ENERGETICS of the MAGNETOSPHERE

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David P. Stern

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Goddard Space Flight Center
Greenbelt, Maryland 20771
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David P. Stern
Planetary Magnetospheres Branch
Goddard Space Flight Center
Greenbelt, Maryland 20771

Abstract

Energy flow in various large-scale processes of the earth's magnetosphere is examined. This energy comes from the solar wind, via the dawn-to-dusk convection electric field, a field established primarily by magnetic merging but with viscous-like boundary interaction as a possible contributor. The convection field passes about $5 \times 10^{11}$ watt to the near-earth part of the plasma sheet, and also moves the plasma earthward. In addition, $1-3 \times 10^{11}$ watt are given to the complex system of the Birkeland currents: about $4 \times 10^{10}$ of this, on the average, goes to parallel acceleration, chiefly of auroral electrons, about 2-3 times that amount to joule heating of the ionosphere, and the rest heats the ring current. The ring current stores energy (mainly as kinetic energy of particles) of the order of $2 \times 10^{13}$ joule, and this value rises and decays during magnetic storms, on time scales ranging from a fraction of a day to several days. The tail can store comparable amounts as magnetic energy, and appreciable fractions of its energy may be released in substorms, on time scales of tens of minutes. The sporadic power level of such events reaches the order of $3 \times 10^{12}$ watt. The role of magnetic merging in such releases of magnetic energy is briefly discussed, as is the correlation between properties of the solar wind and magnetospheric power levels.
Energy has become a key concept in modern society: it is the universal currency in which the cost of almost anything must be paid. In magnetospheric physics too, every process has its energy cost, and therefore an audit of the magnetospheric energy budget reveals a great deal about the processes involved, about their causes and effects. As with finances any discrepancy uncovered here constitutes a strong hint of deeper trouble.

What follows is a brief audit of this sort, with estimates of some fundamental energy and power levels of the magnetosphere.

GLOBAL ENERGY INPUTS

The first quantity considered is $P_1$, the power conveyed by a beam of the solar wind having the same cross-section as the magnetosphere. It is generally agreed that the energy of the magnetosphere comes from the solar wind, and $P_1$ therefore provides a sort of upper limit on what can be extracted from that source—it is the power obtained if all solar wind particles hitting the dayside magnetopause gave up their entire energy.

Take first the dayside magnetosphere as "target", about $15 R_E$ in radius. With a cross section of about $3 \times 10^{14} \text{ m}^2$, density 8 per cc and speed 400 km/sec, one gets

$$P_1 = 1.3 \times 10^{13} \text{ watt} \quad (1)$$

Actually, the preceding estimate is rather conservative: the magnetopause continues to widen past the dawn-dusk plane, attaining an ultimate radius of $30 R_E$ in the distant tail [Slavin et al., 1983; see also Howe and Binsack, 1972]. Thus the flow impinging tailward of the dawn-dusk plane carries 3 times more energy than the one impinging further sunward, and it seems likely (see below) that most of the magnetospheric energy input in fact comes from the tailward segment. Multiplying the preceding by 4 then gives, approximately

$$P_1 = 5 \times 10^{13} \text{ watt} \quad (2)$$
1% of this energy is $5 \times 10^{11}$ watt, and that is a widely cited order of magnitude of the energy extracted.

It is interesting to compare this figure to the radiant solar energy input rate $P_2$ impinging upon the Earth. The area available is about 900 times less, just the cross section of the Earth, but the energy flux, the so-called solar constant, has the large value of 1370 watt/m$^2$. From that

$$P_2 = 1.75 \times 10^{17} \text{ watt} \quad (3)$$

This is about 3500 times $P_1$ and $3.5 \times 10^8$ times the canonical order of energy input into the magnetosphere. This huge factor must weigh against any significant sun-weather coupling arising from the solar wind and from its interaction with the magnetosphere.

Among magnetospheric processes, the one requiring the most energy is probably the cross-tail current. If the strength of the lobe field $B_L$ is 15 nT, the field intensity change $\Delta B$ across the plasma sheet is twice that value, and the corresponding current sheet density $\Delta B/\mu$ comes to about $1.5 \times 10^5$ amperes per $R_E$ of tail length. Taking a length of $40 R_E$ then yields $6 \times 10^6$ amperes, and if the cross tail voltage is 50,000 volts, one needs a power input

$$P_3 = 3 \times 10^{11} \text{ watt} \quad (4)$$

As will be argued below (compare also Figure 5), the plasma sheet serves as a temporary storehouse or transit station for most of the solar wind energy input into the inner magnetosphere, and therefore, $P_3$ should be of the same order of magnitude as that input. The reason why only the nearest $40 R_E$ of the tail's length are taken into account are discussed further below.

This figure is quite uncertain. The potential drop is chosen as 50,000v, because this is the order of $\Delta \phi$, the voltage drop observed across the polar cap. If the polar cap represents open magnetic flux linked to the solar wind, and if that flux (as is believed) makes its exit via the tail lobes, there exists no way for the tail current to close without jumping a gap of
50,000 volts or so, as Figure 1a makes clear. Alternatively, if the magnetosphere is closed, and interplanetary field lines are equipotentials, there exists no voltage and no $P_3$, as Figure 1b shows. The preceding was expressed in terms of electromagnetic theory by Siscoe and Cummings [1969], with similar conclusions.

This energy apparently comes from the kinetic energy of the solar wind flow. In the simplest dynamo, a closed electrical circuit exists in a medium where some regions move relative to others, the motion having a component perpendicular to the magnetic field direction. The magnetic signature of the tail boundary, however, is abrupt, suggesting that the cross-tail current closes through a narrow sheet along the tail boundary, around that tail lobes. The dynamo process, therefore, is also expected to reside on that boundary.

It seems likely that the energy transfer here arises from the fact that the boundary diverges at a small angle, so that it either intercepts or diverts the flow of solar wind. As a crude model, one may take the antisunward motion of the solar wind to extend all the way, until the flow intercepts the boundary [Stern, 1983]. In this approximation, after solar wind particles encounter the boundary, they are generally reflected, because the field magnitude is several times greater inside the tail. In addition, however, the field also tends to bend sharply at the boundary, and in traversals of such sharp bends particles tend to gain or lose energy. This process was studied by Lyons and Speiser [1982] for particles traversing the sharply bent field lines of the plasma sheet, and they deduced appreciable energization there. However, the direction in which field lines bend in order to change from the direction of the lobes to that of the interplanetary field is, on the average, opposed to the one found in the sheet. Thus particles traversing that region do not gain energy but lose it, with the lost energy given to the crosstail current in a way which again agrees with the electrodynamics of Siscoe and Cummings [1969]. If one's imagination is flexible, one can even regard the boundary layer in this region as "spent solar wind" which has given up part of its energy to this circuit, something like the relatively slow stream of water that exits from the blades of a turbine wheel.
The current intensity of $1.5 \times 10^5$ amperes per $R_E$ is derived from $\Delta B$, and it might be argued that it would be more proper to use $\Delta H$, i.e. omit contributions of the magnetization current $V \times M$ due to the gyration of charged particles, in the same way such contributions are omitted in diamagnetic materials. It turns out otherwise, because the definition of the magnetization current is strictly an artifact of guiding center theory. If the current is defined directly from the distribution function without recourse to guiding center theory, it becomes evident that $\Delta B$ has to be used.

Finally, one may question the assumption that only the nearest $40 R_E$ of the tail contribute to the inner magnetosphere. Indeed, ISEE 3 has observed the plasma sheet to distances of over $200 R_E$ though the distant lobe field $B_L$ there is only 9.1 nT [Slavin et al., 1983] and the voltage drop could also be smaller. Indeed, one can derive an estimate for the entire tail and argue for a power level of $2-3 \times 10^{12}$ watt, i.e. 3-5 times larger than $P_s$, as was done by Siscoe and Cummings [1969]. However, one should be cautious in interpreting that figure, because it seems that much of the energy deposited in the distant tail is returned to the solar wind through the flanks, before it can reach the near-earth environment. This may be supported by the following argument.

Consider the inner edge of the plasma sheet [Figure 2]—say, $40 R_E$ wide by $5 R_E$ thick, 0.5 particles/cc at 5 keV each. A drop of 50,000 volt at $B_z=5nT$ gives a drift velocity of $40 \text{ km/sec}$, and a power input

$$P_s = 1.3 \times 10^{11} \text{ watt}$$

(5)

Of course, the tail may store some energy in its magnetic field and release it later in substorms, but this is unlikely to raise the average power input above $3 \times 10^{11}$ watt. Indeed, the primary input from Birkeland (field-aligned) currents is about the same as $P_s$ at quiet times and twice that value at disturbed times (see below). Thus no large energy inputs from the distant tail are evident in the near-earth environment, suggesting that much of this energy is returned to the solar wind and merely causes some heating of the earth's wake.
ENERGY COUPLING

From Figure 1a one would expect that the linkage between the magnetosphere and the interplanetary magnetic field (IMF) is strongest when the IMF is directed southward, or has a strong southward z-component \( B_z \). One also expects that linkage to be weakest when \( B_z \) is northward, since IMF field lines then must turn around to make the appropriate connection. Such a correlation between the IMF and magnetospheric activity has long been established [Foster et al., 1971; Arnoldy, 1971].

Recently attempts have been made to relate \( P \) to interplanetary parameters directly, by Akasofu and by his colleagues [Perreault and Akasofu, 1978; Akasofu, 1979, 1980, 1981]. They have estimated the energy input by means of the "epsilon parameter"

\[
\epsilon = \zeta_0^2 V B^2 \sin^2(\theta/2) \quad (6)
\]

where \( V \) is the solar wind velocity and \( B \) is the magnitude of the IMF, while \( \zeta_0 \) has the dimensions of length and is taken as \( 7 R_E \). The angle \( \theta \) is defined in solar magnetospheric coordinates as the one between the \((y,z)\) component of the IMF and the \( z \) axis: if \( B_z = 0 \), \( \theta = \pi/2 \), and if the IMF is purely (southward, northward) \( \theta \) is \( (\pi, 0) \).

Kan, Lee and Akasofu [Kan and Lee, 1979; Kan et al., 1980] derived a theoretical justification of this dependence, based on earlier work by Sonnerup [1974]. The fundamental relation, derived from reconnection modeling, is that the voltage drop \( \Delta \phi \) across the bundle of open field lines is

\[
\Delta \phi = \xi_R V B \sin^2(\theta/2) \quad (7)
\]

where \( \xi_R \) is the width of the reconnected bundle of field lines. A somewhat similar functional relationship is obtained for the voltage across the tail if (1) The tail is assumed to be a cylinder of radius \( R \); (2) The \((y,z)\) projections of interplanetary field lines are continued straight until they encounter the magnetopause; (3) Only the \((y,z)\) projection of \( B \) is used (\( B_x \) does not contribute to \( y \times B \)) and \( \phi \) is taken as the voltage across the bundle
of field lines entering the tail north of the equator, or leaving the tail south of the equator. In this case $l_R$ is replaced by $2R$, giving a factor much larger than $l_0$, though such a similarly large factor also enters the calculation by Kan et al. [1980], which follows.

If the plasma sheet carries a mean linear density of $J$ amperes/meter over a length $L$, the total power invested is

$$P_3 = \Delta \Phi \ J \ L$$

(8)

Now if $B_L$ is the tail lobe field, $J = 2 B_L / \mu_0$. If $A$ is the average (yz) cross sectional area of each lobe (drawn in Figures 1), each lobe's magnetic flux is approximately $A B_L$ (neglecting the flux of the plasma sheet). Suppose this flux exits through an area $S$, approximated by a rectangle of length $L$ along $x$ and width $W$. If $B_n$ is the (averaged) field component perpendicular to $S$, i.e. the component continuous between the IMF and the lobe (expected to be rather small), then

$$B_L \ A = B_n \ S$$

(9)

Now the electric field just outside $S$, in the solar wind flow, is $E = B_n V$, from which $\Delta \Phi = B_n V W$. Combining everything then gives

$$P_3 = (2L^2/AV\mu_0) \ (\Delta \Phi)^2$$

(10)

This is essentially the result of Kan et al., [1980]; a factor 2 is added by accounting for both lobes. When (7) is substituted, a result proportional to (6) is obtained. Arguments have also been advanced from the standpoint of dimensional analysis, claiming that the interplanetary power input should have a dimensionality which (6) possesses but which some other correlated quantities lack [Kan and Akasofu, 1982]. The greatest variation, however, is contributed by $\sin^4 \theta/2$, which is dimensionless.

The experimental validity of (6) was studied by Akasofu [1979, 1980, 1981] who estimated $P_3$ ($U_T$ in his notation, subscript for "total") as the sum of energy inputs $U_R$ into the ring current, $U_J$ into joule heating of the
ionosphere and $U_A$ into the aurora. As shown below, the ring current energy $W_{RC}$ may be viewed as proportional to the magnetic Dst index (suitably corrected), so that $U_T$ will contain a term proportional to $\Delta \text{Dst}/\Delta t$. However, $U_T$ has at least one additional component, for even if Dst is constant, a certain energy input is required to balance the natural decay of the ring current. Akasofu [1981] assumed for this process a decay time $\tau_R$, set equal to 1 hr during times of rapid ring current growth and to 20 hrs during a storm's decay: the faster decay was assumed to prevail whenever $\epsilon$ exceeded $5 \times 10^{18}$ erg/sec. The two other contributions were viewed as proportional to the auroral electrojet index AE [Rostoker, 1972], yielding

\[
U_T(\text{erg/sec}) = -4 \times 10^2 \left[ \frac{\Delta \text{Dst}}{\Delta t} + \text{Dst}/\tau_R \right] + 3 \times 10^{15} \text{AE} \tag{11}
\]

This was tested by Baker et al. [1983] who derived interplanetary conditions from ISEE-3 data and who deduced a good correlation between $U_T$ of (11) and $\epsilon$ during disturbed times, but a less pronounced one during quiet times. Other interplanetary criteria [e.g. those of Burton et al., 1975] also correlated well, and the best fits required delays of 25-40 minutes during quiet times and up to 15 minutes at disturbed ones.

Many other investigators have examined such correlations [e.g. Murayama, 1982; Holzer and Slavin, 1982; see also Burch, 1983]. The claim is sometimes made that $\epsilon$ correlates not just with the total energy input but also with substorm activity, and that therefore substorms represent, not the release of magnetic energy stored in the tail (see below) but an intensification of a continuous energy flow from the solar wind to the magnetosphere. This issue has not yet been settled [see Baker et al., 1983] but differences may have narrowed: the existence of substorm precursors [e.g. Baker et al., 1981] suggests that energy storage exists in at least some substorms, while storage times now proposed run as short as 1-3 hours, and some of those favoring storage also view increased energy input as playing a role.

The experimental validity of (7) has been examined by Reiff et al. [1981], using polar electric field observations from the AE-C and AE-D
driftmeters and from S3-3 electric field probes. After applying a specific correction to the field [Kan and Lee, 1979, eq. 3], they find that $A^*$ generally consists of 2 components, a constant one of the order of 30 kV (sometimes ascribed to viscous-like interaction between the SW and the magnetosphere) and a variable one which correlated well with interplanetary conditions. The reported fit between this second component and eq. (7) is good [Reiff et al., 1981, end of p. 7645], though some other models also fare well. Wygant et al. [1982], on the other hand, used S3-3 data and reported a relatively poor correlation.

Of particular interest is $A^*$ at times when the IMF is northward ($0 < \theta/2$). At such times Wygant et al. [1982] find that $A^*$ diminishes as the time period increases during which the field has remained northward. Hardy et al. [1981], using a less direct but relatively global method for estimating $A^*$, also seem to find unusually low values of $A^*$ during times of northward $B_z$.

THE RING CURRENT ENERGY

Next we seek the energy $W_{RC}$ contained in the ring current, in the inner magnetosphere.

It is known that the effect of the ring current is to decrease $B$ at the earth. Such a decrease is the main signature of magnetic storms, where $B$ may drop 100 nT or on occasion even more, suggesting that such storms involve a strengthening of the ring current. There exists a remarkable formula due to Dessler, Parker and Skopke [Dessler and Parker, 1959; Parker, 1962; Skopke, 1966, 1971; Olbert et al., 1968] which states that, with certain assumptions, if the ring current causes a decrease $\Delta B$ at the origin, then

$$W_{RC} = 1.5 \frac{(\Delta B/B_e)}{U_e}$$  (12)

where $B_e$ is the surface equatorial field intensity (assuming a dipole) and
is the magnetic field energy of the earth's field contained within the earth itself. Thus if $\Delta B = 100 \text{ nT}$, $W_{RC} = 4 \times 10^{15}$ joule. Four points should be noted here:

(1) The formula assumes a certain model for the ring current: for realistic models, the result is only an approximation. Carovillano and Siscoe [1972] studied this matter and concluded that for small $\Delta B$ the DPS formula was fairly good, but that for large $\Delta B$ the formula overestimated $W_{RC}$ by a factor 1.5-3. Sckopke [1971] compared the value given by the formula to $W_{RC}$ derived independently for certain models, and he concluded that for small $\Delta B$ the estimate was high by about 10%, even if only the kinetic energy of the ring current particles was taken into account. Including the magnetic self-energy of the ring current made the fit worse (by about the same amount) and he therefore recommended omitting such contributions. He also estimated the magnetic interaction energy between the ring current and the magnetopause.

(2) In order to get a baseline for $\Delta B$, one should in principle seek a time when the ring current is completely absent. Such a state of affairs has yet to be observed. The Magsat field analysis, based on two very quiet days in 1979, suggested the existence of an external term, corresponding to a baseline value $\Delta B \sim 20 \text{ nT}$ (see eq.13 below), which implies that even at very quiet times the ring current contains $\sim 10^{15}$ joules.

(3) The value of $\Delta B$ must be corrected for variations in the pressure $p$ of the solar wind. Commonly, the magnetic field change is estimated from the Dst index [Chapman and Bartels, 1939; Rostoker, 1972; Mayaud, 1980], the average magnetic disturbance recorded at a number of near-equatorial observatories, corrected for daily variation and anisotropies, and given a negative sign. If Dst is the pressure-corrected value, obtained by normalizing everything to a time when the SW pressure was $p_o$, then [Siscoe, 1966; Akasofu, 1981]

$$\text{Dst} = \text{Dst} + 1.31 \times 10^4 [p^{1/2} - p_o^{1/2}]$$

(14)
where \( p \) is in dyne/cm\(^2\) and Dst is in nT and negative: thus if \( p \) exceeds \( p_0 \), Dst will appear unusually large, and its normalized value is reduced in magnitude. A quick (if not rigorous) justification of (14) is as follows. The confinement of a dipole of moment \( g \) may be modeled in an axisymmetrical fashion by the addition of a constant northward field \( 2kg \). The combined field satisfies

\[
\mathbf{B} = -\nabla \gamma
\]

\[
\gamma = g \cos \theta \left( \frac{1}{r^2} + 2k r \right)
\]

Here all dipole lines are confined inside a sphere \( r=r_0 \), with \( k=1/r_0^3 \), and at the equatorial boundary \( B = B_0 = 3kg \). If a similar confinement, in the subsolar region only, is produced by an external pressure \( p \), with no external magnetic field, then \( p = B_0^2/2\mu_0 \). The added field at the earth in that case equals the added northward field \( 2kg \), proportional to \( B_0 \) and hence to the square root of \( p \).

Any observational estimate of \( \Delta B \) must be corrected for effects of currents induced in the solid earth. Such currents tend to shield the earth's core and reduce \( \Delta B \) in its interior, and they therefore amplify \( \Delta B \) at the surface and make it larger than it would have been otherwise. It is often stated that about 1/3 of Dst is due to earth currents and that the rest represents \( \Delta B \), the value to be used in (6); this was confirmed by Langel and Estes, [1983], who derived relations for the dawn and dusk meridians along which Magsat orbited, averaging to about

\[
\Delta B = 19.5 \text{ nT} - 0.65 \text{ Dst}
\]

Earth dipole moment = 29990 nT + 0.26 \( \Delta B \)

This however can only serve as a rough estimate, because induced currents depend on \( \Delta B/\Delta t \) and may be larger during the growth of \( \Delta B \), which is relatively fast (below), than during its slower decay.

A rough estimate of the growth time may be obtained from (11). Let a...
magnetic storm have $\Delta B=100$ nT and assume that $W_{RC}$ is only half of the value given by the DPS formula. Then

$$W_{RC} \sim 2 \times 10^{15} \text{ joule} \quad (17)$$

If the energy input from the tail is twice $P_s$, or $2.6 \times 10^{11}$ watt, and all that input goes into $W_{RC}$, with no losses, then the time required is $\sim 4$ hours. Including the second term of (11), which represents the losses, and setting $\tau_R=20$ hrs, makes little difference. However, if $\tau_R$ is reduced to 1 hour (as advocated by Akasofu, [1981], for the initial phase of magnetic storms) increases the loss rate unless it alone exceeds the postulated energy input. An estimate by Kamide and Fukushima [1971] of the rate $P_s$ of energy flow into $W_{RC}$ (i.e. of Akasofu's $U_R$) during very disturbed times comes to $2 \times 10^{11} - 2 \times 10^{12}$ watt. The annual average $<P_s>$, however, is much smaller: Davis [1969] estimated

$$<P_s> = 1.2 - 1.7 \times 10^{10} \quad (18)$$

One gets a comparable value from $<W_{RC}> = 10^{15}$ joule and a decay time $\tau_R=20$ hrs. Figure 3 is taken from Sugiura [1980] and traces Dst for an actual magnetic storm, and it demonstrates that the storm indeed builds up much more slowly than do polar magnetic disturbances.

The explanation of magnetic storm observations is still incomplete, and it could be that events classified as magnetic storms belong to more than one class. Tinsley [1976] has suggested that the differences in $\tau_R$ described above reflect the composition of the ring current, provided the main process removing its particle is charge exchange in collisions with neutral hydrogen. In his view, the injected plasma contains chiefly hydrogen, which is rapidly removed, leaving behind helium, whose charge-exchange lifetime is much longer. Recent composition measurements, however, seem to indicate that helium has only a secondary role, though $O^+$ (also long lived) may be important [Young, 1983, sect 4]. Lyons and Williams [1981] have proposed that most of $\Delta B$ in a magnetic storm is derived not from the injection of fresh particles but by driving closer to earth particles already trapped in the ring current. As such particles move inwards,
adiabatic invariance causes their energy to rise, increasing $W_{RC}$ and producing $\Delta B$. The evidence for this was drawn from the comparison of energy spectra of ring current particles, observed by spacecraft before and during magnetic storms.

A situation in which $W_{RC}$ grows with no new particles being injected was studied by Jaggi and Wolf [1973], Southwood [1977], Siscoe [1982a, b] and Siscoe and Crocker, 1983], and is as follows. The polar electric field of the earth is roughly a constant field directed from dawn to dusk and covering a circle $15^\circ-20^\circ$ in radius, centered slightly nightward of the magnetic pole [Meng, 1980]. As the IMF shifts to become more southward, the polar voltage drop may grow, and so may the radius of the polar cap, and both these cause a growth of the fringing electric field in the polar ionosphere, equatorward of the above circle. The fringing field changes $W_{RC}$ and is in turn modified by the ring current: Vasyliunas [1972] has shown that the effect is similar to what would be produced by adding a term $\gamma_H$ to the ionospheric Hall conductivity, on field lines which thread the ring current, and this weakens and rotates the fringing field. It may then be shown that field lines threading the ring current will indeed move away from the polar boundary, i.e. the ring current moves earthward [Southwood, 1977, eqs. 22-23; Siscoe, 1982a, eq. 5].
BIRKELAND CURRENTS

While the tail current and ring current do not intersect the ionosphere, field aligned Birkeland currents do, in large sheets of approximately constant magnetic latitude. On the average, there exist two sheets (or bundles of sheets) each about $\sim 3^\circ$ wide, adjacent to each other (often, with considerable embedded fine structure), a poleward "region 1" and an equatorward "region 2"; on the dawn side, region 1 flows down and region 2 up, on the dusk side directions are reversed, and there exist interesting overlaps near midnight and near noon. In assessing the power involved, only the poleward "region 1" is considered, because only it seems to be connected to the energy sources. Region 2, closing through the partial ring current [Schield et al., 1969; Vasyliunas, 1972; Stern, 1983a] is linked by the ionosphere to region 1 and thus its energy is drawn from the same account.

The sources of region 1 currents may be on open field lines or field lines threading the boundary layer, in which case the energy is drawn directly from the solar wind, but the main contribution probably comes from the plasma sheet [Stern, 1983a, Fig.6]. When these currents reach the polar ionosphere, they split into two parts. The smaller part ($\sim 1/4$) establishes a direct linkage between the dawn and dusk sheets through the polar ionosphere, either across the middle of the polar cap or through the auroral oval, where conductivity is enhanced by auroral precipitation. This is the part of the current which contributes the observed seasonal variation [Fujii et al., 1981], since ionospheric conductivity is expected to drop considerably during the polar night.

The other, larger part of the current enters region 2 sheets and is closed by convected particles in the earth's partial ring current. Some caution is needed here, because some of this energy may already have been counted as part of $P_n$. The total current $I$ is estimated by Iijima and Potemra [1976] to be

$$I \sim 1.4 \times 10^6 \text{ amp} \quad \text{(quiet times, AE<100nT)}$$

(19a)
I = 2.7 \times 10^6 \text{ amp} \quad (\text{disturbed, AE}>100\text{nT}) \quad (19b)

Multiplying this by a 50,000 volts and again by 2 to account for both polar caps gives the power input $P_6$

\[ P_6 \approx 1.4 \times 10^{11} \text{ watt (quiet)} \quad (20a) \]

\[ P_6 \approx 2.7 \times 10^{11} \text{ watt (disturbed)} \quad (20b) \]

These energy inputs are disbursed among a number of accounts:

1. Energization of particles in the partial ring current (in the circuit of region 2, as discussed above).
2. The power $P_7$, invested in the acceleration, by the parallel electric component $E_p$, of particles which precipitate into the ionosphere.
3. The power invested by $E_p$ in accelerating particles drawn out of the ionosphere.
4. The power $P_8$ invested in joule heating by ionospheric currents, flowing across the short "bridge" between regions 1 and 2 and also across the polar cap.

The auroral power $P_7$ was first derived by Sharp and Johnson [1968] using total energy detectors sensitive down to 80 ev. Their results correlated well with the magnetic disturbance index $K_p$ and typical values were

\[ P_7 = 4 \times 10^9 \text{ watt (Kp = 1)} \]
\[ = 6 \times 10^{10} \text{ watt (Kp = 4)} \]
\[ = 2 \times 10^{10} \text{ watt (average)} \quad (21) \]

Precipitating particles are predominantly electrons, typically with 3-10 keV, and their arrival is also accompanied by auroral displays. Since 1978 NOAA has monitored $P_7$ continually [Evans and Hill, 1980], obtaining values about twice as large as those of (21) (David Evans, private communication). Such an order of magnitude also follows from the analysis of Spiro et al. [1982; see Fig. 7b there], who used AE-C and AE-D data. Those investigators found that $P_7$ tracked the aurora electrojet index AE better
than it did Kp [Fig. 7a there], and was approximately given by

\[ P_7 \propto [1.75(AE/100nT) + 1.6] \times 10^{18} \text{ watt} \] (22)

The power carried by O* ions and by electrons rising from the ionosphere is relatively small and will be neglected here: positive ions have small mobilities and hence relatively low fluxes, while rising electrons support only voltages of the order of 50 volts [Burch et al., 1983]. Since \( P_7 \) is about 30% of \( P_e \), this suggests that the Joule heating power \( P_e \) is 2-3 times larger than \( P_7 \). Independent estimates of \( P_e \) bear this out.

To perform such estimates it is necessary to combine observations of the ionospheric electric field \( E \) with a model of the electrical conductivity. The latter is fairly narrowly peaked in the ionospheric E layer (120-130 km) and contains a Pedersen conductivity \( \sigma_p \) and a Hall conductivity \( \sigma_H \), of comparable magnitudes [Boström, 1961]. It is customary to treat large-scale ionospheric currents in a 2-dimensional approximation, with \((\sigma_p, \sigma_H)\) integrated over the ionosphere's thickness to yield sheet current conductivities \((\Sigma_p, \Sigma_H)\), typically between 0.1 and 2 mho under quiet conditions, dependent on sun angle etc. In the auroral zone, both conductivities are enhanced by auroral precipitation, and Spiro et al. [1982; tables A2-A3] have used satellite data to map this effect under varying conditions, using earlier results cited there. They deduce that the auroral enhancement of the integrated Pedersen conductivity is

\[ \Sigma_p = (20E_o/4+E_o^2)^{1/2} \] (23)

where \( E_o \) is the mean electron energy in keV and \( \psi \) (\( \phi \) in the article) is the electron energy flux in ergs/cm² sec.

The existence of \( \Sigma_H \) gives rise to a Hall current and tends to rotate the polar field pattern around the pole [Vasyliunas, 1970]: the larger the ratio \( \Sigma_H/\Sigma_p \), the larger the deformation. For aurora-enhanced conductivity, Spiro et al. [1982] find (no factor needed if \( E_o \) is in keV)

\[ \Sigma_H/\Sigma_p \propto E_o^{5/8} \] (24)
The large Pedersen currents linking regions 1 and 2 give rise to a pair of concentrated Hall currents, flowing along the auroral oval from both sides of midnight towards ±22 hrs MLT—the auroral electrojets. These currents are responsible for most of the ground-level disturbance due to the Birkeland current system, since the ground signatures of other contributions tend to cancel [Fukushima, 1976]. However, since the ratio in (24) does not depend on auroral intensity (and the variation of $E_o$ is relatively moderate) all the above currents are roughly proportional, so that the AE index which gauges the intensity of the electrojets is a reasonable measure of the intensity of the entire $j_n$ system [Bleuler et al., 1982, Fig. 8].

The ionospheric current flows and the magnitude of $P_8$ were calculated by Bleuler et al. [1982] deriving $(\zeta_P, I_n)$ from theory, and by Foster et al. [1983] who used (23) above. The results are similar and both predict relatively large seasonal variations. This may be expected, since the seasonably varying cross-polar current involves the entire cross-polar voltage drop: thus, even though the cross-polar current is several times smaller than the current arriving via region 2 sheets, its heating power is comparable. Figure 4, from the latter work, shows $P_8$ for 3 seasons, and also $P_7$, in the lowest of the graphs. As can be seen (and is noted by the authors), $P_8$ exceeds $P_7$ by a factor 2-3.

OTHER PROCESSES

The auroral kilometric radiation (AKR) can be maintained by about 1% of the auroral energy output [Gurnett, 1974]; its peak power was estimated in the above study as $10^9$ watts and its average power as $2 \times 10^7$ watt. The likely origin of AKR is from accelerated beams of auroral electrons.

Greenwald and Walker [1980] have examined the energy output of a large Pc5 pulsation event and conclude that it dissipated $2 \times 10^{13}$ joule in its peak hour, yielding a power input of $\sim 6 \times 10^9$ watt. The likely energy source in this case are flapping motions of the magnetopause, but it should be noted that the long-term average of this source is much smaller.
MAGNETIC MERGING

A tentative summary of all processes is given in Figure 5: overall, the various processes are in reasonable agreement. What may or may not be remarkable here is that so far very little attention was given to magnetic merging (or reconnection), which is often invoked as an important energy release process. Magnetic merging figures prominently in theories of solar flares (where admittedly much less information is available), yet here it appears that energy flow in the magnetosphere may be traced fairly completely without energy release due to merging.

As a working definition, merging here is viewed as the flow of plasma through a neutral (null) point of the magnetic field. Two main types of neutral points (NPs) are possible, known from the form of field lines near them as 0-type and X-type and formed when $\nabla B$ at $B=0$ has either 1 or 3 real eigenvalues, respectively. The NPs formed in daytime merging are expected to be of the X-type, and a 2-dimensional model of such merging appears in Figure 6. Since the configuration is independent of the coordinate perpendicular to the figure, the NP is stretched into a neutral line which, like the electric field $E=-v_\times B$, is perpendicular to the drawing. By continuity, $E$ has to be constant, and therefore $v$ near the neutral line becomes quite large.

In X-type neutral lines like the one drawn, the plasma flow changes the field line sharing among particles—some particles on field lines entering the merging region end up on different field lines afterwards, while some particles which did not share field lines beforehand, do afterwards (all these are low-energy particles, magnetic drifts are ignored). Other properties of merging are uncertain, e.g. whether appreciable energy is released near $B=0$ and whether any particles are selectively accelerated. There even exists controversy about the fundamental flow pattern. The distant flow in Figure 6 obviously follows the drawn arrows in the plane of the figure, but the fast "jetting" associated with plasma energization in the merging region itself may flow either in the plane of the figure or orthogonally to it [Vasyliunas, 1975, Table 2]. The first possibility is advocated by MHD theories, which maintain that the X-pattern becomes very
flattened, and that flow discontinuities in it separate inflow from outflow. The acceleration then takes place at the discontinuities, and the magnetic field in the narrow wedge between them is weak. "Collisionless" theories, on the other hand, hold that acceleration is associated with non-adiabatic particle acceleration by $E$ along the neutral line.

As already noted, merging on the dayside is essential for the formation of an open magnetosphere, and unless the configuration is open, interplanetary field lines on the dawn and dusk boundaries have the same potentials. This, however, does not require merging to be an energy releasing process. Rather, it regards merging merely as the opening of a door through which energy then streams from the solar wind into the magnetosphere, remaining open for about one hour, the order of the contact time between a parcel of solar wind and the magnetosphere.

Whether neutral points or lines in the magnetosphere also produce local plasma energization is still controversial. Some theories, in particular those developed to explain solar flares, view merging as "magnetic field annihilation", a process in which plasma enters the NP (or neutral line, or neutral sheet) strongly magnetized, but leaves with only a weak magnetic field embedded in it and with the surplus energy converted to the kinetic energy of particles.

If such "annihilation" occurs in the magnetosphere, it is most likely to be in the tail, since the magnetic energy $W_T = \int B^2/2\mu_0 \, dV$ of the high-latitude tail lobes appears to be lowered by substorms. The lobe energy may be approximated by regarding the lobes as two half-cylinders $20 R_E$ in radius and $50 R_E$ long (more distant parts may be decoupled from the near-earth environment). If the lobe field at quiet times is $B_L = 15$ nT, then

$$W_T = 1.45 \times 10^{15} \text{ joule}$$

(25)

i.e. about as much as $W_{RC}$ of the quiet-time ring current. At disturbed times the area of the polar cap increases, the open magnetic flux (most of which threads the lobes) does likewise, and so does $B_L$, the intensity of the lobe field. If $B_L$ doubles, $W_T$ increases 4-fold. Caan et al. [1973]
have estimated, from a before-and-after comparison, that about 10% of $W_T$ is given up in a typical substorm: if that is spread out over 1000 seconds, (25) gives

$$P_s = 1.45 \times 10^{11} \text{ watt} \quad (26)$$

In a case studied by Baker et al. [1981], $B_L$ dropped from the rather high value of 38 nT to about 27 nT in ~40 minutes, leading the authors to deduce (from a tail length $100 R_E$)

$$P_s \sim 3 \times 10^{12} \text{ watt} \quad (27)$$

It is interesting to note that simultaneous IMP 8 observations suggest that the location of the tail boundary did not change significantly. Here $P_s$ is rather large, due to the high field intensity and to the fact that apparently around 50% of $W_T$ was given up.

ANNIHILATION OF STORED MAGNETIC ENERGY

The preceding section adds no new arrows to Figure 5. Rather, it implies that the energy flow from the plasma sheet is not steady but contains bursts of high activity. It may be interesting to speculate how the "annihilation" of part of the tail's magnetic energy $W_T$ can accelerate particles.

A feature of substorms is the "thinning" of the plasma sheet, its constriction to a very narrow thickness. There exists a wide belief [Hones, 1979] that at such times an X-type neutral point or line are formed as in Figure 7, detaching an island "plasmoid" with an embedded O-type point or line. In what follows merging will be treated by the "collisionless" approach, since the maintenance of the cross-tail current in a thinned sheet suggests accelerated flow perpendicular to the figure, and observations suggest this too [Fairfield et al., 1981]. This should no be taken to mean, however, that MHD merging does not occur in some situations, e.g. on the dayside magnetopause.
In the "collisionless" approximation, the electric field near an X-type line (arrows) can accelerate only the few particles whose motion carries them into the immediate vicinity of B=0. Furthermore, even those particles which enter the acceleration region can easily leave it again, unless the line is collapsed into a sheet (in which case any type of neutral point configuration appears similar to the particle). Thus X-type points or lines accelerate particles only locally and not too efficiently. A similar conclusion was reached by Scudder [1981] who examined dayside merging between arriving solar wind plasma and the magnetosphere. In explaining so-called "flux transfer events" [Russell and Elphic, 1979], Scudder noted that X-type neutral points produced only moderate jetting and only in limited regions, in agreement with the "patchy" nature of such events.

Assuming that the structure of Figure 7 stretches into the 3rd dimension, it will be realized that plasma heading for the O-type line is accelerated far more efficiently. For that line acts as a sinkhole, sucking particles inward until they are trapped around the axis, where their motion is nonadiabatic and their acceleration is rapid [Stern, 1979]. A similar situation exists near a neutral sheet, where Sonnerup [1971] used an adiabatic invariant characteristic of the motion to derive particle behavior.

Vasyliunas [1979] has argued that the accelerating voltage here will not be large, and his argument can be rephrased as follows. The O-type line is in fact an annihilation mechanism for magnetic energy: magnetized plasma is drawn into the sinkhole, but when it comes out (perpendicular to Figure 7) it has hardly any magnetic energy left. That energy is now shared among all plasma particles, and the average share is determined by $\beta$ of the entering plasma, by its ratio of kinetic to magnetic energy density. If $\beta>1$ (a good approximation for the plasma sheet), then even if all magnetic energy is converted, the average particle's energy merely doubles, and as pointed out by Vasyliunas [1979], no great acceleration occurs. But if during the "thinning" which accompanies magnetic substorms the entire plasma sheet is squeezed out, then the two high latitude lobes merge directly. These lobes are rarefied (n~10^2 cm^-3) and $\beta$~1/100, hence a hundredfold energization is possible. In different words, the lobe plasma contains so few particles, that if the energy of the ambient magnetic field
is transformed, each particle's share is quite large. This could explain the bursts of high energy particles occasionally seen in the tail.

Two additional points follow. First, if the plasma sheet reaches its end at some distance $x_n$ from earth, beyond which the two lobes adjoin each other, then field lines forming the plasma sheet boundary are linked to $x_n$. One then expects relatively high-energy particles on such lines, and this may be related to the observation that the "plasma sheet boundary layer" is more energetic than the interior of the plasma sheet.

Secondly, the preceding does not resolve the controversy alluded to before, on whether the release of magnetic energy is "directly driven" by the solar wind or is an "unloading process" of stored magnetic energy. Both energy sources may contribute. On one hand, the dawn-to-dusk electric field perpendicular to Figure 7 may reflect at least in part the potential drop of the open magnetosphere: this drives plasma towards $z=0$ and earthward, even at places where the plasma sheet is constricted as drawn. A fundamental question here, perhaps, is why such flow does not sweep earthward the entire plasma sheet, but instead the sheet renews itself continually, and does not thin out on its own accord.

On the other hand, an additional process may occur during substorms. As "thinning" suggests, the plasma sheet may be squeezed out, perhaps (in the prevalent view) by a relatively high amount of magnetic flux in the tail, caused by more efficient merging on the day side when the IMF has a strong southward component. It is then hard for the rarefied lobe plasma to supply sufficient particles to carry the cross-tail flow required to maintain $B_L$. As a result $B_L$ drops, inducing an added dawn-to-dusk voltage, accelerating the particles more than would happen otherwise: this helps maintain the current, but at the expense of $W_T$. In both cases, the circuit in which energy is released contains not just the plasma sheet but also the boundaries of the lobes, and it has been noted by Meng et al. [1981] that energetic ions and electrons are prevalent in both those regions.
CONCLUSIONS

In the early days of space research scientists were uncertain whether the aurora was a secondary effect of the ring current ("leaky bucket model") or whether, on the contrary, the trapping of ring current particles was secondary to a much larger energy flow into the aurora ("splash catcher"). The comparison of (18) and (22) suggests that both processes have comparable energy inputs, though the turnover in the high-energy part of the ring current (which was all the early observers knew about) is much smaller. They both, however, seem to be just by-products of a much larger energy input into the tail: 2-3 times the auroral energy goes into joule heating by Birkeland currents, and a large (if uncertain) amount of energy leaks out again into the solar wind and does not reach the earth's vicinity. Recent results on all those processes, and attempts to correlate them with interplanetary conditions, have been described, and estimates of the various power are listed in Table 1. There still remain unresolved controversies about substorms, their mechanism, and about the roles which magnetic merging and magnetic energy storage play in them.

NOTE

This work is an expanded and updated version of an earlier article by the same title [Stern, 1980]. Most of the original notation and layout were retained, but some figures were omitted or replaced, and a large number of new references was added.

ACKNOWLEDGMENT

The author thanks T.G. Birmingham for his helpful comments on the draft of this article.
CAPTIONS OF FIGURES

Figure 1 — Schematic tail cross section in a closed (2a) and an open (2b) magnetosphere.

Figure 2 — Energy flow past the inner edge of the plasma sheet.

Figure 3 — The variation of Dst in a magnetic storm [Sugiura, 1980].

Figure 4 — Joule heating rate \( P \), for various levels of magnetic activity and for different seasons [Foster et al., 1983]. The lowest graph gives \( P \), from Spiro et al. [1982].

Figure 5 — Energy flowchart for the earth's magnetosphere.

Figure 6 — Schematic configuration of an X-type neutral line.

Figure 7 — Suggested substorm merging process [Hones, 1979].

CAPTION TO TABLE 1

Energy flow rates related to the earth's magnetosphere, in units of \( 10^{10} \) watt.
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Average</th>
<th>Large events</th>
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<td>$P_1$</td>
<td>SW energy impinging on the dayside</td>
<td>1300</td>
<td></td>
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<tr>
<td>$P_2$</td>
<td>Sunlight power hitting earth</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td>$P_3$</td>
<td>Tail input per $40 \text{R}_E$ length</td>
<td>1.75 $10^7$</td>
<td></td>
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<tr>
<td>$P_4$</td>
<td>Inflow from the plasma sheet</td>
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<td></td>
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<tr>
<td>$P_5$</td>
<td>Ring current injection rate</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>$P_6$</td>
<td>Birkeland current input</td>
<td>1.5</td>
<td>20</td>
</tr>
<tr>
<td>$P_7$</td>
<td>Aurora</td>
<td>14</td>
<td>27+</td>
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<tr>
<td>$P_8$</td>
<td>Joule heating</td>
<td>4</td>
<td>10+</td>
</tr>
<tr>
<td>$P_9$</td>
<td>Tail energy release in substorms (sporadically)</td>
<td>15</td>
<td>300</td>
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<td></td>
<td>Auroral kilometric radiation</td>
<td>0.002</td>
<td>0.1</td>
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<tr>
<td></td>
<td>Pc5 micropulsations</td>
<td>0.6</td>
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Figure 2

PLASMA SHEET

$n \sim 0.5 \text{ cm}^{-3}$

$\langle W \rangle \sim 5 \text{ keV}$

$2Y_0 = 40 R_E$

$2Z_0 = 5 R_E$

40 km/sec
Figure 3
Figure 4
Kilometric waves
$P_g \sim 2 \times 10^7 \, \text{w}$

Solar Wind
Energy hitting the Magnetosphere
$P_1 \sim 1.3-5 \times 10^{13} \, \text{watt}$

$0^+$ ions on open field lines

Production of $E_{II}$ and wave heating
$P_7 = 1-3 \times 10^{11} \, \text{w}$

Joule Heating $\sim 70\%$

Aurora $P_8 \sim 4 \times 10^{10} \, \text{w}$ $\sim 30\%$

Region 1
Trapped $O^+$

Convected Earthwards
Region 2

Trapped in the Ring Current

Lost through flanks
Exit via dayside boundary
Charge Exchange

Diffuse Aurora

Cusp precipitation

Charge as neutrals

Figure 5
This is a revision of TM 82039, of the same name, published November 1980. To appear in Space Science Reviews.

**Abstract**

The approximate magnitudes of power inputs and energies associated with the earth's magnetosphere are derived. The solar wind impinging on the dayside magnetosphere carries about $1.3 \times 10^{13}$ watt, on the night side about 3 times that amount. The nearest 40 RE of the plasma sheet current receive some $3 \times 10^{11}$ watt, and much of this goes to the Birkeland currents, which require $1-3 \times 10^{11}$ watt. Of that energy, about 30% appears as the energy of auroral particles and most of the rest as ionospheric joule heating. The ring current contains about $10^{15}$ joule at quiet times, several times as much during magnetic storms, and the magnetic energy stored in the tail lobes is comparable. Substorm energy releases may range at $1.5-30 \times 10^{11}$ watt.

Compared to these, the local energy release rate by magnetic merging in the magnetosphere is small. Merging is however essential for the existence of open field lines, which make such inputs possible. Merging also seems to be implicated in substorms: there, too, most of the released energy only becomes evident far from the merging region, though some particles may gain appreciable energy in that region itself, if the plasma sheet is squeezed out completely and the high latitude lobes interact directly.