THEORY AND MEASURE OF CERTAIN IMAGE NORMS IN SAR

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ABSTRACT

This paper summarizes principal properties of SAR imagery of point and distributed objects. Against this background, the response of a SAR (Synthetic Aperture Radar) to the moving surface of the sea is considered. Certain conclusions are drawn as to the mechanism of interaction between microwaves and the sea surface.

It has been established for a "well-behaved" SAR (as for other radars) that a principle of conservation of energy is satisfied. This means that the energy of the response (in the image) is constant under conditions of changing phase of the signal. Phase errors may arise systematically, such as focus mis-match to parameters appropriate to a specular scattering centre, or randomly, as from complex motion of the sea surface. Of course, focus errors reduce the peak and spread the impulse response of the image of a point target. Focus errors do not, however, change the speckle spectrum of a truly random "uniform" Gaussian scene.

Focus and speckle spectral tests may be used on selected SAR imagery for areas of the ocean. When this is done, it is observed that the fine structure of the sea imagery is sensitive to processor focus and adjustment. Furthermore, there is frequently correlation between nominally statistically independent looks. Therefore, the ocean reflectivity mechanism must include point-like scatterers of sufficient radar cross-section to dominate the return from certain individual resolution elements. Furthermore, both specular and diffuse scattering mechanisms are observed together, to varying degree. The effect is sea state dependent, of course. This mechanism would explain the evident diversity of theoretical opinion on the subject of SAR wave response.

Several experiments are proposed based on imaging theory that could assist in the investigation of reflectivity mechanisms.
1. INTRODUCTION

It is a reasonable requirement of SAR (Synthetic Aperture Radar) ocean reflectivity theories that they be consistent in every regard with the facts of life of SAR image formation. To the knowledge of this author, there are no theories of ocean reflectivity that pass this test.

A SAR, even operating in a partially coherent mode, is a special kind of linear system. Several fundamental properties for such systems have been rigourously proven (Harger, 1970, Raney, 1983). The observation by the SAR of particular scenes, such as an ocean surface, cannot change these facts.

The purpose of this paper is two-fold. First, pertinent properties of SAR operation are presented and succinctly discussed. Whereas many of these properties have been known for some years, they seem not to have been fully apprehended by workers in the ocean reflectivity field. Second, there follow from the first discussion consequences of importance to the oceanographic application. Several of these are highlighted and discussed. Furthermore, it is possible to design certain data processing and field experiments that may be used to take advantage of these properties, and so to shed some light on the reflectivity mechanisms involved. Suggestions are made for such tests.

The major thrust of Raney (1983) is that for SAR (as for all radar systems) there exist fundamental properties such as conservation of energy. These properties, if suitably employed, can be helpful in deriving quantitative information about the reflectivity mechanism from the imagery. The matter is complicated by (1) the partial coherence of most radar systems, (2) non-linearities and temporal variations found in all practical radar systems, and (3) the presence of both specular and diffuse scatterers in the input scene. This work deals directly with (1), disregards (2), and makes some observations based on (3) that have direct impact on SAR reflectivity models of the ocean surface.

Following the introduction, Section 2 of the paper considers suitable model representation of a SAR. Basic properties are identified in signal processing considerations.

Section 3 considers the "impulse response" of a SAR, the way in which the system images an idealized point object. Section 4 reviews the way in which a SAR images an idealized uniform random distributed scene such as the classic wheat field. In each of these discussions, radar system parameters (such as nominal resolution and bandwidth) and processing system parameters (such as focus and multi-looking) are considered.

In Section 5 basic properties of speckle are reviewed. As is well known, radar speckle is an unwelcome but ever present characteristic of quasi-coherent imagery, and it, too, obeys certain well established principles.
In Section 6, the properties previously presented are revisited, with the objective of seeing them from the point of view of applications to the oceanic imaging problem, leading to a discussion of recommended experiments. The paper has a brief concluding section.

2. SAR Model

The basic objective of an imaging radar system is deceptively simple: we wish to derive an "image" which is a mapping of the reflectivity of a scene observed by microwave probing of the real world, expressed in photographic form. In model language, real world reflectivity $\sigma_0(x,y)$ is estimated as $\hat{\sigma}_0(x,y)$ through a microwave transducer (MT) thus

$$\sigma_0(x,y) \rightarrow \hat{\sigma}_0(x,y)$$

... and at the outset, complexities are apparent. Let us confine the discussion to synthetic aperture radar (SAR) systems. Then the following are true:

i) SAR systems "work" because of the different mechanisms used to form the azimuth (a) and the range (r) dimensions of the image (Harger, 1970). Range scanning is at one half the speed of light (thus in effect instantaneous) and continuous. Azimuth scanning is at the speed of the carrier vehicle (thus at velocities sensitive to possible changes in the scene itself) and is dependent on the pulsed nature of the radar. For the moment, we ignore these fundamental range and azimuth differences, and treat the two "channels" of a SAR in like manner, a satisfactory approximation for the first sections of this paper. The differences in time scale between these channels lie at the core of the SAR ocean imaging problem, however.

ii) The transducer "MT" is not perfect. That is, it is not able to image all of the detail inherent in the scene. This characteristic is typical of any imaging system: resolution (in range or azimuth) is a measure of this limitation. There is rather little subtlety in this consideration, except that...

iii) The microwave probe (transmitted signal of the SAR) is essentially monochromatic and coherent. There are many important implications that follow from this simple fact. It means that we do not measure the reflected energy density directly (as one might visualize, for example, using the polychromatic and non-coherent sun as an illumination source, deriving thus an analog of "reflectivity"). The monochromatic radar illumination makes the radar behave as an interferometer, whose input is a linear sum of complex amplitude signals, each of the form

$$|\sigma_0(x,y)| e^{j\phi(x,y)} e^{j(2\pi ft + \theta)}$$
Thus the input includes the (square root of) reflectivity, but also the phase $\phi(x,y)$ of each reflecting element, which of course is directly affected by the (accidental) distance and aspect angle between that reflector and the radar. The phase is a mixed blessing, for it allows “synthetic aperture” resolution (Brown and Porcello, 1969) (beam sharpening) to occur; it also gives rise to “speckle” (described below).

iv) Whereas the probe of the scene is at complex (microwave) amplitude, the image is in terms of amplitude magnitude squared. Thus, in numerical terms, the image $\hat{\sigma}_0(x,y)$ is a real non-negative variable in contrast to the radar’s observation in the scene, which is in terms of complex amplitude. (This seeming non-linear transformation is the key to the radar principles of “conservation of energy”.)

v) A SAR works because the radar’s motion imposes a structure on the phases of the received signals that can be used to “focus” the resulting imagery to a specific resolution (Brown and Porcello, 1969; Harper, 1970; Raney, 1983). Once this is accomplished, then the phase information becomes irrelevant, and now the disadvantages of the remaining phase structure (speckle) become important. Speckle can be reduced (at the expense of resolution – see below) in either of the two dimensions, range or azimuth, by one of two linear techniques, frequency domain filtering (subapertures) (Bennett and McConnell, 1980; Porcello, 1976) or by adjacent cell averaging (Zelenka, 1976). These two techniques, frequency domain and image domain, are mathematically equivalent for stationary inputs and SAR type systems (Raney, 1983), an important consideration for users who may have to deal with pre-formed imagery. (Non-linear speckle reduction techniques may also be employed, but are not of interest in this paper.)

vi) Finally, all of the above deals with systems that present imagery in $\hat{\sigma}_0(x,y)$ form, i.e., amplitude squared. There are systems, such as the MDA G-SAR processor, that (a) do a square-root or other amplitude mapping, and (b) perform a “most significant bit” or other automatic gain control function, both with the intent to improve image cosmetics or data volume compression. It is important to note that the considerations of this paper apply to the “unscaled amplitude square” image data format. For those wishing to pursue experiments in this area, either access to such data is essential, or suitable transformations are required.

Given all of the above, there exists a “model” of a SAR system that incorporates these characteristics, in terms of a generalized quadratic filter theory (Raney, 1983). In this language, a SAR is described (for either the range or the azimuth channel) as a simple sequence of operations.
... in which the input is in complex amplitude, the range coding or azimuth Doppler modulation is represented by the pre-filter w, multiplicative random phase perturbation, additive (complex receiver) noise n enters, the data is focussed, amplitude squared, and speckle smoothed (Q) to arrive at the image g.

For the following, we assume that the system is "linear", but not "perfect". There may occur focus errors, or variations in the coherence of the processor (i.e. intentional speckle smoothing) or in the scene (unintentional, resulting from sea surface motion). We will be interested in measures of g (the image) as they relate to properties of the scene and the processor.

The system is linear in a special sense. A SAR, like other types of radars, includes filters linear in complex amplitude (pre-detection or coherent integration), square law detection, and image smoothing (post-detection or non-coherent integration, i.e., "multi-looking" in popular SAR terminology). For such partially coherent systems, the input/output relationship may be expressed in terms of a modulation transfer function (O'Neill, 1963) which is linear in intensity (spatial reflectivity density). This is valid, no matter the degree of partial coherence of the (radar) system or the scene (Raney, 1983).

3. IMPULSE RESPONSE

The response of a SAR to a small specular point scatterer such as a corner reflector gives rise to an image pattern that is of fundamental importance in system characterization, analogous to the "point-spread function" of non-coherent optics (O'Neill, 1963). The impulse response is the classic test signal for radars.

Let the impulse response be represented by $g_s(u)$, a non-negative function with units of voltage squared. A well behaved impulse response will be sharply peaked

![Impulse Response Diagram]

... has a width $\Delta$ at the half-power level, and has "reasonable" side lobes. The width $\Delta$ is (loosely) referred to as the resolution of the radar, in either the range or azimuth dimension.

The following properties may be proven for the impulse response:
1) The energy $\int g_t(u)du$ in the response is constant in the face of focus variations in the SAR (processor), random phase perturbations in the reflector, and for differing degrees of non-coherent integration (multi-looking). There is no coherent gain on the mean reflectivity of a point (coherent) target. This is the conservation of energy principle for point targets.

ii) The fully coherent correctly focussed impulse response has width $\Delta_0$, which is the minimum width obtainable from the system. The inverse width $<\Delta_0>^{-1}$ is a measure of the effective system bandwidth in the pertinent channel, suitably scaled from spatial coordinates to Hertz. Focus errors, or partial coherence in the point target, result in broadening of the impulse responses and reduction of its peak.

iii) For a given system differing amounts of non-coherent integration may be employed in the processor. Increased non-coherent integration degrades resolution. For $N$ statistically independent looks, the corresponding impulse response width $\Delta_N = N\Delta_0$.

iv) It follows that for an $N$-look response, the peak is reduced $N$-fold. This occurs because specular scatterers maintain their coherence as the radar observes them, so that the coherent gain due to processor focus is reduced as less of the signal is used coherently. Note that the famous coherent gain of a SAR is applicable to the peak value of reflectivity of a single coherent scattering centre (in one resolution cell), and is dependent on scene and system coherence.

v) As an obvious but important generalization, it follows that for a specular scatterer, there is very high correlation of the response between any two looks of a multi-look set.

vi) In the event that there are deterministic phase perturbations on a point scatterer, azimuth shift (proportional to the linear phase term) and azimuth defocus (proportional to the quadratic phase error) plague the affected impulse response (Raney, 1971). The radar processor may be retuned to match these perturbations, but at the expense of becoming mis-matched to all other signals in the processed field. (It follows that if there are a variety of different shift and focus perturbations affecting various scatterers in the scene, they cannot all be optimally processed simultaneously.)

4. RESPONSE TO DISTRIBUTED SCENES

For many remote sensing applications, the response of a SAR to distributed scenes is of more interest than the point target response. One can show (Raney, 1983) that the input/output relationship is $\delta_0(x,y) = R(x,y)**\delta_0(x,y)$ where ** denotes convolution on the $x$ and $y$ coordinates respectively and $R(x,y)$ is the appropriate impulse response of the SAR. The following properties are satisfied:

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1) The average value of the output, for nominally constant input, is a constant, independent of system focus, system coherence, or scene coherence, assuming that all available data is used in each case. This is the most fundamental feature of the system, the principle of conservation of energy. It means that there is no coherent gain by the radar or processor for the mean reflectivity of diffuse distributed scenes. Furthermore, between multi-look sets, relative gain can be normalized since total average response is not a function of the number of looks.

ii) From i) it follows that a SAR's response to a (Gaussian) distributed scene relative to receiver noise is not a function of processing. Thus, for a given radar and scene, the SNR is constant as processor focus and coherence are varied, assuming that all the available data is used.

iii) The two-dimensional Fourier transform of \( g(x,y) \), written as \( \tilde{g}(\omega,\lambda) \), is the modulation transfer function (MTF) of the system. If the system focus is incorrect, partial coherence is used in the processor, or if there is loss of temporal coherence in the scene, then the width of the MTF is reduced, thereby limiting the ability of the system to image scene detail (Raney, 1983, 1980). This is of central importance in the response of a SAR to distributed dynamic phenomena, such as ocean reflectivity. It has the heavy consequence that the appropriate impulse response for the SAR may not be the same for all parts of the image simultaneously.

5. SPECKLE CONSIDERATIONS

The output of the system is deeply modulated even for nominally constant input \( \sigma_0(x,y) \). This phenomenon is known as speckle (Zelenka, 1976; Porcello, 1976; Bennett and McConnell, 1980), and is a natural consequence of coherent illumination by the radar of a Gaussian scene. (By definition, a Gaussian scene is one in which, for each (nominal) resolution cell, there are many effective scatterers of statistically independent amplitude and phase.)

1) To first and second order, speckle statistics (for a uniform Gaussian distributed scene) are not a function of system focus.

ii) Speckle statistics are not a function of scene temporal coherence. Thus, for Gaussian scattering, one cannot use speckle measures to estimate scene coherence.

iii) For a Gaussian random input, the several "looks" separated by a multi-look processor are largely uncorrelated. The correlation properties observed are a measure of the bandwidth and frequency weighting of the SAR/processor combination.

iv) One effective measure of speckle is its variance. The amount of non-coherent integration in the processor, that is, the effective number of statistically independent looks \( N \), may be estimated by the ratio

\[
N = \frac{\text{mean value } \sigma_0}{\text{variance } \sigma_0} = (\text{VHS})^{-1}
\]

for a nominally uniform region of a given scene.
v) As a consequence of the preceding four properties, speckle can be used to estimate the potential resolution of a SAR/processor. The nominal impulse response of the radar is closely approximated by the square root of the speckle covariance function for correct focus and scene coherence. However, speckle cannot be used to estimate actual SAR performance against particular (possibly dynamic) objects unless focus and scene coherence can be (independently) ascertained as correct for the dynamics of those objects.

vi) There is evidently a direct trade-off between resolution ($A_N$ proportional to N) and speckle reduction (variance inversely proportional to N). This can be stated as the principle of conservation of confusion:

$$\int [R_g(x) - R^2] dx = \text{constant}$$

independent of the degree of partial coherence, where $R_g(x)$ is the spatial correlation function, under the assumption of uniform Gaussian input.

vii) In the event that the scene is not Gaussian at the nominal resolution cell level, then these properties do not necessarily hold. In particular, if there are dominant scattering centres, then image behavior will be described more appropriately by articles in Section 4 above, even if the so-called image resembles speckle in appearance.

6. OBSERVATIONS AND IMPLICATIONS

The properties of SAR imaging behavior introduced above should be incontrovertible. Any observation, theoretical or experimental, that purports to "explain" the content of SAR ocean imagery, or to go even further and to "explain" the scattering mechanism, must be consistent with these principles.

There seem to be two general issues in active discussion in the theory of SAR ocean wave imaging: Gaussian versus non-Gaussian scattering; and the causes (and possible remedy) of azimuth directional spectral narrowing (Hasselman et al., 1984). Therefore, it would be helpful to organize the foregoing SAR facts of life accordingly.

Table 1 compares the response of Gaussian scatterers and a specular scatterer for eleven measurable SAR image properties. These properties in turn are organized into two groups, General Considerations, having to do with a nominally uniform average reflectivity (of which a wheat field is the classic example), and Two-scale Considerations, for which there is assumed a low (spatial) frequency modulation of the reflectivity, as by a swell or more fully developed sea. The Table is filled in under the assumption that the SAR azimuth response is of interest.
**Table 1**

**Comparison of Image Properties**

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<tr>
<th>General Considerations</th>
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<tr>
<td><strong>GAUSSIAN SCATTERING</strong></td>
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<tr>
<td>1. High correlation between looks (azimuth sub-aperture filtering)</td>
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<tr>
<td>2. Average image (intensity) dependent on N-look processing</td>
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<tr>
<td>3. Peak values of image (intensity) dependence on N</td>
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<tr>
<td>4. Focus sensitivity (uniform reflector)</td>
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<td>5. Image sensitivity to scene coherence time</td>
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<tr>
<td>6. Fourier transform of &quot;image&quot; a measure of SAR/processor (resolution)$^{-1}$</td>
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<tr>
<td>7. Fourier transform of &quot;image&quot; a measure of SAR/processor bandwidth</td>
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<tr>
<th>Two-scale Considerations</th>
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<tbody>
<tr>
<td><strong>GAUSSIAN SCATTERING</strong></td>
</tr>
<tr>
<td>8. Velocity bunching</td>
</tr>
<tr>
<td>9. Velocity spreading</td>
</tr>
<tr>
<td>10. Focus sensitivity</td>
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<tr>
<td>11. Coherence time limitation</td>
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</table>
The issue of Gaussian versus non-Gaussian scattering is an important one, both technically and philosophically. From a technical point of view, there is ample evidence that non-Gaussian scattering is characteristic of sea radar return (Trunk and George, 1970; Barkeshli and Moore, 1983), and indeed for many applications it is the reflectivity characteristic of central concern, as in target detection (Trunk and George, 1970), or in explaining the difference between airborne and tower based scatterometer results (Barkeshli and Moore, 1983). It seems well established that as the nominal resolution cell size decreases, the importance of "spikey" non-Gaussian reflectivity elements increases (Trunk, 1972, and especially Jakeman and Pusey, 1976).

There are various "explanations" for non-Gaussian scattering. Most of these explanations are based on statistics that are closely related to the expected result of a very small average number of effective scatterers per resolution cell, where this value ranges from 0.1 or less (Jakeman and Pusey, 1976) to (less than ) 5, the accepted threshold for Rayleigh - hence Gaussian - scattering. For this reason, in Table 1 the non-Gaussian case is represented by a single (dominant) specular scattering centre in a resolution cell. There may or may not be such a scatterer in an adjacent cell in a typical situation, indeed usually not. Hence the point target properties of SAR response apply to such a case.

For Gaussian scattering, it is assumed that there are "many" effective scattering centres per resolution cell.

As an aside, it is important to note that for a SAR, the number of effective scatterers per resolution cell is the spatial scale of interest, not the instantaneous field of view (antenna width by pulse length) of the radar.

From a philosophical point of view, the existence of and differences between Gaussian and non-Gaussian scatterers is important in that agreement on the veracity or significance of theoretical or experimental results is impossible unless assumptions about the underlying scattering, either explicitly presented or implicitly employed, are clearly explored and consistently followed. From the point of view of this writer, most of the controversy in the "focus" area is potentially resolvable if first there would be agreement on the type of scattering being considered.

Finally, some remarks on the nature of "a specular scatterer". It is obvious that a solid corner reflector is an example of a specular scatterer, as is a facet whose plane is orthogonal to the radar range line. It is likely that instances of coherent specular reflection arise from such geometrics accidentally simulated by the sea surface. Likewise, point scattering by a cusp or other surface discontinuity could provide a source of specular reflection.
There is another source of "specular" reflection that seems to have been overlooked to date; Bragg scattering. Whereas Bragg scattering, the geometry in which the radar \( \frac{1}{2} \) wavelength projected onto a wavy surface picks out preferentially the matching surface spatial frequency, is usually taken to be the epitomy of Gaussian scattering (Hasselmann et al., 1984), it is by definition just the opposite! Bragg scattering, for one set of resonant scatterers in a local region, results in a coherent specular signal. It is only if there is an ensemble of many such Bragg scattering cells in one resolution cell that Gaussian statistics again apply. The modelling issue then reduces to representation of the expected size of a Bragg region as compared to a radar resolution cell.

Differentiation between these two specular reflection concepts should be possible experimentally as they are modulated by quite different portions of the ocean Doppler spectrum. The first typically move at nominal phase speed of the longer waves, whereas the second are dominated by orbital advection, hence much more slowly.

Regardless of "the cause" of specular events in SAR ocean imagery, they do exist. Their observable properties are in many cases different from proper Gaussian scattering. Search for and observation of these features is worthwhile.

Turn now to the considerations of the Table. The first group (items 1-7) apply directly to a nominally idealized scene, and may be visualized as being analogous in the uniform Gaussian scattering case to determination of system response by white random noise or in the specular scatterer case to the optical "point spread function" (O'Neill, 1963).

It is of central importance to this discussion that unlike conventional imaging \( \rightarrow \) systems analysis using purely non-coherent illumination, for partially coherent systems there is not a one-to-one equivalence between frequency domain, and time domain norms. Independent measures of frequency structure (e.g., bandwidth) and temporal structure (e.g., coherence) are required. One purpose of the Table is to suggest approaches to this question. In this sense, the first seven items carry over directly and impact the final four items.

The Two-scale Considerations are meant to be those of first order relevance under the assumption that one is attempting to "image" azimuth waves, and thus to understand the azimuth wave spectral response of a SAR.

In order to get a bit more depth into the implications of the Table, consider an experiment using existing data. We need to have examples of SAR ocean imagery with different qualities of azimuth waves visible. There should also be available a control, an example of imagery (from the same radar and processor) that includes a large random field, and if possible at least one point reflector. The experimental procedure is to perform on both the control and the sea images the series of tests suggested in the Table. From the Control, general considerations 1,2,3,4,6 and 7 may be verified. The same measures should then be performed on the sea images, with the results used to classify regions of sea scattering as dominantly Gaussian or
specular.

For example, it has been observed that the apparent speckle in certain SEASAT scenes is elongated in azimuth by several resolution cells. This feature has been called "streakle**, a most descriptive terminology. The occurrence of streakle seems to be correlated with reduction of the azimuth bandwidth of associated directional spectra. The streakle events should be processed according to the tests of Table I. From this one may conclude whether or not they correspond to specular or Gaussian scattering.

Again, one may search for focus sensitivities. Certain investigators have reported finding focus dependence in SAR data. In the context of the Table, this corresponds to item 10, in which there is a value judgement required. The case may be strengthened, by subjecting the area in question to the test of item 1. For those areas in which both strong focus dependence is found and specular correlation between looks is satisfied, then more presumptive steps (such as wave height estimation) can be hazarded.

It is true that an ensemble of scatterers, or a point scatter, should they have a Doppler (linearly changing phase) component will suffer an azimuth position shift (item 8 in the Table). This is difficult to observe confidently on the ocean, however, due to the complexity of the full spectrum of motions present.

In the event that the sea spectrum is nearly pure swell, as is the case for ocean waves in a field of floating ice, and the problem of scattering coherence time is avoided, then the velocity bunching mechanism may be directly observed (Raney, 1981). There are interesting questions that have to do with identifying the cause of loss of azimuth wave sensitivity in a SAR. Perhaps the issue could be addressed by observation of a wave field as it propagates from open water into an ice covered region, progressively attenuating the higher frequency portion of the wave spectrum. Again, any experimental observations should be verified by a control frame of SAR data, and judicious use of the measurable norms of Table I.

7. CONCLUSIONS

This paper has attempted to present and tabulate imaging properties of a SAR that are facts of life. These have been selectively explored as they apply to the ocean imaging problem.

It is suggested that most SAR ocean imagery has properties of both Gaussian and specular scattering. Existing theories and experimental observations are controversial largely due to inconsistencies in the assumed scatterin mechanism. Logical approach to the problem requires that the rules of SAR image formation be utilized as guidance to localize and interpret ocean imagery phenomena. An experimental procedure is suggested to realize this goal.

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REFERENCES


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