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RESEARCH ON OUTPUT FEEDBACK CONTROL OF SYSTEMS WITH ILL-CONDITIONED DYNAMICS

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SECTION 1

INTRODUCTION

This report summarizes our research activities in the first half of 1984. During this period issues pertaining to the well-posedness of a two time scale approach to the output feedback regulator design problem have been examined. An approximate quadratic performance index which reflects a two time scale decomposition of the system dynamics was developed. It is shown that, under mild assumptions, minimization of this cost leads to feedback gains providing a second-order approximation of optimal full system performance.

A sequential numerical algorithm was defined which obtains output feedback gains minimizing a broad class of performance indices, including the standard LQ case. We have proven that the algorithm converges to a local minimum under nonrestrictive assumptions. This procedure was adopted to and demonstrated for the two time scale formulation. As an additional demonstration of the breadth of the class of performance indices minimized by this algorithm, a procedure for optimally zeroing selected gain elements in an output feedback gain matrix was developed and demonstrated.

This report summarizes the main theoretical results for this period. A contractor's report is currently being prepared that will detail all the major developments, and will provide computational algorithms and numerical results that substantiate the theoretical results. A summary of conference and journal publications that have resulted from this research is provided at the end.

SECTION 2

OPTIMAL OUTPUT FEEDBACK DESIGN

In this section, the optimal output feedback problem is formulated for a class of problems which includes the standard LQ case. A convergent sequential numerical algorithm for solving the necessary conditions for optimality is described. Because the algorithm provides a sequence of monotonically improving gains, the solution obtained at convergence is locally optimal.

2.1 Problem Formulation and Necessary Conditions for Optimality

We consider systems of the form

$$\dot{x} = Ax + Bu \qquad x(0) = x_o \tag{2.1}$$

where $x \in \mathcal{R}^n$ and $u \in \mathcal{R}^m$, with output

$$y = Cx \tag{2.2}$$

where $y \in \mathcal{R}^p$. The control has the form

$$u = -Gy \tag{2.3}$$

The gain G is to be chosen to minimize

$$J = \int_o^\infty x^T Qx + u^T Ru \, dt + \gamma(G) \tag{2.4}$$

where $Q = \Gamma^T\Gamma$ such that the pair (Γ, A) is detectable, and $R > 0$. In addition, it will be seen that, in order to avoid singularity in the necessary conditions for optimization problem, we must have

$$\rho(C) = p \tag{2.5}$$

In (2.4), $\gamma(G)$ is any scalar function having a continuous gradient in G, and for which J is bounded below, for all G which render the closed loop dynamics (2.1-2.3) asymptotically stable.

It is well known that the integral portion of J satisfies the relation

$$\int_0^\infty x^T Q x + u^T R u \; dt = tr\{K x_o x_o^T\} \tag{2.6}$$

where $K > 0$ is the unique solution of

$$S(G,K) = \hat{A}^T K + K\hat{A} + Q + C^T G^T R G C = 0 \tag{2.7}$$

$$\hat{A} = A - BGC \tag{2.8}$$

and A is asymptotically stable. It is customary to relieve (2.6) of its dependence on x_o by assuming that it is uniformly distributed on the unit sphere; then the problem statement is modified slightly to that of minimizing $E\{J\}$. This amounts to replacing $x_o x_o^T$ in (2.6) by I.

The minimization of (2.4) is now cast, as a static optimization problem, in which the Lagrangian

$$\mathcal{L}(G;K,L) = tr\{K\} + \gamma(G) + tr\{S(G,K)L^T\} \tag{2.9}$$

is minimized with respect to G, $\bar{K}$ and L, where L is a matrix of Lagrange multipliers. If the system (2.1-2.3) can be stabilized by output feedback, the first order necessary conditions for optimality are

$$\left.\frac{\partial \mathcal{L}}{\partial G}\right|_* = 0 \qquad \left.\frac{\partial \mathcal{L}}{\partial K}\right|_* = 0 \qquad \left.\frac{\partial \mathcal{L}}{\partial L}\right|_* = 0 \tag{2.10}$$

where the *'s mean that the gradients are evaluated at the optimal values of G, K and L. In the sequel, the * notation is suppressed since the gradients are assumed evaluated at their optimal values unless specified otherwise. Defining the gradient of $\gamma(G)$

$$\frac{\partial \gamma(G)}{\partial G} = \gamma_G(G) \tag{2.11}$$

the expansion of (2.10) is

$$RGCLC^T - B^T KLC^T + \frac{1}{2}\gamma_G(G) = 0 \tag{2.12}$$

$$\hat{A}L + L\hat{A}^T + I = 0 \tag{2.13}$$

$$S(G,K) = 0 \quad , \tag{2.14}$$

From (2.12), the optimal value of G will satisfy

$$G^* = R^{-1}[B^TKLC^T - \gamma_G(G)](CLC^T)^{-1} \tag{2.15}$$

where $(CLC^T)^{-1}$ exists because of (2.5) and the fact that $L > 0$ in (2.13).

2.2 A Convergent Numerical Algorithm

The following algorithm suggests itself for solving (2.12-2.14):

0. Choose any G such that $\hat{A}$ is Hurwitzian. Set $i = 0$.
1. Solve (2.13,2.14) for K_i and L_i.
2. On the basis of (2.15), evaluate

$$\Delta G_i = R^{-1}[B^TK_iL_iC^T - \tfrac{1}{2}\gamma_G(G_i)](CL_iC^T)^{-1} - G_i \tag{2.16}$$

3. Set

$$G_{i+1} = G_i + \alpha\Delta G_i \tag{2.17}$$

where $\alpha \in (0,1]$ is chosen to ensure that

$$J_{i+1} < J_i = tr\{K_i\} + \gamma(G_i) \tag{2.18}$$

4. Set $i = i + 1$ and go to 1.

This is a very simple procedure to implement, since it only involves the solution of two Lyapunov equations. The unfortunate necessity of supplying an initial stabilizing gain for step 0 is shared by other sequential algorithms currently available.

The following theorem has been proven:

Theorem 2.1: For the optimal output feedback problem defined in (2.1-2.4), let the following conditions be satisfied:

i) $\mathcal{G} = \{G : A \text{ is Hurwitzian}\} \neq$

ii) $\rho\{C\} = p$

iii) $Q = \Gamma^T\Gamma$ such that (Γ, A) is detectable; $R > 0$

iv) $\gamma(G)$ is C^1 for all $G \in \mathcal{G}$

v) If $\gamma(G) \to -\infty$ for all $\|G \varepsilon \mathcal{G}\| \to \infty$, then it does so in such a way that $|\gamma(G)|/\mathrm{tr}\{K\} < 1$

If (i-v) are true, then the sequence $\{G_i : i = 0,1,\ldots\}$ of stabilizing gains defined by (17) exists for any $G_0 \varepsilon \mathcal{G}$, such that (2.18) is satisfied at each iteration. Moreover, the sequence converges to a stationary point in J.

Note that (i-iii) are the standard conditions required for solving the LQ optimal output feedback problem. Loosely speaking, (v) means that, in choosing $\gamma(G)$, one must be certain that it does not become negatively unbounded at a faster rate than $\mathrm{tr}\{K\}$ becomes positively unbounded for $\|G\| \to \infty$. Recall that, because of (iv), $\gamma(G)$ cannot assume unbounded values for finite G. It should also be noted that, while the theorem does not rule out the theoretical possibility of convergence to a saddle point in J, encountering a saddle in practice would only slow the convergence to a local minimum of J, since the saddle point would be unstable in G.

SECTION 3

SPT IN OUTPUT FEEDBACK

In this section SPT is employed to decompose an ill-conditioned closed-loop output feedback system into its slow and fast subsystems. In the process of doing so, we gain some insight into the well-posedness of the SPT-approximate design problem.

3.1 Problem Formulation

Consider the system

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u \quad x_1(0) = x_{10} \quad x_1 \in R^{n1} \tag{3.1}$$

$$\varepsilon\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2u \quad x_2(0) = x_{20} \quad x_2 \in R^{n2} \tag{3.2}$$

where $0 < \varepsilon << 1$, with output

$$y = C_1x_1 + C_2x_2 \qquad y \varepsilon R^p \tag{3.3}$$

The feedback law is

$$u = -Gy \qquad u \varepsilon R^m \tag{3.4}$$

If A_{22} is invertible, a reduced order approximation of (3.1-3.3) can be obtained by setting $\varepsilon = 0$ in (3.2):

$$\dot{\xi} = A_o + B_ou \qquad \xi \varepsilon R^{n1} \tag{3.5}$$

$$\bar{y} = C_o\xi + D_ou \tag{3.6}$$

where

$$A_o = A_{11} - A_{12}A_{22}^{-1}A_{21} \qquad B_o = B_1 - A_{12}A_{22}^{-1}B_2$$

$$C_o = C_1 - C_2A_{22}^{-1}A_{21} \qquad D_o = -C_2A_{22}^{-1}B_2 \tag{3.7}$$

Substituting (3.4) in (3.1,3.2) and setting $\varepsilon = 0$, the reduced feedback control is expressed as

$$\bar{u} = -G^oC_o\xi \tag{3.8}$$

$$G^o = (I + GD_o)^{-1}G \tag{3.9}$$

which necessitates the assumption

$$\rho(I + GD_o) = m \tag{3.10}$$

The inverse of (9) is

$$G = G^{o}(I - D_{o}G^{o})^{-1} \tag{3.11}$$

The following lemma was proven, which states that satisfaction of the invertibility conditions for (3.9) and (3.11) is simultaneous, and that this guarantees local one-to-one correspondence between G^{o} and G.

Lemma 3.1:

$$\rho(I - D_{o}G^{o}) = p \quad \text{iff} \quad \rho(I + GD_{o}) = m;$$

furthermore, these conditions are necessary and sufficient for G^{o} and G to be locally one-to-one.

The next lemma was proven, which assures that (3.10) will hold for any G not rendering the fast closed-loop system singular.

Lemma 3.2: Given that A_{22} is nonsingular,

$$\rho(I + GD_{o}) = m \quad \text{iff} \quad \rho(A_{22} - B_{2}GC_{2}) = n_{2}$$

In summary, Lemmas 3.1 and 3.2 assure that the inverses in (3.9,3.11) exist for any realistic design problem. Indeed, if $A_{22} - B_{2}GC_{2}$ were singular, the fast subsystem dynamics would not be "fast". It should be noted that if (3.9) and (3.11) did not define a unique correspondence between G^{o} and G, reduced order approximations would have very little utility in output feedback design.

3.2 Asymptotic Properties

The closed-loop system matrix for (3.1-3.4) takes the form

$$\hat{A} = \begin{bmatrix} A_{11} - B_{1}GC_{1} & A_{12} - B_{1}GC_{2} \\ (A_{21} - B_{2}GC_{1})/\varepsilon & (A_{22} - B_{2}GC_{2})/\varepsilon \end{bmatrix} \tag{3.12}$$

Next, construct an invertible transformation which block diagonalizes A:

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = T(\varepsilon) \begin{bmatrix} x_{1} \\ x_{2} \end{bmatrix} \tag{3.13.a}$$

$$T(\varepsilon) = \begin{bmatrix} I-\varepsilon HN & -\varepsilon H \\ N & I \end{bmatrix} \qquad T^{-1}(\varepsilon) = \begin{bmatrix} I & \varepsilon H \\ -N & I-\varepsilon NH \end{bmatrix} \tag{3.13.b}$$

In (3.13), ξ is exclusively the slowly varying portion of the closed loop state and η is the fast transient. After some algebra, it can be shown that

$$N(\varepsilon) = A_{22}^{-1}(A_{21} - B_2G^oC_o)$$
$$+ \varepsilon(I+A_{22}^{-1}B_2G^oC_2)A_{22}^{-2}(A_{21}-B_2G^oC_o)(A_o-B_oG^oC_o) + O(\varepsilon^2) \tag{3.14}$$
$$H(\varepsilon) = (A_{12} - B_oG^oC_2)A_{22}^{-1} + O(\varepsilon) \tag{3.15}$$

These expressions can easily be verified if one recalls the definitions in (3.7) and uses the fact that, if A_{22}^{-1} exists,

$$(A_{22} - B_2GC_2)^{-1} = (I + A_{22}^{-1}B_2G^oC_2)A_{22}^{-1} \tag{3.16}$$

Using (3.13) in (3.12), the dynamics are decoupled:

$$\dot{\xi} = [(A_o-B_oG^oC_o)+O(\varepsilon)]\xi \qquad \xi(0) = x_{10} \tag{3.17}$$
$$\varepsilon\dot{\eta} = [(A_{22}-B_2GC_2)+O(\varepsilon)]\eta \qquad \eta(0) = x_{20}-A_{22}^{-1}(A_{21}-B_2G^oC_o)x_{10}+O(\varepsilon) \tag{3.18}$$

so that, for ε sufficiently small,

$$\xi(t) = \exp[(A_o - B_oG^oC_o)t]\xi(0) + O(\varepsilon) \tag{3.19}$$
$$\eta(t) = \exp[(A_{22} - B_2GC_2)t/\varepsilon]\eta(0) + O(\varepsilon) \tag{3.20}$$

Employing $T^{-1}(\varepsilon)$ from (3.13) to transform back to x_1, x_2, we obtain

$$x_1(t) = \xi(t) + O(\varepsilon) \tag{3.21}$$
$$x_2(t) = -A_{22}^{-1}(A_{21} - B_2G^oC_o)\xi(t) + \eta(t) + O(\varepsilon) \tag{3.22}$$

Similarly, $T^{-1}(\varepsilon)$ transforms u as defined by (3.3,3.4):

$$u(t) = -G^oC_o\xi(t) - GC_2\eta(t) + O(\varepsilon) \tag{3.23}$$

This development is summarized in the following theorem:

Theorem 3.1: If $A_{22} - B_2GC_2$ is Hurwitzian, then (3.21-3.23) describe the full order system and control trajectories for all finite $t \geq 0$. Additionally, if $A_o - B_oG^oC_o$ is Hurwitzian, then (3.21-3.23) are true for all $t \geq 0$.

An immediate (and crucial) consequence of this theorem is that, for sufficiently small ε, output feedback stabilizability of the full system (3.1-3.4) is equivalent to joint output feedback stabilizability of both subsystems. Note that the output feedback problem does not naturally decompose into separate slow and fast designs. Instead, G^o and G must stabilize the separate systems (3.17,3.18) while satisfying the hard constraint (3.9).

SECTION 4

NEAR-OPTIMAL OUTPUT FEEDBACK REGULATION

In this section, for the ill-conditioned system dynamics of Section 3, the block diagonalizing transformation $T(\varepsilon)$ from (3.13) is applied to the quadratic performance criterion of Section 2. If the slow subsystem measurements are nonredundant Minimizing the transformed criterion at $\varepsilon = 0$ results in a gain solution which yields a second order approximation to optimal full system performance, while eliminating the dimensionality and ill conditioning difficulties of minimizing directly for the full system dynamics.

4.1 Definition of the Approximate Problem

The performance index for the full order system (3.1-3.4) is

$$J = \int_0^\infty [x_1^T, x_2^T] \, Q \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + u^T R u \; dt \tag{4.1}$$

where $R > 0$ and $Q = \Gamma^T\Gamma$ such that (Γ, A) is detectable. Q is compatibly partitioned as

$$Q = \begin{bmatrix} Q_1 & Q_2^T \\ Q_2 & Q_3 \end{bmatrix} \tag{4.2}$$

Assuming that the closed-loop system matrix $\hat{A}$ in (3.12) is asymptotically stable, then (4.1) is equivalent to

$$J = \mathrm{tr}\{K x_o x_o^T\} \tag{4.3}$$

where x_o^T is $[x_{10}^T, x_{20}^T]$, and $K > 0$ is the unique solution of

$$\hat{A}^T K + K\hat{A} + \hat{Q} = 0 \tag{4.4}$$

$$K = \begin{bmatrix} K_1 & \varepsilon K_2^T \\ \varepsilon K_2 & \varepsilon K_3 \end{bmatrix} \tag{4.5}$$

$$\hat{Q} = \begin{bmatrix} Q_1 + C_1^T G^T R G C_1 & Q_2^T + C_1^T G^T R G C_2 \\ Q_2 + C_2^T G^T R G C_1 & Q_3^T + C_2^T G^T R G C_2 \end{bmatrix} \tag{4.6}$$

The problem of minimizing (4.3) with respect to G can be decomposed by using $T^{-1}(\varepsilon)$ from (3.13) to transform the coordinates from x_1, x_2 to ξ and η. After transformation, (4.4) decouples into:

$$S_1(G_o, K_1, \varepsilon) = \bar{A}_o^T \bar{K}_1 + \bar{K}_1 \bar{A}_o + \bar{Q}_1 = 0 \tag{4.7}$$

$$\bar{A}_{22}^T \bar{K}_2 + \bar{K}_2 \bar{A}_o + \bar{Q}_2 = 0 \tag{4.8}$$

$$S_3(G, K_3, \varepsilon) = \bar{A}_{22}^T \bar{K}_3 + \bar{K}_3 \bar{A}_{22} + \bar{Q}_3 = 0 \tag{4.9}$$

$$\bar{A}_o = A_o - B_o G^o C_o + O(\varepsilon) \tag{4.10}$$

$$\bar{A}_{22} = A_{22} - B_2 G C_2 + O(\varepsilon) \tag{4.11}$$

$$\bar{Q}_1 = Q_1 - N^T Q_2 - Q_2^T N + N^T Q_3 N + C_o^T G^{oT} R G^o C_o + O(\varepsilon) \tag{4.12}$$

$$\bar{Q}_2 = Q_2 - Q_3 N + C_2^T G^T R G^o C_o + O(\varepsilon) \tag{4.13}$$

$$\bar{Q}_3 = Q_3 + C_2^T G^T R G C_2 + O(\varepsilon) \tag{4.14}$$

As in Section 2, we could remove the dependence of (4.3) on initial conditions by assuming that they are uniformly distributed on the unit sphere. The problem statement is then modified slightly to that of minimizing $E\{J\}$, which amounts to replacing $x_o x_o^T$ in (4.3) by the identity matrix. For the two time scale problem, we instead assume that $[\xi^T(0), \eta^T(0)]$ is uniformly distributed on the unit sphere. This is because, under transformation by $T(\varepsilon)$ at $\varepsilon = 0$, the former assumption leads to

$$E\left\{\begin{matrix}\xi(0)\\ \eta(0)\end{matrix} \quad [\xi^T(0)\eta^T(0)]\right\} = T(0)E\{x_o x_o^T\}T^T(0)$$

$$= \begin{bmatrix} I & N^T \\ N & I + NN^T \end{bmatrix} \tag{4.15}$$

which is inconveniently complicated. It should be noted from (4.3,4.5) and (3.13) that the difference between the costs resulting from either assumption is only $O(\varepsilon)$; further, the results from this section can be extended to any assumption on the initial condition.

The transformed cost for this problem is

$$J = tr\{\overline{K}_1\} + \varepsilon tr\{\overline{K}_3\} \tag{4.16}$$

Now, note that the fast subsystem performance measure is, not unexpectedly, $O(\varepsilon)$. At $\varepsilon = 0$, where we would like to approximate the system dynamics, there is no cost associated with fast dynamics. On the other hand, minimization of $tr\{\overline{K}_1(\varepsilon = 0)\}$ with respect to G^o must be done over the set of gains which would also stabilize $\overline{A}_{22}$, subject to (3.11). In order to do this in a rational way, we instead minimize

$$J^o = tr\{\overline{K}_1(\varepsilon = 0)\} + \varepsilon^o tr\{\overline{K}_3(\varepsilon = 0)\} \tag{4.17}$$

where ε^o is fixed as the value of ε in (3.2). In fact, minimizing (4.17) allows simultaneous near-optimization of the slow and fast dynamics for essentially the same level of computational effort that would have been required to minimize $tr\{\overline{K}_1(\varepsilon = 0)\}$ alone, subject to the asymptotic stability of the fast subsystem. This situation contrasts dramatically with that seen in the singularly perturbed state feedback optimization problem. There, because of the complete decoupling of the slow and fast subsystems, the control designer has the option of only calculating gains for the slow dynamics, if the fast dynamics are open-loop stable and if an $O(\varepsilon)$ approximation to optimal system performance is satisfactory. Even if the fast dynamics require stabilization, this is done as a task totally divorced from the slow subsystem design, and without using information about ε. Here, in the output feedback problem, the constraint (3.9) inseparably links the slow and fast subproblems.

It is fairly obvious that a gain $\tilde{G}$ minimizing J^o, when applied to the full-order dynamics (3.1-3.4), will provide an $O(\varepsilon)$ approximation to actual optimal performance. In cases where $\rho\{C_o\} = p$, however, it is possible to make a stronger statement about the near-optimality of the approximate gain:

Theorem 4.1: Given that $\rho\{C\} = p$, assume that $\rho\{C_o\} = p$. Let G^* be such that $J(G^*) \leq J(G)$ for J given by (4.16) and the dynamics (3.1-3.4). Let $\tilde{G}$ be such that $J^o(\tilde{G}) \leq J^o(G)$ for J^o given by (4.17) and the dynamics (3.17,3.18) at $\varepsilon = 0$. Then,

$$J(\tilde{G}) = J(G^*) + O(\varepsilon^2) \tag{4.18}$$

PUBLICATIONS

CONVERGENCE

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* Accepted for publication in IEEE Trans. on A.C.

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