Mode II Fatigue Crack Growth
Specimen Development

Robert J. Buzzard, Bernard Gross,
and John E. Srawley
Lewis Research Center
Cleveland, Ohio

Prepared for the
Seventeenth National Symposium on Fracture Mechanics
sponsored by the American Society for Testing and Materials
Albany, New York, August 7-9, 1984
MODE II FATIGUE CRACK GROWTH SPECIMEN DEVELOPMENT

Robert J. Buzzard, Bernard Gross, and John E. Srawley

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

A novel Mode II test specimen has been developed which has potential application in understanding phenomena associated with mixed mode fatigue failures in high performance aircraft engine bearing races. The attributes of the specimen are that it contains one single-ended notch, which simplifies data gathering and reduction; the fatigue crack grows in-line with the direction of load application; a single axis test machine is sufficient to perform testing; and the Mode I component is vanishingly small.

NOMENCLATURE

- \( a \): crack length
- \( B \): specimen thickness
- \( E' \): plane stress, \( = E/(1 - v^2) \)
- \( F \): pin shear reaction force
- \( H \): specimen arm height
- \( J \): strain energy release rate (path independent integral)
- \( \Delta J \): variation of \( J \) from \( P_{\text{min}} \) to \( P_{\text{max}} \)
- \( K_I \): Mode I stress intensity factor
- \( K_{II} \): Mode II stress intensity factor
- \( P \): applied end load
- \( R \): minimum fatigue load/maximum fatigue load
- \( r \): polar coordinate referred to crack tip
- \( S \): extent of end reaction parabolic shear distribution
- \( U \): total relative crack mouth displacement in \( x \) direction
- \( W \): specimen width
- \( x, y \): Cartesian coordinate referred to crack tip
- \( Y_I \): dimensionless stress intensity coefficient for Mode I
- \( Y_{II} \): dimensionless stress intensity coefficient for Mode II
- \( \theta \): polar coordinate referred to crack tip
- \( v \): Poisson's ratio
- \( \sigma \): applied end stress \( = P/BH \)
- \( \chi \): stress function

INTRODUCTION

Current development of high performance rolling element bearings for aircraft engines (up to 3 million DN, where DN is the product of shaft diameter in mm and speed in rpm) has aroused concern about fatigue crack growth in the inner bearing race, leading to catastrophic failure of the bearing and the engine.
The following basic model is suggested. During a period of steady engine speed, such as cruise operation, the hoop stress in the inner race (shrink plus centrifugal stresses) is substantial but steady. Consider now a microscopic region of the inner race close to the bearing surface where there is a defect. Each time the race rotates relative to the bearing cage this region will be contacted once by every rolling element in the cage. The Hertzian contact pressure during a particular encounter will depend on the angular position of the defective region in relation to the direction of the overall radial bearing load. The rotational speed is so high that the frequency of significant contacts is of the order of kHz. Eventually a microcrack will develop at this site, so that a fully reversed cyclic stress intensity field may be postulated. The Mode II component changes abruptly in sign as the center of Hertzian pressure moves across the crack. The magnitude of the alternating Mode II field will follow a predictable spectrum which will be essentially repeated at intervals of a certain number of shaft rotations, as can readily be deduced from section 24 of reference 1. The steady hoop stress component of Mode I stress intensity is much greater in magnitude than the cyclic Mode II component, and will tend to keep the crack faces apart, thus avoiding rubbing which would inhibit Mode II growth.

To address this problem in the laboratory, initially a test specimen and loading method is required whereby fatigue crack growth under predominantly Mode II loading may be isolated and studied. The present work is directed toward developing such a specimen and method.

BACKGROUND

A fatigue specimen has been developed at NASA-LeRC (Lewis Research Center) by which fatigue data relative to the aforementioned bearing cracking problem may be obtained. This specimen was developed after an extensive literature search revealed that a specimen meeting all the following requirements did not exist. The requirements of a suitable specimen are:

1. The specimen contains a single notch, which simplifies analysis and the monitoring of crack propagation;
2. The fatigue crack grows in-line with the centerline of the machined notch at or near a zero degree angle to the notch axis;
3. A uniaxial testing machine is sufficient for performing tests;
4. The Mode I loading stresses are insignificant compared to the Mode II stresses.

Prior to this development work, several of the various Mode II specimens described in the literature were examined using acrylic plastic and 7075-T6 aluminum as specimen materials. The results of these examinations are described briefly:

A vee-notched specimen type (fig. 1(a)) designed by Iosipescu has been used by several investigators as a monotonically loaded shear fracture test specimen (ref. 2). Specimens of this type fabricated from plastic and analyzed photoelastically using polarized lighting did not exhibit the classical Mode II stress pattern; that is, concentric semiellipses centered at the notch tip (e.g., see fig. 2). Nor was this expected by Iosipescu, who believed this type of pattern signified compressive rather than shear forces. When loaded monotonically to failure, the vee notched specimens exhibited cracking at 45° to the
notch tip. By revising the notches from the vee configuration to a slot configuration, a weak elliptical pattern was observed at very high loads; however, the centerline of the pattern was not in line with the specimen notch centerline. The most nearly successful test of this general specimen type was with a specimen which was fully restrained from bending by redesign of the test fixture: this specimen failed at the notch centerline but contained additional large cracks starting at the notch tips and propagating at about a 70° angle to the notch centerline toward the tensile loaded zones of the specimen.

Many variations of the specimen and method of loading met with the same generally unsatisfactory results. Consequently further efforts with this specimen type were discontinued. A decided advantage of this specimen type over other designs would have been its simplicity and economy of material. However, even had the preliminary tests been successful, the monitoring of two cracks simultaneously and the resulting fatigue data reduction may have been problematic.

A specimen type designed by Richard (ref. 3) for Mode II fatigue crack propagation was also investigated. This specimen design and fixturing is shown in figure 1(b). A very good Mode II photoelastic stress pattern was observed with this specimen design. However, at high loads this specimen tends to fail at the grip holes, while at moderate loads the fatigue crack propagates from the notch root toward the tensile-loaded leg of the specimen at an angle of about 70°.

Variations to this specimen included notching at both the top and bottom, as well as the use of side grooves to reduce the web cross-section. These modifications, however, did not improve the results.

A double-notched fatigue crack propagation specimen developed by Chisholm (ref. 4) is shown in figure 1(c). This specimen exhibited a very good Mode II pattern at both notch tips, and the monotonic fracture paths were in-line with the machined notches (that is, a crack angle of near 0°). At high loads, the fatigue cracks in 7075-T6 Al specimens proceeded also at near zero degree angles. At low fatigue loads, however, fatigue cracks ran at approximately 70° to the machined notches. It should be noted that according to principal stress or strain energy density criteria crack growth is predicted to occur at an angle of 70° to 80° from the crack line. However, by applying a large enough ΔJ (plastic deformation is observed) crack propagation is constrained along the crack line.

Although the double notched specimen is an attractive candidate for fatigue work a single-notch specimen with simpler fabrication and testing considerations is preferable.

Various other specimen designs were also investigated (about nine major designs, with about 20 variations) with unfavorable results. The primary failure mode of most of these specimens was a 70° crack moving into the tensile loaded side of the notch, indicative of a Mode I failure. These specimen designs are as described in references 5 to 7, plus several original NASA designs.
SPECIMEN AND LOADING METHOD

The final design of the LeRC Mode II test specimen is shown in figure 3(a). It's overall dimensions were initially chosen as 76 mm (3 in.) by 102 mm (4 in.) because this was a convenient size for preliminary photoelastic examination of the specimen's stress field. Photoelastic models were fabricated from both 19.1 mm (3/4 in.) and 6.4 mm (1/4 in.) thick acrylic plastic. Crack growth specimens were made from 6.4 mm (1/4 in.) and 3.2 mm (1/8 in.) thick 7075-T6 aluminum sheet. This thickness was chosen so that loading would not exceed the maximum capacity of the testing machine (a 10-kip, hydraulically operated servocontrolled facility).

The testing fixture is shown in figure 3(b). It was fabricated from type 300 maraging steel and allowed clearance for a 19.1 mm (3/4 in.) thick specimen. Vee-notches were used to engage the top loading pins so that adjustments in horizontal pin spacing could be accomplished by use of steel shims.

The testing machine applies a compressive load to the load train. As the ram of the testing machine is raised, the lower part of the test fixture, along with the specimen, moves upward. The left upper pin experiences a downward reaction force exerted by the upper part of the testing fixture. Rotation of the specimen is prevented by the lower central pin. This pin passes through and is held in place by the specimen, and contacts the upright leg of the test fixture. This pin retains its position within the specimen, but can move in a vertical direction along the upright leg of the test fixture. The slot in the specimen at the lower pin hole allows the right half of the specimen to move upward, with no pin interference, as the specimen is deformed during testing.

SPECIMEN ANALYSIS

The objectives of this analysis were to qualitatively obtain the magnitudes of $K_{II}$ and $K_I$ and their variation with respect to crack length, and to compare analytically obtained displacements with experimental results. The specimen stress intensity and displacements were analyzed by a method of analysis described in detail by Gross and Mendelson (ref. 8). This method consists of finding a stress function $\chi$ that satisfies the biharmonic equation $\nabla^4 \chi = 0$ and the specimen boundary conditions. The biharmonic equation and the boundary conditions along the crack line are satisfied by the Williams stress function (ref. 9). Because of the load asymmetry (fig. 4), the stress function consists of an infinite series of even and odd functions as follows:

$$\chi (r, \theta) = \sum_{n=1}^{\infty} d_{4n-3} r^{n+1/2} \left[ \cos \left( n - \frac{3}{2} \right) \theta - \frac{2n - 3}{2n + 1} \cos \left( n + \frac{1}{2} \right) \theta \right]$$

$$+ d_{4n-2} r^{n+1} \left[ \cos (n - 1) \theta - \cos (n + 1) \theta \right]$$

$$+ d_{4n-1} r^{n+1/2} \left[ \sin (n - 3/2) \theta - \sin (n + 1/2) \theta \right]$$

$$+ d_{4n} r^{n+1} \left[ \sin (n - 1) \theta - \frac{n - 1}{n + 1} \sin (n + 1) \theta \right]$$
The boundary values of $x$ and its normal derivative to the boundary are obtained from the assumed model as shown in figure 4 and are as follows:

along A B  
$$\begin{align*}
&x = - \frac{\sigma y^2}{2} \\
&\frac{\partial x}{\partial x} = \left(\frac{3\sigma}{WH} - \frac{6F}{BH^3}\right) \left(\frac{y^3}{3} - \frac{Hy^2}{2}\right)
\end{align*}$$

along B C  
$$\begin{align*}
&x = - \frac{\sigma H^2}{2} \left(1 + \frac{a}{W} + \frac{x}{W}\right) + \frac{F(x + a)}{B} \\
&\frac{\partial x}{\partial y} = - \sigma H
\end{align*}$$

along C D'  
$$\begin{align*}
&x = - \sigma H y + \frac{FW}{B} \\
&\frac{\partial x}{\partial x} = - \frac{\sigma H^2}{2W} + \frac{F}{B}
\end{align*}$$

along D' D  
$$\begin{align*}
&x = - \sigma H y + \frac{FW}{B} \\
&\frac{\partial x}{\partial x} = \frac{\sigma H^2}{2W} - \frac{12}{5} \left(\frac{y^3}{3} - \frac{Sy^2}{2}\right) + \frac{F}{B}
\end{align*}$$

along D E  
$$\begin{align*}
&x = - \sigma H y + \frac{FW}{B} \\
&\frac{\partial x}{\partial x} = \frac{\sigma H^2}{2W} + \frac{F}{B}
\end{align*}$$

along E F  
$$\begin{align*}
&x = \frac{\sigma H^2}{2} \left(1 + \frac{x}{W} + \frac{a}{W}\right) + \frac{F(x + a)}{B} \\
&\frac{\partial x}{\partial y} = - \sigma H
\end{align*}$$

along F G  
$$\begin{align*}
&x = \frac{\sigma y^2}{2} \\
&\frac{\partial x}{\partial x} = \left(\frac{3\sigma}{HW} + \frac{6F}{BH^3}\right) \left(\frac{y^3}{3} + \frac{Hy^2}{2}\right)
\end{align*}$$

The pin reaction loads $P_F$ and $P+F$ were adjusted to satisfy the observed condition that the crack mouth opening was small.
Preliminary $K_{II}$ and $K_I$ calculations indicate that the specimen type presented here is subjected to a predominantly Mode II condition when under load. Values of $K$ are expressed as $K_{(I, II)} = Y_{(I, II)} \left( \frac{p}{Ba^{1/2}} \right)$, and values for the stress intensity coefficient $Y$ for both modes are presented in Table I for various crack length-to-specimen width ($a/W$) ratios. For example, a specimen used in preliminary fatigue testing had an $a/W$ ratio of 0.679: the corresponding $Y_{II}$ and $Y_I$ values indicate a $K_{II}/K_I$ ratio of about 65 to 1 at this $a/W$ ratio. The solutions obtained for the Mode II specimen shown in Figure 4 are given in Table I for $S=H$. When $S=0.8H$, an overall reduction of less than one percent was obtained for both the stress intensity and displacement coefficients.

While the analytical model does not accurately simulate the actual complex load conditions a good agreement of crack mouth displacement $U$ was obtained when compared with the experimental result. A computed displacement of $U = 0.184$ mm (7.25 mils) compares favorably with 0.187 mm (7.37 mils) obtained experimentally for a 3.18 mm (1/8 in.) thick aluminum plate specimen having an $a/W$ of 0.60 and subjected to a 8.896 kN (2000 lb) force.

RESULTS AND DISCUSSION

Photoelastic examination of 9.5 mm (3/8 in.) thick plastic specimens shows a symmetrical Mode II pattern at the notch tip. The axis of symmetry is in-line with the notch centerline, that is, at an angle of 0° to the centerline (Figure 2). This symmetry and alignment is stable with increasing load and increases in size as load is increased. This suggests that the direction of applied load remains constant during loading. Supportive of this, the notch width measured by use of a feeler gage remains constant during loading. This was true also for tests on aluminum sheet specimens.

Aluminum specimens were initially fabricated from 6.4 mm (1/4 in.) thick sheet, however, the loads required to fracture them monotonically were near the limit of the testing facility. The test section of such specimens was therefore reduced to a thickness of 3.2 mm (1/8 in.) and later specimens were made from 3.2 mm (1/8 in.) sheet entirely. Even at this thickness, no buckling or out-of-plane movement of the specimen tangs was observed.

Initial test data were obtained as monotonic load versus time plots, using a ram movement rate of 1.27 mm (0.05 in.) per minute. A typical test record is shown in Figure 5(a). The photograph of a failed specimen in Figure 5(b) shows that the fracture path is at an angle near 0° to the machined notch.

A partially failed specimen which had been blackened and scribed (Figure 6(a)) shows the type of displacement which occurs under load. The photograph of Figure 6(b) shows the same specimen after partial cracking caused by monotonic loading.

Having established by several additional test runs that the fracture path of this type of specimen is in the desired direction, preliminary fatigue tests were performed to determine the crack propagation direction in Mode II fatigue loading. As predicted by the principal stress and strain energy density criteria, at low loads the crack propagates at about 70° from the notch toward the
tensile-loaded leg of the specimen. However, at increased loads (about 50 percent of the monotonic breaking load), the fatigue crack path was at or near zero degrees to the machined notch direction. The "threshold" load and mechanism responsible for one or the other types of crack propagation has yet to be investigated more thoroughly. It was observed, however, by use of scribed lines on a specimen as in figure 6, that if the specimen is fatigued at a maximum load which is great enough to cause one side of the specimen's entire web section to be displaced vertically relative to the other side, the crack grows in-line with the machined notch. At lower loads — those at which the bottom horizontal scribe line indicates no relative displacement — the crack grows at approximately 70° to the machined notch.

This would at first suggest that for in-line Mode II crack propagation to be operative the overall displacement resulting from the applied load must be great enough to cause one half of the specimen to shear past the other half (here in a vertical direction) in its entirety. If the load or displacement is too low, the lower section of the specimen is comparatively rigid and the tensile leg fails as a tensile specimen would — at the first stress-raising irregularity along its length, which in this case is the upper zone of the "vee" at the notch root. Further work must be done in this area.

The specimen's satisfactory performance under cyclic loading is verified by the preliminary data shown in figure 7. The specimen was cycled at 2 Hz at a maximum load of 13.789 kN (3100 lb). All cyclic loading was performed at an R ratio of 0.1. After approximately 2000 cycles, the maximum load was reduced to 13.344 kN (3000 lb). Crack propagation was monitored with reference to scribe lines spaced at 1.27 mm (0.050 in.) intervals along the web centerline. The test was interrupted after 4000 cycles and subsequently restarted and loaded to a maximum cyclic load of 13.522 kN (3040 lb). At about 4800 total cycles, a second crack was observed emanating at an angle of about 70° from the top of the "vee" at the notch root. The test was discontinued, as the in-line (Mode II) crack growth stopped with the advent of this secondary crack growth. The exact time of secondary crack initiation is not known. However, it is possible that it initiated during initial stages of the restart procedure as the cyclic load was being increased gradually to its maximum level.

The fatigue data obtained from this initial test run (fig. 7) indicate that for each load the crack growth as a function of the number of cycles is fairly linear, simplifying conversion to crack growth rates.

Photographs of the failed specimen are shown in figure 8. The initial direction of cracking was at 0° to 2° to the notch centerline, and gradually changed to about 10° after attaining a length of about 2.5 mm (0.1 in.) (fig. 8(a)). This agrees with the observation of Chisholm for this material (ref. 4), and with data obtained for a Chisholm-type specimen used in this study. A photograph of the fracture surface (fig. 8(b)) shows a dark coloration most likely resulting from the formation of oxide during rubbing of the mating surfaces. This would be expected, since opening mode forces are negligible for this specimen.

Scanning electron microscope photographs (fig. 9) reveal the same evidence of oxidized and rubbed surfaces. That the presence of some Mode I is desirable to prevent such rubbing is debatable: a truly Mode II situation would perhaps by its nature also include the effects of such rubbing in the characterization
of a material's properties. Should the intentional introduction of Mode I forces be desired, it could probably be accomplished by dimensional adjustment of the loading points of the specimen or by altering the length of the machined notch (a).

Further development of this type of specimen will be associated with determining a suitable compliance calibration method, and in further analyses of this specimen type by use of finite element methods. An optimum design of both the specimen and the test fixture is also required, as the information presented herein is based upon an initial design of both.

REFERENCES

TABLE I. - STRESS INTENSITY FACTOR AND CRACK MOUTH
DISPLACEMENT COEFFICIENTS FOR MODE II SPECIMEN
OF PROPORTIONS H/W = 0.5 and S=H

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Figure 1. - Three candidate Mode II test specimens.

(a) Losipescu specimen.

(b) Richard specimen.

(c) Chisholm specimen.
Figure 2. - Mode II photoelastic pattern at notch tip of Columbia resin model of NASA LeRC Mode II specimen.
(a) Mode II test specimen.

\[ a = \text{VARIABLE} \]
\[ W = 74 \text{ mm (2.9 in.)} \]
\[ H = 38 \text{ mm (1.5 in.)} \]
\[ B = 3.18 \text{ mm (0.125 in.)} \]
\[ d = 12.7 \text{ mm (0.500 in.)} \]

(b) Specimen pin-mounted in loading fixture.

Figure 3. - NASA LeRC Mode II test specimen and loading arrangement.
Figure 4. - Analytical model of the NASA LeRC Mode II specimen subject to asymmetric loading; H/W = 0.5.

Figure 5. - Test record (a) and failed specimen (b).

(a) Load versus crosshead displacement test record for NASA LeRC Mode II specimen No. 3-10-83.

(b) Mode II specimen monotonically loaded to fracture. Material: 3.2 mm (1/8 in.) thick 7075-T6 aluminum.
(a) Deformation at 13.344 kN (3000 lb) load (~57% of failure load). Vertical grid spacing = 2.5 mm (100 mils).

(b) Permanent deformation after unloading from 23.57 kN (5300 lb); crack length = 1.3 mm (50 mils).

Figure 6. - Examples of Mode II specimen deformation while under load (a) and after unloading from 98% of failure load (b) for 3.18 mm (0.125 in.) thick 7075-T6 aluminum specimen.
Figure 7. - Results of Mode II cyclic loading of 7075-T6 aluminum NASA LeRC Mode II specimen.

Figure 8. - Notch zone of 7075-T6 aluminum Mode II test specimen after 4800 cycles (total) at 13.788 kN (3100 lb), 13.344 kN (3000 lb), and 13.477 kN (3040 lb).
Figure 9. - Scanning electron microscope photographs of Mode II crack surface. Note white particles believed to be abraded oxides.
**Abstract**

A novel Mode II test specimen has been developed which has potential application in understanding phenomena associated with mixed mode fatigue failures in high performance aircraft engine bearing races. The attributes of the specimen are that it contains one single-ended notch, which simplifies data gathering and reduction; the fatigue crack grows in-line with the direction of load application; a single axis test machine is sufficient to perform testing; and the Mode I component is vanishingly small.

**Key Words**

Mode II  
Fatigue  
Edge sliding mode  
Anti-plane shear

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