PATTERN RECOGNITION FOR SPACE APPLICATIONS
CENTER DIRECTOR'S DISCRETIONARY FUND
FINAL REPORT

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**ABSTRACT**

Results and conclusions are presented on the application of recent developments in pattern recognition to spacecraft star mapping systems. Sensor data for two representative starfields were processed by an adaptive shape-seeking version of the Fc-V algorithm with good results. Also, some newly proposed cluster validity measures were evaluated, but not found especially useful to this application. Recommendations are given for two system configurations worthy of additional study.
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INTRODUCTION

A study was made in 1977 by Gunderson, NASA summer fellow, and the results were presented at the IFAC World Congress in Helsinki, Finland, in 1978 [1]. The purpose of that investigation was to evaluate the feasibility of using a so-called fuzzy clustering algorithm (the fuzzy ISODATA algorithm developed by Bezdek [2]) to recognize apparent star patterns in the data provided by means of a spacecraft-mounted star-mapping device. Several NASA spacecraft have been equipped with star-sensing devices for use in experiments requiring a pointing reference. This function was accomplished by locking onto one or two bright stars in the field of view. The premise leading to this study was that such a reference might be more reliably obtained by recognizing a group or cluster of stars. Previously known attempts to apply clustering algorithms for recognizing apparent star groups have not been successful. However, the "fuzzy" clustering approach now beginning to appear in the scientific literature might provide a new and more promising alternative.

The scope of the initial investigation was limited; however, the results were very encouraging. It only considered the one clustering problem presented by the fifty-one bright stars about Polaris. A hierarchical implementation of Bezdek's algorithm was used along with a cluster splitting criteria based upon a specialized cluster validity measure. In particular, it did appear that an important factor in the success achieved was due to the tendency of the fuzzy ISODATA algorithms to eliminate or "tune out" noisy and ambiguous data.

The conclusions reached by the end of the investigation suggested the following candidates for follow-on studies:

1) The fuzzy ISODATA algorithms failed to recognize clusters consisting of a mixture of different shapes. Progress was needed in developing a family of clustering algorithms that could reliably seek out and identify these shapes.

2) Cluster validity measures that were designed to assess relative cluster quality from fuzzy membership matrices (Reference 2 gives a summary) tended to be biased toward selection of multiple clusters. Progress was needed in either developing more effective measures or a better understanding of existing ones.

Also, a much more comprehensive study would be required before recommendations could be made regarding use of specific star-clustering systems and their performance capabilities for spacecraft attitude determination.

As a result of the initial study, a family of algorithms was developed and reported in References 3 and 4 which were capable of detecting clusters other than round shapes. These results were later extended in References 5 and 6 to an
adaptive family of algorithms which could seek out patterns involving a mixture of cluster shapes. Therefore, this seemed to be a prudent time to re-open the star-sensor study and evaluate the effectiveness of these new developments in that application. The results and conclusions of this new project are presented in this paper.

The first star tracking devices were designed for pointing and control of telescopes for large earth-bound observatories [7]. Applications were also found in aircraft and early missile systems where star directions were used to obtain information for determining true heading. Star tracking devices have been, or are currently being readied for use on such NASA projects as Space Station, Spacelab, High Energy Astronomy Observatories, Skylab, Space Shuttle and several of its experiments. These projects use information from star-trackers for vehicle guidance and control systems and in instrument pointing and control systems. Application of the results of this study, i.e., using star clusters to establish the location of reference stars, can easily be seen in all of these systems.

NASA projects to date have employed star trackers of the image dissector type, such devices providing good sensitivity for tracking single stars. Charge coupled devices (CCD star trackers) have been found to be superior in multiple star tracking or mapping applications and would be the type for use in systems considered in this study. Implied design requirements are not out of line with more advanced CCD's currently available or those that could reasonably be expected in the near future. For example, the current state-of-the-art for CCD's is a matrix of sensing elements of about 200 x 200 elements. Keeping in mind that extreme accuracy would not be required for clustering the viewed starfield, such an array should permit a field of view that would be adequate for purposes necessary in this study.

A SHAPE-SEEKING FAMILY OF FcV CLUSTERING ALGORITHMS

Only a brief summary of the Fc-V algorithms will be given here. A more general discussion can be found in Reference 11, and details in References 2 and 6. Let \( x = (x_1, x_2, \ldots, x_n) \) in \( r_s \) represent a data set consisting of the measurements of \( s \) features on each of \( n \) samples. Also let the set of functions \( u_1, u_2, \ldots, u_n \) each defined for \( u_i: x \to [0,1] \) and the constraint \( \sum u_i(x_k) = 1 \) for each \( x_k \in X \). Membership functions \( u_i \) \( (i = 1, 2, \ldots, c) \) define a fuzzy clustering of \( X \) into \( c > 2 \) clusters. The membership values \( u_{ik} = u_i(x_k) \) define for each \( i \) and \( k \) the elements of a real, \( c \times n \), membership matrix \( U \). Assume a principal component model, for each cluster \( c \), of dimension \( r_i \), that is defined by the linear variety

\[
V_i(v_i,d_{ij}) = \left( y \in R^2 \mid y = v_i + \sum_{j=1}^{n} t_j d_{ij}, t_j \in R \right)
\]

and let the distance of a sample vector \( x_k \) to the linear variety \( V_i \) be defined by
\[ D_{ik} = \left[ |x_k - v_i|^2 / 2 - \alpha \sum_{j=1}^{r} (x_k - v_i, d_{ij})^2 / A \right]^{1/2} \]  

(2)

where \( 0 < \alpha < 1 \) and

\[ (x,y)_A = x^T Ay \]  

(3)

for some positive definite matrix \( A \). If \( A = I \), the identity matrix, and \( \alpha = 1 \), then equation (2) is just the orthogonal euclidean distance to \( V_i \). By setting \( \alpha \) at a fixed value \( 0 < \alpha < 1 \), it is possible to discriminate against samples which lie close to the linear variety, but are relatively distant from the center, \( v_i \). The distance, equation (2), is used in the following equations, which define an Fc-V (fuzzy c-varieties) clustering of the data set \( X \). It is assumed that \( D_{ij} \neq 0 \) for any \( i,k \) and that the parameters \( \alpha, m \) and \( c \) are fixed at values \( 0 < \alpha < 1, 1 < m \) and \( c \geq 2 \):

\[ u_{ik} = \frac{1}{\sum_{j=1}^{c} (D_{ik}/D_{ij})^{1/m-1}} \]  

(4a)

\[ v_i = \frac{\sum_{k=1}^{n} (u_{ik})^m (x_k)}{\sum_{k=1}^{n} (u_{ik})^m} \]  

(4b)

and where the vector \( d_{ij} \) in equation (2) is the \( j \)-th unit eigenvector corresponding to the \( j \)-th largest eigenvalue of the matrix \( S_i A \), where \( S_i \) is the weighted (fuzzy) within-cluster scatter matrix;

\[ S_i = \sum_{k=1}^{n} (u_{ik})^m (x_k - v_i) (x_k - v_i)^T \]  

(4c)

The singular case \( D_{ij} = 0 \) occurs only rarely, but it is easily taken care of by a slight modification of equation (4a) (cf. [4]). The case \( \alpha = 1, D_{ij} = 0 \) happens only if the sample \( x_k \) lies directly on the linear variety, \( v_i \), defining the principle component model for that cluster. Thus, the Fc-V algorithm will assign a membership value of unity to \( x_k \) for that cluster, and zero for the remaining clusters. Therefore, if a data vector \( x_k \) has a membership value close to unity for a particular
cluster, it can be thought of as correspondingly similar to the prototypical vectors which define the linear variety. If $\alpha < 1$, the singular case occurs only if $x_k = v_i$ for some $k$ and some $i$. Prototypical vectors lying on the linear variety defining the principal component model, but not at the center, will have non-zero values and the maximum values are taken on by all of the vectors lying at an equal distance $D_{ij}$ from the center $v_i$. Equations (4a) and (4b) were shown in Reference 4 to be necessary conditions for minimizing the sum-of-squared objective functional

$$J(U,V_1,V_2,\ldots,V_c) = \sum_{i=1}^{c} \sum_{k=1}^{n} (U_{ik})^m (D_{ik})^2$$

over all families of the linear varieties $V_1, V_2, \ldots, V_c$ of a common dimension $r = r_i$ and all possible membership matrices $U = (u_{ik})$. The Fc-V algorithms were initiated by specifying three conditions: (1) the desired number of clusters $c$; (2) setting parameters, $m$ and $r$; and (3) selecting either a starting matrix $U_0$ or a starting family of cluster centers $(v_1, v_2, \ldots, v_c)$. The algorithms then loop through equations (4a) and (4b) until a prescribed stopping condition is met. This procedure may be followed without requiring the dimension $r_i$ to be the same for each of the clusters.

It is this shape-seeking version of the Fc-V algorithms which are of interest for this application. It was shown in Reference 5 that conditions of (4b) could be interpreted as saying that the linear varieties of fixed dimensions $0 < r < s-1$ which minimize the functional, equation (5), are the ones on which there is maximum scatter. The direction vectors $d_{ij}$ $(j = 1, 2, \ldots, r_i)$ defining the maximizing linear varieties are just the principal components of the scatter matrices, $S_i$, pre-multiplied by the norm defining matrix $A$, i.e., the eigenvectors of $AS_i$, which are independent of the dimension $r_i$. Since the directions of maximum scatter are independent of the dimensions of the disjoint linear varieties and are computed after computation of the disjoint cluster centers $v_i$ and $v_j$, it was shown in Reference 6 that the choice of dimension $r_i$ for the cluster defined by the center $v_j$ could be made independently of that center $v_i$ provided $j \neq i$. The eigenvalues of the $i$-th fuzzy within-cluster scatter matrix may be used to provide "shape" information for selecting a value $r_i$ for the next iteration. Therefore, the algorithm can be made to adapt to the structure of the data it encounters.

The shape seeking algorithm used is as follows:

I. Initialize by

i) Fixing $2 \leq c < n$ and $1 < m < \infty$

ii) Selecting "shaping coefficients" $a_k, \in (0,1)$ for each $k = 1, 2, \ldots, s-1$
iii) Selecting a starting membership matrix $U_0$

II.

i) Compute cluster centers $v_i$ from (4b) $i = 1, 2, \ldots, c$

ii) Compute the within-cluster scatter matrices $S_i$ from (4c) for each $i = 1, 2, \ldots, c$

iii) Compute the eigenvalues ordered smallest to largest and the corresponding eigenvectors for all $i = 1, 2, \ldots, c$

iv) If a least integer exists for $k = 1, 2, \ldots, s-1$ such that

$$\lambda_{i,k+1} / \lambda_{ik} < \sigma_k$$

then set $r_i = k$; otherwise set $r_i = 0$

III. Update the membership matrix $U$ using (4a) with cluster-dependent distances

$$D_{ik} = D(x_k, v_i) = (|x_k - v_i|^2 - \sum_{j=1}^{r} (<x_k - v_i, d_{ij}>)^2)^{1/2}$$

IV. If stopping criteria not met then return to II.

Some experimentation was required before selecting values for the shaping coefficients of the preceding algorithm. The rule followed was to set all sigma to fixed value between 0.1 and 0.2 and equal, that choice resulted in good performance. Roughly speaking, the algorithm is forced to seek out "fatter" cluster configurations by raising the values of the shaping coefficients. If the data is of distinctly separated clusters then the membership feature is not particularly essential. The value of fuzzy clustering algorithms becomes useful when this is not the case, especially in the case of a shape seeking algorithm and where sensitivity to noisy or ambiguous data could be disastrous. However, a kind of "indecisiveness" appears that results in an excessive number of iterations funding a final configuration when too great an extent of membership sharing is allowed. The value of $m$ must be set to a number smaller than $m = 2$ which is most effective for non-shape seeking algorithms.

CLUSTER VALIDITY

The membership matrices, which are outputs of the Fc-V algorithm, provide a measure of the extent to which individual samples are similar to the prototypical samples which make up the disjoint principal component models of the cluster. It appears that the closer the final membership matrix is to having all 0's and 1's, the better the corresponding cluster configuration identifies the structure in the data. This conclusion may be arguable, but the validity measures which proceed on that assumption have obtained a degree of popularity in the scientific literature, so they should at least be evaluated. Two such measures were studied in the initial investigation and found to be unsatisfactory. Both measures, partition coefficient and
classification entropy, Bezdek [2], showed a definite functional dependency upon the number of clusters that was independent of any structure encountered in the data. Windham proposed two other measures, proportion exponent and the uniform data function, References 7 and 8 respectively. The uniform data function has only been developed for detecting uniformly round and compact clusters; therefore, applying it to mixed shapes encountered in this study would be inappropriate. The proportion exponent is defined on the columns of the membership matrix by finding the maximum element in each column, greatest integer of the maximum's inverse, and then the proportion exponent is defined. The proportion exponent ends up showing the same unwanted functional dependency upon the number of clusters as did the measures proposed by Bezdek; thus, it can not be used in this investigation either.

RESULTS

The objectives of this study were as follows:

1) Does a structure exist in starfields that would allow a reference to be determined that could be used by a spacecraft?

2) How critical is a cluster validity measure to this application and how effective are some recently proposed clustering measures?

3) How effective are the Fc-V algorithms in using clusters found in starfields?

STARFIELDS

A visual inspection of a good star atlas will show that there is an apparent structure in starfields so a reference may be established given a good pattern recognition algorithm. Figure 1 was constructed from sector charts taken from the Smithsonian star atlas of reference stars [10]. The star mapper was assumed to have a field of view of 18 x 18 deg; thus, the angle of declination for the stars shown range from 9 to 27 deg and the right ascension from 3 hr 00 min to 4 hr 12 min (45 to 63 deg). The stars shown are part of the constellation Taurus in the northern sky (sectors 50 and 51 in Figure 2). Figure 3 is a chart constructed from the same atlas and shows stars from the constellation Scorpius in the southern sky (sectors 119 and 120 in Figure 4). These two groups were more or less casually chosen, but are representative of sectors which would exhibit the kind of cluster structure expected to be seen by a star mapper. That is, both contain open or chain-like clusters, also, closed or ball like clusters of the type sufficiently distinct to be detected by a clustering algorithm providing it could avoid the distraction presented by the remaining spurious stars.

Figure 1 seems to have a tight, round, cluster of medium brightness stars in the upper right center about one-third of the way down from the top. There is also a looser round cluster in the vicinity of the bright star at middle left and an open cluster of medium bright stars at the lower right. Thus, there appears to be sufficient structure available to acquire a reference for use by a spacecraft.
Figure 1. Stars in the constellation Taurus.
Figure 2. SAO chart numbers and constellation boundaries (northern sky).
Figure 3. Stars in the constellation Scorpius.
Figure 4. SAO chart numbers and constellation boundaries (southern sky).
Figure 3 presents a somewhat different structure, here there are several very bright stars and not much in an intermediate range. Visually, the bright stars seem to resemble the shape of a scorpion's tail, an open cluster, and the body, a closed cluster. Thus, the dominant pattern is defined by a few bright stars as opposed to the other starfield of many different magnitude stars.

**FEATURE SELECTION**

The difficult problem of feature selection is not bad in this case, since the algorithms can only cluster data that a CCD mapper can provide. Thus, the features available are the coordinates, \((x, y)\), of the star and the visual magnitude, \(M\). The equal importance of all three features is obvious by an inspection of Figures 1 and 3. The idea of using a form of weighted 3-dimensional feature space of the form \((x, y, M)\) was tried and rejected after a little experimentation. The data was then presented by a 2-dimensional vector \((x, y)\) of planar coordinates, weighted according to magnitude by concentrating an equivalent number of fifth magnitude stars at the position of the star of brighter magnitude. A star of magnitude 1 was replaced by 100 stars of magnitude 5, thus reflecting the accepted scale factor of 100:1 in every five magnitudes of brightness. These weights were found to be extreme for the study; however, a weighting was obviously needed and so a preliminary weighting of 1, 2, 7, 15, 30 was used for stars of magnitude 5, 4, 3, 2, 1 respectively and was found to perform satisfactorily.

**FcV CLUSTERING**

Figures 5 and 6 were obtained by clustering the starfield of Figure 1 to \(c = 2\) and \(c = 5\) clusters, respectively, using the weighting scheme described in the preceding section. In each case, the shape seeking adaptive version of the clustering algorithms were used with initialization parameters set at \(\sigma_k = 0.2\), \(m = 2\), and \(\alpha = 0.95\). The principal component model for each cluster is shown, with the direction vectors scaled to indicate the "shape" defined by the eigenvalues of the scatter matrix. Cluster membership values for some representative stars are located near the star, with the parentheses opening toward the cluster of maximum membership. When clustered to either \(c = 2\) or \(c = 5\), the data was found to be very stable with respect to starting guesses at the cluster centers. Even for other values of \(c\) investigated the two principal clusters that were established at \(c = 2\) moved only slightly.

The cluster models shown in Figures 7 and 8 were also found to be quite insensitive to initial choices of starting centers; thus, a pattern recognition system might serve well in recognizing or establishing a reference based on a few bright stars. Since the clusters would have a distinct signature provided by their individual principal component models, including a scatter or "shape" information, it should be feasible to verify that the desired reference has in fact been obtained.

Figure 7 is also of interest because it provides a good example of the shape seeking operation of the adaptive form of the Fc-V algorithms in recognizing the linear tail and the round body of the scorpion. When the same data was clustered with the \(r\) set at a fixed value \(r_1 = 0\) for each cluster, the effect was to draw the
Figure 5. Stars in the constellation Taurus clustered to $c = 2$. 
Figure 6. Stars in the constellation Taurus clustered to $c = 5$. 
Figure 7. Stars in the constellation Scorpius clustered to $c = 2$. 
Figure 8. Stars in the constellation Scorpius clustered to $c = 5$. 
center of the cluster close to the bright star of magnitude 1 at the tip of the scorpion tail. The open cluster was lost, as the other stars of the tail were given membership values proportional only to the distances to the respective centers.

CONCLUSIONS

It seems clear enough that structure does exist in starfields and can be presented in a form available from the output of a star sensing device suitable for detection by the Fc-V algorithms. It also seems clear that cluster validity measures based only on the membership matrix continue to be unsatisfactory. Instead, a validity measure based upon disjoint principal component models appeared to provide more of the needed information. Therefore, based on the results of this study, it seems reasonable to propose two schemes that should be investigated as follow-on studies. The first, a star pattern recognition system would function with data from individual star-tracker systems that are now being used. In this scheme, centers of clusters would be placed at points in the projected star data where certain pre-selected clusters are expected to exist. Experience thus far shows that these centers could have a significant displacement from the true centers and the algorithms would still rapidly find the true centers. This kind of configuration operation does not require the use of a cluster validity functional, the area where least satisfactory results have been found. Second, it may be possible to use pattern recognition in a more ambitious configuration. The star mapper would be allowed to search until a sufficiently distinct cluster configuration is detected by the pattern recognition algorithms. Then, a star catalogue would be searched matching the detected principal component model with the known models. In this manner, the pattern recognition system may be able to acquire an unsupervised reference for use by a spacecraft in attitude determination or pointing purposes. However, success of this method would be more dependent upon developing a satisfactory cluster validity measure.
REFERENCES


APPROVAL

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The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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Director, Systems Analysis and Integration Laboratory