The Physics of a Single-Event Upset in Integrated Circuits — A Review and Critique of Analytical Models for Charge Collection

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Abstract

When an energetic particle (kinetic energy > 0.5 MeV) originating from a radioactive decay or a cosmic ray traverses the active regions of semiconductor devices used in integrated circuit (IC) chips, it leaves along its track a high density electron-hole plasma. The subsequent decay of this plasma by drift and diffusion leads to charge collection at the electrodes large enough in most cases to engender a false reading, hence the name single-event upset (SEU). The problem of SEU's is particularly severe within the harsh environment of Jupiter's radiation belts and constitutes therefore a matter of concern for the Galileo mission.

In the following we shall analyze the physics of an SEU event in some detail. Our main conclusion is this: Owing to the predominance of nonlinear space charge effects and the fact that positive (holes) and negative (electrons) charges must be treated on an equal footing, analytical models for the ionized-charge collection and their corresponding currents as a function of time prove to be inadequate even in the simplest case of uniformly doped, abrupt p-n junctions in a one-dimensional geometry. The necessity for full-fledged computer simulation of the pertinent equations governing the electron-hole plasma therefore becomes imperative.
1 Introduction

It is well known that an energetic particle (electron, α-particle, cosmic ray, etc.) produces electron-hole pairs when traversing a semiconductor. The number of pairs produced per cm of track length is given by

\[ N_0 = - \frac{dE}{w dx} \]  

(1)

where \(-dE/dx\) is the stopping power or linear energy transfer (LET) of the particle and \(w\) the mean energy spent to produce one electron-hole pair. For Si at 300°K, this number is \(w = 3.6\) eV independent of the type of particle. For example, a 5 MeV α-particle produces \(N_0 = 4 \times 10^8\) electron-hole pairs per cm, a number which can easily be calculated using standard stopping power tables.

The track of the energetic particle or, more specific, the volume in which electron-hole pairs are generated can well be approximated by a cylinder of length \(R\), where \(R\) is the range of the fast particle, possessing a circular base area corresponding to a radius of \(0.1\ \mu m = 10^{-5}\) cm. In the above mentioned example, this gives for the number density of electron-hole pairs generated by the fast α-particle

\[ N = N_0 / \pi \times 10^{-10} = 1.25 \times 10^{18} \text{ cm}^{-3} \]  

(2)

a number considerably larger than the usual doping levels of extrinsic material \((N_A = 10^{16}\) cm\(^{-3}\)). Since \(dE/dx\) of eq. (1) depends on \(x\), \(N_0\) will in general also be a function of \(x\), a fact which has been analyzed in more detail by Yaney et al. for α-particles.

Here is not the place to delve into this particular matter any further. The fact that energetic particle beams generate copious amounts of electron-hole pairs in semiconductors is being used extensively in high energy particle detectors.

\(\text{*)If the range is larger than the semiconductor sample size, the latter must of course be used}\)
in materials research (determination of diffusion lengths by means of the scanning electron microscope for instance).

The property of energetic particles to readily ionize semiconductor material used so advantageously in the examples just given becomes a detriment in many other cases. For instance RAM cells of modern computers consist of a number of transistors and diodes which in turn are fabricated from Si p-n junctions. Information is encoded as different voltages or different charge states at appropriate nodes of the circuitry. These voltages and charge states are constantly changing as information is processed. It has been realized for some time that an energetic particle crossing a RAM cell may upset the circuitry by virtue of the generation of electron-hole pairs which when collected at sensitive nodes may give rise to false readings.

Since then a fairly large body of literature has arisen dealing with this problem which is now generally known under the name of "single-event upset" (SEU).

On the next pages we shall formulate the problem, show its inherent difficulties and show that analytical models are inadequate. This state of affairs is caused by an enhanced collection of carriers at the nodes via drift in the originally space charge neutral region of a p-n junction. This is referred to in the literature as "funnelling." Incidentally, funnelling, which is a typical high level injection effect, was discovered by computer modelling rather than analytical methods and came as a surprise to the community.

2. Formulation of the problem. We consider for simplicity here a p-n junction consisting of a narrow N⁺ region (donor concentration \( N_D \approx 10^{20} \text{ cm}^{-3} \)) joined to a wider P region (acceptor concentration \( N_A \approx 10^{15} \text{ cm}^{-3} \)). Again for simplicity we take as the energetic particle a 5 MeV α-particle, which penetrates the planar

\(^{5}\) 5 MeV α-particles were chosen because both analytical and computer models exist for this case (see below).
device normally A 5 MeV α-particle possesses a range of 24 μm in Si. Its speed is \( c = \frac{v}{c} = 0.052 \). A device 20 μm thick will be traversed in \( 1.28 \times 10^{-12} \) sec.

During this time electron-hole pairs will be created along the track in the amount of \( N_0 \approx 4 \times 10^8 \) cm\(^{-1}\). At most \( Q = 8 \times 10^5 \) elementary charges can be collected from this ion track at any node. For static RAM's, both the total charge and the rise time (the current \( dQ/dt \)) are important. Consequently, the time history of the excess carriers must be known. For times much greater than thermalization times \((\approx 10^{-13} \text{ sec})\), carrier transport proceeds via drift and diffusion. The electron and hole currents are then given by

\[
\begin{align*}
\frac{\partial n}{\partial t} &= q \mu_n n E + q D_n \frac{\partial n}{\partial x} \\
\frac{\partial p}{\partial t} &= q \mu_p p E - q D_p \frac{\partial p}{\partial x}
\end{align*}
\]

(3a) and

(3b)

Here \( q \) is the elementary charge \( 1.6 \times 10^{-19} \) A sec, \( \mu \) and \( D \) are the mobility and diffusion constant for electrons and holes, \( E \) is the electric field. The number densities of electrons \( n \) and of holes \( p \) satisfy the continuity equations.

\[
\begin{align*}
\frac{\partial n}{\partial t} - q^{-1} \frac{\partial}{\partial x} (q \mu_n n E) &= G_n - R_n \\
\frac{\partial p}{\partial t} + q^{-1} \frac{\partial}{\partial x} (q \mu_p p E) &= G_p - R_p
\end{align*}
\]

(4a) and

(4b)

Here \( G \) and \( R \) signify total generation and recombination rates. In Si, recombination proceeds predominantly via recombination centers with energy levels within the forbidden gap. According to the Shockley-Read-Hall theory, \( G-R \), both for electrons and holes \( G_n - R_n \) and \( G_p - R_p \), respectively, become functions of the density of charged recombination centers in such a manner that

\[
\frac{3}{2} (n_+^+ - n_-^-) = R_p - R_n + G_n - G_p
\]

(5)
In eq (5) \( G_n \) and \( G_p \) are the generation rates of electrons and holes associated with recombination centers whereas the difference

\[
G_n - G_p = G_n - G_p
\]

signifies generation rates by other mechanisms (for instance fast \( \alpha \)-particles) where electrons and holes are generated at an equal rate \( n_T^+ \) and \( n_T^- \) are the number densities of positively and negatively charged recombination centers.

To close the systems of equations (3) to (6), the electric field \( \mathbf{E} \) must be related self-consistently to \( n \) and \( p \). For time-varying currents and electron-hole concentrations with characteristic time constants \( \tau \sim 10^{-12} \) sec, the electric field is approximately irrotational and we can put

\[
\mathbf{E} = -\nabla \phi
\]

where \( \phi \) is an electrostatic potential, a function of both position and time. The final connection is made through Poisson's equation

\[
\mathbf{E} = -\nabla \phi = -\frac{\mathbf{E}}{\varepsilon_0} (n - p + N_A^- - N_D^+ + n_T^- - n_T^+)
\]

which constitutes a relationship between the number density of charged carriers, charged acceptors and donors, charged recombination centers, and the gradient of the electric field, and is nothing more than a manifestation of Gauss's law \( \nabla \cdot \mathbf{E} = 0 \). \( \varepsilon_0 = 8.854 \times 10^{-14} \) A sec/V cm is the dielectric constant of vacuum, \( \varepsilon = 10.8 \) for Si signifies the relative dielectric constant of the medium. \( N_A^- \) and \( N_D^+ \) are assumed to be known time-independent functions of position. At room temperature and for non-degenerate material they are equal to the total acceptor \( N_A \) and donor \( N_D \) densities respectively. From eqs (4), (5), (7) and (8) it follows that the total current density \( \mathbf{j} \) is conserved.
The set of equations (3) to (9) forms the most general basis from which to
attack the SEU problem. However, before this can be even contemplated, boundary
conditions and initial conditions must be formulated. As we have seen earlier,
the i-particle generates electron-hole pairs in a narrow column rather suddenly,
i.e., in a time short compared to current rise times due to drift and diffusion
of the excess carriers. The appropriate initial condition may then be formulated
in the following way. Initially, at time t = 0, the electric field distribution
and the electron as well as the hole concentrations are those prevailing before
the i-particle strike except for an additional density of electron-hole pairs
within a right circular cylinder along the i-particle track with a radius of about
0.1 \mu m and a magnitude for the excess carrier density computed from eq (1).
Although the formulation of an appropriate initial condition is rather straight-
forward as we have seen, boundary conditions are less easily accessible. This
is because, as we shall see in the next section, we are dealing here with
phenomena which have an appreciable content of amplitudes in the 10^{10} to 10^{11} Hz
frequency range of their Fourier spectrum and it is not at all clear that
boundary conditions applicable to static or low frequency situations are valid
in the high frequency regime. The static boundary conditions for the ohmic con-
tacts of a transistor^{12) have however been recently applied to a c signal
analyses with seeming success^{13) The static boundary conditions at ohmic
contacts assume equilibrium^{12)}

\[ \text{np} = n_i^2 \]  

(10)

where \( n_i \) is the intrinsic carrier concentration, and charge neutrality
Furthermore the line integral of the electric field between any two contacts must be equal to the potential difference existing between the contacts\(^*\)

\[
\oint F \cdot dS = V_2 - V_1
\]  

At free surfaces of the semiconductor the usual boundary conditions in the steady state are

\[
\begin{align*}
\frac{1}{n} \quad & \left. \frac{n_{12}}{n_0} = -S(n - n_0) \right| \\
\frac{1}{p} \quad & \left. \frac{n_{12}}{p_0} = -S(p - p_0) \right|
\end{align*}
\]  

with \(n_{12}\) the outward normal and \(S\) the surface recombination velocity \(n_0\) and \(p_0\) are the equilibrium values for electron and hole concentrations respectively.

Now, according to Many et al.,\(^{14}\) \(S\) is a function of the number density of recombination centers at the surface and in a time-dependent situation, particularly for high frequencies, these states will empty and fill according to certain rate equations governed among other things by the time dependence of electron and hole densities at the surface.\(^{10}\) As a result, the common surface recombination velocity \(S\) of eqs. (13) splits into two different time-dependent expressions \(S_n(t)\) and \(S_p(t)\) valid for electrons and holes separately. Fortunately, Si surfaces used in computer hardware are passivated and their surface recombination velocity is small. It is possible to use as a first approximation

\[
\begin{align*}
\frac{1}{n} \quad & \left. \frac{n_{12}}{n_0} = 1_p \quad \frac{n_{12}}{n_0} = 0 \right|
\end{align*}
\]  

\(^*\)The line integral (12) must include the built-in potentials of any junction crossed by the path of integration.
or in other words, the normal component of electron and hole currents vanishes at free surfaces. Eqs (10), (11), (12) and (14) constitute the necessary boundary conditions to implement a solution of the basic equations (3) to (9), given the initial condition discussed in the text.

Methods of solution. The set of equations and boundary conditions, eqs (3) to (14), constitutes indeed a formidable non-linear system which cannot be solved in full generality, even with sophisticated computers. Therefore approximations are inevitable. In order to find sensible approximations we must look at the physical system described by the equations in more detail. Confining ourselves just to one p-n junction in reverse bias as an element of a static RAM cell, we know that within the depletion layer approximation, the junction consisting of a narrow n+ layer (the emitter) and a wide, moderately doped p-type bulk (the base), admits of three distinct regions: The quasi-neutral emitter, a depletion layer and finally a neutral (field free) base region. For a one-dimensional abrupt junction, the depletion layer width \( \delta \) is given by

\[
\delta = \left( \frac{2 q_0}{e N_A} \right)^{1/2} (V_b + \Phi)^{1/2}
\]

where \( V_b \) is the built-in voltage and \( V \) the reverse bias voltage. \( V_b \) is in turn given by

\[
V_b = \frac{kT}{q} n \left( \frac{N_A \lambda_D}{n_i^2} \right)
\]

The depletion layer contains a strong electric field which is due to the polarized dipole layer. If an \( x \)-coordinate is introduced positive in the direction from the n+ type to the p-type material, the electric field may be expressed as

\[
E = E_m (1 - \frac{x}{\delta})
\]
where

\[ E_m = \left( \frac{2q N_A}{\varepsilon_0} \right)^{1/2} (V_d + V)^{1/2} \]  

(18)

Eqs (17) and (18) are again valid in the depletion layer approximation. For typical Si junctions, assuming an acceptor density of \( N_A = 10^{15} \text{ cm}^{-3} \), a donor density of \( N_D = 10^{19} \text{ cm}^{-3} \), room temperature \( T = 300^\circ \text{K} \) and a reverse bias \( V = 8 \text{ volt} \), we find for \( W \) and \( E_m \)

\[ W = 3.24 \text{ cm}, \quad E_m = 5.43 \times 10^4 \frac{\text{V}}{\text{cm}} \]  

(19)

As we see, the electric field drops linearly from a rather high value to zero at the edge of the depletion layer.

\( 15 \text{ MeV} \) \( \alpha \)-particle striking the p-n junction at normal incidence will traverse the device and leave behind a column of a high density electron-hole plasma as already discussed. Part of this plasma finds itself in the depletion layer where it is subjected to a large electric field, part of it has been generated within the field-free bulk of the device. The generation of free carriers happens almost instantaneously, so that initially the field configuration remains what it was before the \( \alpha \)-strike. But immediately following the \( \alpha \)-strike that part of the plasma generated by the \( \alpha \)-particle which occupies the depletion layer will be torn apart by the strong electric field and drift toward the electrodes, while the other part within the base will diffuse toward the electrodes. The charges will be collected by the electrodes and it can be seen that there are two mechanisms, which are represented by two time constants, governing the flow of carriers drift and diffusion. The mobility of holes \( \mu_p \) is about twice as small as the mobility of electrons \( \mu_n \) in Si. Therefore
M will determine the time constant for prompt collection *). Since the speed of holes is equal to \( J_p E \), we have as an estimate for the time constant governing drift:

\[
\tau_r = \frac{1}{J_p E_m} = 10^{-11} \text{ sec}
\]  

(20)

using the values for \( W \) and \( E_m \) given before (eq (19)) and using for the hole mobility \( J_p = 600 \text{ cm}^2/\text{V} \text{ sec} \) valid for the doping density of \( N_A = 10^{15} \text{ cm}^{-3} \). The diffusion exhibits another time constant which is considerable larger and may be estimated by:

\[
\tau_S = \frac{q^2}{4D_p \ln 2} = 2 \times 10^{-8} \text{ sec}
\]  

(21)

where we used for the width of the base \( t = 10 \text{ \mu m} \) and for the diffusion constant for holes \( D_p = 15 \text{ cm}^2/\text{sec} \). We see therefore, that charge collection proceeds fast by drift (prompt charges) and slowly by diffusion (delayed charges). This qualitative picture has been borne out both by experiment and computer simulation.

Because of the anticipated small time constant (20) for prompt charge collection, the computer program could be simplified drastically by neglecting the recombination-generation terms \( G - R \) of eqs (4). In this case numerical solutions of the pertinent equations of the previous section become possible, but require extensive use of computer time. During the course of these calculations the effect of "field funnelling" was discovered. As the electron-hole pairs situated along the \( \alpha \)-track in the depletion layer are separated by the strong electric field (18), space charges are set up which interact with the rest of the electron-hole pairs along the \( \alpha \)-track located within the originally field free base in such a manner.

*) This is so because charge collection by drift ceases when the slower carriers have been picked up. Only then will diffusion become predominant.

**) In good Si devices, recombination times are \( > 10^{-8} \) sec. The influence of recombination centers on charge collection may therefore be neglected as far as prompt charges are concerned. It cannot be neglected for the collection of delayed charges.
as to set up a strong electric field along the ion-track within the base. At the same time the original field of the depletion layer collapses. Within a very short time the electric field of the depletion layer has moved into the base along the ion-track, a "funnel" has opened up as it were, hence the name "field funnelling". Carriers within the base are therefore collected by drift rather than diffusion, a fact which was not anticipated before. Earlier work on SE emphasized diffusion as the main collection mechanism.16) Once the prompt charges have been collected (which takes about $10^{-10}$ sec) the original field of the depletion layer will be restored and diffusion becomes gradually the more important collection mechanism. Field funnelling, being an essentially nonlinear space charge effect, comes about when two conditions are met: (1) the original plasma column generated by the energetic particle must be of high density (\( N \cong N_0 \)), and (2) part of the column must traverse the depletion layer. These are the prerequisites for the occurrence of field funnelling or enhanced prompt charge collection.

The funnelling may be qualitatively described by examining the one-dimensional junction case governed by equations (15) through (21). Fig. 1a shows the energy-band diagram for a one-dimensional p-n junction (\( t = 0^- \)). An electric field given by equation (17) is present in the space-charge region. We recall from semiconductor junction theory that the electric field in the fixed space-charge region establishes equilibrium between diffusive and drift current flow across the junction. For the situation in Fig. 1a where the applied bias potential is zero (\( V = 0 \)), the net current flow is also zero. With no current flow there is no ohmic drop in the p and n regions and therefore the field is zero outside the space-charge region. If, now, an ion-track traverses the junction, an electron-hole plasma (\( \cdot p = \cdot n \)) is superimposed on the picture in Fig. 1a at \( t = 0^- \). The electron-hole plasma is instantaneously affected by the electric field in the original space-charge region. In this region, the plasma holes and electrons are accelerated.
to the left and to the right, respectively, by the space-charge field. This initiates the anti-symmetric injection of holes and electrons into the p and n regions, respectively. This sets up a double-injection plasma distribution, emanating from the space-charge region. The initial flow of carriers will serve to neutralize the space-charge, thereby cancelling the original field by some time $t=0^+$ (Fig 1(b)). However, the excess plasma charge injected into the adjacent regions will establish an electric field where it was zero before. For the zero-bias case, the fields established will be commensurate with Poisson’s equation (eq (8)) as depicted in Fig 1(b). Even with zero-bias, a current will flow through the contacts of the device in the opposite direction of positive-bias current flow (i.e., as in a solar cell). The field distribution undergoes further changes as time progresses, during which time the charge created by the ion track is collected in the form of current flow in the device. The total collection of ionized charge is implemented by the combined effects of drift and diffusion. However, the extension of the electric field into the otherwise field-free regions of the device (i.e., what is termed “funnelling”) by the initial action of the original field in the space-charge region on the ion-produced plasma, serves to accelerate in time the collection of the charge created by the ion track.

The question naturally arises, “Why has the effect of field funnelling not been discovered much earlier?” After all, p-n junction devices have been used routinely since the early sixties as high energy particle detectors. However, the structure of these devices is such that the energetic particle track is confined entirely to the depletion layer which has been made deliberately wide by using high resistivity material (a low doping concentration) and a large reverse bias. As can be seen from eq (15), a 1300 μm sample corresponding to $N_A = 10^{13}$ cm$^{-3}$ and a bias of 300 volts gives a depletion layer width of $W = 190$ μm, rather large compared to the 2 μm of IC devices. At the same time the field free region of the base
is kept small, the back electrode being placed right next to the depletion layer. Nonlinear space charge effects in particle detectors, somewhat akin to funnelling, (here called "plasma effects") have nevertheless been observed and investigated by a number of authors\textsuperscript{17}. The plasma effect may be described as follows: The almost instantaneously generated electron-hole plasma along the track will be polarized at the outer edges of the plasma column owing to the electric field present in the depletion layer. Electrons and holes will be separated in a narrow sheet and a nonzero space charge will be set up. The space charge in turn creates an electric field inside the column which essentially screens out the original field. For the field configuration of eq (17), holes will escape at the end of the track and move toward the cathode and a steady current will flow. This process will continue until the whole plasma column is eroded.

The plasma time, the time it takes for the whole plasma to decay, can be computed with this model (Seibt et al\textsuperscript{17}) and turns out to be

\[ t_p = 1.32 \times 10^{-10} \left( \frac{N_0 E_0}{E_m} \right)^{1/3} \frac{2W}{R} \text{ [sec]} \]  

where \( E_m \) is given by eq (18) (in V/cm), \( W \) by eq. (15) (in cm), \( N_0 \) by eq (1) (in cm\(^{-1}\)), \( R \) is the range of the plasma-generating fast particle (in cm) and \( E_0 \) is the initial energy of the incident particle (in eV). The model assumed a constant \( N_0 \) along the track. This limitation has been removed by Finch\textsuperscript{17}.

It is here not the place to delve into all these considerations and analyses; suffice it to say that these attempts at dealing with nonlinear space charge effects suffer from serious drawbacks. The assumption of steady state erosion and the neglect of field distortion are but two examples of somewhat dubious simplifications made to obtain an analytically tractable model. In fact, in searching the literature, we have found that there are scarce examples of analytically amenable problems dealing with the full nonlinearity of space
Fig. 1(a) Band diagram for an equilibrium p-n junction in a one-dimensional configuration (V=0, t=0-).

\[ E = -\frac{d\phi}{dx} \]

Fig. 1(b) Band diagram for non-equilibrium p-n junction after strike by single high energy ion (V=0, t=0+).

\[ \frac{d^2\phi}{dx^2} = -\rho_{\text{plasma}} \]
charge effects in a time-dependent setting. All problems of this kind which have been solved analytically have exclusively dealt with single injection, i.e., considering only the motion of one kind of carrier. But field funnelling and related effects are basically double injection phenomena. Electrons and holes must be treated on an equal footing. No analytical methods are known to this author which can handle nonlinear double injection phenomena. Therefore, computer programs have been devised to handle these situations. One such program, developed by the IBM group, has already been mentioned. Another code, developed by SKA, has also successfully tackled the field funnelling problem. The reader is referred to the literature for details. Like all computer codes dealing with complex nonlinear mathematical relationships, those devised recently for dealing with SEII's also have their drawbacks. They are slow and expensive and it is therefore difficult to extract information in parametric form. One would like for instance to know the time at which the maximum current induced by an i-strike occurs, its magnitude, etc., as a function of doping concentration, applied voltages, etc. To extract this type of information from a computer code is of course rather awkward. Consequently, attempts have been made in the recent past to incorporate field funnelling in heuristic models which are conceptually simple and at the same time accurate enough for engineering purposes. These models will be discussed in the next section.

4 Comparison of heuristic SFU modelling with computer calculations. Ever since field funnelling has been discovered by computer simulation, people have been trying to construct simple theories incorporating the funnelling effect in one way or another. The theories by Hu, McLean et al., and Messenger are notable examples. Let us first discuss the theory by Hu. Hu assumes that the funnelling effect produces a depletion region reaching far into the base.
although the original depletion layer has collapsed. Actually, the computer simulations of both Hsieh et al.15) and Grubin et al.19) show that, although drift in the base constitutes the major means of charge collection for prompt charges, the base is far from depleted owing to the large residual carrier concentration within the energetic particle track during prompt charge collection. Nevertheless, assuming a depleted region of width

$$W = \frac{u_p}{u_n + u_p} \frac{Q(t)}{qN_0}, \quad (23)$$

where

$$Q(t) = \int_0^t I(\tau) d\tau, \quad (24)$$

is the prompt charge collected at time $t$ and $N_0$ is given by eq (1), Hu assumes that a potential difference develops, given by

$$V(t) = V_b + V - \frac{q}{\varepsilon_N} \frac{W(t)^2}{2\tau_0}, \quad (25)$$

where we recognize the last term of eq (25) to be the potential drop across a depletion layer valid for low level injection conditions. Eq (25) constitutes therefore a rather bold assumption. Given the usual expression for the conductance $G$ of a semiconductor, eqs (23) to (25) lead to an expression for the collected charge as a function of time

$$Q(t) = Q_H(1 - e^{-t/\tau_H}) \quad (26)$$

where

$$Q_H = (2q \varepsilon_0 (V_b + V) \kappa_{\varepsilon}^{-1})^{1/2} \frac{u_p + u_n}{u_p} N_0, \quad (27)$$

and

$$15$$
are the prompt charge and the collection time respectively. Fig. 2 shows a comparison between eq. (26) and the computer solution by Hsieh et al. As can be seen, Hu's simple model predicts a smaller collection time than the exact theory.

In their model for SEU's McLean and Oldham do not address themselves to the time history of charge collection but proceed to derive an expression for the total collected prompt charge. They assume a funnel length \( L_c \) determined by the average drift velocity of electrons toward the junction and a charge collection time \( T^c \). The latter was estimated from lateral diffusion of the initial charge column and is given by

\[
T^c = \left[ 9x^2/42 \right]^{1/3} \left[ q^2/\nu (V_b + V) \right]^{1/3} \quad (29)
\]

where \( \nu \) is the mbipolar diffusion constant. The total collected prompt charge was determined as \( Q^c = q N_0 \left( V_b + V \right) \) and has the value

\[
Q^c = \left( 3 x^4 \nu^2 (V_b + V)/8 \right)^{1/3} \left( 2 \nu_0^2 q^5/\nu \right)^{1/6} \quad (30)
\]

a rather unwieldy expression. Using values for the various quantities occurring in eqs. (27) to (30) valid for our example (\( N_A = 10^{15} \text{ cm}^{-3} \), 5 MeV \( n \)-particle strike, etc.) we find \( Q_H = 21 \text{ fC} \) and \( Q_{ML} = 65 \text{ fC} \) for the prompt charges collected according to Hu's model \( ^{20} \) and McLean-Oldham's model \( ^{21} \), respectively. The charge collection times for the two models turn out to be \( \tau_H = 180 \text{ psec} \) and \( \tau_{ML} = 64 \text{ psec} \) respectively. Incidentally, the total charge collected amounts to 100 fC for the 5 MeV

\( ^{a}) \) In eq. (27) \( R \) signifies the range of the fast particle or the length of the device, whichever is smaller.
Fig. 2 The normalized charge collected at the emitter electrode of a p-n as a function of time in ns. Solid curve according to the model by Hu20). Dashed curve according to computer calculations by Hsieh et al.15)
α-strike and the collection time was 750 psec—according to the computer calculations of Hsieh et al. As can be seen from the values for Q and T just quoted, for Hu's model, a third of the charge is collected in a time three times longer than for the model of McLean and Oldham. But since both models are based on different assumptions, they do not necessarily agree. Furthermore, since the underlying assumptions, a fully developed depletion region in Hu's model on the one hand, and a funnel length determined by lateral diffusion in McLean-Oldham's model on the other hand—assumptions which are not borne out by computer calculations—are used, we must conclude that the two models discussed so far are of heuristic value at best.

Yet another model has been proposed by Messenger. He first proceeded to solve eq. (4a) for the case in which the fast ion penetrates the junction at normal incidence and is stopped within the depletion layer. He assumed that the column of electron-hole pairs is subjected to the full electric field of the junction. Neglecting recombination, assuming that recombination times are much longer than transit times, he showed that the excess carrier density decays as

$$\delta p \sim e^{-(\mu E_m/\omega)t}$$

(31)

where $\mu$ is a suitably averaged hole mobility, taking into account the dependence of $\mu$ on $E_m$. $E_m$ and $W$ are given by eqs. (18) and (19) respectively. Another time constant $\beta^{-1}$ is then introduced ad hoc to describe the thermalization of hot electrons. In the end, the transient current generated by the fast ion is shown to be proportional to the difference of two exponentials, viz.,

$$I \sim \left( e^{-\left(\mu E_m/\omega\right)t} - e^{-\beta t} \right)$$

(32)

* This has been confirmed by exact computer calculations.
Since the mobility decreases with increasing field strength, we may use
\[ \bar{\mu} = 300 \text{ cm}^2/\text{V sec} \] rather than the value of 600 cm²/V sec used earlier. The characteristic time constant, eq (20), now becomes
\[ \tau_F = \frac{\bar{\mu}}{\mu} E_m = 2 \times 10^{-11} \text{ sec} \] (33)
for our example. Defining \( t_F = \tau \), the relationship (32) goes over into
\[ I \sim \left( e^{-\tau} - e^{-\tau \lambda} \right) = F(\lambda) \] (34)
with \( \lambda = \frac{t}{\tau_F} \). The function \( F(\lambda) \) exhibits the characteristics of the transient current qualitatively. The current, initially being zero, rises fast toward a maximum and then decreases to become zero again, just as the computer simulations of Hsieh ¹⁵ and Ribien ¹⁹ predict. According to Hsieh ¹⁵, the current maximum occurs for our example (8 V reverse bias, \( N_A = 10^{15} \text{ cm}^{-3} \), etc) at \( t = t_m = 0.06 \mu\text{sec} \), corresponding to \( \lambda = \lambda_m = 3 \). The maximum of \( F(\lambda) \), eq (34), occurs at
\[ \lambda_m = \frac{n}{\mu-1} \] (35)
Now, \( \lambda \) must be greater than one in order for \( F(\lambda) \) to be positive. Also, \( \lambda > 1 \) implies thermalization times less than \( \tau_F \), which is reasonable on physical grounds. However, with \( \lambda_m = 3 \), eq (35) has no solution for \( \lambda < 1 \). The root for this inconsistency lies in the neglect of nonlinear effects which are of prime importance here, as repeatedly stressed in this report. The depletion layer collapses in the neighborhood of the \( \lambda \)-track, electric fields are generated in the formerly field-free bulk by space charges developed during the collapse of the originally strong built-in junction field. This sequence of events, being caused by nonlinear interactions between electrons and holes of the dense plasma created by ionization, cannot be modelled easily with the aid of simple pictures. After all, the funnelling effect has been discovered by computer analyses, taking into account
the full nonlinearity of the system, which analytic methods were not then and are not now able to do.

5 Conclusion An energetic particle traversing a p-n junction leaves in its wake a high density electron-hole plasma. The subsequent decay of this plasma due to drift and diffusion leads to charge collection at pertinent electrodes which may upset the circuitry unduly. Since such events happen randomly and frequently given an adverse space environment, the necessity for understanding these events is obvious. The basic equations governing the fate of the plasma (section 2) have been attacked for the problem at hand both by numerical computations and heuristic (analytical) methods. Numerical methods are relatively slow and expensive. The need for simple models which capture the gist of the pertinent numerical work is felt quite vividly. Owing to the nonlinearity of the formalism describing double injection phenomena (electrons and holes) as discussed in section 3, ab initio analytical methods cannot be devised successfully. Models, which have been discussed in section 4, have been shown to be inadequate. This is due to excessive oversimplification, without which however, the models would become quite cumbersome, defeating their purpose. The conclusion is inevitable. Only numerical computations give reliable information on the time history of currents and collected charges generated by an energetic particle. Only numerical computations take properly into account the nonlinearities which determine the shape of a current pulse.
References

1) G Restelli and A Rota in Semiconductor Detectors, edited by G Bertolini and A Coce, North-Holland Publ Amsterdam 1968, p 75


