Cyclic Torsion Testing

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August 1984
The stress-strain response of a material due to a single, monotonic load application is typically not representative of that when repeated, cyclic loads are applied. In the latter case, progressive to-and-fro slip on planes of maximum shear stress introduces the time-dependent phenomenon known as fatigue. Mechanical testing and analysis to determine materials' properties under cyclic loading has evolved primarily around uniaxial stress states (refs. 1 to 4) due to the relative ease of dealing with this stress state. The methodology of baseline fatigue testing today concentrates on repeated axial loading of coupon specimens, as do most life prediction techniques.

Torsional fatigue testing may sometimes be deemed necessary in the case of prototypical tests of actual machine components that experience cyclic torsional loading in service. More frequently such testing is included as part of multiaxial fatigue research programs which incorporate the torsional stress state as one (among others) of interest. It is the need for multiaxial life prediction capabilities that has stimulated experimental efforts in multiaxial fatigue response. While engineering approaches duplicating specific multiaxial histories of particular components are not uncommon, fundamental research programs targeted at cyclic torsional response have been sparse.

Indeed, there are currently no ASTM standards governing cyclic torsion testing. Where torsional properties are required, it is quite common to see axial cyclic response extrapolated to the torsional regime. Typically, this

transition is accomplished via an "effective" stress or strain parameter (often incorporating the von Mises or Tresca criteria) which can readily be determined from axial response (refs. 5 to 7). The effective stress/strain versus life approach is frequently seen not only in torsional fatigue, but in many multi-axial loading situations in which the three dimensional stresses and strains can be resolved.

Originally conceived as yield criteria to characterize monotonic response, effective stress/strain estimations have been quite useful for purposes of extrapolating from one simple stress state to another (i.e., completely reversed axial fatigue to completely reversed constant amplitude torsional fatigue). However, in more complicated loading environments, this extrapolation is not straight forward. Hence there is motivation for experimental and analytical efforts in torsional fatigue as a subset of the general multiaxial environment.

Given the current immature status of torsional fatigue, it would be misleading to dictate specific testing and analysis procedures. Rather, this article will point out the various options, and associated ramifications, available to the experimentalist and will emphasize testing procedures to characterize baseline materials response in torsion rather than component history simulation. Probably the most crucial parameter to establish in planning or evaluating a cyclic torsion test program is the control mode. There are three basic choices: load/torsional moment, stroke or strain control. In essence, the control mode governing the test may impose certain limitations on one's ability to resolve stable stress/strain response, and on the life regime (high cycle or low cycle) in which the results may be applied. One must bear these limitations in mind when planning cyclic torsion tests, and certainly when applying test results to specific applications.
HIGH CYCLE TORSIONAL FATIGUE

The terminology "High Cycle Fatigue" (HCF) refers to material response in the long life regime (e.g., greater than 100,000 cycles.) While many investigators have encompassed cyclic torsional response as a subset of multiaxial fatigue, testing methods for high cycle torsional fatigue have not been standardized. The extensive work of Sines (refs. 7 to 9) is representative of contemporary approaches to long life multiaxial fatigue situations.

In the high cycle regime, stress and strain amplitudes are low, and the material response is primarily elastic. That is, of the total strain range imposed on the test specimen, the predominant portion reflects recoverable work, with shear stress and shear strain being linearly related through Hooke's law. Hence the relationships between torsional moment, angular deflection, shear stress and shear strain can be assumed to be linear throughout most of the test. While the choice of control mode is less critical here than in circumstances where lower lives are of interest, torsional load or stroke control is common. ASTM E466 (ref. 10) is a Standard Recommended Practice for performing constant amplitude, axial fatigue tests of metallic materials in air at room temperature. Transposed into the torsional stress state, one might find portions of this document useful as guidelines for high cycle torsional fatigue testing procedures. For any one specimen, the torsional load is cycled around zero with a constant amplitude. This infers a zero mean shear stress, and a fully reversed amplitude, $\pm \tau_a$ representing one cycle. (Note that the sign of the shear stress in this instance reflects only a reversal in direction of load application, whereas in axial fatigue it corresponds to tensile or compressive loads.) The controlled cycling is continued until some predetermined failure condition is observed and recorded. Each specimen tested would contribute one data point relating shear stress amplitude, $\tau_a$, to cycles to failure, $N_f$. 

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ASTM E468 is the Standard Practice for Presentation of Constant Amplitude Fatigue Test Results for Metallic Materials (ref. 11), including torsional fatigue tests in air at room temperature. It establishes the desirable and minimum information deemed necessary for reporting purposes. Data reduction suggested varies from an empirical fit of the stress-life results on a linear-log, or log-log coordinates, to a least squares regression, straight line fit on a log-log graph. Such a regression represents a power law relationship, such that

$$\tau_a \propto (N_f)^b$$

where b is an exponent characteristic of a particular material. (see fig. 1).

When very long lives are of interest (e.g., around $10^7$ cycles) the concept of an endurance limit is still popular. The idea of such a defined quantity is to indicate a stress level below which fatigue failure will never occur. The reader should be aware that a very low stress level which may appear to represent such a limit in laboratory testing may be eradicated in actual applications through a few cycles of overstrain. Many materials never exhibit even an apparent limit. Hence such defined quantities must only be used guardedly.

LOW CYCLE TORSIONAL FATIGUE

Low cycle fatigue (LCF) response encompasses those instances when the cyclic stress and strain amplitudes are sufficiently high to result in relatively short lives (i.e., ... less than around 10,000 cycles). Plastic response dominates in this life regime, hence the approach to testing and analysis focuses on the very local stress-strain behavior within the deforming region.

Since the relationships between torsional moment, local stress and local strain are not necessarily linear, there are considerably different mechanical
ramifications of each control mode. Controlling torsional load (moment) presents experimental and analytical difficulties in establishing stable response. Cyclic hardening or softening of the material may be reflected by changes in local strain measurements. Any such deviations in response could cause extreme changes in the width of (and area enclosed by) the hysteresis (stress-strain) loop. Conversely, in a constant amplitude strain controlled test, such changes in cyclic behavior result in relatively minor fluctuations. For all practical purposes, the loop encompasses a constant area throughout most of the test, which is indicative of the plastic work imparted to the specimen on each cycle. In these circumstances, a material will settle into a "stable" stress-strain response, making it possible to characterize baseline material behavior. Hence, in strain control, a known strain amplitude (resolvable into elastic and plastic components) is applied, and the stress response measured.

Shear strain and stroke control indicate control of the angular deflection between two planes along the length of the specimen. Stroke control infers control of the angular deflection between the gripped regions of the specimen. Shear strain control infers control of the angle of twist within the gage length of deformation, and is therefore the preferred operational mode for low cycle torsional fatigue testing.

Local Strain Approach

The underlying purpose for most LCF research or applied engineering efforts is to attain or improve finite life prediction capabilities. With regards to this goal, the analytical aspects of the local stress strain approach and the experimental methods of axial, strain controlled low cycle fatigue testing have been well established. References 1 to 5 and 11 to 13 cite only a few of the excellent documents on these subjects.
While transposition of this same methodology for use in the area of shear strain controlled torsional fatigue is not standardized, it has been demonstrated with several engineering metals (refs. 14 to 17). To characterize baseline torsional fatigue response, each individual test specimen is cycled at a constant, fully reversed total shear strain, $\pm \gamma_T$, until some predetermined failure condition occurs, recorded as the cycles to failure, $N_f$ or the reversals to failure, $2N_f$. Torsional load is monitored for input (along with specimen geometry) into shear stress calculations.

Cyclic changes due to hardening and softening typically occur early in life. With most wrought metals of engineering significance, it is technically sound to assume that cyclic stabilization has occurred by the half-life of a test specimen. Given a stable half-life hysteresis loop, one can measure or calculate the magnitude of the crucial parameters that characterize response at that particular total strain range, including the shear stress amplitude, $\tau_a$, and the elastic and plastic shear strain amplitudes, $\gamma_e$ and $\gamma_p$, respectively. There are a number of methods of resolving the total shear strain amplitude into its elastic and plastic components. Perhaps the preferred and most straightforward is to calculate the elastic component using the linear Hooke’s law relationship:

$$\gamma_e = \frac{\tau_a}{G}$$

(1)

where $G$ is the elastic shear modulus of the material. Assuming that the total shear strain is composed only of elastic and plastic components, one can determine the plastic shear strain magnitude using the difference of the known quantities:

$$\gamma_p = \gamma_T - \gamma_e$$

(2)

Alternately, the loop width at zero stress may be measured as an indicator of the plastic strain range.
The reported test data should include the following parameters for each specimen:

- \( \Delta \gamma, \gamma_f \) or \( \gamma_a \) total shear strain range, or amplitude, respectively
- \( G \) shear modulus of elasticity
- \( \Delta \tau, \tau_f \) shear stress range or amplitude, respectively
- \( \Delta \gamma, \gamma_e \) elastic shear strain range or amplitude, respectively
- \( \Delta \gamma, \gamma_p \) plastic shear strain range or amplitude, respectively
- \( N_f, 2N_f \) cycles or reversals to failures, respectively.

As an example, hysteresis loops from the first cycle and half-life of a 1045 HR and normalized steel specimen cycled at \( \gamma_a = \pm 0.025 \) are shown in figure 2. Note the upper and lower yield point behavior on the first quarter cycle (typical of this class of materials). On ensuing cycles, the response generates smooth hysteresis loops. One should also appreciate the graphical representation of the parameters listed above, as labeled on the "generic" hysteresis loop in figure 3.

Upon completion of a series of cyclic torsion tests, one should have data pairs relating \( \gamma_e \) and \( 2N_f \), \( \gamma_p \) and \( 2N_f \), \( \tau_f \) and \( \gamma_p \) for each specimen. As in the axial fatigue case with analogous parameters, these can be related with the following power law relationships:

\[
\gamma_e = \frac{\tau_f}{G} (2N_f)^b \quad (3)
\]

\[
\gamma_p = \gamma_f^l (2N_f)^c \quad (4)
\]

\[
\tau_f = K' (\gamma_p)^{n'} \quad (5)
\]

When graphing these relationships on logarithmic coordinates, \( b, c \) and \( n' \) are the slopes of straight lines, and \( \tau_f' / G \) and \( \gamma_f' \) are characteristic intercepts at \( 2N_f = 1 \). Here, \( K' \) is defined at \( \gamma_p = 1 \).
Since the representation of such results has not been standardized, one effective way to communicate the torsional fatigue results is to modify the nomenclature of axial fatigue properties to indicate the analogous torsional fatigue quantities. For example:

\( \tau_f \)  
- torsional fatigue strength coefficient
\( b \)  
- torsional fatigue strength exponent
\( \gamma_f \)  
- torsional fatigue ductility coefficient
\( c \)  
- torsional fatigue ductility exponent
\( K' \)  
- cyclic torsional strength exponent
\( n' \)  
- cyclic torsional strain hardening exponent

These coefficients and exponents can be established by linear regression of the logarithmic values of the raw data pairs, as indicated by equation (3) to (5). Results may be summarized through the total shear strain versus life and cyclic shear stress versus shear strain relationships. The total shear strain - life relationship is merely a summation of the elastic and plastic components:

\[
\gamma_T = \gamma_e + \gamma_p
\]  
\[
\frac{\Delta \gamma}{2} = \frac{\tau_f}{G} (2N_f)^b + \gamma_f (2N_f)^c
\]

Similarly with the torsional cyclic stress-strain relationship:

\[
\gamma_T = \gamma_e + \gamma_p
\]  
\[
\frac{\Delta \gamma}{2} = \frac{\tau_a}{G} + \left( \frac{\tau_a}{K'} \right)^{1/n'}
\]

Figures 4 and 5 illustrate these relationships using the data from the same 1045 steel as in figure 2.

It must be stressed that the above is a direct translation of axial LCF methodology to the torsional case. There are experimental complications due to
the nature of torsional loading that have not been accounted for. These will be discussed in the next section. There are also limitations of this approach particular to the local stress-strain low cycle fatigue concepts, which have been shown valid primarily for wrought metals at room temperature in laboratory air.

EXPERIMENTAL CONSIDERATIONS AND COMPLEXITIES

Among the equipment necessary for torsional fatigue tests is, of course, hardware capable of imparting a known and controllable torsional load to the test specimen. Generally, this involves either offset arms carrying equal and opposite loads thereby producing known torsional moments, or a rotary actuator and torsional load cell coupled directly in line with with the specimen. In either case, the most suitable equipment for low cycle fatigue testing is closed-loop, servo-controlled, electrohydraulic test systems. Given that such equipment, as well as the philosophy of closed-loop testing, is well documented elsewhere (ref. 18), this discussion will address some of the complications peculiar to the nature of torsional loading. While these topics are not all inclusive, they are probably the most obvious issues that arise when a cyclic torsion test program is undertaken or evaluated.

Extensometry

Extensometers are commercially available for use in measuring and/or controlling axial strain within a specified gage length in low cycle fatigue tests. While today's commercial equipment is certainly capable of measuring and/or controlling torsional load and stroke (overall angular deflection), "shear strain extensometers" for purposes of torsional strain control have neither been marketed nor gained general acceptance on a widespread basis.

The role of the shear strain extensometer is to measure and/or control the angle of twist (hence the shear strain) within a gage length of a torsional fatigue specimen itself. Such hardware typically consists of some type of
transducer whose electrical output (reflecting the local shear strain) can be incorporated into the closed loop control signal in the test system. Difficulties in attachment to the specimen, signal stability, mechanical and electrical isolation, resolution and compatibility with existing equipment have paced the development of such devices. The most desirable extensometer would be supported on the specimen without introducing geometric stress raisers such as notches and/or indentations, and would also expose most of the specimen surface for observation. Illustrative examples of such devices are described in references 14 to 17 and 19 to 21.

Specimen Design

Just as there are no standard cyclic torsion test methods, neither are there standard test specimen geometries. Of the typical specimens used, the working section is most frequently designed with a uniform gage length and a round cross section. The key geometric factor to consider is whether the gage length cross section constitutes a solid round, a thick-walled cylinder or a thin-walled tube.

Solid rounds and thick walled tubes may present complications after the test material yields in calculating surface shear stress in a strain controlled test. After the onset of plasticity the shear stress distribution across the radius is nonlinear and hence cannot be solved directly. This is not an insurmountable problem, however. The reader is referred to references 22 to 24, where analytical methods have been documented to solve for surface shear stress given directly measurable test parameters.

Thin-walled tubular specimens are frequently used in low cycle torsional fatigue specimens. Using thin-walled theory, one assumes that in the regime of plastic response, the shear stress is uniform throughout the wall thickness, hence can be calculated directly from the known geometry and torsional moment.
The accuracy of this approach is dependent upon having a sufficiently small wall thickness relative to the gross dimensions of the specimen. Typically, the ratio of wall thickness to outer diameter is less than 0.1. Of course, prior to plastic yielding, the exact elasticity solutions (available in most mechanics textbooks and handbooks) are applicable to either specimen design (refs. 25 to 27).

Grips

Specimens must be securely mounted into the test fixture in such a manner to avoid slip and backlash during loading, unloading and load reversals. Many gripping systems designed for axial loading applications are inadequate for torsion. For example, mated threaded ends would merely tighten and release under cyclic torsional loading. Frequently, squared specimen shoulders and matching holders are employed to prevent rotation within the grips. However, these designs often prove impractical for several reasons, including costs of machining specimens, accuracy of alignment required, and excessive wear of the reusable parts in the gripping assemblies. Collet gripping systems, either mechanical or hydraulic, are gaining the most popularity due to ease of use and functionality.

Failure Criteria

This most fundamental issue is perhaps the most confusing when discussing torsional LCF. However, fatigue data is rendered useless without a clear definition of the condition considered failure.

Failure or crack initiation in axial strain controlled LCF tests is defined as some predetermined drop in the tensile load required to enforce the controlled strain amplitude of the test. One can easily see how a load drop reflects a decreased cross-sectional area, hence the presence of a crack. From a practical viewpoint, this criteria is easy to incorporate into automated test...
schemes. The crack indicated is normal to the loading direction, and hence, normal to the tensile stress.

Such a convenient, criteria is not obvious in torsional fatigue. Especially apparent in ductile metals, there may be surface cracks on the two planes of maximum shear stress, that are quite large relative to the gage length dimensions, throughout most of full load carrying lifetime of the specimen. There may also be multiple crack systems throughout the gage length. Hence the question arises of what physical condition represents failure or meaningful crack initiation in torsional fatigue.

While it is agreed that a description of the condition regarded as failure or crack initiation is essential in reporting torsional fatigue results, there is no widely accepted answer to this question within the technical community. Such a description should be quantitative (e.g., crack length and plane) and also include the method of failure determination (e.g., visual inspection, surface replication, measured bulk parameters, etc.)

SPECIAL CONSIDERATIONS

Deviating from the room temperature, laboratory, air environment is likely to affect a material's fatigue response regardless of the cyclic stress state imposed. Changes in atmospheric conditions, such as testing in a vacuum or inert gas, may be reflected by decreased chemical interaction on the surface layer of a specimen, thought to enhance resistance to fatigue crack initiation. Conversely, the detrimental effects of harsh conditions, such as highly corrosive environments (e.g., high humidity and/or temperature, atmospheres promoting chemical oxidation of the specimen) are of continual interests for practical applications.

Mechanical response at elevated temperatures, often involving fatigue and creep interactions is a particularly important engineering problem. Testing and analytical methods for dealing with low cycle fatigue/creep have been
devised for the axial loading situation (ref. 28), most notably Strain Range Partitioning (ref. 29). While theoretical extensions to multiaxial cyclic loading (e.g., torsion) have been proposed, (ref. 30) this area has been relatively untouched by experimentalists.

This discussion has mentioned only some of the many complications faced in applying laboratory results, or in designing experiments to simulate service conditions rather than achieve baseline materials response. Complexities of an entirely different nature may be introduced when nonmetals are of concern. Whereas the amount of cyclic torsion testing performed on metals is limited, it is even scarcer on other classes of materials, such as ceramics, polymers and composites. Practical applications of ceramics are usually limited by their tendency towards brittle fracture, rather than by low cycle fatigue response. Similarly, there is little information available concerning torsional fatigue testing of polymer or composite materials systems.

SUMMARY

One's approach to cyclic torsional testing must be a strong reflection of the eventual use of the results. In the same light, using torsional fatigue data requires an understanding of the testing procedures and their consequences on the results. Three most crucial areas requiring care and consistency when exploiting torsional fatigue data are life regimes (HCF or LCF), control mode and failure criteria.

At the date of this writing, there are no standards governing cyclic torsion tests. However, standard recommended practices for axial fatigue tests published by ASTM may provide useful guidelines for experimental procedures and data reduction for baseline fatigue characterization. Experimental (e.g., extensometry) and theoretical (e.g., failure definitions) difficulties have paced refinement and general acceptance of test procedures. Hence, cyclic
torsion testing is, in general, not part of routine engineering or material evaluations. Rather, it is directed towards specific areas of research, such as multiaxial fatigue response and life prediction. As these research needs expand, so will testing abilities and procedures in torsional fatigue, as well as fatigue in other multiaxial stress states.

REFERENCES


Figure 1. - Torsional stress vs life in the high cycle regime, wrought aluminum alloy (from ref. 8).

Figure 2. - Torsional hysteresis loops of a 1045 HR and normalized steel.
Figure 3. - Stable torsional hysteresis loop.

Figure 4. - Shear strain vs life of a 1045 HR and normalized (from ref. 15).
Figure 5. Cyclic and monotonic shear stress vs shear strain of a 1045 HR and normalized steel (from ref. 15).
Cyclic Torsion Testing

Torsional fatigue testing and data analysis procedures are described. Since there are no standards governing cyclic torsion testing that are generally accepted on a widespread basis by the technical community, the different approaches that dominate current experimental activity, and the ramifications of each are discussed. Particular attention is given to the theoretical and experimental difficulties that have paced refinement and general acceptance of test procedures. Finally, specific quantities and nomenclature modelled after analogous axial fatigue properties are suggested as an effective way to communicate torsional fatigue results until accepted standards are established.
