamic memory is utilized to eliminate any need to redimension any DIMENSION statement. These and other ideas have been combined with those algorithms of reference 1 as described herein to develop a truly interactive program with scratchpad capability.

The program was developed using that MODAL synthesis package written by Srinathkumar. That program, however, suffered greatly in two important points. First, it was based only on equations (1) through (12) and equations (23) through (25). This limitation eliminated spectral synthesis as a viable option within the program. As an interactive tool, this option may not only be useful but also essential. Second, MODAL was a card-oriented program (subroutine) possessing numerous options which required the user to resubmit the job for processing with each change in either options or model. The interactive program eliminates these time-consuming requirements.

Srinathkumar's program (or, more accurately, subroutine) was extremely flexible and well written. It was capable of estimating either eigenvectors, eigenvalues, or gains for either state feedback or output feedback control systems given those matrices indicated by either control system. (See eqs. (1) and (2) and eqs. (23) and (24).) Although the modal matrix would normally be synthesized by using equation (10), another option computes this matrix, along with its associated matrix of eigenvalues \( \Lambda \), for the given system matrix \( A \). The gain matrix \( K \) was estimated by using equation (11). It was also possible to compute \( K \) on entering both the modal matrix and its associated \( \Lambda \) matrix as input to the program; the gains would be estimated using both equations (10) and (11). Other options were also available to the user. But, the semimodular nature of the batch-oriented subroutine MODAL imposed a requirement by the control logic for several different codes, each of which could assume several different values.

Those code words used in MODAL along with their possible values are given in table I. As can be seen, one can be easily confused by the many possible combinations required to solve a particular problem. These codes, however, were necessary if the program was to be both efficient and useful in a batch computational mode. They are also essential to a lesser degree in the interactive program. They activate that control logic essential to provide the flexibility needed to accomplish many different objectives by using the same source code.
IMAGES uses a modified version of Srinathkumar's program. The interactive program does not require the user to provide that FORTRAN logic needed for a calling program. Neither is it required to maintain a close watch over returning output as well as an ever-changing data deck. Punched card input has been replaced by either keyboard or disk file entries. Control words and other variables associated with control logic are automatically assigned within IMAGES and are no longer the responsibility of the user. These objectives are accomplished through a series of question and answer diagnostics, which are described in greater detail later. These questions and answers determine which codes will be set along with the appropriate entry point within MODAL. This has been achieved by restructuring the existing subroutine by using entry points. Hence, the original subroutine MODAL has essentially been decomposed into four modules: (1) an input module, (2) a synthesis module, (3) a gain module, and (4) a response module. These modules are described graphically by the flow charts shown in figures 1 through 4. They represent the major entry points within the present version of MODAL.

These charts are somewhat self-explanatory. The various entries and operations have been grouped into those modules which best describe their functions. Addresses of selected READ's and other transfer points have also been included to aid the user who may have a listing of the program. (Both the program and the listing are available through COSMIC - Computer Software Management and Information Center, Suite 112, Barrow Hall, University of Georgia, Athens, GA 30602.) For example, the numbers 1030, 1040, 1037, and 1045 in the gain module (see fig. 3) indicate addresses of transfer points within MODAL. Numbers less than 10 on the figures are associated with those codes selected automatically by the various interactive responses. These responses are shown in figure 5 and are discussed later. Hence, that path taken within each module depends on the particular option chosen by the user.

One should not be confused by the subroutine names but should be continuously aware of the differences between modal and spectral synthesis. The spectral synthesis subroutine SPECTRL - which is called from the synthesis module within MODAL - was not included within the original version of the pole placement package. It is based on that discussion centered around equations (13) through (22) and depends on subroutine SYNTHS. The original subroutine was named to reflect its primary function of modal synthesis. But, with this addition of the spectral synthesis algorithm, both the spectral synthesis and the modal synthesis are options within the synthesis module. Hence, MODAL is now an executive subroutine, and SYNTHS is that subroutine on which both algorithms depend. Both algorithms require the user to provide m elements of each eigenvector along with its corresponding eigenvalue. Modal synthesis will not adjust these design values. But, spectral synthesis permits the user to perturb these values in such a way as to satisfy within a certain tolerance (denoted by $\epsilon$) the condition of mutual independence.

The remaining subroutines will neither be described graphically nor discussed herein, since the program listing is not included as a part of this document. These subroutines are, however, listed in table II along with a brief description of their functions. Hence, the remaining discussion is restricted to operational considerations.

IMAGES is a user-oriented, interactive, digital computer program. Many different questions, messages, and other diagnostics are included within this program. The various matrices and other critical variables are printed on the terminal screen as they are generated, with sufficient pauses to permit the user to examine his or her results. In many instances, the user is permitted to modify these values. This is
particularly true with those matrices and vectors serving as input. The user may also permute the A, B, C, and D matrices. These, as well as the several other options, provide considerable flexibility in the design and analysis of control systems.

The interactive discourse is given in the flow chart shown in figure 5. Most of these diagnostics are included within the executive program IMAGES and the executive subroutine MODAL. Other diagnostics are also present, but these are not considered to be essential to the discussion. That path taken by the program in response to a given command is indicated by its number. Clearly, the user has many different choices.

The program begins with the "master menu." It is indicated by the bold border in figure 5. All other blocks, with only one exception, are subordinate to this block and must ultimately return to it. Hence, each of those options within the master menu is a major program branch. Moreover, options within the submenus on one or more of those program modules are described elsewhere herein.

The user is essentially free to choose among the several options as needed. Each is, hopefully, self-explanatory, and the order of selection is usually unimportant. But, the user should be aware that the printed order of options is suggestive of that sequence of decisions a researcher would ordinarily make during the synthesis of an eigensystem for gain estimation. This is particularly true for new problems. The user must always "begin problem," "enter input," "enter computational mode," and "stop," or terminate, the program. The remaining options are utilities which enhance the overall flexibility and operational capability of the program. The "save problem" option, in particular, allows the user to automatically save all those program variables needed to restart the program in the solution for a particular problem at some future date without penalty. This is provided as a safety feature as well as for computational flexibility. The save option automatically files all the current program variables and other data onto STRTUP - a random-access mass-storage disk file - under a name chosen by the user for future reference. It is also provided within each of the two remaining branches: (1) enter input and (2) enter computational mode.

On beginning a problem, the user must first specify whether the problem is an old or a new problem and supply a file name, which can be as many as seven characters. (Note: A first-time user must first establish STRTUP as a viable storage file within the computer complex.) The program will then determine if the name has been used previously. If so, the user must decide if the name is to be retained. In the event his problem is to be a new example, the user may either purge the existing file or rename the new file. If the problem is an old one, the program will automatically find and load all those values associated with all steps completed up to the most recent save command using that file name.

System matrices can be reviewed, modified, or both reviewed and modified on entering the enter input module. The user may also choose to go directly to the computational mode. If the problem is a new one, however, the user must enter those matrices required for his problem. Diagnostics specify the expected format for these new data. Provision is also made for correcting the data. All these diagnostics are self-explanatory and follow in a natural sequence to minimize the demand on the user.

The user need not be concerned with array dimension statements. The program uses dynamic memory. Program dimensions and memory requirements are automatically allocated during execution. Moreover, they are automatically adjusted during a
session to accommodate any other problem the user may choose to analyze. This capability is possible through the use of the Control Data Corporation (CDC) Common Memory Manager, which is a part of their operating system.

The Common Memory Manager is a set of several FORTRAN-callable subroutines, each of which begins with the prefix CMM. These subroutines return to the calling program the first word address (FWA) of any array name. Another returns the offset needed to reposition this array at the end of the program, while a third routine sets aside that memory needed to execute the program. A fourth routine releases this space, and a fifth routine extends it. These routines are extremely useful within a variable dimension program like IMAGES and are used extensively therein.

All array variables are listed in table III along with their memory assignments. This table provides the user with a quick index to each program variable and its position within the memory table. The variable name $A$ is used extensively within both IMAGES and its executive subroutine MODAL. Other variables are listed in table III along with their algebraic equivalents. These variables can be correlated with those algebraic expressions discussed earlier.

The "computational mode" is much more complex than either of the other modes. It is the backbone of IMAGES. All computations are initiated from this module. Some options require the user to choose between other suboptions, as can be seen in figure 5. In some instances, the user is required to provide additional input. For example, the synthesis option will usually require the user to provide both the modal and the $A$ matrices. In fact, the spectral synthesis returns to these inputs repeatedly until the specified tolerance $\sigma$ is satisfied. The "response" option is another example where additional input is required. In this instance, the user must specify either a step input or initial condition input as well as other parameters such as time increments. Other options will simply provide the specified computational result. The user should be cautioned, however, to save needed results in all instances before entering the response option. In fact, the user should save results after each operation if it is desired to retain the data from completed steps for future use.

The response option is provided as an analysis aid only. This option will provide the user with a graphical display of system response to various types of input. These results may not be saved by the user. They are intended only as design aids. Hence, a particular design can be modified repeatedly until the desired response is obtained.

The program is designed to provide the user with considerable numerical results. Hence, automatic scrolling could present difficulties for some terminals. Line limitations vary considerably from terminal to terminal. This is controlled internally, to some extent, within the program by using a PAUSE command. This command stops execution until the user enters a carriage return. The user can employ this command to control the rate of execution. It also allows the user to examine the returning output. If the numerical results are not progressing as expected, the user can return to an earlier step to modify his input.

An example session is given in the appendix. It is not an example of a complete design problem. This example represents the end result of several sessions where the desired eigenvalues and eigenvectors have been refined to achieve the desired results. The user is encouraged to try this example. The program is extremely fast and requires less than 70K octal memory words to run. Response time is essentially instantaneous for most options. The various diagnostics will quickly become second
nature with only a few practice sessions such as this. Moreover, it will provide a valuable check on program compatibility. The reader is referred to the references for that logic used in selecting desired eigenvalues and eigenvectors in a practical design problem.

CONCLUDING REMARKS

An interactive digital computer program for modal analysis and gain estimation through eigensystem synthesis has been described. The program is modular in design and is extremely flexible. It is capable of both modal and spectral synthesis of multi-input control systems. The numerous options along with the incorporation of both memory management and random access have been combined to produce a truly user friendly, scratchpad program for the design of multi-input control systems.

The program is very fast and requires less than 70K octal memory words to run. Some facilities impose limitations on programs to maximize the number of jobs which can be processed. Hence, these characteristics can be an added advantage to potential users.

Langley Research Center
National Aeronautics and Space Administration
Hampton, VA 23665
November 30, 1983
APPENDIX

EXAMPLE OF AN INTERACTIVE SESSION

13.43.21.RETURN(IMAGES)
13.43.22.ATTACH(FTNMLIB\UN=LIBRARY)
13.43.26.REWIND(LMAP)
13.43.27.LDSET(LIB=FTNMLIB,PRESETA=INDEF,MAP=SBEX\LMAP)

MASTER MENU
CHOOSE OPTION:
  1--BEGIN PROBLEM
  2--ENTER INPUT
  3--ENTER COMPUTATIONAL MODE
  4--SAVE PROBLEM
  5--RENAME FILE
  6--DUPLICATE FILE
  0--STOP

? 1
THERE ARE 2 PROBLEMS ON FILE. THESE ARE AS FOLLOWS:

SAMPLE
TEST

IS THIS A NEW EXAMPLE?
CHOOSE ONE:
  1--NEW
  2--OLD
  3--NEITHER

? 1

ENTER SEVEN (7) OR LESS CHARACTER FILE NAME:
? sample

PROBLEM NAME IS ON FILE!!
DO YOU WISH TO PURGE?
  CHOOSE ONE:? 
    1--YES
    2--NO

? 2

IS THIS A NEW EXAMPLE?
CHOOSE ONE:
  1--NEW
  2--OLD
  3--NEITHER

? 1
APPENDIX

ENTER SEVEN (7) OR LESS CHARACTER FILE NAME:
? example

MASTER MENU
CHOOSE OPTION:
1--BEGIN PROBLEM
2--ENTER INPUT
3--ENTER COMPUTATIONAL MODE
4--SAVE PROBLEM
5--RENAME FILE
6--DUPLICATE FILE
0--STOP

? 2

PROBLEM IS A NEW ONE. ARRAYS MUST BE DIMENSIONED!
DO YOU WANT A GRAPHIC DISPLAY OF MATRICES & THEIR DIMENSIONS
CHOOSE ONE:
1--YES
2--NO

? 1

***
*****
***

*   *   *   *   *   *****   *   *
*   *   *   *   *   *   *   *   *   *   *   *
*   *   *   *   *****   *   *   *   *   *   *   *
*   *   *****   *****   *   * *****   *****   *   *   *   *   *
*   *   ****   *   *   *   *   *   *   *   *   *   *   *   *   *   *
*   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
*   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *

N X 1  N X N  N X 1  N X M  M X 1
VECTOR  SYSTEM  STATE  INPUT  INPUT
MATRIX  VECTOR  MATRIX  VECTOR

PLEASE ENTER THE NUMBER OF STATE ELEMENTS, N.
? 4
APPENDIX

THE CONTROL LAW MAY BE EITHER AN INPUT OR AN OUTPUT CONTROL:

\[ \begin{align*}
\text{M} \times \text{1} & \quad \text{M} \times \text{N} & \quad \text{N} \times \text{1} & \quad \text{M} \times \text{1} \\
\text{INPUT} & \quad \text{FEEDBACK} & \quad \text{STATE} & \quad \text{INPUT} \\
\text{VECTOR} & \quad \text{MATRIX} & \quad \text{VECTOR} & \quad \text{VECTOR}
\end{align*} \]

OR

\[ \begin{align*}
\text{M} \times \text{1} & \quad \text{M} \times (\text{R} = \text{N}) & \quad (\text{R} = \text{N}) \times \text{1} & \quad \text{M} \times \text{1} \\
\text{INPUT} & \quad \text{FEEDBACK} & \quad \text{OUTPUT} & \quad \text{INPUT} \\
\text{VECTOR} & \quad \text{MATRIX} & \quad \text{VECTOR} & \quad \text{VECTOR}
\end{align*} \]

BUT, THE NUMBER OF INPUTS WILL REMAIN UNCHANGED.
APPENDIX

PLEASE ENTER THE NUMBER OF INPUTS, M.

? 2

PLEASE ENTER THE NUMBER OF OUTPUTS, R.

? 4

DESCRIBE YOUR INPUT (CHOOSE ONE)
1--THE SYSTEM MATRICES A & B
2--OUTPUT MATRICES (I.E., Y=CX+DU)
3--BOTH "1" &"2"
4--EIGENVALUE/EIGENVECTOR MATRICES
5--MODAL COUPLING MATRIX
6--ENTER COMPUTATIONAL MODE
7--SAVE INPUT
8--PRINT A & B MATRICES
9--PRINT C & D MATRICES
0--EXIT

? 1

...EIGENVALUE/EIGENVECTOR ASSIGNMENT ALGORITHM ...

...SYSTEM PARAMETERS...
NO OF STATES =  4
NO OF INPUTS =  2

READ SYSTEM MATRICES A AND B
ENTER A MATRIX
ENTER BY ROWS:

? -.56758,.859871,-2.192935,0.0
? -.006922,-.147531,.230069,0.0
? -195034,-.970592,-.079899,.123380
? 1.0,.140541,0.0,0.0

ENTER B MATRIX
ENTER BY ROWS:
**APPENDIX**

<p>| | | | | |</p>
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PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION.

PAUSE

<p>| | |</p>
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<td>-.115903</td>
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<td>0.000000</td>
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PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION.

PAUSE
DO YOU LIKE A & B?
CHOOSE ONE:
1--YES
2--NO

? 1

DO YOU WISH TO PERMUT A & B MATRICES?
CHOOSE ONE:
0--NO
1--YES

? 0

DESCRIBE YOUR INPUT (CHOOSE ONE)
1--THE SYSTEM MATRICES A & B
2--OUTPUT MATRICES (I.E., Y=CX+DU)
3--BOTH "1" &"2"
4--EIGENVALUE/EIGENVECTOR MATRICES
5--MODAL COUPLING MATRIX
6--ENTER COMPUTATIONAL MODE
7--SAVE INPUT
8--PRINT A & B MATRICES
9--PRINT C & D MATRICES
0--EXIT

? 7

DESCRIBE YOUR INPUT (CHOOSE ONE)
1--THE SYSTEM MATRICES A & B
2--OUTPUT MATRICES (I.E., Y=CX+DU)
3--BOTH "1" &"2"
4--EIGENVALUE/EIGENVECTOR MATRICES
5--MODAL COUPLING MATRIX
6--ENTER COMPUTATIONAL MODE
7--SAVE INPUT
8--PRINT A & B MATRICES
9--PRINT C & D MATRICES
0--EXIT

? 0

MASTER MENU
CHOOSE OPTION:
1--BEGIN PROBLEM
2--ENTER INPUT
3--ENTER COMPUTATIONAL MODE
4--SAVE PROBLEM
5--RENAME FILE
6--DUPLICATE FILE
0--STOP
APPENDIX

COMPUTATIONAL MODE:
CHOOSE ONE--
1--CONSTRAINING RELATION
2--SYNTHESIS
3--GAINS
4--TIME RESPONSE
5--SAVE
6--EXIT

? 1
CURRENT EPS IS 0. WHAT IS ITS NEW VALUE?
? 0.000001

: B1 INVERSE :

1.242133  2.256626
-.144251  -8.889970

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION.
PAUSE
?

: DETERMINANT OF B1 = -.93310E-01

: S : MATRIX

-.009670  -.112908
0.000000  0.000000

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION.
PAUSE
?
APPENDIX

: F : MATRIX

\begin{align*}
0.075129 & \quad -0.123380 \\
0.000000 & \quad 0.000000
\end{align*}

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION.
PAUSE
?

: G : MATRIX

\begin{align*}
0.188764 & \quad -0.978934 \\
1.000000 & \quad 0.140541
\end{align*}

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION.
PAUSE
?

COMPUTATIONAL MODE:
CHOOSE ONE--
1--CONSTRAINING RELATION
2--SYNTHESIS
3--GAINS
4--TIME RESPONSE
5--SAVE
6--EXIT
?

5
COMPUTATIONAL MODE:
CHOOSE ONE--
  1--CONSTRAINING RELATION
  2--SYNTHESIS
  3--GAINS
  4--TIME RESPONSE
  5--SAVE
  6--EXIT

? 2

CHOOSE ONE:
  1--SPECTRAL SYNTHESIS
  2--MODAL SYNTHESIS
  3--EXIT

? 2

CHOOSE ONE:
  0--MODAL MATRIX
  1--MODAL COUPLING MATRIX

? 0

SYSTEM MODES (AND THE OPTIONAL MODAL MATRIX) ARE REQUIRED.
ENTER DESIRED MODES

A MAXIMUM OF 4 REAL MODES ARE EXPECTED
COMPLEX MODES WILL AUTOMATICALLY DIMINISH THIS NUMBER

ENTER EACH IN TABLE FORM WITH COMMA (,) SEPARATORS:

  TYPE , REAL PART , IMAG PART

SPACING IS NOT IMPORTANT AND TYPE CODES ARE AS FOLLOWS:
  0--IMPLIES REAL MODE; 1--IMPLIES COMPLEX MODE; 2--ENDS INPUT

? 0,-.72,0.0
? 1,-.15,.4
? 0,-.031,0.0
APPENDIX

DESIRED MODES

<table>
<thead>
<tr>
<th>TYPE</th>
<th>REAL PART</th>
<th>IMAG PART</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-.7200E+00</td>
<td>0.</td>
</tr>
<tr>
<td>1</td>
<td>-.1500E+00</td>
<td>.4000E+00</td>
</tr>
<tr>
<td>0</td>
<td>-.3100E-01</td>
<td>0.</td>
</tr>
</tbody>
</table>

CHOOSE ONE:
1--CORRECT MODES
2--CONTINUE

? 2

ENTER THE M X N DESIGN MATRIX " Z " BY ROWS

DO YOU WANT A GRAPHIC DISPLAY OF MATRICES & THEIR DIMENSIONS
CHOOSE ONE:
1--YES
2--NO

? 1

N X 1
EIGEN-
VECTOR
ENTER ROW 1
? 1.0,0.0,0.0,-.045841
ENTER ROW 2
? 0.0,.5,.5,.10927

24
APPENDIX

DESIGN PARAMETER MATRIX "Z"

\[
\begin{bmatrix}
1.000000 & 0.000000 & 0.000000 & -.045841 \\
0.000000 & .500000 & .500000 & .109270
\end{bmatrix}
\]

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION.

PAUSE

? CHOOSE ONE:
1--CORRECT "Z"
2--CONTINUE

? 2

COMPUTATIONAL MODE:
CHOOSE ONE--
1--CONSTRAINING RELATION
2--SYNTHESIS
3--GAINS
4--TIME RESPONSE
5--SAVE
6--EXIT

? 5

COMPUTATIONAL MODE:
CHOOSE ONE--
1--CONSTRAINING RELATION
2--SYNTHESIS
3--GAINS
4--TIME RESPONSE
5--SAVE
6--EXIT

? 3
APPENDIX

FEEDBACK GAINS

CHOOSE ONE:
1--ENTER STATE FEED BACK GAINS
2--CALCULATE FEEDBACK GAINS
3--EXIT

? 2

CHOOSE SYSTEM:
1--STATE FEEDBACK
2--OUTPUT FEEDBACK

? 1

MATRIX OF EIGENVALUES

-0.720000 0.000000 0.000000 0.000000
0.000000 -0.150000 0.400000 0.000000
0.000000 -0.400000 -0.150000 0.000000
-0.000000 0.000000 0.000000 -0.031000

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION.
PAUSE

: V : MATRIX (MODAL MATRIX)

0.584162 0.000000 0.000000 -0.045841
0.000000 0.269093 0.269093 0.109271
-0.022072 -0.579930 0.709355 .137647
-0.811337 .051806 -0.113974 .983368
APPENDIX

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION.
PAUSE

DETERMINANT OF : V : = \(0.19181 \times 10^0\)

: V-\(\text{INVERSE}\) :

\[
\begin{align*}
1.827418 & \quad 0.007014 & \quad 0.010662 & \quad 0.082917 \\
-0.203078 & \quad 2.034071 & \quad -0.791655 & \quad -0.124680 \\
-0.394926 & \quad 1.645819 & \quad 0.736485 & \quad -0.304382 \\
1.472654 & \quad 0.089380 & \quad 0.135863 & \quad 1.056615
\end{align*}
\]

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION.
PAUSE

NUMERICAL CONDITION OF SOLUTION
(NORM OF RECIPROCAL VECTORS)

<table>
<thead>
<tr>
<th>MODE</th>
<th>CONDITION NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.72000 \times 10^0) (+/-)</td>
<td>0.18293 \times 10^1</td>
</tr>
<tr>
<td>(-0.15000 \times 10^0) (+/-)</td>
<td>0.28846 \times 10^1</td>
</tr>
<tr>
<td>(-0.31000 \times 10^-1) (+/-)</td>
<td>0.18198 \times 10^1</td>
</tr>
</tbody>
</table>

: A + BK : MATRIX

\[
\begin{align*}
-0.766513 & \quad -0.002823 & \quad -0.004291 & \quad -0.033373 \\
-0.001501 & \quad -0.190628 & \quad 0.166251 & \quad 0.005603 \\
0.196346 & \quad -0.957383 & \quad -0.093858 & \quad 0.124335 \\
1.000000 & \quad 0.140541 & \quad 0.000000 & \quad 0.000000
\end{align*}
\]
APPENDIX

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION. PAUSE

? 

: K : MATRIX

\[
\begin{bmatrix}
-0.234868 & -1.168835 & 2.574575 & -0.054098 \\
-0.019499 & 0.507576 & 0.251622 & 0.054626 \\
\end{bmatrix}
\]

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION. PAUSE

? 

: ERROR MATRIX :

\[
\begin{bmatrix}
-0.000000 & 0.000000 & 0.000000 & -0.000000 \\
-0.000000 & 0.000000 & 0.000000 & 0.000000 \\
\end{bmatrix}
\]

PROGRAM EXECUTION IS TEMPORARILY INTERRUPTED. PRESS 'ENTER' TO CONTINUE EXECUTION. PAUSE

? 

...MESSAGE ...

IF ELEMENTS OF ERROR MATRIX HAVE NOT APPROACHED ZERO
SIGNIFICANT ROUND OFF ERRORS HAVE OCCURRED IN COMPUTATION.
PROBABLE CAUSES ... 1) MATRIX B1 IS ILL CONDITIONED
TRY PERMUTING SYSTEM MATRICES TO GENERATE BETTER B1 MATRIX

2) MODAL MATRIX IS ILL CONDITIONED
CHANGE DESIGN VECTORS

28
APPENDIX

COMPUTATIONAL MODE:
CHOOSE ONE--
  1--CONSTRAINING RELATION
  2--SYNTHESIS
  3--GAINS
  4--TIME RESPONSE
  5--SAVE
  6--EXIT

? 5

COMPUTATIONAL MODE:
CHOOSE ONE--
  1--CONSTRAINING RELATION
  2--SYNTHESIS
  3--GAINS
  4--TIME RESPONSE
  5--SAVE
  6--EXIT

? 4

CHOOSE OPTION:
  0--EXIT
  1--STATE RESPONSE
  2--STATE+CONTROL RESPONSE
  3--STATE+OUTPUT RESPONSE
  4--STATE+OUTPUT+CONTROL RESPONSE

? 1

ENTER THE NUMBER OF TIME STEPS:
? 50
ENTER NUMBER OF OUTPUT RESPONSES, R
? 4
WHAT RUN NUMBER IS THIS?
? 1
ENTER THE TIME INCREMENT
? .2
CHOOSE THE DESIRED TYPE OF RESPONSE:
  0--INITIAL CONDITION RESPONSE
  1--STEP RESPONSE
? 0
ENTER THE " 4" X " 1" INITIAL STATE VECTOR
? 0.0,0.0,-.1,0.0
APPENDIX

STATE VARIABLE RESPONSE

TIME RESPONSE PARAMETERS

NO OF STEPS = 50
TYPE OF RESPONSE = 0
TIME INCREMENT = .200 SECONDS

...INITIAL CONDITION RESPONSE ...

... I.C. VECTOR ...

0.000 0.000 -.100 0.000
APPENDIX

CHART 1

<table>
<thead>
<tr>
<th>Value</th>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
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31
APPENDIX

CHOOSE OPTION:
0--EXIT
1--STATE RESPONSE
2--STATE+CONTROL RESPONSE
3--STATE+OUTPUT RESPONSE
4--STATE+OUTPUT+CONTROL RESPONSE

? 0

COMPUTATIONAL MODE:
CHOOSE ONE--
1--CONSTRAINING RELATION
2--SYNTHESIS
3--GAINS
4--TIME RESPONSE
5--SAVE
6--EXIT

? 6

MASTER MENU
CHOOSE OPTION:
1--BEGIN PROBLEM
2--ENTER INPUT
3--ENTER COMPUTATIONAL MODE
4--SAVE PROBLEM
5--RENAME FILE
6--DUPLICATE FILE
0--STOP

? 0
HAS THE FILE BEEN SAVED?
(YES/NO)
? yes
14.16.55.REWIND(LGO)
14.16.56.RETURN(FTNMLIB)
14.16.57.REPLACE, STRTUP.
REFERENCES


### TABLE I.- CONTROL WORDS AND POSSIBLE VALUES

<table>
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<tr>
<th>Name</th>
<th>Function or description</th>
<th>Possible values</th>
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<tr>
<td><strong>EPS</strong></td>
<td>Tolerance parameter</td>
<td>If $</td>
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| **ENTRY** | Control parameter | -2 - Computes time response for a specified $V$ and $A$,  
-1 - Computes eigenvalues/eigenvectors of matrix $A$,  
numerical condition (sensitivity) of modal matrix, and time response  
0 - Normal setting for both modal and spectral synthesis  
1 - Normal setting for gain estimation  
2 - Setting for feedback synthesis |
| **INPUT** | Controls type of response                       | 0 - Initial condition response  
1 - Step response |
| **IPRMT** | Reorders state variables to improve numerical condition of $B_1$ matrix | 0 - No  
1 - Yes |
| **IPUNCH** | Controls print                                  | 0 - No  
1 - Yes |
| **ITSOLN** | Controls time response                          | 0 - No time response  
1 - State variable response  
2 - State + control response  
3 - State + output response  
4 - State + control + output response |
| **MODE(I)** | Type of mode (complex modes occur in pairs) | 0 - Real mode  
1 - Complex mode |
| **NPHASE** | Control parameter                               | -2 - Permutes $A$ & $B$ and $C$ & $D$  
-1 - Computes $B_1^{-1}$, $F$, $S$, and $G$ matrices  
0 - Selects modal synthesis  
1 - Computes modal coupling matrices $C_i$ for each specified mode $\lambda_i$  
2 - Selects spectral synthesis |
<p>| <strong>NRUN</strong> | Identification number                           | |</p>
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<tr>
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<td>CONDN</td>
<td>Computes numerical ill-conditioning (sensitivity) of modal matrix ( V )</td>
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<td>System routine for generating the quasi-diagonal equivalent of the eigenvalue and eigenvector matrices</td>
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<td>System routine for finding eigenvalues and eigenvectors</td>
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<td>Solves a system of linear equations</td>
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<td>MATEQN</td>
<td>Inverts a square matrix</td>
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<td>System routine for inverting matrices</td>
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<td>MULT</td>
<td>Multiplies two matrices</td>
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<td>Normalizes a vector</td>
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<td>Computes the output feedback gains approximation ( \bar{K} ) to a state feedback design ( K )</td>
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<tr>
<td>PERMUT</td>
<td>Permutes rows and columns of a matrix</td>
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<td>System routine for generating printed plots</td>
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<td>PRT</td>
<td>Prints a matrix</td>
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<td>SGAIN</td>
<td>Computes the equivalent state feedback gains ( \bar{K} ) for an output feedback design</td>
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<td>SPECTRL</td>
<td>Interactive subroutine for eigensystem placement</td>
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<td>SYNTHS</td>
<td>Computes modal coupling matrices and eigenvectors for specified ( \lambda_i ) and ( z_i ) ( i = 1, 2, \ldots, n )</td>
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<td>TFUNC</td>
<td>Evaluates ( e^{At} )</td>
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<td>Computes the closed-loop system after applying output feedback</td>
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<td>Computes actuator response ( u = K_x + u_{\text{ref}} )</td>
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<td>Computes state variable response time histories for ( x(t) = ve^{At}v^{-1}x(0) )</td>
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<tr>
<td>YSOLN</td>
<td>Computes designated output response ( y_o = C_o x + D_o u )</td>
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TABLE III.- Concluded

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Figure 1.- Flow chart for input module.
Figure 2.- Computational flow for eigensystem synthesis.
Figure 3.- Computational flow for gain estimation.
Figure 4.- Flow of response computations.
Figure 5.— Flow chart of iterative discourse.
Figure 5.— Concluded.
An interactive digital computer program for modal analysis and gain estimation for eigensystem synthesis has been written. Both mathematical and operational considerations are described; however, the mathematical presentation is limited to those concepts essential to the operational capability of the program. The program is capable of both modal and spectral synthesis of multi-input control systems. It is user friendly, has scratchpad capability and dynamic memory, and can be used to design either state or output feedback systems.