SATELLITE TECHNIQUES FOR DETERMINING THE GEOPOTENTIAL
FOR SEA-SURFACE ELEVATIONS

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ABSTRACT

Spaceborne altimetry with measurement accuracies of a few centimeters has the potential for determining sea-surface elevations necessary to compute accurate three-dimensional geostrophic currents from traditional hydrographic observations. The limitation in this approach is the uncertainties in our knowledge of the global and ocean geopotentials which produce satellite and geoid height uncertainties about an order of magnitude larger than the goal of about 10 cm. This paper begins with a description of the quantitative effects of geopotential uncertainties on processing altimetry data. This is followed by a review of existing models which can shown to be inadequate. Potential near-term improvements, not requiring additional spacecraft, are discussed. However, even though there would be substantial improvements at the longer wavelengths, the oceanographic goal would not be achieved. The potential NASA Geopotential Research Mission (GRM) is described. This mission should produce geopotential models that are capable of defining the ocean geoid to 10 cm and near-earth satellite positions significantly better. For completeness, the state-of-the-art and the potential of spaceborne gravity gravimetry is described as an alternative approach to improve our knowledge of the geopotential.

1. INTRODUCTION

This paper addresses the importance of an accurate representation of the geopotential in physical oceanography, its current state of knowledge and possible near-term and long-term improvements.

If the oceans were static and subject only to gravitational and centrifugal forces, they would conform to a conceptual surface called the geoid on which the gravity potential function (or geopotential) is a constant. The oceans are not static, and transport of heat, salt, and momentum have a profound effect on climate. As a result, study of the temporal and steady-state circulation of the oceans is important. These motions are governed by the equations of fluid dynamics and solution is dependent on conditions at the boundaries and throughout the medium. Velocities at the ocean surface are primarily induced by wind stress and differences in pressure. On the rotating earth, ocean pressure differences give rise to geostrophic velocities resulting from a balance between pressure and Coriolis forces such that the velocity is normal to the pressure gradient. Surface pressure differences are caused by departures of the ocean surface from the ocean geoid. These departures, called sea-surface elevations, can be as large as 1 to 2 meters in the broad ocean areas and significantly larger at the land-sea interfaces.

The relation between the surface geostrophic velocity and change in sea-surface elevation is given by

\[ v = (g/f)(\alpha h/\alpha x) \]

where \( g \) is the local acceleration of gravity (9.8 m s\(^{-2}\)), \( f = 2\alpha \sin \text{ (latitude)} \) where \( \alpha \) is the angular velocity of the earth (7.27 \times 10^{-5} \text{ s}^{-1}), \) and \( \alpha h/\alpha x \) is the horizontal slope of the sea-surface elevation. An uncertainty in sea-surface elevation of 10 cm
over a distance of 1000 km would result in a velocity uncertainty of 1 cm·s⁻¹ at a latitude of 45 degrees. This would be sufficient for a better understanding of current velocities.

A gross quantitative understanding of large-scale ocean circulation has resulted from shipboard measurements. Velocities based on density distributions inferred from in situ measurements can be determined to within a local constant if the sea-surface elevations are unknown. Determination of this constant is possible by in situ measurement of velocity. However, in practice this is not usually feasible as it would take months to average the small-scale velocity variations. Alternatively, the constant can be determined from the suspect assumption that the velocity vanishes at some level in the ocean called the level-of-no-motion. The inability to accurately determine this constant has been a limitation to the quantitative description of the geostrophic current systems.

With the advent of spaceborne microwave altimetry a solution is possible. With its inherent global coverage and measurement accuracy, altimetry has the potential for determining sea-surface elevations to a few centimeters. This, together with traditional hydrographic observations, would make possible determination of the three-dimensional geostrophic currents unencumbered by the assumption of a level-of-no-motion. However, the effectiveness of this approach is limited by uncertainties in our knowledge of the global geopotential and the ocean geoid.

2. SATELLITE ALTIMETRY

Spaceborne nadir pointing high resolution microwave radar altimetry has been an exciting source of data for ocean topography (Chovitz, 1983; and Marsh, 1983). Instrument range measurement precision has improved steadily from the 1 to 2 m for Skylab and the 30 to 40 cm for GEOS-3 to the 5 to 7 cm for SEASAT. Two new spaceborne altimeter missions are planned. The U.S. Navy program, GEOSAT, is undergoing fabrication at The Johns Hopkins University Applied Physics Laboratory with launch scheduled in the fall of 1984 (Pisacane and DeBra, 1983). Primary purpose of the mission is to better determine the ocean geoid by essentially completing the SEASAT mission. By collecting data at widely spaced intervals of time errors caused by time dependent sea-surface elevations, can be minimized. Instrument accuracy should be the same as SEASAT because it is essentially of the same design with improvements of an engineering nature. The second program is TOPEX which is a NASA venture now in the planning and instrument development stage. The primary purpose of TOPEX is to determine sea-surface elevations. An official new start is projected for 1985 with launch expected in 1988 or 1989.

Figure 1 shows the geometrical configuration of satellite altimetry. From the altimetry data, h can be inferred where \( \hat{H} \) is the distance from the center of mass of the spacecraft to that point on the ocean nearest to it. Sea-surface elevation \( H \) is given in terms of \( \hat{H} \) by

\[
\hat{H} = \hat{T} - \hat{R} - \hat{N} - \hat{G}
\]

where \( \hat{N} \) is geoid height vector and \( \hat{T} \) and \( \hat{R} \) are the position vectors to the center of mass of the spacecraft and to the subsatellite point on the reference surface respectively. Errors in \( \delta H \) are

\[
\delta H = \frac{\hat{H}}{H} (\delta \hat{T} - \delta \hat{R} - \delta \hat{N})
\]
Because the vectors are near parallel, 

\[ dH = dr - dh \]
so that errors in the satellite altitude, the measurements, and the geoid height have the same sensitivity. Because of the large footprint of the altimeter, i.e., \( 1 \) km, and the large orbit, all need be determined to the same degree of accuracy.

**Fig. 1** Satellite altimetry geometry.

<table>
<thead>
<tr>
<th>Vectors</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{r} )</td>
<td>Satellite position from center of mass reference frame</td>
</tr>
<tr>
<td>( \vec{r}_g )</td>
<td>Satellite reference spheroid</td>
</tr>
<tr>
<td>( \vec{N} )</td>
<td>Geoid height</td>
</tr>
<tr>
<td>( \vec{H} )</td>
<td>Sea surface elevation</td>
</tr>
</tbody>
</table>

Because of the large footprint, the satellite and geoid height have the same sensitivity. Consequently, all need to be determined to the same degree of accuracy.
Contributions to δh are uncertainties in instrumentation delays, distance from the center of mass to the electrical center of the antenna, spacecraft attitude, propagation velocity, the effect of ocean surface characteristics, and random noise. These are discussed in depth by Marsh (1983) and Tapley et al (1982) and will not be considered here as other errors dominate. Errors that contribute to δr, i.e., satellite altitude, are of two types: those from the spacecraft tracking system (e.g., station location, propagation velocity, and instrumentation errors) and modeling of the forces that act on the spacecraft, (e.g., gravity, radiation pressure, and drag) that are necessary to correlate tracking data taken at different times. Today, the dominant error in determining satellite ephemerides from tracking systems such as laser and radiofrequency doppler is the uncertainty in the global geopotential. This uncertainty manifests itself in tracking station position errors and errors in the gravity forces. Errors in the geoidal height, δN, follow directly from both errors in and truncation of models of the geopotential. Because of density inhomogeneities, primarily in the earth's crust, the geoid is not a smooth surface and can depart from the reference oblate spheroid surface by as much as 100 m.

The total gravitational potential, V*, can be represented in terms of spherical harmonics by

\[ V^*(r, \lambda, \phi) = V_0 + V \]

where

\[ V_0 = \frac{GM}{r} \text{, the Newtonian potential;} \]

\[ V = \sum_{\lambda=2}^{\infty} \sum_{m=0}^{\lambda} V_{\lambda m} \]

\[ V_{\lambda m} = \frac{GM}{R} \left( \frac{R}{r} \right)^{\lambda+1} (C_{\lambda m} \cos m\lambda + S_{\lambda m} \sin m\lambda) P_{\lambda m}(\cos \phi) \]

G = universal gravitational constant

M = mass of earth

R = normalizing radius, generally the mean equatorial radius

r, λ, φ = radius, longitude and colatitude

\[ P_{\lambda m} = (1 - t^2)^{m/2} \frac{d^m}{dt^m} P_{\lambda}(t) \text{, associated Legendre function of the first kind} \]

\[ P_{\lambda}(t) = \frac{1}{2^\lambda \lambda!} \frac{d^\lambda}{dt^\lambda} (t^2 - 1)^{\lambda} \text{, Legendre polynomials} \]

\[ C_{\lambda m} = \frac{1}{M} \int \frac{(r')^2}{R} P_{\lambda m}(\cos \phi) dM \]
\[
\begin{bmatrix}
C_{\ell m} \\
S_{\ell m}
\end{bmatrix} = \frac{2(\ell-\nu)}{M(\ell+\nu)!!} \int \left( \frac{r'}{R} \right)^\ell P_{\ell m}(\cos \phi') \left( \frac{\cos m\phi'}{\sin m\phi'} \right) dM \quad m \neq 0
\]

\[r', \lambda', \phi' = \text{integration parameters over the mass of the earth}\]

\(C_{\ell m}, S_{\ell m}\) are the harmonic coefficients which are unknown integrals of the mass distribution of the earth. Following Dunnell et al (1977), the spherical harmonic \(V_{\ell m}\) can be represented in Kepler elements \((a, e, i, \omega, \nu, M)\) for near zero eccentricity by

\[V_{\ell m} = \frac{GM}{R} \sum_{p=0}^{\ell} \frac{(\ell+1)^2}{\ell} I_{\ell m p} S_{\ell m p}\]

(3)

Where

\[S_{\ell m p} = \begin{cases}
C_{\ell m}(\ell-\nu) \text{even} & \cos \phi_{\ell m p}
S_{\ell m}(\ell-\nu) \text{odd} & \sin \phi_{\ell m p}
\end{cases} + \begin{cases}
S_{\ell m}(\ell-\nu) \text{even} & \cos \phi_{\ell m p}
C_{\ell m}(\ell-\nu) \text{odd} & \sin \phi_{\ell m p}
\end{cases}
\]

(4)

\[\phi_{\ell m p} = (\ell-2p)\beta + m(\nu - \phi).\]

(5)

\(a\) is the semimajor axes, \(I_{\ell m p}\) is a function of the inclination, \(\beta = M+\omega\) is the argument of latitude, \(\nu\) is the longitude of the ascending node, and \(\phi\) is the right ascension of Greenwich relative to Aries. This provides a convenient representation of the geopotential for deriving both the geoid height and the effect of the harmonic coefficients on satellite motion. Geoid undulations can be determined from the Bruns theorem as

\[N = \frac{V-U}{g} = \frac{V-U}{g} \bigg|_{\ell=2}
\]

(6)

where \(g = GM/R^2\) is the local acceleration of gravity and \(U\) is the difference of the potential of the ellipsoidal reference surface with the Newtonian potential for which all terms for \(\ell > 2\) can be neglected. Substituting for \(V\) and approximating \(a\) by \(R\) gives

\[N_{\ell} = \frac{V_{\ell}-U}{g} = \frac{\ell}{m=0} \frac{\ell}{p=0} R I_{\ell m p} S_{\ell m p} \quad \ell > 2
\]

(7)

Perturbations in the radial direction of a satellite in a near-circular orbit, following Dunnell et al (1977), are

\[\delta r = \frac{3}{2} \bigg|_{\ell=2} \delta r_{\ell}
\]

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Estimates of the relative values of $\delta r_\ell$ and $N_\ell$ can be determined as follows.

If the earth were essentially nonrotating such that $\Omega - \omega$ is a constant then

$$\delta r_\ell = \frac{-GM}{R^2} \left( \frac{R}{a} \right)^{\ell+2} \frac{1}{\ell} \sum_{m=0}^{\ell} \sum_{p=0}^{\ell} \frac{2n_\ell + 2n_\ell - 2n(n-2p)}{2n+1} \frac{I_{\ell mp} S_{\ell mp}}{(n^2-\ell^2)^{1/2}}$$  

where

$$\delta r_\ell = \frac{-GM}{R^2} \left( \frac{R}{a} \right)^{\ell+2} \frac{1}{\ell} \sum_{m=0}^{\ell} \sum_{p=0}^{\ell} \frac{2n_\ell + 2n_\ell - 2n(n-2p)}{2n+1} \frac{I_{\ell mp} S_{\ell mp}}{(n^2-\ell^2)^{1/2}}$$  

Because of the linear nature of the expressions for $N_\ell$ and $\delta r_\ell$ in terms of the harmonic coefficients $C_{\ell m}$ and $S_{\ell m}$, equations (7) and (8) can also be interpreted as determining the effect of uncertainties in the harmonic coefficients.

The maximum contribution to the perturbation from the harmonic coefficients at a given frequency $\pm (\ell-2)p n$ occurs when $\ell$ is a minimum for which $p$ must be either 0 or $\pm n$. For the highest frequency in $\delta r_\ell$, denoted by $|\delta r_\ell|_\phi = \pm \ell n$, the perturbation is

$$\delta r_\ell |_{\phi = \pm \ell n} = R \left( \frac{R}{a} \right)^{\ell-1} \left[ I_{\ell m o} S_{\ell m o} + I_{\ell m z} S_{\ell m z} \right]$$

Similarly, for $N_\ell$

$$N_\ell |_{\phi = \pm \ell n} = R \left[ I_{\ell m o} S_{\ell m o} + I_{\ell m z} S_{\ell m z} \right]$$

Then the amplitude or the uncertainty of the ratio of the perturbation in satellite height to the geoid undulation at a frequency $\ell n$ is

$$\frac{\delta r_\ell}{N_\ell} |_{\phi = \pm \ell n} = \left( \frac{R}{a} \right)^{\ell-1} (\ell + 1)^{-1}$$

This ratio as a function of harmonic degree for two satellite altitudes is given in Figure 2. Attenuation of the ratio as $\ell$ increases demonstrates the difficulty in determining small-scale variations in the geopotential by measurement of orbit perturbations. This also demonstrates that the orbit is less affected by contributions of the harmonic coefficients at higher degree $\ell$. Rotation of the earth introduces frequencies that are smaller than orbital, $\phi_{\ell m p} < n$, such that the magnitude of the orbital perturbations are enhanced by the factor $\phi_{\ell m p}^{-1}$. Consequently, the more significant perturbations occur at the longer wavelengths and small-scale
Altimeter variations are primarily a result of the topography of the ocean surface. Equations (10) and (11) demonstrate that small variations in the mean Kepler elements produce second-order effects in both the satellite altitude perturbation and the ocean geoid. This is the mathematical justification of using an orbit with a repeating ground track so that the geopotential uncertainties result in highly correlated errors. Variations in the altimeter measurements can then be interpreted as time dependent components of sea-surface elevations.

To determine the absolute sea surface elevation requires a global geopotential model to determine the satellite altitude which is far less detailed than the model required to define the ocean geoid.

3. GEOPOTENTIAL MODELING STATUS

A review of the current status of modeling the geopotential is available (Lerch 1983). These models use various sources of data: orbit perturbations through laser and radio frequency doppler observations, satellite altimetry, and measurements of terrestrial gravity anomalies. Resolutions are as small as a half-wavelength of one degree.

Over the years, NASA has generated the Goddard Earth Model (GEM) series which has been the accepted standard. The latest in the series are GEM-9 and 10 (Lerch et al, 1979), GEM-10B and 10C (Lerch et al, 1981), and GEM-L2 (Lerch et al, 1983). Characteristics of these models are given in Table 1. GEM-9 and GEM-L2 are based solely on satellite tracking data, GEM-10 is based on the data used in GEM-9 but augmented by terrestrial gravity anomaly measurements, and GEM-10B and 10C use satellite tracking, surface gravimetry and GEOS-3 altimetry. Various measures of accuracy for GEM-9, 10, 10B and 10C are given in Table 2.
For orbit determination GEM-9, 10, 10B and 10C are comparable with a radial accuracy of about 1 to 2 meters. This compares favorably to the 1 to 2 meters estimated for the satellites of the Navy Navigation Satellite System at about 1000 km. The GEM-L2 model is reported to be superior to the earlier models by more accurate determination of the harmonic coefficients up to degree and order 4. The orbit errors for the LAGEOS spacecraft at an altitude of almost 5900 km were reduced from 2m with GEM-9 to 0.30 cm for GEM-L2. There has not yet been evaluation of this model for low altitude satellites.

The measures of accuracy given in Table 2 indicate that the GEM-10C model has an edge in reproducing the ocean geoid. An accuracy of 1 to 2 meters in geoid height is suggested, and an anomaly accuracy of 4 to 7 mgal for 1 degree regions has been reported (Lerch et al, 1983).

Table 1

Goddard Earth Models (Lerch et al 1981 1983)

<table>
<thead>
<tr>
<th>Name</th>
<th>Year</th>
<th>Degree complete</th>
<th>No. of Coef's</th>
<th>Minimum wavelength (km deg)</th>
<th>Satellite tracking</th>
<th>Surface grav. metry</th>
<th>GEOS-3 altimetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEM-9</td>
<td>1979</td>
<td>20</td>
<td>566</td>
<td>2000 9</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>GEM-10</td>
<td>1979</td>
<td>22</td>
<td>594</td>
<td>1820 8.2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>GEM-10B</td>
<td>1981</td>
<td>36</td>
<td>1296</td>
<td>1110 5</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>GEM-10C</td>
<td>1981</td>
<td>180</td>
<td>32,400</td>
<td>222 1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>GEM-L2</td>
<td>1983</td>
<td>20</td>
<td>566</td>
<td>2000 9</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2

Accuracy assessment of ocean geoid (Lerch et al 1981)

<table>
<thead>
<tr>
<th>Comparisons</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GEM-9</td>
</tr>
<tr>
<td>Geoid heights (m)</td>
<td></td>
</tr>
<tr>
<td>GEOS-3 altimetry - trench areas*</td>
<td>2.90</td>
</tr>
<tr>
<td>GEOS-3 altimetry - nontrench areas*</td>
<td>1.92</td>
</tr>
<tr>
<td>Skylab altimetry</td>
<td>3.2</td>
</tr>
<tr>
<td>Seasat altimetry</td>
<td></td>
</tr>
<tr>
<td>GEOS-3 radial position from crossing analysis</td>
<td>~1</td>
</tr>
<tr>
<td>Anomalies (mGal)</td>
<td></td>
</tr>
<tr>
<td>GEOS-3 altimeter, 5° blocks</td>
<td>4.7</td>
</tr>
<tr>
<td>Terrestrial, 5° blocks</td>
<td>9.4</td>
</tr>
</tbody>
</table>

*After bias and tilt have been fit to remove orbit errors.
4. NEAR TERM IMPROVEMENTS

A workshop was held in February 1982 to address future directions for developing improved models of the geopotential (NASA, 1982). One recommendation was to develop improved models that did not require additional spacecraft to be launched. The consensus was that perhaps up to a 50 percent reduction in errors up to degree 10, i.e., half-wavelengths of 2000 km, with lesser improvements at higher degree could be achieved. These improvements would be realized through two activities. The first would be to develop improved modeling tools, i.e., software incorporating improved physical, mathematical and statistical models taking advantage of the enormous computational speeds and storage now available. The second is to improve the quality and quantity of the existing data by reprocessing, to add additional data from satellites not previously used, to incorporate the SEASAT altimetry data, and to collect additional laser and radio frequency doppler observations in new campaigns.

This effort has not yet been formally undertaken. However, even if all expectations were realized, it would not satisfy the requirements for either orbit determination or the ocean geoid to use altimetry data for the time-invariant surface geostrophic currents.

5. GEOPOTENTIAL RESEARCH MISSION (GRM)

Significant improvements in the global geopotential will be possible as a result of the GRM, formerly GRAVSAT, which is under study by NASA (Pisacane et al., 1982; and Keating, 1983). Accuracy of the global geopotential should be adequate for determining the sea-surface elevations to 10 cm from altimetry data. This program is still in the study phase with the possibility of a new start in 1988 or later and launch in 1992 or later.

Terrestrial tracking of near-earth spacecraft to refine the geopotential is limited. Uncertainties in the propagation velocity, in the ionosphere and especially the atmosphere, can induce data reduction errors larger than the orbital perturbations of interest. To increase the magnitude of the orbital perturbation it is necessary to decrease the altitude as low as possible. At the 160 km altitude of the GRM spacecraft, it would require about 276 stations uniformly distributed to provide global coverage. At this low altitude, the drag force uncertainty is about three orders of magnitude larger than the gravity forces of interest. These limitations are overcome in the GRM by satellite-to-satellite tracking between two satellites in near circular polar orbits separated by distances of 100 to 300 km. The disturbing effects of drag and radiation pressure and in addition, orbit altitude maintenance can be accomplished by the Disturbance Compensation System (DISCOS). This device which was successfully demonstrated on an advanced navigation satellite in 1972, TRIAD, uses a mass expulsion system to force the spacecraft to follow the motion of a free proof mass in a cavity which is shielded from the atmosphere and solar radiation.

Studies have defined the system characteristics given in Table 3. A mission of about 7 months, 6 of which will be operational, will require about 1400 kg of hydrazine fuel for each spacecraft, just under half the total mass. An artist's conception is shown in Figure 3. The two spacecraft will be launched by a single shuttle mission from the Western Launch Facility. Range-rate measurements between the two proof masses will be made to 1 \( \mu \text{m} \cdot \text{s}^{-1} \) at 4 s intervals. This will be accomplished in part by a radio frequency satellite-to-satellite doppler system. A laboratory instrument has been developed and tested. Differences with measurements from a laser interferometer were significantly less than the goal, i.e., about 0.03 \( \mu \text{m} \cdot \text{s}^{-1} \) rms. With a measurement precision of 1 \( \mu \text{m} \cdot \text{s}^{-1} \) at 160 km altitude, the global geod as a function of degree should be recovered to the accuracy depicted in Figure 4.
Table 3
GRM spacecraft characteristics

<table>
<thead>
<tr>
<th>Orbit</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude</td>
<td>≈ 160 km</td>
</tr>
<tr>
<td>Inclination</td>
<td>90 ± 1 deg</td>
</tr>
<tr>
<td>Lifetime</td>
<td>1/2 year (operational)</td>
</tr>
<tr>
<td>Launch</td>
<td>Shuttle</td>
</tr>
<tr>
<td>Separation</td>
<td>Variable 100 to 300 km</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Physical characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>4.8 m</td>
</tr>
<tr>
<td>Body diameter</td>
<td>0.9 m</td>
</tr>
<tr>
<td>Solar panels (2)</td>
<td>1.5 m x 3.5 m</td>
</tr>
<tr>
<td>Mass</td>
<td>2900 kg (1400 kg hydrazine)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Power</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar panels (2) and body mounted cells</td>
<td>400 W (average)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tracking systems</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Proof mass to proof mass</td>
<td>1 μm · s⁻¹</td>
</tr>
<tr>
<td>Ground based</td>
<td>100 m</td>
</tr>
</tbody>
</table>

Fig. 3 Artist concept of the Geopotential Research Mission.

Fig. 4 Geopotential Research Mission (GRM) geoid height uncertainty.
Total geoid height error should be less than 10 cm and satellite altitude error at 850 km should be about one order of magnitude less.

The GRM should be able to provide a global geopotential with sufficient accuracy to limit the geopotential modeling error of the geoid to less than 10 cm.

6. SATELLITE GRADIOMETERS

Satellite-borne gravity gradiometers have been proposed to effect further improvements in determining the geopotential. This approach is currently envisaged as a follow-on to the GRM. To satisfy the objectives of the GRM, measurement precision of about $5 \times 10^{-3} \text{E} \left(1 \text{E} = 10^{-9} \text{s}^{-2}\right)$ is necessary at the same spacecraft altitude, 160 km. As a result, a measurement accuracy of $10^{-4} \text{E}$ over a 4 to 8 s interval is necessary to provide an improvement. Reviews of the current state-of-the-art of gravity gradiometry are given by Wilcox and Scheibe (1983) and Piscacane (1983).

Current mobile gravity gradiometers can measure to about $1 \text{E}$. Fundamental limitations are the instability of the materials, thermal distortion, stability of the scale factor and Brownian noise. To achieve the required accuracy it will be necessary to take advantage of cryogenic technology which should reduce each of the errors described above and most significantly the Brownian contribution. An instrument under development at the University of Maryland uses two opposed superconducting proof masses on a soft suspension and two superconducting sensing coils in a pancake shape (Paik, 1981). Two SQUID amplifiers are used to detect the motion from which the gravity gradient is determined. A single-axis instrument has been tested and a three-axis vector gradiometer should be completed in the near term. A design using a superconducting cavity oscillator accelerometer also appears to have promise (Reinhardt et al., 1982). Other instruments have been proposed as an interim step to achieve $10^{-2} \text{E}$. These are the Bell Aerospace Miniature Electrically Suspended Accelerometer (MESA) and the ONERA proposed CACTUS instrument.

A spaceborne gravity gradiometer mission in the late 1990's is a possibility. The specific improvement in the geopotential will depend on the extent of the reduction of errors below $5 \times 10^{-3} \text{E}$.

7. CONCLUSION

Spaceborne microwave altimetry has the potential for measuring sea surface elevations to a few centimeters. To properly interpret these data for surface geostrophic currents, it is necessary to make significant advances in modeling the geopotential. The NASA GRM has the potential for meeting the 10 cm accuracy requirement for the global geoid. The success of this mission will be a significant achievement in the three-dimensional determination of the geostrophic currents.

8. ACKNOWLEDGMENT

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