Optimization of Cooled Shields in Insulations

J. C. Chato
J. M. Khodadadi
J. Seyed-Yagoobi

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J. C. Chato
J. M. Khodadadi
J. Seyed-Yagoobi
Department of Mechanical and Industrial Engineering
University of Illinois at Urbana-Champaign
Urbana, IL

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NASA
National Aeronautics and Space Administration
Ames Research Center
Moffett Field, California 94035
ABSTRACT

A relatively simple method has been developed to optimize the location, temperature, and heat dissipation rate of each cooled shield inside an insulation layer. The method is based on the minimization of the entropy production rate which is proportional to the heat leak across the insulation. The results show that the maximum number of shields to be used in most practical applications is three. However, cooled shields are useful only at low values of the overall, cold wall to hot wall absolute temperature ratio. The performance of the insulation system is relatively insensitive to deviations from the optimum values of temperature and location of the cooling shields.

Design curves are presented for rapid estimates of the locations and temperatures of cooling shields in various types of insulations, and an equation is given for calculating the cooling loads for the shields.
NOMENCLATURE

A Area of heat flow, m²
Cₚ Specific heat of the boiloff vapor, kJ/kg*K
D Functional defined by Eq. (14)
F Functional defined by Eq. (13)
hₙ Latent heat of vaporization of the boiloff liquid, kJ/kg
k Thermal conductivity, W/m-K; with subscripts, coefficients in Eq. (1)
L Overall thickness of insulation, m
m,n Exponents in conductivity function, Eq. (1)
P Tₛ/Tᵥ, temperature ratio
q Heat flow rate, W
R Tᵥ/Tᵥ, overall temperature ratio
s Dimensionless entropy production rate defined by Eq. (5)
S Entropy production rate, W/K
t Thickness between walls with single shield between, m
T Absolute temperature, K
x Distance from cold wall, m
x' Distance from cold wall in a multi-shield configuration, m
X x/t, dimensionless distance
X' x'/L, dimensionless distance
γ Defined by Eq. (8)

Subscripts
C Cold wall
H Hot wall
i i-th shield
min Minimum
opt Optimum
S Shield

*For systems with single shield L = t, x = x', X = X'.
INTRODUCTION

The search for the ultimate, energy efficient insulation system has led in the past few years to a fascinating rediscovery and application of some fundamental concepts of thermodynamics: specifically, the second law and the use of entropy production rates and availability (or exergy) for design optimization purposes. The classical approach has been to minimize the heat flow between surfaces at different temperatures.

The concept of a single vapor-cooled shield in an insulation has been treated theoretically as far back as 1959 in Scott's classic textbook on cryogenics [1] and designs employing them were described not much later [2]. Paivanas, et al., obtained a patent [3] and later reported on the use of uniformly spaced multiple shields which were cooled by the boil-off from the insulated dewar [4]. Eyssa and Okasha [5] considered only radiative heat exchange between shields and minimized the total refrigeration power required. Hilal, et al., [6,7] used a similar minimization of refrigeration power as the design basis. Related works were reported by Bejan, et al., [8-11].

Recently, Bejan [12] proposed a new point of view, based on the second law of thermodynamics, which considers thermal insulations as dissipators of useful mechanical power (i.e. the availability or exergy) or, alternately, as generators of irreversibility or entropy. Thus, in this method, optimization of an insulation corresponds to minimization of either the entropy production rate or the irreversibility, or the decrease of availability. Various applications of this concept to insulation systems have been documented subsequently [13,14].

Our work grew out of an examination of Cunnington's paper [13] who utilized a numerical technique to find optimum temperatures at given locations for one and two shields for a thermal conductivity function of the form
Although several equations seemed to be incorrectly printed we have found two of the design curves to be essentially correct. Thus, our purpose was

1. To develop a simple optimization technique;
2. To generalize the results to a broader class of insulations; and
3. To develop simple design methods for cooled shields.

The essentials of this report were already published [15].
ANALYSIS

We accept the previously developed concept that to optimize an insulation system is equivalent to minimizing the entropy production rate. In addition, we assume one-dimensional heat flow and that the heat capacity of the boil-off gas is adequate to do the cooling for all shields and does not impose a restriction on the optimization. In contrast to Rejan [9,11] who has developed a constrained optimization based on the heat capacity of the boiloff we employ the argument that in all practical systems the boil-off is generated by cooling of some equipment in addition to the heat leakage across the insulation.

Parallel heat paths, e.g. supports, have not been considered. However, each path can be optimized separately using its own thermal conductivity function. Then a design decision has to be made whether the two structures should be independently cooled at their respective optimum conditions.

We examine the general situation of an insulation where equivalent thermal conductivity, \( k \), can be expressed as a two-term function of the absolute temperature

\[
   k = k_1 T^m + k_2 T^n
\]

where, typically, the first term represents actual conduction with \( m = 1 \) and the second term represents radiation with \( n \geq 3 \). In the following, \( m \) and \( n \) can be any value except -1.

The heat flow across a layer of insulation can be expressed in terms of Fourier's law

\[
   q \, dx = A k \, dT
\]
Substituting $k$ from Eq. (1) and integrating across a layer from one end at 1, to the other at 2, yields

$$q = \frac{A}{x_2 - x_1} \left[ \frac{k_1}{m+1} (T_{2}^{m+1} - T_{1}^{m+1}) + \frac{k_2}{n+1} (T_{2}^{n+1} - T_{1}^{n+1}) \right].$$ \hfill (3)

Now consider the insulation with a cooled shield at $T_s$ located at $x$ between a hot surface at $T_H$ and a cold one at $T_C$, separated by the insulation thickness, $t$, as shown in Fig. 1a. The entropy production rate for the insulation can be determined from the heat flows and temperatures as follows

$$S = -\frac{q_H}{T_H} + \frac{q_C}{T_C} + \frac{q_S}{T_S}$$ \hfill (4)

where $q_S = q_H - q_C$.

The heat flow terms can be expressed in the form of Eq. (3) and the resulting expression can be non-dimensionalized using the following terms

$$s \equiv \frac{S t}{k_H} \quad \text{where} \quad k_H = k \text{ at } T_H,$$ \hfill (5)

$$P \equiv \frac{T_s}{T_C},$$ \hfill (6)

$$R \equiv \frac{T_C}{T_H},$$ \hfill (7)

$$\gamma \equiv \frac{k_2(m+1)}{k_1(n+1)} \frac{T_H^{n-m}}{T_H},$$ \hfill (8)

and

$$X \equiv \frac{x}{t}.$$ \hfill (9)
The resulting equation is

\[ s(m + 1)(1 + \gamma \frac{n + 1}{m + 1}) \]

\[ = \frac{1}{1 - \gamma} \left\{ [(PR)^m + 1 - (PR)^m - 1 + (PR)^{-1}] \right\} \]

\[ + \gamma[(PR)^n + 1 - (PR)^n - 1 + (PR)^{-1}] \]

\[ + \frac{1}{1} \left\{ R^m[p^{m+1} - p^m - 1 + p^{-1}] \right\} \]

\[ + \gamma R^n[p^{n+1} - p^n - 1 + p^{-1}] \}

(10)

Since \( R \), the overall temperature ratio, is generally known, \( s \) is a function of \( P \) and \( X \), and its extreme value can be found by differentiating it with respect to each variable separately and setting the results equal to zero. This procedure yields two equations to be solved simultaneously: \( \frac{\partial s}{\partial P} = 0 \) and \( \frac{\partial s}{\partial X} = 0 \). Because of the regular form of the expressions, one of the final two equations contains only a single unknown as follows:

\[ \frac{R^m F(m,P) + \gamma R^n F(n,P)}{[R^{m-1} D(m,P) + \gamma R^{n-1} D(n,P)]^2} \]

\[ = \frac{F(m,PR) + \gamma F(n,PR)}{[D(m,PR) + \gamma D(n,PR)]^2} \]

(11)

\[ \frac{X}{1-X} = - \frac{R^{m-1} D(m,P) + \gamma R^{n-1} D(n,P)}{D(m,PR) + \gamma D(n,PR)} \]

(12)

where the following functionals were used:
Thus, to find the optimum temperature and location for a shield, Eq. (11) can be solved for \( P \), and then \( X \) can be calculated from Eq. (12). The heat to be removed by the shield, \( q_S = q_H - q_C \), can be found, as before, from Eq. (3). In dimensionless form the equation becomes

\[
\frac{q_S}{Ak_H^{-1}} = \left( 1 - \left( \frac{PR}{n+1} \right)^{m+1} \right) \left[ 1 - \frac{\gamma \left[ 1 - \left( \frac{PR}{n+1} \right)^{m+1} \right]}{1 - X} \right]
\]

For multiple shields, \( t_i \) represents the distance between the two surfaces surrounding the \( i \)-th shield on either side, \( T_{Hi} \) and \( T_{Ci} \) are the temperatures of these two surfaces, \( x_i = x_i/t_i \) is the location of the shield relative to \( t_i \), and \( x_i^1 \) is the location of the shield relative to the cold wall as shown in Fig. 1b. To determine the optimum temperatures and locations for multiple shields, first we assumed a temperature for the first shield next to the cold wall, then we used Eqs. (11) and (12) to find the temperature and location of the second shield. This process was repeated for the rest of the shields and the hot wall. Thus, each shield was optimized consecutively with respect to the two surfaces on either side. With given values of the overall temperature ratio, \( R \), and of the number of shields, the process requires iterative solution.
To put the results into proper perspective, the entropy production rates can be compared to the thermodynamically minimum rate obtainable through spatially continuous cooling. According to Bejan [12], this rate is

\[ S_{\text{min}} = \frac{A}{t} \int_{T_C}^{H} (k)^{1/2} T^{-1} \, dT \]  

(16)

This expression was evaluated analytically for the single-term functions of k, i.e. for \( \gamma = 0 \), and numerically otherwise.
RESULTS AND DISCUSSION

The first set of curves, Figs 2 through 9, show the relative entropy production rates for various thermal conductivity functions and for up to four optimally cooled shields as functions of the overall temperature ratio \( R = T_C/T_H \). The curves show that the entropy production rate increases with decreasing values of the temperature ratio, \( R \), and with increasing values of the exponent, \( m \) and \( n \). Adding shields, of course, reduces the entropy production rate; but for most of the practical temperature range, say \( 0.01 < R < 0.4 \), only three shields contribute to significant decreases and adding a fourth shield can be considered unnecessary. No shields are useful at high values of \( R \); but this "high" range is strongly dependent on the exponent of the temperature. The curves developed with \( k = k_1 T^{0.6} \) for one and two shields were very close to those given by Cunnington [13], converted appropriately.

Study of the results of two-term conductivities reveals that the curves fall between those obtained for each of the two terms alone. If \( \gamma \) is small the first term, \( T^m \), dominates; whereas if \( \gamma \) is large (>10), the second term, \( T^n \), controls. Thus, general conclusions can be drawn from examining the results of the single-term conductivities.

The second set of curves, Figs. 10 through 31, show the optimum temperature ratios, \( T_S/T_H \), and optimum locations, \( x'/L \), of cooled shields as functions of the overall temperature ratio, \( T_C/T_H \), for various thermal conductivity functions and with different number of cooled shields.

Figures 10 and 11 show the optimum single shield temperature ratios, \( PR = T_S/T_H \), and locations, \( X = x/L \), for five conductivity functions. Both of these functions generally decrease with decreasing \( R \). The other figures in this set show shield temperatures and locations for systems with up to three.
shields and for both single-term and two-term conductivities. The results are strongly non-linear. For example, for $k_1T^3$ and $R = 0.01$, the optimum temperature ratios for three shields are about 0.09, 0.3, and 0.6 and the optimum locations are about 0.05, 0.2, and 0.5. As is to be expected, our unconstrained optimization yields a somewhat better performance per shield than Bejan's [9,11] constrained method.

The sensitivities of the entropy production rates to deviations from the optimum values of PR and X are demonstrated in the last set of curves, Figs. 32 through 35, for single shields. The sensitivity increases with the value of the exponents, m and n, but the curves are relatively flat near the minima. A ±20 percent change from optimum, for example, has negligible effect. Thus, the system is relatively tolerant of deviations from the optimum design conditions.

Calculations with two different conductivities on the two sides of a cooled shield show that using the better insulator on both sides always yields the optimum condition. However, if for some reason two types of insulations have to be used, then the better insulator should be placed on the warm side of the shield.
REFERENCES


Figure 1 Schematic of the Nomenclature for (a) Single and (b) Multiple Shields
Curve Set 1: Figures 2 through 9

The effect of optimally cooled shields on the entropy production rate for various thermal conductivities.
Figure 2

\( k = k_1 \)

No. of Shields

- 0
- 1
- 2
- 3
- 4

\[ S/S_{\text{min}} \]

\[ T_{\text{cold}}/T_{\text{hot}} \]
Figure 3

$k = k_1 T_{0.6}^{No. \text{ of } \text{Shields}}$

$\frac{T_{\text{cold}}}{T_{\text{hot}}}$
$k = k_1 T^{3.0}$

No. of Shields

$\frac{S}{S_{\text{min}}}$

$T_{\text{cold}} / T_{\text{hot}}$

Figure 6
\[ k = k_1 T + k_2 T^3.0 \quad \gamma = 0.5 \]

No. of Shields

\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} \]

Figure 7
$k = k_1^T + k_2 T^3.0 \gamma = 5.0$

$\text{No. of Shields}$

Figure 9
Curve Set 2: Figures 10 through 31
Optimal shield temperatures and locations for various thermal conductivity functions with different number of shields.
Figure 10

No. of Shields = 1

- $k = k_1$
- $k = k_1 T^{0.6}$
- $k = k_1 T$
- $k = k_1 T^{2.0}$
- $k = k_1 T^{3.0}$
No. of Shields = 1

\[
\frac{T_{\text{cold}}}{T_{\text{hot}}} = \frac{k}{k_1} \frac{T_0}{T_{10}} \quad \text{when } k = k_1 T_{10} \quad \text{and } k = k_1 T_{20} \quad \text{and } k = k_1 T_{30}
\]
$k = k_1 T^{0.6}$
No. of Shields = 2

Figure 12
$k = k_1 T_0^{0.6}$

No. of Shields = 3

Figure 14
\[ k = k_1 T^{0.6} \]

No. of Shields = 3

---

**Figure 15**

\[ \frac{X'}{L} \text{ vs. } \frac{T_{\text{cold}}}{T_{\text{hot}}} \]
\[ k = k_1 T \]

No. of Shields = 3

\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} \]

\[ \frac{T_{\text{hot}}}{T_{\text{hot}}^o} \]

Figure 16
\[ k = k_1 T \]

No. of Shields = 3

\[ T_{\text{cold}} / T_{\text{hot}} \]

\[ \frac{1}{d_0} \left( \frac{\tau}{X} \right) \]

Figure 17
$k = k_1 T^{3.0}$

No. of Shields = 2

---

SHIELD 1

---

SHIELD 2

Figure 18
$k = k_1 T^{3.0}$  No. of Shields = 2

- SHIELD 1
- SHIELD 2

$[X/L]_{opt}$ vs. $T_{cold}/T_{hot}$

Figure 19
\[ k = k_1 T^{3.0} \]

No. of Shields = 3

- SHIELD 1
- SHIELD 2
- SHIELD 3

Figure 20
\[ k = k_1 T^{3.0} \]

No. of Shields = 3

\[ I_0 \left( \frac{7}{X} \right) \]

\[ T_{\text{cold}} / T_{\text{hot}} \]
Critical figure 22
of poor quality

No. of Shields = 1

\[ k = k_1 T + k_2 T^3.0 \]
\[ k = k_1 T + k_2 T^3.0 \]
\[ k = k_1 T + k_2 T^3.0 \]

\[ \frac{T_{cold}}{T_{hot}} \]

Figure 22
No. of Shields = 1

\[ k = \frac{k_1 T + k_2 T_{3.0}}{T} \]

\( \gamma = 0.5 \)

\( \gamma = 2.0 \)

\( \gamma = 5.0 \)

\[ T_{\text{cold}} / T_{\text{hot}} \]
\[ k = k_1 T + k_2 T^{3.0} \]

No. of Shields \( \gamma = 0.5 \)

Shields 1
Shields 2
Shields 3

\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} \]

Figure 26
\[ k = k_1 T + k_2 T^{3.0} \]

\[ \gamma = 0.5 \]

No. of Shields = 3

\[ [X/L]_{opt} \]

\[ T_{cold}/T_{hot} \]

Figure 27
\[ k = k_1 T + k_2 T^{3.0} \quad \gamma = 2.0 \]

No. of Shields = 2

---

**Figure 28**
\[ k = k_1 T + k_2 T^3 \]

\[ \gamma = 2.0 \]

No. of Shields = 3

\[ \frac{T_{\text{cold}}}{T_{\text{hot}}} \]

Figure 30
Curve Set 3: Figures 32 through 35

System sensitivity to deviations from the optimum shield temperatures and locations for two overall temperature ratios with one cooled shield
\( T_{\text{cold}} / T_{\text{hot}} = 0.006 \)

\[ k = \frac{k_1}{k_1} T_{3.0} \]

\[ k = \frac{k_1}{k_1} \]

\[ k = \frac{k_1}{k_1} \]

\[ k = \frac{k_1}{k_1} \]

\[ k = \frac{k_1}{k_1} \]

Figure 33
The diagram shows a plot of $S/S_{min}$ against $T_{shield}/T_{hot}$ for $T_{cold}/T_{hot} = 0.060$. The curves represent different values of $k$, which are:

- $k = k_1$
- $k = k_1T$
- $k = k_1T^{3.0}$

Figure 34
SEPARS and SHIELD

These two programs are essentially identical, but SEPARS is written in PASCAL whereas SHIELD is in BASIC.

To allow for consecutive calculations of different systems, the program always recycles to the starting point. Consequently, the first input requested is either a 1, if a calculation is to be performed, or a 0, if no more work is to be done.

Next the program requests input of the insulation's characteristics, specifically, the two exponents of the temperatures in the two-term conductivity function, the maximum number of cooled shields (<10) to evaluate, the value of γ, and the temperature ratio of the first shield to the cold wall, \( P(1) = \frac{T_{S1}}{T_C} \). The program calculates and presents the characteristics of all optimal systems of cooled shields from one shield to the maximum number specified in the input.

The flow chart and a program sample follows.
TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC

IS PFC = 1?

ENTER INSULATION CHARACTERISTICS, NUMBER OF SHIELDS AND PI(1)

I = 1

IM1 = I - 1
IM2 = I - 2

SOLVE R(I) ITERATIVELY

CALCULATE S(I), X(I)

IS I > 1?

L2(I) = (1.0 - X(I-1)) / X(I)

ONE SHIELD ARRANGEMENT
ASSIGN L3(I), TCH(I), TSH(I), L4(I) AND XPL(I)

TWO SHIELDS ARRANGEMENT
ASSIGN L4(2) AND TCH(2)

THREE SHIELDS ARRANGEMENT
ASSIGN L4(3) AND TCH(3)

FOUR OR MORE SHIELDS ARRANGEMENT
ASSIGN B AND L4(1)

J = 2
\[ B = B / X1(J) \]
\[ L4[I] = L4[I] + B \]
\[ J = J + 1 \]

\[ \text{IS } J > IM2 ? \]
\[ \text{NO} \]
\[ \text{YES} \]

\[ \text{ASSIGN } L4[I] \text{ AND } L0 \]
\[ J = 2 \]
\[ L0 = L0 / X1(J) \]
\[ J = J + 1 \]

\[ \text{IS } J > IM1 ? \]
\[ \text{NO} \]
\[ \text{YES} \]

\[ \text{ASSIGN } L0, L4[I] \text{ AND } TCH[I] \]
\[ J = 1 \]
\[ TCH[I] = TCH[I] / P(I) \]
\[ J = J + 1 \]

\[ \text{IS } J > IM1 ? \]
\[ \text{NO} \]
\[ \text{YES} \]

\[ L3[I] = L4[I] \]
\[ XPL[I] = X1(I) / L3[I] \]
\[ J = 2 \]

\[ \text{ASSIGN } L3[I] \text{ AND } XPL[I] \]
\[ J = J + 1 \]

\[ \text{IS } J > I ? \]
\[ \text{NO} \]
\[ \text{YES} \]

\[ TSH[I] = TCH[I] * P(I) \]
\[ J = 2 \]

\[ TSH[J] = TSH[J-1] * P(J) \]
\[ J = J + 1 \]

\[ \text{IS } J > I ? \]
\[ \text{NO} \]
\[ \text{YES} \]

\[ \text{YES} \]
ORIGINAL PAGE IS OF POOR QUALITY

Y

TSH(I+1) = 1.0
T2(I) = TCH(I)

IS I () 1 ?

NO

YES

J = 2

T2(J) = TSH(J-1)
J = J + 1

IS J () 1 ?

NO

YES

ASSIGN L3(I+1) AND I3(I+1)
J = 1

CALCULATE Q(J), SISH(J)

OUTPUT THE HEAT REMOVAL RATE AND ENTROPY PRODUCTION RATE AT EACH SHIELD ALONG WITH OPTIMUM LOCATION AND OPTIMUM TEMPERATURE FOR THE SHIELDS' ARRANGEMENT

J = J + 1

IS J () 1 ?

NO

YES

CALCULATE QHOT, QCOLD AND SCOLD

CALCULATE STOTAL(I)

CALCULATE SMIN(I), STOTMIN(I), SMAI(I), SMAIN(I)

IS I () NS ?

NO

YES

P(I+1) = 1.0 / P1

2
OUTPUT TCH(I), QCOLD, QHOT, SCOLD, SMIN(I), SMAX(I), STOTAL(I), SMAXN(I) AND STOTMIN(I)

I = I + 1

IS I > NS ?

YES

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC

IS PFC = 1 ?

YES

NO

END SEPARS
This Pascal program was developed to optimize the location, temperature and heat dissipation rate of each cooled shield inside an insulation layer. The thermal conductivity of the insulation has the general form:

\[ k = k_1(T^m) + k_2(T^p) \]

The method is based on the minimization of the entropy production rate which is proportional to the heat leak across the insulation.

The size of the arrays determines the maximum number of shields. The size of array is equal to \( n-1 \).

The spacing between (i-1)-th and (i+1)-th shields of the insulation thickness ratio and local cold temperature ratio is always 1. The heat dimensionless entropy production rate for (i-1)-th shield is always 1. The maximum dimensionless entropy production rate is always 1. The dimensionless entropy production rate is always 1. The total dimensionless entropy production rate is always 1. The cold wall / hot wall temperature ratio is always 1.

The distance from local cold shield / thickness ratio is always 1. The distance from cold wall / overall insulation thickness is always 1.
PROCEDURE INPUT.

BEGIN

(* INPUT OF DATA HEADING *)

WRITELN.

ENTER ----> M  N  NS  GAMMA  P(1)  (----).  

WHERE

M ----> 1ST POWER IN THE THERMAL CONDUCTIVITY EQUATION',

N ----> 2ND POWER IN THE THERMAL CONDUCTIVITY EQUATION',

NS ----> NUMBER OF SHIELDS',

GAMMA ----> 0 IF USING ONE TERM THERMAL CONDUCTIVITY EQUATION',

(*) 0 IF USING TWO TERM THERMAL CONDUCTIVITY EQUATION',

P(1) ----> 1ST SHIELD (COLD WALL TEMPERATURE RATIO, ALWAYS > 1').

END.

(* INPUT OF DATA HEADING *)

PROCEDURE PFCH.

BEGIN

(* PFCH *)

WRITELN.

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

END.

PROCEDURE SINGLESPACE.

BEGIN

(* SINGLE SPACE IN OUTPUT *)

WRITELN.

END.

(* SINGLE SPACE IN OUTPUT *)

FUNCTION PW(X:REAL) REAL.

VAR

A

REAL.

BEGIN

(* COMPUTE X**E *)

A := 1.0**X;

PWR := 1.0**A;

END.

(* COMPUTE X**E *)

FUNCTION D(E:REAL) REAL.

BEGIN

(* FUNCTIONAL D *)

D := (E := 0.0)*PWR-E/(PWR+E**(-1.0)+SQR(E))

END.

(* FUNCTIONAL D *)

FUNCTION PFCH.

BEGIN

(* PFCH *)

WRITELN.

END.

(* PFCH *)

PROCEDURE INPUT.

BEGIN

(* INPUT OF DATA HEADING *)

WRITELN.

ENTER ----> M  N  NS  GAMMA  P(1)  (----).  

WHERE

M ----> 1ST POWER IN THE THERMAL CONDUCTIVITY EQUATION',

N ----> 2ND POWER IN THE THERMAL CONDUCTIVITY EQUATION',

NS ----> NUMBER OF SHIELDS',

GAMMA ----> 0 IF USING ONE TERM THERMAL CONDUCTIVITY EQUATION',

(*) 0 IF USING TWO TERM THERMAL CONDUCTIVITY EQUATION',

P(1) ----> 1ST SHIELD (COLD WALL TEMPERATURE RATIO, ALWAYS > 1').

END.

(* INPUT OF DATA HEADING *)

PROCEDURE PFCH.

BEGIN

(* PFCH *)

WRITELN.

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

END.

PROCEDURE SINGLESPACE.

BEGIN

(* SINGLE SPACE IN OUTPUT *)

WRITELN.

END.

(* SINGLE SPACE IN OUTPUT *)

FUNCTION PW(X:REAL) REAL.

VAR

A

REAL.

BEGIN

(* COMPUTE X**E *)

A := 1.0**X;

PWR := 1.0**A;

END.

(* COMPUTE X**E *)

FUNCTION D(E:REAL) REAL.

BEGIN

(* FUNCTIONAL D *)

D := (E := 0.0)*PWR-E/(PWR+E**(-1.0)+SQR(E))

END.

(* FUNCTIONAL D *)

FUNCTION PFCH.

BEGIN

(* PFCH *)

WRITELN.

END.

(* PFCH *)

PROCEDURE INPUT.

BEGIN

(* INPUT OF DATA HEADING *)

WRITELN.

ENTER ----> M  N  NS  GAMMA  P(1)  (----).  

WHERE

M ----> 1ST POWER IN THE THERMAL CONDUCTIVITY EQUATION',

N ----> 2ND POWER IN THE THERMAL CONDUCTIVITY EQUATION',

NS ----> NUMBER OF SHIELDS',

GAMMA ----> 0 IF USING ONE TERM THERMAL CONDUCTIVITY EQUATION',

(*) 0 IF USING TWO TERM THERMAL CONDUCTIVITY EQUATION',

P(1) ----> 1ST SHIELD (COLD WALL TEMPERATURE RATIO, ALWAYS > 1').

END.

(* INPUT OF DATA HEADING *)

PROCEDURE PFCH.

BEGIN

(* PFCH *)

WRITELN.

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

END.

PROCEDURE SINGLESPACE.

BEGIN

(* SINGLE SPACE IN OUTPUT *)

WRITELN.

END.

(* SINGLE SPACE IN OUTPUT *)

FUNCTION PW(X:REAL) REAL.

VAR

A

REAL.

BEGIN

(* COMPUTE X**E *)

A := 1.0**X;

PWR := 1.0**A;

END.

(* COMPUTE X**E *)

FUNCTION D(E:REAL) REAL.

BEGIN

(* FUNCTIONAL D *)

D := (E := 0.0)*PWR-E/(PWR+E**(-1.0)+SQR(E))

END.

(* FUNCTIONAL D *)

FUNCTION PFCH.

BEGIN

(* PFCH *)

WRITELN.

END.

(* PFCH *)
```
FUNCTION F(X, Y, REAL). REAL;
BEGIN (* FUNCTIONAL F *)
  F = FWHM(SQRT(X-1.0))+FWHM(Y)+SQRT((X-1.0)/X)
END. (* FUNCTIONAL F *)

FUNCTION SIMPSON(T, X, REAL). REAL;
TYPE
  ARR = ARRAY[1..10] OF REAL;
VAR
  C, T
  DELTAT, REAL;
  N, REAL;
  K, L, INTEGER;
BEGIN (* COMPUTE MINIMUM ENTROPY PRODUCTION RATE USING SIMPSON'S NUMERICAL INTEGRATION SCHEME *)
  DELTAT = (1.0-TCHEB)/100.0;
  FOR i = 1 TO 10; DO
    BEGIN
      C = TCHEB*DELAT*(L-1);
      TIC = FWHM(TCHEB)*C + C + FWHM(TCHEB)*C + C;
      END;
    FOR i = 1 TO 100; DO
      BEGIN
        IF X = (K DIV 2) + 1 THEN
          H = H + C + C;
        ELSE
          H = H + C + C;
        END;
      END;
      SIMPSON -= SQRT(DELAT/SQRT(1.0 + C + C))/DELAT;
END. (* COMPUTE MINIMUM ENTROPY PRODUCTION RATE USING SIMPSON'S NUMERICAL INTEGRATION SCHEME *)

(* MAIN PROGRAM BODY *)
BEGIN
  PCL.;
  READ.;
  READ(XF, SF, DF, CF);
  WHILE PFC = 1 DO
    BEGIN
      (* THIS BLOCK IS USED TO INPUT THE INSULATION THERMAL CONDUCTIVITY, NUMBER *)
      (* OF SHIELDS AND 1ST. SHIELD / COLD WALL TEMPERATURE RATIO *)
      INPUT;
      READ:
      READ:(XIN, XNF, XDF, XCF);
      SINGLESPE.
      IF XIN = 0 THEN
        BEGIN
          THERMAL CONDUCTIVITY OF THE INSULATION IS X = EXP**' .3 .1)
          ELSE
            BEGIN
              THERMAL CONDUCTIVITY OF THE INSULATION IS X = EXP**'.3 .1) + EXP**'.3 .1).
              EXP**'X(N-1))/(EXP**'X(N))\*THOT**'X(N-M) = ,CAMA V 2)
          END.
          SINGLESPE.
          SINGLESPE.
          IF XIN = 0 THEN
            BEGIN
              Y = 0.0;
              FOR i = 1 TO N-1 DO
                BEGIN
                  TM = Y + 1.
                  TM = TM - 2.
                  RC(i) = 0.00001.
                  CC = C + 1.
                  DD = 1.0.
                  COUNT = 0.
                END.
```
(* THIS BLOCK CALCULATES K(I) ITERATIVELY *)

REPEAT

IF K(I) = K(I-1) THEN

V1 = F(K(I-1), M) * F(M, P(I)) * GAMP * PV(K(I), M) * F(M, P(I))

V2 = PH(K(I), M) * F(M, P(I)) * GAMP * PV(K(I), M) * F(M, P(I))

U = V1 + V2

END

IF U = 0 THEN GOTO 200

CC = 1

IF CC = 0 THEN GOTO 200

CC = 0

GOTO 100

END

100 COUNT = COUNT + 1

UNTIL (G(I) = 0) OR (ABS(CC) = 0)

END

(* IN THIS BLOCK VARIABLES ARE ASSIGNED FOR DIFFERENT SHIELD CONFIGURATIONS *)

IF I = 1 THEN

BEGIN

L(I) = 0

END

IF I = 1 THEN

BEGIN

FOR J = 2 TO IM1 DO

BEGIN

B = 0

END

END

ELSE

BEGIN

FOR J = 2 TO IM1 DO

BEGIN

TCH(J) = TCH(J-1)

END

END

ELSE

BEGIN

END

END
BEGIN
LINE 1 = 0.
TCH(J) = 0.
TSH(J) = TCH(J) + P(J).
EPL(J) = EPL(J) + P(J).
END.
TSH(I+1) = E.
T(J) = TCH(J).

SIMPLESPACE.

WRITELN: 4 NUMBER OF SHIELDS = ',' I 2 '.
WRITELN: 4 NUMBER OF ITERATIONS = ',' COUNT 1 '.
WRITELN: 4 HEAT REMOVAL RATE
WRITELN: 4 ENTHROPY PRODUCTION RATE
WRITELN: 4 OPTIMUM LOCATION
WRITELN: 4 OPTIMUM TEMPERATURE.

IF (J > 1) THEN
FOR J = 2 TO I DO T(J) = TSH(J-1).
END.

(* IN THIS BLOCK DIMENSIONLESS HEAT REMOVAL AND ENTHROPY PRODUCTION RATES *)
(* ARE CALCULATED FOR EACH SHIELD *)

FOR J = 1 TO I DO
BEGIN
Z = (PWR(TSH(J-1),NP1) - PWR(TSH(J),NP1) * TSH(J)) / (EPL(J-2) / EPL(J) - 1).
QHOT = PWR(TCH(J-1),NP1) - PWR(TCH(J),NP1) * TCH(J) / TCH(J-1).
QGOLD = PWR(TCH(J-1),NP1) - PWR(TCH(J),NP1) * TCH(J) / TCH(J-1).
SNMIN = (S(TCH(J),NP1) + S(TCH(J-1),NP1)) / 2.
STOTMIN = (S(TCH(J),NP1) + S(TCH(J-1),NP1)) / 2.
END.

(* FINALLY, OTHER QUANTITIES OF INTEREST ARE CALCULATED IN THIS BLOCK *)

SIMPLESPACE.

QHOT = (S(TCH(J),NP1) + S(TCH(J-1),NP1)) / 2.
QGOLD = (S(TCH(J),NP1) + S(TCH(J-1),NP1)) / 2.
SNMIN = (S(TCH(J),NP1) + S(TCH(J-1),NP1)) / 2.
STOTMIN = (S(TCH(J),NP1) + S(TCH(J-1),NP1)) / 2.

WRITELN: 4 COLD WALL / HOT WALL TEMPERATURE RATIO = ',. TCH(J) / TCH(J-1) '.
WRITELN: 4 HEAT IN AT COLD WALL = ',. QGOLD '.
WRITELN: 4 HEAT IN AT HOT WALL = ',. QHOT '.
WRITELN: 4 ENTHROPY PRODUCTION RATE AT COLD WALL = ',. SNMIN '.
WRITELN: 4 ENTHROPY PRODUCTION RATE AT HOT WALL = ',. STOTMIN '.
WRITELN: 4 MINIMUM ENTHROPY PRODUCTION RATE = ',. SNMIN '.
WRITELN: 4 MAXIMUM ENTHROPY PRODUCTION RATE = ',. STOTMIN '.
WRITELN: 4 TOTAL ENTHROPY PRODUCTION RATE WITH ',. I 2 '. SHIELDS = ',. STOTMIN '.
WRITELN: 4 MAXIMUM / MINIMUM ENTHROPY PRODUCTION RATIO = ',. SNMIN '.
WRITELN: 4 TOTAL / MINIMUM ENTHROPY PRODUCTION RATIO = ',. STOTMIN '.

SIMPLESPACE.
SIMPLESPACE.
SIMPLESPACE.

END.
PFCH.
READLN.
READPF.
END.

*/
TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

ENTER M N NS GAMMA [1] <-----

WHERE: M ---- 1ST. POWER IN THE THERMAL CONDUCTIVITY EQUATION
N ---- 2ND. POWER IN THE THERMAL CONDUCTIVITY EQUATION
NS ---- NUMBER OF SHIELDS
GAMMA -- = 0 IF USING ONE TERM THERMAL CONDUCTIVITY EQUATION
> 0 IF USING TWO TERM THERMAL CONDUCTIVITY EQUATION
P[1] -- 1ST. SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS > 1

? 1.0 0.90 2 0.0 25.0

THERMAL CONDUCTIVITY OF THE INSULATION IS K = k1*T**1.0 + k2*T**3.0

[2.50] pc13 = M + GAMA - NS

NUMBER OF SHIELDS = 1
NUMBER OF ITERATIONS = 35

HEAT REMOVAL ENTROPY PRODUCTION OPTIMUM OPTIMUM
RATE RATE LOCATION TEMPERATURE

SHIELD 1 0.43837 1.85659 0.36744 0.23611

COLD WALL / HOT WALL TEMPERATURE RATIO = 0.015741
HEAT OUT AT COLD WALL = 0.014350
HEAT IN AT HOT WALL = 0.452719
ENTROPY PRODUCTION RATE AT COLD WALL = 0.911631
ENTROPY PRODUCTION RATE AT HOT WALL = -0.452719
MINIMUM ENTROPY PRODUCTION RATE = 1.000503
MAXIMUM ENTROPY PRODUCTION RATE = 18.236148
TOTAL ENTROPY PROD. RATE WITH 1 SHIELDS = 18.226962
MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO = 2.314340
TOTAL / MINIMUM ENTROPY PRODUCTION RATIO = 2.315503

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

ENTER M N NS GAMMA [1] <-----

WHERE: M ---- 1ST. POWER IN THE THERMAL CONDUCTIVITY EQUATION
N ---- 2ND. POWER IN THE THERMAL CONDUCTIVITY EQUATION
NS ---- NUMBER OF SHIELDS
GAMMA -- = 0 IF USING ONE TERM THERMAL CONDUCTIVITY EQUATION
> 0 IF USING TWO TERM THERMAL CONDUCTIVITY EQUATION
P[1] -- 1ST. SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS > 1

? 1.0 0.90 2 0.0 25.0

THERMAL CONDUCTIVITY OF THE INSULATION IS K = k1*T**1.0

NUMBER OF SHIELDS = 1
NUMBER OF ITERATIONS = 23
### Heat Removal Rate and Entropy Production Rate

<table>
<thead>
<tr>
<th>Shield</th>
<th>Heat Removal Rate</th>
<th>Entropy Production Rate</th>
<th>Optimum Location</th>
<th>Optimum Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shield 1</td>
<td>0.75466</td>
<td>7.03151</td>
<td>0.35870</td>
<td>0.10732</td>
</tr>
<tr>
<td>Shield 2</td>
<td>0.05470</td>
<td>2.71297</td>
<td>0.17645</td>
<td>0.02016</td>
</tr>
</tbody>
</table>

**COLD WALL / HOT WALL Temperature Ratio**
- 0.004293
- 0.016030
- 0.770687
- 3.734070
- -0.770687
- 3.504633
- 115.966533

**Maximum / Minimum Entropy Production Ratio**
- 9.994873
- 33.089491
- 2.851908

**Number of Shields** = 2
**Number of Iterations** = 36

### Heat Removal Rate and Entropy Production Rate

<table>
<thead>
<tr>
<th>Shield</th>
<th>Heat Removal Rate</th>
<th>Entropy Production Rate</th>
<th>Optimum Location</th>
<th>Optimum Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shield 1</td>
<td>0.05470</td>
<td>2.71297</td>
<td>0.17645</td>
<td>0.02016</td>
</tr>
<tr>
<td>Shield 2</td>
<td>0.8841</td>
<td>4.70678</td>
<td>0.48690</td>
<td>0.18786</td>
</tr>
</tbody>
</table>

**COLD WALL / HOT WALL Temperature Ratio**
- 0.000806
- 0.001162
- 0.940073
- 3.921467
- 3.921467
- 619.47774

**Maximum / Minimum Entropy Production Ratio**
- 7.920388
- 157.970919
- 2.019751

To perform computation, enter 1. Otherwise, enter 0.

? 0

0.175 CP SECS, 124158 CM USED.
PROGRAM SHIELD

1 00010 REM THIS IS A "BASIC" PROGRAM TO CALCULATE OPTIMUM TEMPERATURES.
2 00020 REM LOCATIONS AND COOLING LOADS FOR COOLED SHIELDS IN A CRYOCENIC
3 00030 REM INSULATION SYSTEM WHOSE THERMAL CONDUCTIVITY FOLLOWS THE RELATION
4 00040 REM \[ \frac{\text{K}}{\text{W} \cdot \text{m} \cdot \text{K}} \] \text{E}^{(\frac{T_H - T_C}{T_H - T_C})}
5 00045 REM MODIFIED IN LATE NOV. 1982.
6 00050 REM
7 00060 REM DEFINITION OF SYMBOLS USED
8 00070 REM
9 00080 REM COLD-SIDE WALL TEMPERATURE TO
10 00090 REM WARM-SIDE WALL TEMPERATURE \( T_L \)
11 00100 REM SPACING BETWEEN SHIELDS AT \( I \) AND \( I-1 \) \( \ell(I) \)
12 00110 REM OVERALL THICKNESS OF INSULATION \( L \)
13 00120 REM LOCAL SPACING RATIO \( \ell(I)/\ell(I-1) \) \( L(I) \)
14 00130 REM OVERALL SPACING RATIO \( \ell(I)/\ell(I-1) \) \( \ell(I) \)
15 00140 REM DISTANCE FROM COLD WALL TO \( \ell(I) \)
16 00150 REM \( I \)-TH SHIELD TEMPERATURE \( T(I) \)
17 00160 REM \( I \)-TH SHIELD POSITION RATIO \( \ell(I)/\ell \)
18 00170 REM \( I \)-TH SHIELD TEMPERATURE RATIO \( \frac{T(I)}{T_C} \)
19 00180 REM \( I \)-TH COLD-WARM TEMPERATURE RATIO \( \frac{\ell(I)}{\ell(I)} \) (ALWAYS \( 1 \))
20 00190 REM \( I \)-TH LOCAL SPACING RATIO \( 1-1 \) \( \ell(I) \)
21 00200 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
22 00210 REM OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
23 00220 REM \( I \)-TH DISTANCE FROM COLD WALL \( \ell(I) \)
24 00230 REM \( I \)-TH LOCAL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
25 00240 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
26 00250 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
27 00260 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
28 00270 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
29 00280 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
30 00290 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
31 00300 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
32 00310 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
33 00320 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
34 00330 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
35 00340 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
36 00350 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
37 00360 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
38 00370 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
39 00380 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
40 00390 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
41 00400 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
42 00410 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
43 00420 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
44 00430 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
45 00440 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
46 00450 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
47 00460 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
48 00470 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
49 00480 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
50 00490 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
51 00500 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
52 00510 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
53 00520 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
54 00530 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
55 00540 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
56 00550 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
57 00560 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
58 00570 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
59 00580 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
60 00590 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
61 00600 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
62 00610 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
63 00620 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
64 00630 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
65 00640 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
66 00650 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
67 00660 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
68 00670 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
69 00680 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
70 00690 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
71 00700 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
72 00710 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
73 00720 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
74 00730 REM \( I \)-TH OVERALL SPACING RATIO \( \ell(I)/\ell(I) \) \( \ell(I) \)
ORIGINAL PAGE IS OF POOR QUALITY
FOR n=1 TO n0
10 X=n+1
20 G0=G0+1
30 M=M+1
40 IF M=0 THEN 60
50 PRINT "X=",X
60 NEXT n
70 PRINT "G0=",G0
80 PRINT "M=",M
90 END
NEWRAF

This program solves the original, complete, constrained optimization equations developed in Ref. [9] without the simplifying assumption suggested there which eliminated the dimensionless parameter, \( h_{fg}/C_p T_H \). Only single-term thermal conductivity functions were considered in this analysis.

This program also recycles to the starting point. Consequently, the first input is either a 1, if a calculation is to be performed, or a 0, if no more work is to be done.

Next the program requests input of the insulation's characteristics, specifically, the exponent of temperature in the thermal conductivity function, the number of cooled shields, the dimensionless parameter \( h_{fg}/C_p T_H \) for the boiloff from the insulated container, and \( R = T_C/T_H \).

The output specifies the optimal characteristics of the given number of shields with the constraint that the cooling capacity is limited to the boiloff of the liquid due only to the heat leak through the insulation itself.

The flow chart and a program sample follows.
BEGIN NEWRAT

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC.

IS PFC = 1? NO

YES

ENTER INSULATION CHARACTERISTICS, NUMBER OF SHIELDS, HFG / (CP • THOT) AND TCH

ASSIGN TSHG(I)'S WHICH ARE THE INITIAL GUESSES FOR TSH(I)'S

COUNT = 0
EPS = 1.0E-10

COUNT = COUNT + 1

ASSIGN ELEMENTS OF THE TRIDIAGONAL MATRIX A(I), B(I), C(I) AND ALSO D(I) BASED ON TSHG(I)'S AND OTHER DATA

SOLVE THE SYSTEM OF EQUATIONS FOR THE TSH(I)'S USING THE TRIDIAGONAL MATRIX SOLVER

I = 1

TSH(I) = TSHG(I) - TSH(I)
I = 1 + 1

IS I > NS? NO

YES

CALCULATE DIFFMAX, THE MAXIMUM OF |TSH(I) - TSHG(I)| I = 1, ..., NS

IS DIFFMAX > EPS? NO

YES
I = 1

TSHG[1] = TSH[1]
I = I + 1

IS I > NS

YES

ASSIGN IR[I]'S

SOLVE FOR XPL[I]'S

I = 1

CALCULATE Q[I], SISH[I]

OUTPUT THE HEAT REMOVAL RATE AND ENTROPY PRODUCTION RATE AT EACH SHIELD ALONG WITH OPTIMUM LOCATION AND OPTIMUM TEMPERATURE FOR THE SHIELDS' ARRANGEMENT

I = I + 1

IS I > NS

NO

YES

CALCULATE QHOT, QCOLD, SCOLD, STOTAL, SMIN, STOTMIN, SMAI AND SMAXIN

OUTPUT TCH, QCOLD, QHOT, SCOLD, SMIN, SMAI, STOTAL, SMAXIN AND STOTMIN

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFCT

IS PFCT = 1

YES

B

NO

A

END NEWRAF
**PROGNIAT** NEVRAPHUNPrr/.OSTPUT.AKN).  

This Pascal program was developed to optimize the location, temperature and heat dissipation rate of each cooled shield inside an insulation layer. The thermal conductivity of the insulation has the general form:

\[ K = K_0(T^{m-1}) \]


**LABEL** :EC.  
**TYPE** :REAL.  
**VAR** :A - ARRAYS.  
**VAR** :B - ARRAYS.  
**VAR** :C - ARRAYS.  
**VAR** :D - ARRAYS.  
**VAR** :Q - ARRAYS.  
**VAR** :G - ARRAYS.  
**VAR** :SMAI - REAL.  
**VAR** :SHAI - REAL.  
**VAR** :STOTAL - REAL.  
**VAR** :STOTIN - REAL.  
**VAR** :TSH - ARRAYS.  
**VAR** :TSCH - ARRAYS.  
**VAR** :WORK - ARRAYS.  
**VAR** :I - ARRAYS.  
**VAR** :JPL - ARRAYS.  
**VAR** :JK - ARRAYS.  
**VAR** :AEK - TEST.  
**VAR** :BTAT - REAL.  

The size of ARRAYS determines the maximum number of shields. The size of ARRAYS is NS-1. The size of ARRAYS should be twice the number of shields.

**IPL** computes the entropy production rate for each shield. **SHAI**/SHIN is always less than 1. The spacing between neighboring shields is determined by the insulation thickness. The output file to be used if desired is PARAMETER DEFINED IN PROCEDURE IMPUT.
PROCEDURE INPUT.
BEGIN (* INPUT OF DATA HEADING *)
WRITE
ENTER M MS BETA TCH (* * * * *)
WHILE
WHERE M ---- POWER IN THE THERMAL CONDUCTIVITY EQUATION (* * * * *)
WHILE
WHERE MS ---- NUMBER OF SHIELDS (* * * * *)
WHILE
WHERE BETA ---- HFC / (CP*THOT) (* * * * *)
WHILE
WHERE TCH ---- HEAT OF VAPORIZATION (J/KG) (* * * * *)
WHILE
WHERE CP ---- SPECIFIC HEAT AT CONSTANT PRESSURE (J/KG K) (* * * * *)
WHILE
WHERE THOT ---- COLD WALL / HOT WALL TEMPERATURE RATIO; ALWAYS (1) (* * * * *)
WHILE
WRITE
END (* INPUT OF DATA HEADING *)

PROCEDURE PFCCH.
BEGIN (* PFCCH *)
WRITE
TO PERFORM COMPUTATION. ENTER 1. OTHERWISE. ENTER 0 (* * * * *)
WHILE
WRITE
END (* PFCCH *)

PROCEDURE SINGLESPACE.
BEGIN (* SINGLE SPACE IN OUTPUT *)
WRITE (' ')
INC (* SINGLE SPACE IN OUTPUT *)

FUNCTION FWR(E, E REAL) REAL.
VAR
A DELTA: REAL
BEGIN (* COMPUTE E=E *)
A = FWR(E)
B = FWR(E)
END (* COMPUTE E=E *)

FUNCTION MAI(T, T REAL).
BEGIN (* DETERMINES THE LARGEST OF THE TWO GIVEN NUMBERS *)
IF T1 > T2 THEN
MAI(T1, T2)
ELSE
MAI(T2, T1)
END (* DETERMINES THE LARGEST OF THE TWO GIVEN NUMBERS *)
FUNCTION MINOF1 (NO1, NO2) REAL, REAL:
BEGIN  (* DETERMINES THE SMALLEST OF THE TWO GIVEN NUMBERS *)
  IF NO1 < NO2 THEN
    MINOF1 = NO1
  ELSE
    MINOF1 = NO2
  END.  (* DETERMINES THE SMALLEST OF THE TWO GIVEN NUMBERS *)

BEGIN
  READK.
  READR.
  WHILE PG = 1 DO
    BEGIN
      (* THIS BLOCK IS USED TO INPUT THE INSULATION THERMAL CONDUCTIVITY, NUMBER *)
      (* OF SHIELDS, NEC/(CP*THOT) AND COLD WALL / HOT WALL TEMPERATURE RATIO *)
      INPUT
      READK.
      READING AS BETA * CH.
      SINGLESPACE.
      WRITEK: NEC/(CP*THOT) = BETA * S.
      SINGLESPACE.
      SINGLESPACE.
      MW: = M; G.
      MN: = M; D.
      (* INITIAL GUESSED VALUES FOR TSHI: S ARE ENTERED *)
      DELTATC = C - TCH / (NEC: 8).
      FOR j = 1 TO NS DO TSHC1 = C + DELTATC * TCH.
      (* VARIABLE USED TO CHECK CONVERGENCE CRITERION IS SET AND THE ITERATIVE PROCEDURE *)
      (* OF NEWTON-RAPHSON METHOD IS STARTED *)
      IFS = 01-10.
      COUNT = 0.
      IFS = 01.
      REPEAT
        COUNT = COUNT + 1.
        FOR 1 = 1 TO NS DO
          BEGIN
            GI: = TSHC1;
            IF NS = 1 THEN
              IF 0 THEN
                IF INE THEN
                  BEGIN
                    GI: = TSHC1 - 1;
                    CIP = TSHC1 + 1
                  END.
                ELSE
                  BEGIN
                    GI: = TCH;
                    CIP: = TSHC1 + 1
                  END.
            ELSE
              BEGIN
                CIP = TCH;
                GI: = TSHC1 + 1
              END.
          END
        END.
      END.
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ORIGINAL PAGE 73

73

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GIP = 1.0

END.

(*) ELEMENTS OF THE TRIDIAGONAL MATRIX ARE COMPUTED *)

AII = FWR(CIP(MP)+FWR(GI,M)+N+G1+MP+TCH-BETA));

BII = MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+

BETA-TCH+G1+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+

(TCH-BETA));

CII = MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+

BETA-TCH+G1+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+MP+

(TCH-BETA));

END.

AII = 0.

(*) THE TRIDIAGONAL MATRIX SOLVER IS SHOWN IN THIS BLOCK *)

(*) SEE WESTLAKE, J. R., A HANDBOOK OF NUMERICAL MATRIX *)

(*) INVERSION AND SOLUTION OF LINEAR EQUATIONS, SECTION *)

(*) 17. PP 34-35. JOHN WILEY & SONS, INC., NY, 1968 *)

IF I=1=0 THEN GOTO 100.

BOLD = BOLD.

OLD = OLD.

WORKING = WORKING.

MIN = MIN.

FOR I = 1 TO NS DO BEGIN

D1W = BOLD. 

IF DiW = 0 THEN GOTO 100.

D1W = MIN + OLD.

GOOD = WORKING = WORKING + NS.

END.

FOR I = 1 TO NS DO BEGIN

TSHC! = GOOD.

END.

(*) NEWLY CALCULATED VALUES OF TSHCI)’S ARE COMPUTED *)

(*) CONVERGENCE IS CHECKED. IF THE CRITERION IS SATISFIED, THE ITERATION IS *)

(*) TERMINATED. OTHERWISE THE NEWLY CALCULATED TSHCI)’S ARE USED AS NEW *)

(*) GUESSES FOR ANOTHER ROUND OF ITERATION *)

DIFFMAX = 1 EPS.

FOR I = 1 TO NS DO BEGIN

DIFF = ABS(TSHC(I) - TSHC(I)).

DIFFMAX = MAX(DIFF, DIFFMAX).

END.

IF DIFFMAX = EPS THEN ITERIN = I.

ELSE FOR I = 1 TO NS DO TSHC(I) = TSHC(I).

UNTIL ITERIN =.

(*) IN THIS BLOCK QUANTITIES USED IN DETERMINING THE SHIELDS’ SPACINGS ARE COMPUTED *)

FOR I = 1 TO NS DO BEGIN

T = TSHC(I).

FOR I = 1 TO NS DO FOR J = 1 TO NS DO IF T(I,J) THEN IF I(J) THEN IF I = NS THEN TIP = TSHC(I-1)

ELSE ELSE TIP = TSHC(I-1)

ELSE
TIMI = TCH
ELSE
TIMI = TCH
END.

DEN = 1
FOR J = 1 TO NS DO DEN = DEN * X(NS(J) + 1) / 1.0;
NSP = NS Pix;

(* FINALLY, SPACINGS BETWEEN SHIELDS AND OTHER QUANTITIES OF INTEREST ARE CALCULATED *)
FOR J = 1 TO NSP DO BEGIN
XI(J) = J / DEN.
XPI(J) = XI(J).
END.
TOTAL = XI.
FOR J = 1 TO NSP DO BEGIN
XI(J) = XI(J) + XI(J - 1).
IF (NSP) THEN XPI(J) = XPI(J - 1).
END TOTAL = TOTAL + XI.
END.
IF (ABS(TOTAL - 1.0) > 1.0E-5) THEN GOTO 100.
QHI = (PWR(TSHI(J), MPJ)/PWR(TCH(MPJ)) / (XI(J) + MPJ).
QHNSP1 = (0.0 - PWR(TSHI(NSP), MPJ)) / (XI(NSP) + MPJ).
FOR J = 1 TO NS DO BEGIN
XPI(J) = PWR(TSHI(J), MPJ)/PWR(TSHI(J - 1), MPJ) / (XI(J) + MPJ).
END SINGLESPACE.
BEGIN PRINTHEAT REMOVAL ENTROPY PRODUCTION OPTIMUM OPTIMUM
--- --- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ---- ****
To perform computation, enter 1. Otherwise, enter 0.

? 1

Enter: M NS BETA TCH

Where:

M ———— Power in the thermal conductivity equation
NS ———— Number of shields
BETA ———— MFG / (CP*THOT)
MFG ———— Heat of vaporization [J/kg]
CP ———— Specific heat at constant pressure [J/kg K]
THOT ———— Hot wall temperature [K]
TCH ———— Cold wall / hot wall temperature ratio, always = 1

1.0 3 0.0145 0.001

Thermal conductivity of the insulation is K = K1*1.0
MFG / (CP*THOT) = 0.01450

Number of shields = 3
Number of iterations = 9

<table>
<thead>
<tr>
<th>Shield</th>
<th>Heat Removal Rate</th>
<th>Entropy Production Rate</th>
<th>Optimum Location</th>
<th>Optimum Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10438</td>
<td>1.56143</td>
<td>0.09719</td>
<td>0.06685</td>
</tr>
<tr>
<td>2</td>
<td>0.25983</td>
<td>1.12595</td>
<td>0.28870</td>
<td>0.23076</td>
</tr>
<tr>
<td>3</td>
<td>0.47781</td>
<td>0.89782</td>
<td>0.58568</td>
<td>0.53219</td>
</tr>
</tbody>
</table>

Cold wall / hot wall temperature ratio = 0.001000
Heat out at cold wall = 0.022985
Heat in at hot wall = 0.864998
Entropy production rate at cold wall = 22.984544
Entropy production rate at hot wall = -0.864998
Minimum entropy production rate = 3.751018
Maximum entropy production rate = 499.499501
Total entropy prod. rate with 3 shields = 25.704743
Maximum / minimum entropy production ratio = 133.163725
Total / minimum entropy production ratio = 6.852738

To perform computation, enter 1. Otherwise, enter 0.

? 1

Enter: M NS BETA TCH

Where:

M ———— Power in the thermal conductivity equation
NS ———— Number of shields
BETA ———— MFG / (CP*THOT)
MFG ———— Heat of vaporization [J/kg]
CP ———— Specific heat at constant pressure [J/kg K]
THOT ———— Hot wall temperature [K]
TCH ———— Cold wall / hot wall temperature ratio, always = 1

1.0 2 0.0154 0.000808
THERMAL CONDUCTIVITY OF THE INSULATION IS $K = k_B T \approx 1.0$

MFG / (CP*THOT) = 0.01540

NUMBER OF SHIELDS = 2
NUMBER OF ITERATIONS = 8

<table>
<thead>
<tr>
<th>SHIELD</th>
<th>HEAT REMOVAL RATE</th>
<th>ENTROPY PRODUCTION RATE</th>
<th>OPTIMUM LOCATION</th>
<th>OPTIMUM TEMPERATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.19732</td>
<td>1.97595</td>
<td>0.16252</td>
<td>0.09986</td>
</tr>
<tr>
<td>2</td>
<td>0.39037</td>
<td>1.48999</td>
<td>0.48495</td>
<td>0.39623</td>
</tr>
</tbody>
</table>

COLD WALL / HOT WALL TEMPERATURE RATIO = 0.000806
HEAT OUT AT COLD WALL = 0.030677
HEAT IN AT HOT WALL = 0.818366
ENTROPY PRODUCTION RATE AT COLD WALL = 38.061092
ENTROPY PRODUCTION RATE AT HOT WALL = -0.018366
MINIMUM ENTROPY PRODUCTION RATE = 3.776103
MAXIMUM ENTROPY PRODUCTION RATE = 619.846992
TOTAL ENTROPY PROD. RATE WITH 2 SHIELDS = 40.708665
MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO = 164.149921
TOTAL / MINIMUM ENTROPY PRODUCTION RATIO = 10.780603

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0.

? 0
0.072 CP SECS, 11471B CM USED.
/BYE:

3KMUFTC COSTS: 255.028 SRUS AT $0.0059 = $1.50
DESINS

This program optimizes the characteristics of a single cooled shield with different insulations on the two sides. Only one-term thermal conductivity functions are considered.

This program also recycles to the starting point; thus the first input is 1, if a calculation is to be performed, or 0 if no more work is to be done.

Next inputs are the characteristics of the two insulations, specifically, the exponents of temperature in the thermal conductivity functions on the hot and cold sides of the shield, a coefficient ratio ALFA (defined in the program), the shield to cold wall temperature ratio, $P = T_S/T_C$, and the hot wall temperature, $T_H$.

The output specifies the optimal characteristics of the cooled shield as well as other, related information.

The flow diagram and a program sample follows.
BEGIN DESIGNS

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC.

IS PFC = 1 ? NO

A

B

YES

ENTER THE TWO INSULATIONS' CHARACTERISTICS, \( P \) AND HOT WALL TEMPERATURE

SOLVE TCH ITERATIVELY

CALCULATE \( I_1 \) AND \( I \)

CALCULATE \( Q_{HOT}, Q_{COLD}, S_{COLD}, S_{TOTAL}, S_{ISH}, S_{MIN1}, S_{MIN2}, S_{MAX1} \) AND \( S_{MAX2} \)

OUTPUT TCH, \( T_{SH} \), \( Q_{COLD} \), \( Q_{HOT} \) \( S_{COLD} \), \( S_{ISH} \), \( S_{MIN1} \), \( S_{MIN2} \), \( S_{TOTAL} \), \( S_{MAX1} \) AND \( S_{MAX2} \)

TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC

IS PFC = 1 ? YES

B

NO

A

END DESIGNS
**Program Diffcond**

---

This Pascal program was developed to optimize the location, temperature and heat dissipation rate for a cooled shield in a cryogenic insulation system where thermal conductivity was the primary factor. The method is based on the minimization of the entropy production rate which is proportional to the heat leak across the insulation.

---

<table>
<thead>
<tr>
<th>Label</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>REAL</td>
</tr>
<tr>
<td>SHI</td>
<td>REAL</td>
</tr>
<tr>
<td>SMAI</td>
<td>REAL</td>
</tr>
<tr>
<td>SMAII</td>
<td>REAL</td>
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<tr>
<td>SM1</td>
<td>REAL</td>
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<td>REAL</td>
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<td>SSH</td>
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<td>TC</td>
<td>REAL</td>
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<tr>
<td>TSH</td>
<td>REAL</td>
</tr>
<tr>
<td>I</td>
<td>REAL</td>
</tr>
<tr>
<td>T0</td>
<td>REAL</td>
</tr>
</tbody>
</table>

---

**Variables:**

- P: Shield / Cold Wall Temperature Ratio. Always > 1.
- SHI: Maximum Entropy Production Rate Based on H(T+M).
- SMAI: Maximum Entropy Production Rate Based on E(T+M).
- SMAII: Minimum Entropy Production Rate Based on E(T+M).
- SM1: Minimum Entropy Production Rate Based on E(T+M).
- SNI: Total Dimensionless Entropy Production Rate.
- SSUM: Dimensionless Entropy Production Rate at Shield.
- TC: Cold Wall / Hot Wall Temperature Ratio. Always > 1.
- TSH: Shield / Hot Wall Temperature Ratio. Always > 1.
- I: Distance from Cold Wall / Thickness Ratio.
- T0: Heat Out at Cold Wall.

---

This program includes variables such as COUNT, D0, ALFA, CI, IN0, N, MP1, MP2, PIC, G0D1, G0DT, SCOLD, SEM, and TH0T.

---

**Constraints:**

- Dummy variables and constants used in the program for various calculations.
- Equations for optimization and thermodynamic properties.

---

J.C. Chato & J.M. Khodadadi

Dept. of Mechanical & Industrial Engineering

Univ. of Illinois at Urbana-Champaign

1364 W. Green Street

Urbana, IL 61801

July 1989

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**Note:**

The program's purpose is to optimize the location, temperature, and heat dissipation rate for a cooled shield in a cryogenic insulation system, focusing on the entropy production rate as a key factor in the optimization process.
PROCEDURE INPUT:
BEGIN
WHITELN(* INPUT OF DATA HEADING *)
WHITELN ENTER M M ALFA P THOT
WHITELN WHERE  M ----- POWER OF THE THERMAL CONDUCTIVITY EQUATION ON THE HOT SIDE
P ----- POWER OF THE THERMAL CONDUCTIVITY EQUATION ON THE COLD SIDE
ALFA -- EXP((R-1)/2)/EXP((R-1)/2)
THOT -- SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS 1
WHITELN END (* INPUT OF DATA HEADING *)

PROCEDURE PCH:
BEGIN
WHITELN (* PCH *)
WHITELN TO PERFORM COMPUTATION. ENTER 1 OTHERWISE, ENTER 0 *)
WHITELN END (* PCH *)

PROCEDURE SINGLESPACE:
BEGIN
WHITELN (* SINGLE SPACE IN OUTPUT *)
WHITELN (* SINGLE SPACE IN OUTPUT *)

FUNCTION PWR II.
VAR A
BEGIN
A = EXP(II)
PWR = EXP(II)
END (* FUNCTION PWR II *)

FUNCTION D.
BEGIN
FUNCTION C = (1 - EXP(-1/II))/2
END (* FUNCTION D *)

FUNCTION F.
BEGIN
FUNCTION F = ((EXP(II) - 1)/(1 + 0/II))
END (* FUNCTION F *)

BEGIN (* MAIN PROGRAM BODY *)
PCH
READLN
WHILE PCH = 0 DO
BEGIN
(* THIS BLOCK IS USED TO INPUT THE TWO INSULATION THERMAL CONDUCTIVITIES *)
(* SHIELD / COLD WALL TEMPERATURE RATIO AND HOT WALL TEMPERATURE *)

INPUT:
READLN
READIN M ALFA P THOT)
SINGLESPACE
WHITELN THERMAL CONDUCTIVITY OF THE INSULATION ON THE HOT SIDE IS K = EXP((II - 1)/2)
WHITELN THERMAL CONDUCTIVITY OF THE INSULATION ON THE COLD SIDE IS K = (EXP(II - 1)/2)
WHITELN ALFA (E2*(II - 1)/E1*(II - 1))
ORIGINAL PAGE IS OF POOR QUALITY

(* THIS BLOCK CALCULATES TCH ITERATIVELY *)

REPEAT

TSH = TCH.

G = (N,P)***F(TCH/(1-0*N))D(M,TSH)**D(M,TSH)/D(M,TSH)/ALFA.

CC = (0,1)**CC.

IF (TCH = 0) THEN GOTO 100.

CC = (0,1)**CC.

IF ABS(CC) = 0 THEN GOTO 100.

CC = (0,1)**CC.

IF TCH = 0 THEN GOTO 100.

CC = (0,1)**CC.

IF ABS(CC) = 0 THEN IND = 1.

END.

COUNT = COUNT+1.

UNTIL (IND = 1) OR (ABS(CC) = 0) OR (IND = 1).

IF IND = 1 THEN

BEGIN

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ENTROPY PROD W/O SHIELD BASED ON EXP**W = SMALL 14 41.

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To perform computation, enter 1. Otherwise, enter 0.

<table>
<thead>
<tr>
<th>Enter</th>
<th>M</th>
<th>N</th>
<th>ALFA</th>
<th>P</th>
<th>THOT</th>
<th>---</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.0</td>
<td>20.0</td>
<td>4.5</td>
<td>300.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Where:
- M ---- Power of the thermal conductivity equation on the hot side
- N ---- Power of the thermal conductivity equation on the cold side
- ALFA = \( \frac{(K2^*(M+1))}{(K1^*(N+1))} \)
- P ---- Shield / Cold wall temperature ratio, always: 1
- THOT -- Hot wall temperature [K]

Thermal conductivity of the insulation on the hot side is \( K = K1^*(T^{1.0}) \).
Thermal conductivity of the insulation on the cold side is \( K = K2^*(T^{0.0}) \).
\( \frac{(K2^*(M+1))}{(K1^*(N+1))} = 20.00 \)
Hot wall temperature = 300.00 [K]

| Number of iterations | = | 36 |
| Cold wall / Hot wall temperature ratio | = | 0.001666 |
| Shield / Hot wall temperature ratio | = | 0.007497 |
| Shield location | = | 0.397055 |
| Heat out at shield | = | 0.820144 |
| Heat out at cold wall | = | 0.000497 |
| Heat in at hot wall | = | 0.820641 |
| Entropy production rate at cold wall | = | 0.295568 |
| Entropy production rate at hot wall | = | -0.820641 |
| Entropy production rate at shield | = | 109.396253 |
| Minimum entropy production rate based on \( K1^*T^{1.0} \) | = | 3.680131 |
| Minimum entropy production rate based on \( K2^*T^{0.0} \) | = | 40.925828 |
| Total entropy production rate | = | 178.307751 |
| Entropy prod. w/o shield based on \( K1^*T^{1.0} \) | = | 299.619216 |
| Entropy prod. w/o shield based on \( K2^*T^{0.0} \) | = | 598.241762 |

To perform computation, enter 1. Otherwise, enter 0.

| 0.044 CP secs, 10233B CM used. |
A relatively simple method has been developed to optimize the location, temperature, and heat dissipation rate of each cooled shield inside an insulation layer. The method is based on the minimization of the entropy production rate which is proportional to the heat leak across the insulation. The results show that the maximum number of shields to be used in most practical applications is three. However, cooled shields are useful only at low values of the overall, cold wall to hot wall absolute temperature ratio. The performance of the insulation system is relatively insensitive to deviations from the optimum values of the temperature and location of the cooling shields.

Design curves are presented for rapid estimates of the locations and temperatures of cooling shields in various types of insulations, and an equation is given for calculating the cooling loads for the shields.
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