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A COMPARISON OF TWO PHOTOGRAMMETRIC ALGORITHMS FOR THE MEASUREMENT OF MODEL DEFORMATION IN THE NATIONAL TRANSONIC FACILITY

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INTRODUCTION

Information relative to model deformation is an important factor in structural and aeronautical research. Many times environmental or model constraints preclude techniques which require a displacement sensor on the model, and remote measurement techniques must be used (ref. 1). Photogrammeteric measurement techniques can provide a solution to this problem as long as suitable targets can be implemented. Unless the model is restricted to planar motions, two or more cameras are required to provide information regarding three-dimensional target locations. Essential to any photogrammetric technique is the determination of the camera parameters necessary for the triangulation operations. Two of several computational techniques for making these determinations are:

1. Direct Linear Transformation algorithm (DLT) (refs. 2-3)
2. Bundle algorithm (refs. 5-7).

Both techniques utilize photogrammetric triangulation, but the methods of achieving the camera parameters differ.

There is considerable literature available on the theoretical aspects of each algorithm (refs. 2-8), but without testing both algorithms using the same data and with the same physical constraints, accuracy comparisons are speculative. Since the NTF (National Transonic Facility)(ref. 9) at the NASA Langley Research Center will be employing photogrammetric techniques to determine model deformation, a comparison was made of the two algorithms for this case. Locations for two cameras in the NTF test section were used, and geometric projection of selected targets on the camera image planes was used to generate simulated data. In addition to comparison of the two algorithms, the timing and accuracy of using them with various computational precisions were examined.

PHOTOGRAMMETRIC TRIANGULATION

The minimum requirement of any photogrammetric technique is the direct or indirect determination of nine projective parameters at each camera (ref. 8). These are given here and illustrated in figure 1.

(a) Three translations X_0, Y_0, Z_0 which define the location of the center of projection in object space.

(b) Three translations x_p, y_p, c which define the location of the center of projection in image space (elements of interior orientation).

(c) Three parameters which uniquely define the orientation of image space axes with respect to those of object space (rotation angles ω, ϕ, κ).

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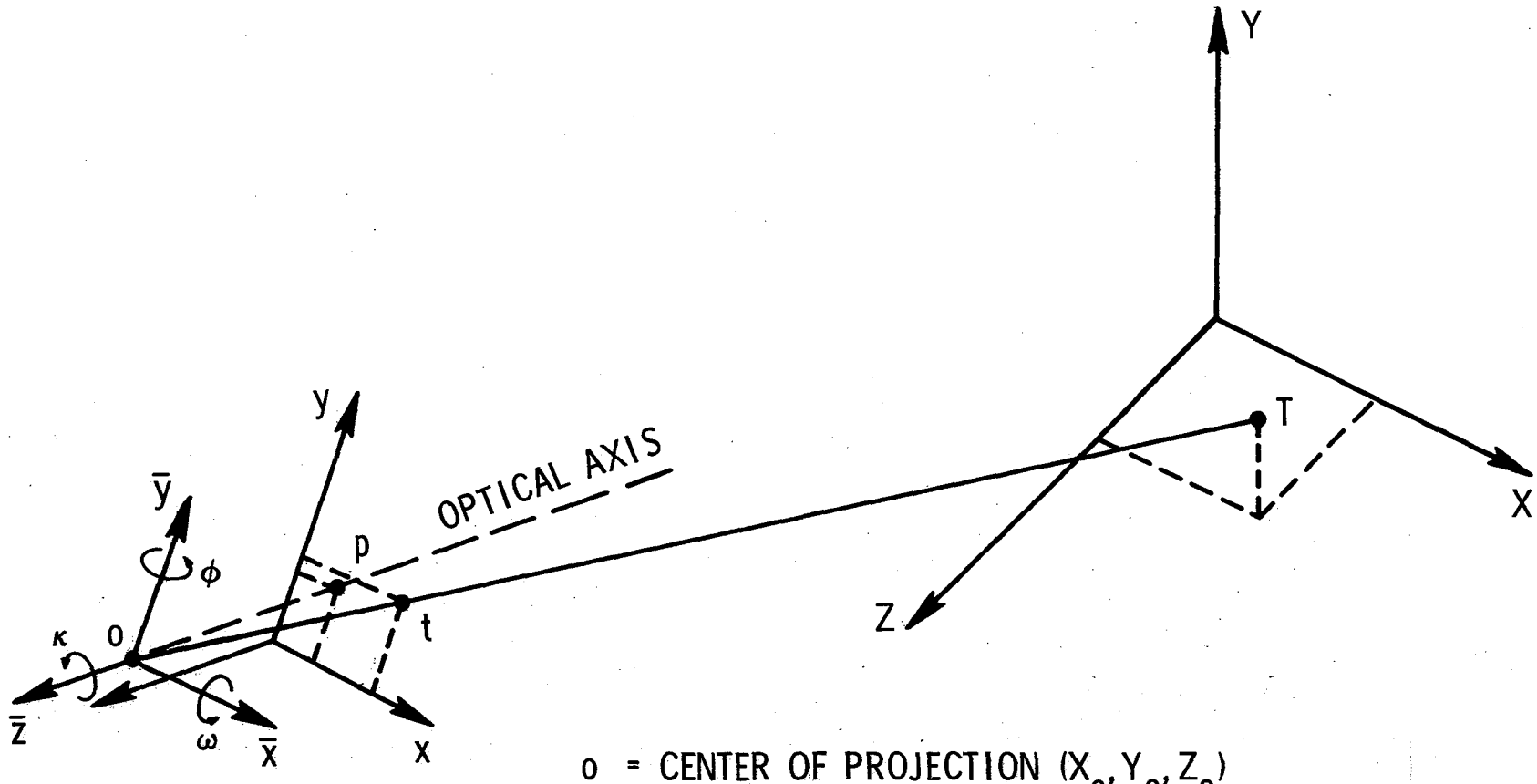

o = CENTER OF PROJECTION (X_0, Y_0, Z_0)

c = PRINCIPAL DISTANCE O - P

p = PRINCIPAL POINT (x_p, y_p)

ω, ϕ, κ = ANGLES OF ROTATION

T = TARGET OBJECT SPACE COORDINATES (X, Y, Z)

t = TARGET IMAGE COORDINATES (x, y)

Figure 1. - Photogrammetric projections.

These parameters are then used in a set of equations (collinearity equations) to relate a target's image location (x, y) to its corresponding three-dimensional spatial coordinates (X, Y, Z).

where

$$x + \Delta x = x_p - c \frac{A(X - X_0) + B(Y - Y_0) + C(Z - Z_0)}{D(X - X_0) + E(Y - Y_0) + F(Z - Z_0)} \tag{1}$$

$$y + \Delta y = y_p - c \frac{A'(X - X_0) + B'(Y - Y_0) + C'(Z - Z_0)}{D(X - X_0) + E(Y - Y_0) + F(Z - Z_0)} \tag{2}$$

and

x, y are target image location in camera coordinates

x_p, y_p are principal point (camera axis intersection on the image plane) coordinates

c is principal distance (distance from the center of projection to the image plane)

X, Y, Z are target spatial coordinates

X_0, Y_0, Z_0 are spatial coordinates of center of projection

A, B, C, A', B', C', D, E, F are parameters derived from image space relative to object space orientation (the elements of the rotation matrix which are functions of the three rotation angles: ω, ϕ, κ), and Δx, Δy are distortion correction terms.

These equations are based on two assumptions:

(a) Collinearity - any object point, its image point, and the center of projection lie on a straight line.

(b) All image points lie in a common plane.

Two computational techniques available for the determination of these parameters are the DLT (Direct Linear Transformation) and the Bundle algorithms. Both utilize photogrammetric triangulation, but their methods of obtaining and using the camera parameters are different.

DIRECT LINEAR TRANSFORMATION (DLT)

The DLT is an algorithm which was developed to perform a variation of the method of photogrammetric resection (determination of the nine camera parameters). While simplifying the computational process, it increases the number of unknowns from nine to eleven which are complex combinations of the nine original unknowns.

Each target point produces two equations (one from the x-image and one from the y-image location).

where

$$x = f(L_1, L_2, L_3, \ldots, L_{11}, X_n, Y_n, Z_n) \quad (3)$$

$$y = f(L_1, L_2, L3, \ldots, L_{11}, X_n, Y_n, Z_n) \quad (4)$$

and

x, y is the image location of target n

X_n, Y_n, Z_n are object space coordinates of target n, and

L_1 - L_{11} are DLT camera parameters.

Thus, if the image and the object space coordinates of six control points are known, 12 equations can be generated with only the 11 unknown camera parameters to be determined. A control point is a target which has its three-dimensional spatial coordinates known to some specified accuracy. These points should have good spatial distribution (cannot be coplaner). Generally, 10 to 20 control points are used to increase the number of equations and strengthen the least squares solution. While the same control points can be used (if seen) by each camera, the determination of the unknown camera parameters (L_1 - L_{11}) is performed independently. (If it is desired to make lens distortion corrections, then a greater number of control points would be needed to satisfy the additional unknowns introduced by the correction terms.) Once the camera parameters are determined for all cameras, a triangulation procedure is implemented incorporating these parameters and the image data to compute all target object point locations.

BUNDLE METHOD

This method determines camera parameters by employing data from all cameras simultaneously, and the nine actual camera parameters are determined rather than the 11 complex ones developed in the DLT. A major advantage of this technique is the ability to apply constraints to those camera parameters or any object space coordinates of targets which are exactly known or known to some degree. This feature is not available in the DLT. Least squares adjustments are used in both the resection and triangulation calculations in the program. The whole procedure is an iterative solution with sequential resections (determination of camera parameters) and triangulations (determination of target object space coordinates) until one of two results occur:

1. A prescribed number of iterations have transpired.
2. The adjustments have converged to within a prescribed value.

A minimum of two and one-third control points are required for this technique, but once again more can be used to increase the number of equations and strengthen the least squares solution. This technique requires that estimates be provided for the elements of interior orientation (x_p, y_p, and c) for each camera. They can

be obtained from calibrations of the cameras and be constrained to those values during the algorithm's adjustments. If they are not known through calibration, a reasonable estimate can be used and the algorithm can be allowed to adjust the parameters. Distortion correction coefficients can be determined by the algorithm by a process termed "bundle adjustment with self-calibration" (ref. 5).

TEST PROCEDURES

Camera Locations

An electro-optic camera system will be utilized in the NTF with up to four cameras. These cameras can be located at any of the six locations shown in figure 2. Since error determinations under minimum operating conditions were desired, only a two-camera operation was examined in this study (camera stations 3 and 4).

Figure 2 also shows these camera locations relative to an object-space coordinate system having its origin at the model and located midway between the cameras along the tunnel centerline.

Target Locations

The NTF wind tunnel test models will generally have poor spatial distribution of control point targets (targets which have known spatial coordinates and are not expected to move relative to each other during the measurements of other targets). For this reason, the control-point target locations in these tests were restricted to a nearly planar distribution (0.5 mm separation in the X direction) along the fuselage of the theoretical model geometry as shown in figures 2 and 3. No attempt was made to provide a realistic model geometry or deflections which were consistent with such geometry. The intent of the study was to test the ability to locate target positions. Target locations and deflection magnitudes are, however, typical of what could be expected in actual model tests. There were 11 control points used, and their locations are given as the first 11 entries of Table 1. Non-control target locations were selected along the wings to cover the area of overlap seen by the two cameras. Three chord locations on each wing were selected with three targets used to define each chord. Three sets of data were provided for each target:

1. Normal undeflected position ($X = 0$ mm)
2. Positive deflections with magnitudes proportional to outboard wing location ($0 \text{ mm} < X \leq 10 \text{ mm}$), and
3. Opposite or negative deflections ($-10 \text{ mm} \leq X < 0 \text{ mm}$).

Table I lists the target locations (11 control points and 54 displacement points, 2 wings . 3 span . 3 chord . 3 conditions). To provide some measure of the effect of the poor spatial distribution of control points, another set of data was obtained with the 11 control points having 50 mm separation in the X direction. The coordinates of these points are shown in Table 1 in parenthesis. A coordinate system, which differs from the standard tunnel definition, was used in these tests. A standard coordinate system has X as the tunnel centerline, and Z as the vertical axis.

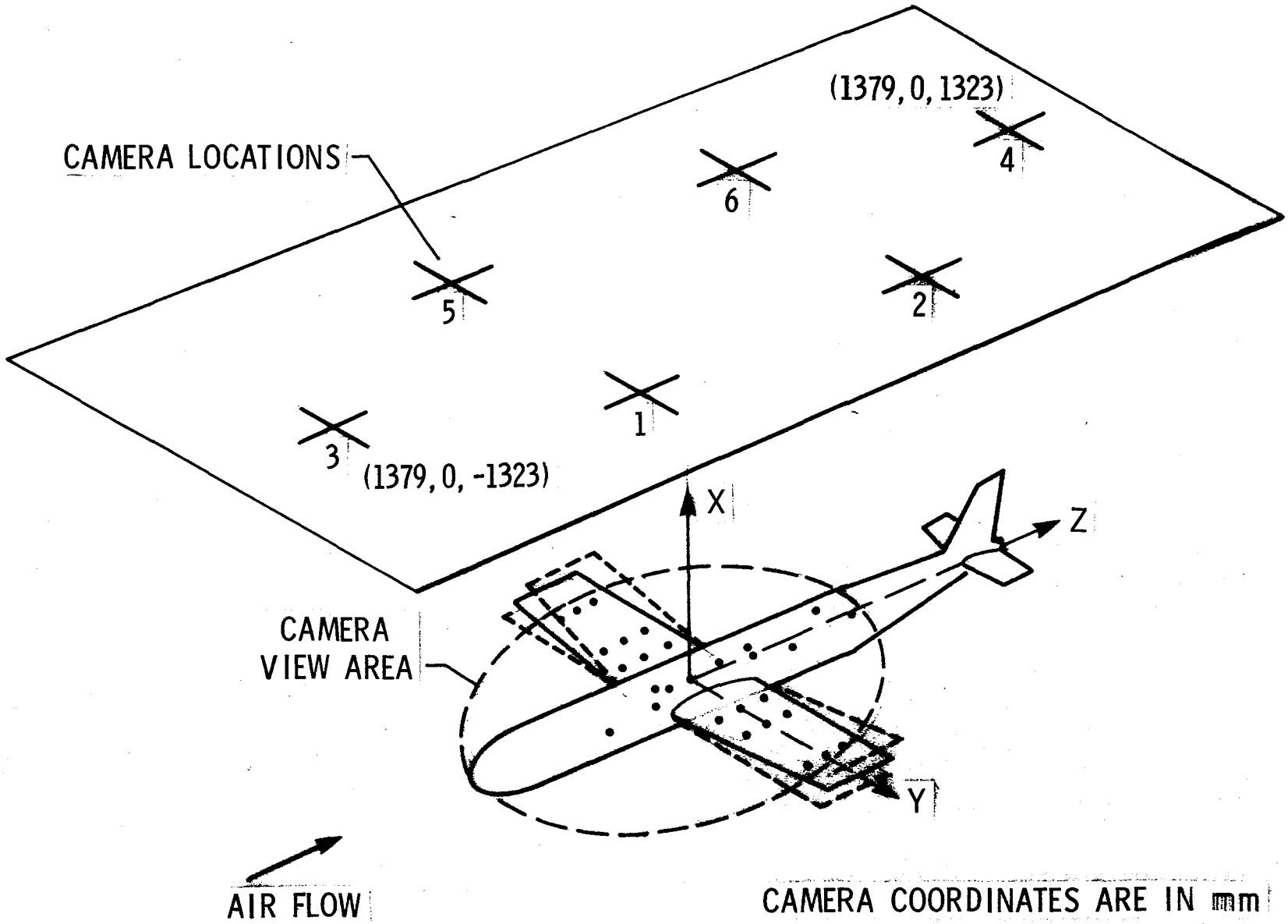

Figure 2. - Model-tracker locations.

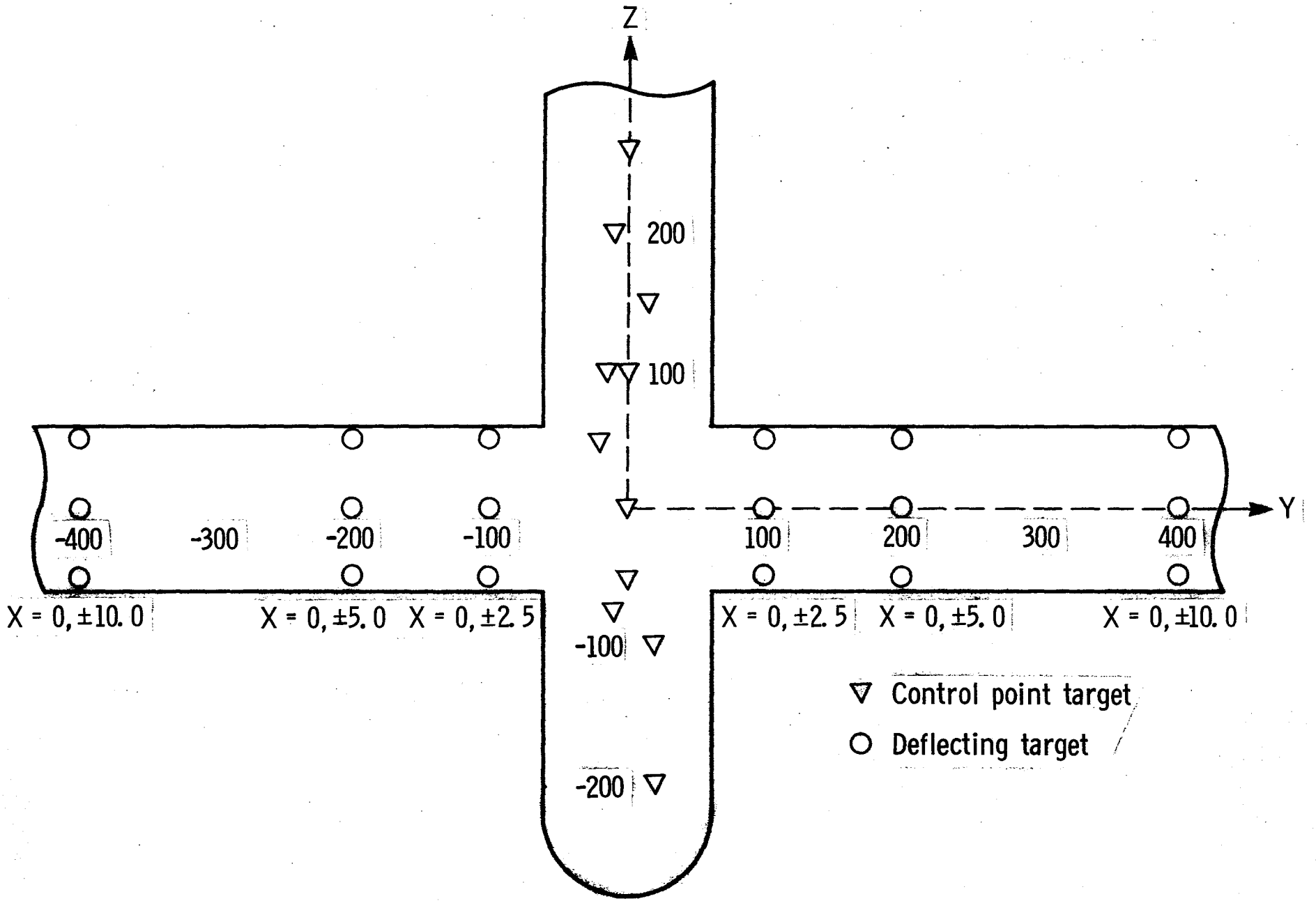

Figure 3. - Target locations (mm).

TARGET	X		Y	Z		TARGET	X	Y	Z
1 *	0.000	(50.000)	20.000	-200.000		33	-5.000	200.000	-50.000
2 *	0.000	(50.000)	0.000	-50.000		34	-5.000	200.000	0.000
3 *	0.000	(50.000)	-20.000	50.000		35	-5.000	200.000	50.000
4 *	0.000	(50.000)	-10.000	200.000		36	0.000	-200.000	-50.000
5 *	0.000	(50.000)	-10.000	-75.000		37	0.000	-200.000	0.000
6 *	0.000	(50.000)	0.000	250.000		38	0.000	-200.000	50.000
7 *	.500	(50.000)	15.000	150.000		39	0.000	200.000	-50.000
8 *	.500	(50.000)	-15.000	100.000		40	0.000	200.000	0.000
9 *	.500	(50.000)	0.000	0.000		41	0.000	200.000	50.000
10 *	.500	(50.000)	20.000	-100.000		42	5.000	-200.000	-50.000
11 *	.500	(50.000)	0.000	100.000		43	5.000	-200.000	0.000
12	-2.500		-100.000	-50.000		44	5.000	-200.000	50.000
13	-2.500		-100.000	0.000		45	5.000	200.000	-50.000
14	-2.500		-100.000	50.000		46	5.000	200.000	0.000
15	-2.500		100.000	-50.000		47	5.000	200.000	50.000
16	-2.500		100.000	0.000		48	-10.000	-400.000	-50.000
17	-2.500		100.000	50.000		49	-10.000	-400.000	0.000
18	0.000		-100.000	-50.000		50	-10.000	-400.000	50.000
19	0.000		-100.000	0.000		51	-10.000	400.000	-50.000
20	0.000		-100.000	50.000		52	-10.000	400.000	0.000
21	0.000		100.000	-50.000		53	-10.000	400.000	50.000
22	0.000		100.000	0.000		54	0.000	-400.000	-50.000
23	0.000		100.000	50.000		55	0.000	-400.000	0.000
24	2.500		-100.000	-50.000		56	0.000	-400.000	50.000
25	2.500		-100.000	0.000		57	0.000	400.000	-50.000
26	2.500		-100.000	50.000		58	0.000	400.000	0.000
27	2.500		100.000	-50.000		59	0.000	400.000	50.000
28	2.500		100.000	0.000		60	10.000	-400.000	-50.000
29	2.500		100.000	50.000		61	10.000	-400.000	0.000
30	-5.000		-200.000	-50.000		62	10.000	-400.000	50.000
31	-5.000		-200.000	0.000		63	10.000	400.000	-50.000
32	-5.000		-200.000	50.000		64	10.000	400.000	0.000
						65	10.000	400.000	50.000

* CONTROL POINTS

TABLE 1. TARGET LOCATIONS.

Image Data

Given the camera and target locations, the true image locations can be determined through geometric projections. Rays were traced from their target object space location through the camera's center of projection to find their image plane location. The camera parameters used were:

$c = 22.0$ mm

$x_p = y_p = 0.0$ mm

These are the nominal values of the cameras that will be used at the NTF. All the data used by the transformation algorithms were generated in this way. To simulate the fact that image data will have errors, a random error term was added to each x and y image value. These error values were generated having a Gaussian amplitude distribution, and the standard deviation magnitude was set to the level which was on the order of the camera system' target image location uncertainty ($\pm$0.001 mm). It was assumed that all systematic and distortion errors had been removed. It was also assumed that there were no errors in the control point object space coordinate values. However, the control point image values were generated with errors in the same manner as the other target points. This same image data were processed through the DLT and the Bundle. Ten separate sets of image data were processed for each control point configuration ($\Delta x = 0.5$ mm and $\Delta x = 50$ mm). Each set had different image errors, but the standard deviation of each error set was the same.

Data Processing

In processing the data with the DLT and Bundle algorithms only the object space coordinates of the control points were considered as known. All other parameters were allowed to adjust to those values determined by the algorithm. The Bundle algorithm, however, requires starting estimates for the camera parameters. The camera location and orientation parameters are not critical, but should be within 20 percent of their true values. The inner parameters are a little more sensitive, and starting values should be as good as possible. Calibrations should provide camera inner parameters, but a reasonable estimate should allow the algorithm to converge. The values used in this study were:

Function	True Value	Bundle Start Value
c	22.0 mm	22.1 mm
x_p	0.0 mm	0.1 mm
y_p	0.0 mm	0.1 mm

Test Results

With no error in the image data, both the DLT and the Bundle determined camera parameters and object space coordinates of targets with negligible errors; however, both the DLT and the Bundle methods produced errors in transformed object points when image errors were introduced. In the transformation of each set of targets there were two sources of error in the transformed data. First, the errors in the

image data for the control points caused errors in the determination of the camera parameters. The second source of error was that the target points being transformed also had image errors. The total error of transformation was due to the combination of the two. The average and standard deviation errors for each coordinate (X_n, Y_n, Z_n) at each non-control target ($n = 12 - 65$) were determined from the 10 data sets for the weak control point distribution ($\Delta X = 0.5$ mm). The same error determinations were made for the 10 data sets corresponding to the stronger control point distribution ($\Delta X = 50.0$ mm). The calculation for average error was

$$\left.\begin{matrix} X_{AVG_n} \\ Y_{AVG_n} \\ Z_{AVG_n} \end{matrix}\right\} = \sum^{10} (\tilde{c}_n - c_n)/10$$

and standard deviation was

$$\left.\begin{matrix} X_{SIG_n} \\ Y_{SIG_n} \\ Z_{SIG_n} \end{matrix}\right\} = \left[\frac{\sum^{10} (\tilde{c}_n - c_n)^2}{10 - 1}\right]^{1/2}$$

where

$\tilde{c}_n$ equals measured coordinate value

c_n equals true coordinate value

n equals target number

Table 2 lists the results of the 0.5 mm data sets, and Table 3 lists results of the 50.0 mm sets. Since the measurements of model wing deflections will be important at the NTF, the standard deviation errors for X were examined at each of three absolute wing locations ($|Y| = 100$ mm, 200 mm, 400 mm). There were 18 target values for each of these locations, and an average was taken of the X standard deviation errors. These averages are shown plotted in figures 4(a) and 4(b).

Computation Precision and Timing

The Bundle program was tested in two configurations using a minicomputer. One version used extended double precision variables having a mathematical precision of 16 significant digits. The other version used standard, single precision with six to seven digit precision. It required about eight times longer to do the extended double precision math. The 65 object points used in the previous test were processed in about 2 minutes using single precision, while the extended precision required about 16 minutes. There was no apparent significant accuracy penalty for using the faster single precision. Image data with errors were processed using both precisions and differences in transformed values were less

TARGET	TARGET COORD.			BUNDLE ERRORS						DLT ERRORS					
NO.	X	Y	Z	XAVG	YAVG	ZAVG	XSIG	YSIG	ZSIG	XAVG	YAVG	ZAVG	XSIG	YSIG	ZSIG
12	-2.500	-100.000	-50.000	-.023	-.014	-.203	.169	.164	.302	-.005	.084	.045	.458	.442	.339
13	-2.500	-100.000	0.000	-.023	-.018	-.149	.211	.177	.262	.079	.060	-.045	.372	.397	.317
14	-2.500	-100.000	50.000	.054	-.004	-.201	.199	.178	.282	.052	.063	-.050	.288	.394	.378
15	-2.500	100.000	-50.000	-.046	.012	.099	.173	.135	.206	.377	.041	-.273	.873	.547	1.075
16	-2.500	100.000	0.000	-.060	-.017	.090	.199	.154	.218	.345	-.008	-.343	.826	.636	.953
17	-2.500	100.000	50.000	-.022	-.044	.104	.198	.174	.185	.318	.009	-.347	.866	.619	.836
18	0.000	-100.000	-50.000	.037	-.017	-.112	.163	.161	.238	-.079	.011	.180	.447	.308	.504
19	0.000	-100.000	0.000	.009	-.012	-.147	.169	.189	.284	-.099	-.014	.097	.355	.293	.299
20	0.000	-100.000	50.000	-.004	-.060	-.121	.199	.217	.220	-.044	.006	.069	.287	.299	.167
21	0.000	100.000	-50.000	-.041	-.015	.167	.179	.148	.263	.213	.128	-.224	.495	.343	.504
22	0.000	100.000	0.000	.000	-.005	.087	.207	.155	.197	.123	.108	-.175	.452	.357	.388
23	0.000	100.000	50.000	-.025	-.011	.134	.170	.142	.264	.137	.105	-.181	.434	.399	.421
24	2.500	-100.000	-50.000	.029	-.049	-.175	.150	.177	.270	-.358	-.130	.208	.737	.471	.965
25	2.500	-100.000	0.000	.029	.002	-.128	.199	.196	.232	-.271	-.119	.222	.691	.480	.781
26	2.500	-100.000	50.000	.077	-.037	-.131	.209	.126	.231	-.266	-.126	.304	.618	.430	.669
27	2.500	100.000	-50.000	-.034	-.019	.122	.168	.209	.207	-.010	.209	-.173	.375	.393	.278
28	2.500	100.000	0.000	-.018	-.035	.112	.149	.157	.232	-.017	.178	-.088	.290	.383	.232
29	2.500	100.000	50.000	-.063	-.026	.093	.189	.200	.232	-.056	.217	-.057	.268	.422	.203
30	-5.000	-200.000	-50.000	-.002	-.110	-.367	.327	.350	.551	.015	.506	.173	.813	1.370	.582
31	-5.000	-200.000	0.000	.038	-.133	-.271	.391	.350	.470	.145	.484	.051	.713	1.336	.559
32	-5.000	-200.000	50.000	.095	-.156	-.245	.341	.359	.442	.205	.432	-.056	.618	1.270	.638
33	-5.000	200.000	-50.000	-.076	-.068	.233	.330	.376	.408	.736	-.057	-.496	1.607	2.073	2.128
34	-5.000	200.000	0.000	-.058	-.113	.256	.345	.321	.462	.703	-.042	-.575	1.575	2.157	1.783
35	-5.000	200.000	50.000	-.071	-.061	.176	.310	.340	.405	.555	-.034	-.654	1.570	2.268	1.615
36	0.000	-200.000	-50.000	.052	-.096	-.326	.332	.362	.486	-.308	.121	.358	.831	1.153	.921
37	0.000	-200.000	0.000	.086	-.149	-.300	.337	.366	.501	-.231	.108	.306	.697	1.169	.671
38	0.000	-200.000	50.000	.076	-.123	-.245	.305	.347	.443	-.142	.050	.246	.589	1.138	.511

TABLE 2. ERROR ANALYSIS FOR POOR CONTROL POINT DISTRIBUTION ($\Delta X = .5$ MM).

TARGET	TARGET COORD.			BUNDLE ERRORS						DLT ERRORS					
NO.	X	Y	Z	XAVG	YAVG	ZAVG	XSIG	YSIG	ZSIG	XAVG	YAVG	ZAVG	XSIG	YSIG	ZSIG
39	0.000	200.000	-50.000	-.042	-.119	.284	.315	.325	.450	.344	.330	-.395	.869	1.275	.986
40	0.000	200.000	0.000	-.003	-.100	.234	.311	.332	.460	.255	.376	-.318	.799	1.334	.862
41	0.000	200.000	50.000	-.063	-.088	.223	.362	.336	.418	.194	.331	-.323	.745	1.361	.649
42	5.000	-200.000	-50.000	.094	-.136	-.245	.277	.364	.438	-.644	-.331	.461	1.487	1.753	2.028
43	5.000	-200.000	0.000	.028	-.120	-.315	.310	.330	.471	-.655	-.336	.453	1.434	1.793	1.662
44	5.000	-200.000	50.000	.054	-.116	-.243	.375	.373	.425	-.545	-.395	.541	1.380	1.849	1.366
45	5.000	200.000	-50.000	-.055	-.111	.241	.299	.327	.426	-.033	.751	-.304	.773	1.406	.627
46	5.000	200.000	0.000	-.058	-.119	.265	.334	.325	.479	-.016	.711	-.161	.705	1.344	.468
47	5.000	200.000	50.000	-.049	-.085	.210	.309	.372	.425	-.111	.726	.019	.581	1.383	.531
48	-10.000	-400.000	-50.000	.082	-.429	-.602	.711	.829	.956	.131	2.325	.472	1.610	5.248	1.189
49	-10.000	-400.000	0.000	.067	-.465	-.534	.591	.818	.924	.258	2.172	.115	1.383	5.091	.980
50	-10.000	-400.000	50.000	.083	-.428	-.467	.641	.835	.875	.389	2.035	-.184	1.231	4.946	1.166
51	-10.000	400.000	-50.000	-.166	-.392	.559	.595	.777	.956	1.518	-.365	-.989	3.142	7.932	4.174
52	-10.000	400.000	0.000	-.124	-.419	.460	.614	.749	.859	1.292	-.384	-1.110	3.040	8.206	3.557
53	-10.000	400.000	50.000	-.070	-.357	.477	.606	.711	.881	1.175	-.379	-1.192	3.047	8.460	3.069
54	0.000	-400.000	-50.000	.098	-.452	-.554	.658	.790	.951	-.558	.698	.721	1.646	4.687	1.878
55	0.000	-400.000	0.000	.162	-.460	-.504	.634	.826	.887	-.412	.587	.649	1.444	4.683	1.405
56	0.000	-400.000	50.000	.143	-.511	-.482	.606	.889	.877	-.311	.494	.523	1.252	4.629	1.074
57	0.000	400.000	-50.000	-.176	-.428	.513	.677	.731	.897	.587	1.188	-.871	1.743	5.035	2.037
58	0.000	400.000	0.000	-.109	-.377	.516	.627	.735	.884	.503	1.127	-.700	1.597	5.092	1.541
59	0.000	400.000	50.000	-.152	-.348	.492	.625	.683	.862	.370	1.117	-.632	1.477	5.188	1.263
60	10.000	-400.000	-50.000	.083	-.469	-.595	.612	.861	.997	-1.408	-1.042	.832	3.039	7.248	4.080
61	10.000	-400.000	0.000	.094	-.480	-.581	.608	.835	.956	-1.277	-1.113	.985	2.887	7.385	3.422
62	10.000	-400.000	50.000	.222	-.466	-.534	.655	.845	.871	-1.132	-1.191	1.259	2.768	7.515	2.991
63	10.000	400.000	-50.000	-.131	-.381	.567	.585	.723	.921	.015	2.887	-.624	1.622	5.512	1.280
64	10.000	400.000	0.000	-.098	-.377	.479	.628	.729	.898	-.105	2.775	-.265	1.397	5.325	.898
65	10.000	400.000	50.000	-.090	-.332	.415	.649	.706	.856	-.236	2.686	.067	1.258	5.234	1.081

TABLE 2.(CONT.) ERROR ANALYSIS FOR POOR CONTROL POINT DISTRIBUTION (ΔX=.5 MM).

TARGET	TARGET COORD.			BUNDLE ERRORS						DLT ERRORS					
NO.	X	Y	Z	XAVG	YAVG	ZAVG	XSIG	YSIG	ZSIG	XAVG	YAVG	ZAVG	XSIG	YSIG	ZSIG
12	-2.500	-100.000	-50.000	-.017	.003	-.092	.094	.102	.161	-.283	-.070	.021	.386	.268	.569
13	-2.500	-100.000	0.000	-.018	-.006	-.047	.094	.087	.144	-.181	-.060	-.006	.251	.261	.473
14	-2.500	-100.000	50.000	.057	.002	-.103	.118	.091	.187	-.173	-.010	.061	.244	.236	.359
15	-2.500	100.000	-50.000	-.039	-.011	-.010	.156	.112	.120	.354	.085	-.030	.545	.307	.679
16	-2.500	100.000	0.000	-.054	-.036	-.026	.181	.063	.137	.315	.022	-.055	.456	.303	.616
17	-2.500	100.000	50.000	-.017	-.058	-.016	.150	.097	.100	.257	.012	-.043	.418	.270	.595
18	0.000	-100.000	-50.000	.042	.002	-.000	.112	.081	.171	-.322	-.035	-.045	.436	.248	.633
19	0.000	-100.000	0.000	.013	-.002	-.045	.074	.072	.148	-.178	-.021	.031	.275	.245	.475
20	0.000	-100.000	50.000	-.002	-.054	-.023	.104	.120	.119	-.126	-.022	-.024	.250	.177	.327
21	0.000	100.000	-50.000	-.035	-.039	.059	.087	.097	.093	.375	.043	-.073	.522	.269	.668
22	0.000	100.000	0.000	.005	-.024	-.030	.126	.115	.110	.283	.047	-.022	.421	.302	.596
23	0.000	100.000	50.000	-.021	-.025	.014	.168	.069	.078	.292	.013	-.043	.421	.279	.609
24	2.500	-100.000	-50.000	.033	-.032	-.063	.163	.092	.138	-.314	-.099	.026	.434	.270	.605
25	2.500	-100.000	0.000	.032	.014	-.026	.079	.106	.116	-.203	-.049	.012	.281	.247	.466
26	2.500	-100.000	50.000	.079	-.031	-.034	.107	.092	.137	-.168	-.046	.031	.249	.212	.313
27	2.500	100.000	-50.000	-.029	-.043	.014	.156	.113	.115	.392	.023	-.005	.562	.319	.622
28	2.500	100.000	0.000	-.013	-.054	-.005	.129	.094	.146	.317	.005	-.035	.448	.285	.586
29	2.500	100.000	50.000	-.060	-.041	-.027	.165	.130	.120	.260	.030	-.065	.408	.313	.556
30	-5.000	-200.000	-50.000	.004	-.070	-.147	.125	.195	.277	-.655	-.084	.007	.855	.865	1.246
31	-5.000	-200.000	0.000	.043	-.103	-.061	.116	.155	.141	-.470	-.101	.007	.676	.808	1.036
32	-5.000	-200.000	50.000	.097	-.137	-.039	.155	.212	.203	-.348	-.114	.031	.496	.772	.874
33	-5.000	200.000	-50.000	-.068	-.108	.015	.196	.176	.157	.717	.066	-.011	.986	.943	1.288
34	-5.000	200.000	0.000	-.051	-.143	.031	.220	.193	.148	.616	.023	-.045	.839	.947	1.124
35	-5.000	200.000	50.000	-.065	-.080	-.052	.207	.153	.152	.456	.044	-.029	.706	1.003	1.059
36	0.000	-200.000	-50.000	.056	-.055	-.104	.145	.165	.199	-.683	-.112	.001	.918	.842	1.165
37	0.000	-200.000	0.000	.089	-.118	-.089	.141	.181	.250	-.494	-.110	.071	.637	.799	.967
38	0.000	-200.000	50.000	.078	-.103	-.038	.131	.216	.211	-.358	-.097	.022	.516	.818	.811

TABLE 3. ERROR ANALYSIS FOR GOOD CONTROL POINT DISTRIBUTION (ΔX=50 MM).

TARGET	TARGET COORD.			BUNDLE ERRORS						DLT ERRORS					
NO.	X	Y	Z	XAVG	YAVG	ZAVG	XSIG	YSIG	ZSIG	XAVG	YAVG	ZAVG	XSIG	YSIG	ZSIG
39	0.000	200.000	-50.000	-.036	-.159	.065	.188	.199	.164	.720	.012	-.031	.979	.930	1.217
40	0.000	200.000	0.000	.003	-.131	.008	.253	.174	.152	.599	.059	.015	.819	.919	1.150
41	0.000	200.000	50.000	-.058	-.108	-.005	.182	.142	.154	.437	.028	-.093	.667	.987	1.038
42	5.000	-200.000	-50.000	.095	-.094	-.023	.146	.175	.152	-.637	-.120	.028	.914	.796	1.172
43	5.000	-200.000	0.000	.029	-.089	-.103	.078	.175	.184	-.503	-.114	.040	.696	.840	.988
44	5.000	-200.000	50.000	.054	-.096	-.036	.150	.162	.189	-.302	-.113	.072	.495	.820	.814
45	5.000	200.000	-50.000	-.049	-.152	.022	.174	.204	.177	.681	.045	-.042	.966	.930	1.194
46	5.000	200.000	0.000	-.053	-.151	.038	.154	.192	.152	.605	.028	-.077	.843	.919	1.100
47	5.000	200.000	50.000	-.044	-.107	-.020	.197	.157	.163	.400	.051	-.102	.647	.994	1.034
48	-10.000	-400.000	-50.000	.088	-.336	-.165	.213	.460	.377	-1.324	-.253	-.021	1.793	3.263	2.390
49	-10.000	-400.000	0.000	.072	-.392	-.107	.213	.488	.359	-1.013	-.244	-.016	1.366	3.210	2.119
50	-10.000	-400.000	50.000	.086	-.376	-.045	.253	.490	.345	-.711	-.194	-.007	1.042	3.165	1.811
51	-10.000	400.000	-50.000	-.155	-.455	.123	.343	.544	.326	1.423	.052	-.076	1.959	3.521	2.552
52	-10.000	400.000	0.000	-.114	-.462	.018	.304	.535	.242	1.139	.027	-.045	1.597	3.533	2.214
53	-10.000	400.000	50.000	-.062	-.379	.035	.263	.444	.334	.899	.039	-.035	1.291	3.561	2.097
54	0.000	-400.000	-50.000	.101	-.355	-.113	.248	.440	.369	-1.315	-.243	-.003	1.759	3.240	2.407
55	0.000	-400.000	0.000	.164	-.384	-.074	.274	.468	.357	-1.039	-.251	.065	1.380	3.218	2.010
56	0.000	-400.000	50.000	.143	-.456	-.057	.258	.539	.353	-.754	-.234	.076	1.098	3.200	1.769
57	0.000	400.000	-50.000	-.167	-.494	.074	.355	.577	.305	1.411	.036	-.082	1.945	3.501	2.469
58	0.000	400.000	0.000	-.102	-.423	.071	.272	.486	.299	1.130	.039	-.127	1.542	3.523	2.228
59	0.000	400.000	50.000	-.145	-.374	.046	.374	.439	.267	.917	.048	-.130	1.377	3.567	1.985
60	10.000	-400.000	-50.000	.081	-.369	-.150	.210	.482	.376	-1.319	-.272	.069	1.801	3.271	2.401
61	10.000	-400.000	0.000	.092	-.401	-.148	.219	.499	.341	-1.029	-.254	.140	1.379	3.258	2.035
62	10.000	-400.000	50.000	.219	-.408	-.106	.305	.478	.297	-.862	-.236	.152	1.216	3.243	1.705
63	10.000	400.000	-50.000	-.125	-.450	.125	.316	.519	.365	1.382	.051	-.139	1.902	3.494	2.402
64	10.000	400.000	0.000	-.093	-.427	.031	.345	.505	.310	1.114	.085	-.113	1.509	3.523	2.091
65	10.000	400.000	50.000	-.085	-.363	-.034	.303	.417	.271	.824	.066	-.106	1.256	3.534	1.868

TABLE 3.(CONT.) ERROR ANALYSIS FOR GOOD CONTROL POINT DISTRIBUTION (ΔX=50 MM).

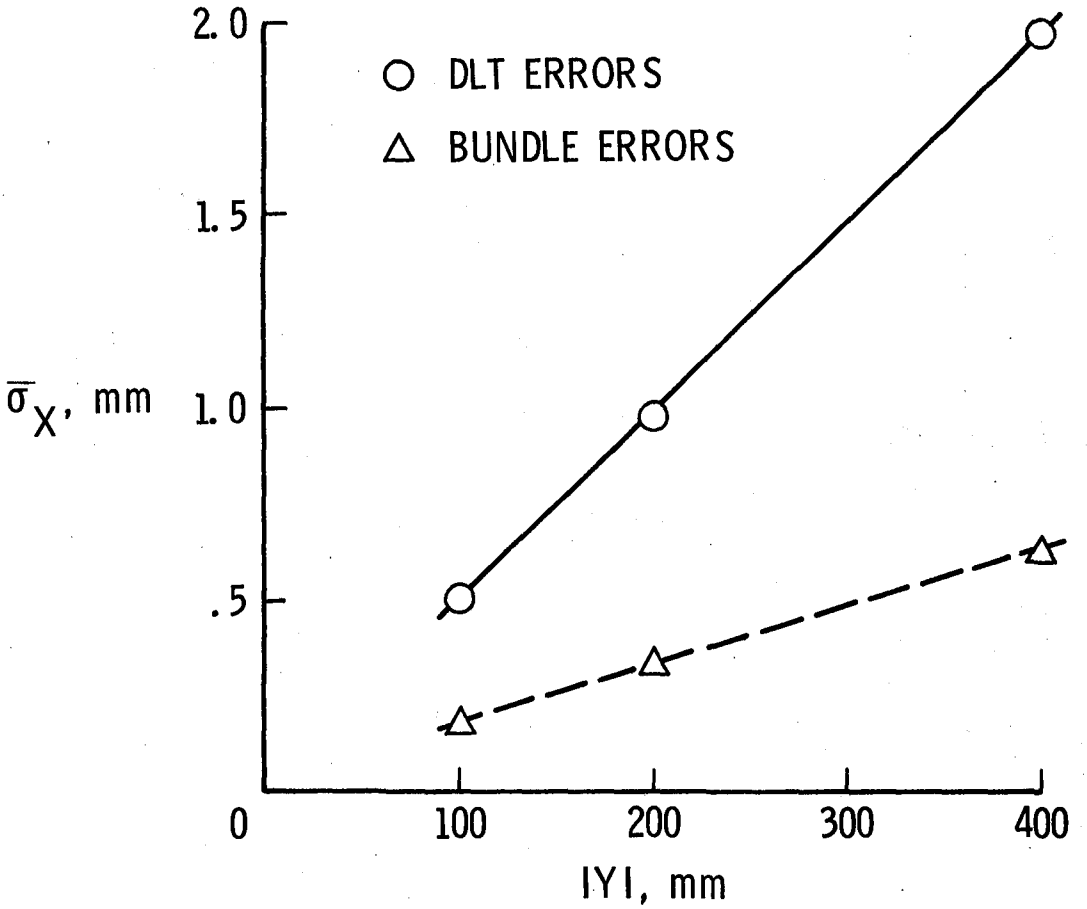

Figure 4a. - Wing deflection errors (control point ΔX = 0.5 mm).

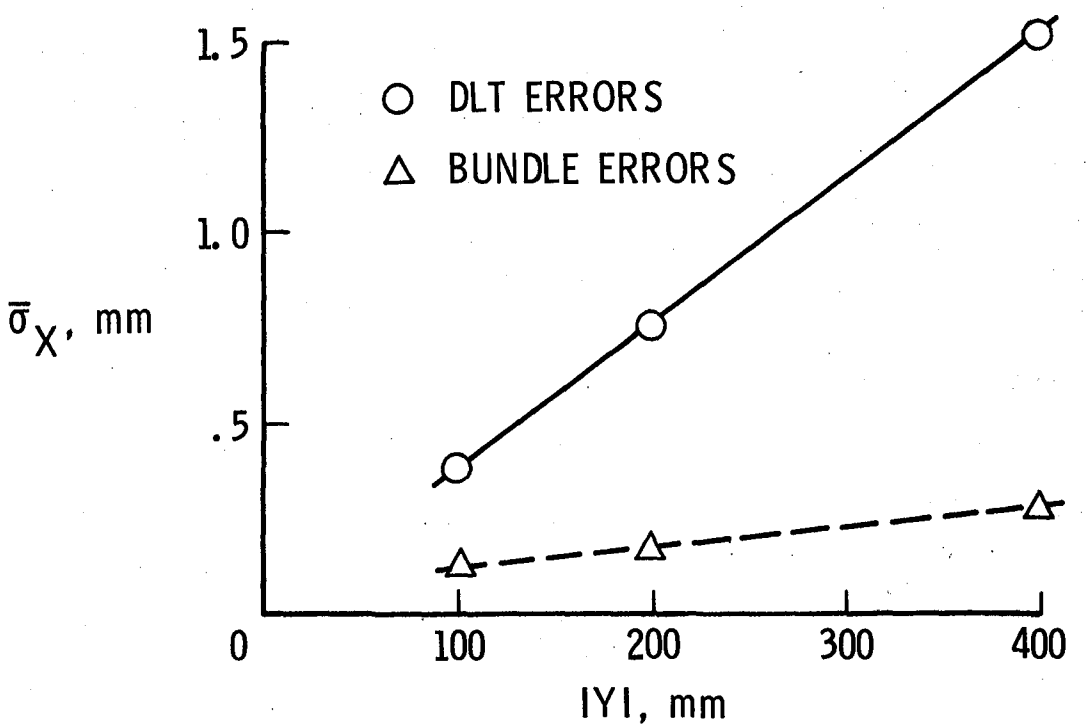

Figure 4b. - Wing deflection errors (control point ΔX = 50 mm).

than 0.01 mm. All error comparisons with DLT were determined with Bundle data that were computed using single precision.

The DLT program used double precision variables with mathematical precision of 11 significant digits in that portion of the program which performed the matrix operations used in the least squares solution. All other parts of the program used single precision variables (six to seven digit precision). This resulted in a process time of about 1 minute for the 65 targets.

CONCLUSIONS

This study has shown that when target-camera geometry is defined with the NTF configuration, the Bundle transformation technique can be expected to produce smaller model deformation errors than those obtained using the DLT. The Bundle needs a minimum of two and one-third control points; whereas, the DLT needs six non-coplaner points. There were 11 control points used in this study in two configurations. The first case represented a "worst case" configuration where control point distribution would be poor (nearly coplaner), while the second case utilized control points which were in a much better spatial distribution. In both cases the Bundle errors were less than DLT errors. The Bundle technique is somewhat slower than the DLT due to its iterative processes, but these tests have shown that quick-look answers can be obtained using minicomputer single precision computation with little loss in accuracy.

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15. Supplementary Notes

16. Abstract

A comparison was made of two photogrammetric measurement techniques which can be used for the determination of the spatial locations of targets. One technique is known as the "Direct Linear Transformation (DLT)," and the other is referred to as the "Bundle" method. Each technique utilizes triangulation of image data from two or more cameras, but they differ in their method of determining camera parameters. While the study was performed with simulated data, the geometries, image data accuracies and other parameters were set to simulate conditions for a two-camera, photogrammetric system which will be used at the National Transonic Facility at the NASA's Langley Research Center.

For the conditions studied the "Bundle" technique was found to have smaller errors. Both methods were sensitive to the spatial distribution of control-point targets, but this effected the accuracy of the "Bundle" algorithm less than that of the "DLT" algorithm.

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