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Intrinsic Adaptive Mesh Techniques

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Final Report

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During the grant period, an alternating direction adaptive grid movement code was developed and, in addition, the thesis research of Gordon Erlebacher was directed on adaptive triangular meshes. Subsequently, the thesis research was successfully completed and Gordon Erlebacher joined the Computational Methods Branch at NASA Langley Research Center. The alternating direction code was also established on the NASA Langley computer system and is available for use there.

The development of the alternating direction code was based upon a theoretical study initiated under AFOSR sponsorship (AFOSR-82-0176) and continued under this NASA grant. The basic theory along with some modest applications were presented at the AIAA Computational Fluid Dynamics Conference in Danvers, MA in June, 1983 as AIAA paper 1937 on pages 339-348 of the proceedings [1]. The paper has also been accepted for publication in the AIAA Journal, pending some modest revisions.

In essence, grid points are moved on an "abstract surface" above physical space by means of alternating coordinate directions. The abstract surface is formed with the salient solution properties if they can be extracted by a priori physical reasoning; or otherwise, in the absence of such reasoning, by the use of error estimates in some chosen norm. Upon formulation, all important driving properties for adaptive purposes are consolidated into one object - the abstract surface. At a basic level, a uniform distribution of surface points is equivalent to gradient resolution. This arises from a projection back down into physical space. At a higher level, a more accurate view of the abstract surface is obtained when changes in surface direction are also resolved. The appropriate measure for direction changes is normal curvature. It is defined as the rate of change of surface tangent planes as a surface coordinate curve is transversed in uniform increments of arc length. Such variations in surface tangents clearly pick up the desired direciton changes which can become extremely tilted about the specific coordinate curve employed. Should only curve information be used, the tilting effect of the tangent planes would be discarded, resulting in a severe loss in the detection of surface direction changes. The curves used in the algorithm are coordinate curves on the surface. Taken one at a time, the points on each are redistributed in cycles which sweep through each direction in succession. This alternation of directions and splitting into one dimensional pieces is similar in spirit to that of ADI numerical schemes.

The alternating direction algorithm represents a substantial synthesis of the generalization beyond previous methods. In one-dimension, White [2] and Ablow and Schecter [3] used the arc length of the actual solution by deriving an auxiliary differential equation for movement which subsequently was simultaneously solved by implicit methods. The differential equations added complexity particularly in the case of Ablow and Schecter when curvature was used. Even in one-dimension, the use of an abstract surface and the explicit decoupling of movement is advantageous. Not only does the abstract surface allow a greater freedom of choice to simplify the movement process, but also it can be smoothed independently of the solution. In cases where the actual solution contains discontinuities, the computation of curvature would be greatly in error; thereby, rendering it useless as a clustering property. To smooth the actual solution in order to retrieve curvature clustering would be a cure which could kill the solution. By contrast, smoothing the abstract surface gives the desired movement while retaining the solution. In multidimensional problems, to my knowledge, there have been no prior studies that use a solution surface formulation. If the abstract surface, however, is trivially taken to correspond with physical space, then the alternating direction method reduces to that of Gnoffo [4] when gradient clustering is used as a property in the weight function. When further specialization is given by restricting movement to just one direction, the method of Dwyer et al. [5] is obtained where unlike Gnoffo, second derivatives of the solution are also used in the weight function. Since the second derivatives use only curve information, the critical tilting measurements of tangent planes are lost.

Returning to the use of general abstract solution generated surfaces, the alternating direction development served as the basis for the development of the adaptive triangular mesh algorithm with Gordon Erlebacher for his thesis[6]. In his thesis, Dr. Erlebacher formed an abstract surface for a plasma equilibrium study in a toroidally symmetric tokamak configuration. Over the tokamak cross section, he prescribed the abstract surface as a linear combination of toroidal current and inverse flux gradient squared. The current part resolved the plasma boundary which could be taken as a free boundary. The flux gradient part resolved the magnetic axis. By contrast, the solution itself, which for historical reasons was a first try, had failed to resolve any of the desired properties. The mesh movement on the abstract surface was accomplished with a geometrically constructed molecule which was used in a point iterative sense. Due to the use of a general connectivity triangular mesh, curvature was more conveniently represented with an approximation of mean curvature as a Laplacian suitably normalized with the magnitude of the gradient, rather than by the direct use of normal curvatures. The finite difference representation of gradient and Laplacian as exemplified by Fritts and Boris [7] was also improved upon and analyzed in the sense of local truncation errors.

In the alternating direction algorithm, the effect of normal curvature was examined through a sequence of test cases. In the first instance, a circular bump was examined with a conflicting rectalinear mesh topology. The use of normal curvature was seen there to cause coordinate curves to wrap around the base and top rim of the bump in correspondence with the regions where direction changes occurred. Upon projection, the curvature regions were added on to both sides of the gradient regions; thereby, giving a broader band of resolution.

Since the use of normal curvature caused coordinate curves to follow along surface folds as in the case of the bump, it seemed reasonable that it could also be used to automatically push coordinate curves into alignment with disturbances such as shock waves. As a second sequence of test cases, the alignment issue was examined with an artifically prescribed moving bow wave in front of a biconvex airfoil. The motion was from a vectical position to one which bent over and extended downstream from the airfoil. Grid movement with and without normal curvature was then examined for the same evolutionary abstract surface. In comparison between these two cases, grid alignment appeared only with curvature and there it appeared quite well; substantiating our claim.

When a nontrivial boundary in physical space must be resolved, a lifted form of it appears in the abstract solution generated surface as a boundary. Since normal curvature measures direction changes of surface tangent planes, changes within such planes are undetected and are those which belong to direction changes of curves <u>within</u> the surface. These latter changes, however, are measured by geodesic curvature. In particular, geodesic curvature is required to detect the above mentioned surface boundary. An initial examination was begun on the combined effect of both normal and geodesic curvature. The primary properties of each were observed in the case of a disturbance over an airfoil with an O-type grid. However, the correct balancing between the two has yet to be established in that case. With rectalinear topologies, by contrast, the balance is readily done.

REFERENCES

[1] Eiseman, P.R. (1983) AIAA CFD Conf., Danvers, MA, 339-348.

[2] White, A.B., (1982) SIAM J. Num. Anal. 19, 683-697.

[3] Ablow, C.M. and S. Schecter (1978) J. Comp. Phys. 27, 351-362.

[4] Gnoffo, P. (1982) Numerical Grid Generation, Ed. by J.F. Thompson, North Holland , 819-836.

[5] Dwyer, H.A., M.D. Smooke and R.J. Kee (1982) Numerical Grid Generation, Ed. by J.F. Thompson, North Holland, 339-356.

[6] G. Erlebacher, (1984) Columbia University Ph.D. Thesis.

[7] Fritts, M.J. and J.P. Boris (1979) J. Comp. Phys. 31, 173-215.