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Unsteady Transonic Flow in Cascades

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INTRODUCTION

There is a need for unsteady transonic air load prediction methods suitable for aeroelastic analysis of turbomachine blading. Considerable progress has been made in the development of cascade analysis for incompressible, subsonic and supersonic flows. Comparatively the progress on development of such methods for transonic flow has been slow. Verdon and Caspar[1] and Kerlick and Nixon[2] used numerical methods to represent aerodynamic phenomena in transonic cascade flow. The present work presents an analytical solution for unsteady transonic cascade flow valid for large reduced frequencies. This solution has been obtained by using the Wiener-Hopf technique. The details of the mathematical development of the model will be presented in another report[3]. The model formulation and analysis are briefly described in this paper. Based on this analysis, a parametric study is conducted to find the effects of reduced frequency, solidity, stagger angle, and position of pitching axis on the flutter.

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*NASA Resident Research Associate; this work performed while he was an NRC-NASA Research Associate.

MODEL FORMULATION AND ANALYSIS

An elementary turbomachine stage is replaced by a rectilinear two-dimensional cascade of thin airfoils. The steady relative flow approaching the cascade is assumed to be transonic, irrotational, isentropic, and inviscid. The blades are allowed to undergo a small amplitude harmonic oscillation which generates a small unsteady flow superimposed on the existing steady flow field. The blades are assumed to oscillate with a prescribed motion of constant amplitude and interblade phase angle. Since transonic fan and compressor blades are thin, it is assumed that steady flow deviates slightly from uniform base flow.

The unsteady transonic small perturbation equation can be linearized when the free stream Mach number M_1 is close to 1 and the reduced frequency k satisfies the following requirements: (i) $k >> 1- M_1$, (ii) $k >> (\delta)^{2/3}$ where δ is thickness ratio. The thickness camber and mean angle of attack of the blades only influence steady flow perturbations. Therefore, for the purpose of this analysis, the blades can be replaced by a set of zero thickness plates as shown in Fig. 1. The governing equation used in this analysis is the unsteady small perturbation equation given by Landahl[4] for $M_1 = 1$.

$$k^2 \frac{\partial^2 \phi}{\partial \eta^2} - 2 \frac{\partial^2 \phi}{\partial \xi \, \partial \tau} - \frac{\partial^2 \phi}{\partial \tau^2} = 0 \qquad (1)$$

where

ϕ	nondimensional unsteady perturbation potential
b	semi chord
U	free stream velocity
x,y,t	space and time coordinates
ξ,η,τ	nondimensional coordinates
k	reduced frequency, $\omega b/U$
ω	frequency of blade oscillation

(2)

We assume that disturbances are prohibited from propagating upstream in transonic flow. Hence the flow downstream of the Mach wave emanating from the trailing edge of each blade does not influence the flow upstream of this wave. For this upstream portion of the flow, the cascade may be replaced by a row of semi-infinite plates as shown in Fig. 2. An analytical solution ϕ_1 to Eq. (1) for this region is obtained by using the Wiener-Hopf procedure[5]. The solution downstream of the trailing edge Mach wave is obtained by considering a backward-facing row of semi-infinite plates as shown in Fig. 3. It is assumed that the velocity component normal to the plate vanishes and that the jump in pressure across the wake region is equal and opposite to that given by the upstream solution. The analytical solution ϕ_2 to Eq. (1) for this region is again obtained by using the Wiener-Hopf procedure. Hence, when the downstream solution is added to the upstream solution, an exact solution that satisfies all of the boundary conditions for an oscillating staggered cascade is obtained

$$\phi_T = \phi_1 + \phi_2 \tag{3}$$

The nondimensional unsteady lift L and moment M acting about midchord of the reference blade (i.e, m = 0) can be expressed in terms of perturbation potential ϕ_T as

$$L = iK\int_0^2 \Delta\phi_T d\xi - [\Delta\phi_T]_0^2 \tag{4}$$

$$M = ik\int_0^2 \xi\Delta\phi_T d\xi + \int_0^2 \Delta\phi_T d\xi - [\xi\Delta\phi_T]_0^2 - L \tag{5}$$

where

$$\Delta\phi_T = \{\phi\}_{\eta=0} - \{\phi\}_{\eta=kS^-} e^{-i\sigma}$$

σ - interblade phase angle

The work done by the fluid on the cascade over a cycle of motion is a direct measurement of the susceptibility of the cascade to flutter. For a single degree of freedom, pitching or plunging motion, the nondimensional work done by the fluid on the blade is given as

$$W_1 = \pi\ \alpha_0\ \text{Im}\ (M) \text{ - for pitching motion}$$

$$W_2 = \pi\ h_0\ \text{Im}\ (L) \text{ - for plunging motion} \tag{6}$$

where α_0 and h_0 are amplitudes of pitching and plunging motions respectively, and Im means the imaginary part of bracketed quantity.

If the work done over a cycle is positive, the cascade receives energy from the flow and becomes unstable. But if the sign of the work is negative, the cascade is doing work on the fluid and is thus stable. This stability criteria is studied over a range of reduced frequencies, stagger angles, solidities and locations of pitching axis.

RESULTS AND DISCUSSION

A computer code was developed to obtain the unsteady lift and moment acting on a cascade of blades at Mach number equal to 1. A transonic similarity law was used to compare the results of the present analysis at Mach numbers other than 1 to the results obtained from other linearized cascade analyses. The absolute value of unsteady lift ($|L/\pi k^2|$) and unsteady moment ($|M/\pi k^2|$) are plotted for various values of reduced frequency in Figs. 4 and 5. The present analysis results are compared with the results of Smith's[6] analysis in Fig. 4 for Mach number = 0.9 and with the results of Adamczyk and Goldstein[7] for Mach number = 1.1 in Fig. 5. It is observed that the results obtained from the present analysis are in good agreement with the results from the other analyses.

The work W_2 done by the fluid on the cascade over a cycle for plunging motion always remained negative and cascade remains stable. Hence the results of plunging motion are not presented here. The work W_1 per cycle for a cascade undergoing simple harmonic pitching motion is plotted for various values of interblade phase angles in Fig. 6. The parameter varied in Fig. 6(a) is reduced frequency based on semichord from a value of 0.25 to 2.0. It is observed that the cascade tends to become more stable as reduced frequency is increased. Fig. 6(b) shows the effect of solidity of the cascade on stability. Increasing the solidity has a slight destabilizing effect on cascade. Stagger angle also plays an important role in setting the stability boundary. Its influence on the stability is shown in Fig. 6(c). It is noted that the stability of the cascade can be increased by reducing stagger angle. The effect of pitching axis location is shown in Fig. 6(d). It is observed that a rearward shift of pitching axis decreases the stability of the cascade. The aerodynamic model developed in this analysis can be coupled with a structural model allowing one to study the forced vibration characteristics of the blades in transonic flow.

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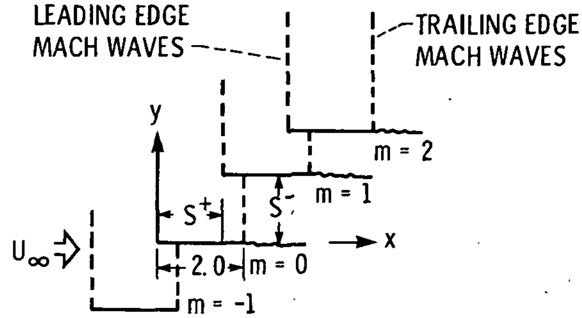

Figure 1. - Cascade configuration.

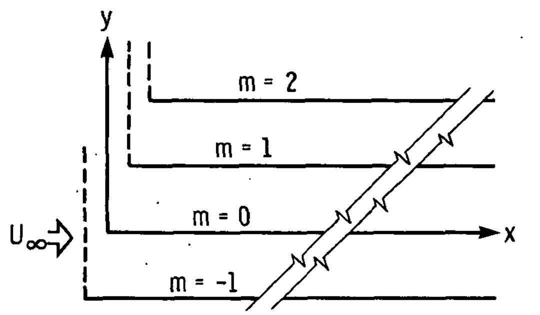

Figure 2. - Configuration for upstream solution.

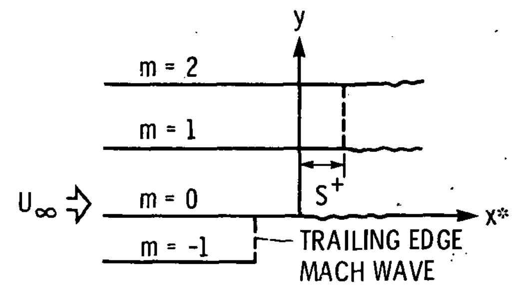

Figure 3. - Configuration for downstream solution.

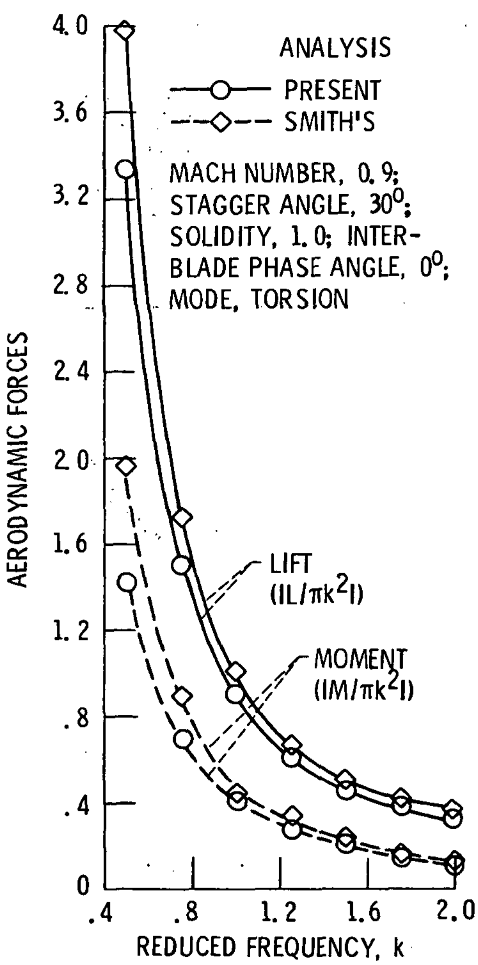

Figure 4. - Comparison of results with Smith analysis.

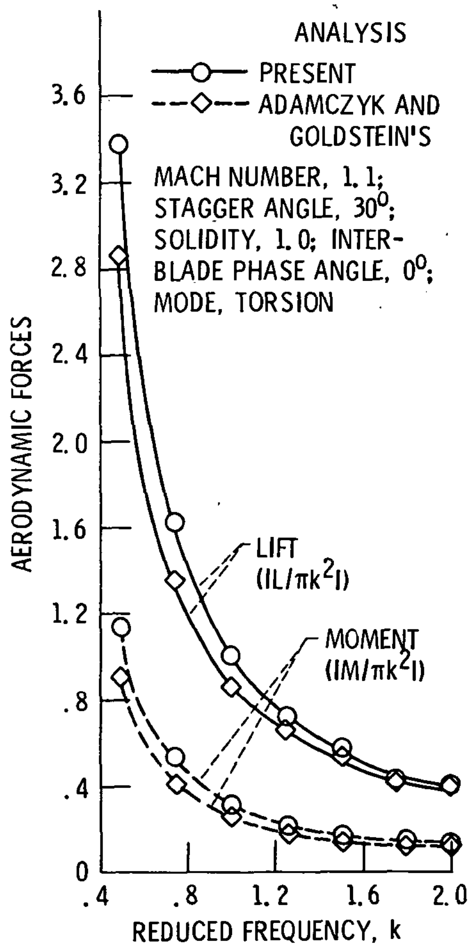

Figure 5. - Comparison of results with Adamczyk and Goldstein analysis.

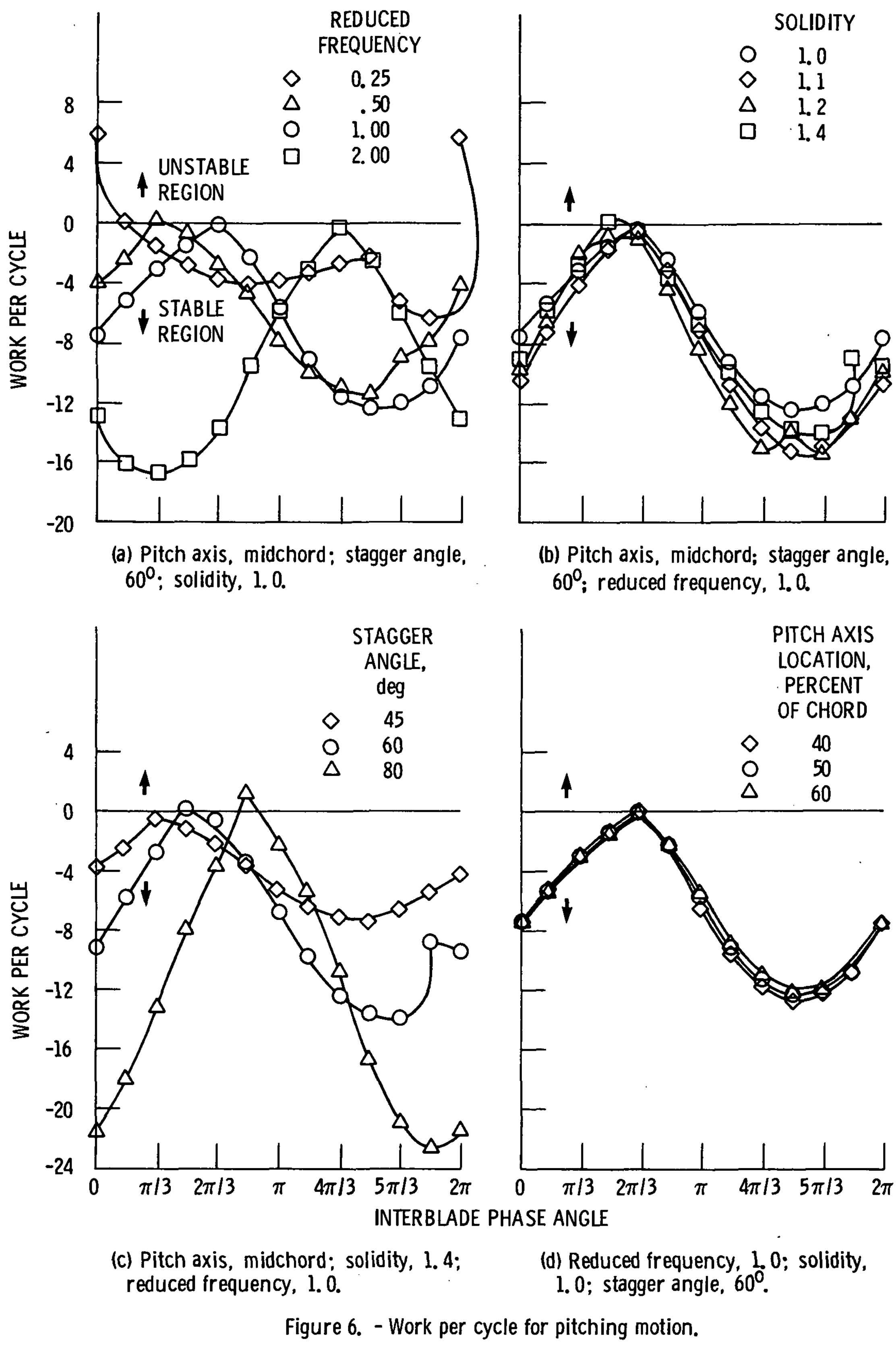

Figure 6. - Work per cycle for pitching motion.

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15. Supplementary Notes

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16. Abstract

There is a need for methods to predict the unsteady air loads associated with flutter of turbomachinery blading at transonic speeds. The present report presents the results of such an analysis in which the steady relative flow approaching a cascade of thin airfoils is assumed to be transonic, irrotational, and isentropic. The blades in the cascade are allowed to undergo a small amplitude harmonic oscillation which generates a small unsteady flow superimposed on the existing steady flow. The blades are assumed to oscillate with a prescribed motion of constant amplitude and interblade phase angle. The equations of motion are obtained by linearizing about a uniform flow the inviscid nonheat-conducting continuity and momentum equations. The resulting equations are solved by employing the Weiner-Hopf technique. The solution yields the unsteady aerodynamic forces acting on the cascade at Mach number equal to 1. Making use of an unsteady transonic similarity law, these results are compared with the results obtained from linear unsteady subsonic and supersonic cascade theories. A parametric study is conducted to find the effects of reduced frequency, solidity, stagger angle, and position of pitching axis on the flutter.

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