Optimization of Cooled Shields in Insulations

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ABSTRACT

A relatively simple method has been developed to optimize the location, temperature, and heat dissipation rate of each cooled shield inside an insulation layer. The method is based on the minimization of the entropy production rate which is proportional to the heat leak across the insulation. The results show that the maximum number of shields to be used in most practical applications is three. However, cooled shields are useful only at low values of the overall, cold wall to hot wall absolute temperature ratio. The performance of the insulation system is relatively insensitive to deviations from the optimum values of temperature and location of the cooling shields.

Design curves are presented for rapid estimates of the locations and temperatures of cooling shields in various types of insulations, and an equation is given for calculating the cooling loads for the shields.
NOMENCLATURE

A  Area of heat flow, m²
Cₚ  Specific heat of the boiloff vapor, kJ/kg*K
D  Functional defined by Eq. (14)
F  Functional defined by Eq. (13)
h_fg  Latent heat of vaporization of the boiloff liquid, kJ/kg
k  Thermal conductivity, W/m*K; with subscripts, coefficients in Eq. (1)
L  Overall thickness of insulation, m
m,n  Exponents in conductivity function, Eq. (1)
P  Tₑ/Tₑ, temperature ratio
q  Heat flow rate, W
R  Tₑ/Tₑ, overall temperature ratio
s  Dimensionless entropy production rate defined by Eq. (5)
S  Entropy production rate, W/K
t  Thickness between walls with single shield between, m
T  Absolute temperature, K
x  Distance from cold wall, m
x'  Distance from cold wall in a multi-shield configuration, m
X  x/t, dimensionless distance
X'  x'/L, dimensionless distance
γ  Defined by Eq. (8)

Subscripts
C  Cold wall
H  Hot wall
i  i-th shield
min  Minimum
opt  Optimum
S  Shield

*For systems with single shield L = t, x = x', X = X'.
INTRODUCTION

The search for the ultimate, energy efficient insulation system has led in the past few years to a fascinating rediscovery and application of some fundamental concepts of thermodynamics: specifically, the second law and the use of entropy production rates and availability (or exergy) for design optimization purposes. The classical approach has been to minimize the heat flow between surfaces at different temperatures.

The concept of a single vapor-cooled shield in an insulation has been treated theoretically as far back as 1959 in Scott’s classic textbook on cryogenics [1] and designs employing them were described not much later [2]. Paivanas, et al., obtained a patent [3] and later reported on the use of uniformly spaced multiple shields which were cooled by the boil-off from the insulated dewar [4]. Eyssa and Okasha [5] considered only radiative heat exchange between shields and minimized the total refrigeration power required. Hilal, et al., [6,7] used a similar minimization of refrigeration power as the design basis. Related works were reported by Bejan, et al., [8-11].

Recently, Bejan [12] proposed a new point of view, based on the second law of thermodynamics, which considers thermal insulations as dissipators of useful mechanical power (i.e. the availability or exergy) or, alternately, as generators of irreversibility or entropy. Thus, in this method, optimization of an insulation corresponds to minimization of either the entropy production rate or the irreversibility, or the decrease of availability. Various applications of this concept to insulation systems have been documented subsequently [13,14].

Our work grew out of an examination of Cunnington’s paper [13] who utilized a numerical technique to find optimum temperatures at given locations for one and two shields for a thermal conductivity function of the form
For $k_1T^{0.6}$, although several equations seemed to be incorrectly printed, we have found two of the design curves to be essentially correct. Thus, our purpose was

1. To develop a simple optimization technique;
2. To generalize the results to a broader class of insulations; and
3. To develop simple design methods for cooled shields.

The essentials of this report were already published [15].
ANALYSIS

We accept the previously developed concept that to optimize an insulation system is equivalent to minimizing the entropy production rate. In addition, we assume one-dimensional heat flow and that the heat capacity of the boil-off gas is adequate to do the cooling for all shields and does not impose a restriction on the optimization. In contrast to Rejan [9,11] who has developed a constrained optimization based on the heat capacity of the boiloff we employ the argument that in all practical systems the boil-off is generated by cooling of some equipment in addition to the heat leakage across the insulation.

Parallel heat paths, e.g. supports, have not been considered. However, each path can be optimized separately using its own thermal conductivity function. Then a design decision has to be made whether the two structures should be independently cooled at their respective optimum conditions.

We examine the general situation of an insulation where equivalent thermal conductivity, $k$, can be expressed as a two-term function of the absolute temperature

$$k = k_1 T^m + k_2 T^n$$  \hspace{1cm} (1)

where, typically, the first term represents actual conduction with $m = 1$ and the second term represents radiation with $n = 3$. In the following, $m$ and $n$ can be any value except -1.

The heat flow across a layer of insulation can be expressed in terms of Fourier's law

$$q \ dx = Ak \ dT$$  \hspace{1cm} (2)
Substituting $k$ from Eq. (1) and integrating across a layer from one end at 1, to the other at 2, yields

$$q = \frac{A}{x_2 - x_1} \left[ \frac{k_1}{m + 1} (T_2^{m+1} - T_1^{m+1}) + \frac{k_2}{n + 1} (T_2^{n+1} - T_1^{n+1}) \right].$$  (3)

Now consider the insulation with a cooled shield at $T_S$ located at $x$ between a hot surface at $T_H$ and a cold one at $T_C$, separated by the insulation thickness, $t$, as shown in Fig. 1a. The entropy production rate for the insulation can be determined from the heat flows and temperatures as follows

$$s = -\frac{q_H}{T_H} + \frac{q_C}{T_C} + \frac{q_S}{T_S}$$  (4)

where $q_S = q_H - q_C$.

The heat flow terms can be expressed in the form of Eq. (3) and the resulting expression can be non-dimensionalized using the following terms

$$s \equiv \frac{St}{k_H} \text{ where } k_H = k \text{ at } T_H,$$  (5)

$$P \equiv \frac{T_S}{T_C},$$  (6)

$$R \equiv \frac{T_C}{T_H},$$  (7)

$$\gamma \equiv \frac{k_2(m + 1)}{k_1(n + 1)} T_H^{n-m},$$  (8)

and

$$X \equiv \frac{x}{t}.$$  (9)
The resulting equation is

\[ s(m + 1)(1 + \gamma \frac{n + 1}{m + 1}) \]

\[ = \frac{1}{1 - \lambda} \{[(PR)^m + 1 - (PR)^m - 1 + (PR)^{-1}] \]

\[ + \gamma[(PR)^{n+1} - (PR)^n - 1 + (PR)^{-1}] \}

\[ + \frac{1}{\lambda} \{R^m[p^{m+1} - p^m - 1 + p^{-1}] \]

\[ + \gamma r^n[p^{n+1} - p^n - 1 + p^{-1}] \} \] (10)

Since \( R \), the overall temperature ratio, is generally known, \( s \) is a function of \( P \) and \( X \), and its extreme value can be found by differentiating it with respect to each variable separately and setting the results equal to zero. This procedure yields two equations to be solved simultaneously: \( \frac{\partial s}{\partial P} = 0 \) and \( \frac{\partial s}{\partial X} = 0 \). Because of the regular form of the expressions, one of the final two equations contains only a single unknown as follows:

\[ \frac{R^m F(m, P) + \gamma R^n F(n, P)}{[R^{m-1} D(m, P) + \gamma R^{n-1} D(n, P)]^2} \]

\[ = \frac{F(m, PR) + \gamma F(n, PR)}{[D(m, PR) + \gamma D(n, PR)]^2} \] (11)

\[ \frac{X}{1-X} = -\frac{R^{m-1} D(m, P) + \gamma R^{n-1} D(n, P)}{D(m, PR) + \gamma D(n, PR)} \] (12)

where the following functionals were used:
Thus, to find the optimum temperature and location for a shield, Eq. (11) can be solved for $P$, and then $X$ can be calculated from Eq. (12). The heat to be removed by the shield, $q_s = q_{H} - q_{C}$, can be found, as before, from Eq. (3). In dimensionless form the equation becomes

$$\frac{q_s}{A_{kH}^{-1}}(m+1)(1 + \gamma \frac{n+1}{m+1})$$

$$= \frac{1 - (PR)^{m+1} + \gamma[1 - (PR)^{n+1}]}{1 - X}$$

$$- \frac{(PR)^{m+1} - R^{m+1} + \gamma[(PR)^{n+1} - R^{n+1}]}{X}.$$  \hspace{1cm} (15)

For multiple shields $t_i$ represents the distance between the two surfaces surrounding the $i$-th shield on either side, $T_{H,i}$ and $T_{C,i}$ are the temperatures of these two surfaces, $X_i = x_i/t_i$ is the location of the shield relative to $t_i$, and $x_i'$ is the location of the shield relative to the cold wall as shown in Fig. 1b. To determine the optimum temperatures and locations for multiple shields, first we assumed a temperature for the first shield next to the cold wall, then we used Eqs. (11) and (12) to find the temperature and location of the second shield. This process was repeated for the rest of the shields and the hot wall. Thus, each shield was optimized consecutively with respect to the two surfaces on either side. With given values of the overall temperature ratio, $R$, and of the number of shields, the process requires iterative solution.
To put the results into proper perspective, the entropy production rates can be compared to the thermodynamically minimum rate obtainable through spatially continuous cooling. According to Bejan [12], this rate is

$$S_{\text{min}} = \frac{A}{t} \left[ \int_{T_C}^{T_H} (k)^{1/2} T^{-1} \, dT \right]^2. \quad (16)$$

This expression was evaluated analytically for the single-term functions of $k$, i.e. for $\gamma = 0$, and numerically otherwise.
RESULTS AND DISCUSSION

The first set of curves, Figs 2 through 9, show the relative entropy production rates for various thermal conductivity functions and for up to four optimally cooled shields as functions of the overall temperature ratio \( R = \frac{T_C}{T_H} \). The curves show that the entropy production rate increases with decreasing values of the temperature ratio, \( R \), and with increasing values of the exponent, \( m \) and \( n \). Adding shields, of course, reduces the entropy production rate; but for most of the practical temperature range, say \( 0.01 < R < 0.4 \), only three shields contribute to significant decreases and adding a fourth shield can be considered unnecessary. No shields are useful at high values of \( R \); but this "high" range is strongly dependent on the exponent of the temperature. The curves developed with \( k = k_1 T^{0.6} \) for one and two shields were very close to those given by Cunnington [13], converted appropriately.

Study of the results of two-term conductivities reveals that the curves fall between those obtained for each of the two terms alone. If \( \gamma \) is small the first term, \( T^m \), dominates; whereas if \( \gamma \) is large (>10), the second term, \( T^n \), controls. Thus, general conclusions can be drawn from examining the results of the single-term conductivities.

The second set of curves, Figs. 10 through 31, show the optimum temperature ratios, \( T_S/T_H \), and optimum locations, \( x'/L \), of cooled shields as functions of the overall temperature ratio, \( T_C/T_H \), for various thermal conductivity functions and with different number of cooled shields.

Figures 10 and 11 show the optimum single shield temperature ratios, \( PR = T_S/T_H \), and locations, \( X = x/L \), for five conductivity functions. Both of these functions generally decrease with decreasing \( R \). The other figures in this set show shield temperatures and locations for systems with up to three
shields and for both single-term and two-term conductivities. The results are strongly non-linear. For example, for $k_1T^3$ and $R = 0.01$, the optimum temperature ratios for three shields are about 0.09, 0.3, and 0.6 and the optimum locations are about 0.05, 0.2, and 0.5. As is to be expected, our unconstrained optimization yields a somewhat better performance per shield than Bejan's [9,11] constrained method.

The sensitivities of the entropy production rates to deviations from the optimum values of PR and X are demonstrated in the last set of curves, Figs. 32 through 35, for single shields. The sensitivity increases with the value of the exponents, m and n, but the curves are relatively flat near the minima. A ±20 percent change from optimum, for example, has negligible effect. Thus, the system is relatively tolerant of deviations from the optimum design conditions.

Calculations with two different conductivities on the two sides of a cooled shield show that using the better insulator on both sides always yields the optimum condition. However, if for some reason two types of insulations have to be used, then the better insulator should be placed on the warm side of the shield.
REFERENCES


Figure 1 Schematic of the Nomenclature for (a) Single and (b) Multiple Shields
Curve Set 1: Figures 2 through 9

The effect of optimally cooled shields on
the entropy production rate for various thermal conductivities.
Figure 2

$k = k_1^{\text{No. of Shields}}$

$\frac{T_{\text{cold}}}{T_{\text{hot}}}$

$S/S$

$10^0, 10^1, 10^2, 10^3, 10^4$
\[ k = k_1 T^{0.6} \]

No. of Shields

\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} \]

Figure 3
$k = k_1 T$

No. of Shields

$S/S_{\text{min}}$

$T_{\text{cold}}/T_{\text{hot}}$

Figure 4
Figure 5

$k = k_1 T^{2.0}$

No. of Shields

$S/S$
\[ k = k_1 T + k_2 T^{3.0}, \gamma = 2.0 \]
Figure 9:

The graph depicts a plot with the following axes:

- X-axis: \( S/S \)
- Y-axis: \( T_{\text{cold}}/T_{\text{hot}} \)
- X-axis: No. of Shields

The equation for the graph is:

\[ k = k_1 T + k_2 T^{3.0} \]

where \( \gamma = 5.0 \).
Curve Set 2: Figures 10 through 31
Optimal shield temperatures and locations for various thermal conductivity functions with different number of shields.
Figure 10

No. of Shields = 1

\[ k = \frac{k_1 T_0.6}{k_2 T_{0.0}} = \frac{k_1 T_{2.0}}{k_3 T_{3.0}} \]

\( T_{\text{cold}} / T_{\text{hot}} \) vs \( \text{Shields} / T_{\text{cold}} \)
$k = k_1 T^{0.6}$

No. of Shields = 2

- SHIELD 1
- SHIELD 2

Figure 12
Figure 13

$k = k_1 T^{0.6}$

No. of Shields = 2

$\frac{T_{\text{cold}}}{T_{\text{hot}}}$

$[\frac{1}{l/X}]$
$k = k_1 T^{0.6}$

No. of Shields = 3

---

SHIELD 1

SHIELD 2

SHIELD 3

Figure 14
The diagram shows a plot with the equation $k = k_1 T$.

It states that the number of shields is 3.

The plot is labeled Figure 16.
Figure 17

\[ k = k_1 T \]

No. of Shields = 3

\[ 1.0 \]

\[ 0.9 \]

\[ 0.8 \]

\[ 0.7 \]

\[ 0.6 \]

\[ 0.5 \]

\[ 0.4 \]

\[ 0.3 \]

\[ 0.2 \]

\[ 0.1 \]

\[ 0.005 \]

\[ 0.010 \]

\[ 0.100 \]

\[ 1.000 \]

\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} \]

\[ \Phi^0 \left[ \frac{\Pi}{X} \right] \]
$k = k_1 T^{3.0}$

No. of Shields $= 2$

$\left[ \frac{T_{\text{shield}}}{T_{\text{hot}}} \right]_{\text{opt}}$

$T_{\text{cold}} / T_{\text{hot}}$

Figure 18
\[ k = k_1 T^{3.0} \]

No. of Shields = 2

[Diagram showing plots for SHIELD 1 and SHIELD 2 with axes labeled: \( \frac{T_{\text{cold}}}{T_{\text{hot}}} \) and \( \frac{\ell_0}{X} \).]
\[ k = k_1 T^{3.0} \]

Number of Shields = 3

\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} \]
\[ k = k_1 T^{3.0} \text{ No. of Shells} = 3 \]

[Diagram showing a graph with curves labeled Shield 1, Shield 2, and Shield 3, and axes labeled \( T_{\text{cold}} / T_{\text{hot}} \) and \( \frac{1}{d_0} \frac{\gamma}{X} \).]
Optimal Case 2
of Poor Quality

No. of Shields = 1

\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} \]

\[ \frac{T_{\text{shield}}}{T_{\text{hot}}} \]

\( k = k_1 T + k_2 T^3 \)

\( \gamma = 0.5 \)

\( \gamma = 2.0 \)

\( \gamma = 5.0 \)

Figure 22
No. of Shields = 1

\[ k = k_1 T + k_2 T^{3.0} \quad \gamma = 0.5 \]
\[ k = k_1 T + k_2 T^{3.0} \quad \gamma = 2.0 \]
\[ k = k_1 T + k_2 T^{3.0} \quad \gamma = 5.0 \]
\[ k = k_1 T + k_2 T^{3.0} \quad \gamma = 0.5 \]

No. of Shields = 2

---

SHIELD 1

SHIELD 2

Figure 24
The diagram shows a plot with the following information:

- The equation $k = k_1 T + k_2 T_3.0$ is given, where $\gamma = 0.5$.
- The number of shields is $n$. 

The graph includes two lines labeled "SHIELD 1" and "SHIELD 2". The y-axis represents $T_{cold}/T_{hot}$ ranging from $0.010$ to $0.100$. The x-axis represents $[L/X]$ ranging from $1.0$ to $0.0$. 

A label at the bottom of the graph reads "Figure 25."
Figure 26

\[ k = k_1 T + k_2 T^{3.0} \]

\[ \gamma = 0.5 \]

No. of Shields = 3

\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} \]
\[ k = k_1 T + k_2 T^3 \]

\[ \gamma = 2.0 \]

No. of Shields = 2

Figure 28
\[ k = k_1 T + k_2 T^{3.0} \]

\[ \gamma = 2.0 \]

No. of Shields = 3

Figure 30
$k = k_1 T + k_2 T^2$  \( \gamma = 2.0 \)  No. of Shields = 3

$[7/X]$
Curve Set 3: Figures 32 through 35

System sensitivity to deviations from the optimum shield temperatures and locations for two overall temperature ratios with one cooled shield
Figure 32

\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} = 0.006 \]

\[ k = k_1 T \]

\[ k = k_2 T_{\text{3.0}} \]
$T_{\text{cold}} / T_{\text{hot}} = 0.006$

\[
\begin{align*}
S / S_{\text{min}} & \\
T_{\text{shield}} / T_{\text{hot}} & \\
\end{align*}
\]

Figure 33
\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} = 0.060 \]

\[ k = k_{L}^{1.5} \]

\[ k = k_{L}^{1.3} \]

\[ k = k_{L}^{1} \]

\[ T_{\text{shield}}/T_{\text{hot}} \]
APPENDIX

COMPUTER PROGRAMS
SEPARS and SHIELD

These two programs are essentially identical, but SEPARS is written in PASCAL whereas SHIELD is in BASIC.

To allow for consecutive calculations of different systems, the program always recycles to the starting point. Consequently, the first input requested is either a 1, if a calculation is to be performed, or a 0, if no more work is to be done.

Next the program requests input of the insulation's characteristics, specifically, the two exponents of the temperatures in the two-term conductivity function, the maximum number of cooled shields (<10) to evaluate, the value of $\gamma$, and the temperature ratio of the first shield to the cold wall, $P(1) = \frac{T_{S1}}{T_C}$. The program calculates and presents the characteristics of all optimal systems of cooled shields from one shield to the maximum number specified in the input.

The flow chart and a program sample follows.
TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC

IS PFC = 1?

ENTER INSULATION CHARACTERISTICS, NUMBER OF SHIELDS AND P(I)

I = 1

IM1 = 1 - 1
IM2 = 1 - 2

SOLVE R(I) ITERATIVELY

CALCULATE S(I), X(I)

IS I > 1?

ONE SHIELD ARRANGEMENT
ASSIGN L3(I), TCH(I), TSH(I), L4(I) AND XPL(I)

L2(I) = (1.0 - X(I-1)) / X(I)

TWO SHIELDS ARRANGEMENT
ASSIGN L4(2) AND TCH(2)

IS I > 2?

THREE SHIELDS ARRANGEMENT
ASSIGN L4(3) AND TCH(3)

IS I > 3?

FOUR OR MORE SHIELDS ARRANGEMENT
ASSIGN B AND L4(I)

J = 2
Y

TSH(I+1) = 1.0
T2(I) = TCH(I)

IS I > 1 ?

NO

YES

J = 2

T2(J) = TSH(J-1)
J = J + 1

IS J > 1 ?

NO

YES

ASSIGN L3(I+1) AND I(I+1)
J = 1

CALCULATE Q(J), SISH(J)

OUTPUT THE HEAT REMOVAL RATE AND ENTROPY PRODUCTION RATE AT EACH SHIELD ALONG WITH OPTIMUM LOCATION AND OPTIMUM TEMPERATURE FOR THE SHIELDS' ARRANGEMENT

J = J + 1

IS J > 1 ?

NO

YES

CALCULATE QHOT, QCOLD AND SCOLD

CALCULATE STOTAL(I)

CALCULATE SMIN(I), STOTMIN(I), SMA(I), SMAIN(I)

IS I > NS ?

NO

YES

P(I+1) = 1.0 / P1

2
THIS PAPER WAS DEVELOPED TO OPTIMIZE THE LOCATION, TEMPERATURE AND HEAT DISSIPATION RATE OF EACH COATED SHEILD INSIDE AN INSULATION LAYER. THE THERMAL PROPERTIES OF THE INSULATION HAVE BEEN GENERALIZED BASED ON THE MINIMIZATION OF THE ENTHALPY PRODUCTION RATE WHICH IS PROPORTIONAL TO THE HEAT LEAK ACROSS THE INSULATION. THE "ERR-0" IS BASED ON THE MINIMIZATION OF THE ENTROPY PRODUCTION RATE WHICH IS PROPORTIONAL TO THE "ERR-0".
PROCEDURE INPUT.
BEGIN
(* INPUT OF DATA HEADING *)
WRITELN.
WRITELN.
WRITELN.
WRITELN.
WHERE N ---- 1ST POWER IN THE THERMAL CONDUCTIVITY EQUATION)
BEGIN
(* FUNCTION D *)
VAR D REAL.
BEGIN
(* FUNCTIONAL D *)
END.
FUNCTION PW(X,E REAL) REAL.
VAR A REAL.
BEGIN
(* COMPUTE X**E *)
END.
BEGIN
(* FUNCTIONAL D *)
END.
BEGIN
(* INPUT OF DATA HEADING *)
WRITELN.
WRITELN.
WRITELN.
WRITELN.
BEGIN
(* FUNCTIONAL D *)
END.
FUNCTION PW(X,E REAL) REAL.
VAR A REAL.
BEGIN
(* COMPUTE X**E *)
END.
BEGIN
(* FUNCTIONAL D *)
END.
BEGIN
(* INPUT OF DATA HEADING *)
WRITELN.
WRITELN.
WRITELN.
WRITELN.
BEGIN
(* FUNCTIONAL D *)
END.
FUNCTION PW(X,E REAL) REAL.
VAR A REAL.
BEGIN
(* COMPUTE X**E *)
END.
BEGIN
(* FUNCTIONAL D *)
END.
BEGIN
(* INPUT OF DATA HEADING *)
WRITELN.
WRITELN.
WRITELN.
WRITELN.
BEGIN
(* FUNCTIONAL D *)
END.
FUNCTION F(X, X: REAL): REAL.
BEGIN (* FUNCTIONAL F *)
F := PW(X, (1.0/21))-PW((X-1.0)-1.0/21) (* FUNCTIONAL F *)
END.

FUNCTION SIMPSON(TCH: REAL); REAL.
TYPE
ARR = ARRAY(1..101) OF REAL;
VAR
C, T
DELTA
N
K, L
INTEGER.
BEGIN (* COMPUTE MINIMUM ENTROPY PRODUCTION RATE USING SIMPSON'S NUMERICAL INTEGRATION SCHEME *)
DELTA := (TCH-100.0)/100.0.
FOR I = 1 TO 100 DO
BEGIN
CIL := TCH-DELTA*(L-1).
YC := PW(CIL).N+CAMA*KPI/KPI*(PW((CIL)-1.0)).0.5/CIL
END.
BEGIN
H := CIL-TCH;
FOR K = 1 TO 100 DO
BEGIN
IF K*(K DIV 2)*2 THEN
H := H+D*K
ELSE
H := H+D*K
END.
SIMPSON := SQRT(DELTA/3)*H/(D+CAMA*KPI/KPI); (* COMPUTE MINIMUM ENTROPY PRODUCTION RATE USING SIMPSON'S NUMERICAL INTEGRATION SCHEME *)
END.

(* MAIN PROGRAM BODY *)
BEGIN
FILE
READ;
READ(IFC).
WHILE FIC<>1 DO
BEGIN
(* THIS BLOCK IS USED TO INPUT THE INSULATION THERMAL CONDUCTIVITY, NUMBER *)
(* OF SHIELDS AND 1ST. SHIELD / COLD WALL TEMPERATURE RATIO *)
BEGIN
INPUT;
READ;
READ(IFC).
SINGLES;
IF CAMA<>0 THEN
BEGIN
WRITE("THERMAL CONDUCTIVITY OF THE INSULATION IS K = E1*T**.N.3.1").
BEGIN
WRITE("THERMAL CONDUCTIVITY OF THE INSULATION IS K = E1*T**.N.3.1."))
END.
SINGLES;
END;
BEGIN
NP := N-1.
BEGIN
FOR I = 1 TO NS DO
BEGIN
IM := I-1.
INC := IM-1.
RR := 0.0.
CC := 0.1.
DO := 1.8.
COUNT := 0.
END.
(* THIS BLOCK CALCULATES R(I) ITERATIVELY *)

REPEAT

P = F[R(I)]

V1 = F(R(I)) * F(M, P(I)) * F(M, P(I))

V1 = F(R(I)) * F(M, P(I))

V3 = F(R(I)) * F(M, P(I))

C = F(W/M) / V3

G = F(V/D)

IF (G < 0) THEN GOTO 100.

IF (G > 0) THEN GOTO 200.

CC = A / G

IF ABS(CC) < 0.00001 THEN GOTO 1000.

IF V < 0.00001 THEN GOTO 300.

RETURN.

CC = ABS(V)

IF ABS(ABS(ABS(V))) < 0.00001 THEN GOTO 100.

IF U < 0.00001 THEN GOTO 200.

COUNT = COUNT + 1.

UNTIL (G < 0) OR (ABS(ABS(V)) < 0.00001).

(* IN THIS BLOCK VARIABLES ARE ASSIGNED FOR DIFFERENT SHIELD CONFIGURATIONS *)

IF (C) THEN

BEGIN

L(I,J) = (C - X(I,J)) / X(J). 

IF (C) THEN

BEGIN

B = 0.

L(I,J) = C.

FOR J = 1 TO IM DO

BEGIN

L(I,J) = L(I,J) / X(J).

END.

ELSE

BEGIN

L(I,J) = (C - X(I,J)) / X(J).

TCH(I,J) = TCH(I,J) / P(I).

END.

ELSE

BEGIN

L(I,J) = (C - X(I,J)) / X(J).

TCH(I,J) = TCH(I,J) / L(I,J) / P(I).

END.

END.
BEGIN
L1[1] = .0.
LCH[1] = .0.
TSH(1) = TCH(1).1*P(1).
LCH[1] = .0.
END.
TSH(1+1) = .0.
TCH(1) = TCH(1).

SINGLESPACE.
WRITE(' NUMBER OF SHIELDS = ', I);
WRITE(' NUMBER OF ITERATIONS = ', COUNT).

SINGLESPACE.
WRITE(' HEAT REMOVAL RATE
ENTROPY PRODUCTION RATE
OPTIMUM LOCATION
OPTIMUM TEMPERATURE ');

SINGLESPACE.
IF (I) THEN FOR J = 2 TO I DO T(J) = TSH(J-1).
L1[1-1] = L1[1];
L1[1-1] = 0.0.

(* IN THIS BLOCK DIMENSIONLESS HEAT REMOVAL AND ENTROPY PRODUCTION RATES *)
(* ARE CALCULATED FOR EACH SHIELD *)

FOR J = 1 TO I DO BEGIN
Z1 = (PWR(TSH(J-1),MP1) - PWR(TSH(J),MP1)) * L1[J-1] / L1[J] * PWR(TSH(J),MP1) / PWR(TSH(J),MP1) * L1[J] / L1[J-1] * PWR(TSH(J),MP1) / PWR(TSH(J),MP1).
Z2 = (PWR(TSH(J),MP1) - PWR(TSH(J-1),MP1)) * L1[J-1] / L1[J] * PWR(TSH(J-1),MP1) / PWR(TSH(J),MP1) * L1[J] / L1[J-1] * PWR(TSH(J-1),MP1) / PWR(TSH(J),MP1).

SISH(J) = Z2 - Z1.

WRITE(' SHIELD J ', J, ' VS. TSH(J) ', 0.9, ' VS. TSH(J) ', 0.9, ' VS. TSH(J) ', 0.9, ' VS. TSH(J) ', 0.9).
END.

(* FINALLY OTHER QUANTITIES OF INTEREST ARE CALCULATED IN THIS BLOCK *)

SINGLESPACE.
QHOT = (1.0 - PWR(TSH(J),MP1)) * CASH - CASH * PWR(TSH(J),MP1) * CASH / CASH * PWR(TSH(J),MP1) / CASH.
QCOLD = PWR(TSH(J),MP1) * CASH - CASH * PWR(TSH(J),MP1) * CASH / CASH * PWR(TSH(J),MP1) / CASH.
QCOLD = QCOLD / TCH(J).
STOTAL(J) = QCOLD - QHOT.
FOR J = 1 TO I DO STOTAL(J) = STOTAL(J) + SISH(J).

SISH(J) = STOTAL(J) / SISH(J).
TCH(J) = SISH(J) = SISH(J).

SISH(J) = STOTMIN / SISH(J).
SISH(J) = STOTMIN / SISH(J).

IF (I) THEN P(I) = 0.0.

SINGLESPACE.
WRITE(' COLD WALL / HOT WALL TEMPERATURE RATIO ');
WRITE(' HEAT OUT AT COLD WALL ');
WRITE(' HEAT IN AT HOT WALL ');
WRITE(' ENTROPY PRODUCTION RATE AT COLD WALL ');
WRITE(' ENTROPY PRODUCTION RATE AT HOT WALL ');
WRITE(' MINIMUM ENTROPY PRODUCTION RATE ');
WRITE(' MAXIMUM ENTROPY PRODUCTION RATE ');
WRITE(' TOTAL ENTROPY PRODUCTION RATE WITH ', I, ' SHIELDS ');
WRITE(' MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO ');
WRITE(' TOTAL / MINIMUM ENTROPY PRODUCTION RATIO ');

SINGLESPACE.
SINGLESPACE.
SINGLESPACE.

END.
PFIN.
READ.
READ(PFIN).
END.
TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

ENTER ----> M N NS GAMA P[1] ---->

WHERE:  M ---- 1ST. POWER IN THE THERMAL CONDUCTIVITY EQUATION
        N ---- 2ND. POWER IN THE THERMAL CONDUCTIVITY EQUATION
        NS ---- NUMBER OF SHIELDS
        GAMA -- >0 IF USING ONE TERM THERMAL CONDUCTIVITY EQUATION
               ==0 IF USING TWO TERM THERMAL CONDUCTIVITY EQUATION
        P[1] ---- 1ST. SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS > 1

? 1.0  3.0  1  2.5  15.0

THERMAL CONDUCTIVITY OF THE INSULATION IS K = K1*T**1.0 + K2*T**3.0
(K2*(N+1))/(K1*(N+1))*THOT**(N-M) = 2.50

NUMBER OF SHIELDS = 1
NUMBER OF ITERATIONS = 35

<table>
<thead>
<tr>
<th>HEAT REMOVAL RATE</th>
<th>ENTROPY PRODUCTION RATE</th>
<th>OPTIMUM LOCATION</th>
<th>OPTIMUM TEMPERATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHIELD 1</td>
<td>0.43837</td>
<td>1.85659</td>
<td>0.36744</td>
</tr>
</tbody>
</table>

COLD WALL / HOT WALL TEMPERATURE RATIO = 0.015741
HEAT OUT AT COLD WALL = 0.014350
HEAT IN AT HOT WALL = 0.452719
ENTROPY PRODUCTION RATE AT COLD WALL = 0.911631
ENTROPY PRODUCTION RATE AT HOT WALL = -0.452719
MINIMUM ENTROPY PRODUCTION RATE = 1.000503
MAXIMUM ENTROPY PRODUCTION RATE = 18.236148
TOTAL ENTROPY PROD. RATE WITH 1 SHIELDS = 2.315503
MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO = 18.226962
TOTAL / MINIMUM ENTROPY PRODUCTION RATIO = 2.314340

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

? 1

ENTER ----> M N NS GAMA P[1] ---->

WHERE:  M ---- 1ST. POWER IN THE THERMAL CONDUCTIVITY EQUATION
        N ---- 2ND. POWER IN THE THERMAL CONDUCTIVITY EQUATION
        NS ---- NUMBER OF SHIELDS
        GAMA -- >0 IF USING ONE TERM THERMAL CONDUCTIVITY EQUATION
               ==0 IF USING TWO TERM THERMAL CONDUCTIVITY EQUATION
        P[1] ---- 1ST. SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS > 1

? 1.0  0.9  2  0.0  25.0

THERMAL CONDUCTIVITY OF THE INSULATION IS K = K1*T**1.0

NUMBER OF SHIELDS = 1
NUMBER OF ITERATIONS = 23
### Heat Removal Rate and Entropy Production Rate

<table>
<thead>
<tr>
<th>Shield</th>
<th>Heat Removal Rate</th>
<th>Entropy Production Rate</th>
<th>Optimum Location</th>
<th>Optimum Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shield 1</td>
<td>0.75466</td>
<td>7.03151</td>
<td>0.35870</td>
<td>0.10732</td>
</tr>
<tr>
<td>Shield 2</td>
<td>0.8841</td>
<td>4.70678</td>
<td>0.48690</td>
<td>0.18786</td>
</tr>
</tbody>
</table>

- **Cold Wall / Hot Wall Temperature Ratio**:
  - Shield 1: 0.004293
  - Shield 2: 0.000806

- **Heat Out at Cold Wall**:
  - Shield 1: 0.016030
  - Shield 2: 0.001162

- **Heat In at Hot Wall**:
  - Shield 1: 0.770687
  - Shield 2: 0.940073

- **Entropy Production Rate at Cold Wall**:
  - Shield 1: 3.734070
  - Shield 2: 1.440716

- **Entropy Production Rate at Hot Wall**:
  - Shield 1: -0.770687
  - Shield 2: -0.940073

- **Minimum Entropy Production Rate**:
  - Shield 1: 3.504633
  - Shield 2: 3.921467

- **Maximum Entropy Production Rate**:
  - Shield 1: 115.966533
  - Shield 2: 619.477774

- **Total Entropy Prod. Rate with 1 Shields**:
  - Shield 1: 9.994873
  - Shield 2: 7.920388

- **Maximum / Minimum Entropy Production Ratio**:
  - Shield 1: 33.089491
  - Shield 2: 157.970919

- **Total / Minimum Entropy Production Ratio**:
  - Shield 1: 9.994873
  - Shield 2: 33.089491

---

**To Perform Computation, Enter 1. Otherwise, Enter 0.**

- 0.175 CP SECS, 124158 CM USED.
PROGRAM SHIELD

1 00010 REM THIS IS A "BASIC" PROGRAM TO CALCULATE optimum TEMPERATURES.
2 00020 REM LOCATIONS, AND COOLING LOADS FOR COOL SHIELDS IN A CRYOCENIC
3 00030 REM INSULATION SYSTEM WHOSE THERMAL CONDUCTIVITY FOLLOWS THE RELATION
4 00040 REM K=E^((T-HC+CIF-TM)
5 00050 REM MODIFIED IN LATE NOV 1982.
6 00060 REM
7 00070 REM DEFINITION OF SYMBOLS USED
8 00080 REM
9 00090 REM COLD-SIDE WALL TEMPERATURE T
10 00100 REM WARM-SIDE WALL TEMPERATURE T
11 00110 REM SPACING BETWEEN SHIELDS AT I-i AND I-1 L(I)
12 00120 REM OVERALL THICKNESS OF INSULATION L
13 00130 REM OVERALL SPACING RATIO. L/L(1)-L(1)
14 00140 REM LOCAL SPACING RATIO. L/I(I)-L(1)
15 00150 REM DISTANCE FROM COLD WALL/VE L(I)
16 00160 REM I-TH SHIELD TEMPERATURE T(I)
17 00170 REM I-TH POSITION RATIO R(I)
18 00180 REM I-TH COOL-SIDE TEMPERATURE TO
19 00190 REM I-TH WARM-SIDE TEMPERATURE T(I)
20 00200 REM I-TH SHIELD POSITION R(I)
21 00210 REM I-TH DIMENSIONLESS ENTROPY PRODUCTION RATE S(I)
22 00220 REM I-TH DIMENSIONLESS HEAT REMOVAL RATE Q(I)
23 00230 REM I-TH TOTAL ENTROPY PRODUCTION RATE S(I)
24 00240 REM I-TH ENTROPY PRODUCTION RATE WITHOUT SHIELDS S(I)
25 00250 REM I-TH LOCAL SPACING RATIO. L/I(I)-L(1)
26 00260 REM I-TH SPACING RATIO. L/L(1)-L(1)
27 00270 REM I-TH DISTANCE FROM COLD WALL/VE L(I)
28 00280 REM I-TH SHIELD TEMPERATURE T(I)
29 00290 REM I-TH COOL-SIDE TEMPERATURE TO
30 00300 REM I-TH WARM-SIDE TEMPERATURE T(I)
31 00310 REM I-TH SHIELD POSITION R(I)
32 00320 REM I-TH DIMENSIONLESS ENTROPY PRODUCTION RATE S(I)
33 00330 REM I-TH DIMENSIONLESS HEAT REMOVAL RATE Q(I)
34 00340 REM I-TH TOTAL ENTROPY PRODUCTION RATE S(I)
35 00350 REM I-THENTROPY PRODUCTION RATE WITHOUT SHIELDS S(I)
36 00360 REM I-TH LOCAL SPACING RATIO. L/I(I)-L(1)
37 00370 REM I-TH SPACING RATIO. L/L(1)-L(1)
38 00380 REM I-TH DISTANCE FROM COLD WALL/VE L(I)
39 00390 REM I-TH SHIELD TEMPERATURE T(I)
40 00400 REM I-TH COOL-SIDE TEMPERATURE TO
41 00410 REM I-TH WARM-SIDE TEMPERATURE T(I)
42 00420 REM I-TH SHIELD POSITION R(I)
43 00430 REM I-TH DIMENSIONLESS ENTROPY PRODUCTION RATE S(I)
44 00440 REM I-TH DIMENSIONLESS HEAT REMOVAL RATE Q(I)
45 00450 REM I-TH TOTAL ENTROPY PRODUCTION RATE S(I)
46 00460 REM I-THENTROPY PRODUCTION RATE WITHOUT SHIELDS S(I)
47 00470 REM I-TH LOCAL SPACING RATIO. L/I(I)-L(1)
48 00480 REM I-TH SPACING RATIO. L/L(1)-L(1)
49 00490 REM I-TH DISTANCE FROM COLD WALL/VE L(I)
50 00500 REM I-TH SHIELD TEMPERATURE T(I)
51 00510 REM I-TH COOL-SIDE TEMPERATURE TO
52 00520 REM I-TH WARM-SIDE TEMPERATURE T(I)
53 00530 REM I-TH SHIELD POSITION R(I)
54 00540 REM I-TH DIMENSIONLESS ENTROPY PRODUCTION RATE S(I)
55 00550 REM I-TH DIMENSIONLESS HEAT REMOVAL RATE Q(I)
56 00560 REM I-TH TOTAL ENTROPY PRODUCTION RATE S(I)
57 00570 REM I-THENTROPY PRODUCTION RATE WITHOUT SHIELDS S(I)
58 00580 REM I-TH LOCAL SPACING RATIO. L/I(I)-L(1)
59 00590 REM I-TH SPACING RATIO. L/L(1)-L(1)
60 00600 REM I-TH DISTANCE FROM COLD WALL/VE L(I)
61 00610 REM I-TH SHIELD TEMPERATURE T(I)
62 00620 REM I-TH COOL-SIDE TEMPERATURE TO
63 00630 REM I-TH WARM-SIDE TEMPERATURE T(I)
64 00640 REM I-TH SHIELD POSITION R(I)
65 00650 REM I-TH DIMENSIONLESS ENTROPY PRODUCTION RATE S(I)
66 00660 REM I-TH DIMENSIONLESS HEAT REMOVAL RATE Q(I)
67 00670 REM I-TH TOTAL ENTROPY PRODUCTION RATE S(I)
68 00680 REM I-THENTROPY PRODUCTION RATE WITHOUT SHIELDS S(I)
69 00690 REM I-TH LOCAL SPACING RATIO. L/I(I)-L(1)
70 00700 REM I-TH SPACING RATIO. L/L(1)-L(1)
71 00710 REM I-TH DISTANCE FROM COLD WALL/VE L(I)
72 00720 REM I-TH SHIELD TEMPERATURE T(I)
73 00730 REM I-TH COOL-SIDE TEMPERATURE TO
74 00740 REM I-TH WARM-SIDE TEMPERATURE T(I)
ORIGINAL PAGE IS OF POOR QUALITY
"FOR k=2 TO 100"
150 01104 IF k=1=INT(k/1) THEN 01207
151 01105 H=H+2*Y(i)
152 01106 GO TO 01208
153 01107 H=H+4*Y(i)
154 01108 MEIT K
155 01110 S(i)=((D/3*H(i))2/(1+G8*N(i)/(1)))
156 01120 S(i)=S2(i)/S0(i)
157 01130 SF(i)=-S(i)-R0(i)-R1(i)-R2(i)/(1+B0(i)-1)/M1/(1+G8*N(i)/(1))
158 01140 S(i)=S(i)/S0(i)
159 01150 IF i=M THEN 01270
160 01160 P(i+1)=P(1):
161 01170 PRINT "".
162 01180 PRINT "". P(i), R(i), E(i), "". E(i), S(i), S(i)
163 01190 PRINT "".
164 01200 PRINT "". COLD WALL/HOT WALL TEMPERATURE RATIO, TO/T=",R0(i)
165 01210 PRINT "". HEAT IN AT COLD WALL=",R0(i)
166 01220 PRINT "". ENTROPY PRODUCTION RATE AT COLD WALL=",S0(i)
167 01230 PRINT "". MINIMUM ENTROPY PRODUCTION RATE. S0=",S0(i)
168 01240 PRINT "". ENTROPY PRODUCTION RATE AT WARM WALL=",S0(i)
169 01250 PRINT "". MAXIMUM ENTROPY PRODUCTION RATE. S0=",S0(i)
170 01260 PRINT "". ENTROPY PRODUCTION RATIO. S0=",S0(i)
171 01270 NEXT k
172 01280 END
NEWRAF

This program solves the original, complete, constrained optimization equations developed in Ref. [9] without the simplifying assumption suggested there which eliminated the dimensionless parameter, \( h_{fg}/C_p T_H \). Only single-term thermal conductivity functions were considered in this analysis.

This program also recycles to the starting point. Consequently, the first input is either a 1, if a calculation is to be performed, or a 0, if no more work is to be done.

Next the program requests input of the insulation's characteristics, specifically, the exponent of temperature in the thermal conductivity function, the number of cooled shields, the dimensionless parameter \( h_{fg}/C_p T_H \) for the boiloff from the insulated container, and \( R = T_C/T_H \).

The output specifies the optimal characteristics of the given number of shields with the constraint that the cooling capacity is limited to the boil-off of the liquid due only to the heat leak through the insulation itself.

The flow chart and a program sample follows.
BEGIN NEWRAF

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PTC.

IS PTC = 1?

YES

B

NO

A

ENTER INSULATION CHARACTERISTICS, NUMBER OF SHIELDS, HFC / (CP * THOT) AND TCH

ASSIGN TSHC(i)'S WHICH ARE THE INITIAL GUESSES FOR TSH(i)'S

COUNT = 0
EPS = 1.0E-10

D

COUNT = COUNT + 1

ASSIGN ELEMENTS OF THE TRIDIAGONAL MATRIX A(i), B(i), C(i) AND ALSO D(i) BASED ON TSHC(i)'S AND OTHER DATA

SOLVE THE SYSTEM OF EQUATIONS FOR THE TSH(i)'S USING THE TRIDIAGONAL MATRIX SOLVER

I = 1

TSH(i) = TSH(i) - TSH(i)
I = I + 1

IS I > NS?

NO

IS DIFFMAX > EPS?

NO

C

YES

YES

CALCULATE DIFFMAX, THE MAXIMUM OF |TSH(i) - TSHC(i)|, I = 1, ..., NS

W
TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC.
**THIS PASCAL PROGRAM WAS DEVELOPED TO OPTIMIZE THE**

**LOCATION, TEMPERATURE AND HEAT DISSIPATION RATE**

**OF EACH COOLED SHIELD INSIDE AN INSULATION LAYER**

**THE THERMAL CONDUCTIVITY OF THE INSULATION HAS**

**THE GENERAL FORM:**

\[ K = E_1^T(T^T)^N \]

**THE OBJECTIVE HAS BEEN TO SOLVE THE SET OF 1+MS-1 NON-LINEAR EQUATIONS OBTAINED BY BEJAN, A. "DIS-**

**CRETE COOLING OF LOW HEAT LEAK SUPPORTS TO 4.2 K."**

**CRYOCOGENICS, VOL 15, 1975, PP 290-292.**

**SOLUTION IS BASED ON THE NEWTON-RAPHSON TECHNIQUE**

**DISCUSSED BY STOECKER, W. F , DESIGN OF THERMAL**

**SYSTEMS.2ND EDITION, SECTION 4-11, PP 117-119, M**

**MCGRAW-HILL BOOK CO. NY, 1980**

**THE SIZE OF ARRAYS DETERMINES THE MAXIMUM NUMBER OF SHIELDS**

**THE SIZE OF ARRAYP IS N5-1**

**THE SIZE OF ARRAT Should BE TWICE THE NUMBER OF SHIELDS**

**LOWER-DIAGONAL ELEMENTS OF THE TRIDIAGONAL MATRIX**

**DIAGONAL ELEMENTS OF THE TRIDIAGONAL MATRIX**

**UPPER-DIAGONAL ELEMENTS OF THE TRIDIAGONAL MATRIX**

**RIGHT-HAND SIDE OF THE SET OF EQUATIONS DURING ITERATIONS**

**I-TH DIMENSIONLESS HEAT REMOVAL RATE**

**DIEMENSIONLESS HEAT TRANSFER BETWEEN SHIELDS**

**DIEMENSIONLESS ENTROPY PRODUCTION RATE FOR I-TH LATER**

**MAXIMUM DIMENSIONLESS ENTROPY PRODUCTION RATE**

**MINIMUM DIMENSIONLESS ENTROPY PRODUCTION RATE**

**SMAX**

**Smin**

**TOTAL DIMENSIONLESS ENTROPY PRODUCTION RATE**

**DIMENSIONLESS ENTROPY PRODUCTION RATE FOR I-TH LATER**

**MAXIMUM DIMENSIONLESS ENTROPY PRODUCTION RATE**

**MINIMUM DIMENSIONLESS ENTROPY PRODUCTION RATE**

**SPACING BETWEEN NEIGHBORING SHIELDS / INSULATION THICKNESS**

**DISTANCE FROM COLD WALL / INSULATION THICKNESS**

**E11 / (E11-1)**

**OUTPUT FILE TO BE USED IF DESIRED**

**PARAMETER DEFINED IN PROCEDURE INPUT**
PROCEDURE INPUT:
BEGIN
  (* INPUT OF DATA HEADING *)
  WRITE: 'ENTR -- M NS BETA TCH (-----).
  WRITE: 'WHERE M ----- POWER IN THE THERMAL CONDUCTIVITY EQUATION'.
  WRITE: 'NS ----- NUMBER OF SHIELDS').
  WRITE: 'BETA -- HFC / (CP*THOT)',.
  WRITE: 'HFC --- HEAT OF VAPORIZATION (J/KG/K).
  WRITE: 'CP ---- SPECIFIC HEAT AT CONSTANT PRESSURE (J/KG/K).
  WRITE: 'THOT -- HOT WALL TEMPERATURE (K).
  WRITE: 'TCH ---- COLD WALL / HOT WALL TEMPERATURE RATIO, ALWAYS (1).
END

PROCEDURE PITCH:
BEGIN
  (* PITCH *)
  WRITE: 'TO PERFORM COMPUTATION. ENTER 1. OTHERWISE. ENTER 0 '.
END

PROCEDURE SINGLESPACE:
BEGIN
  (* SINGLE SPACE IN OUTPUT *)
  WRITE: '('
  WRITE: '.
  END

FUNCTION PWR(II,E REAL) REAL.
VAR
A
BEGIN
A = E**II.
RETURN
END

FUNCTION RAIIOR(MO1,MO2 REAL) REAL.
BEGIN
(* DETERMINES THE LARGEST OF THE TWO GIVEN NUMBERS *)
IF MO1>MO2 THEN
  MAXO1 = MO1.
ELSE
  MAXO1 = MO2.
END

FUNCTION RATIO(MO1,MO2 REAL) REAL.
BEGIN
(* DETERMINES THE LARGEST OF THE TWO GIVEN NUMBERS *)
IF MO1>MO2 THEN
  RATIO = MO1.
ELSE
  RATIO = MO2.
END

FUNCTION PWR(II,E REAL) REAL.
VAR
A
BEGIN
A = E**II.
RETURN
END

FUNCTION RAIIOR(MO1,MO2 REAL) REAL.
BEGIN
(* DETERMINES THE LARGEST OF THE TWO GIVEN NUMBERS *)
IF MO1>MO2 THEN
  MAXO1 = MO1.
ELSE
  MAXO1 = MO2.
END

FUNCTION RATIO(MO1,MO2 REAL) REAL.
BEGIN
(* DETERMINES THE LARGEST OF THE TWO GIVEN NUMBERS *)
IF MO1>MO2 THEN
  RATIO = MO1.
ELSE
  RATIO = MO2.
END

FUNCTION PWR(II,E REAL) REAL.
VAR
A
BEGIN
A = E**II.
RETURN
END

FUNCTION RAIIOR(MO1,MO2 REAL) REAL.
BEGIN
(* DETERMINES THE LARGEST OF THE TWO GIVEN NUMBERS *)
IF MO1>MO2 THEN
  MAXO1 = MO1.
ELSE
  MAXO1 = MO2.
END

FUNCTION RATIO(MO1,MO2 REAL) REAL.
BEGIN
(* DETERMINES THE LARGEST OF THE TWO GIVEN NUMBERS *)
IF MO1>MO2 THEN
  RATIO = MO1.
ELSE
  RATIO = MO2.
END
FUNCTION MINOF1(NO1, NO2) REAL REAL;
BEGIN (* DETERMINES THE SMALLEST OF THE TWO GIVEN NUMBERS *)
  IF NO1(=NO1 THEN
  IF NO1(=NO2 THEN
  MINOF1 =NO1
  ELSE
  MINOF1 =NO2
  END. (* DETERMINES THE SMALLEST OF THE TWO GIVEN NUMBERS *)

BEGIN
FUNCTION
READK.
READT.
WHILE PGE=1 DO
BEGIN
(* THIS BLOCK IS USED TO INPUT THE INSULATION THERMAL CONDUCTIVITY, NUMBER *)
(* OF SHIELD, NFG/(CP*THOT) AND COLD WALL / HOT WALL TEMPERATURE RATIO *)

INPUT
READK.
READ M AS BETA TCH.
SINGLESPACE.
WRITE T.
THERMAL CONDUCTIVITY OF THE INSULATION IS K = K1*T**M 3-1.
WRITE.
SINGLESPACE T
SINGLESPACE.

ME. =M-1.0.
MM. =M-1.0.

(* INITIAL GUESSED VALUES FOR TSH(II)'S ARE ENTERED *)

DELTA T = ( K-TCH) / (ME** M).
FOR J = 1. TO ME DO TSH(J) = J*DELTA T + TCH.

(* VARIABLE USED TO CHECK CONVERGENCE CRITERION IS SET AND THE ITERATIVE PROCEDURE *)
(* OF NEWTON-RAPHSON METHOD IS STARTED *)

IFS = 0.0-1C.
COUNT = 0.
ITERM = 0.
REPEAT
COUNT = COUNT -1.
FOR J = 1. TO ME DO
BEGIN
G1 = TSH(J).
IF ME/J THEN IF I/O THEN
BEGIN
G1 = TSH(J-1).
G1 = TSH(J+1).
END.
ELSE
BEGIN
G1 = TSH(J).
G1 = TSH(J+1).
END.
ELSE
BEGIN
G1 = TCH.
G1 = TSH(J+1).
END.
ELSE
BEGIN
G1 = TCH.
END.

END.
**ORIGINAL PAGE**

OF POOR QUALITY

```
GIP = 1.0
END.

(*) ELEMENTS OF THE TRIDIAGONAL MATRIX ARE COMPUTED *)

A11 = PWR(C1P1,M1) - PWR(C1,M1) + PWR(C1P1,M1)*(TCH-BETA)
B11 = M1 + PWR(C1,M1) + M1 + PWR(C1P1,M1)*(TCH-BETA)
C11 = M1 + PWR(C1P1,M1)
D11 = (M1 + PWR(C1P1,M1)) - PWR(C1,M1) + PWR(C1,M1) + PWR(C1P1,M1)
END.

(*) THE TRIDIAGONAL MATRIX SOLVER IS SHOWN IN THIS BLOCK *)
(*) SEE WESTLAKE, J. R., A HANDBOOK OF NUMERICAL MATRIX *)
(*) INVERSION AND SOLUTION OF LINEAR EQUATIONS, SECTION *)
(*) 2.7, PP 14-35, JOHN WILEY & SONS, INC., NY, 1948 *)

IF EI = 0 THEN GOTO 100.
BOLD = 0.0
OLD = 0.0
WORK = 0.0
WORKNS = 0.0
DATA = 0.0
CMIN = 0.0
FOR i = 1 TO NS DO
BEGIN
D1W = B11 - A11*OLD.
IF D1W = 0 THEN GOTO 100.
DATA = MAXD1(DATA,ABS(D1W))
DMIN = MIN(D1W,MIN(A11,ABS(D1W))
OLD = D1W
WORK = 0.0
OLD = C11/D1W
WORKNS = B11/D1W
END.
TSHC = OLD
FOR i = 1 TO NS DO
BEGIN
OLD = 0.0
WORK = OLD: WORKNS = OLD
TSHC = OLD
END.

(*) NEWLY CALCULATED VALUES OF TSH(1)'S ARE COMPUTED *)

FOR i = 1 TO NS DO TSH(i) = TSHC(i) - TSH(i)

(*) CONVERGENCE IS CHECKED. IF THE CRITERION IS SATISFIED, THE ITERATION IS *)
(*) TERMINATED, OTHERWISE THE NEWLY CALCULATED TSH(1)'S ARE USED AS NEW *)
(*) GUESSES FOR ANOTHER ROUND OF ITERATION *)

DIFFMAX = 1.0
FOR i = 1 TO NS DO
BEGIN
DIFF = ABS(TSH(i) - TSHC(i))
DIFFMAX = MAX(DIFF,DIFFMAX)
END.
IF DIFFMAX < EPS THEN
ITERN = 1
ELSE
FOR i = 1 TO NS DO TSHC(i) = TSH(i)
UNTIL ITERN = 1.

(*) IN THIS BLOCK QUANTITIES USED IN DETERMINING THE SHIELDS' SPACINGS ARE COMPUTED *)

FOR i = 1 TO NS DO
BEGIN
T = TSH(i)
IF NS(i) THEN
IF i(i) THEN
IF (NS THEN
TIP1 = TSH(i-1)
ELSE
TIP1 = TSH(i-1)
```
END.

DEN = 0.
FOR J = 1 TO NS DO DEN = DEN + X(NSP,J - 1) + 1.0.
NSP = NSP + 1.

(* FINALLY, SPACINGS BETWEEN SHIELDS AND OTHER QUANTITIES OF INTEREST ARE CALCULATED *)

IF (MSP) THEN IF(I) = XPL(P - 1) + 1.
END = END + X.

FOR I = 1 TO NSP DO BEGIN

IF (HST(I)) THEN IF(I) = XPL(P - 1) + 1.0.
END = END + X.

IF (ABS(TOTAL - 1.0) < 0.0E-5) THEN GOTO 100.
Q(!) = MPW(TSH(I), NP) = MPW(TCH, NP) + (H(I) + MP).
Q(NSP) = 0.0 = MPW(TSH(NP) - MPW(TCH, NP)) + (I(MSP)) + MP).
FOR I = 1 TO NS DO END = MPW(TSH(I), NP) = MPW(TCH, NP) + (I(MSP)) + MP).

SINGLESpace.
WRITE: NUMBER OF SHIELDS = NSP.
WRITE: NUMBER OF ITERATIONS = COUNT.

SINGLESpace.
WRITE: HEAT REMOVAL RATE.
WRITE: ENTROPY PRODUCTION RATE.
WRITE: OPTIMUM LOCATION.
WRITE: OPTIMUM TEMPERATURE.

SINGLESpace.
FOR I = 1 TO NS DO BEGIN

Q(!) = Q(!) + 1.0.
S(I) = S(I) + 1.0.
WRITE: SHEILD .1.2.
S(I) = S(I) + 1.0.
END.

SINGLESpace.
WRITE: COLD WALL / HOT WALL TEMPERATURE RATIO.
WRITE: COLD WALL / STRTING WALL TEMPERATURE RATIO.
WRITE: HEAT IN AT COLD WALL.
WRITE: ENTROPY PRODUCTION RATE AT COLD WALL.
WRITE: HEAT IN AT HOT WALL.
WRITE: ENTROPY PRODUCTION RATE AT HOT WALL.
WRITE: MAXIMUM ENTROPY PRODUCTION RATE.
WRITE: TOTAL ENTROPY PRODUCTION RACH WITH "NS 2" SHIELDS.
WRITE: MAXIMUM ENTROPY PRODUCTION RATE.
WRITE: TOTAL / MINIMUM ENTROPY PRODUCTION RACH.

SINGLESpace.
IF (DIW = 0) OR (B(I-1) = 0) THEN BEGIN
WRITE: CHECK THE ASSEMBLY OF COEFFICIENTS TO BE USED IN TRIDIAGONAL MATRIX.
WRITE: CHECK THE TRIDIAGONAL MATRIX SOLVER.
END.

SINGLESpace.
IF (ABS(TOTAL - 1.0) < 0.0E-5) THEN BEGIN
WRITE: TOTAL IS NOT EQUAL TO 1.0.
WRITE: COMPUTATIONS ARE NOT CORRECT.
END.

SINGLESpace.
SINGLESpace.
To perform computation, enter 1. Otherwise, enter 0.

? 1

Enter: M NS BETA TCH

Where: M ----- Power in the thermal conductivity equation
       NS ----- Number of shields
       BETA = MFG / (CP*THOT)
       MFG ----- Heat of vaporization [J/kg]
       CP ----- Specific heat at constant pressure [J/kg K]
       THOT ----- Hot wall temperature [K]
       TCH ----- Cold wall / hot wall temperature ratio, always = 1

? 1.0 3 0.0145 0.001

Thermal conductivity of the insulation is K = K1*T**1.0
MFG / (CP*THOT) = 0.01450

| NUMBER OF SHIELDS = 3 |
| NUMBER OF ITERATIONS = 9 |

<table>
<thead>
<tr>
<th>Heat removal rate</th>
<th>Entropy production rate</th>
<th>Optimum location</th>
<th>Optimum temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHIELD 1</td>
<td>0.10438</td>
<td>1.56143</td>
<td>0.09719</td>
</tr>
<tr>
<td>SHIELD 2</td>
<td>0.25963</td>
<td>1.12595</td>
<td>0.28870</td>
</tr>
<tr>
<td>SHIELD 3</td>
<td>0.47781</td>
<td>0.88732</td>
<td>0.58586</td>
</tr>
</tbody>
</table>

Cold wall / hot wall temperature ratio = 0.001000
Heat out at cold wall = 0.022985
Heat in at hot wall = 0.864998
Entropy production rate at cold wall = 22.984544
Entropy production rate at hot wall = -0.864998
Minimum entropy production rate = 3.751018
Maximum entropy production rate = 499.499501
Total entropy prod. rate with 3 shields = 25.704743
Maximum / minimum entropy production ratio = 133.163725
Total / minimum entropy production ratio = 6.852738

To perform computation, enter 1. Otherwise, enter 0.

? 1

Enter: M NS BETA TCH

Where: M ----- Power in the thermal conductivity equation
       NS ----- Number of shields
       BETA = MFG / (CP*THOT)
       MFG ----- Heat of vaporization [J/kg]
       CP ----- Specific heat at constant pressure [J/kg K]
       THOT ----- Hot wall temperature [K]
       TCH ----- Cold wall / hot wall temperature ratio, always = 1

? 1.0 2 0.0154 0.000806
THERMAL CONDUCTIVITY OF THE INSULATION IS $k = 1.0$

$\text{MFG} / (\text{CP}*\text{THOT}) = 0.01540$

NUMBER OF SHIELDS = 2

NUMBER OF ITERATIONS = 8

<table>
<thead>
<tr>
<th>SHIELD</th>
<th>HEAT REMOVAL RATE</th>
<th>ENTROPY PRODUCTION RATE</th>
<th>OPTIMUM LOCATION</th>
<th>OPTIMUM TEMPERATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.19732</td>
<td>1.97595</td>
<td>0.16252</td>
<td>0.09986</td>
</tr>
<tr>
<td>2</td>
<td>0.39037</td>
<td>1.46999</td>
<td>0.48495</td>
<td>0.39623</td>
</tr>
</tbody>
</table>

COLD WALL / HOT WALL TEMPERATURE RATIO = 0.000806

HEAT OUT AT COLD WALL = 0.030677

HEAT IN AT HOT WALL = 0.818366

ENTROPY PRODUCTION RATE AT COLD WALL = 38.061092

ENTROPY PRODUCTION RATE AT HOT WALL = -0.818366

MINIMUM ENTROPY PRODUCTION RATE = 3.776103

MAXIMUM ENTROPY PRODUCTION RATE = 619.846992

TOTAL ENTROPY PROD. RATE WITH 2 SHIELDS = 40.708665

MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO = 164.19921

TOTAL / MINIMUM ENTROPY PRODUCTION RATIO = 10.780603

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

? 0

0.072 CP SECS, 11471B CM USED.

/BYE:

3KMUFTC COSTS: 255,028 SRUS AT 0.0059 = $1.50
DESINS

This program optimizes the characteristics of a single cooled shield with different insulations on the two sides. Only one-term thermal conductivity functions are considered.

This program also recycles to the starting point; thus the first input is 1, if a calculation is to be performed, or 0 if no more work is to be done.

Next inputs are the characteristics of the two insulations, specifically, the exponents of temperature in the thermal conductivity functions on the hot and cold sides of the shield, a coefficient ratio ALFA (defined in the program), the shield to cold wall temperature ratio, \( P = T_S/T_C \), and the hot wall temperature, \( T_H \).

The output specifies the optimal characteristics of the cooled shield as well as other, related information.

The flow diagram and a program sample follows.
BEGIN DESIGNS

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC.

IS PFC = 1? NO

B

YES

ENTER THE TWO INSULATIONS' CHARACTERISTICS, P AND HOT WALL TEMPERATURE

SOLVE TCH ITERATIVELY

CALCULATE X1 AND X

CALCULATE QHOT, QCOLD, SCOLD, STOTAL, SISH, SMIN1, SMIN2, SMAX1 AND SMAX2

OUTPUT TCH, TSH, QCOLD, QHOT, SCOLD, SISH, SMIN1, SMIN2, STOTAL, SMAX1 AND SMAX2

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC

IS PFC = 1? YES

B

NO

A

END DESIGNS
PROGRAM DIFFCOND(input, output, sem):

(* THIS PASCAL PROGRAM WAS DEVELOPED TO OPTIMIZE THE *)
(* LOCATION, TEMPERATURE AND HEAT DISSIPATION RATE *)
(* FOR A COOLED SHIELD IN A CRYOGENIC INSULATION *)
(* SYSTEM WHOSE THERMAL CONDUCTIVITY HAS THE FORM *)
(* K = 1/(T^2 + M) ON THE HOT SIDE *)
(* K = 2/(T^2 + N) ON THE COLD SIDE *)
(* THE METHOD IS BASED ON THE MINIMIZATION OF THE *)
(* ENTROPY PRODUCTION RATE WHICH IS PROPORTIONAL TO *)
(* THE HEAT LEAK ACROSS THE INSULATION *)
(* TOTAL DIMENSIONLESS ENTROPY PRODUCTION RATE *)
(* DIMENSIONLESS ENTROPY PRODUCTION RATE AT SHIELD *)
(* COLD WALL / HOT WALL TEMPERATURE RATIO, ALWAYS < *)
(* SHIELD / HOT WALL TEMPERATURE RATIO, ALWAYS < *)
(* DISTANCE FROM COLD WALL / THICKNESS RATIO *)
(* E / (1.8 - E) *)
(* DUMMY VARIABLE *)
(* NUMBER OF ITERATIONS NEEDED TO DETERMINE TCH *)
(* DUMMY VARIABLE *)
(* [E^2*(M-1)/E^2*(M-1)] *)
(* DUMMY VARIABLES *)
(* INDEX TO TERMINATE THE SEARCH FOR TCH *)
(* POWER OF THE THERMAL CONDUCTIVITY ON HOT SIDE *)
(* EQUALS [M-1] *)
(* POWER OF THE THERMAL CONDUCTIVITY ON COLD SIDE *)
(* EQUALS [N-1] *)
(* PROGRAM FLOW CONTROLLER *)
(* HEAT IN AT HOT WALL *)
(* ENTROPY PRODUCTION RATE AT COLD WALL *)
(* OUTPUT FILE TO BE USED IF DESIRED *)
(* NOT VALL TEMPERATURE *)

VAR

P REAL
SHAI REAL (* SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS ) 1 *)
SHAI REAL (* MAXIMUM ENTROPY PRODUCTION RATE BASED ON 1/(T^2 + M) *)
SHAI REAL (* MAXIMUM ENTROPY PRODUCTION RATE BASED ON 2/(T^2 + N) *)
SHAI REAL (* MINIMUM ENTROPY PRODUCTION RATE BASED ON 1/(T^2 + M) *)
SHAI REAL (* MINIMUM ENTROPY PRODUCTION RATE BASED ON 2/(T^2 + N) *)
SHAI REAL (* TOTAL DIMENSIONLESS ENTROPY PRODUCTION RATE *)
SHAI REAL (* DIMENSIONLESS ENTROPY PRODUCTION RATE AT SHIELD *)
TCH REAL (* COLD WALL / HOT WALL TEMPERATURE RATIO, ALWAYS < *)
TCH REAL (* SHIELD / HOT WALL TEMPERATURE RATIO, ALWAYS < *)
T REAL (* DISTANCE FROM COLD WALL / THICKNESS RATIO *)
N REAL (* E / (1.8 - E) *)

CC REAL (* DUMMY VARIABLE *)
COUNT INTEGER (* NUMBER OF ITERATIONS NEEDED TO DETERMINE TCH *)
DD REAL (* DUMMY VARIABLE *)
ALPHA REAL (* [E^2*(M-1)/E^2*(M-1)] *)
G REAL (* DUMMY VARIABLES *)
IND INTEGER (* INDEX TO TERMINATE THE SEARCH FOR TCH *)
M REAL (* POWER OF THE THERMAL CONDUCTIVITY ON HOT SIDE *)
MF REAL (* EQUALS [M-1] *)
N REAL (* POWER OF THE THERMAL CONDUCTIVITY ON COLD SIDE *)
NF REAL (* EQUALS [N-1] *)
PIC INTEGER (* PROGRAM FLOW CONTROLLER *)
QCHD REAL (* HEAT OUT AT COLD WALL *)
QHOT REAL (* HEAT IN AT HOT WALL *)
SCOLD REAL (* ENTROPY PRODUCTION RATE AT COLD WALL *)
SEM TEXT (* OUTPUT FILE TO BE USED IF DESIRED *)
NIGHT REAL (* NOT VALL TEMPERATURE *)

PROCEDURE INPUT: (* INPUT OF DATA HEADING *)
BEGIN
WRITELN;
WRITELN('ENTER ----) M M ALFA P THOT (----)');
WRITELN('WHERE: M ---- POWER OF THE THERMAL CONDUCTIVITY EQUATION ON THE HOT SIDE.);
WRITELN('M ---- POWER OF THE THERMAL CONDUCTIVITY EQUATION ON THE COLD SIDE.);
WRITELN('ALFA -- (EXP((M+1)/2(1)))/(EXP((M+1)/2)).
WRITELN('P ---- SHIELD / COLD WALL TEMPERATURE RATIO. ALWAYS > 1).
WRITELN('THOT -- HOT WALL TEMPERATURE (K)
END. (* INPUT OF DATA HEADING *)

PROCEDURE FCH. (* FCH *)
BEGIN
WRITELN;
WRITELN('TO PERFORM COMPUTATION. ENTER I OTHERWISE. ENTER N ');
WRITELN;
END. (* FCH *)

PROCEDURE SINGLESPACE.
BEGIN (* SINGLE SPACE IN OUTPUT *)
WRITELN;
END. (* SINGLE SPACE IN OUTPUT *)

FUNCTION PWR II.E REAL); REAL.
VAR A.
BEGIN (* COMPUTE II**E *)
A = EXP2(II).
END. (* COMPUTE II**E *)

FUNCTION DIE.II REAL); REAL.
BEGIN (* FUNCTIONAL D *)
D = PWR2(II,(1.6-II)) - PWR2(II,(4.016II)) - PWR2(II,(1.6-II)) - PWR2(II,(4.016II))
END. (* FUNCTIONAL D *)

FUNCTION F(E.II REAL); REAL.
BEGIN (* FUNCTIONAL F *)
F = PWR2(II,(1.6-II)) - PWR2(II,(1.6-II)) - PWR2(II,(1.6-II))
END. (* FUNCTIONAL F *)

* MAIN PROGRAM BODY *
BEGIN
FCH.
READLN.
WHILE FCH = 'D' DO
BEGIN
(* THIS BLOCK IS USED TO INPUT THE TWO INSULATION THERMAL CONDUCTIVITIES. *)
(* SHIELD / COLD WALL TEMPERATURE RATIO AND NOT WALL TEMPERATURE *)

INPUT.
READLN.
READLN.M ALFA.P THOT).
SINGLESPACE.
WRITELN('THERMAL CONDUCTIVITY OF THE INSULATION ON THE HOT SIDE IS E = EXP((M+1)/2(1))')
WRITELN('THERMAL CONDUCTIVITY OF THE INSULATION ON THE COLD SIDE IS E = EXP((M+1)/2(1)).')
WRITELN('ALFA.P THOT '):
[Original page is of poor quality]

(* THIS BLOCK CALCULATES TCH ITERATIVELY *)

REPEAT

IF CI=0.0 THEN GOTO 100.

CC = (-0.1)*CC.

IF ABS(CC) > 0.000001 THEN GOTO 100.

DO = DO.

100 TCH = TCH - CC.

IF TCH < 0.000001 OR (TCH > 0.000001) THEN BEGIN

TCH = TCH - CC.

END.

105 COUNT = COUNT + 1.

UNTIL (|CI|= 0.0) OR (|ABS(CC)| > 0.000001) OR (IND = 1).

END

(* OTHER QUANTITIES OF INTEREST ARE COMPUTED IN THIS SECTION *)

I: = ALPHA*PWR(TCH, (N-1)*D(M,P)/D(M,TSH)).

QHOT = (I - PWR(TCH, MP)) - (I - PWR(TCH, MP)).

QCOLD = ALPHA*PWR(TCH, MP) - (PWR(TCH, MP)).

QCORE = QCOLD/TCH.

QHOT - QCOLD = TSH.

IF N=0 THEN

S'M.: = S'M.: + 1/TCH.

ELSE

S'M.: = S'M.: - 1/TCH.

IF N=0 THEN

S'M.: = S'M.: + 1/TCH.

ELSE

S'M.: = S'M.: + 1/TCH.

SMAI = F(I, TCH, MP).

SMAI - F(I, TCH, MP).
300 SINGLESPACE.
SINGLESPACE.
SINGLESPACE.
PECH.
READIN.
READ(PFC).
END
END

ORIGINAL PAGE IS OF POOR QUALITY
TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

ENTER -----> M N ALFA P THOT <-----

WHERE: M ----- POWER OF THE THERMAL CONDUCTIVITY EQUATION ON THE HOT SIDE
N ----- POWER OF THE THERMAL CONDUCTIVITY EQUATION ON THE COLD SIDE
ALFA -- \[\frac{K_2(T^{(M+1)})}{K_1(T^{(N+1)})}\]
P ----- SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS = 1
THOT -- HOT WALL TEMPERATURE [K]

1.0 0.0 20.0 4.5 300.0

THERMAL CONDUCTIVITY OF THE INSULATION ON THE HOT SIDE IS K = \(K_1(T^{1.0})\).
THERMAL CONDUCTIVITY OF THE INSULATION ON THE COLD SIDE IS K = \(K_2(T^{0.0})\).
\[\frac{K_2(T^{M+1})}{K_1(T^{N+1})}\] = 20.00
HOT WALL TEMPERATURE = 300.00 [K]

NUMBER OF ITERATIONS = 36
COLD WALL / HOT WALL TEMPERATURE RATIO = 0.001666
SHIELD / HOT WALL TEMPERATURE RATIO = 0.007497
SHIELD LOCATION = 0.390755
HEAT OUT AT SHIELD = 0.820144
HEAT OUT AT COLD WALL = 0.000497
HEAT IN AT HOT WALL = 0.820641
ENTROPY PRODUCTION RATE AT COLD WALL = 0.298568
ENTROPY PRODUCTION RATE AT HOT WALL = -0.820641
ENTROPY PRODUCTION RATE AT SHIELD = 109.396253
MINIMUM ENTROPY PRODUCTION RATE BASED ON K_1 T^M = 3.680131
MINIMUM ENTROPY PRODUCTION RATE BASED ON K_2 T^N = 40.925828
TOTAL ENTROPY PRODUCTION RATE = 178.307751
ENTROPY PROD. W/O SHIELD BASED ON K_1 T^M = 299.619216
ENTROPY PROD. W/O SHIELD BASED ON K_2 T^N = 598.241762

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

0.044 CP SECS, 102338 CM USED.
A relatively simple method has been developed to optimize the location, temperature, and heat dissipation rate of each cooled shield inside an insulation layer. The method is based on the minimization of the entropy production rate which is proportional to the heat leak across the insulation. The results show that the maximum number of shields to be used in most practical applications is three. However, cooled shields are useful only at low values of the overall, cold wall to hot wall absolute temperature ratio. The performance of the insulation system is relatively insensitive to deviations from the optimum values of the temperature and location of the cooling shields.

Design curves are presented for rapid estimates of the locations and temperatures of cooling shields in various types of insulations, and an equation is given for calculating the cooling loads for the shields.