DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

COMPONENT MODE SYNTHESIS AND
LARGE DEFORMATION VIBRATIONS OF
COMPLEX STRUCTURES

By
Chau Mei, Principal Investigator

Final Report
for the period November 1, 1982 to October 31, 1983

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23685

Under
Research Grant NAG1-301
Joseph E. Malz, Technical Monitor
Structural Dynamics Branch (SDB)

February 1984
(NASA-CR-173338) COMPONENT MODE SYNTHESIS
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STRUCTURES Final Report, 1 Nov. 1982 - 31
Oct. 1983 (Old Dominion Univ., Norfolk, Va.) Uncias
16 p HC A02/HE A01
CSCL 20K 63/39 18509
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Prepared for the National Aeronautics and Space Administration Langley Research Center Hampton, Virginia 23665

Under Research Grant NAG1-301 Joseph E. Walz, Technical Monitor Structural Dynamics Branch (SDD)

Submitted by the Old Dominion University Research Foundation P. O. Box G369 Norfolk, Virginia

February 1984
FOREWORD

The material presented in this final report was performed under Grant No. NAG-1-301 entitled "Component Mode Synthesis and Large Deflection Vibrations of Complex Structures." This report summarizes the research results on modal synthesis and nonlinear forced vibrations of beams. The study was performed at the NASA/Langley Research Center during the period from November 1, 1982 to October 31, 1983. The work was monitored under the supervision of Joseph E. Walz and Dr. Jerrold M. Housner, Structural Dynamics Branch, Structures and Dynamics Division, NASA/Langley Research Center.
COMPONENT MODE SYNTHESIS AND LARGE DEFORMATION VIBRATIONS
OF COMPLEX STRUCTURES

By
Chuh Hei*

Part 1. Dynamic Analysis of Large Complex Structures
Using Component Mode Methods in NASTRAN

The complexity of aerospace structures has been increased enormously
during the past decade. A new challenge has confronted the structural
dynamists by the proposed space station to be in service by the year 1990.
It will be an evolving structure (ref. 1), and it will not be possible for
it to be ground tested because the final configuration may not be known when
the first component is put into space. The component mode method, there­
fore, may be employed for the dynamic analyses for determining frequency,
mode shape and transient response of such a large structure system in space.

The NASTRAN computer program, a structural analysis tool widely used in
the aerospace industry, contains a modal synthesis capability. Other than
the nine-bay truss structural problem presented in the NASTRAN demonstration
manual, little is publicly known about its capabilities. Preliminary
assessment of the accuracy of the NASTRAN modal synthesis analysis is accom­
plished by making a comparison of the NASTRAN modal synthesis with full
structure NASTRAN and nine other modal synthesis results (ref. 2) using the
nine-bay truss shown in Figure 1. Figures 2-4 show the relative accuracy
obtained using the various modal synthesis procedures. The limited study
indicates that the fixed-interface method in NASTRAN, fixed-interface

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Old Dominion University, Norfolk Virginia 23508.
Figure 1. Truss Model.
Figure 2. Comparison of Methods with Frequency Error of 0.1%.
Figure 3. Comparison of Methods with Frequency Error of 0.5%.
Figure 4. Comparison of Methods with Frequency Error of 5.0%
method of Hurty (ref. 3) and the equivalent Craig-Bampton method (ref. 4) produce the most accurate results for a given number of degrees-of-freedom. Slightly better accuracy was achieved by the procedures introduced by Benfield and Hruda (ref. 5). But these latter methods (ref. 5) suffer the disadvantage that the modes of one substructure are not independent of the modes of other substructures. More detailed results are documented in the progress report entitled, "NASTRAN Modal Synthesis Capability (ref. 6)."

A NASTRAN component mode transient response analysis was also performed on the free-free truss structure. A concentrated force was applied at grid point 42 of component B for 0.12 seconds and then removed. Tables 1 and 2 give the displacements and stresses and are compared with the full structure NASTRAN results. It demonstrates that excellent transient response can be obtained using component modal synthesis. Detail results, DMAP alters, solution sequences and computer CPU time can be found in reference 6.
### Table I. Transient Response and Percent Error in Displacement

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PART 2. Large Deflection Vibrations of Beams using Finite Element Methods

Since space structures will be mostly large, lightweight and flexible, large deflection analysis methods are urgently needed to study the vibratory responses of complex structures.

A finite element method has been developed for nonlinear vibrations of beam structures subjected to harmonic excitation. Longitudinal deformation and inertia are both included in the formulation. A harmonic force matrix \([h]\) was developed for a beam element for nonlinear oscillations under uniform harmonic excitation. Formulation of the harmonic force matrix is based on the mathematical basis (ref. 7) that the simple harmonic force \(P_0 \cos \omega t\) is simply the first order approximate solution of the simple elliptic forcing function \(BA \text{cn}(px,k)\). Also the well known perturbation solution

\[
\left( \frac{\omega}{\omega_L} \right)^2 = 1 + \frac{3}{4} \beta A^2 - \frac{P_0}{A}
\]

of a Duffing system \(q_{tt} + q + \beta q^3 = P_0 \cos \omega t\) is the first order approximate solution of the simple elliptic response \(q = A \text{cn}(px,k)\). Derivation of the element harmonic force and nonlinear stiffness matrices are given in detail in progress report entitled, "Finite Element Analysis of Nonlinear Free and Forced Vibrations of Beams (ref. 8)."

Table 3 shows the finite element free vibration results with and without considering effects of longitudinal deformation and inertia (ELDI). It clearly demonstrates the remarkable agreement between the present finite element with ELDI and Rayleigh-Ritz solutions.

Table 4 shows the frequency ratios for a simply supported and a clamped beam (\(L/R = 100\)) subjected to an uniform harmonic force of \(P_0 = 2.0\) (\(F_0 = 1322\) lb/in.) and 1.0 (3277 lb/in.), respectively. It demonstrates the
### TABLE III
Free Vibration Frequency Ratios $\omega/\omega_L$
for a Simply Supported Beam with Immovable Axial Ends

<table>
<thead>
<tr>
<th>$A$</th>
<th>Elliptic Function Solution (ref. 9)</th>
<th>Finite Element First Iteration</th>
<th>Finite Element Final Result</th>
<th>Rayleigh Ritz Solution (ref. 10)</th>
<th>Finite Element First Iteration</th>
<th>Finite Element Final Result</th>
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<td>1.0</td>
<td>1.0392</td>
<td>1.0395</td>
<td>1.0388</td>
<td>1.0607</td>
<td>1.0613</td>
<td>1.0613(3)</td>
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<td>2.0</td>
<td>1.3178</td>
<td>1.3203</td>
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<td>3.0</td>
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<td>4.0</td>
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<td>1.9761</td>
<td>1.9216</td>
<td>1.7309</td>
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<td>2.2544</td>
<td>2.0389</td>
<td>2.0393</td>
<td>2.0378(7)</td>
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a. Effects of longitudinal deformation and inertia.
b. Number in brackets denotes the number of iterations to get a converged solution.
TABLE IV

Forced Vibration Frequency Ratios $\omega / \omega_L$
for a Simply Supported and a Clamped Beam with Immovable Axial Ends

<table>
<thead>
<tr>
<th>$A$</th>
<th>Simple Elliptic Response (ref. 9)</th>
<th>Perturbation Solution</th>
<th>Finite Element First Iteration</th>
<th>Finite Element Final Result</th>
<th>Without HLDX</th>
<th>With HLDX</th>
</tr>
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<tr>
<td>$-$ 1.0</td>
<td>1.7852</td>
<td>1.7854</td>
<td>1.7852</td>
<td>1.7656</td>
<td>1.7682(3)$^b$</td>
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<td>$\pm$ 2.0</td>
<td>0.8672</td>
<td>0.8660</td>
<td>0.8621</td>
<td>0.8460</td>
<td>0.7160(4)</td>
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<tr>
<td>1.6557</td>
<td>1.6563</td>
<td>1.6512</td>
<td>1.5829(4)</td>
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<td>$\pm$ 3.0</td>
<td>1.4003</td>
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<td>$\pm$ 5.0</td>
<td>2.2606</td>
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Simply Supported Beam Subjected to $P_o = 2.0$ ($F_o = 1322$ lb/in.)

<table>
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<th>$A$</th>
<th>Simple Elliptic Response (ref. 9)</th>
<th>Perturbation Solution</th>
<th>Finite Element First Iteration</th>
<th>Finite Element Final Result</th>
<th>Without HLDX</th>
<th>With HLDX</th>
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<tr>
<td>$\pm$ 1.0</td>
<td>0.2118</td>
<td>0.2165</td>
<td>0.2096</td>
<td>0.2091</td>
<td>0.1772(3)</td>
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<td>1.4307</td>
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<td>1.4297</td>
<td>1.4251(3)</td>
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<tr>
<td>$\pm$ 2.0</td>
<td>0.8279</td>
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Clamped Beam Subjected to $P_o = 1.0$ ($F_o = 3277$ lb/in.)

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<th>Finite Element Final Result</th>
<th>Without HLDX</th>
<th>With HLDX</th>
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a. Effects of longitudinal deformation and inertia.
b. Number in brackets denotes the number of iterations to get a converged solution.
closeness between the earlier finite element results without ELDO, the simple elliptic response and the perturbation solution. The present finite element results indicate clearly that the ELDO are to reduce the nonlinearity.

Beams with various boundary conditions, including movable axial ends, are given in reference 8. Results obtained will be presented at the Second International Conference on Recent Advances in Structural Dynamics, to be held April 9-13, 1983, at the Institute of Sound Vibration Research, Southampton, England.
REFERENCES


End of Document