Preliminary Experiments on Phase Conjugation for Flow Visualization

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August 1984
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Summary

This report on experiments utilizing a barium titanate single crystal discusses the procedure for polarizing a crystal; a test for phase conjugation; transients in the production of phase conjugation; real time readout by a separate laser of a hologram induced within the crystal, including conjugation response times to on-off switching of each beam; and a demonstration of a Twyman-Green interferometer utilizing phase conjugation.

Introduction

Disturbances induced in optical wavefronts by nonuniformities in fluids are often used for visualizing and evaluating those nonuniformities. Interference methods typify this approach. The application of phase conjugation of waves may permit the development of new devices for flow visualization and evaluation which possess advantages, particularly simplicity and increased sensitivity, over their forerunners.

This report describes some preliminary experiments using a polarized single crystal of barium titanate

(1) to obtain phase conjugation,
(2) to study some characteristics of a phase conjugator (barium titanate crystal) exposed to a low power laser beam,
(3) to consider consequences of using two lasers in a phase conjugation experiment,
(4) to construct an interferometer utilizing phase conjugation, and
(5) to discover difficulties inherent in using a conjugator.

The present report is mainly a progress report concerned with basic experiments leading to the application of phase conjugation in flow visualization. Some of the experiments, namely preparation of the crystal and the phase conjugation test are not original. However, they do include new data which may be of value considering the variability in nonlinearities among crystals. The experiments on transient effects are believed to be original and may be of value in establishing how the crystal functions as a holographic
optical element. The intended application of the crystal was in interferometry. Subsequent to the interferometer experiment it was found to have been preceded by a similar preliminary experiment in Israel. Both devices require further development to achieve practical usefulness.

The work reported herein was performed while the first author served as a 1983 Summer Faculty Fellow at the Lewis Research Center.

PHASE CONJUGATION (ref. 1)

Consider a propagating electric wave \( \hat{E} \) given by

\[
\hat{E}(x,y,z,t) = \hat{E}(x,y,z) \cos\left\{ \frac{2\pi}{\lambda} \left[ ct - \psi(x,y,z) \right] \right\}
\]

where \( \hat{E} \) is the amplitude of the field, \( c \) is the speed of light in vacuum, \( \psi \) is the optical path length traversed by a wavefront, and \( \lambda \) is the wavelength. A wavefront is progressively located on a surface determined by

\[
\phi(x,y,z,t) = ct - \psi(x,y,z) = 0
\]

Now consider a wave

\[
\hat{E}^X(x,y,z,t) = \hat{E}(x,y,t) \cos\left\{ \frac{2\pi}{\lambda} \left[ ct + \psi(x,y,z) \right] \right\}
\]

where \( \hat{E}^X \) denotes the phase conjugate of \( \hat{E} \). The phase conjugate of \( \hat{E} \) differs from \( \hat{E} \) with regard to the sign of \( \psi \). Whereas, on the original wavefront

\[
\psi = ct
\]

on the phase conjugate wavefront,

\[
\psi = -ct
\]

This implies that if a propagating wave becomes phase conjugated, the conjugate wave will propagate backward along the path of the original wave. Alternatively, the conjugate wave can be interpreted as propagating backward in time.

Phase conjugation of light can be achieved by propagation through any medium capable of producing nonlinear optical effects. Electrically polarized, single-crystal, barium titanate (BaTiO₃) is especially suited for this purpose because it is capable of producing phase conjugation at very low light levels in the absence of an imposed, steady electric field. A BaTiO₃ crystal was, therefore, selected and prepared for preliminary experiments.

PREPARATION OF CRYSTAL

First, the barium titanate crystal (dimensions, 4 by 4 by 4 mm) was examined in polarized light to locate the two faces perpendicular to the
optical axis (C axis) (ref. 2). Silver paint was applied to these two faces to act as electrodes. A crystal holder (shown in fig. 1) was assembled. The crystal in its holder was placed in an oven with precise temperature control and heated slowly to about 128° C. An electric field of 3.5 kV/cm was applied along the C axis (as specified in ref. 3). (The crystal temperature must be kept less than 133° C and more than 5° C to prevent undesired structural changes in the crystal.) The aforementioned electric field corresponds to a potential of 1.4 kV across the 4 mm crystal. The temperature-time history is shown in figure 2.

The existence of permanent electric polarization (poling) was subsequently tested by measuring energy transfer between interacting beams (as described in ref. 3). The test geometry and symbolism are shown in figures 3 and 4. (Without poling energy transfer could not be obtained.) By means of a half-wave plate, horizontally polarized light from a 50 mW helium-neon laser was imposed on the crystal along directions A and B at the arbitrarily chosen angles shown in figure 4 using the test configuration shown in figure 3. The illumination along directions A and B was measured in front of and behind the crystal using a Luna-Pro exposure meter with a spherical diffusing screen. The exposure meter was convenient for making relative radiant power determinations.

Results of measurements are shown in figure 4. Energy transfer from one beam to the other, say from A to B' or B to A', was indicated by the fact that the more intense beam preceding the crystal was often the less intense beam upon emergence (denoted by the prime). Also, if beam A was blocked, after having been on for at least 30 sec, the beam in direction A' persisted but slowly died out. With the direction of the C axis pointed more nearly in the sense of the A axis rather than the B axis and the angle between axes A and B equal to 34°, the relative radiant power along A' took 9 sec to decay from 416 to 16.

PHASE CONJUGATION TEST

The test for phase conjugation was performed several times with some variation in the geometry of the components and different means for measuring the beam intensities. Conjugation was accomplished using the geometry for four-wave mixing (ref. 1, Chap. 2). The experimental configuration is shown in figure 5.

In four-wave mixing a laser beam is transmitted through the crystal and then upon emergence is reflected back in its original direction by an ordinary mirror. This beam is called the pump beam. A second, coherent beam is also imposed on the conjugator at an angle to the first beam. This beam is called the probe beam. As a consequence of interaction of the beams, a phase conjugate of the probe beam is reflected back along the original path of the probe beam. This beam is separated from the probe beam by a semi-reflecting plate.

The preceding result can be explained from the viewpoint of real-time holography. Specifically, the incident pump and probe beams intersect in the crystal to produce an interference pattern and consequently a steady refractive index variation produced by migrated charges (ref. 3). The reflected pump beam is diffracted by the refractive index grating and appears
as the phase conjugate of the probe wave transmitted backward along the path of the probe wave.

In the conjugation experiment the crystal was placed on a rotatable platform with allowable translation along three orthogonal coordinates. Figure 6 shows the mount and reflectors with the crystal in a cuvette containing castor oil (n = 1.47). Initial tests were performed without the cuvette of oil, but it was included later in order to increase the effective aperture of the system.

The experimental incidence angles for maximum power in the phase conjugate beam are shown in figure 7 along with power measurements for the crystal in oil. The variation in power of the phase conjugate beam as a function of angle of incidence θ of the pump beam at the crystal is shown in figure 8 for the crystal in air. These measurements were made with a photo transistor calibrated relative to the Luna-Pro exposure meter. There was considerable scatter in the power measurements of the phase conjugate beam partly because the adjustment of mirrors M1 and M2 (shown in fig. 5) was found to be very critical.

The existence of phase conjugation was confirmed by an aberration correction experiment (ref. 4). The aberrator consisted of a glass cover slide with Duco cement dabbed on it. As shown in figure 9, the aberrator distorts lettering viewed through it. The aberrator was placed at plane B (fig. 5), and the consequent phase conjugate beam was recorded on film at plane B'. Without the aberrator in place the phase-conjugate beam appeared at plane B' (as shown in fig. 10(a)). With the aberrator in place the phase-conjugate beam appeared unchanged at plane B' (as shown in fig. 10(b)). This implied the existence of phase conjugation because the aberration of the beam reflected back through the crystal was cancelled by the aberrator. This was confirmed by replacing the phase conjugator by a plane mirror and recording the aberrated light at plane B'. The resultant unconjugated beam after passage through the aberrator remains aberrated, (as shown in fig. 10(c)).

**TRANSIENT IN PHASE CONJUGATION**

In the four-wave mixing configuration a strong transient overshoot in the power from the phase conjugate beam was observed when beam 3 (fig. 11), between the crystal and pump mirror, was suddenly turned on.

A shutter at C was used to block beam 3. First, the crystal was exposed to beams 1 and 2 for more than 30 sec. This exposure allowed a grating to form within the crystal. Then, the shutter was opened allowing beam 3 to reflect back through the crystal to produce the phase conjugate beam observed at B'. This procedure resulted in a sharp initial pulse of phase-conjugated light (downward deflection of oscilloscope trace in figs. 12 and 13) followed by a subsequent exponential-like rise to a lower steady-state value after 3 or 4 sec. Apparently, the exposure of the crystal to beam 3 leads initially to efficient diffraction of beam 3 by the grating formed by beams 1 and 2. However, beam 3 subsequently forms a hologram with beam 2 so that some energy comprising conjugate beam 2 is transferred elsewhere.
Beam power was determined using the phototransistor. The maximum peak, relative radiant power of the phase-conjugate beam was found to be (fig. 12)

\[ I_{pc}(t_p) = 600 \]

for which \( I_{pc}(t_s) = 90 \), so that

\[ I_{pc}(t_p)/I_{pc}(t_s) = 6.7 \]

where \( t_p \) denotes the time of occurrence of the peak and \( t_s \) is the time of occurrence of the steady-state value. This maximum peak value was obtained for the beam ratio

\[ I_2/I_1 = 0.15 \]

The maximum steady-state, relative radiant power of the phase-conjugate beam was found to be (fig. 13)

\[ I_{pc}(t_s) = 250 \]

for which \( I_{pc}(t_p) = 450; I_{pc}(t_p)/I_{pc}(t_s) = 1.8; \)

and the beam ratio was

\[ I_2/I_1 = 0.36 \]

These results, wherein the hologram was formed before the reflected pump beam was turned on, contrast with those reported in reference 5, wherein a slow rise in power of the phase conjugate beam occurred when all beams were turned on simultaneously.

The slow response of the crystal to the steady state condition might constitute a serious limitation on its use for interferometry of time dependent phenomena, at least at the low levels of illumination provided by a helium-neon laser.

**SEPARATE LASER FOR READOUT**

Using the configuration shown in figure 11, the mirror in beam 3 was replaced by a second, 2 mW, helium-neon laser, as shown schematically in figure 14. This laser was slightly misaligned to prevent reflection of beam 1 back into the crystal. The beam from the second laser was diffracted in the crystal by the grating, as before. Shutters were located at A and B to block the beams from the 50 mW and 2 mW laser, respectively. Beam 1 could also be blocked at D, and beam 2 could be blocked at C. Quantitative measurements of the radiant power in the conjugate beam were precluded by the low power of the 2 mW laser. However, the conjugate beam could be detected visually, so that the following qualitative observations were made by opening and closing the aforementioned apertures.
O = open  
C = close  

<table>
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<tr>
<th>No.</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>After 30 s</th>
<th>Conjugate beam at B'</th>
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<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>Reached steady state in 4 s (approx.)</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Close B</td>
<td>Immediate erasure</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Close C</td>
<td>Faded out in 6 s (approx.) then Reopen C</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Close D</td>
<td>Faded out in 4 s (approx.)</td>
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<tr>
<td>5</td>
<td>0</td>
<td>C</td>
<td>0</td>
<td>0</td>
<td>Close A</td>
<td>Conjugate formed, then Open B</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Open A</td>
<td>Reached steady state in 2 s (approx.)</td>
</tr>
</tbody>
</table>

All of the above results can be explained in terms of the time taken to construct and/or obliterate a grating formed within the crystal. The first experiment indicates the time (4 sec) for the grating to form and for the conjugate beam to reach steady-state. When the pump beam 3 was turned off in experiment 2 the conjugate beam was immediately extinguished. This indicated that the conjugate beam was derived from the pump beam 3. When beam 2 was blocked, as in experiment 3, the grating slowly weakened so that its diffraction efficiency, and hence the power in the conjugate beam, declined. It is noteworthy that the time to reconstruct the grating upon reintroducing beam 2 was only half that for its obliteration. When beam 1 was blocked (experiment 4) instead of beam 2, the conjugate beam faded out more rapidly. This is the expected result if beam 1 were more intense than beam 2, as was the case. Experiment 5 illustrates the apparent persistence of a grating produced by a strong electric field when the grating was subsequently illuminated by a weak pump beam. First, the grating was formed by beams 1 and 2 with beam 3 turned off. Then, beam 3 was turned on as beams 1 and 2 were turned off. Beam 3, which was very weak relative to beam 1 and 2, produced the conjugate beam while obliterating the grating very slowly (15 sec). In the final experiment 6, the pump beam 3 was turned on first. Then, the beams 1 and 2 were turned on to produce the grating in one-half the time (2 sec) required in experiment 1.

**INTERFEROMETER USING PHASE CONJUGATION**

The achievement of phase conjugation leads to the possibility of new interferometers possessing certain advantages over existing ones. For example, in flow measurements the Mach-Zehnder interferometer is favored. This interferometer requires a reference beam, usually consisting of plane waves, to form interference fringes at an image plane. At this plane the intensity \( I \) is given by

\[
I = 1 + \cos \left( \frac{2\pi}{\lambda} \Delta \psi \right)
\]

where \( \Delta \psi \) is the difference, or change, of optical path lengths between the interfering beams. However, if a phase conjugator is used, then interference
can be produced between one beam and its phase conjugate. The plane-wave reference beam can be eliminated, and the time-average intensity becomes

\[ I_{pc} = 1 + \cos \left( \frac{4\pi}{\lambda} \psi \right) \]

This interferometer is, therefore, twice as sensitive to optical path length as the Mach-Zehnder interferometer.

The interferometer shown in Figures 15 and 16 was assembled to demonstrate interference using the phase conjugate beam. It consisted of the configuration for four-wave mixing with a beam splitter \( S_2 \) and mirror \( M_3 \) added to obtain a Twyman-Green interferometer with an un conjugated beam interfering at \( B' \) with the conjugate beam. In this configuration the test section would be in that section of beam 2 preceding splitter \( S_2 \). In this preliminary experiment a 35 mm focal length lens was introduced preceding the film plane to expand the previously unexpanded beam. The divergence of beam 2 required introduction of a 25 \( \mu \)m-diameter spatial filter and a collimating lens preceding the first beam splitter \( S_2 \). This improved the parallelism of the interfering beams to yield the fringes shown in Figure 17. Considerable difference between the intensities of the interfering beams existed so that a secondary reflection off the uncoated side of splitter \( S_2 \) was used to produce interference with the conjugate beam. A satisfactory alternative was to insert a neutral density filter between \( M_3 \) and \( S_2 \). No interferograms were recorded with this arrangement. The relatively low conjugation efficiency of the crystal was not a serious limitation for purposes of interferometry.

Following completion of these tests it was found that this interferometer had been reported previously in reference 6.

**CONCLUSION**

Through the use of a barium titanate single crystal it was found that optical phase conjugation could easily be achieved using a helium-neon laser by methods previously reported. Various time dependent effects on the intensity of the conjugate beam could be explained in terms of the effect of the imposed laser beams on the diffraction (holographic) grating formed within the crystal. As in ordinary holography, the phase conjugate beam could be reconstructed using a laser independent of that used to form the grating, and in real time within the limitation of the response time of the crystal.

A Twyman-Green interferometer was assembled using a probe (test) beam and its phase conjugate and having twice the sensitivity of a Mach-Zehnder interferometer.

The main drawback in using the crystal conjugator at low power levels was its slow response. The relatively low conjugation efficiency of the crystal was not regarded as a serious limitation for purposes of interferometry.
REFERENCES


Figure 1. - Crystal in holder.

Figure 2. - Oven temperature history for poling BaTiO₃ crystal.
Vertically polarized 50 mW HeNe laser

Figure 3. - Arrangement for two-beam energy transfer.

Figure 4. - Beam relative radiant power near the poled crystal.
Figure 5. Arrangement for phase conjugation.
Figure 6. - Crystal mounted in cuvette.

Figure 7. - Crystal orientation in cuvette of oil for conjugation experiment.
Figure 8. - Variation of relative radiant power in phase-conjugate beam as a function of angle of incidence of probe beam. (Crystal in air.) (Note: Relative radiant power in beam (3) (figure 5) is =1400 at the crystal.)

Figure 9. - Printed page viewed through beam aberrator.
(a) Phase-conjugated beam without aberrator.

Figure 10.

(b) Phase-conjugated beam after passing through aberrator.

Figure 10. - Continued.
(c) Non-conjugated beam after passing through aberrator (crystal replaced by plane mirror).

Figure 10. - Concluded.

Figure 11. - Arrangement for studying phase-conjugate beam overshoot.
Figure 12. Oscilloscope trace at maximum peak power of phase conjugate beam.

Figure 13. Oscilloscope trace at maximum steady-state power of phase conjugate beam.
Figure 14. Arrangement using separate laser for read-out.

Figure 15. Schematic of optical paths for interferometer.
Figure 16. - Arrangement of elements for phase-conjugated interferometer.

Figure 17. - Interference fringes.
This report on experiments utilizing a barium titanate single crystal discusses the procedure for polarizing a crystal; a test for phase conjugation; transients in the production of phase conjugation; real time readout by a separate laser of a hologram induced within the crystal, including conjugation response times to on-off switching of each beam; and a demonstration of a Twyman-Green interferometer utilizing phase conjugation.