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Universal Single Level Implicit Algorithm for Gasdynamics

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Universal Single Level Implicit Algorithm for Gasdynamics

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Abstract

We present a new single level effectively explicit implicit algorithm for gasdynamics. The method meets all the requirements for unconditionally stable global iteration over flows with mixed supersonic and subsonic zones including blunt body flow and boundary layer flows with strong interaction and streamwise separation. For hyperbolic (supersonic flow) regions the method is automatically equivalent to contemporary space marching methods. For elliptic (subsonic flow) regions, rapid convergence is facilitated by alternating direction solution sweeps which bring both sets of eigenvectors and the influence of both boundaries of a coordinate line equally into play. Point by point updating of the data with local iteration on the solution procedure at each spatial step as the sweeps progress not only renders the method single level in storage but, also, improves nonlinear accuracy to accelerate convergence by an order of magnitude over related two level linearized implicit methods. The method derives robust stability from the combination of an eigenvector split upwind difference method (CSCM) with diagonally dominant ADI (DDADI) approximate factorization and computed characteristic boundary approximations. The properties and performance of the technique are demonstrated in a variety of quasi 1-D nozzle flows including completely subsonic or supersonic or mixed subsonic/supersonic with sonic points and shocks. The performance of the new method in 2-D and axisymmetric flows are compared against the precursor two level linearized scheme in an inviscid inlet problem for which the exact solution is known and in a viscous transonic nozzle problem previously studied both experimentally and computationally.

Acknowledgements

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Research Report

Universal Single Level Implicit Algorithm for Gasdynamics

by

C. K. Lombard and Ethiraj Venkatapathy

I. Background

The motivation for the present work stems from the following reasoning. Parabolized Navier Stokes (PNS) procedures can effectively solve steady supersonic and boundary layer flow problems with small adverse pressure gradient at a fraction of the cost required by conventional time dependent procedures. But PNS procedures are ill-posed for subsonic and streamwise reversed flow problems from the theoretical point of view, a failing that is manifested in grid sensitive numerical stability problems. On the

other hand, certain eigenvector split upwind differenced implicit time dependent procedures are unconditionally stable, practically robust and their application is not restricted with respect to the nature or type of compressible flow. A solution procedure that has the speed and the storage requirements of PNS schemes and the stability and applicability of the implicit upwind time marching schemes will be very useful and desirable. We give such a procedure here. Before we go on to describe this novel algorithm, we present a brief background to PNS and other related techniques.

The Beam-Warming factored implicit algorithm[1] with the Baldwin-Lomax thin layer viscous approximation[2] has provided the basis for two similar space marching (PNS) procedures[3,4] for the compressible Navier-Stokes equations. These PNS methods which are highly efficient from the points of view of both data storage and computer time have proven effective for flows[5,6] with favorable streamwise pressure gradient or with relatively small adverse pressure gradients. However, in the presence of strong adverse pressure gradient such as occurs in a wing or fin root regions the contemporary PNS methods suffer numerical stability problems and may infer streamwise separation even where separation doesn't occur[7]. In such unseparated (perhaps weakly separated) regions, numerical stability may be maintained at the price of employing large amounts of artificial viscosity with a resulting loss in predictive accuracy and knowledge of the actual state of the flow. Where strong streamwise separation occurs the methods

are unstable and cannot proceed. Particularly for the increasingly relevant laminar flow situation that will be encountered at very high altitude by aerodynamic systems such as orbital transfer vehicles (OTV's) and space shuttle, streamwise separation becomes a likely occurrence[8] in compression corners associated with canopies, pods, flared bodies, wing or fin roots and deflected control surfaces. Thus a more general technique is needed that is inherently stable for all types of upstream influence. At a minimum the mixed elliptic hyperbolic problem requires global iteration, preferably with type dependent differencing. Various steps in this direction have recently been taken by Rakich[9], by Rubin and Reddy[10] and by Rizk and Chaussee[11].

Rakich[9] utilized global iteration with Vigneron's Mach number dependent upwind and downwind splitting[4] of the pressure gradient for the streamwise momentum equation. The approach provides an improvement in both accuracy and stability for both weakly interactive boundary layer flow and strongly interactive flow without (significant) streamwise separation. Global iteration is carried out by marching the PNS equation repeatedly only along the downstream direction.

For incompressible flow Rubin and Reddy[10] (also Lin and Rubin[12] for supersonic flow) introduced type dependent (upwind) differencing of the streamwise velocity along with pressure splitting in an implicit method involving a staggard grid - dependent variable location

scheme. This method was extended to compressible flow by Khosla and Lai[13]. The method admits strong interaction with streamwise separation but is not homogeneous across the spectrum of Mach number regimes of interest here. Other PNS related methods for subsonic flow and boundary layer flow with streamwise reversal have recently been reviewed by Brown[14] in conjunction with a new staggered grid scheme.

Most immediately relevant, Rizk and Chaussee[11] presented a hybrid technique of space marching in supersonic zones and relaxation with a Beam-Warming time dependent central difference method over zones of strong upstream influence with streamwise separation. They have two variations of the time dependent method: one fully implicit requiring two levels of storage and matrix inversion procedures in all space directions, the other explicit in all space directions but the thin layer direction and requiring only one level of storage as in a space marching algorithm. Both procedures require the same several hundred iterations minimum to reasonably converge, with the semi explicit procedure requiring substantially less machine time. However, both procedures are inherently unstable, except for the use of artificial dissipation. The marginal stability coupled with the lack of type dependent differencing and well posed boundary approximations all contribute to slow convergence. In balance, while workable, the hybrid approach leaves something to be desired from the points of view of convenience, computer time and internal consistency of the global

solution procedure.

A new globally iterated scheme related to that which will be presented here, has been presented by Moretti[15]. In his procedure for steady inviscid flows, the Euler equations are cast in Riemann variables and the resulting uncoupled equations are solved by integrating each Riemann variable separately, sweeping back and forth alternatively along each coordinate line. The coupling between the equations and, thus, the non-linearity are introduced only through the boundaries and the updating of state after complete sweeps. As can be inferred from the results to be presented here, the reduced coupling inherently limits the rate of convergence of Moretti's method and the procedure is not extendable to the compressible Navier-Stokes equations.

II. New Universal Single Level Scheme CSCM-S

The CSCM flux difference eigenvector split upwind implicit method[16,17,18] for the inviscid terms of the compressible Navier-Stokes equations provides the natural basis for an unconditionally stable space marching technique through regions of subsonic and streamwise separated flow. In such regions the split method can be likened to stable marching of each scalar characteristic wave system in the direction of its associated eigenvalue (simple wave velocity). In supersonic flow, where all eigenvalues have the same sign, the method automatically becomes similar to the referenced PNS techniques based on the Beam-Warming fac-

tored implicit method with the Baldwin-Lomax thin layer viscous approximation.

Compared to contemporary central difference methods, the CSCM characteristics based upwind difference approximation with its inherent numerical stability leads to greatly reduced oscillation and greater accuracy in the presence of captured discontinuities such as shocks, contacts and physical or computational boundaries. The correct mathematical domains of dependence that correspond with physical directions of wave propagation are coupled with well posed characteristic boundary approximations[17] naturally consistent with the interior point scheme. The result is faster sorting out of transient disturbances and substantially more rapid convergence to the steady state. The splitting and the associated time dependent implicit method have been described in detail in references (16) and (18) for quasi 1-D and 2-D planar or axisymmetric flow.

In the following, we will sketch the differences between the time dependent method and the new space marching technique which we designate CSCM-S. The discussion will begin with the quasi 1-D inviscid formulation, present some results elucidating the properties and performance of the method, then give additional details entering into multidimensional inviscid and thin layer viscous procedures and, lastly, present early 2-D solutions obtained with the new single level scheme in problems solved previously[18] with the time dependent method.

Quasi 1-D Formulation

The general jth interior point difference equations for the time dependent CSCM upwind implicit method is written

$$(I + \widetilde{A}^{+} \nabla + \widetilde{A}^{-} \Delta)\delta q_j = -\widetilde{A}^{+} \Delta q)_{j-1}^{n} - \widetilde{A}^{-} \Delta q)_{j}^{n} \qquad (1)$$

where ∇ and Δ are backward and forward spatial difference operators. In the notation the interval averaged matrices between node points j and $j+1$ are labled j. The right hand side of equation (1) is written for the first order method. Higher order methods in space are given with results in references 16 and 18.

Central to its accurate shock capturing capability, the CSCM conservative flux difference splitting has the "property U" put forth by Roe[19]

$$(\widetilde{A}^{+} + \widetilde{A}^{-})\Delta q)_j \equiv \Delta F)_j = F_{j+1} - F_j \qquad (2)$$

Here q is the conservative dependent variable vector and F is the associated flux vector. The matrices $\widetilde{A}^{+}$ and $\widetilde{A}^{-}$ are the splittings of the CSCM interval averaged Jacobian matrix according to the signs of the averaged eigenvalues. Thus in the equation for the jth grid point, $\widetilde{A}^{+} \Delta_{\xi} q)_{j-1}$ represents stable characteristic spatial differencing backward for positive eigenvalue contributions and $\widetilde{A}^{-} \Delta_{\xi} q)_j$, forward for negative ones.

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With $\delta q = q^{n+1} - q^n$, equation (1) defines a two level linearized coupled block matrix implicit scheme that can be solved by a block tridiagonal procedure. In reference (18) a new (DDADI) approximately factored alternating sweep bidiagonal solution procedure for equation (1) is presented that is shown to be very robust and is effectively explicit, i.e. requires only a decoupled sequence of local block matrix inversions rather than the solution of the coupled set. For the forward sweep the bidiagonal solution procedure can be written

$$(I + \widetilde{A}^+ - \widetilde{A}^-)\delta q^*{}_j = RHS + \widetilde{A}^+ \delta q^*{}_{j-1} \tag{3}$$

For the linear problem, i.e. constant coefficient case of stability analysis, equation (3) is equivalent to the single level space marching procedure

$$(I + \widetilde{A}^+ - \widetilde{A}^-)\delta q^*{}_j = \widetilde{A}^+ q^*{}_{j-1} - \widetilde{A}^+ q^n_j - \widetilde{A}^- \Delta q)^n_j \tag{4}$$

Nonlinearity enters in the single level space marching form (4) in that at each step of the forward sweep the matrices $\widetilde{A}^+$ are averaged between $q^*{}_{j-1}$ and q^n_j rather than homogeneously at the old iteration level n. Similarly, companion backward space marching sweep that is symmetric to equation (4) and that is intimately related to the backward sweep of the alternating bidiagonal algorithm of reference (18) is

$$(I + \widetilde{A}^+ - \widetilde{A}^-)\delta q_j = -\widetilde{A}^+ \Delta q^*)_{j-1} + \widetilde{A}^- q^*_j - \widetilde{A}^- q^{n+1}_{j+1} \tag{5}$$

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As will be shown in a forthcoming paper[20], the method given by equations (4) and (5) is von Neumann unconditionally stable for the scalar wave equation. The analysis shows the significance of DDADI approximate factorization in rendering both the forward and backward sweeps separately stable regardless of eigenvalue sign. Consequently as the local Courant number becomes very large, the robust method becomes a very effective (symmetric Gauss-Seidel) relaxation scheme for the steady equations, a fact which substantially contributes to the very fast performance that will be demonstrated.

At a right computational boundary on the forward sweep we solve the characteristic boundary point approximation

$$(\tilde{A}^{+} + \tilde{A}^{+})\delta q_N = \tilde{A}^{+} q^{*}_{N-1} - \tilde{A}^{+} q^{n}_{N} \tag{6}$$

$q^{n+1}_{N} = q^{*}_{N}$ and at a left, on the backward sweep

$$(\tilde{A}^{-} - \tilde{A}^{-})\delta q_1 = \tilde{A}^{-} q^{n}_{1} - \tilde{A}^{-} q^{n+1}_{2} \tag{7}$$

Following the solution of equations (6) and (7) the conservative state vector is iteratively corrected[17] to maintain the accuracy of prescribed boundary conditions while not disrupting the representation of the computed characteristic variables running to the boundary from the interior. Analysis of a model system with upwind differenced scalar equations and coupled boundary conditions was related to the linearized bidiagonal scheme[18] by Oliger and Lombard[21]; the analysis also strongly supports the

numerically confirmed robust stability of the present nonlinear method for gasdynamics.

With the updating at each step, where in equation (5) $\delta q_j = q_j^{n+1} - q_j^*$, it is clear that the symmetric pair of equations (4) and (5) serve to advance the solution two pseudo time (iteration) levels; whereas, the linear alternating bidiagonal sweep algorithm of reference (18) advances the solution only one level. To maintain conservation to a very high degree, in single sweep marching in supersonic zones we iterate (at least) once locally at each space marching step. The local iteration serves to make the eigenvectors in the coefficient matrices consistent with the advanced state and, thus, provides improved accuracy for the nonlinear system. It appears effective to do this inner iteration everywhere, i.e. in both subsonic and supersonic regions, as the number of global iteration steps to convergence with two inner iterations has been found reduced by a factor of three to four. Since the computational work per two steps is about the same for the single level and two level schemes and beyond the fact that one saves a level of storage in the space marching algorithm, the question arises: Can one get solutions in less computational work through faster convergence with the nonlinear space marching algorithm?

One Dimensional Results

First, we present results for supersonic flow with no shock in Shubin's diverging nozzle. In purely supersonic zones, the experience with the

present method is that the solution can be marched accurately in one global iteration, as ought to be the case. Figure 1 shows the exact solution (in solid line) and the computed result from the first forward sweep. It is evident that the method correctly predicts the solution to plotting accuracy in one forward sweep. With subsequent sweeps the error (the difference between the exact and the computed solution) reduces to machine accuracy in less than three global iterations. In fact, by increasing (from two) the number of inner iterations on the solution procedure at each space marching step one can guarantee convergence to prescribed accuracy in one forward sweep. This is also true of contemporary locally linearized unsplit methods in supersonic flow.

With the globally iterative nonlinear space marching formulation, early experience in two quasi 1-D nozzle problems with mixed supersonic-subsonic zones is that solutions are obtained in roughly an order of magnitude fewer iteration steps than had been required with the previously fast pseudo time dependent technique and block tridiagonal solving.

The two nozzle problems which are described and solved by Yee, Beam and Warming[22] and solved with the CSCM time dependent technique in references (16) and (17) are Shubin's diverging nozzle flow and Blottner's converging-diverging nozzle flow. Both problems involve unmatched overpressures at the outflow which result in inter-

nal shock terminated supersonic zones and subsonic outflow. For the experiments involving flow of mixed type the same initial data given by Yee, Beam and Warming – a linear interpolation between inflow and outflow values for effectively exact solutions of the problems – is used that was used previously with the time dependent approach.

For flows of mixed type, in Figures 2 and 3 respectively, results are shown for successive forward and backward sweeps for five global iteration steps with Shubin's and Blottner's nozzle flows. In both cases, the exact solution as given by Yee, Beam and Warming is shown in solid line and the present computational results solved on a 51 point mesh, in boxes. Blottner's nozzle flow is shown converged after 10 global iteration steps. There is substantial evidence in other results not shown that with further work the number of global iterations required to compute flows such as Blottner's will be reduced by a factor of two, to about five.

In Figure 4, we show a subcritical, i.e. completely subsonic, flow solution computed in only two global iteration steps for the Blottner nozzle geometry with different inflow conditions. Here the exact analytical solution derived by Venkatapathy is shown in solid line and our computed results in boxes.

The alternating direction sweeps in our method have been derived directly out of theory for solving the implicit set of difference equations.

However, with a moments thought, one can see mechanistically, numerically speaking, that omitting the backward sweep from the pair and globally iterating only with the forward sweep equation (4) will result in permitting the influence of a subsonic outflow boundary (or interior disturbance) to propagate upstream only one grid point per global iteration. In such a case, which relates to other global iteration methods found in the literature and that sweep only in the main flow direction, rate of convergence is greatly inhibited relative to symmetric sweeping by a factor of order roughly the number of grid points in the subsonic zone. Mathematically, this inhibition is the result of the failure to include the effect of the eigenvectors governing upstream influence in the implicit process but to treat these waves explicitly with effective CFL unity.

In Figure 5 we illustrate the progress of the transient solution to the subcritical nozzle problem after 15 forward sweeps, with the backward sweeps omitted. One can clearly see that the wave influence of the outflow boundary has progressed only 15 mesh points forward of the outflow boundary. In Figure 6 the transient solution is shown after 60 steps which is beyond one characteristic transit time (equivalent to 50 mesh intervals) for the upwind wave to reach the inflow boundary. In Figure 7 we show the history of the RMS error in the primitive variables. The solution is found to converge to roughly the same RMS error after three characteristic times (150 steps) as the solution obtained

with the symmetric alternating sweep sequence after only 3 global iterations.

Blottner's supercritical nozzle problem which involves subsonic inflow accelerating through a sonic point to a supersonic zone terminated by a shock to subsonic outflow is the most computationally demanding of the test cases and indicates the capability for the method to compute simply and consistently over the subsonic forebody and base regions of blunt bodies in supersonic flow. Thus the need for separate time dependent codes will be obviated by this new method.

Finally, in Figure 8 and 9, we present the convergence history for the present nonlinear scheme and the linearized time dependent scheme for completely subsonic and supersonic nozzle flows. The x-axis shows the number of iterations each scheme requires to reduce the exact error to five orders of magnitude for various Courant numbers. It is evident that the present scheme converges extremely fast at all CFL numbers compared with the method based on the linearized block tridiagonal solver.

Two Dimensional Formulation

For two dimensional flow, assuming a marching coordinate ξ, inviscid terms

$$\tilde{B}^{+}\nabla_{\eta} + \tilde{B}^{-}\Delta_{\eta} \tag{6a}$$

and

$$-\widetilde{B}^{+}\Delta_{\eta}q)_{k-1} - \widetilde{B}^{-}\Delta_{\eta}q)_{k} \tag{6b}$$

are added to the left and right hand sides respectively of both the forward and backward sweep equations (4) and (5). For viscous flow, second centrally differenced, thin layer viscous terms are also added in the η direction as is conventionally practiced, e.g. Steger[23]. With the terms for the η cross marching coordinate direction, the technique now becomes an implicit method of lines. Along each η coordinate line, one can solve the equations coupled with a block tridiagonal procedure. Alternatively, a further DDADI bidiagonal approximate factorization can be employed in the η direction and solved either linearly as in reference (18) or nonlinearly as here in the ξ direction. As shown in the quasi 2-D numerical experiments of reference (18), DDADI bidiagonal approximate factorization is stable for viscous as well as inviscid terms. Finally in reference (18) there is a relevant discussion of the reduced approximate factorization error that attends using DDADI in one or more space directions. The variety of multidimensional solution strategies derived from DDADI bidiagonal approximate factorization will be presented in greater detail, along with extension of the method to higher order, in a forthcoming paper[20].

While extensions to 3-D are not given in detail here, we note such extensions to a developing two level 3-D CSCM upwind scheme are possible and will be adopted in the future. For 3-D, the inviscid terms

similar to (6a) and (6b) will again be added for the ς cross flow direction. For efficiency in solving each resulting marching plane, the implicit operators for the η and ς coordinate directions can be approximately factored using DDADI as described in reference (18) for two space directions.

Two Dimensional Results

We present results for a 45° - 15° axisymmetric transonic nozzle flow previously studied experimentally by Cuffel, Back and Massier[24] and computationally by Cline, Prozan, Serra and Shelton (all referenced in (24)) and ourselves[18]. In Figure 10 we show results after 10 steps of an early computation run at a local CFL number of 20 with the present first order single level scheme. Except for the addition of an error correction procedure[17] to counter numerical inflow boundary condition drift, a factor which has improved the present solution in the vicinity of the axis, the effectively converged results found here are the same as those given for the two level scheme in reference 18. (As long as the problem has a unique solution, the two schemes must give equivalent results since the right hand side difference equation sets, including boundary approximations, are the same.)

For the solution given in Figure 10, we noted a very rapid rate of reduction in residual, three orders of magnitude in ten steps. This compares with 60 steps given in reference (18) for the solution obtained with the two level scheme. The rapid convergence found in this transonic problem for the CSCM-S method with viscous terms provides the reasonable expectation of similar fast results to be obtained without viscous effects. Thus the method in multidimensions appears to have attractive potential for an improved transonic Euler solver as well as Navier-Stokes solver.

Next, we present first order inviscid and viscous results for an inlet problem shown in Figure 11. The pressure contours for the first order inviscid solutions are shown in Figure 12. Figure 13 shows the first order viscous results. The viscous computation shows the presence of the leading edge shock. The flow structure compares very well with the theoretical (for the inviscid case) and other computational results. In Figure 14, the inviscid and viscous wall pressure are compared with the exact solution (inviscid). Figure 15 shows the convergence history of the RMS residue of all the consevative flow variables for the inviscid problem solved at CFL = 100 with 4 inner iterations at every axial location. For the inviscid case, only forward marching was carried out and backward marching was omitted. The solution has converged for practical purposes at the end of the first sweep. The residue reaches machine accuracy in 10 iterations. In a later paper[25], we will show the residual reduction versus inner iteration number in single sweep solutions for supersonic flow and compare results with contemporary PNS procedures.

III. Summary and Conclusions

From both theory and computational experiments we observe that the new single level eigenvector split upwind method is unconditionally stable and practically robust for all types of flow. Thus the method removes the restrictions suffered by contemporary PNS methods and the resulting need to use other complimentary methods. As a result, we conclude, the present method will permit the solution of complete flowfields about complex configurations in a comparatively convenient, efficient and consistent manner. The new method, in addition to requiring only half the dependent variable storage, has been found in test problems to obtain converged solutions in fewer iterations and less net work – through improved treatment of nonlinear effects – than the linearized implicit methods from which it evolved. For flows with subsonic zones or boundary layers with strong upstream influence, the use of the present alternating direction implicit procedure that is related to symmetric Gauss-Seidel relaxation greatly accelerates convergence (by order the number of grid points in a subsonic zone) relative to global iterations (and other schemes) that repetitively sweep only in the predominantly streamwise direction. Hence for flows with elliptic influence in the flow direction, we conclude that such unidirectional sweep strategies, regardless of other factors, will not result in very effective solution algorithms.

References

[1.] Beam, R.M. and Warming, R.F.: "An Implicit Factored Scheme for the Compressible Navier-Stokes Equations," AIAA Paper 77-645, June, 1977.

[2.] Baldwin, B.S. and Lomax, H.: "Thin Layer Approximation and Algebraic Model for Separated Turbulent Flows," AIAA 73-257, 1973.

[3.] Schiff, L.B. and Steger, J.L.: "Numerical Simulation of Steady Supersonic Viscous Flow," AIAA-79-0130, January, 1979.

[4.] Vigneron, Y.C., Rakich, J.V. and Tannehill, J.C.: "Calculation of Supersonic Viscous Flow over Delta Wings with Sharp Subsonic Leading Edges," NASA TM-78500, 1978.

[5.] Schiff, L.B. and Sturek, W.B.: "Numerical Simulation of Steady Supersonic Flow Over an Ogive-Cylinder-Boattail Body," AIAA-80-0066, January, 1980.

[6.] Chaussee, D.S., Patterson, J.L., Kutler, P., Pulliam, T.H. and Steger, J.L.: "A Numerical Simulation of Hypersonic Viscous Flows over Arbitrary Geometries at High Angle of Attack," AIAA Paper 81-0050, Jan. 1981.

[7.] Rai, M.M., Chaussee, D.S. and Rizk, Y.M.: "Calculation of Viscous Supersonic Flows Over Finned Bodies," AIAA-83-1667, July, 1983.

[8.] Chaussee, D.S. and Rizk, Y.M.: "Computation of Viscous Hypersonic Flow Over Control Surfaces," AIAA Paper 82-0291, Jan. 1982.

[9.] Rakich, J.V.: "Iterative PNS Method for Attached Flows with Upstream Influence," AIAA-83-1955, July 1983.

[10.] Rubin, S.G. and Reddy, D.R.: "Global PNS Solutions for Laminar and Turbulent Flow," AIAA Paper 83-1911, July 1983.

[11.] Rizk, Y.M. and Chaussee, D.S.: "Three-Dimensional Viscous-Flow Computations Using a Directionally Hybrid Implicit-Explicit Procedure," AIAA Paper 83-1910, July 1983.

[12.] Lin, A. and Rubin, S.G.: "Three-Dimensional Supersonic Viscous Flow over a Cone at Incidence," AIAA J., Vol 20, no. 11, Nov. 1982, pp. 1500-1507.

[13.] Khosla, P. K. and Lai H. T.: "Global PNS Solutions for Subsonic Strong Interaction Flow over a Cone-Cylinder-Bottail Configuration," Computers and Fluids,.

[14.] Brown, James L.: "Parabolized Navier-Stokes Solutions of Separation and Trailing-Edge Flows," NASA TM 84378, June 1983.

[15.] Moretti, Gino: "A Fast Euler Solver for Steady Flows," AIAA Paper 83-1940, 1983.

[16.] Lombard, C.K., Oliger, J. and Yang, J.Y.: "A Natural Conservative Flux Difference Splitting for the Hyperbolic Systems of Gasdynamics," AIAA Paper 82-0976, 1982.

[17.] Lombard, C.K., Oliger, J., Yang, J.Y. and Davy, W.C.: "Conservative Supra-Characteristics Method for Splitting the Hyperbolic Systems of Gasdynamics with Computed Boundaries for Real and Perfect Gases," AIAA Paper 82-0837, 1982.

[18.] Lombard, C.K., Bardina, J., Venkatapathy, E. and Oliger, J.: "Multi-Dimensional Formulation of CSCM - An Upwind Flux Difference Eigenvector Split Method for the Compressible Navier-Stokes Equations," AIAA-83-1895, Danvers, Mass.

[19.] Roe, P.L.: "The Use of the Riemann Problem in Finite- Difference Schemes," Seventh International Conference on Numerical Methods in Fluid Dynamics, Lecture Note in Physics, 141, pp. 354-359, 1981.

[20.] Lombard, C.K. and Venkatapathy, Ethiraj: "Universal Single Level Implicit Algorithm for Gasdynamics," Abstract submitted for the AIAA 17th Fluid Dynamics Conference, Snowmass Colorado, June 25-27, 1984.

[21.] Oliger, Joseph and Lombard, C.K.: "Boundary Approximations for Alternating Sweep Implicit Upwind Methods for Hyperbolic Systems," SIAM 1983 Fall Meeting, Norfolk, VA.

[22.] Yee, H.C., Beam, R.M. and Warming, R.F.: "Stable Boundary Approximations for a Class of Implicit Schemes for the One-Dimensional Inviscid Equations of Gas Dynamics," AIAA Paper 81-1009-CP, 1981.

[23.] Steger, J.L.: "Implicit Finite Difference Simulation of Flow About Arbitrary Geometries with Application to Airfoils," AIAA Paper 77-665, 1977.

[24.] Cuffel, R.F., Back, L.H. and Massier, P.F.: "Transonic Flowfield in a Supersonic Nozzle with Small Throat Radius of Curvature," AIAA J., Vol. 7, no. 7, 1969, pp 1364-1372.

[25.] Venkatapathy, Ethiraj and Lombard, C.K.: "Universal Single Level Implicit Algoirth for Gasdynamics," Abstract submitted for the NIneth International Conference on Numerical Methods in Fluid Dynamics, Saclay, France, June 25-29, 1984.

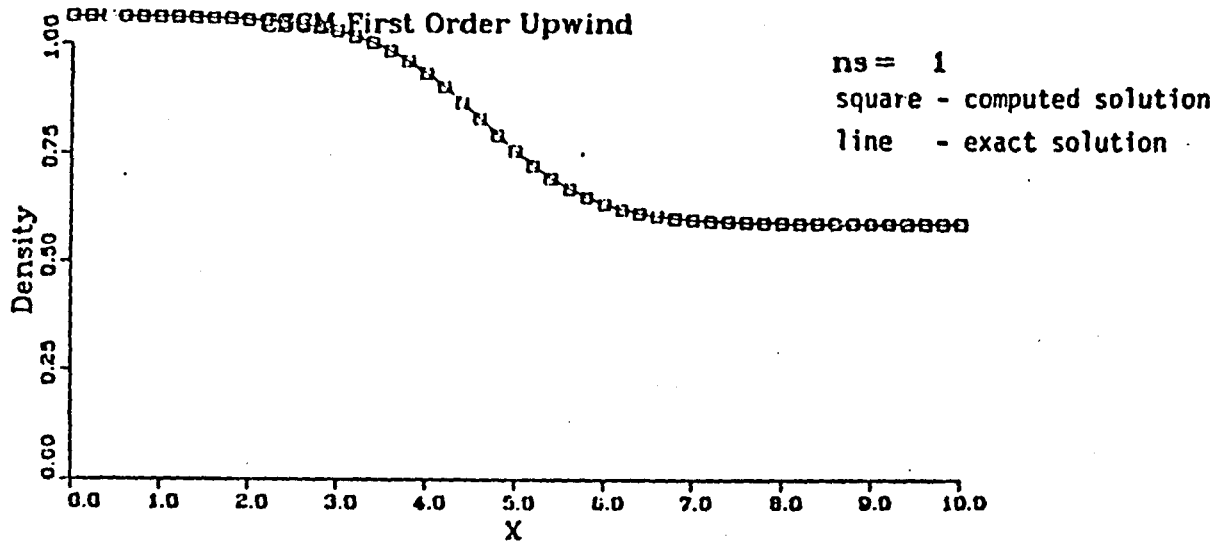

Figure 1. Shubin's diverging nozzle supersonic flow solution developed in one forward sweep from supersonic initial data.

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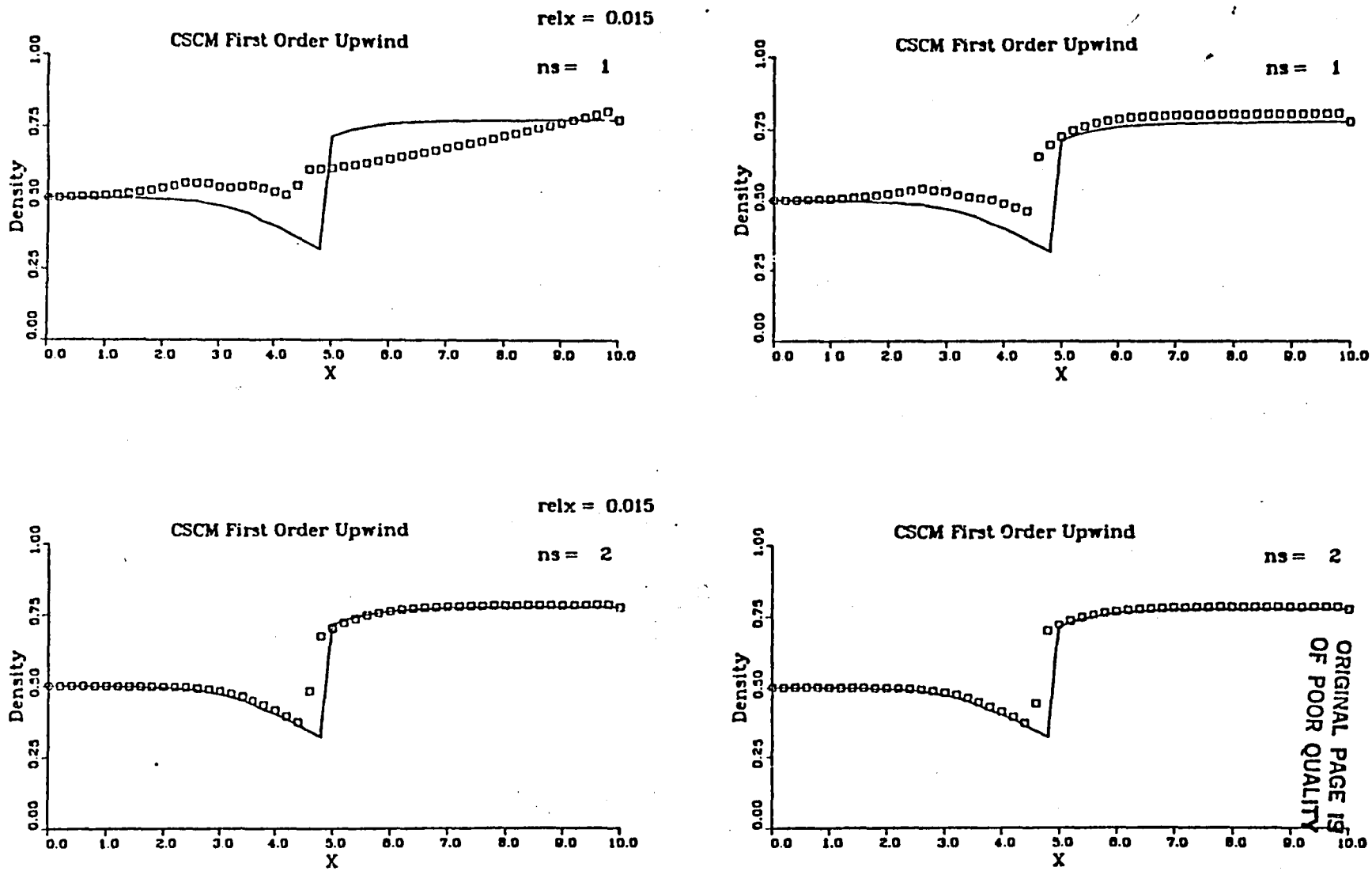

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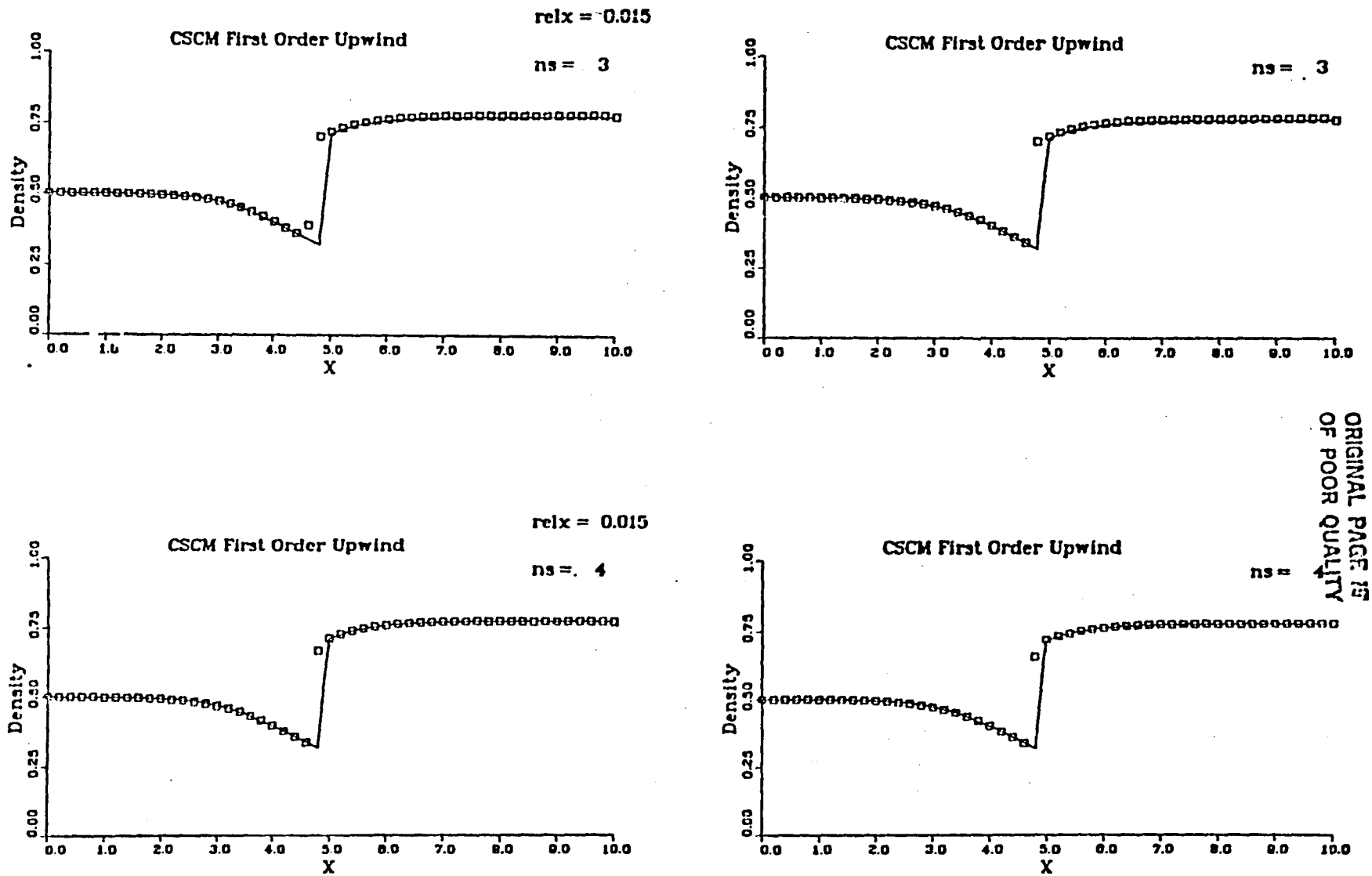
Figure 2. Shubin's diverging nozzle flow solution developed in alternating forward and backward sweeps, one each per global iteration step for five steps. square - computed results , line - exact solution



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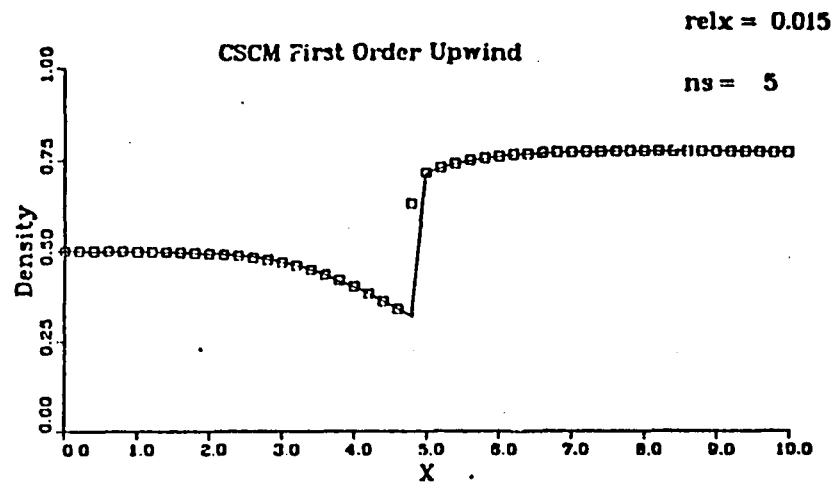

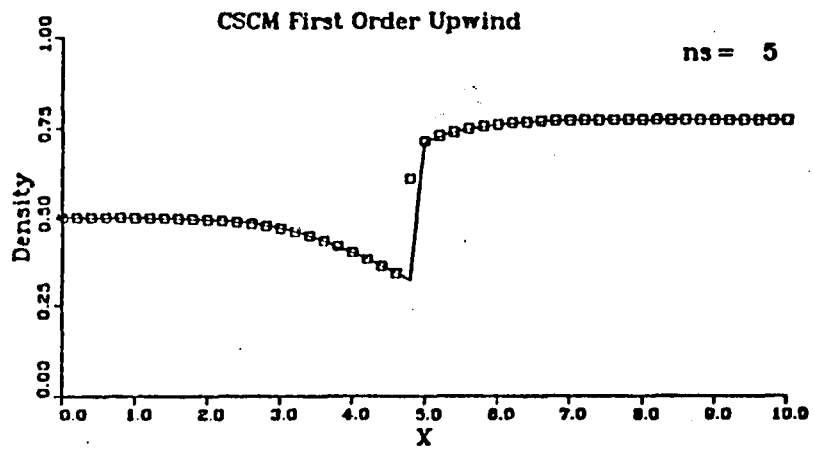

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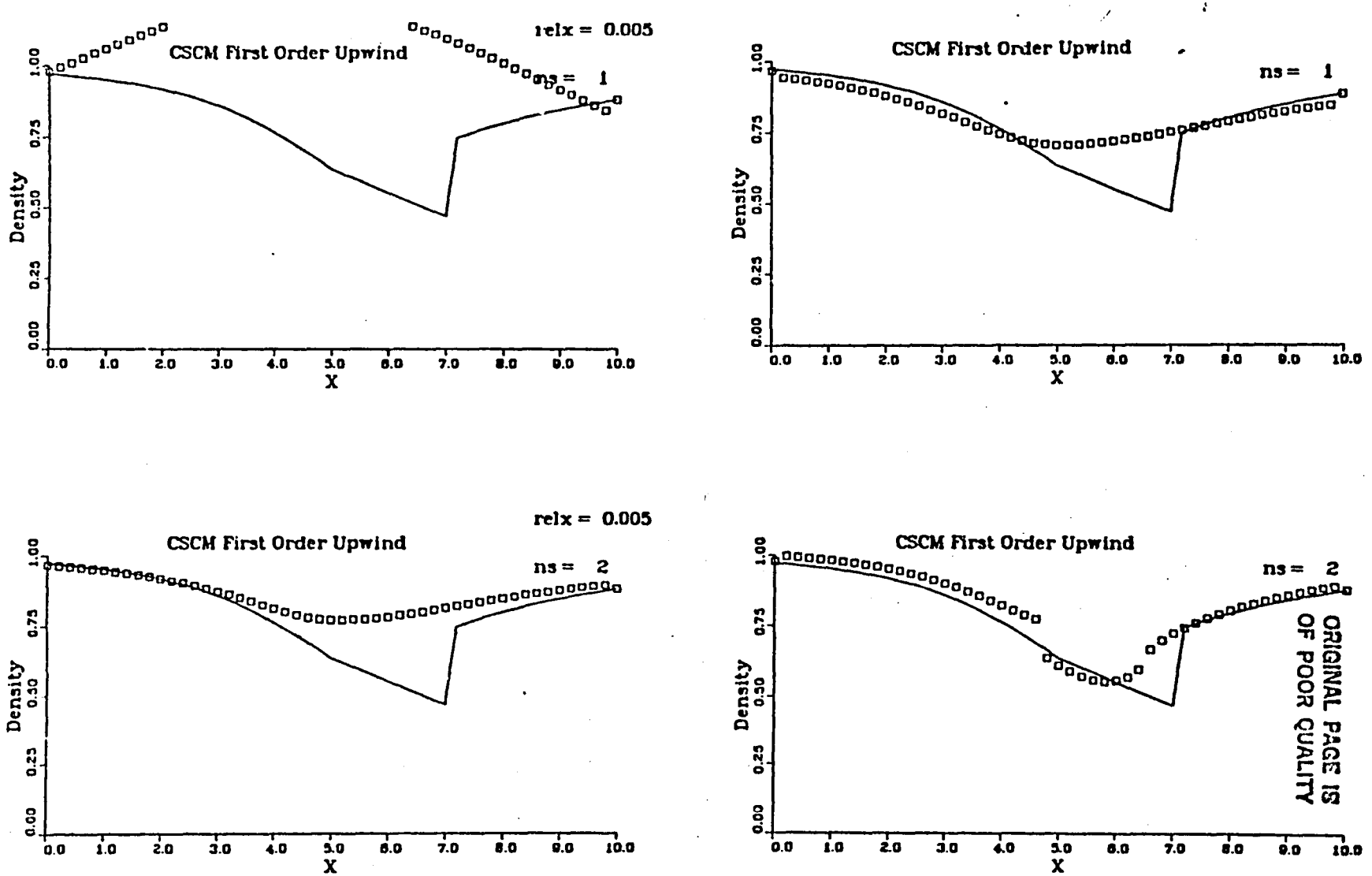

Figure 3. Blottner's converging-diverging supercritical nozzle flow solution developed in alternating direction sweeps for ten global iteration steps. square - computed results , line - exact solution

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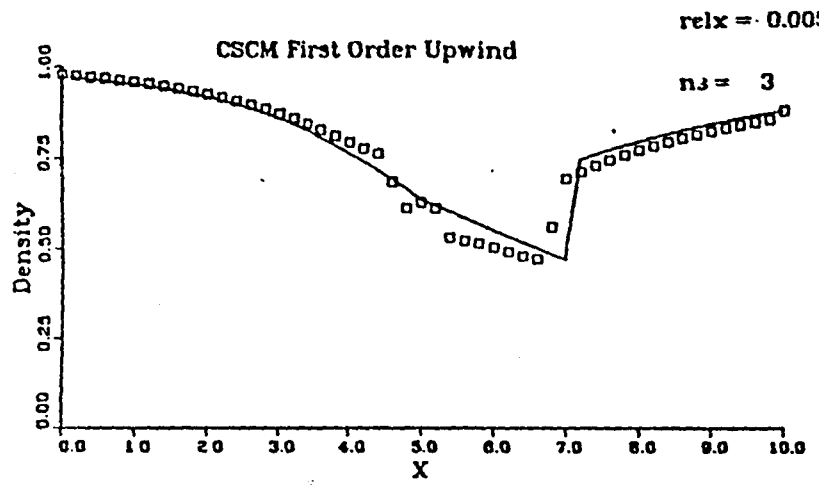

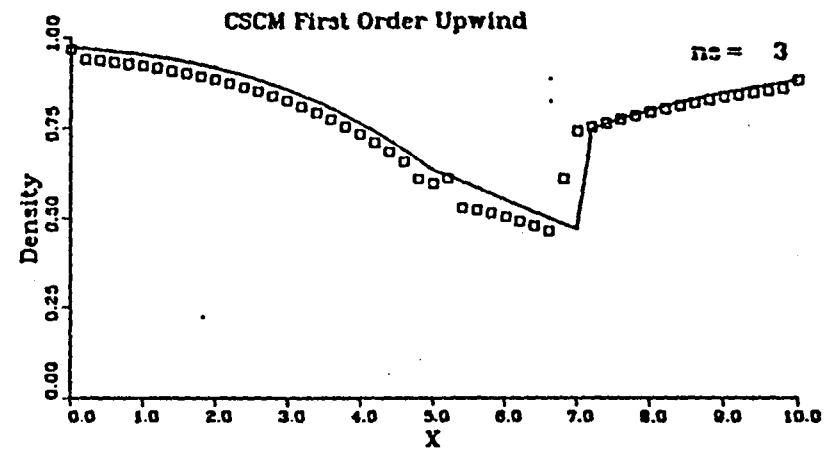

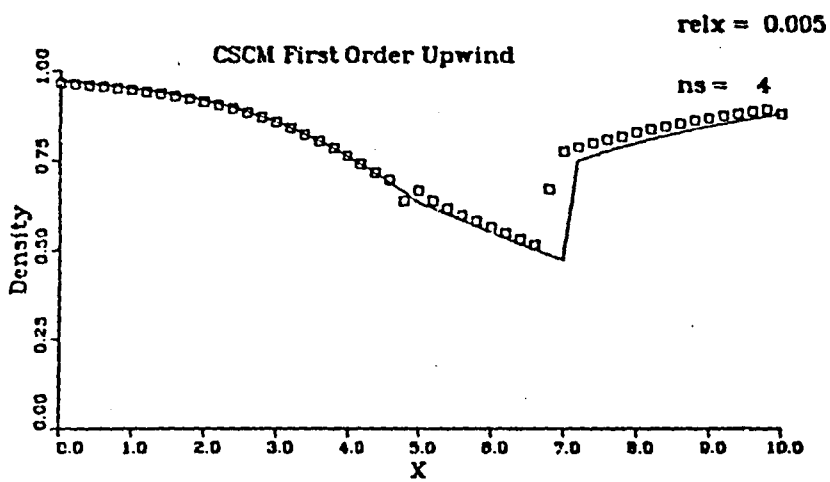

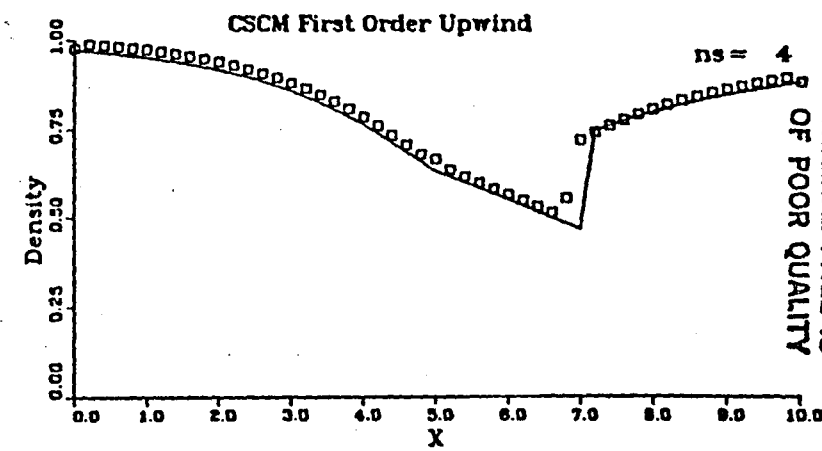

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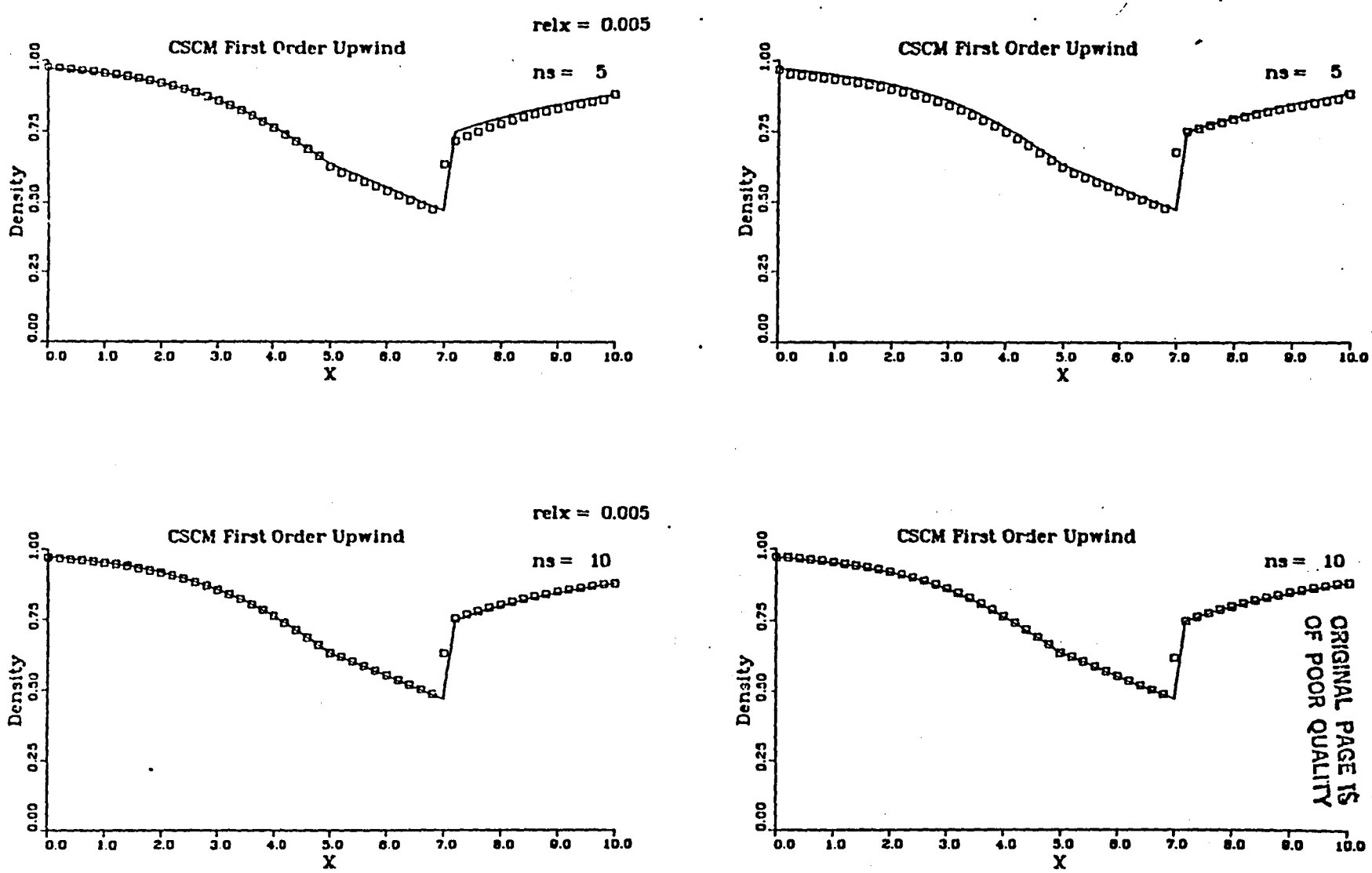

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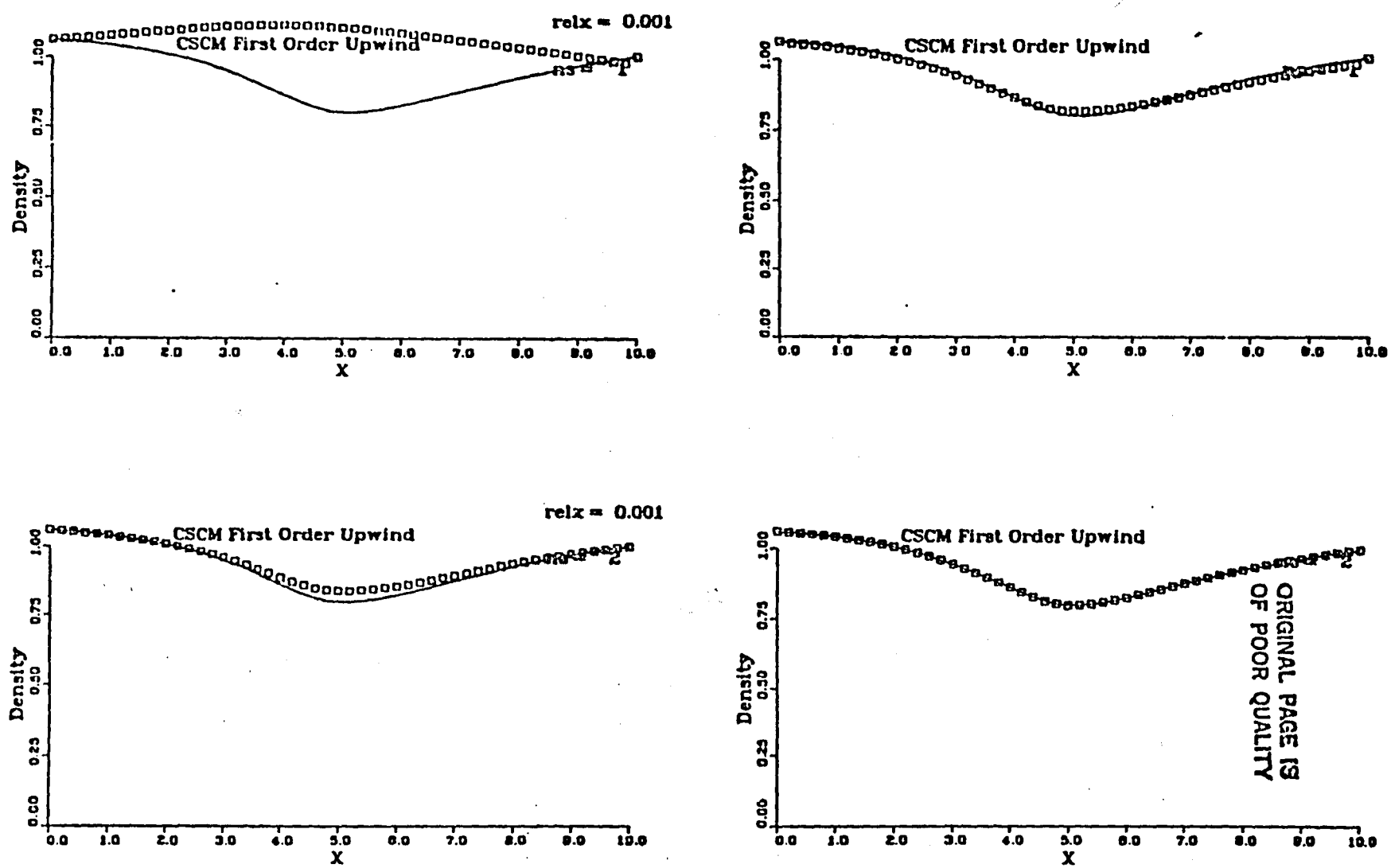

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Figure 4. Subcritical solution to Blottner's converging-diverging nozzle developed with alternating sweeps in two global iteration steps. square - computed results , line - exact solution

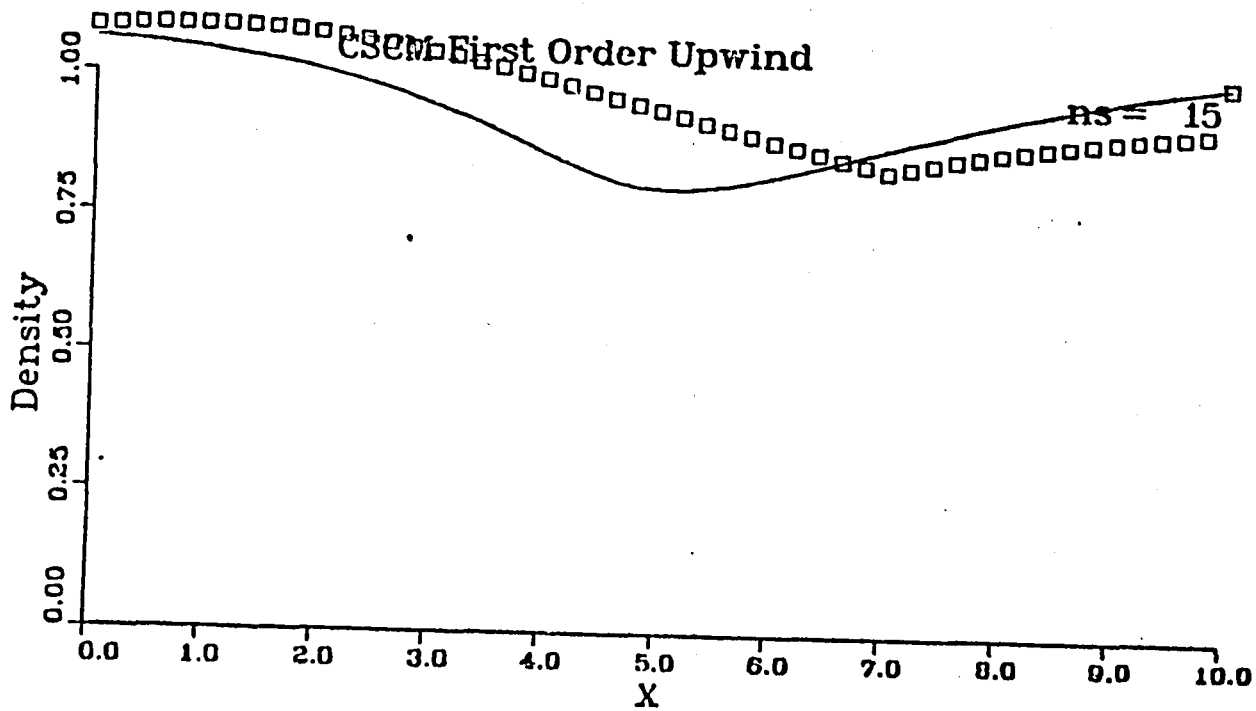

Figure 5. Solution to subcritical nozzle problem after 15 global iterations with forward marching sweeps only.

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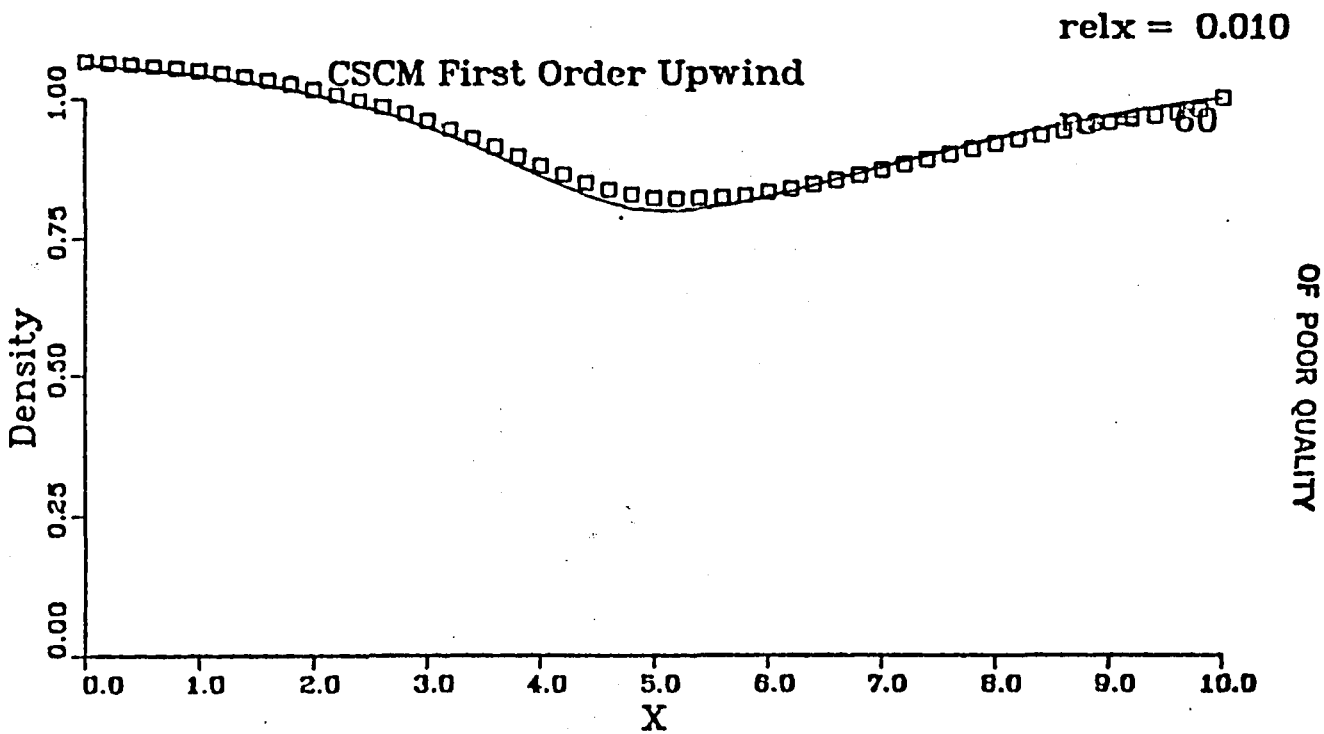

Figure 6. Solution to subcritical nozzle problem after 60 global iterations with forward marching sweeps only.

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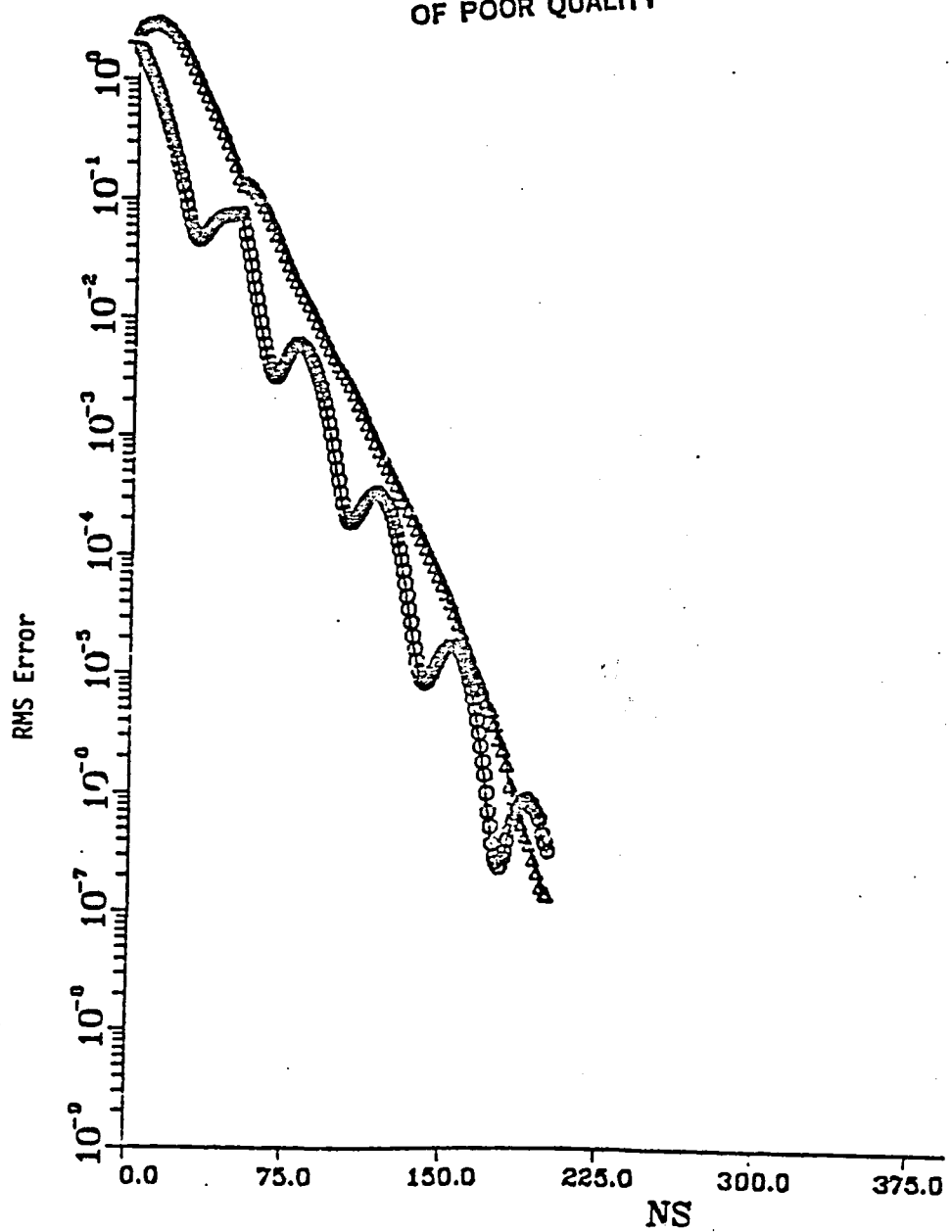

Figure 7. Convergence history of RMS error based on the exact steady solution for the subcritical nozzle flow solved with forward sweeps only.

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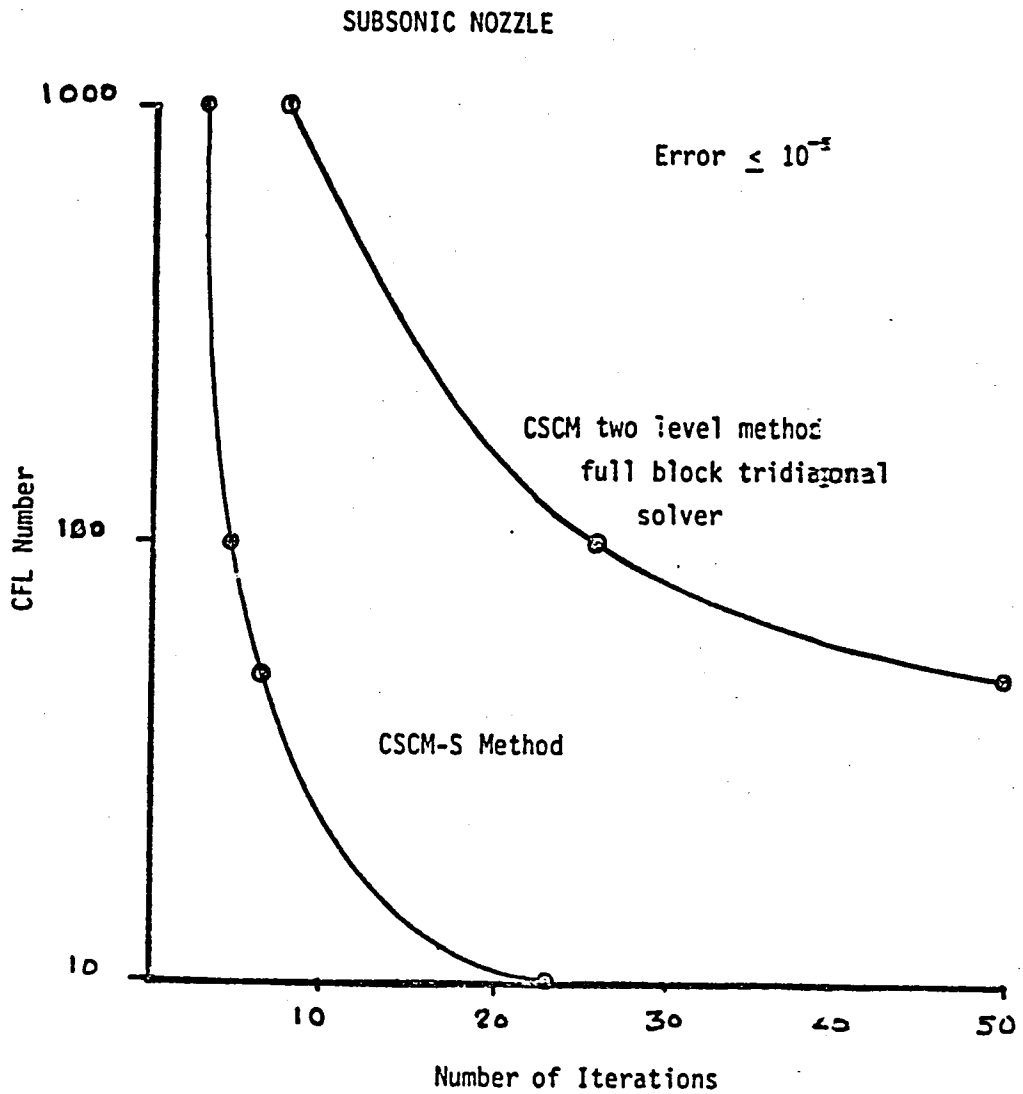

Figure 8. Convergence history comparison between the full block tridiagonal CSCM Solver and the CSCM-S method. The solutions have reached an RMS error less than 1×10^{-5}.

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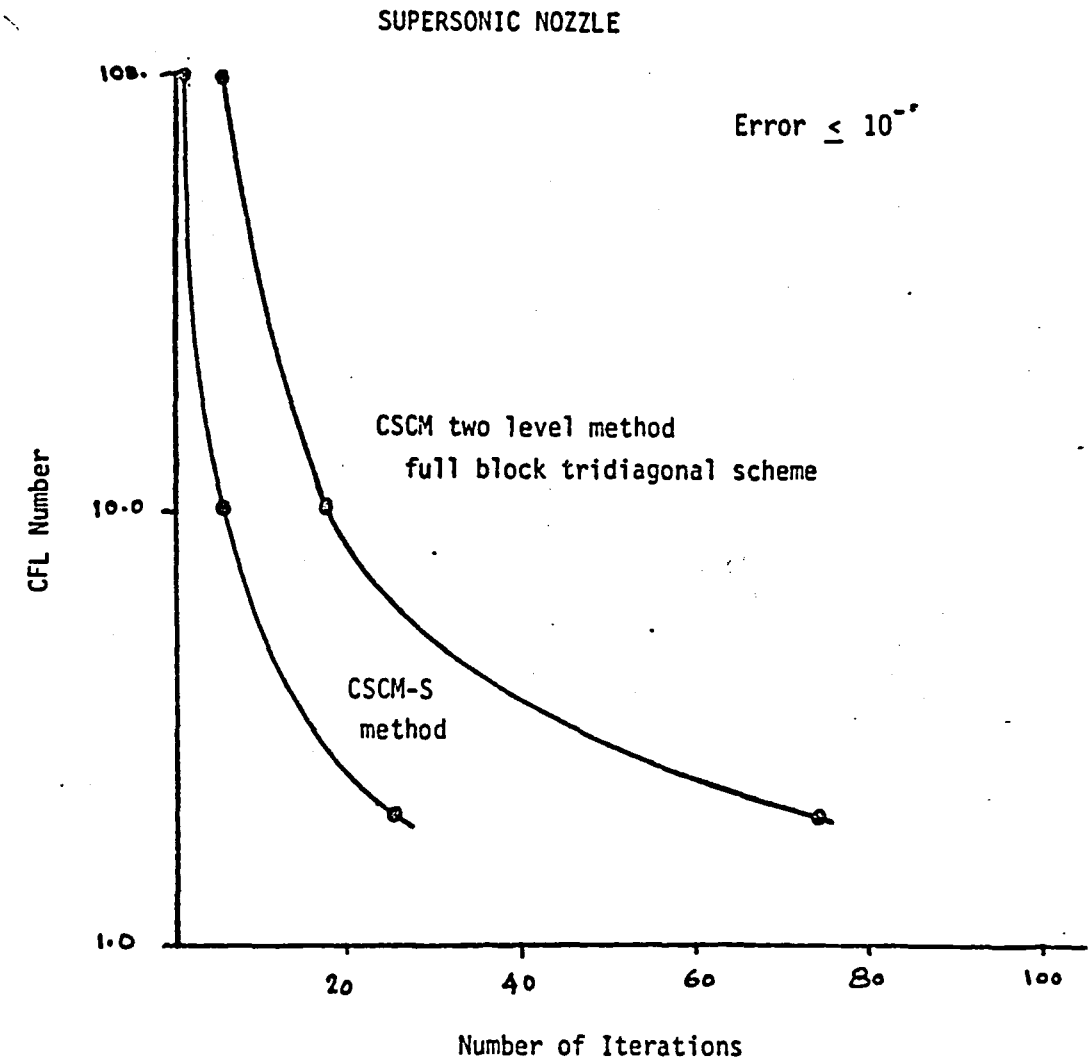

Figure 9. Convergence history comparison between the full block tridiagonal CSCM Solver and the CSCM-S method. The solutions have reached an RMS error less than 1×10^{-5}.

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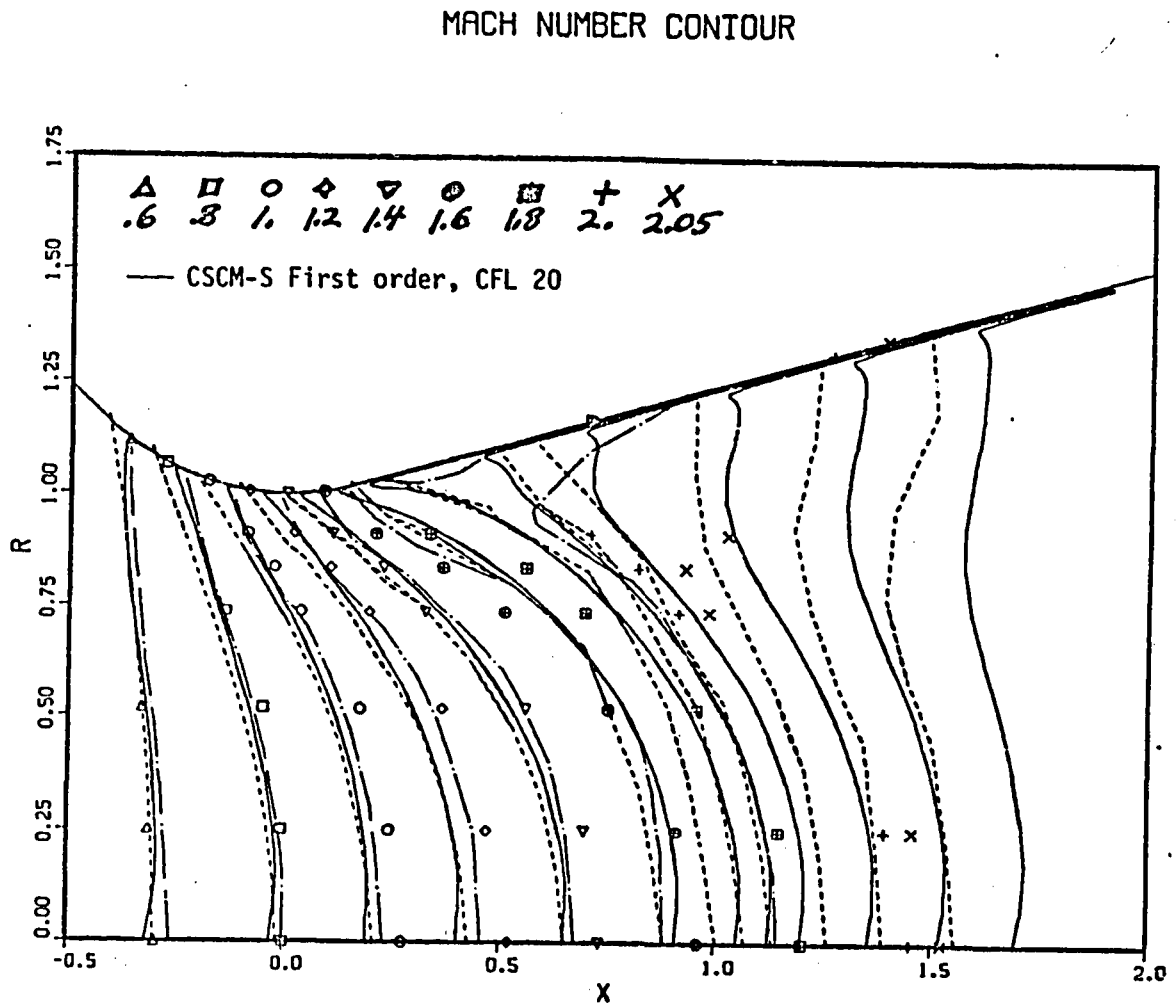

Figure 10. Mach number contours in vicinity of the throat for the axisymmetric transonic nozzle problem solved in ten global iterations. Other computed results of Cline (dashed line) and Prozan (chain dot) and experiment of Cuffel, et al (symbols) are discussed in references 18 and 24.

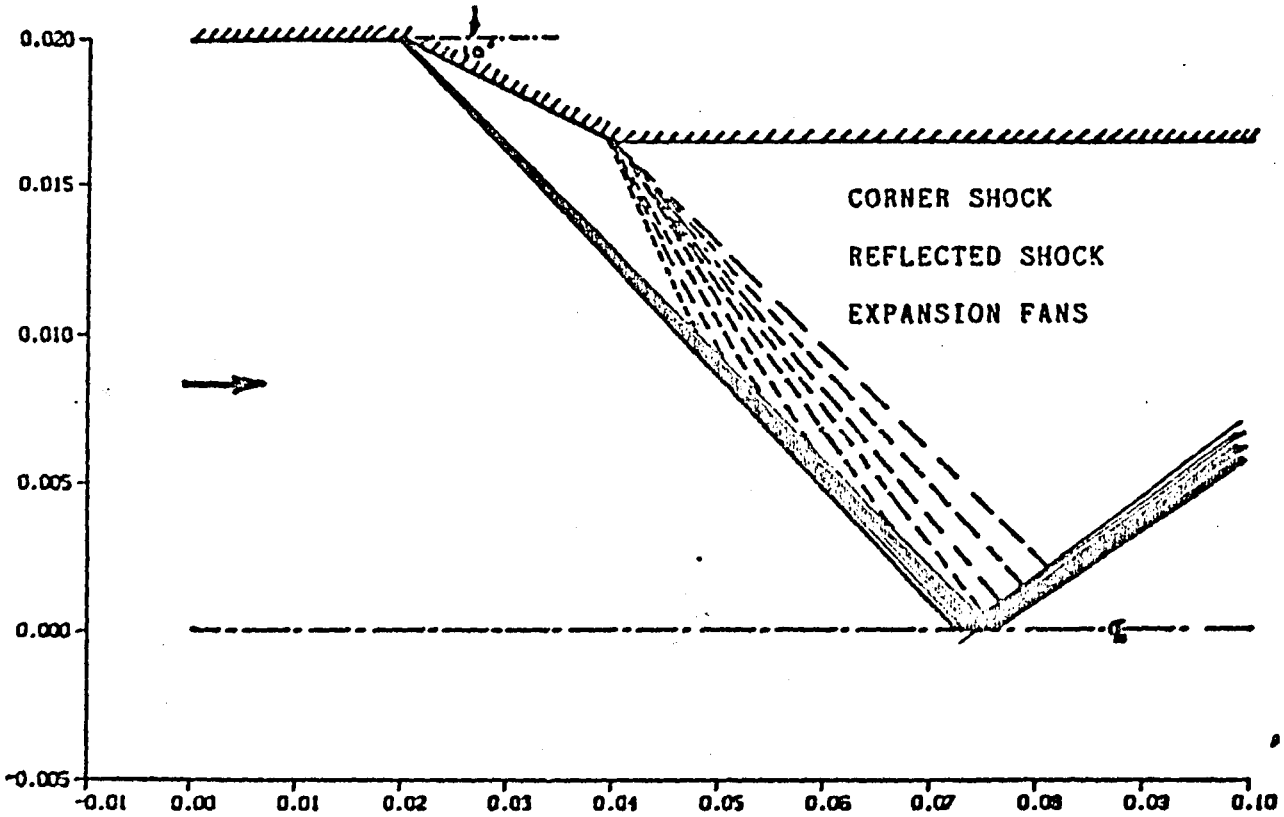

Figure 11. Schematics of the supersonic inlet flow problem.

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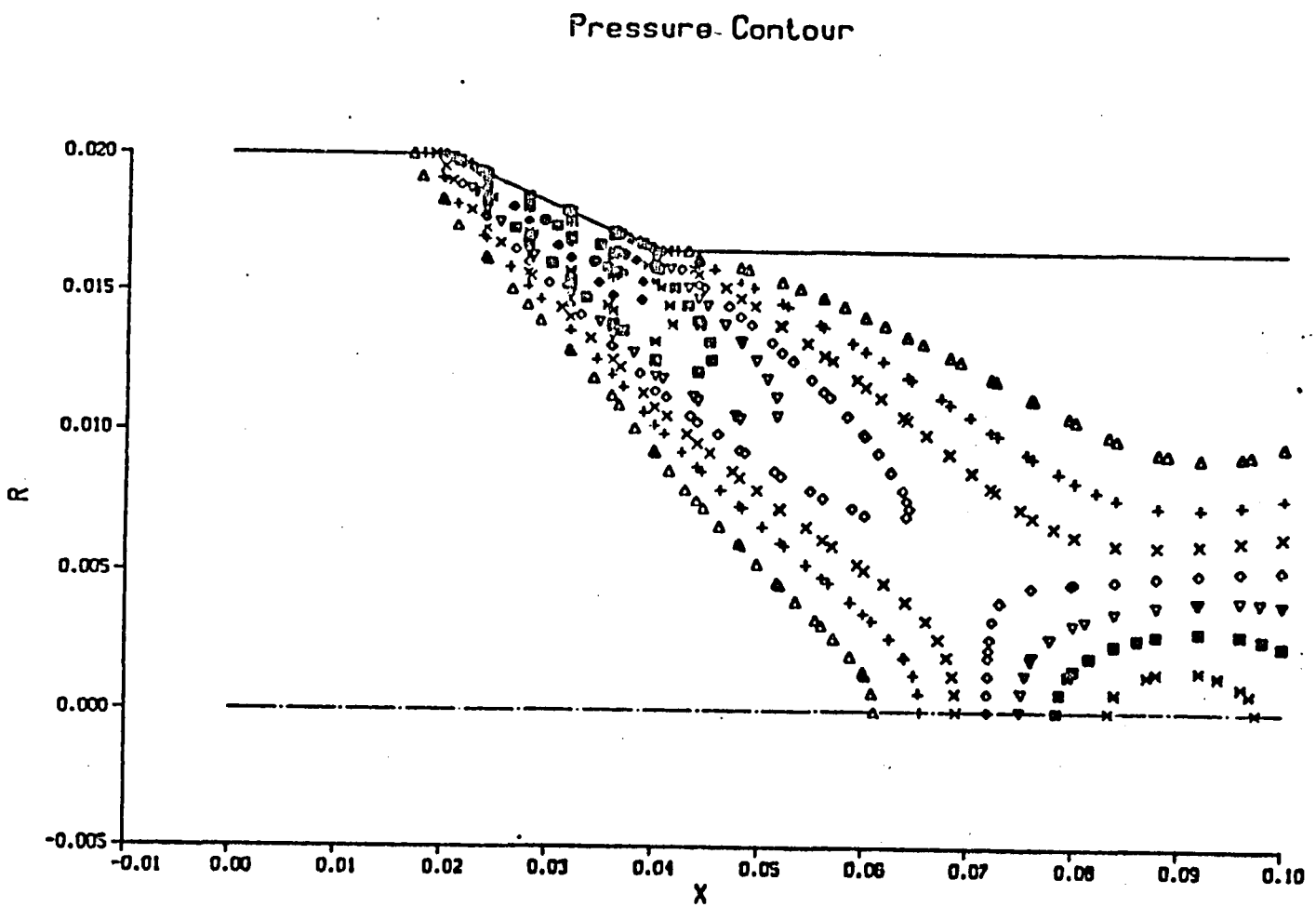

Figure 12. Pressure contours from the first order inviscid solution for the inlet problem after 10 forward sweeps with a 26 by 26 unstretched grid.

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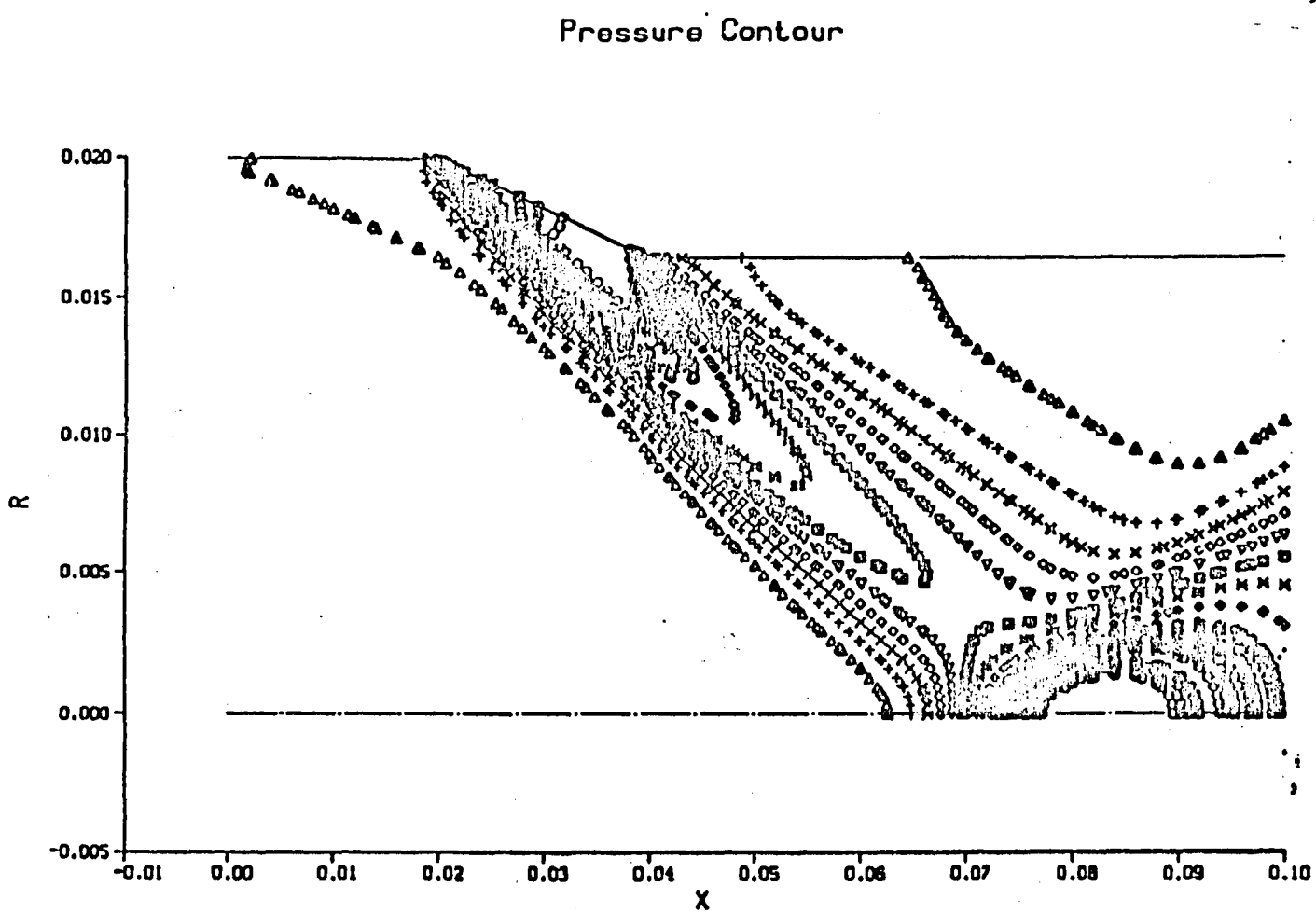

Figure 13. Pressure contours from the first order viscous solution for the inlet after 20 global sweeps with a 51 by 51 stretched mesh. Note the leading edge shock.

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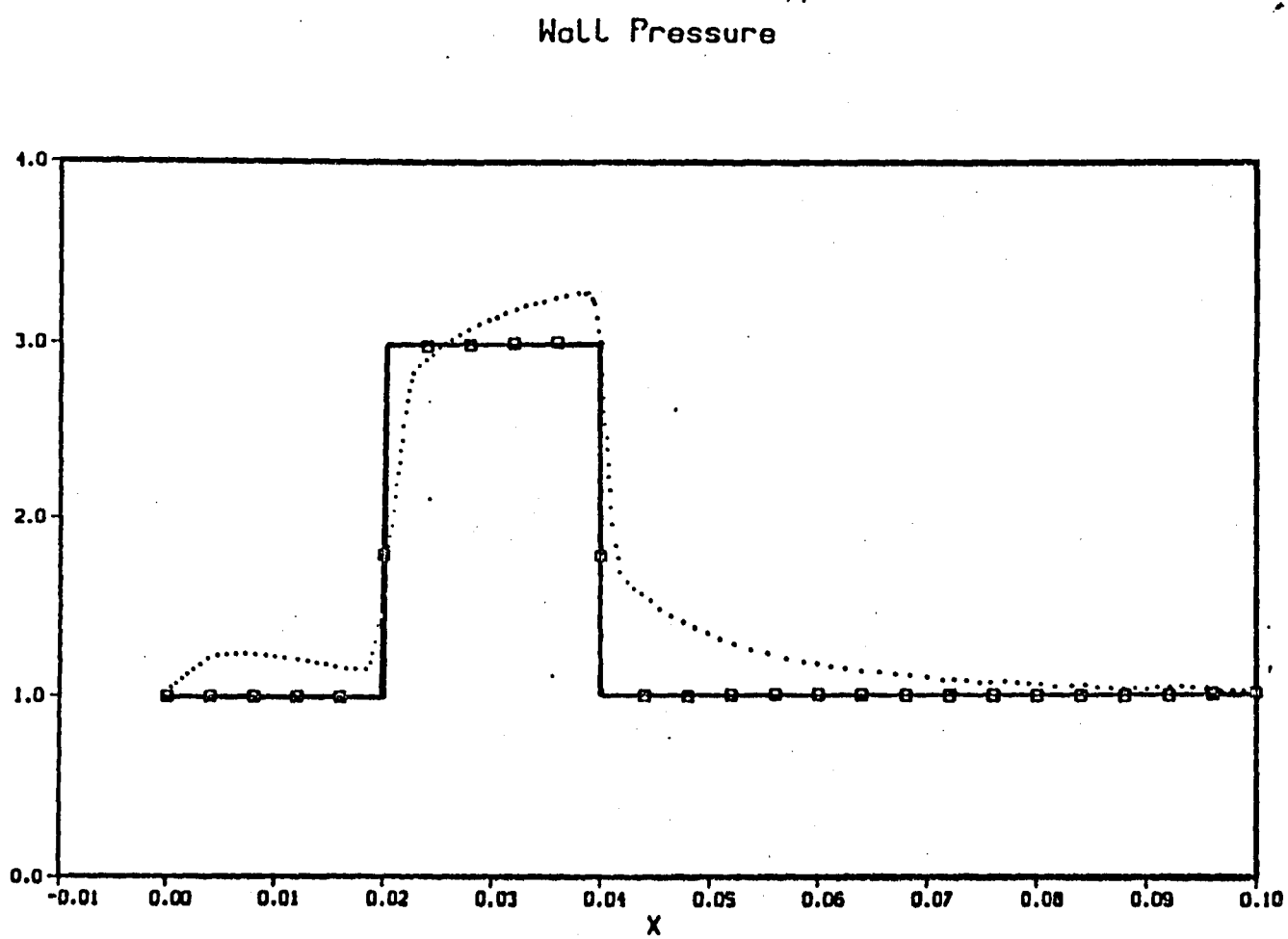

Figure 14. Wall pressure comparison. Line - exact inviscid solution, square - first order inviscid solution, dot - first order viscous solution

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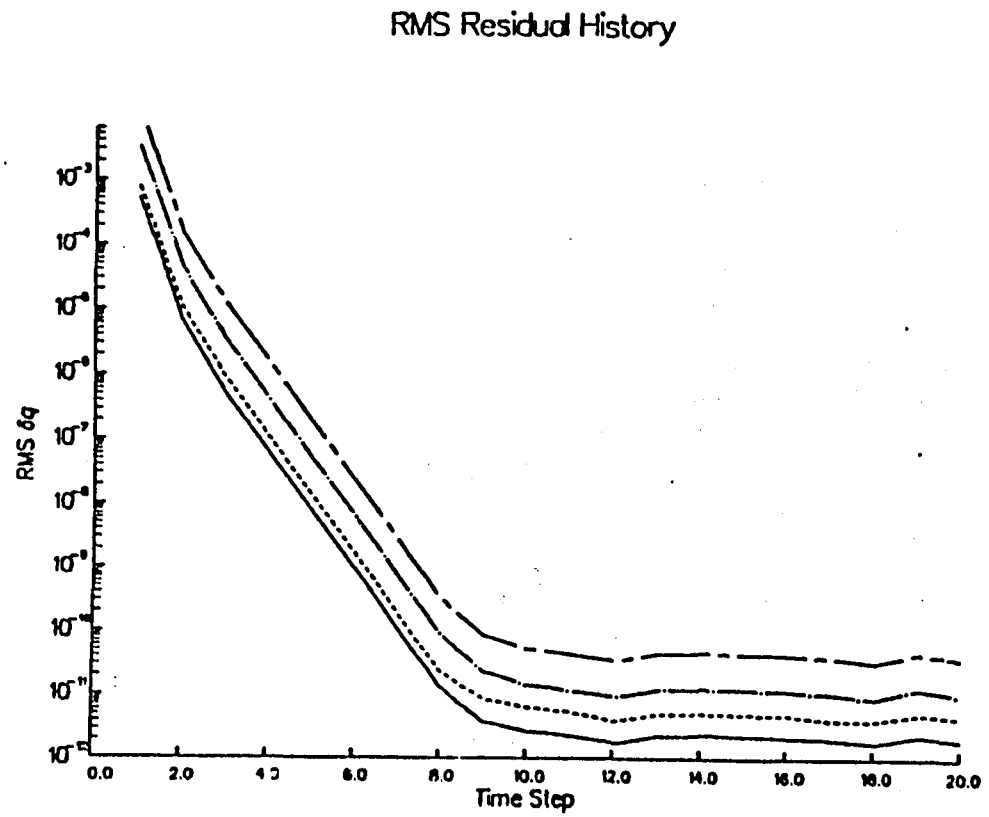

Figure 15. Convergence history of the RMS of the residuals for the inviscid first order inlet problem with 4 inner iteration per global sweep. Note at the end of first sweep the residue has reduced and the solution has converged for all practical purposes.

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