EXPRESSIONS TO DETERMINE TEMPERATURES AND EMISSION MEASURES FOR SOLAR X-RAY EVENTS FROM GOES MEASUREMENTS

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SOLAR X-RAY EVENTS FROM GOES MEASUREMENTS

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Abstract

We have developed expressions which give the effective color temperatures and corresponding emission measures for solar X-ray events observed with instruments onboard any of the GOES satellites. Since 1976, these satellites have been used to monitor continuously the full-Sun X-ray emission in two broadband wavelength intervals (approximately 0.5-4 Å and 1-8 Å) with a time resolution of 3 seconds. To simulate the solar X-ray input at a variety of plasma temperatures, we used theoretical spectra provided by D. L. McKenzie. These spectra were folded through the wavelength dependent transfer functions for the two GOES detectors as given by R. F. Donnelly, R. N. Grubb, and F. C. Cowley. The resulting detector responses and their ratio as a function of plasma temperature were then fit with simple analytic curves. Over the entire range between 5 and 30 million degrees, these fits reproduce the calculated color temperatures within 2% and the calculated emission measures within 5%. With the theoretical spectra provided by McKenzie, we can determine similar expressions for any pair of broadband X-ray detectors whose sensitivities are limited to wavelengths between 0.2 and 100 Å.
Since 1976, the solar X-ray flux has been monitored continuously with instruments on board a series of Geostationary Operational Environmental Satellites (GOES). The measurements are reported as the full-Sun X-ray emission in two broadband wavelength intervals (1/2 to 4 Å and 1 to 8 Å) with a time resolution of 3 seconds. They are routinely published in the Solar–Geophysical Data Bulletin (Comprehensive Reports) and are widely available from the National Oceanic and Atmospheric Administration (NOAA) by request. We have developed expressions which give the effective color temperatures, T, and corresponding emission measures, EM, for solar X-ray bursts from these GOES data. Here, the effective color temperature is defined to be the temperature of an isothermal plasma which would produce the observed ratio of responses in the two GOES detectors.

The output current, $A_i$, of detector $i$ is related to the incident radiant flux, $F(EM,T,\lambda)$, by means of a wavelength dependent transfer function, $G_i(\lambda)$, as follows:

$$A_i = C_i \cdot \int_0^\infty G_i(\lambda) \cdot F(EM,T,\lambda) \, d\lambda,$$

where $C_i$ is a constant near unity. The measured currents are converted into the reported X-ray flux values, $B_i$, according to the expression:

$$B_i = A_i / (C_i \cdot \bar{G}_i),$$

where $\bar{G}_i$ is the wavelength-averaged transfer function for detector $i$. Because the radiant flux from an isothermal plasma can be expressed as:

$$F(EM,T,\lambda) = EM \cdot f(T,\lambda),$$

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we can combine Equations (1) through (3) to give:

$$B_i = EM \cdot \int_0^\infty G_i(\lambda) \cdot f(T, \lambda) \, d\lambda / \bar{G}_i .$$

Equation (4) can be written as:

$$B_i = EM \cdot b_i(T) .$$

It is clear that, for an isothermal plasma, the ratio $R$ of these detector responses is independent of $EM$ and is a function only of $T$, so that:

$$R(T) = B_4 / B_8 = b_4(T) / b_8(T) .$$

(Here, $i = 4$ denotes the 1/2 to 4 Å GOES detector and $i = 8$ the 1 to 8 Å detector.) Once the effective color temperature is determined from the ratio of detector responses, $b_8(T)$ can be found and Equation (6) can then be inverted to give the corresponding emission measure:

$$EM = B_8 / b_8(T) .$$

Values of $\bar{G}_i$ and $G_i(\lambda)$ for each of the GOES detectors are shown in Figure 1 as given by Donnelly et al. (1977), who also describe the satellite instrumentation and
available data base in detail. The isothermal spectra, \( f(T, \lambda) \), for plasmas with unit emission measure were provided by McKenzie (private communication) from a variety of sources as follows. The continuum and some line fluxes were taken from unpublished calculations originally done by Walker and used in Walker et al. (1974a) and Walker et al. (1974b). Satellite-line fluxes also were based on Walker's calculations, but with scaling factors from Bhalla et al. (1975). Results for Fe XVII lines were from Walker et al. (1974c), which used analyses of Loulergue and Nussbaumer (1975). Additional line fluxes were derived from the data of Kato (1976) using formulas from Tucker and Koren (1971). Finally, ionization equilibrium results were taken from Jordan (1969) and Jordan (1970). Representative spectra at three different temperatures are shown in Figure 2.

All of the above results were combined to calculate \( b_4(T) \) and \( b_8(T) \) from Equation (5) for a number of plasma temperatures between \( 4 \times 10^6 \) K and \( 30 \times 10^6 \) K. In addition, the ratio \( R \) of these responses as a function of plasma temperature was also found. To simplify the use of this procedure, analytic fits to our calculations were determined which can be written as follows:

\[
T(R) = 3.15 + 77.2 R - 164 R^2 + 205 R^3 , \tag{9}
\]

and:

\[
b_8(T) \cdot 10^{55} = -3.86 + 1.17 T - 1.31 \times 10^{-2} T^2 + 1.78 \times 10^{-4} T^3 . \tag{10}
\]

The effective emission measure can then be found directly from these fits by writing Equation (8) in the following form:

\[
EM = 10^{55} \cdot B_8 / ( b_8(T) \cdot 10^{55} ) . \tag{11}
\]
In these expressions, each $B_i$ is the reported X-ray flux enhancement above the pre-flare background in units of $W \, m^{-2}$ as measured with the appropriate GOES detector, $R$ is the ratio of the reported flux enhancements from the two detectors as defined by Equation (7), $T$ is in units of $10^6$ K, and $EM$ is in units of cm$^{-3}$. The calculated points and our fits to them for $T(R)$ and for $b_g(T)$ are shown in Figure 3 and Figure 4, respectively. Over the entire range between 5 and 30 million degrees ($0.025 \leq R \leq 0.548$), these fits reproduce the calculated color temperatures within 2% and the calculated response per unit emission measure within 5%. The fits should be relatively insensitive to details of the theoretical line-flux calculations because the response of both detectors is dominated by continuum emission, except for the response of the 1 to 8 Å detector to plasma radiation at temperatures below $10 \times 10^6$ K. Even in the latter case, however, the sum of all line emission contributes no more than 54% to the total, as shown in Figure 5. Therefore, our results should not be seriously affected by simplifying assumptions such as, for example, neglecting density sensitive effects in certain satellite lines.

Although these results were derived from parameters for the detectors on the GOES-1 satellite, they can be applied to measurements from any in the series of GOES satellites due to the designed similarity and stability of the instrumentation. The wavelength dependent transfer functions used here are estimated to be accurate within 20% (Donnelly et al., 1977). All other instrumental uncertainties and sources of error are significantly smaller.

The procedure described above is quite general. With this procedure and the theoretical spectra provided by McKenzie, we can determine similar expressions for any pair of broadband X-ray detectors whose sensitivities are limited to wavelengths between 0.2 and 100 Å.
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References


Figure Captions

Figure 1: The transfer functions for each of the two GOES detectors with nominal wavelength ranges of 1/2 to 4 Å and 1 to 8 Å.

Figure 2: Examples of the theoretical spectra provided by McKenzie for isothermal plasmas at a variety of temperatures. Emission lines were given an arbitrary width of 0.01 Å in these graphs.

Figure 3: Calculated points for the effective plasma temperatures as a function of detector response ratios, and our analytic fit to them given by Equation (9).

Figure 4: Calculated points for the response per unit emission measure of the 1 to 8 Å GOES detector as a function of temperature of the emitting plasma. Our analytic fit to these points as given by Equation (10) is also shown.

Figure 5: The contribution of all line emission to the total response of each GOES detector as a function of plasma temperature.
Figure 2
Figure 4
We have developed expressions which give the effective color temperatures and corresponding emission measures for solar X-ray events observed with instruments onboard any of the GOES satellites. Since 1976, these satellites have been used to monitor continuously the full-Sun X-ray emission in two broadband wavelength intervals (approximately 0.5-4 Å and 1-8 Å) with a time resolution of 3 seconds. To simulate the solar X-ray input at a variety of plasma temperatures, we used theoretical spectra provided by D. L. McKenzie. These spectra were folded through the wavelength dependent transfer functions for the two GOES detectors as given by R. F. Donnelly, R. N. Grubb, and F. C. Cowley. The resulting detector responses and their ratio as a function of plasma temperature were then fit with simple analytic curves. Over the entire range between 5 and 30 million degrees, these fits reproduce the calculated color temperatures within 2% and the calculated emission measures within 5%. With the theoretical spectra provided by McKenzie, we can determine similar expressions for any pair of broadband X-ray detectors whose sensitivities are limited to wavelengths between 0.2 and 100 Å.