Static and Dynamic Structural-Sensitivity Derivative Calculations in the Finite-Element-Based Engineering Analysis Language (EAL) System

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INTRODUCTION

Researchers at the Langley Research Center are currently developing and applying the latest optimization capability to the multidisciplinary optimization of aircraft and spacecraft (ref. 1). A key part of this effort is to efficiently calculate structural-sensitivity derivatives which quantify the change in a behavior variable with respect to a structural parameter and which are used in the following applications: (1) to act as input to optimization algorithms; (2) to enhance response analysis programs which aid engineering judgment leading to design modifications; (3) to guide the modification of a finite-element model to better correlate analytical and test results; and (4) to approximate structural response by using Taylor series expansions.

The most basic and straightforward approach to sensitivity analysis is the finite-difference method; however, it is computationally slow and its accuracy must be verified by convergence checks. Analytical methods for calculating exact derivatives from the governing equations have been developed (refs. 2 to 10). These methods greatly reduce the computational effort but are somewhat cumbersome to implement for bending-type elements (ref. 4). Most recently, a semianalytical method for calculating derivatives has been developed which has the generality and programing ease of the finite-difference method while retaining much of the efficiency of the analytical method (ref. 11).

The purpose of this paper is to describe the implementation and verification of the aforementioned methods for computing sensitivity derivatives in the structural finite-element computer program currently used in multidisciplinary optimization studies. The program, denoted the Engineering Analysis Language (EAL) System (ref. 12), is similar to its predecessor, SPAR (ref. 13). It is a modular system of individual analysis processors which may be used in any appropriate sequence to perform a variety of analyses. The EAL System differs from SPAR by providing FORTRAN-like commands which permit branching, testing data, looping, and calling runstreams (similar to calling FORTRAN subroutines). These capabilities permit the implementation of sensitivity calculations without changing the basic program or requiring user-written subroutines in separate programs (as in ref. 4). Further, use of the EAL System avoids the need for extensive operating system control commands as used previously (ref. 4) and thus assures machine independence of the resulting system.

This paper draws on results from references 4, 8, and 11 for the basic methodology and presents EAL input runstreams which calculate derivatives of displacements, stresses, vibration frequencies and mode shapes, and buckling loads and mode shapes with respect to structural variables in the model. The variables are sectional properties including thicknesses, cross-sectional areas, and moments of inertia. Results are presented and comparisons are made among analytical, semianalytical, and finite-difference methods for the following four structural configurations: a swept wing, a box beam, a stiffened cylinder with a cutout, and a space radiometer-antenna truss.
SYMBOLS AND ABBREVIATIONS

A  cross-sectional area
E  modulus of elasticity
{\mathbf{F}}  pseudo load vector
\mathbf{[F]}  matrix of pseudo load vectors (eq. (21))
f  applied load
{\mathbf{f}}  applied load vector
\mathbf{[G]}  matrix which relates stresses and temperatures (eq. (26))
\mathbf{[I]}  identity matrix
I_1, I_2  moments of inertia
J_o  polar moment of inertia
\mathbf{[K]}  stiffness matrix
\mathbf{[K_g]}  geometric stiffness matrix
\mathbf{[M]}  mass matrix
P_j  structural definition parameter
{\mathbf{Q}}  particular solution of equation (35)
r  independent design variable for tube element (eq. (40))
\mathbf{r_i, r_o}  inner and outer radii of tube element
\mathbf{[S]}  stress-displacement matrix
T  temperature
T_o  stress-free temperature
t_i  element thickness
u, v, w  displacements in x, y, and z directions, respectively
{\mathbf{u}}  displacement vector
{\mathbf{v}}  vector of independent design variables
{\mathbf{v}}  vector of dependent design variables
x, y, z  Cartesian coordinates
{\alpha}  vector of linear thermal expansion coefficients
\( \{\lambda, \omega^2\} \) vectors of buckling loads and vibration frequencies, respectively

\( \nu \) Poisson's ratio

\( [\Phi], [\phi] \) matrices of buckling and vibration mode shapes, respectively

\( \rho \) density

\( \{\sigma\} \) stress vector

Superscripts:

\( n \) new (perturbed)

\( o \) original (unperturbed)

Abbreviations:

NDDV number of dependent design variables

NIDV number of independent design variables (NDV in computer listing)

NPOL degree of polynominal used in linking (eq. (1))

NSDP number of structural definition parameters

DESCRIPTION OF THE ENGINEERING ANALYSIS LANGUAGE (EAL) SYSTEM

The EAL finite-element analysis system (ref. 12) evolved from the SPAR computer program (ref. 13). As indicated in figure 1(a), the EAL System contains individual processors which communicate through a data base consisting of one or more libraries of data sets. The data sets typically contain data describing the finite-element model of the structure (e.g., node point coordinates and material properties) as well as response information such as displacements and stresses. All data base communications between processors are in terms of data sets. A set of data-handling utilities transfers data between the processors in central memory of the computer and the data base on auxiliary storage (ref. 14). Contents of individual data sets for SPAR are compiled and listed in reference 15. (In some instances these data sets may be slightly different from corresponding EAL data sets.)

A list of the EAL processors is shown in the uppermost block of figure 1(a). The functions of the various processors are described in table 1. The processors may be executed in any appropriate sequence. A sequence of processor executions is denoted a runstream and may be defined and stored in the data base as a runstream data set. Runstreams and runstream data sets may be nested within an input file.

The EAL system differs from SPAR in its use of a set of flexible FORTRAN-like statements, denoted executive control system (ECS) commands, which allow branching, testing data, looping, and calling runstreams (similar to calling FORTRAN subroutines). The ECS commands are always preceded by an asterisk (*) and are used to execute processors; for example, the ECS command to execute the TAB processor is *XQT TAB. A list of the ECS commands and their functions is given in table 8-1 of reference 12. The ECS commands are also used to test variables (denoted registers). Registers may be input data, output data, or simple variables defined and manipulated
by the EAL user. If the value of a register is regarded as a variable which changes
during an EAL analysis, the current value of the register is defined as a surrogate.
A surrogate is always enclosed in quotation marks. Registers and surrogates
are defined and manipulated by the use of register action commands (RAC). A RAC is
always preceded by an exclamation point (!) in the EAL input. For example,
!A=ABS("STR") assigns the absolute value of the surrogate "STR" to register A. A
complete list of the register action commands is given in table 7.3-1 of
reference 12.

The makeup of a sample EAL input file is illustrated in figure 1(b). The file
begins with an ECS command to execute processor TAB using appropriate input data
following the command; second, there is a call to execute runstream data set CHNG DV;
third, there is execution of processor ELD using appropriate input data; and finally,
there is execution of processors of K and INV, neither of which require user input.

The makeup of the sample runstream data set CHNG DV is shown in figure 1(c).
The function of this runstream is to loop over the membrane elements in the model
(looping is controlled by the JLZ command), extract the membrane thicknesses from
data set DV DFN and store them in register DV (using the DS register action command),
and finally update that part of the input file in which the membrane thicknesses are
defined. The updated portion of the file follows the SA identifier and contains the
membrane element numbers ("NI") and the corresponding thicknesses ("DV"). Following
the satisfaction of the loop and the arrival at LABEL 2, control returns to the calling
runstream by means of the RETURN command.

SCOPE OF CAPABILITY FOR CALCULATING STRUCTURAL-SENSITIVITY DERIVATIVES

Methods Used and Overall Capability

In the present work, structural behavior quantities include displacements \{u\},
stresses \{\sigma\}, buckling loads and mode shapes \{\lambda\} and \{\varphi\}, and vibration fre-
quencies and mode shapes \{\omega^2\} and \{\phi\}. Structural variables include element
thicknesses, cross-sectional areas, and moments of inertia. The following methods
for calculating derivatives of structural behavior quantities have been implemented:
(1) an analytical method, (2) a semianalytical method (also called the indirect
method in refs. 11 and 16), and (3) a finite-difference method (also called the
direct method in ref. 11). A list of applicable EAL finite elements is given in
table 2. The analytical method was not implemented for the elements which include
bending deformation (E32, E33, E42, and E43) because of the algebraic complexity
(discussed in the following section).

Design-Variable Definition

Structural modifications are specified as changes to certain structural quanti-
ties called design variables, which are related to section properties or mass proper-
tries of the finite-element model. A structural definition parameter (ref. 4) is
defined to be a parameter which has a linear contribution to the stiffness matrix or
the mass matrix, or both, of individual finite elements in the structural model. The
design variables can be identical to or have a one-to-one relationship to the struc-
tural definition parameters. For example, the cross-sectional area of a rod element
is a structural definition parameter, and the areas of several elements in the structure
could be equal to a single design variable. In some instances, there is a non-
linear relationship between the structural definition parameters and the design
variables, for example, when the moment of inertia per unit width of a plate
\( I_1 = \frac{t^3}{12} \) is a structural definition parameter and the plate thickness \( t \) is a
design variable. Finally, in some optimization techniques (e.g., ref. 3), design-
variable linking is used to reduce the number of independent design variables. In
this instance independent design variables can be linearly or nonlinearly related to
the dependent design variables, which in turn can be linearly or nonlinearly related
to the structural definition parameters. Chain-rule partial differentiation is then
used to compute the required derivatives (ref. 4).

**Design-Variable Linking**

When linking is used, the number of dependent design variables (NDDV) is larger
than the number of independent design variables (NIDV), and the two are related via a
mathematical relationship which is often linear (ref. 17). In the present work the
following nonlinear design-variable linking algorithm is used:

\[
v_i = C_{oi} + \sum_{m=1}^{NPOL} \sum_{k=1}^{NIDV} C_{mk,i} V_k^m \quad (i = 1, 2, ..., NDDV) \tag{1}
\]

where \( NPOL \) is the degree of the polynomial expression for each dependent design vari-
able, \( C_{mk,i} \) is the ith linking coefficient, \( C_{oi} \) is the ith additive constant,
\( v_i \) is the ith dependent design variable, and \( V_k \) is the independent design
variable. With matrix notation, equation (1) may be written

\[
\{v\} = \{C\}_o + \sum_{m=1}^{NPOL} \{C\}_m \{v\}^m \tag{2}
\]

**STIFFNESS AND MASS MATRIX DERIVATIVES**

The derivatives of the stiffness matrix \([K]\) and of the mass matrix \([M]\) with
respect to the independent design variable \( V_k \) are needed in the subsequent calcu-
lations of sensitivity derivatives. Two methods are used for calculating these
matrix derivatives: analytical and finite differences.

**Analytical Method**

When the stiffness or the mass matrix of a finite element is linearly related to
a design variable, the analytical derivative of the matrix is obtained by simply set-
ting the design variable to unity and calculating the corresponding matrix. When a
nonlinear relationship exists between the stiffness or the mass matrix and the design
variables or when design-variable linking is used, or both, the situation is more
complicated and chain-rule partial differentiation is needed to formulate generalized
analytical expressions. For a beam element the stiffness matrix is linearly related
to four structural definition parameters (\( A, I_1, I_2, \) and \( J_0 \)). Analytical
expressions may be written to relate the design variables to these structural
definition parameters, and the derivative of the stiffness matrix can be calculated
by using

$$\frac{\partial \{K\}}{\partial \{v\}_i} = \frac{\partial \{K\}}{\partial A} \frac{\partial A}{\partial \{v\}_i} + \frac{\partial \{K\}}{\partial I_1} \frac{\partial I_1}{\partial \{v\}_i} + \frac{\partial \{K\}}{\partial I_2} \frac{\partial I_2}{\partial \{v\}_i} + \frac{\partial \{K\}}{\partial J_o} \frac{\partial J_o}{\partial \{v\}_i}$$

(3)

or

$$\frac{\partial \{K\}}{\partial \{v\}_i} = \sum_{j=1}^{NSDP} \frac{\partial \{K\}}{\partial P_j} \frac{\partial P_j}{\partial \{v\}_i}$$

(4)

where NSDP is the number of structural definition parameters, \(\frac{\partial \{K\}}{\partial P_j}\) is computed
by substituting unity for \(P_j\), and \(\frac{\partial P_j}{\partial \{v\}_i}\) is calculated by differentiating the
analytical expression relating \(P_j\) to \(\{v\}_i\). The analogous expression for mass matrix
derivatives is

$$\frac{\partial \{M\}}{\partial \{v\}_i} = \sum_{j=1}^{NSDP} \frac{\partial \{M\}}{\partial P_j} \frac{\partial P_j}{\partial \{v\}_i}$$

(5)

If design-variable linking is used, the independent and dependent design vari-
ables are related by equation (1) and the expressions for derivatives with respect to
the independent design variables are

$$\frac{\partial \{K\}}{\partial \{v\}_k} = \sum_{i=1}^{NDDV} \sum_{j=1}^{NSDP} \frac{\partial \{K\}}{\partial P_j} \frac{\partial P_j}{\partial \{v\}_i} \frac{\partial \{v\}_i}{\partial \{v\}_k}$$

(6)

and

$$\frac{\partial \{M\}}{\partial \{v\}_k} = \sum_{i=1}^{NDDV} \sum_{j=1}^{NSDP} \frac{\partial \{M\}}{\partial P_j} \frac{\partial P_j}{\partial \{v\}_i} \frac{\partial \{v\}_i}{\partial \{v\}_k}$$

(7)

Example of Analytical Derivative of Stiffness Matrix

To crystallize the ideas and terms involved, differentiation of the stiffness
matrix for a beam modeled by three E23 channel-section elements (fig. 2) is per-
formed. The structural definition parameters consist of the cross-sectional area \(A\)
and the moments of inertia \(I_1\), \(I_2\), and \(J_o\). The dependent design variables are the
dimensions of the channel \(B_1\), \(B_2\), and \(t\). There are three independent design
variables, each associated with an element and denoted \( V_1 \), \( V_2 \), and \( V_3 \). The dependent design variables are linked to the independent design variables by a 9 \( \times \) 3 matrix \([C]\). Thus, for the structure in figure 2,

\[
\{p\}^T = \begin{bmatrix} A & I_1 & I_2 & J_o \end{bmatrix}
\]

\( (8)\)

\[
\{v\}^T = \begin{bmatrix} B_1 P_1 t & B_1 P_1 t & B_1 P_1 t \end{bmatrix}
\]

Element 1 Element 2 Element 3

\( (9)\)

\[
\{v\}^T = \begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix}
\]

\( (10)\)

\[
\{v\} = [C]\{v\}
\]

\( (11)\)

The structural definition parameters are related to the dependent design variables by equations given in reference 4, as follows:

\[
P_1 = A = (2B_1 + B_2)t
\]

\[
P_2 = I_1 = \frac{B_1(B_2 + 2t)^3}{12} - \frac{(B_1 - t)B_2^3}{12}
\]

\[
P_3 = I_2 = \frac{2tB_1^3}{12} + 2tB_1\left(\frac{B_1^2}{2} - c\right)^2 + \frac{B_2^3}{12} + B_2t\left(c - \frac{t}{2}\right)^2
\]

\( (12)\)

\[
P_4 = J_o = \frac{1}{3}(2B_1 + B_2)t^3
\]

\[
c = \left(\frac{B_1^2t}{2} + \frac{B_2^2t^2}{2}\right)
\]

\[
\frac{\partial[K]}{\partial V_k} = \sum_{i=1}^{9} \sum_{j=1}^{4} \frac{\partial[K]}{\partial P_j} \frac{\partial P_j}{\partial v_i} \frac{\partial v_i}{\partial V_k}
\]

\( (13)\)
where

\[ \frac{\partial [K]}{\partial p_j} = [K]_{p_j=1} \]  
\[ (j = 1, 2, 3, 4) \]  

\[ \text{and} \]

\[ \frac{\partial p_j}{\partial v_i} \]

is obtained from differentiating equations (12) and noting equation (9), and

\[ \frac{\partial v_i}{\partial v_k} \]

is obtained from differentiating equation (11).

Finite-Difference Method

The stiffness and mass matrix derivatives can also be computed by finite differences, as follows:

\[ \frac{\partial [K]}{\partial v_k} = \frac{\Delta [K]}{\Delta v_k} = \frac{[K]_k^n - [K]_k^o}{v_k^n - v_k^o} \]  
\[ (15) \]

\[ \frac{\partial [M]}{\partial v_k} = \frac{\Delta [M]}{\Delta v_k} = \frac{[M]_k^n - [M]_k^o}{v_k^n - v_k^o} \]  
\[ (16) \]

where \([K]_k^n\) and \([M]_k^n\) are the perturbed stiffness and mass matrices (formed by incrementing the kth independent design variable), \([K]_k^o\) and \([M]_k^o\) are the original (unperturbed) stiffness and mass matrices, and \(v_k^n\) and \(v_k^o\) are the perturbed and original values of the kth independent design variable. Since this method operates with the independent design variables directly, it does not require looping over the dependent design variables or the structural parameters indicated by equations (6) and (7).

Finite Differences With LSK Processor

In some instances the finite-difference calculation of the stiffness matrix may be improved by use of the LSK processor (table 1). This processor selects appropriate combinations of elements (submatrices) of the global stiffness matrix \([K]\) in a sparse matrix form (called LS-format) and performs the necessary finite differencing. For this case the differencing must be with respect to the dependent design variables, as follows:

\[ \frac{\partial [K]}{\partial v_i} = \frac{\Delta [K]}{\Delta v_i} = \frac{[K]_i^n - [K]_i^o}{v_i^n - v_i^o} \]  
\[ (17) \]
If design-variable linking is used (eq. (1)),

$$\frac{\partial[K]}{\partial V_k} = \sum_{i=1}^{NDDV} \frac{\partial[K]}{\partial V_i} \frac{\partial V_i}{\partial V_k}$$  \hspace{1cm} (18)

where $\frac{\partial V_i}{\partial V_k}$ is obtained by differentiating equation (1). Using the LSK processor to operate on selected portions of the global stiffness matrix is more efficient than summing and multiplying stiffness matrices having the dimensions of the entire structure. Hence, depending on the ratio of the number of dependent design variables (NDDV) to the number of independent design variables (NIDV), the finite-difference methods of equation (15) or of equations (17) and (18) may be more efficient than the analytical method of equation (6). A large ratio of dependent to independent design variables and a large number of structural parameters needed to specify a particular element favor the use of one of the finite-difference methods for calculating $\partial[K]/\partial V_k$.

DISPLACEMENT DERIVATIVES

Analytical Method

In the analytical method, derivatives are computed from the governing finite-element equations. For static finite-element structural analysis, the equilibrium equation is

$$[K]{u} = {f}$$  \hspace{1cm} (19)

where $\{u\}$ is the vector of nodal displacements and $\{f\}$ is the applied load vector. Differentiating equation (19) with respect to the independent design variable $V_k$ gives

$$[K]\frac{\partial\{u\}}{\partial V_k} = \frac{\partial\{f\}}{\partial V_k} - \frac{\partial [K]\{u\}}{\partial V_k} = \{F\}_k$$  \hspace{1cm} (20)

or

$$[K]\left[\frac{\partial u}{\partial V}\right] = \{F\}$$  \hspace{1cm} (21)

where $\{F\}_k$ is the kth pseudo applied load vector and $[\partial u/\partial V]$ and $\{F\}$ are, respectively, matrices for which the columns are the displacement derivatives and pseudo load vectors which correspond to individual independent design variables. If
the applied load vector \{f\} is not a function of the design variables, then the term \(\frac{\partial \{f\}}{\partial V_k}\) is equal to zero.\(^1\) For this case the pseudo load vector becomes

\[
\{F\}_k = - \frac{\partial \{u\}}{\partial V_k}
\]  

(22)

The analytical method consists of solving equation (20) for \(\frac{\partial \{u\}}{\partial V_k}\) using analytically computed derivatives for \(\frac{\partial [K]}{\partial V_k}\) (eq. (6)). Equation (20) is solved by the same solution algorithm used for solving equation (19), taking advantage of the fact that the factored form of [K] is available from the solution of equation (19). Substitution of equation (6) into equation (20) leads to

\[
[K] \frac{\partial \{u\}}{\partial V_k} = \frac{\partial \{f\}}{\partial V_k} - \sum_{i=1}^{NDDV} \sum_{j=1}^{NSDP} \frac{\partial [K]}{\partial P_j} \frac{\partial P_j}{\partial V_i} \frac{\partial \{u\}}{\partial V_k}
\]  

(23)

**Semianalytical Method**

The semianalytical method contains the analytical expression for the displacement derivatives (eq. (20)) with finite-difference derivatives of the stiffness matrix (eq. (15) or eqs. (17) and (18)). If equations (15) and (20) are used, the following expression results:

\[
[K] \frac{\partial \{u\}}{\partial V_k} = \frac{\partial \{f\}}{\partial V_k} - \left( \frac{[K]^n_k - [K]^o_k}{V^n_k - V^o_k} \right) \{u\} = \{F\}_k
\]  

(24)

where \(\frac{\partial \{f\}}{\partial V_k}\) can be calculated analytically or by finite differences and the term in parentheses may be obtained by finite differences from equation (15) or by the LSK processor (eqs. (17) and (18)). Derivatives are calculated by creating individual pseudo load vectors \{F\}_k and solving equation (24) using the factored [K] matrix from equation (19). This formulation is a significant simplification of the analytical approach of reference 4 in that it avoids the need for element-dependent manipulations to handle bending-type elements. It also eliminates the complications in the analytical method when design-variable linking is included (eqs. (1) and (23)).

**Finite-Difference Method**

The simplest method to implement, but the most time consuming computationally (especially for large finite-element models), is the finite-difference method. In addition, the accuracy of the finite-difference method depends upon the perturbation

\(^1\)One practical case when this term is not zero is when thermal loads are included.
step size. In this method, the original structure is analyzed; the structure is then modified by perturbing a design variable and reanalyzed. The displacement derivative is

$$\frac{\partial \{u\}}{\partial V_k} = \frac{\{u\}^n_k - \{u\}^0}{V^n_k - V^0_k}$$

(25)

where \(\{u\}^n_k\) is the displacement vector due to a perturbation in the kth independent design variable and \(\{u\}^0\) is the displacement vector of the original structure.

**STRESS DERIVATIVES**

**Analytical Method**

Element stresses \(\{\sigma\}\) are related to joint displacements and element temperatures by the equation

$$\{\sigma\} = [S]\{u\} - [G]\{\alpha\}(T - T^o)$$

(26)

where \([S]\) is the stress-displacement matrix, \([G]\) is the stress-temperature matrix, \(\{\alpha\}\) is the vector of thermal expansion coefficients, \(T^o\) is the stress-free element temperature, and \(T\) is the actual element temperature. Upon differentiation with respect to the kth independent design variable, the general expression for stress derivatives is

$$\frac{\partial \{\sigma\}}{\partial V_k} = [S] \frac{\delta \{u\}}{\partial V_k} + \sum_{i=1}^{NDDV} \sum_{j=1}^{NSDP} \left[ \frac{\partial [S]}{\partial \beta_j} \right] \frac{\partial \beta_j}{\partial V_i} \frac{\partial \{u\}}{\partial V_k}$$

(27)

Equation (27) neglects any dependence of temperature on the design variables.

For rods (E23), membranes (E31 and E41), and shear panels (E44), the matrix \([S]\) is independent of the section-property design variables; thus, the second term on the right side of equation (27) is zero. For bending-type elements, beams (E21) and plates (E32, E33, E42, and E43), the stress-displacement matrices are dependent on the element cross-sectional geometry, and this dependence must be included in the stress derivative calculations. This was done in reference 4 for the channel-section beam in figure 2 by using specific, user-defined subroutines.

**Semianalytical Method**

In this method (reported in ref. 11), the perturbed displacement vector \(\{u\}^n_k\) is approximated by a truncated Taylor series expansion as follows:

$$\{u\}^n_k = \{u\}^0 + \frac{\delta \{u\}}{\partial V_k}(V^n_k - V^0_k)$$

(28)
where $\partial \{u\} / \partial V_k$ is calculated with equation (24). The perturbed stress vector is given by

$$\{\sigma\}^n_k = \{S\}_{k}^n \{u\}^n_k - \{G\}\{\alpha\}\{G\}\{\alpha\}(T - T_o)$$

(29)

and the derivative is

$$\frac{\partial \{\sigma\}}{\partial V_k} = \frac{\{\sigma\}^n_k - \{\sigma\}^0}{V_k^n - V_k^0}$$

(30)

where

$$\{\sigma\}^0 = \{S\}\{u\}^0 - \{G\}\{\alpha\}(T - T_o)$$

(31)

Finite-Difference Method

The stress derivatives are calculated in a manner analogous to the displacement derivatives. The structure is modified and reanalyzed and the derivatives are approximated as

$$\frac{\partial \{\sigma\}}{\partial V_k} = \frac{\{\sigma\}^n_k - \{\sigma\}^0}{V_k^n - V_k^0}$$

(32)

where $\{\sigma\}^n_k$ is the stress vector due to a perturbation in the $k$th independent design variable calculated with equation (29) and $\{\sigma\}^0$ is the original stress vector.

DERIVATIVES OF VIBRATION AND BUCKLING EIGENVALUES AND EIGENVECTORS

Methodologies for calculating derivatives of eigenvalues and eigenvectors have been presented in references 4, 6, 8, and 9. The methods which have been implemented in EAL are an analytical method (ref. 8) and the finite-difference method.

Analytical Method

This method is implemented in EAL using runstreams based on the development in reference 4 and the theory in reference 8. The matrix equation for free vibrations is

$$\left( [K] - \omega_j^2 [M] \right) \{\phi\}_j = 0$$

(33)
where $\{\phi\}_j$ and $\omega_j$ are, respectively, the vector of the mode shape and the scalar frequency corresponding to vibration mode $j$, $[M]$ is the mass matrix, and $[K]$ is the stiffness matrix. Normalizing the modes with respect to the mass matrix gives

$$\{\phi\}^T[M]\{\phi\} = [I]$$  \hspace{1cm} (34)

where $[I]$ is the identity matrix. Differentiating equation (33) with respect to $V_k$ gives

$$\left([K] - \omega_j^2[M]\right)\frac{\partial\{\phi\}_j}{\partial V_k} = \frac{\partial\omega_j^2}{\partial V_k}\{\phi\}_j - \frac{\partial[K]}{\partial V_k}\{\phi\}_j + \omega_j^2 \frac{\partial[M]}{\partial V_k}\{\phi\}_j$$  \hspace{1cm} (35)

Premultiplying equation (35) by $\{\phi\}^T$ and using equations (33) and (34) gives


Since $[K] - \omega_j^2[M]$ is singular, a direct solution of equation (35) is not attempted. The value of one component of $\frac{\partial\{\phi\}_j}{\partial V_k}$ is fixed, yielding a particular solution $\{Q\}_j$ to equation (35). This is accomplished in EAL by identifying the component of the eigenvector with the largest absolute value and constraining to zero the corresponding component of the eigenvector derivative. The eigenvector $\{\phi\}_j$ is the complementary solution to equation (35). Thus, the expression for the eigenvector derivative is

$$\frac{\partial\{\phi\}_j}{\partial V_k} = \{Q\}_j + C\{\phi\}_j$$  \hspace{1cm} (37)

The value of the multiplier $C$ is obtained by substituting the expression for $\frac{\partial\{\phi\}_j}{\partial V_k}$ from equation (37) into the following:

$$2\{\phi\}_j^T[M]\{\omega\}_j \frac{\partial\{\phi\}_j}{\partial V_k} = -\{\phi\}_j^T[M]\{\phi\}_j$$

Equation (38) is the derivative of the expression for the normalization of modes with respect to the mass matrix (eq. (34)). The resultant equation for $C$ is

$$C = -\{\phi\}_j^T[M]\{\phi\}_j - \frac{1}{2}\{\phi\}_j^T[M]\{\phi\}_j$$  \hspace{1cm} (39)
For buckling eigenvalue and eigenvector derivatives, the preceding equations apply directly, with the buckling mode shape \( \{\phi\}_j \) and load \( \lambda_j \) substituted for the vibration mode shape \( \{\phi\}_j \) and frequency \( \omega_j^2 \) and with the negative of the geometric stiffness matrix \(-[K_g]\) substituted for the matrix \([M]\). In the present implementation, derivatives of the matrices \([K]\), \([M]\), and \([K_g]\) were calculated by finite differences.

Finite-Difference Method

The equations for finite-difference derivatives of eigenvalues and eigenvectors are analogous to those previously shown for displacement and stress derivatives (eqs. (25) and (32)). When the finite-difference method is used to obtain derivatives of eigenvectors, careful attention must be paid to accuracy of the eigenvectors. Problems arise because eigenvalue routines generally base convergence on accuracy of the eigenvalues, whereas the associated eigenvectors may not be as accurate. Hence, when differencing the eigenvectors, further errors are introduced which may make the derivatives inaccurate.

IMPLEMENTATION OF DERIVATIVE CAPABILITY IN EAL

An outline of the implementation of the derivative capability in EAL is given in the appendix. For brevity, only the implementation of the semianalytical method is discussed in the appendix, which includes descriptions of the overall procedure, descriptions of the key portions of the EAL runstreams, and a listing of the input for the problem of the box beam.

NUMERICAL EXAMPLES AND RESULTS

Four example problems were investigated to verify the derivative implementation and to compare the methods. The problems were a swept wing, a box beam, a stiffened cylinder with a cutout, and a space radiometer-antenna truss. Comparisons of solution times for the various derivative calculation methods are given, and a convergence study was performed for the wing problem to verify the accuracy of the finite-difference results and to establish an appropriate perturbation in the design variables for the remaining finite-difference calculations. Analytical derivatives (when available) were used to assess the accuracy of the other methods. All numerical calculations were performed on the CDC® CYBER 175 computer under NOS 1.4.

Swept Wing

Optimization of the swept wing shown in figure 3 has been investigated and reported by several researchers (refs. 17 to 19). This example was a moderately complex representation of a wing modeled by rods (E23), triangular membranes (E31), and shear panels (E44). Since bending elements were not used, the calculation of derivatives with the analytical method was straightforward, and the results were used to check the accuracy of the semianalytical and finite-difference methods. The geometry and node numbering are shown in figure 3; nodal coordinates are listed in table 3 and design variables are described in table 4. In this problem, 32 design variables controlled the section properties of 150 elements. The wing was aluminum, had a constrained root, and was subjected to two load conditions. (See table 5.) Load condition 1 was approximately equivalent to a uniform pressure loading of
0.556 psi. Load condition 2 had the same total load, but the distribution was changed to move the center of pressure forward. The wing was symmetric with respect to the x-y plane (u and v displacements were zero on the plane z = 0), and hence only half the wing was modeled. The model consisted of triangular membrane elements for the skin, rod elements for the spar caps, and shear panels for the rib and spar webs. (See fig. 4.) Forty-four transverse rod elements (not shown) were added at the vertical edges of the shear panels to provide the necessary stiffness throughout the depth of the wing. The thicknesses t_i were 0.2 in. for E31 elements 1 to 24, 0.1 in. for E31 elements 25 to 60, and 0.2 in. for all E44 elements. The cross-sectional area A_i was 0.02 in² for E23 elements 1 to 20 and 0.2 in² for E23 elements 21 to 64.

A study was performed to determine the perturbation step size needed to obtain sufficiently accurate results for the finite-difference and semianalytical methods. The results are shown in table 6(a). The value of the largest displacement derivative value (\( \frac{\partial w}{\partial V_{14}} \) at joint number 41 for load condition 1) was used to judge convergence. As indicated in table 6(a), the finite-difference result, with a 1-percent increment in the design variable, was within 1 percent of the analytical results. Use of perturbations smaller than 0.01 percent led to degradation of accuracy. The semianalytical method was capable of duplicating the analytical result with a 1-percent perturbation in the design variable. Although the acceptable perturbation sizes are generally problem dependent, this dependence was not significant. In all subsequent calculations, perturbations of 1 percent were used.

Comparisons of solution times for displacement derivatives are also presented in table 6(b). The finite-difference method was slowest (700 sec), the semianalytical method required 419 sec without the LSK processor and 161 sec with the LSK processor, and the analytical method required 135 sec. Hence, the use of the LSK processor makes the semianalytical method significantly more effective and makes it competitive with the analytical method.

Box Beam

The next problem was a cantilevered box beam (fig. 5). This example was used to verify displacement derivative runstreams for all the EAL structural elements listed in table 2.

The geometry and joint numbering are shown in figure 5, and elements are illustrated in figure 6. The thickness of all two-dimensional elements was 0.1 in. The cross-sectional area of the rod elements was 1.0 in², and the beam elements had tube sections with an inner radius of 2.0 in. and an outer radius of 2.5 in. The material was aluminum with the following material properties: \( \rho = 0.096 \text{ lb/in}^3 \), \( E = 10.6 \times 10^6 \text{ psi} \), and \( v = 0.3 \). The applied load was composed of 10 000-lb forces in the positive z direction at nodes 27 and 28. The cross-sectional area or thickness of each group of elements was considered to be a separate design variable. The problem had 10 dependent design variables and 9 independent design variables. The inner and outer radii of the tube elements were linked to an independent variable \( r \) by the equation

\[
\begin{bmatrix}
x_i \\
x_o
\end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} r + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}
\]  
(40)
Displacement derivatives with respect to each independent design variable were calculated with the finite-difference method and the semianalytical method. Results shown in Table 7(a) indicate run times of 133 sec for the finite-difference method and 63 sec for the semianalytical method with the LSK processor. The reduction in solution time for the semianalytical method with LSK over the finite-difference method was smaller for this example than for the swept wing because the swept wing problem had more independent design variables (32 instead of 9). Values shown in Table 7(b) reveal which design variables would be most effective to increment to reduce tip deflections. As expected, the upper and lower covers (coupled plate elements E43 and E33) had the greatest effect on limiting tip deflections. In addition, since the E32 and E42 elements had only bending stiffness and the bending loads in the covers were negligible, the derivatives with respect to thickness of these elements were negligible. The results in Table 7(b) suggest that derivatives can be used directly to guide improvements in a structural design apart from their use in formal optimization. In some instances substantial improvements may be realized (ref. 20).

Stiffened Cylinder With Cutout

Optimization studies of a cantilevered stiffened cylinder with a cutout (Fig. 7) were reported in references 4 and 21. Node point locations are given in Table 8. Element types and locations are shown in Figure 7(b). The cylinder model had 352 degrees of freedom and was stiffened by 5 equally spaced rings along the length and 16 equally spaced stringers around the circumference. Rotations about the Y-axis at all node points and all translations at node points 1 to 16 were constrained to zero. The rings and stringers were modeled by beam (E21 channel sections) and rod elements (E23). Rectangular panels between rings and stringers were modeled by membrane elements (E41). The cross-sectional area of all rod elements was 0.646 in², the thickness of the rectangular membranes was 0.0394 in., and the channel-section dimensions are shown in Figure 7(b). The material was aluminum alloy with \( E = 10.8 \times 10^6 \) psi and \( v = 0.3 \). The independent design variables for this problem included the area of the rods, the thickness of the membrane elements, and a scale factor which controlled the cross-sectional dimensions of the beam elements. The loading consisted of two equal and opposite concentrated axial forces of 20,000 lb applied at the free end.

Derivatives of displacements, stresses, vibration frequencies and mode shapes, and buckling loads and mode shapes were calculated. Computer times for calculating derivatives are presented in Table 9. Use of the semianalytical method without the LSK processor for displacement derivatives resulted in a reduction in solution time of about 25 percent over the finite-difference method. Using the semianalytical method with the LSK processor resulted in a reduction in solution time of 28 percent over the finite-difference method. The reason solution times for the semianalytical methods were so similar for this case was that the ratio of the number of independent to the number of dependent design variables was only 3 to 5. As this ratio decreases, the advantage of the LSK processor increases. (In the problem of the swept wing, the ratio was 32 to 150 and the LSK processor had a much larger effect.)

Stress derivatives were calculated by finite differences (122 sec) and semianalytically (95 sec). The savings in solution time for finite differences relative

2 Derivatives for the membrane and rod elements were computed with the analytical method.
to the semianalytical method were not as dramatic in this problem as in the problems of the swept wing or the box beam because of the smaller number of design variables.

Computer times for calculating derivatives of vibration frequencies, vibration mode shapes, buckling loads, and buckling mode shapes with the finite-difference and analytical methods are also shown in table 9. For the vibration problem, the analytical method was 57 percent faster than finite differences, and for the buckling problem it was 37 percent faster.

Antenna Truss

A finite-element model of a 180-ft radiometer-antenna reflector is shown in figure 8. The reflector, described in reference 22, was made up of tetrahedral truss modules, and the model consisted of 109 structural node points (table 10) and 420 rod elements. The structure was composed of graphite-epoxy composite with an effective modulus of elasticity of $10.6 \times 10^6$ psi and a coefficient of thermal expansion of $0.13 \times 10^{-6}$ per degree Fahrenheit. The antenna was subjected to thermal loading corresponding to Earth orbit at an altitude of 216 miles in an Earth-facing orientation. The thermal loading consisted of a combination of solar, Earth-reflected (albedo), and Earth-emitted heat flux. A transient thermal analysis of the structure for a complete orbit was performed (ref. 23) and node point temperature differences from a worst-case condition (largest temperature gradients, table 11) were used for the present calculations. Three design variables were used for this problem. They were the cross-sectional area of the elements in the upper surface (0.2530 in$^2$), the area of the diagonal elements joining the upper and lower surfaces (0.1741 in$^2$), and the area of the elements in lower surface (0.2530 in$^2$). Both stress and displacement derivatives were calculated, and for this problem the applied load vector $\{f\}$ was a function of the design variables. (See eq. (20).)

Solution times for displacement and stress derivatives are summarized in table 12(a). The semianalytical method was 39 percent faster than the finite-difference method for displacement derivatives and 29 percent faster for stress derivatives for this problem. As indicated by the tabulated derivatives in table 12(b), the most effective way to reduce the center deflection of the reflector would be to decrease the areas of the upper-surface elements.

CONCLUDING REMARKS

A capability for computing structural-sensitivity derivatives has been implemented in the general purpose, finite-element computer program denoted the Engineering Analysis Language (EAL) System. This paper presented the development of runstreams which calculate derivatives of displacements, stresses, vibration frequencies and mode shapes, and buckling loads and mode shapes with respect to structural variables which include thicknesses, areas, and moments of inertia. Linear and nonlinear design-variable linking representations were included. Three methods for computing the derivatives were documented: analytical (which calculates exact structural-sensitivity derivatives from the governing equations), semianalytical (which combines finite-difference derivatives of mass, stiffness, and geometric stiffness matrices with analytical expressions for derivatives), and finite differences. The derivative capability was demonstrated for the following four structures: a swept wing, a box beam, a stiffened cylinder with a cutout, and a space radiometer-antenna truss. Comparisons of solution times for the various methods were described. A convergence study was performed for the swept wing to
verify the accuracy of the finite-difference results and to establish an appropriate increment in the design variables for use in the subsequent finite-difference calculations. Analytical derivatives of displacements for the wing and of vibration frequencies of the cylinder served as benchmarks to assess the accuracy and efficiency of the finite-difference and semianalytical methods. Results indicated that the semianalytical method for calculating displacement and stress derivatives was efficient, general, and straightforward to implement. Further, the semianalytical method was not adversely affected by the presence of bending elements or design-variable linking.

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February 8, 1984
APPENDIX

SYSTEM OF EAL RUNSTREAMS FOR CALCULATING STRUCTURAL-SENSITIVITY DERIVATIVES WITH SEMIANALYTICAL METHOD

Overall Procedure

This appendix describes a system of EAL runstreams used to calculate structural-sensitivity derivatives with the semianalytical method. A flowchart for the overall system is shown in figure 9. Runstreams or groups of runstreams presented at the end of this appendix are keyed by numerals to specific portions of flowcharts shown in figures 10 to 12. The semianalytical method uses equations (15) and (16) to calculate derivatives of the stiffness and mass matrices and uses equations (24) and (28) to (30) to calculate the displacement and stress derivatives.

As shown in figure 10, derivatives of the stiffness, mass, and geometric stiffness matrices are calculated by finite differences. The procedure is as follows: loop over the independent design variables, perturb a design variable, link design variables to form vector of dependent design variables, loop over the dependent design variables and modify the structure, calculate new stiffness and mass matrices and geometric stiffness matrix (if appropriate), differentiate the resulting matrices, and restore the original value of independent design variable $V_k$. The numerals beside specific boxes correspond to runstreams described subsequently.

Displacement derivatives (fig. 11) are calculated by looping over the independent design variables, building a pseudo load matrix (eq. (20)) and solving equation (24). The acquired displacement derivatives are then used to approximate a perturbed displacement vector $\{u\}_k^n$ (eq. (28)) and then to approximate a perturbed stress vector (eq. (29)). The stress derivatives are then calculated by finite differences (eq. (30)).

The flowchart which illustrates the calculation of derivatives of vibration and buckling eigenvalues is shown in figure 12. The outer loop is on the mode for which derivatives are required. The two inner loops are both over the independent design variables. The first inner loop calculates the derivatives of the vibration frequencies and buckling loads with equation (36) and then builds a pseudo load vector (right-hand side of eq. (35)). A particular solution of equation (35) is calculated and used in the second inner loop to calculate derivatives of the eigenvectors with equations (37) and (39).

Descriptions of the key runstreams used in the system of figures 9 to 12 (particularly those which are problem dependent) are given in the following table:
### APPENDIX

#### DESCRIPTION OF KEY RUNSTREAMS

<table>
<thead>
<tr>
<th>Runstream name</th>
<th>Identification no. in figs. 9 to 12</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRIV GRD4</td>
<td>1</td>
<td>Driver runstream which calculates derivatives semianalytically; the entire system can be run by the statement *DCALL (DRIV GRD4)</td>
</tr>
<tr>
<td>INIT MODL</td>
<td>2</td>
<td>Sequence of EAL processors (TAB, ELD, TAN, and AUS) which describes the initial structural model and load cases; the example shown in the runstream listing is for the box beam</td>
</tr>
<tr>
<td>SET PARA</td>
<td>3</td>
<td>Initializes following parameters used by system to determine specifics of derivative information required:</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>NLST</strong> Number of load sets</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>MODT</strong> Number of modes for which derivatives are computed by method of reference 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>MODE</strong> Same as MODT</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>OLIB</strong> Output library for intermediate buckling and vibration derivatives</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>OTIB</strong> Output library for intermediate stress or displacement derivatives</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>NPOL</strong> Degree of polynomial used in linking design variables (eq. 1))</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>DISP</strong> If equal to 1, displacement derivatives are calculated</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>STRD</strong> If equal to 1, stress derivatives are calculated</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>BUCD</strong> If equal to 1, buckling load and node shape derivatives are calculated; if equal to 2, only buckling load derivatives are calculated</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>VIRD</strong> Same as BUCD for vibration derivative</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>NJ</strong> Number of joints in model</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>NDV</strong> Number of independent design variables</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>NDVD</strong> Number of dependent design variables</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>NDVA</strong> Number of rod and membrane dependent design variables (E23, E31, E41, E44)</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>NNDV</strong> Number of beam dependent design variables (E21)</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>NPDV</strong> Number of plate dependent design variables (E32, E33, E42, E43)</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>NDVX</strong> Number of shape-dependent design variables</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>DR</strong> Incrementing factor for design variables for finite-difference calculations</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>DE21, DE23, ..., DE44</strong> Number of dependent design variables for each element</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>NE21, NE23, ..., NE44</strong> Number of independent design variables for each element</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>MODE NUM</strong> Name of table of node numbers for which derivatives are taken</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>TABLE</strong></td>
</tr>
</tbody>
</table>

**Note:** The table above contains identification numbers and descriptions for key runstreams. The descriptions include the purpose and parameters used by each runstream in the context of calculating derivatives for structural analysis.
<table>
<thead>
<tr>
<th>Runstream name</th>
<th>Identification no. in figs. 9 to 12</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DESV DFN</td>
<td>4</td>
<td>Set up tables denoted DVA DFN, DVP DFN, and DVB DFN which define membrane, plate, and beam dependent design variables, respectively; element types are defined by a numeral: E23-1, E31-2, E32-3, E33-4, E41-5, E42-6, E43-7, E44-8, and E21-9; the type of beam used in EAL in section 3.1.9 of reference 12 is denoted by a code as follows: TUBE - 1, BOX - 2, TEE - 3, ANC - 4, WFL - 5, CHN - 6, ZEE - 7, GIVN - 8, DSY - 9; beam design variable numbers are designated by the following codes: b1 - 1, t1 - 2, b2 - 3, t2 - 4, b3 - 5, t3 - 6, l1 - 7, q1 - 8, z2 - 9, z2 - 10, a - 11, l2 - 12, l2 - 13, z1 - 14, z2 - 15, a1 - 16, q1 - 17, q2 - 18, q2 - 19; each of the variables shown is described in section 3.1.9 of reference 12; the order in which the dependent design variables are listed (starting with the membrane, then plate, and then beam design variables) are the actual locations of each design variable in the dependent design variable vector (see INTL DESV); the only restriction is that the structural parameters of beams with the same section property number must be listed together in the DVB DFN table; this permits the linking of several section properties of a beam; this could be used to facilitate the scaling of an entire beam cross section by one design variable or scale factor</td>
</tr>
<tr>
<td>INTL DESV</td>
<td>5</td>
<td>Initializes design variables by forming a table of initial design variables called DESV CNMN whose only limitation is that when linked it will produce the same structural parameters as the initial structural model (see INIT MODL)</td>
</tr>
<tr>
<td>COEF LINK</td>
<td>6</td>
<td>Series of tables which define the coefficients for nonlinear design-variable linking as expressed by equation (1) and rewritten as ( {v} = {C_o} + {C_1}{v} + {C_2}{v^2} + \ldots + {C_m}{v^m} ) where ( {C_o} ) is represented as table COEF DV 1 0, ( {C_1} ) is COEF DV 1 1, ( {C_2} ) is COEF DV 1 2, \ldots, ( {C_m} ) is COEF DV 1 M; each block of the above tables represents a column in the ( [C] ) matrices; this runstream must be included in the system of runstreams even if linking is not used</td>
</tr>
</tbody>
</table>
This section of the appendix contains a listing of the EAL runstreams used to calculate derivatives of displacement, stress, and vibration and buckling eigenvalues and eigenvectors with the semianalytical method.

```
* CHAR $*1111Z
* ECHO 1,2,3,4
* ABORT 1
* QUIT U1

$---------------------------------------------------------- 1
$*(DRY GRD4) ENDDG4 $ DRIVER PROGRAM TO CALCULATE DU/DCAPV BY THE SEMI-ANALYTICAL METHOD
*DCALL(INIT MDL)
* QUIT DCU
DUPLICATE 1 2
DUPLICATE 1 3
DUPLICATE 1 4
DUPLICATE 1 5
*DCALL(SET PARA)
*DCALL(DSV DFN)
*DCALL(INTL DSV)
*DCALL(COEF LINK)
*DCALL(CALC GRD4)
*DCALL(PRT SUB)
* RETURN
*
$---------------------------------------------------------- 2
$*(INIT MDL) ENDDM
$ $ INITIAL MODEL STRUCTURAL DEFINITION
$*
TITLE 'BOX BEAM GRADIENT CALCULATION'
*START 28
*GRADIENT CALCULATION OF A BOX-BEAM WHICH USES
'E2I,E23,E31,E32,E33,E41,E42,E43,AND E44 ELEMENTS'
*INPUT JOINT LOCATIONS
JLOC
1 0. 0. 0. 0. 10. 0. 217
1 0. 10. 0. 0. 10. 0. 217
1 10. 0. 10. 10. 0. 217
1 10. 0. 10. 10. 0. 217
$SPECIFY MATERIAL PROPERTIES
MATERIAL CONSTANTS
1 10.6E6 +3 .096
$DESCRIBE MATERIAL REFERENCE FRAME
MREF
FORMAT=2
1,1,10000,0,10000,
2,1,10000,10,10000,
$BEAM ELEMENT SECTION PROPERTIES
BA
TUBE 1, .25, .3
$ROD ELEMENT AREAS
dC
1,1,
$MEMBRANE ANS PLATE ELEMENT THICKNESSES
SA
NMAT=1
1,1E31 ELEMENTS
2,1E32 ELEMENTS
3,1E33 ELEMENTS
4,1E41 ELEMENTS
5,1E42 ELEMENTS
6,1E43 ELEMENTS
$SHEAR PANEL THICKNESSES
SB
1,1,
$CONSTRAINT DEFINITION
```
APPENDIX

CDN=1
ZERO 1,2,3,4,5,6,11,2
ZERO 1,2,3,4,5,6,15,16
*XOT ELD
*BEAM ELEMENT DEFINITION
E21
NMAT=1
NSECT=1
HREF=1
1 3 1 6
15 17 1 6
HREF=2
2 4 1 6
16 18 1 6
$ROD ELEMENT DEFINITION
E23
NMAT=1
NSECT=1
1 15 1 1 7 2
2 16 1 1 7 2
$TRIANGULAR MEMBRANE ELEMENT DEFINITION
E31
NMAT=1
NSECT=1
8 10 7
7 9 10
9 11 10
10 12 11
12 14 11
11 13 14
$UNCOPLED TRIANGULAR PLATE ELEMENT DEFINITION
E32
NMAT=1
NSECT=2
9 10 7
7 9 10
9 11 10
10 12 11
12 14 11
11 13 14
$UNCOPLED TRIANGULAR PLATE ELEMENT DEFINITION
E33
NMAT=1
NSECT=3
1 3 2
1 4 3
4 6 3
3 5 6
5 7 6
6 8 7
$QUADRILATERAL MEMBRANE ELEMENT DEFINITION
E41
NMAT=1
NSECT=4
21 23 24 22 1 3 1
$UNCOPLED QUADRILATERAL PLATE ELEMENT DEFINITION
E42
NMAT=1
NSECT=5
21 23 24 22 1 3 1
$UNCOPLED QUADRILATERAL PLATE ELEMENT DEFINITION
E43
NMAT=1
NSECT=6
15 17 18 16 1 3 1
$SHEAR PANEL DEFINITION
E44
NMAT=1
NSECT=1
1 3 17 15 1 6 1
2 18 16 1 6 1
*XQT TAN
$APPLIED LOADS DEFINITION
*XQT AUS
SYSVEC$APPLIED FORCE 1
i=3; j=27,28; 10000. 10000.
*RETURN
APPENDIX

* ENDMEM
$
$-------------------------------------------- 3
$

*(SET PARA) ENDPAR

** SET CONTROL PARAMETERS
INLST=1$NUMBER OF LOAD SETS
ILCAS=1$NUMBER OF LOAD CASES
IMOD=2$TOTAL NUMBER OF MODES
IDLIB=1$OUTPUT LIBRARY FOR INTERMEDIATE DYNAMIC DERIVATIVES
INDV=1$OUTPUT LIBRARY FOR INTERMEDIATE STRESS OR DISP. DERIVATIVES
INPLD=1
INDP=1$MODE=2
INPOL=1$DEGREE OF POLYNOMIAL LINKING
IDIS=1$DISPLACEMENT DERIVATIVES
ISTRD=1$STRESS DERIVATIVES
IBUC=1$WUCK DERIVS. (LOAD&FREQ)
IVBD=1$VIB. DERIVS. (LOAD&FREQ)
INJ=2$NO. OF JOINTS
INDV=9$NO. OF INDEPENDENT DESIGN VARIABLES
INDP=1$NO. OF DEPENDENT SHAPE VARIABLES TO BE LINKED
INPA=4$NO. OF DEPENDENT MEMBRANE-TYPE DESIGN VARIABLES TO BE LINKED
INLNX=10$NO. OF LINKED DESIGN VARIABLES
INDV=1
INPA=2$NO. OF PLATE DEPENDENT DESIGN VARIABLES TO BE LINKED
INPDV=4$NO. OF PLATE DEPENDENT DESIGN VARIABLES TO BE LINKED
INDV=NDV+NDV+NDV+NPDV
INDV=NDV
IDK=.001% FACTOR FOR INCREMENTING DESIGN VARIABLE
IDT2=15$NO. OF E21 D.V.'S
IDT31=1$NO. OF E31 D.V.'S
IDT32=1$NO. OF E32 D.V.'S
IDT33=1$NO. OF E33 D.V.'S
IDT41=1$NO. OF E41 D.V.'S
IDT42=1$NO. OF E42 D.V.'S
IDT43=1$NO. OF E43 D.V.'S
IDT44=1$NO. OF E44 D.V.'S
IDT21=2$NO. OF E21 D.V.'S
INCP=1
INX1=D5.4,1,1,1,1,JDF1,BAB1,8
INX2=D5.5,1,1,1,1,JDF1,BAB1,8
INX3=D5.6,1,1,1,1,JDF1,BAB1,8
INX4=D5.7,1,1,1,1,JDF1,BAB1,8
INX5=D5.8,1,1,1,1,JDF1,BAB1,8
INX6=D5.9,1,1,1,1,JDF1,BAB1,8
$

$ TABLE OF MODE NUMBERS
$

* OUT AUS
TABLE(RMODE=2,TYPE=0,NI=1,NJ="MODE");MODE NUM
J=111
J=212
$

$ TABLE OF UNCONSTRAINED DEGREES OF FREEDOM
$

TABLE(RMODE=2,TYPE=0,NI=1,NJ=6);IN EX
J=111;"INX1"
J=222;"INX2"
J=333;"INX3"
J=444;"INX4"
J=555;"INX5"
J=666;"INX6"
INX1=FREE()
INX2=FREE()
INX3=FREE()
INX4=FREE()
INX5=FREE()
INX6=FREE()
*RETURN
$ ENDMEM
$
$-------------------------------------------- 4
$

*(DESV QFN) ENDDVD

** DEPENDENT DESIGN VARIABLE DEFINITION
*OUT AUS

APPENDIX

$MEMBRANE-TYPE DESIGN VARIABLES
   TABLE(RMODE=2,TYPEN=0,NI=4,NJ="NDVA") DVA DFN
   J=1,"NDVA" $ELEMENT TYPE GROUP NO. NO. OF ELEM. SECT. NO.
   1 1 12 1
   2 1 6 1
   5 1 3 4
   8 1 12 1

$PLATE-TYPE DESIGN VARIABLES
   TABLE(RMODE=2,TYPEN=0,NI=4,NJ="NPDV") DVP DFN
   J=1,"NPDV" $ELEMENT TYPE GROUP NO. NO. OF ELEM. SECT. NO.
   3 1 6 2
   4 1 6 3
   6 1 3 5
   7 1 3 6

$BEAM-TYPE DESIGN VARIABLES
   TABLE(RMODE=2,TYPE=0,NI=5,NJ="NBVDV") DOB DFN
   J=1,"NBVDV" $ELEMENT TYPE GROUP NO. NO. OF ELEM. SECT. NO. D.V. NO.
   1 1 24 1 2

*RETURN

$ENDDVD

$--------------------------------------------- 5

*INTL DESV)
*ENDIND

$** $ INITIALIZE DESIGN VARIABLES
**XAT AUS
   TABLE(NJ="NPDV") DESV CMM#DATA SET CONTAINING INITIAL VALUE OF DESIGN VARIABLES
   J=1;1.
   J=2;6;1
   J=9;29
*RETURN
*ENDIND

$--------------------------------------------- 6

$** $COEF LINK)
**ENDCLK

$** $ MATRIX OF COEFFICIENTS FOR LINKING
**XAT AUS
   TABLE(NJ="NDVA") COEF DV 1 1
   BLOCK 1
   I=1J=1;1.
   BLOCK 2
   I=1J=2;1.
   BLOCK 3
   I=1J=3;1.
   BLOCK 4
   I=1J=4;1.
   BLOCK 5
   I=1J=5;1.
   BLOCK 6
   I=1J=6;1.
   BLOCK 7
   I=1J=7;1.
   BLOCK 8
   I=1J=8;1.
   BLOCK 9
   I=1J=9;1.
   I=1J=10;1.
   TABLE(NJ="NBVDV") COEF DV 1 0
   J=1;910.
   J=10;49
*RETURN
*ENDCLK

$--------------------------------------------- 7

$** $CALC GRD4)
**ENGD4

$** $ CALCULATE DERIVATIVES OF U W.R.T. DESIGN VARIABLES USING THE SEMI-ANALYTICAL METHOD
**CALL(CALC KM)
**CALL(DKDV DBDV)
**JNZ(DISP,262)
**JNZ(STRD,262)
**JUMP 265
**LABEL 262
**CALL(CALC DUVD)
**LABEL 264
**JZ(STRD,265)$TEST TO SEE IF STRESS DERIVATIVES ARE REQUIRED
APPENDIX

*DCALL(STAS DERV)CALCULATE STRESS DERIVATIVES
*LABEL 265
*TEST TO SEE IF BUCKLING DERIVATIVES ARE REQUIRED
*JZI(BUCD=266)
IN=BUCD
*JNZ,-1(N=267)
INAM9=*BUCK
INAM7=*BVVL
INAM8=*BKMD
*DCALL(NELS METH)
*JUMP 266
*LABEL 267
*JNZ,-1(N=269)
INAM9=*BUCK
INAM7=*BVVL
INAM8=*BKMD
*DCALL(NELS METH2)
*LABEL 269
*JNZ,-1(N=273)
INAM9=*BUCK
INAM7=*BVVL
INAM8=*BKMD
*DCALL(MODL METH)
*JUMP 266
*LABEL 273
*JNZ,-1(N=285)
INAM9=*BUCK
INAM7=*BVVL
INAM8=*BKMD
*DCALL(NELS METH)
*JUMP 266
*LABEL 285
*QXT EXIT
*LABEL 266
*JZI(VIBD=268)*TEST TO SEE IF VIBRATION DERIVATIVES ARE REQUIRED
IN=VIBD
*JNZ,-1(N=270)
INAM9=*VIBR
INAM7=*VBVL
INAM8=*VBMD
*DCALL(NELS METH)
*JUMP 268
*LABEL 270
*JNZ,-1(N=271)
INAM9=*VIBR
INAM7=*VBVL
INAM8=*VBMD
*DCALL(NELS METH2)
*JUMP 268
*LABEL 271
*JNZ,-1(N=274)
INAM9=*VIBR
INAM7=*VBVL
INAM8=*VBMD
*DCALL(MODL METH)
*JUMP 264
*LABEL 274
*JNZ,-1(N=275)
INAM9=*VIBR
INAM7=*VBVL
INAM8=*VBMD
*DCALL(MODL METH)
*JUMP 264
*LABEL 275
*QXT EXIT
*LABEL 268
*RETURN

ENDG04

ENDCM$PERFORM STATIC AND/DIN DYNAMIC ANALYSES OF INITIAL STRUCTURE

*(CALC KM)
*QXT E
*QXT EKS
*QXT K
*JNZ(DISP,156)
*JNZ(STRO,156)
*JUMP 157
APPENDIX

*LABEL 156
*XQT KSI
IA=NLST
*LABEL 5666
*JLZ=-1(A,5667)
INNST=NLST-A
*XQT SSOL CALCULATE DISPLACEMENTS
RESET SET="NNST"
*JUMP 5666
*LABEL 5667
IA=FRE()1
INNST=FREE()
*LABEL 157
*XQT U1
INN3=TOC,N3(1,K,SPAR,MASK, MASK)
INN4=TOC,N4(1,K,SPAR,MASK, MASK)
*JZ(STRO,150)
IA=NLST
*LABEL 7666
*JLZ=-1(A,7667)
INNST=NLST-A
INAM1=STAT
INAM2=DISP
INAM3="NNST"
INAM4=1
IGLIB=1
IN11=1000 SET N1=1000 FOR UNPERTURBED STRESS CALCULATIONS
*CALL(CALC STRS) CALCULATE STRESSES
*JUMP 7666
*LABEL 7607
IA=FRE()
INNST=FREE()
INAM1=FREE()
INAM2=FREE()
INAM3=FREE()
INAM4=FRE()
IGLIB=DISP
*LABEL 150
*JNZ(DISP,193)
*JNZ(STRO,193)
*XQT RSI
*XQT SSOL
*LABEL 193
*JZ(BUC0,151)
*XQT GSF
RESET EMBED*1
*XQT KG
*XQT EIGS PERFORM BUCKLING ANALYSIS
RESET CONV=1.-10
RESET PKO=BUCK
RESET INIT="MMD8", NREQ="MRQB"
RESET NDY=20
*XQT EIG
RESET CONV=1.-10,FROB=BUCK,NREQ="MRQB",NDY=20
*LABEL 151
*JZ(VIB0,152)
*XQT E4S PERFORM VIBRATION ANALYSIS
RESET NMODES="MMD6", NREQ="MREQ"
*LABEL 152
*XQT DCU
*JZ(DISP,159)
*JNZ(STRO,159)
*JUMP 158
*LABEL 159
IA=NLST
*LABEL 6666
*JLZ=-1(A,6667)
INNST=NLST-A
$RENAME ALL DATA SETS AS INDICATED BELOW
CHANGE 1 STAT DISP "NNST" 1,PREV DISP "NNST" 1
*JUMP 6666
*LABEL 6667
*LABEL 159
IA=FRE()
INNST=FREE()
*LABEL 150
CHANGE 1 DEM DIAG 0 0,DEMP DIAG 0 0
CHANGE 1 K SPAR "NN3" "NN4", KP SPAR "NN3" "NN4"

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APPENDIX

CHANGE 1 INV K 1 0, INV KP 1 0
CHANGE 1 XINV K 1 0, XINV KP 1 0
*JZIBUCD0,154)
INNN3*TOC>311, KG, SPAR, MASK, MASK
INMSA, N4/1, KG, SPAR, MASK, MASK
CHANGE 1 KG SPAR "NNN3", "NNN4", KG SPAR "NNN3", "NNN4"
CHANGE 1 BUCK EVAL 1, 1, PREV BKVL 1 1
CHANGE 1 BUCK MODE 1, 1, PREV BKMD 1 1
*LABEL 154
*JZIVIBD0,155)
CHANGE 1 VIBR EVAL 1, 1, PREV VBVL 1 1
CHANGE 1 VIBR MODE 1, 1, PREV VBMD 1 1
*LABEL 155
*RETURN
*
ENDCKM

#----------------------------- 9
#
#(DKDV DMV) ENODCKM$CALCULATE DK/DM, DM/DM, AND DKG/DM
IIVV=0
IIEV=NDV
*LABEL 803
*JZI-1(IIEV>804)$LOOP OVER DESIGN VARIABLES
IIV=IIV+1
IDV=DSV1,"IIV",11, DESV, CMN3, MASK, MASK
IDV=DR*DVM+DVM
IAG=1, JDR
IAG=AQ/DVM
IBO=-1.*AQ
XQT AUS
TABLE=UNH="NDV"; DESV CMN3 INCREMENT DESIGN VARIABLE
UPEK*XSUM
J="IIV"; MDV"
*DCALL(TRNS DESV)
*DCALL(LINK POLY)
*XQT DCJ
PRINT 1 LNK DM
*XQT TAB
UPDATE=1
*JZI(NDV>800)
IIEV=NDV
*DCALL(IDV FO0)$UPDATES ALL MEMBRANE PROPS. TO CURRENT VALUE
*LABEL 800
*JZI(NPDV>801)
IIEV=NPDV
*DCALL(IDV FO0)$UPDATES ALL PLATE PROPS. TO CURRENT VALUES
*LABEL 801
IIEV=NBDV
*DCALL(IDV FO0)$UPDATES ALL BEAM PROPS. TO CURRENT VALUES
*LABEL 802
INNN3*TOC>311, KG, SPAR, MASK, MASK)
*XQT E
*XQT EKS
*XQT K
KESET OUTLIB= "OLI0"
*XQT AUS
DEFINE KN= "OLI0" K SPAR
DEFINE KD=1 KP SPAR
UK=SUM("AQ", KN, "BQ", KD)
DEFINE A=DEMP DIAG 0 0
DEFINE A=DEMP DIAG 0 0
DMOV DIAG 0 "IIV": SUM("AQ", B, "BQ", A)
*XQT DCJ
SPRINT 15 K SPAR
SPRINT 1 KP SPAR
CHANGE 1 UK MASK MASK MASK, DDKV SPAR "NN3" "IIV"
SPRINT 1 DDKV SPAR "NN3" "IIV"
*JZI(BUCD0294)
*XQT RSI
RESET NUK="OLI0"
*XQT SSJL
RESET KLIB="ULIB"
KESET KILIB=1
*XQT GSF
RESET EMBED=1
*LABEL 292
*JZI(BUCD0293)
APPENDIX

*JNZ(V180,293)
*JUMP 294
*LABEL 293
*XOT KG
*XOT AUS
DEFINE KG=KG SPAR "NNN3" "NNN4"
DEFINE KG=KG SPAR "NNN3" "NNN4"
OKG=SUM("AO" KG,"BO" KG)
*XOT DCU
CHANGE 1 OKG SPAR "NNN3" "NNN4",OKG SPAR "NNN3" "NNN4"
*LABEL 294
*XOT AUS
TABLE:UNJ="NDV"
I DESV CMNN$ SET ORGINAL VALUE DF DESIGN VARIABLE
QPER=XSUM
J=="IV"="POVP"
INIEV=IIENV
*JNZ(INIEV,805)
*DCALL(TPNS DESV)
*DCALL(LINK POLY)
*LABEL 805
*DCALL(DCU EFIL)
*JUMP 803
*LABEL 804
ID1=FREE()
ID2=FREE()
ID3=FREE()
ID4=FREE()
ID5=FREE()
ID6=FREE()
ID7=FREE()
ID8=FREE()
ID9=FREE()
ID10=FREE()
ID11=FREE()
ID12=FREE()
ID13=FREE()
ID14=FREE()
ID15=FREE()
ID16=FREE()
ID17=FREE()
ID18=FREE()
ID19=FREE()
ID20=FREE()
ID21=FREE()
ID22=FREE()
ID23=FREE()
ID24=FREE()
ID25=FREE()
*RETURN

$--------------------------------------------------------------- 10$
$*$
*(CALC OUDV) ENDDUV$CALCULATE DU/DCAPV
IA=NLST
*LABEL 5666
*JLZ=-1(A-5667) $LOOP OVER LOAD SETS
INNSt=NLST-A
IIIV=0
IIIEV=NDV
*LABEL 7777
*JLZ=-1(IIIEV-7778) $LOOP OVER DESIGN VARIABLES
IIIV=IIIV+1
INNSt
*XOT AUS
DEFINE U=PREV DISP "NNST" 1
DEFINE OK=OKDV SPAR "NN3" "IIIV"
OK=PROD(OK1,UL)
*XOT DCU
$PRINT 1 DKU MASK
$PRINT 1 PKEV DISP "NNST" 1
*XOT AUS
IS=IIIV-1
*JGZ(I5,825)
APPL FORCE 50=UNION(-1, DKU)
*XOT DCU
*JUMP 826
*LABEL 825
DEFINE AF2=APPL FORCE 50

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```plaintext
APPENDIX

AF2 TMP=UNION(AF2,-1, DKU)
  *XOT DCU
  CHANGE 1 AF2 TMP 1,APPL FORCE 50 1
  *PRINT 1 APPL FORCE 50 1
  *LABEL 926
  *JUMP 7777
  *LABEL 7778
  *XOT SSUL
  RESET SET*50
  RESLT *=KP
  *XOT DCU
  TOC 1
  CHANGE 1 STAT DISP 50 HASK,DUDV CAPV "NST" 1
  *JNZ(DISP,263)
  *JNZ(SRHD,263)
  *JUMP 265
  *LABEL 263
  *JUMP 5666
  *LABEL 5667
  *RETURN

ENDUV

*TRANS DESV ENDTRS
**TRANSFORMS ROW VECTOR OF D.V.'S INTO A COLUMN VECTOR
*XOT AUS
  DEFINE DVS=DESV CNMN
  TABLE[NJ="NDVI",NJ=1];DESV TMP
  TRANSFER(SOURCE=DVS,NSEQ="NDVI")
  *RETURN
  ENDTRS

*LINK POLY ENDLKP
**$NONLINEAR POLYNOMIAL LINKING
  !I1=0
  !I2=0
  !NPL=NPOL$POWER OF POLYNOMIAL
  !NSEQ=0
  *XOT AUS
  !XX=TOC;NJ11=CDEF,DV,1,0),NSEQ
  !JLZ(NSEQ=1301)
  !I2=I2+1
  DEFINE CO=CDEF DV 1 0
  LNK DV=UNION(CL)
  *LABEL 1301
  !JLZ=-1(NPL,1300)
  !I1=I1+1
  !NSEQ=0
  !XX=TOC;NJ11=CDEF,DV,1,"ILL")',NSEQ
  !JLZ(NSEQ=1301)
  !I2=I2+1
  DEFINE V=DESV TMP
  DEFINE C=CDEF DV 1 "ILL"
  !NIL1=I1+1,
  Z=POWER(V,"NIL1")
  %V=CBR(C,Z)
  TABLE[NJ="NDV",NM])I TMP1
  TRANSFER(SOURCE=DVNC,NSEQ="NDV")
  !NIL2=I2
  *JNZ,-1(NIL2,1302)
  LNK DV=UNION(TMPL)
  *JUMP 1301
  *LABEL 1302
  LNK DV=SUM(TMPL, LNK DV)
  *JUMP 1301
  *LABEL 1300
  !I1=FREE()
  !I2=FREE()
  !NPL=FREE()
  !XX=FREE()
  !NIL1=FREE()
  !NIL2=FREE()
  *RETURN
  ENDLKP

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```
APPENDIX

*(DVP FDO) ENDFDAS UPDATE ALL MEMBRANE DESIGN VARIABLES
INIV=IEV
*LABEL 900
*JL2=1(IEV,901)
INIV=NIV=IEV
ITYP=DS,1,"NIV",111,DVA,DFN,MASK,MASK
!SECT=DS,1,"NIV",111,DVA,DFN,MASK,MASK
!UDVP=DS,1,"NIV",111,LNK,DV,MASK,MASK
IRR=DVP
!DCALL(FD LP1)
*JUMP 900
*LABEL 901
!SECT=FREE()
!IRR=FREE()
!IDDVP=FREE()
INIV=FREE()
*RETURN
ENDFA

*(FD LP1) ENDFDL
$ UPDATE MEMBRANE AND ROD ELEMENT PROPERTIES TO $ VALUE RR WHICH IS DETERMINED PREVIOUSLY
INTYP="TYP"
*JN2=-1(INTYP,1802)$TEST IF E23 ELEMENT
$SET ROD SECTION PROPERTIES
E23 SECTION PROPERTIES
"SECT" "RR"
*JUMP 1803
*LABEL 1802
INTYP="TYP"
*JN2=-2(INTYP,1804)$TEST IF E31 ELEMENT
$SET MEMBRANE SECTION PROPERTIES
SA
"SECT" "RR"
*JUMP 1803
*LABEL 1804
INTYP="TYP"
*JN2=-5(INTYP,1807)$TEST IF E41 ELEMENT
$SET MEMBRANE SECTION PROPERTIES
SA
"SECT" "RR"
*JUMP 1803
*LABEL 1807
INTYP="TYP"
*JN2=-8(INTYP,1810)$TEST IF E44 ELEMENT
$SET SHEET PANEL PROPERTIES
SB
"SECT" "RR"
*JUMP 1803
*LABEL 1810
*RT EXIT
*LABEL 1803
*RETURN
ENDFDL

*(DVP FDO) ENDFDPS UPDATE ALL PLATE DESIGN VARIABLES
INIV=IEV
*LABEL 902
*JL2=1(IEV,903)
INIV=NIV=IEV
ITYP=DS,1,"NIV",111,DVP,DFN,MASK,MASK
!SECT=DS,4,"NIV",111,DVP,DFN,MASK,MASK
!UDVP=DS,1,"NIV",111,LNK,DV,MASK,MASK
IRR=DVP
!DCALL(FD LP2)
*JUMP 902
*LABEL 903
INIV=FREE()
ITYP=FREE()
APPENDIX

1SECT=FREE()
INRIV=FREE()
IDDVP=FREE()
IKR=FREE()
RETURN
*
ENDFD0
$

*(FV LPZ)  ENFD0
1NTYP="TPK"
*JZ=3(1NTYP,600) TEST IF E32 ELEMENT
*JZ=1(1NTYP,600) TEST IF E33 ELEMENT
*JZ=2(1NTYP,600) TEST IF E42 ELEMENT
*JZ=1(1NTYP,600) TEST IF E43 ELEMENT
*EXIT
*LABEL 900
*DCALL(SA TYP)
*RETURN
*
ENDFD0
$

*(SA TYP)  ENOSAT
SA
"SECT" "RR"
*RETURN
*
ENDOSAT
$

*(DV FDD)  ENOFDB$ UPDATE ALL BEAM DESIGN VARIABLES
1SECI=0
INIV=1EV
*LABEL 904
*JZ=1(EIV,90S)
INIV=HIV=1EV
1SECT=US,NS="NHIV",11,DV,DFN,MASK,MASK
INIV=NHIV=NDV-A+NDV
1DDVP=DS,1="NHIV",11,DLK,DV,MASK,MASK
1RR=DDVP
*Gcall(8A TP2)
*JUMP 900
*LABEL 905
1SEL1=FREE()
INIV=FREE()
INIV=FREE()
1SECT=FREE()
INRIV=FREE()
IDDVP=FREE()
IKR=FREE()
*ETLAN
*
ENDFD
$

*(8A TP2)  ENDBA2$ UPDATES BEAM DESIGN VARIABLES
1SEC2=SECT-SEC1
*JZ(SEC2,1000)
1DV1=DS,20,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV2=DS,27,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV3=DS,26,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV4=DS,29,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV5=DS,30,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV6=DS,31,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV7=DS,41,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV8=DS,51,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV9=DS,61,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV10=DS,71,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV11=DS,81,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV12=DS,91,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV13=DS,101,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV14=DS,111,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV15=DS,121,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV16=DS,131,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV17=DS,141,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV18=DS,151,1,"SECT"(1,8A,DTAB,MASK,MASK)
1DV19=DS,161,1,"SECT"(1,8A,DTAB,MASK,MASK)
APPENDIX

ID120=DS,18,1,"SECT"(I,8A,BTAB,MAKd,MASK)
ID121=DS,19,1,"SECT"(I,8A,BTAB,MAKd,MASK)
ID122=DS,20,1,"SECT"(I,8A,BTAB,MAKd,MASK)
ID123=DS,21,1,"SECT"(I,8A,BTAB,MAKd,MASK)
ID124=DS,22,1,"SECT"(I,8A,BTAB,MAKd,MASK)
ID125=DS,23,1,"SECT"(I,8A,BTAB,MAKd,MASK)
ID126=DS,24,1,"SECT"(I,8A,BTAB,MAKd,MASK)
ID127=DS,25,1,"SECT"(I,8A,BTAB,MAKd,MASK)
*LABEL 1000
*XQT 01
*SHQ
*XQT TAB
*UPD=1
IDVB1=DS,1,"NNIV",11,DFN,DFN,MASK,MASK
IDVB2=DS,5,"NNIV",11,DFN,DFN,MASK,MASK
*JNZ,-1(DVB2,1001)
ID1=RR
*JUMP 1007
*LABEL 1001
*JNZ,-1(DVB2,1002)
ID12=RR
*JUMP 1007
*LABEL 1002
*JNZ,-1(DVB2,1003)
ID13=RR
*JUMP 1007
*LABEL 1003
*JNZ,-1(DVB2,1004)
ID14=RR
*JUMP 1007
*LABEL 1004
*JNZ,-1(DVB2,1005)
ID15=RR
ID16=106
*JUMP 1007
*LABEL 1005
*JNZ,-1(DVB2,1006)
ID16=RR
*JUMP 1007
*LABEL 1006
*JNZ,-1(DVB2,1030)
ID17=RR
*JUMP 1007
*LABEL 1030
*JNZ,-1(DVB2,1031)
ID18=RR
*JUMP 1007
*LABEL 1031
*JNZ,-1(DVB2,1032)
ID19=RR
*JUMP 1007
*LABEL 1032
*JNZ,-1(DVB2,1033)
ID10=RR
*JUMP 1007
*LABEL 1033
*JNZ,-1(DVB2,1034)
ID11=RR
*JUMP 1007
*LABEL 1034
*JNZ,-1(DVB2,1035)
ID12=RR
*JUMP 1007
*LABEL 1035
*JNZ,-1(DVB2,1036)
ID13=RR
*JUMP 1007
*LABEL 1036
*JNZ,-1(DVB2,1037)
ID14=RR
*JUMP 1007
*LABEL 1037
*JNZ,-1(DVB2,1038)
ID15=RR
*JUMP 1007
*LABEL 1038
APPENDIX

*JNZ,-1(DVB2,1039)
IDI16*RR
*JUMP 1007
*LABEL 1039
*JNZ,-1(DVB2,1040)
IDI17*RR
*JUMP 1007
*LABEL 1040
*JNZ,-1(DVB2,1041)
IDI18*RR
*JUMP 1007
*LABEL 1041
*JNZ,-1(DVB2,1042)
IDI19*RR
*JUMP 1007
*LABEL 1042
*JNZ,-1(DVB2,1043)
IDI20*RR
*JUMP 1007
*LABEL 1043
*JNZ,-1(DVB2,1044)
IDI21*RR
*JUMP 1007
*LABEL 1044
*JNZ,-1(DVB2,1045)
IDI22*RR
*JUMP 1007
*LABEL 1045
*JNZ,-1(DVB2,1046)
IDI23*RR
*JUMP 1007
*LABEL 1046
*JNZ,-1(DVB2,1047)
IDI24*RR
*JUMP 1007
*LABEL 1047
*JNZ,-1(DVB2,1048)
IDI25*RR
*JUMP 1007
*LABEL 1048
*JNZ,-1(DVB2,1049)
IDI26*RR
*JUMP 1007
*LABEL 1049
*JNZ,-1(DVB2,1050)
IDI27*RR
*JUMP 1007
*LABEL 1050
*XQT EXIT
*LABEL 1057
*JNZ,-1(DVB1,1008)
*DCALL(ANG BEAM)
*JUMP 1016
*LABEL 1008
*JNZ,-1(DVB1,1009)
*DCALL(BOX BEAM)
*JUMP 1016
*LABEL 1009
*JNZ,-1(DVB1,1010)
*DCALL(FZT BEAM)
*JUMP 1016
*LABEL 1010
*JNZ,-1(DVB1,1011)
*DCALL(ANG BEAM)
*JUMP 1016
*LABEL 1011
*JNZ,-1(DVB1,1012)
*DCALL(WFL BEAM)
*JUMP 1016
*LABEL 1012
*JNZ,-1(DVB1,1013)
*DCALL(CHN BEAM)
*JUMP 1016
*LABEL 1013
*JNZ,-1(DVB1,1014)
*DCALL(ZCZ BEAM)
*JUMP 1016
*LABEL 1014
APPENDIX

*JNZ,-1(DVB1,1015)
*DCALL(GIVN BEAM)
*JUMP 1016
*LABEL 1015
*JNZ,-1(DVB1,1020)
*DCALL(0SY BEAM)
*JUMP 1016
*LABEL 1020
*X@T EXIT
*LABEL 1016
ISEC1="SECT"
ISEC2=FREE()
IDVB1=FREE()
IDVB2=FREE()
*RETURN
$-------------------------------------------------------------
$ *(TUBE BEAM) ENDTUB\$ SET TUBE SECTION PROPERTIES
BA TUBE "SECT","D11","D12"
*RETURN
*
$-------------------------------------------------------------
$ *(BOX BEAM) ENDBOB\$ SET BOX SECTION PROPERTIES
BA BOX "SECT","D11","D12","D13","D14"
*RETURN
*
$-------------------------------------------------------------
$ *(TEE BEAM) ENDTEB\$ SET TEE SECTION PROPERTIES
BA TEE "SECT","D11","D12","D13","D14"
*RETURN
*
$-------------------------------------------------------------
$ *(ANG BEAM) ENDANB\$ SET ANG SECTION PROPERTIES
BA ANG "SECT","D11","D12","D13","D14"
*RETURN
*
$-------------------------------------------------------------
$ *(WFL BEAM) ENDWFB\$ SET WFL SECTION PROPERTIES
BA WFL "SECT","D11","D12","D13","D14","D15","D16"
*RETURN
*
$-------------------------------------------------------------
$ *(CHN BEAM) ENDCHB\$ SET CHN SECTION PROPERTIES
BA CHN "SECT","D11","D12","D13","D14","D15","D16"
*RETURN
*
$-------------------------------------------------------------
$ *(ZEE BEAM) ENDZEB\$ SET ZEE SECTION PROPERTIES
BA ZEE "SECT","D11","D12","D13","D14","D15","D16"
*RETURN
*
$-------------------------------------------------------------
$ *(GIVN BEAM) ENDGVB\$ SET GIVN SECTION PROPERTIES
BA
APPENDIX

GIVN "SCGT","D17","D18","D19","D10","D11","D12","D13","D14"% "D15","D16"
*RETURN
* ENDSY

$------------------------------------------------------------------
$ *(DSY BEAM) ENDSY
BA
DSY "SCGT","D17","D18","D19","D10","D11","D12","D13"% "D17","D16","D19","D12","D11","D13","D14"% "D15","D16","D17","D18"
*RETURN
* ENDSY

$------------------------------------------------------------------ 15
$ *(EFIL JCW) ENDEDU
*X GT D1C
*JIZE81,8010) DISABLE 1 E21 EFIL MASK MASK
CHANGE 1 EE21 EEIL 1 "IIV",E21 EFIL 1 2
*LABEL 8010
*JIZE82,8011) DISABLE 1 E23 EFIL MASK MASK
CHANGE 1 EE23 EEIL 2 "IIV",E23 EFIL 2 3
*LABEL 8011
*JIZE83,8012) DISABLE 1 E31 EFIL MASK MASK
CHANGE 1 EE31 EEIL 6 "IIV",E31 EFIL 6 3
*LABEL 8012
*JIZE84,8013) DISABLE 1 E32 EFIL MASK MASK
CHANGE 1 EE32 EEIL 7 "IIV",E32 EFIL 7 3
*LABEL 8013
*JIZE85,8014) DISABLE 1 E33 EFIL MASK MASK
CHANGE 1 EE33 EEIL 8 "IIV",E33 EFIL 8 3
*LABEL 8014
*JIZE86,8015) DISABLE 1 E41 EFIL MASK MASK
CHANGE 1 EE41 EEIL 9 "IIV",E41 EFIL 9 4
*LABEL 8015
*JIZE87,8016) DISABLE 1 E42 EFIL MASK MASK
CHANGE 1 EE42 EEIL 10 "IIV",E42 EFIL 10 4
*LABEL 8016
*JIZE88,8017) DISABLE 1 E43 EFIL MASK MASK
CHANGE 1 EE43 EEIL 11 "IIV",E43 EFIL 11 4
*LABEL 8017
*JIZE89,8018) DISABLE 1 E44 EFIL MASK MASK
CHANGE 1 EE44 EEIL 12 "IIV",E44 EFIL 12 4
*LABEL 8018
*RETURN
* ENDEDU

$------------------------------------------------------------------
$ *(DCU EFIL) ENDEDC
*X GT D1C
*JIZE90,8000) CHANGE 1 E21 EFIL 1 "IIV",E21 EFIL 1 2
*LABEL 8000
*JIZE91,8001) CHANGE 1 E23 EFIL 3 "IIV",E23 EEIL 3 2
*LABEL 8001
*JIZE92,8002) CHANGE 1 E31 EEIL 6 "IIV",E31 EEIL 6 3
*LABEL 8002
*JIZE93,8003) CHANGE 1 E32 EEIL 7 "IIV",E32 EEIL 7 3
*LABEL 8003
*JIZE94,8004) CHANGE 1 E33 EEIL 8 "IIV",E33 EEIL 8 3
*LABEL 8004
*JIZE95,8005)
APPENDIX

CHANGE 1 E41 EFIL 9 4,EE41 EEIL 9 "IIV"
*LABEL 8005
*JZ(DE42*8006)
CHANGE 1 E42 EFIL 10 4,EE42 EEIL 10 "IIV"
*LABEL 8006
*JZ(DE43*8007)
CHANGE 1 E43 EFIL 11 4,EE43 EEIL 11 "IIV"
*LABEL 8007
*JZ(DE44*8008)
CHANGE 1 E44 EFIL 12 4,EE44 EEIL 12 "IIV"
*LABEL 8008
*RETURN
*ENDDEL
$
$------------------------------------------------------------------------------------------------------------------ 16$
$
#(STRS DERV) ENDS
## $ CALCULATE STRESS DERIVATIVES
$IIE1=NLST$LOOP OVER LOAD SETS
*LABEL 2102
*JLZ=-1(IIE1,2103)
IIE1=NLST-IIE1#LOAD SET NUMBER
IIIEV=NDV
IIS=0
IISS=0
*LABEL 2100
*JLZ=-1(IIEV,2101)$LOOP OVER DESIGN VARIABLES
IIIEV=NDV-11IEV
DVP=DS,1,"IIIEV",11,DES,CMN,MASK,MASK)
DCALL(EEIL DCU)
I DELV=UR*DVP
I AQ=1,0DELV
IBQ=-1,AQ
INCA=LCAS$ LOOP OVER LOAD CASES
*LABEL 3000
*JLZ=-1(INCA,3001)
*XQT AUS
INCA=LCAS=NCAS
I1=IIIEV*LCAS=NCAS
DEFINE U1=PREV DISP "NIE1" 1 "NCAS","NCAS"
DEFINE U2=DUUVP CAPV "NIE1" 1 "N11","N11"
*JZI(ISS,3825)
U DELU "NIE1" 1 *SUM(U1, "DELU" U2)
IIS=IIS+1
*JUMP 3026
*LABEL 3026
DEFINE TEMP=U DELU "NIE1" 1
DU=SUM(U1, "DELV" U2)
UDEL TMP=UNION TEMP,DU)
*XQT DCU
CHANGE 1 UDEL TMP 1 1,U DELU "NIE1" 1
*LABEL 3026
*JUMP 3000
*LABEL 3001
INAM1=U
INAM2='DELU
INAM3="NIE1"
INAM4="MASK"
DCALL(CALC STRS)
IIS=0
IIS=IIS+1
DCALL(DCU EFIL)
*JUMP 2100
*LABEL 2101
IIIEV=FREE()
IIIEV=FREE()
IIIEV=FREE()
INCA=FREE()
I1=FREE()
IIIEV=FREE()
INCA=FREE()
INAM1=FREE()
INAM2=FREE()
INAM3=FREE()
INAM4=FREE()
*XQT AUS
OUTLIB=1
*JZ(DE41*2104)
APPENDIX

!I\E-N\E21
INAME=-E21
*DCALL(0SIG EIJ)
*LABEL 2104
*JZ(DE23,2105)
*IIE=DE23
INAME=E23
*DCALL(0SIG EIJ)
*LABEL 2105
*JZ(DE31,2106)
*IIE=DE31
INAME=E31
*DCALL(0SIG EIJ)
*LABEL 2106
*JZ(DE32,2107)
*IIE=DE32
INAME=E32
*DCALL(0SIG EIJ)
*LABEL 2107
*JZ(DE33,2108)
*IIE=DE33
INAME=E33
*DCALL(0SIG EIJ)
*LABEL 2108
*JZ(DE41,2109)
*IIE=DE41
INAME=E41
*DCALL(0SIG EIJ)
*LABEL 2109
*JZ(DE42,2110)
*IIE=DE42
INAME=E42
*DCALL(0SIG EIJ)
*LABEL 2110
*JZ(DE43,2111)
*IIE=DE43
INAME=E43
*DCALL(0SIG EIJ)
*LABEL 2111
*JZ(DE44,2112)
*IIE=DE44
INAME=E44
*DCALL(0SIG EIJ)
*LABEL 2112
*JUMP 2102
*LABEL 2103
!IIE1=FREE()
!IIE1=FREE()
!IIE=FREE()
!INAME=FREE()
*RETURN
*
ENDSTD

$*\CALC STRS)$
$*ENDCCS$
$$\% Calculates stresses of design variables in$$
$$\% ES format to be used in stress derivative.$$$$
$$\% and constraint calculations.$$

*NOT ES
OUTLIB= "OLIB"
U= "NAM1" "NAM2" "NAM3" "NAM4"
PMODE= 2
NDDES= 0
*J2(NE21,2500)
!IIE=NE21
INAME=E21
*DCALL(STRS GRP)
*LABEL 2500
*J2(DE23,2501)
!IIE=DE23
INAME=E23
*DCALL(STRS GRP)
*LABEL 2501
*J2(DE31,2502)
!IIE=DE31
INAME=E31

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APPENDIX

*DCALL (STRS GRP)
*LABEL 2502
*JZ(DE32,2503)
IIE=DE32
INAMS=DE32
*DCALL (STRS GRP)
*LABEL 2503
*JZ(DE33,2504)
IIE=DE33
INAMS=DE33
*DCALL (STRS GRP)
*LABEL 2504
*JZ(DE41,2505)
IIE=DE41
INAMS=DE41
*DCALL (STRS GRP)
*LABEL 2505
*JZ(DE42,2506)
IIE=DE42
INAMS=DE42
*DCALL (STRS GRP)
*LABEL 2506
*JZ(DE43,2507)
IIE=DE43
INAMS=DE43
*DCALL (STRS GRP)
*LABEL 2507
*JZ(DE44,2508)
IIE=DE44
INAMS=DE44
*DCALL (STRS GRP)
*LABEL 2508
*DCALL (CAL STRS)
IIE=FREE()
INAMS=FREE()
*RETURN
*
ENDCCS

*CAL STRS
*DCALL
*JZ(NE21,2500)
IIE=NE21
INAMS=NE21
*DCALL (STRS CALC)
*LABEL 2500
*JZ(DE23,2501)
IIE=DE23
INAMS=DE23
*DCALL (STRS CALC)
*LABEL 2501
*JZ(DE31,2502)
IIE=DE31
INAMS=DE31
*DCALL (STRS CALC)
*LABEL 2502
*JZ(DE32,2503)
IIE=DE32
INAMS=DE32
*DCALL (STRS CALC)
*LABEL 2503
*JZ(DE33,2504)
IIE=DE33
INAMS=DE33
*DCALL (STRS CALC)
*LABEL 2504
*JZ(DE41,2505)
IIE=DE41
INAMS=DE41
*DCALL (STRS CALC)
*LABEL 2505
*JZ(DE42,2506)
IIE=DE42
INAMS=DE42
*DCALL (STRS CALC)
*LABEL 2506
*JZ(DE43,2507)
IIE=DE43

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APPENDIX

INAM5=E43
*DCALL(STRS CALC)
*LABEL 2507
*JZ(IE=E44,2508)
IE=E44
INAM5=E44
*DCALL(STRS CALC)
*LABEL 2508
IE=FREE()
INAM5=FREE()
*RETURN
*
ENDCALL
$
$-----------------------------------------------------------
$
*(STRS CALC)  ENDS
!NAMJ=IE
*LABEL 2500
*JZ(IE=NAMJ,2501)
!IV=IE-NAMJ
*DCALL(STRS BLD)
*JUMP 2500
*LABEL 2501
!NAMJ=IE
*LABEL 2600
*JZ(IE=NAMJ,2601)
!IV=IE-NAMJ
*DCALL(NEW NAME)
*JUMP 2600
*LABEL 2601
!NAMJ=FREE()
!IV=FREE()
*RETURN
*
ENDS
$
$-----------------------------------------------------------
$
*(STRS BLD)  STRS8
*JZ(N11,5000)
*XQT AUS
INLIB="OLIB"
OUTLIB="OLIB"
$115
$IN11
$1OLIB
$INAM5
$INIE1
$1ISS
$IV
DEFINE ENE="OLIB" ES "NAM5" "NIE1" "IV"
$ENE=UNION(ETMP)
*JZ(155,2202)
DEFINE EOLD="OLIB" EST "NAM5" "NIE1" "IV"
$EOLD=UNION(ESSE)
EST"NAMS""NIE1""IV"=UNION(EOLD,ENEW)
*XQT OCU
CHANGE "OLIB" EST "NAMS" "NIE1" "IV",EST "NAM5" "NIE1" "IV"
$ TUC "OLIB"
$PRINT "OLIB" EST "NAM5" "NIE1" "IV"
*JUMP 3001
*LABEL 2202
EST "NAM5" "NIE1" "IV" = UNION(ENEW)
*XQT OCU
$ TUC "OLIB"
$PRINT "OLIB" EST "NAM5" "NIE1" "IV"
*LABEL 3001
*LABEL 5000
*RETURN
*
STRS8
$----------------------------------------------------------
$
*(STRS GRP)  ENDRP
!NAMJ=IE
*LABEL 1000
*JLZ-1(WORD,1001)
!IV=IE-NAMJ
"NAM5" "IV"
APPENDIX

*JUMP 1000
*LABEL 1001
*RETURN
*
ENDGRP
$
$-----------------------------------------------------------------------
$
*[CHNG NAME] ECHG
[NI1]
*JL(N11,5000)
IIV
IIIE
*J2(IIIEV,3000)
*JUMP 5000
*LABEL 3000
*XQT DCU
CHANGE "DLIB" EST "NAMS" "NIE1" "IV", ES "NAMS" "NIE1" "IV"
*LABEL 5000
*RETURN
*
ECHG
$
$-----------------------------------------------------------------------
$
#(SIG EIJ) ENDSIG
## "$ SUBPROGRAM USED TO CALCULATE STRESS DERIVATIVES
## "$ USING PRASAD'S METHOD
IIIEZ=IE
*LABEL 2200
*JL=-1(IEE2201)$$LODP OVER GROUPS
[NI1]I
IIV=IE2-IE$GROUP NUMBER
IIIEV=NDV
*LABEL 2100
IIV=NDV-IIIEV
IIVP=US1,"IV",111,DESV,CNMN,MASK,MASK)
IDELV=DRVDP
IAQ=1,ILDelv
IEG=-1,IEG
INCA=LCAS$$LODP OVER LOAD CASES
*LABEL 3000
*XQT AUS
*JL=-1(INCA,3001)
INCAS=LCAS-NCAS
IN1=IIV*LCAS-LCAS+NCAS
*XQT AUS
DEFINE STRP=1 ES "NAMS" "NIE1" "IV", "NAMS", "NCAS", "NCAS"
DEFINE STRN= "DLIB" ES "NAMS" "NIE1" "IV", "NIE1", "N11"
BB1=SUM("AQM STRN", "BG" STRP)
**XQT DCU
$ TDC 1
$PRINT 1 BB1 MASK MASK MASK
$XQT AUS
*JL=-1(INN1,2202)
[NI1]=INN1+1
DEFINE DSG=DSIG "NAMS" "NIE1" "IV"
$DSG=UNION(DSG)
$DSG=UNION(DSG,BB1)
DTMP "NAMS" "NIE1" "IV"=UNION(DSG,BB1)
*XQT DCU
CHANGE 1 DTMP "NAMS" "NIE1" "IV", DSIG "NAMS" "NIE1" "IV"
$ TDC 1
$PRINT 1 DSIG "NAMS" "NIE1" "IV"
**XQT AUS
*JUMP 3000
*LABEL 2202
INN1=INN1+3
DSIG "NAMS" "NIE1" "IV"=UNION(BB1)
**XQT DCU
$ TDC 1
$PRINT 1 DSIG "NAMS" "NIE1" "IV"
$XQT AUS
*JUMP 3000
*LABEL 3001
*JUMP 2100
*LABEL 2101
*JUMP 2200
*LABEL 2201

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APPENDIX

IIE2=FREE(I)
IIV=FREE(I)
IIIEV=FREE(I)
INCA=FREE(I)
INCA=FREE(I)
INCA=FREE(I)
RETURN
ENDSG

(*NELS METH) ENDNLN
** % CALCULATES DERIVATIVES OF VIB. OR BUCK. MODES
** % AND/OR LOAUS ANALYTICALLY.
*OCALL (BOND CONO)
IJDF=DS,2,1,11,JDF1,6TAB,1,6
INMD=MODE
ITT1=EQUAL(NAM9,MDE1)
ITT2=EQUAL(NAM9,MDE2)
*XQT AUS
INNDV=NNDV
*LABEL 4109
*JLZ2=1(INMD,4108)
IIV=NDV-NNDV
TABLE(NJ=1,NI="MODE");DIVAL "NAM9" 1 "IDV" 1=1;J=110,
*JUMP 4109
*LABEL 4108
*LABEL 2010
*JLZ2=1(INNDV,2000)$LOOP ON MODES
INM=MODE-NNMD
INNI=HM
INDM=DS,1,"NM";11,MODE,NUM,MASK,MASK
ISKP=MIST-MODN
IBOE=MIST-1
** FIND EIGENVALUE
*XQT AUS
DEFINE L=PREV "NAM7" 1 1
TABLE(N1=NJ1=1);LAM1 AUS 1 1
TRANSLUSURE=L,SBASE="IDSE",ILIM=1,SSKIP="ISKP"
*XQT U1
(LAMM=DS,1,111,LAM1,AUS,1,1)
LAMN=5*LAMM
*XQT AUS
DEFINE PP=PREV "NAM8" 1 1 "MODN","MODN"
PHI=UNION(1.0 PP)
*XQT DCU
PRINT 1 PHI
*XQT AUS
*JZT(TT1=100)
LDER DIAG 0 0=UNION("LAMM" DEMP)
DEFINE LDER=LDER DIAG
ASP SPAR=SUM(KFP,-1.0 LDER)
*LABEL 4100
*JZT(TT2=101)
LAMN=5*LAMM
LDER DIAG 0 0=SUM("LAMM" KGP, "LAMM" KGP)
ASP SPAR=SUM(KFP,1.0 LDER)
*LABEL 4101
PONE=UNION("LAMM" PHI)
INNDV=NNDV
*LABEL 2006
*JLZ2=1(INNDV,2001)$LOOP OVER DESIGN VARIABLES
IIV=NDV-NNDV
IJDF=DS,2,1,11,JDF1,6TAB,1,6
INDF=JDF=JDF
*JZT(TT3=2003)
DEFINE DM=OMDV DIAG 0 "IDV"
*LABEL 2003
*JZT(TT2=2004)
DEFINE DM=OKG SPAR "NDF" "IDV"
*LABEL 2004
DEFINE OK=OKDV SPAR "NDF" "IDV"
PTWO=PROD(DM,PONE)
PTRI=PROD(OK,PONE)
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TONE *PRD0(UEMP, VDER)
DEFINE DM=DMOV DIAG O "IDV"
*LABEL 4105
* J (IT1, 1106)
TONE *PRD0(KGP, VDER)
DEFINE DM=UKG SPAR "NDF" "IDV"
*LABEL 4106
TND=XTYO(PHI, TONE)
TTR1=PK1D(DM, PHI)
TFOR=XTYO(PHI,0.5 TTR1)
* J (IT1, 1108)
CEE=SUM(-1, TTWJ,1, TFOR)
*LABEL 4108
* J (IT1, 1109)
CEE=SUM(TTWJ, TFOR)
*LABEL 4109
* XOT U1
ICCE=DS, 1, 1, 11, CEE, MASK, MASK, MASK)
* XOT AUS
TFIV=UNION("CCE" PHI)
DMOD "NAM0" "MODN" "IDV"*SUM(VDER, TIV)
GGG=PRUV(ASP, DMOD "NAM0" "MODN" "IDV")
GGR=SUM(F, -1, GGG)
* XOT AUS
* JUMP 2009
* LABEL 2008
INNM=NM
* JNZ=-1(INNM, 2010)
* JUMP 2010
* LABEL 2000
INM=FREE() INM1=FREE() INNM=FREE() INM4=FREE()
INH=FREE() INH5=FREE() IN1=FREE() IN2=FREE() IN3=FREE()
INJN=FREE() INMM=FREE() INDG=FREE() INNVD=FREE() INNM=FREE()
*RETURN
*
ENDNL M
S
-------------------------------------------------------------------------------
S
*(BOND COND) END BCD
* XOT TAB
CON=1
ZERO 1, 2, 3, 4, 5, 6, 11, 2
ZERO 1, 2, 4, 5, 6, 15, 16
*RETURN
*
END BCD
S
------------------------------------------------------------------------------- 18
S
*(NELS MET) END NL2
** $ CALCULATE DERIVATIVES OF
** $ BUCKLING OR VIBRATION LOADS ANALYTICALLY.
** CALL (BOND COND)
* JDF=DS, 2, 1, 1, 1, JDFI, BTA, 1, 5
INNMD=MODE
* XOT AUS
INNVD=NOV
* LABEL 4111
* LABEL 411
* JLF=-1(INNDV, 4110)
IDV+NDV=NDV

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APPENDIX

TABLE(NJ=1,N1="MODE");DOVAL "NAM9" 1 "IDV"
I=1;J=1:0.
*JUMP 4111
*LABEL 4110
ITI=EQUAL(NAM9,MDE1)
ITZ=EQUAL(NAM9,MDE2)
*LABEL 2801
*JZJ=(NAM9,2800)!LUOP ON MODES
IN=MODE-NNMD
IN1=IN
IND=DS,1,"NM",11,MODE,NUM,MASK,MARK)
!ISKP=MODT-MODN
!IBSE=MODN-1
$ FIND EIGENVALUE
*XQ AUS
DEFINE L=PREV "NAM7" 1 1
TABLE(N1=1,NJ=1);LAM1 AUS 1 1
TRAN(SOURCE=L;SBASE="IBSE",ILIM=1;SSKIP="ISKP")
*XQ U1
!LAMM=DS,1,1;1(1,LAM1,AUS,1,1)
!LAMM=LAMM*.5
*XQ AUS
DEFINE PP=PREV "NAM8" 1 1;MODN,"MODN"
PHI=UNION(1,0 PP)
*JZIT=1.4000
LDEM DIAG 0 0=UNION("LAMM" DEMP)
ASP SPAR=SUM(KP,-1. LDEM)
*LABEL 4000
*JZIT2=1.4001
LDEM DIAG 0 0=SUM("LAMM" KGP,"LAMN" KGP)
ASP SPAR=SUM(KP,LDEM)
*LABEL 4001
PONE=UNION("LAMM" PHI)
IND=MODN
*LABEL 2806
*JZJ=(NAM9,2801)
"IDV"=MODN-INDV
1JDF=DS,2,1,1(1,JDF1,BTAB,1,1)
IND=JDF-1JDF
*JZIT1=1.2803)
DEFINE DM=MODV DIAG 0 "IDV"
*LABEL 2803
*JZIT2=1.2804)
DEFINE DM=KG SPAR "MODN" "IDV"
*LABEL 2804
DEFINE DK=MODV SPAR "MODN" "IDV"
PTW=PKDD(OM,PONE)
PT1=PKDD(OM,PHI)
PF=XTYD(PT1,PT1)
PF=XTYD(PT1,PTW)
*JZIT1=1.4000)
PSIX=SUM(PF=1, PF)
*LABEL 4000
*ZIT2=1.4401
PSIX=SUM(PF=PF, PF)
*LABEL 4401
EDER DIAG "MODN" "IDV"=UNION(1,0 PSIX)
1EDG=DS,1,1,1(1,EDER,DIAG,"MODN" "IDV")
TABLE(N1=1,N1="MODE");DOVAL "NAM9" 1 "IDV"
UPKATION<SUM
1="MODN";J=1;"EDGG"
*JUMP 2806
*LABEL 2807
INM=FREE()
INM=FREE()
IN=FREE()
IND=FREE()
IND=FREE()
!IBSE=FREE()
!LAMM=FREE()
!IDV=FREE()
!IND=FREE()
*JITI=FREE()
*JIT2=FREE()
!*N=FREE()
*JUMP 2801
*LABEL 2800
*LABEL 2807
*RETURN

ENDNLZ

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This runstream has been deleted.

This runstream has been deleted.
APPENDIX

*DCALL(PRIN OSIG)
*LABEL 3111
*JZ(IE44,3112)
IE=GE44
NAM5=DE44
*DCALL(PRIN OSIG)
*LABEL 3112
IE1=FREE()
NIE1=FREE()
I1=FREE()
NAM5=FREE()
*RETURN
ENDPS
$-----------------------------------------------
$ *(PRIN OSIG) ENDPS
IE2=IE
*LABEL 3113
*JLZ=1(IE,3114)$ LOOP OVER GROUP NUMBER
IV=IE2=IE$GROUP NUMBER
IE1=NLST$ LOOP OVER LOAD SETS
*LABEL 3102
*JLZ=1(IE1,3103)$LOAD SET NUMBER
IE1=NLST-IE1
PRINT 1 OSIG "NAM5" "NIE1" "IV"
JUMP 3102
*LABEL 3103
*JUMP 3113
*LABEL 3114
*RETURN
ENDPS
$-----------------------------------------------
$ *(PRIN DMOD) ENDPS
$ SS PRINT VIBRATION AND OR BUCKLING DERIVATIVES
IN1=NDV
*LABEL 3120
*JLZ=1(IN1,3121)$ LOOP OVER DESIGN VARIABLES
IV=NDV=IN1$DESIGN VARIABLE NUMBER
INMD=MODE
*LABEL 3122
IT1=EQUAL(NAM9,MDE1)
IT2=EQUAL(NAM9,MDE2)
*JZ(IT1),3125
IN1=VIBD
*JZ=2(IN1,3123)
*LABEL 3125
*JZ(IT2),3126
IN1=UCD
*JZ=2(IN1,3123)
*LABEL 3126
*JLZ=1(INMD,3123)$ LOOP OVER MODES
INM=MODE-NDV
INMOD=DS,1,"NAM",1,MODE,NUM,NUM,NUM,NUM,NUM,NUM
MODE NUMBER
PRINT 1 DMOD "NAM9" "MODN" "IV"
JUMP 3126
*LABEL 3123
IT1=EQUAL(NAM9,MDE1)
IT2=EQUAL(NAM9,MDE2)
*JZ(IT1),3127
IN1=VIBD
*JZ=4(IN1,3120)
*LABEL 3127
*JZ(IT2),3128
IN1=UCD
*JZ=4(IN1,3120)
*LABEL 3128
PRINT 1 DMOD "NAM9" 1 "IV"
JUMP 3120
*LABEL 3120
IN1=FREE()
IV=FREE()
IT1=FREE()
IT2=FREE()
INN1=FREE()
IN1=FREE()
REFERENCES


### TABLE 1.- FUNCTIONS OF EAL PROCESSORS

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<thead>
<tr>
<th>Processor name</th>
<th>Function</th>
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<tbody>
<tr>
<td>TAB</td>
<td>Creates data sets containing tables of joint locations, section properties, material constants, and so forth</td>
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<tr>
<td>ELD</td>
<td>Defines the finite-element connections in model</td>
</tr>
<tr>
<td>E</td>
<td>Generates sets of information for each element, including connected joint numbers, geometrical data, material, and section property data</td>
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<tr>
<td>EKS</td>
<td>Adds the stiffness and stress matrices for each element to the set of information produced by the E processor</td>
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<tr>
<td>SEQ</td>
<td>Determines joint sequences, i.e., equation numbering sequences to be used in sparse matrix solution methods</td>
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<tr>
<td>TOPO</td>
<td>Analyzes element interconnections and topology and creates data sets used to assemble and factor the system mass and stiffness matrices</td>
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<tr>
<td>K</td>
<td>Assembles the unconstrained system stiffness matrix in a sparse matrix format</td>
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<tr>
<td>M</td>
<td>Assembles the unconstrained system mass matrix in a sparse matrix format</td>
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<tr>
<td>KG</td>
<td>Assembles the unconstrained system initial-stress (geometric stiffness) matrix in a sparse matrix format</td>
</tr>
<tr>
<td>INV</td>
<td>Factors the assembled system matrices</td>
</tr>
<tr>
<td>RSI</td>
<td>Similar to INV</td>
</tr>
<tr>
<td>DRSI</td>
<td>Similar to RSI; factors double precision SPAR format matrices</td>
</tr>
<tr>
<td>EQNF</td>
<td>Computes equivalent joint loading associated with thermal, dislocational, and pressure loading</td>
</tr>
<tr>
<td>SSOL</td>
<td>Computes displacements and reactions due to applied loading at the joints</td>
</tr>
<tr>
<td>TAN</td>
<td>Similar to TOPO</td>
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<tr>
<td>LSK</td>
<td>Forms partial or complete system stiffness (K) and damping (D) matrices in a sparse matrix form called LS-format</td>
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<td>LSU, RMK</td>
<td>Translates arbitrary source K and M data into LS- or SPAR-format and transforms SPAR- and LS-format matrices</td>
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<td>GSF</td>
<td>Generates element stresses and internal loads</td>
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<td>PSF</td>
<td>Prints the information generated by the GSF processor</td>
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<td>ES</td>
<td>Analyzes element interior state, given joint displacements, and initial strains, if present; creates and stores stresses and internal load data</td>
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<td>Processor name</td>
<td>Function</td>
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<td>----------</td>
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<tr>
<td>EIG</td>
<td>Solves linear vibration and bifurcation buckling eigenproblems</td>
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<td>Similar to EIG</td>
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<td>Produces mass and stiffness matrices for systems comprised of interconnected substructures</td>
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<tr>
<td>STRP</td>
<td>Computes eigenvalues and eigenvectors of substructured systems</td>
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<tr>
<td>SSBT</td>
<td>Back-transforms synthesized system results into individual substructure terms used in conjunction with SYN and STRP</td>
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<tr>
<td>AUS</td>
<td>Performs matrix arithmetic functions and is used in construction, editing, and modification of data sets</td>
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<tr>
<td>DCU</td>
<td>Performs data management functions including display of table of contents, data transfer between libraries, changing data set names, printing data sets, and transferring data between libraries and sequential files</td>
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<td>VPRT</td>
<td>Performs editing and printing of data sets which are in the form of vectors on the data libraries</td>
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<td>PLTA, PLTB, PXY</td>
<td>Produce graphic displays of finite-element models and computed results such as vibration and buckling modes, stresses, and response histories</td>
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<tr>
<td>PR</td>
<td>Generates reports of dynamic response analysis</td>
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<tr>
<td>DR</td>
<td>Computes linear transient modal response and back-transforms to determine any required system response details and maximum-minimum and time-of-occurrence data</td>
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<td>U1</td>
<td>Creates, edits, and manipulates runstreams and permits direct table input</td>
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<td>U3/RP2</td>
<td>Produces tabular multipage reports, using formats, headings, and footnotes prescribed by the user at execution time</td>
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<td>U4/VU</td>
<td>Enables vector arithmetic functions</td>
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<td>FSM</td>
<td>Creates SPAR-format matrices for compressible fluid elements</td>
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<td>PS</td>
<td>Prints SPAR-format matrices and factored system matrices produced by RSI or DRSI</td>
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<td>Description</td>
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<td>E23</td>
<td>Rod element</td>
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<td>Triangular and quadrilateral membrane elements, respectively</td>
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<tr>
<td>E32, E42</td>
<td>Triangular and quadrilateral uncoupled (bending only) plate elements, respectively</td>
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<tr>
<td>E33, E43</td>
<td>Triangular and quadrilateral coupled (membrane and bending) plate elements, respectively</td>
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<td>Shear panel element</td>
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TABLE 4. - DESIGN VARIABLE GROUPS FOR SWEPT WING

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TABLE 5.- LOAD DATA FOR SWEPT WING

\[ f_x = 0 \text{ and } f_y = 0 \]

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<th>( f_x )</th>
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TABLE 6.- SUMMARY OF SENSITIVITY ANALYSIS FOR SWEPT WING

88 nodes; 194 elements; 2 load conditions; 32 design variables; 164 degrees of freedom

(a) Convergence for displacement derivatives

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<th>Error, a percent for -</th>
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aRelative to analytical method.

(b) Solution time for displacement derivatives

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<td>Semianalytical with LSK processor</td>
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<td>Analytical</td>
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TABLE 7.- SUMMARY OF SENSITIVITY ANALYSIS FOR BOX BEAM

28 nodes; 77 elements; 1 load condition; 9 independent design variables; 10 dependent design variables; 122 degrees of freedom

(a) Solution time comparison

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<th>Solution time, sec, for displacement derivatives</th>
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(b) Typical values of derivatives

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TABLE 9.- SUMMARY OF SENSITIVITY ANALYSIS FOR STIFFENED CYLINDER WITH A CUTOUT

80 nodes; 190 elements; 1 load condition; 3 independent design variables; 5 dependent design variables; 352 degrees of freedom

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<td>12.7</td>
<td>107</td>
<td>14.3</td>
</tr>
<tr>
<td>34</td>
<td>19.1</td>
<td>71</td>
<td>21.4</td>
<td>108</td>
<td>15.8</td>
</tr>
<tr>
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<td>25.3</td>
<td>72</td>
<td>14.0</td>
<td>109</td>
<td>20.3</td>
</tr>
<tr>
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<td>6.9</td>
<td>73</td>
<td>15.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>13.1</td>
<td>74</td>
<td>16.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 12.- SUMMARY OF ANTENNA REFLECTOR SENSITIVITY ANALYSIS

[109 nodes; 420 elements; 1 load condition; 3 design variables]

(a) Solution time comparison

| Method               | Solution time, sec, for derivatives of | | |
|----------------------|----------------------------------------|--|
|                      | Displacement | Stress |
| Finite difference    | 155          | 160    |
| Semianalytical       | 95           | 114    |

(b) Comparison of derivatives of center deflection

<table>
<thead>
<tr>
<th>Independent variable, cross-sectional area</th>
<th>Value of derivative of center deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper surface</td>
<td>$2.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>Diagonals</td>
<td>$-8.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>Lower surface</td>
<td>$-1.8 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
Figure 1.-- The EAL system terminology and configuration.
(a) Finite-element model of cantilevered channel-section beam.

(b) Section A-A showing cross-sectional geometry and structural variables.

Figure 2.- Cantilevered channel-section beam used to illustrate analytical derivative method.
Figure 3.— Geometry and node numbering for swept wing. All dimensions are in inches.
Figure 4. - Element numbering for swept wing.
Figure 5. - Overall geometry and nodal numbering for box beam.
Figure 6. Elements for box beam.
Figure 7.- Overall geometry and nodal numbering for stiffened cylinder with cutout.
Figure 8.- Finite-element model of radiometer-antenna reflector.
(b) Upper surface.

Figure 8.—Continued.
(c) Lower surface.

Figure 8.— Concluded.
Figure 9.- Flowchart for system of runstreams to calculate structural-sensitivity derivatives with semianalytical method. Numbers indicate appropriate sections of runstreams in appendix.
Perturb independent design variable
\[ V_k = V_k + \Delta V_k \]

Compute dependent design variables \( \{v\} \) using eq. (1)

Loop over dependent design variables \( v_i \)

Modify structure

Calculate
\[
\begin{align*}
[K]_k^n, [M]_k^n, [S]_k^n, \\
[K_g]_k^n
\end{align*}
\]

Calculate derivatives:
\[
\frac{\partial [K]}{\partial V_k} = \frac{[K]_k^n - [K]_k^0}{V_k^n - V_k^0} \\
\frac{\partial [M]}{\partial V_k} = \frac{[M]_k^n - [M]_k^0}{V_k^n - V_k^0} \\
\frac{\partial [K_g]}{\partial V_k} = \frac{[K_g]_k^n - [K_g]_k^0}{V_k^n - V_k^0}
\]

Set original value of design variable

Figure 10.- Flowchart of section of runstream system which calculates derivatives of stiffness, mass, and geometric stiffness matrices. Numbers indicate appropriate sections of runstreams in appendix.
Figure 11.— Flowchart of section of runstream system which details calculation of displacement and stress derivatives. Numbers indicate appropriate sections of runstreams in appendix.
Figure 12.- Flowchart of section of runstream for calculation of derivatives of vibration and of buckling eigenvectors and eigenvalues. Numbers indicate appropriate sections of runstreams in appendix.
This paper describes the implementation of static and dynamic structural-sensitivity derivative calculations in a general purpose, finite-element computer program denoted the Engineering Analysis Language (EAL) System. Derivatives are calculated with respect to structural parameters, specifically, member sectional properties including thicknesses, cross-sectional areas, and moments of inertia. Derivatives are obtained for displacements, stresses, vibration frequencies and mode shapes, and buckling loads and mode shapes. Three methods for calculating derivatives are implemented (analytical, semianalytical, and finite differences), and comparisons of computer time and accuracy are made. Results are presented for four examples: a swept wing, a box beam, a stiffened cylinder with a cutout, and a space radiometer-antenna truss.