VELOCITIES OF GUIDED ULTRASONIC WAVES IN HETEROGENEOUS MEDIUM

M. Touratier


NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON D.C. 20546 APRIL 1984

REPRODUCIBLE COPY
(FACILITY CASEFILE COPY)
VELOCITIES OF GUIDED ULTRASONIC WAVES IN HETEROGENEOUS MEDIUM

Maurice Touratier
Ecole Nationale Superieure d'Arts et Metiers, Paris, France

SCITRAN
Box 5456
Santa Barbara, CA 93108

National Aeronautics and Space Administration
Washington, D.C. 20546

Translation of "Vitesses d'ondes ultrasoniques guidees en milieu heterogene."
Comptes Rendus, Serie II - Mechanique, Physique, Chimie, Sciences de l'Univers,

(A83-38067)

Experimental and theoretical examinations were performed of the longitudinal velocity characteristics of waves in trilaminar and encapsulated waveguides. The study was confined to waveguides with core material that featured transverse wave velocities much worse than the longitudinal wave velocities. The velocities were obtained using a dispersion equation, with consideration given to both the core and encapsulant. Asymptotic velocities were also calculated for bending and twisting in trilaminar waveguides. Trials were run with bimetallic waveguides for comparison with the theoretical predictions. Good agreement was found between the predicted velocity of the propagation of the fundamental mode and the measured velocities. The method was calculated valid for modes above four, confirming that the data were contained in either the core or outer layer, and were insensitive to the encapsulant.
INTRODUCTION.—Heterogeneous media can be of considerable value for containing the propagation of elastic waves. At high frequencies, it is possible to direct certain modes of propagation along a region determined by a heterogeneous wave guide. For this, one can use a facility adapted to a law of desired dispersion or of velocities of ultrasound propagation by adjusting the mechanical and geometric properties of media that comprise the heterogeneous wave guide.

Two heterogeneous symmetrical guides are proposed:

(a) a "trilaminar" guide made by superposition of three layers, the central layer being composed of a different material from that of the two outside layers (Fig. 1).

(b) a "rectangular encapsulated core," made from a rectangular bar (core), covered by an also rectangular but hollow sheath; the material of the core is distinct from the material of the sheath (Fig. 2).

The guides are meant to channel the essence of the signal, either in the central layer of the "trilaminar," or in the core of the rectangular encapsulated core. So that protection of the fundamental modes of propagation which will be collected at the other extreme of the central layer or of the core will be assured, certain superior modes can be transmitted in the exterior layers or in the sheath. Such
Fig. 1

1 external layer
2 central layer

Fig. 2

3 sheath
4 core
conditions will be realizable if the material which constitutes the central region has a transverse spatial wave velocity smaller than that of the material designed to shield the signal.

ASYMPTOTIC VELOCITIES OF PROPAGATION.—The dispersion characteristics of rectangular encapsulated core or "trilaminar" wave guides are presented in [2] and [3]. The formulation used allows a very simple explanation of asymptotic phase velocities at high frequencies, both for the fundamental modes and certain superior modes of symmetrical heterogeneous guides for which longitudinal and flexion-torsion movements are uncoupled. The analysis that follows is limited to longitudinal motion for the "encapsulated" and flexion-torsion motion for the "trilaminar." Also, the study is limited to the case of the core material or of the central layer presenting a transverse wave velocity in space less than that which propagates in the material of the sheath of the exterior layers.

(a) Asymptotic velocities of longitudinal motion in the "rectangular encapsulated core."—These velocities are deduced from the dispersion equation presented in [2] when \( K \to +\infty \) (very short waves) where \( K = 2\pi a/\lambda \) is a reduced wavenumber and \( \lambda \) is the wavelength.

The propagation velocity of the fundamental mode is expressed by

\[
\text{Inf}(v_{1\omega}, v_{2\omega})
\]

and that of the first superior mode is equal to

\[
\text{Sup}(v_{1\omega}, v_{2\omega})
\]

where:
In these formulas, one has successively

\[
\begin{align*}
\sigma &= \frac{b_2}{a_2^2 S_{55} + a_1 S_{55}} + \frac{2b_1}{a'' S_{55}}, \\
c_2 &= \frac{a_2}{b_2 S_{44} + b_1 S_{44}} + \frac{2a_1}{b'' S_{44}},
\end{align*}
\]

\[
\begin{align*}
a''' &= \frac{56}{15 \pi}, \\
a'' &= \frac{56}{15 \pi}, \\
b'' &= b_1 + b_2, \\
\epsilon &= \rho a_2 b_2 + 2 \rho (a b_1 + a_1 b_2).
\end{align*}
\]

The geometric characteristics \(a_1, a_2, a, b_1, b_2, b\) are defined in Fig. 2; \(S_{44}, S_{55}, S_{44}^{S}, S_{55}^{S}, S_{33}^{S}\) are the compressibilities used in this calculation, respectively for the materials of the sheath and of the core, such that \(\rho\) and \(\rho^0\) are the mass densities of the sheath and the core. The longitudinal movement in the core could be guided in the core according to a longitudinal spatial wave of velocity \(v_2\) in the core material if the geometric and material characteristics of the guide are adjusted in a proper fashion. The displacement distribution, calculated from the short wavelengths by the exact three-dimensional theory for two coaxial cylinders, confirms this interpretation of asymptotic velocities [1].
(b) Asymptotic flexion-torsion velocities in the "trilaminar." The formulas presented were obtained with the aid of the dispersion equation which is explained in [3]. The flexion-torsion motions give rise to three fundamental modes of propagation, uncoupled only if \( K \to +\infty \): a mode of shearing in thickness of velocity \( \omega \frac{v_1}{c_0} \), a mode of shearing in width of velocity \( \omega \frac{v_2}{c_0} \), and a mode of torsion about the axis of the guide of velocity \( \omega \frac{v_3}{c_0} \). The first superior mode has as a velocity \( \sup \frac{v_1}{c_0}, \omega \frac{v_2}{c_0} \). These velocities from [3] are formulated as follows:

\[
\begin{align*}
\omega \frac{v_1}{c_0} &= \frac{1}{\sqrt{\varepsilon} c_0}, \\
\omega \frac{v_2}{c_0} &= \frac{1}{\sqrt{\varepsilon} c_0}, \\
\omega \frac{v_1}{c_0} &= \frac{1}{\sqrt{\varepsilon} c_0}, \\
\omega \frac{v_2}{c_0} &= \frac{1}{\sqrt{\varepsilon} c_0}, \\
\omega \frac{v_3}{c_0} &= \frac{1}{\sqrt{\varepsilon} c_0}.
\end{align*}
\]

where:

\[
\begin{align*}
c_1^2 &= \frac{a^2 b}{a_2 S_{55} + a_1 S_{55}}, \\
c_2^2 &= \frac{a^2 b^2}{b''(a_2 S_{44} + 2a_1 S_{44})}, \\
\varepsilon &= (\rho a_2 + 2 \rho a_1) b, \\
c_3^2 &= a_2 \left\{ \frac{b_2}{a_2 S_{55} + a_1 S_{55}} + \frac{2b_1}{a'' S_{55}} \right\}, \\
c_4^2 &= \frac{b''}{b} \left\{ \frac{a_2}{S_{44}} + \frac{2a_1}{S_{44}} \right\}.
\end{align*}
\]

The geometric characteristics \( a_1, a_2, a_3, b_1, b_2, b \) are indicated in Fig. 1, the mechanical characteristics of the central layer being noted \( \rho, S_{55}, S_{44}, S_{33} \) and those of the exterior layers by \( \rho, S_{44}, S_{33} \).

To obtain one of the fundamental modes guided in the central layer following a transverse spatial wave, one would adjust the geometric and material characteristics of the
guide in a fashion to require:

\[
\begin{pmatrix}
00 & 00 \\
V_{1\omega} & V_{2\omega}
\end{pmatrix}
\begin{pmatrix}
001 \\
010
\end{pmatrix}
\rightarrow
\text{Inf}(V_{1\omega}, V_{2\omega}) \equiv v^2_T
\]

where \( v^2_T \) is the velocity of the transverse spatial wave in the material of the central layer.
BIMETALLIC GUIDE EXPERIMENT.—The formulas recalled above have been applied to a bimetallic "rectangular encapsulated core" and a "trilaminar," in which the longitudinal transverse modes are guided in the core and the central layer respectively. For that, the rectangular encapsulated core guide presents the following geometric and material characteristics:

\[
\begin{align*}
2a &= 6 \text{ mm}, \quad 2b = 12 \text{ mm}, \quad 2a_2 = 3 \text{ mm}, \quad 2b_2 = 9 \text{ mm}, \quad L = 684 \text{ mm}, \\
\sigma_0 &= 9600 \text{ kg/m}^3, \quad S_{33} = 4.167 \times 10^{-11} \text{ m}^2/\text{N}, \quad \sigma_{44} = \sigma_{55} = 8.667 \times 10^{-11} \text{ m}^2/\text{N}, \\
p &= 8900 \text{ kg/m}^3, \quad S_{33} = 6.803 \times 10^{-12} \text{ m}^2/\text{N}, \quad S_{44} = S_{55} = 1.782 \times 10^{-11} \text{ m}^2/\text{N}.
\end{align*}
\]

The "trilaminar" guide was obtained with the same materials; the geometric characteristics were, on the other hand:

\[
\begin{align*}
2a &= 4 \text{ mm}, \quad 2b = 8 \text{ mm}, \quad 2a_2 = 3 \text{ mm}, \quad L = 608 \text{ mm}.
\end{align*}
\]

The experimental verification of the modes of guided propagation at the center of the heterogeneous media considered was effected by putting the cylinders in resonance in the middle of piezoelectric transducers. Portions of distant longitudinal displacements of extremities of the cylinder were taken at the lateral surface to a length of 127 mm and analyzed by Fourier transform. This operation allows detection of the wavelengths that compose the signal observed.
at the lateral surface to a length of 127 mm and analyzed by Fourier transform. This operation allows detection of the wavelengths that compose the signal observed at the lateral exterior surface of the cylinder for a fixed resonance frequency. This method shows the modes of propagation that are propagated up to the exterior surface of the guide. The experimental analysis was limited to about 700 kHz. Such frequencies are sufficient, in fact, to obtain asymptotic velocities, at least on the fundamental mode and the first superior mode. The results of the experimental propagation velocities are shown in Fig. 3 for a longitudinal excitation of the section of the guide (○) or of the core (●), of the rectangular encapsulated core, and Fig. 4 for a transverse excitation of the section of the guide (○) or of the central layer (●), of the "trilaminar." In both cases, the fundamental modes of propagation are no longer detected at the exterior of the cylinder if $K > 4$, that is to say, for frequencies somewhat greater than 350 kHz. The velocities $v$ are dimensionless, scaled by $c^2 = \rho S_{33}$. 

![Graph](image1)

![Graph](image2)
EXPERIMENT-THEORY COMPARISON. CONCLUSION.—In Fig. 5 and 6, the experimental results (○) are compared to numerical results (—) deduced from the previously evoked model. Note that these latter agree asymptotically on the fundamental modes of propagation and the first superior mode. The passage of experimental propagation velocities from the fundamental mode to the superior mode for $K > 4$ confirms that the essence of the signal is contained in the core or in the central layer and is no longer detectable on the protective exterior surface. Besides, the measures marked by the asterisk (*) in Fig. 6 and obtained for a summary statement on the lateral surface $x_2 = 4$ mm and $x_1 = 0$ mm of the "trilaminar," thus following one of the two free faces of the central layer, permits one to find the perturbation on the fundamental mode, quite confined between the external layers of the guide.
Submitted March 21, 1983.

