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Reliability Considerations for the Total Strain Range Version of Strainrange Partitioning

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Tucson, Arizona

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SUMMARY

A total strainrange version of strainrange partitioning (SRP) was proposed by Halford and Saltsman to enhance the manner in which SRP is applied to life prediction. This report describes, for the SRP model, how advanced reliability technology can be applied to (a) perform risk analysis and (b) to derive safety check expressions.

Uncertainties existing in the design factors associated with life prediction of a component which experiences the combined effects of creep and fatigue can be identified; (a) inherent uncertainty in material behavior, (b) statistical uncertainty associated with parameter estimates resulting from small samples of fatigue specimens, (c) modelling error associated with the SRP model, (d) data scatter in the environment, e.g., loads, temperatures, hold times, (e) modelling error associated with service strain analysis. Examples are presented which illustrate how reliability analyses of such a component can be performed when all design factors in the SRP model are random variables reflecting these uncertainties.

Using the Rackwitz-Fiessler and Wu algorithms, estimates of the safety index β and the probability of failure p_f are demonstrated for an SRP problem. Methods of analysis of creep-fatigue data with emphasis on procedures for producing synoptic statistics are presented. An attempt was made to demonstrate the importance of the contribution of the uncertainties associated with small sample sizes (fatigue data) to risk estimates. In the example presented, the influence of such statistical uncertainty was small.

Finally, an illustration of the procedure for deriving a safety check expression for possible use in a design criteria document was presented. The format employs partial safety factors (PSF) which are derived from reliability analyses. The safety check inequality has the appearance of a "conventional" design requirement, and therefore is familiar to designers.

NOTATION

a	Constant in linear model; defined by Eq. A.2
$\hat{a}$	Least squares estimator of a
a_i	Coefficients; Eq. D.6
A	Coefficient of strain-life curve; See Eq. A.1
A_i	Coefficient of strain-life curves; defined in Eq. 6
$\tilde{A}_i$	Median of A_i
b	Constant in linear model; defined by Eq. A.2; also exponent in inelastic strain-life curve
$\hat{b}$	Least squares estimator of b
B_{PP}	Coefficient of elastic strain-life curve in which only PP strain is present; defined by Eq. 10
$\tilde{B}_{PP}$	Median value of B_{PP}
c	1/b
CC	Hysteresis loop in which tensile creep reversed by compressive creep
CP	Hysteresis loop in which tensile creep reversed by compressive plasticity
C_{A_i}	COV of A_i
COV	Coefficient of variation
d	Exponent of elastic strain-life curves; defined by Eqs. 10 and 11
D_i	Coefficient of $B_{PP} - B_i$ relationship; defined in Eq. 12
EVD	Type I extreme value distribution of maxima
f_i	Fraction of the total of each strain range type i = PP, PC, CP, CC
f_{ε_T}	Probability density function of $\Delta\varepsilon_T$
f_{ε_S}	Probability density function of $\Delta\varepsilon_S$
$\overline{f}_{PP}$	Mean value of f_{PP}

NOTATION - (continued)

$\bar{f}_{PC}$	Mean value of f_{PC}
g	Function which accounts for statistical scatter; defined in Eq. A.8; also Eq. B.3
G	A random variable which quantifies modelling error in computing service strain range
H	A random variable which quantifies material behavior uncertainties in computing service strain range
J	Coefficient of inelastic strain-life relationship; defined by Eqs. 8 & 9
LN	Lognormal distribution
n	Sample size
N	Cycles to failure; also normal distribution
N_o	Service life
N_i	Cycles to failure for i^{th} strain range type; i = PP, CP, PC, CC
p_f	Probability of failure in service life N_o
p_o	Target risk or probability of failure
$P(\cdot)$	Probability of
PC	Hysteresis loop in which tensile plasticity reversed by compressive creep
PP	Hysteresis loop in which tensile plasticity reversed by compressive plasticity
PSF	Partial safety factors
Q	Load (or nominal stress) range on the component
s	Sample standard deviation; estimate of σ
SRP	Strain range partitioning
t	Time
$t_{\alpha;n-1}$	Students' t variate
T	Hold time; in general a random variable
$\bar{T}$	Mean value of T

NOTATION (continued)

u_i	Reduced coordinate; defined in Eq. D.3
X	$\mathrm{Log}_{10}\Delta\varepsilon$
X_i	$\mathrm{Log}_{10}\Delta\varepsilon_i$ where i refers to i^{th} specimen
Y	$a + bx$; defined by Eq. A.2; also $\log_{10}N$
$\hat{v}$	Least squares line
Y_i	$\mathrm{Log}_{10}N_i$ where i refers to i^{th} specimen
z_α	Standard normal variate
*	As a superscript, refers to design point for that variable
α	Reference level for g; See Eqs. A.7 and A.8 and Refs. 5 and 11
β	Safety index
β_o	Target safety index
γ	Empirical function of Θ and T; defined by Eq. 2 and Fig. 5
γ_X	Partial safety factor for variable X
δ	Exponent of $B_{PP} - B_i$ relationship; defined in Eq. 12
$\Delta\varepsilon$	Total strain range
$\Delta\varepsilon_i$	Strain range; i = PP, CP, PC, CC
$\Delta\varepsilon_{in}$	Inelastic strain range
$\Delta\varepsilon_{PP}$	PP strain range
$\Delta\varepsilon_{CP}$	CP strain range
$\Delta\varepsilon_{PC}$	PC strain range
$\Delta\varepsilon_{CC}$	CC strain range
$\Delta\varepsilon_S$	Total service strain range
$\overline{\Delta\varepsilon_S}$	Mean value of $\Delta\varepsilon_S$

NOTATION - (continued)

$\Delta\varepsilon_T$	Total strain range to produce failure at life N; describes the strength of the material; defined in Eq. 14
Θ	Temperature; in general a random variable
η	Empirical function of Θ and T; defined by Eq. 2 and Fig. 5
μ_{Ni}	Equivalent normal mean
σ	Standard deviation of $Y\|X$
σ_{Ni}	Equivalent normal standard deviation
σ_o	Equivalent standard deviation
ϕ	Standard normal density function; also empirical constant defined by Eq. 2 and Fig. 5
Φ	Standard normal cumulative distribution function

TABLE OF COMPARISONS OF NOTATION

The authors of this paper used notation which differs from that of the authors (Halford and Saltsman [2]) of the advanced version of SRP. The intent of this change of notation was to simplify the presentation in a reliability format. Following is a comparison of notation of some key parameters

Symbol

Halford Saltsman	This Paper	
t	T	hold time
$\Delta\varepsilon_{el}$	$\Delta\varepsilon_{e}$	elastic strain range
$\Delta\varepsilon_{in}$	$\Delta\varepsilon_{in}$	inelastic strain range
$\Delta\varepsilon_{T}$	$\Delta\varepsilon_{T}$	total Strain range
C^{1}	J	coefficient of the inelastic strain life curve
c	c	exponent of the inelastic strain life curve
B_i	B_i	coefficient of the elastic strain life curve for the ith strain type; i = PP, PC, CP, CC
b	d	exponent of the elastic strain life curve
F_i	f_i	fraction of ith strain type to total inelastic strain
A_i	D_i	coefficient of relationship between B_{PP} and B_i
a	δ	exponent to hold time in relationship between B_{PP} and B_P

1. INTRODUCTORY COMMENTS ON STRAIN-RANGE PARTITIONING

The method of strainrange partitioning (SRP) for predicting high temperature low cycle fatigue was introduced a decade ago by Manson, Halford, and Hirschberg [1]. This scheme for making life predictions is based on explicit knowledge of the magnitudes of the inelastic creep and plastic strains present in a cycle of loading. Unfortunately for typical engineering applications, the magnitudes of the plastic strains are small and they cannot be calculated reliably from nonlinear structural analysis methods.

Halford and Saltsman have proposed a method which enhances the manner in which SRP is applied to life prediction [2]. They developed the basic Manson-Coffin plastic strain-range power law of low cycle fatigue into a total strain-range representation by the addition of the elastic and plastic strain life relationships. It is argued that this method, a total strain-range version of SRP, has the promise of more accurately estimating cyclic lifetimes over a much broader range of strains and lives than was possible on the basis of either the plastic or elastic strain-range alone.

Many uncertainties exist in the process of employing SRP for life prediction. In a broad sense, these would include (a) scatter in environmental data, and uncertainty in the computations of the environment, e.g., temperature, (b) modelling error associated with the procedures for computing loads on the components and then computing responses (stresses), (c) uncertainty in the responses of the material to the environment, (d) scatter in fatigue data, (e) modelling error of the theoretical strength model, i.e., SRP. The general goal of this study is to demonstrate how modern probabilistic design theory can be employed to predict reliabilities of components subjected to high temperature low cycle fatigue. SRP will be the basic prediction method used.

For reference purposes, the following basic definitions and descriptions of SRP are included from Ref. 3. First consider a hysteresis loop as shown in Fig. 1. Defined are the inelastic ($\Delta\varepsilon_{in}$), elastic ($\Delta\varepsilon_e$) and total ($\Delta\varepsilon$) strainranges. The basic premise for SRP is that in any hysteresis loop there are combinations of just two directions of straining and two types of inelastic strain. The two directions are, of course, tension (associated with a positive inelastic strain rate) and compression (associated with a negative inelastic strain rate); the two types of inelastic strain are time dependent (creep) and time independent (plastic). It should be noted that only a portion of transient creep strain should be considered as plastic strain and only the steady-state component be considered as creep strain [4]. By combining the two directions with the two types of strain, we arrive at four possible kinds of strainranges that may be used as basic building blocks for any conceivable hysteresis loop. These define the manner in which a tensile component of strain is balanced by a compressive component to close a hysteresis loop. The types of strain are illustrated in Fig. 2 and are described as follows:

(a) Tensile plasticity reversed by compressive plasticity is designated a PP strainrange and represented by $\Delta\varepsilon_{PP}$.

(b) Tensile creep reversed by compressive plasticity is designated a CP strainrange and represented by $\Delta\varepsilon_{CP}$.

(c) Tensile plasticity reversed by compressive creep is designated a PC strainrange and represented by $\Delta\varepsilon_{PC}$.

(d) Tensile creep reversed by compressive creep is designated a CC strainrange and represented by $\Delta\varepsilon_{CC}$.

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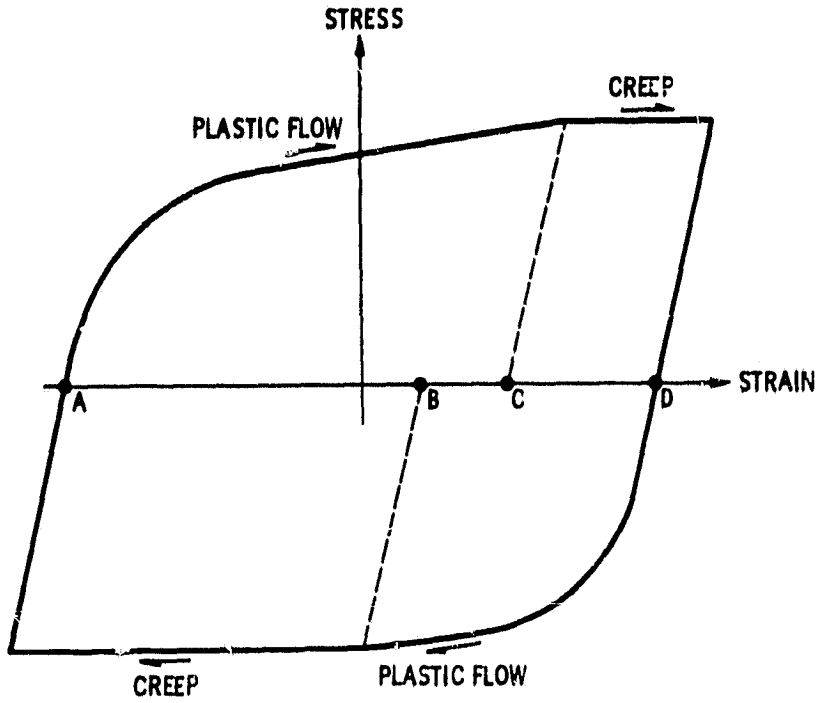

INELASTIC STRAINRANGE, $\Delta\varepsilon_{in}$ ELASTIC STRAINRANGE, $\Delta\varepsilon_e$

TOTAL STRAINRANGE, $\Delta\varepsilon$

Fig. 1. Hysteresis Loop.

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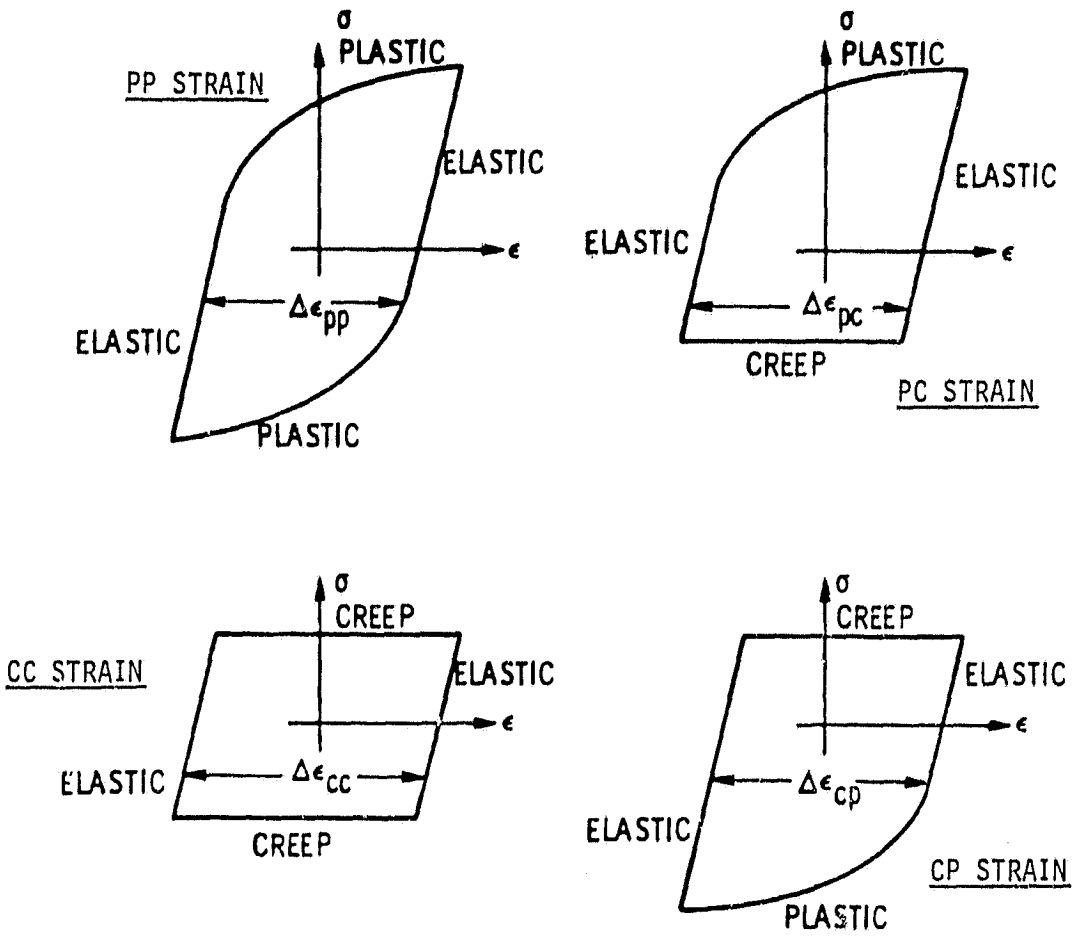

Fig. 2. Idealized hysteresis loops for the four basic types of inelastic strainrange.

The notation for the subscripts for the strainranges uses the type of tensile strain first, followed by the type of compressive strain. The name strainrange partitioning was chosen because it represented the premise that, in order to handle a complex high-temperature, low-cycle fatigue problem, the inelastic strainrange must first be partitioned into its components.

The strength of the material is described by strain life curves, an example of which is shown in Fig. 3. These relationships follow the basic Manson-Coffin law.

Given a stable hysteresis loop under constant amplitude oscillatory loading, as shown in Fig. 1, the fraction of each strainrange type f_i, a component of the total inelastic strainrange, is identified using an algorithm as described in Refs. 1 and 3. For example, $f_{PP} = \Delta\varepsilon_{PP}/\Delta\varepsilon_{in}$.

$$\Delta\varepsilon_{in} = \sum_{i=1}^{4} \Delta\varepsilon_i \qquad \sum_{i=1}^{4} f_i = 1 \qquad i = PP,\ CP,\ PC,\ CC \tag{1}$$

Finally, it should be noted that notation of the original SRP work has been changed somewhat herein. This was done for mathematical convenience in applying reliability theory.

2. UNCERTAINTIES IN THE LIFE PREDICTION PROCESS

For typical designs in a high temperature environment, the present state-of-the-art precludes an accurate deterministic definition of the environments and associated material responses. Moreover, fatigue behavior under carefully controlled conditions is characterized by significant uncertainty as evidenced by the large scatter in fatigue failure data.

The goal of this study is to cast the total strain range version of SRP into a reliability format. All sources of uncertainty will be identified. Techniques for quantifying uncertainty will be addressed. Mechanisms for

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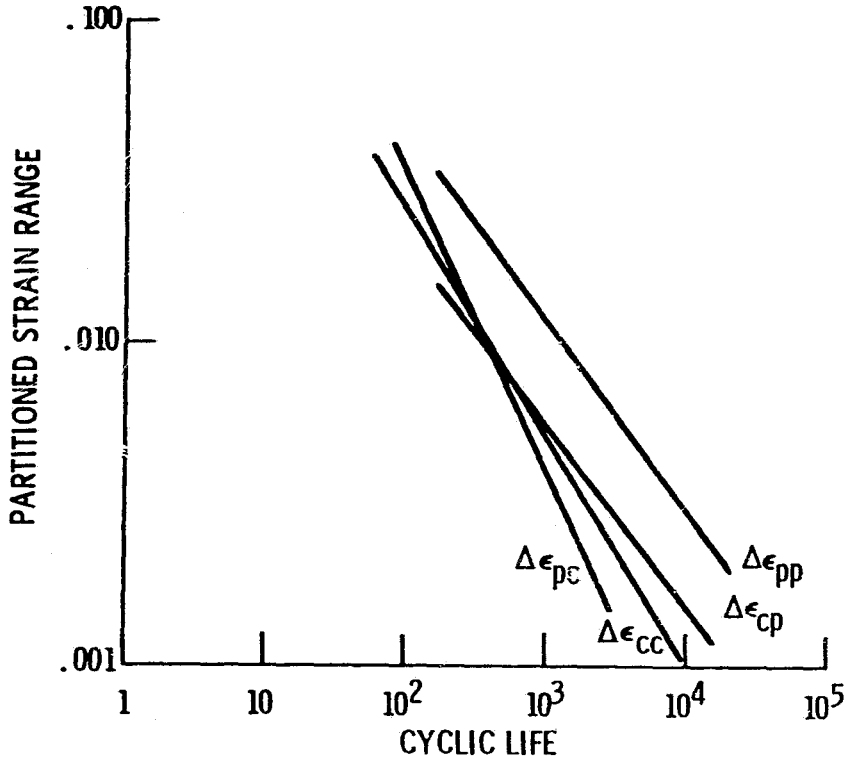
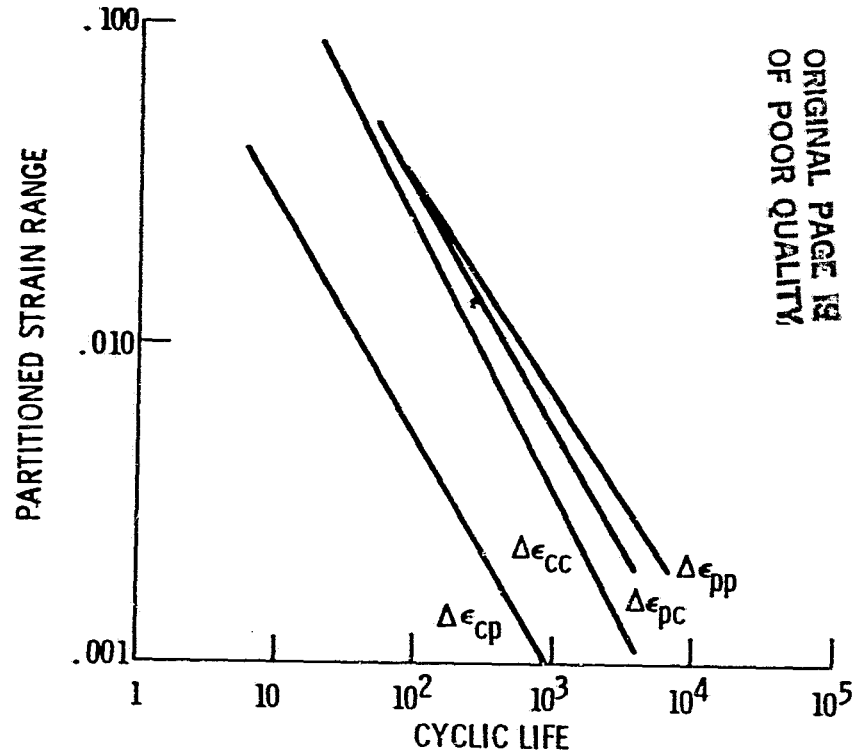


Summary of partitioned strain-life relations. $2\frac{1}{4}$ Cr - 1 Mo steel, 1100 F (865 K).



Summary of partitioned strain-life relations. Type 316 stainless steel, 1300 F (980 K).

Fig. 3. Examples of the Inelastic Strain Life Relationships for the Four Strainrange Types (after [1])

formal introduction into the limit state function will be described as will modern methods for performing the reliability analyses.

The sources of uncertainty in the process of fatigue life estimation can be identified as follows:

1. Environment. There is uncertainty in temperatures, hold times and pressures, static and centrifugal loads, etc. Stress producing environments may be random processes or deterministic processes with random magnitudes. Statistical descriptions of the environment may be available.

2. Response to the Environment. The computational methods for computing stresses will contain modelling error resulting from the assumptions made.

3. In general, strain at the fatigue critical point will be a random process. Strain range and mean strain for each hysteresis loop will be a random variable, reflecting uncertainties in material properties (e.g., Young's modulus) as well as environmental and analysis uncertainties.

4. Dividing hysteresis loops into strain types. The process of identifying the fractions of each hysteresis loop associated with each strain type (PP, PC, CP, CC) will likely contain uncertainty. The method used may not accurately reflect real strain behavior of the material.

5. Linear damage rule. The interaction damage rule used in SRP may not accurately describe fatigue behavior of the material [5]. This uncertainty is referred to as modelling error, and is associated with the theoretical model which is assumed to define strength.

6. Material behavior. Fatigue data is typically characterized by "large" scatter. Moreover, parameters used to provide statistical summaries

of the data are themselves random variables when the estimators are used to represent the parameters.

7. Material behavior, . . . other uncertainties. The fatigue strength of a material may be influenced by processing operations, e.g., cold working and heat treating), . . . and assembly operations (e.g., bolting, shrink fits). Uncertainties in material strength may result. Moreover, material strength may be influenced by time and/or by corrosion and/or extreme thermal environments to a degree which is not accurately known.

Following are discussions of the components of the SRP model and a demonstration of how modern methods can be used to perform reliability analysis on a high temperature low cycle fatigue problem.

3. SERVICE STRAIN

It is assumed that the component operates at constant temperature and that the temperature is high enough so that creep deformation must be considered. Also, it is assumed that the loading is constant amplitude. The physical problem is illustrated in Fig. 4. Assume that the load (or nominal stress) range, Q, is a random variable reflecting (a) uncertainties in the environment, and (b) modelling error in translating the effect of environment to loads on the component.

The total service strainrange $\Delta\varepsilon_S$ at the notch will be a function of Q, temperature Θ and hold time T as shown in Fig. 5. An analytical model for service strain range can be formulated as

$$\Delta\varepsilon_S = \gamma Q + (\phi Q)^{\eta} \qquad (2)$$

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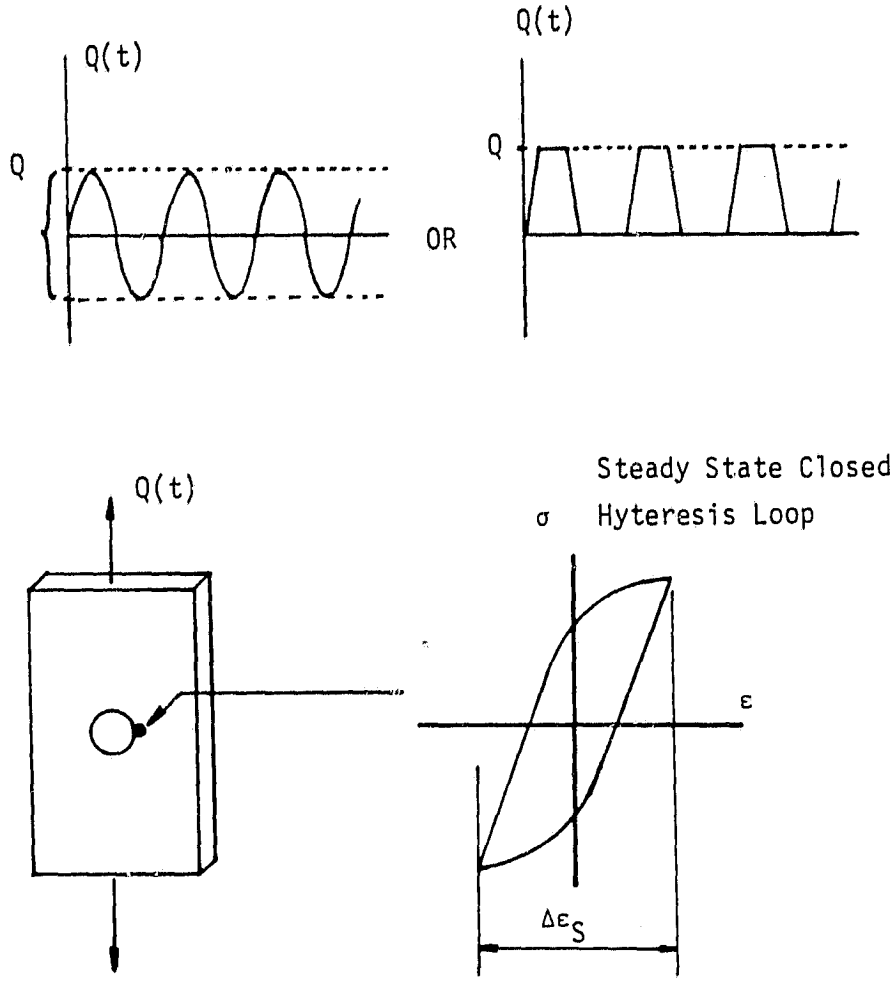

Fig. 4. Stress-Strain Behavior at a Notch.

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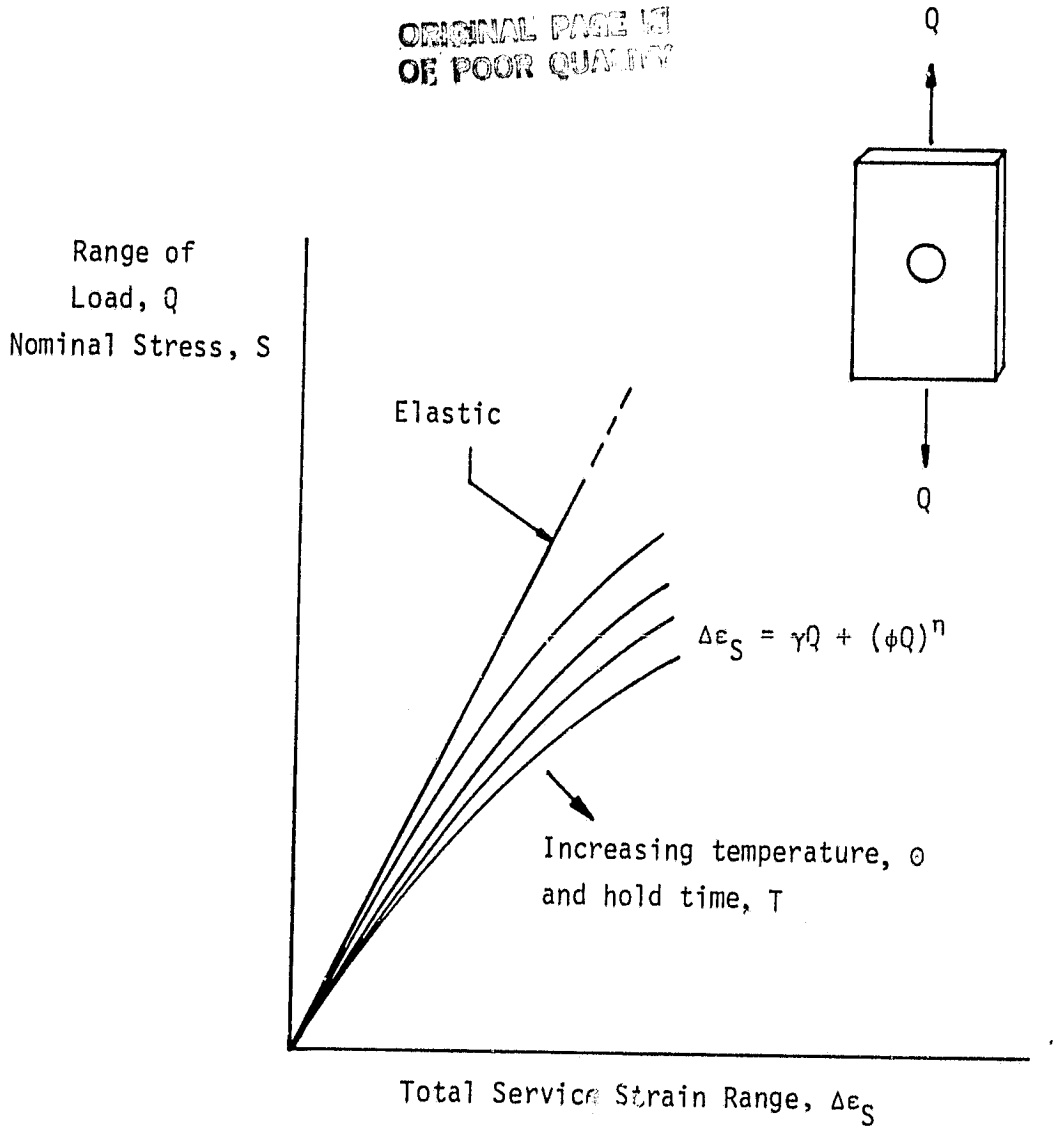

Fig. 5. Relationship Between Service Load Ranges and Notch Strain Range.

where the parameters γ, ϕ, and η may be functions of Θ and T.

$$\gamma = \gamma(\Theta,T)$$
$$\phi = \phi(\Theta,T) \qquad (3)$$
$$\eta = \eta(\Theta,T)$$

Both hold time and temperature can be random variables by virtue of uncertainties in the operating environment and perhaps the codes used for their prediction. Thus γ, ϕ, and η will, in general, be random variables.

There are two other sources of uncertainty here. First, the method by which strains are computed from load will contain modelling error. Then there will be uncertainties in the material response as could be measured from experimental data. A more general form of $\Delta\varepsilon_S$ would be

$$\Delta\varepsilon_S = GH\ \{\gamma Q + (\phi Q)^{\eta}\} \qquad (4)$$

where G and H are random variables which account for modelling error and material behavior respectively.

4. IDENTIFYING THE STRAINRANGE COMPONENTS

The SRP literature describes the mechanical procedure for quantifying the partitioned strainrange components of a complex hysteresis loop [3]. It is possible to have only three of the four types in the same loop. Let f_i; i = PP, PC, CP, CC denote the fractions of each partitioned strainrange. The sum of the fractions is unity and as an example consider

$$1 = f_{PP} + f_{PC} + f_{CC} \tag{5}$$

Each term can be considered as a random variable. Three sources of uncertainty can be identified. First, there may be uncertainty in the way that the loop is analyzed (modelling error) or there could be some error in the basic algorithm for dividing the plastic strains. It is expected that this error may be small and difficult to quantify. Nevertheless, if the f_i's can be modelled as random variables, no problem is presented to the reliability method.

But the f_i's will also depend upon hold time T and temperature Θ. If T and Θ are known random variables, and if their functional relationship to f_i can be described, then in theory, the distribution of each f_i can be derived. Fig. 6 illustrates the relationship which must be established from testing.

In the design equation, the f_i's are clearly not independent as seen from Eq. 2. For three strainrange types, two f_i's can be specified independently, and the third f_i expressed as a function of the other two. A demonstration of how to handle this in a reliability format is provided in the examples below.

5. THE STRAIN-LIFE RELATIONSHIPS. HOW SCATTER IN FATIGUE DATA IS TREATED

The inelastic strainrange-life curves are established by conventional SRP techniques. It is assumed that the data will follow a linear trend on log-log paper (the Manson-Coffin law) and that the techniques of basic linear model analysis apply. Methods of analysis for ε-N data for design purposes are summarized in Appendix A.

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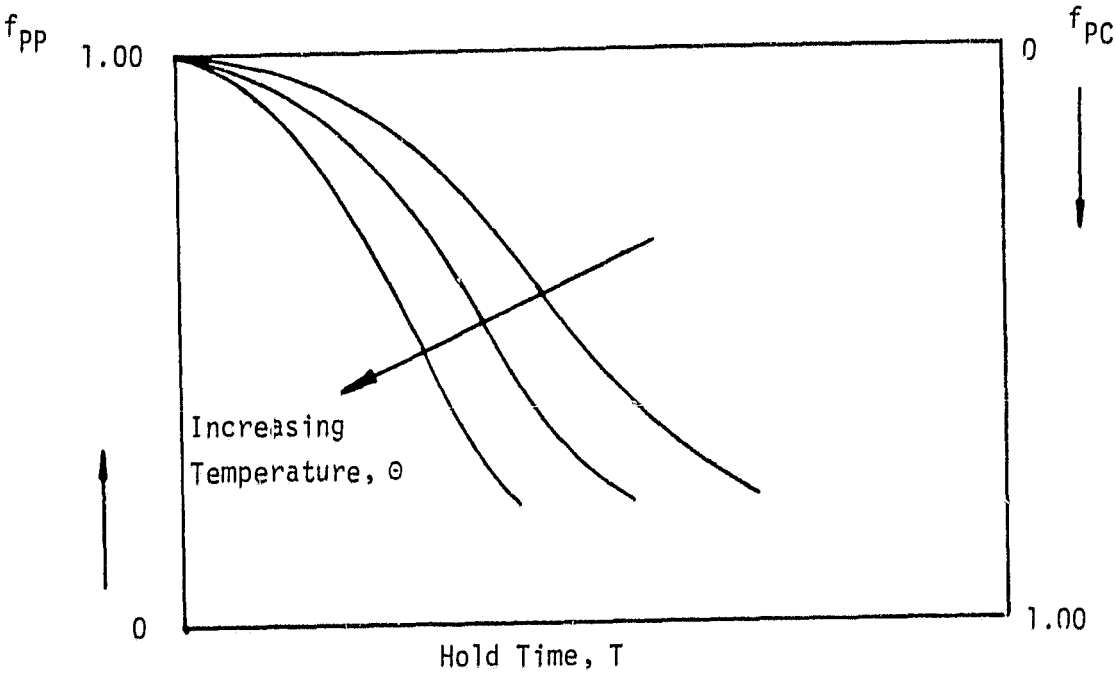

Fig. 6. An Example of How Strain Type Fractions Depend Upon Temperature and Hold Time.

As shown in Fig. 7, it is also assumed that the slope, b, of each ε-N curve is the same and equal to the PP curve (for which the sample sizes are usually much larger). Fig. 3 illustrates how this assumption may be in error, but these curves were based on a small number of points.

The empirical relationship for each strainrange type is given as

$$N_{PP} = A_1(\Delta\varepsilon_{PP})^b \tag{6}$$

$$N_{PC} = A_2(\Delta\varepsilon_{PC})^b$$

$$N_{CP} = A_3(\Delta\varepsilon_{CP})^b$$

$$N_{CC} = A_4(\Delta\varepsilon_{CC})^b$$

The ε-N curves of Fig. 7 are <u>median</u> curves through the data and are defined by the relationships given on the figure. The tildes indicate median value.

Scatter in observed fatigue data is accounted for by treating the A's as random lognormally distributed variables. The exponent b is considered to be constant. Appendices A and B describe the process of translating ε-N data into statistical parameters of the random variables A_i.

In order to construct the appropriate inelastic strain-life curve, the basic SRP model is employed [1]. The total cycles to failure, N, is

$$\frac{1}{N} = \sum_{i=1}^{4} f_i/N_i \qquad i = PP, PC, CP, CC \tag{7}$$

$$= \frac{1}{(\Delta\varepsilon_{in})^b} \sum_{i=1}^{4} f_i/A_i$$

Rearranging, the resulting ε-N relationship becomes,

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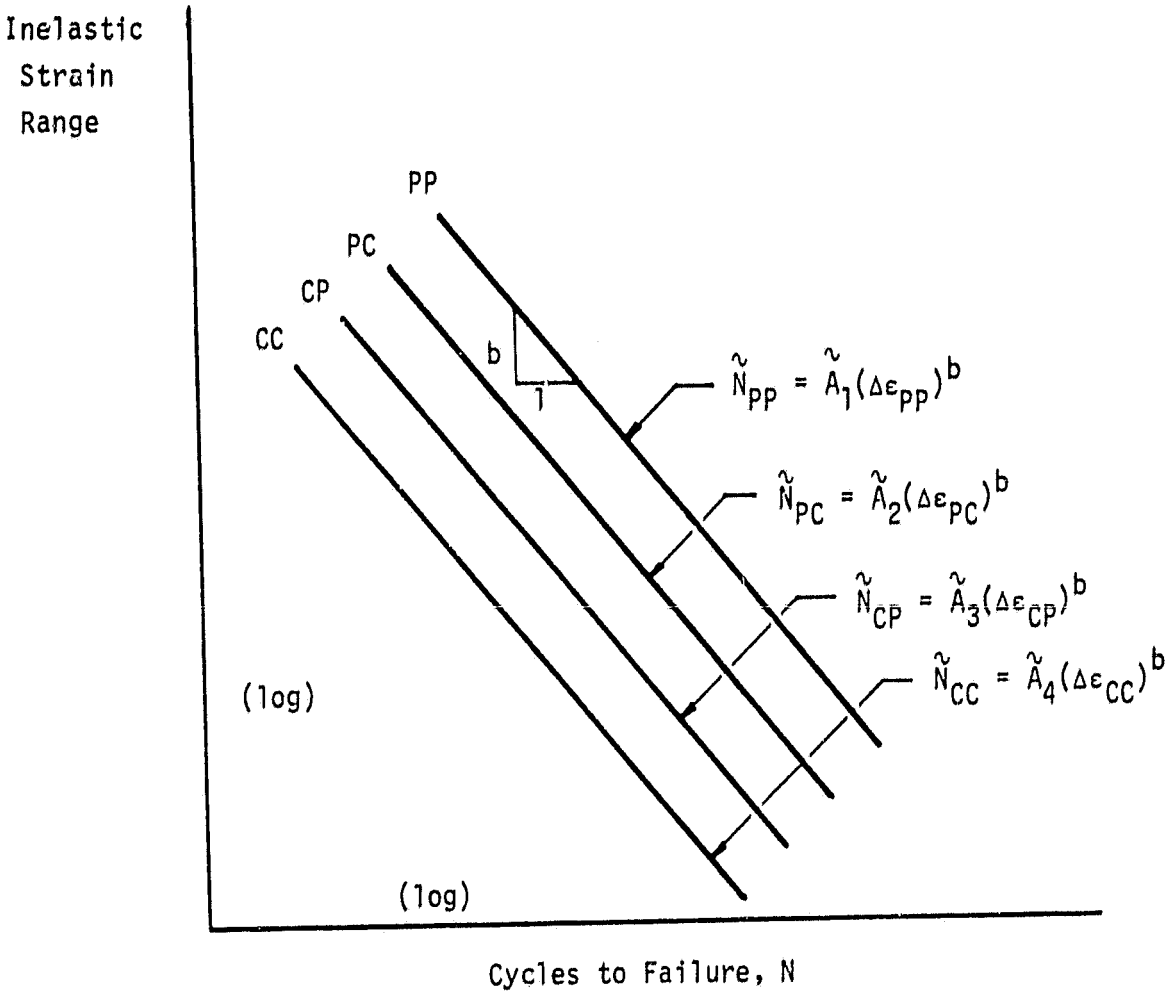

Fig. 7. Inelastic Strain-Life Curves for the Total Strainrange Version of SRP.

$$\Delta\varepsilon_{in} = JN^{c} \tag{8}$$

where,

$$J = \left[\sum_{i=1}^{4} f_i/A_i \right]^{c} \tag{9}$$

$$c = 1/b$$

Eq. 8 then provides an expression of the inelastic strain life curve. An example is given in Fig. 8 in which it is assumed that only PP and PC strain are present.

The elastic strain range-life curves are established as follows. First, the PP line is defined from the data using the method of Appendix A. The strain life model is,

$$\Delta\varepsilon_e = B_{PP}N^{d} \tag{10}$$

in which B_{PP} is a random variable and the exponent d is a constant. This curve is illustrated in Fig. 9. Then the elastic strain life curve for a given hold time and for a given constant amplitude load can be established from experimental data in the same way.

As the hold time is increased, the ε-N curves will indicate lower fatigue strength as suggested by Fig. 9. The curves will be parallel to the PP curve and will have the form

$$\Delta\varepsilon_e = B_i N^{d} \qquad i = \text{PC, CP, CC or some combination} \tag{11}$$

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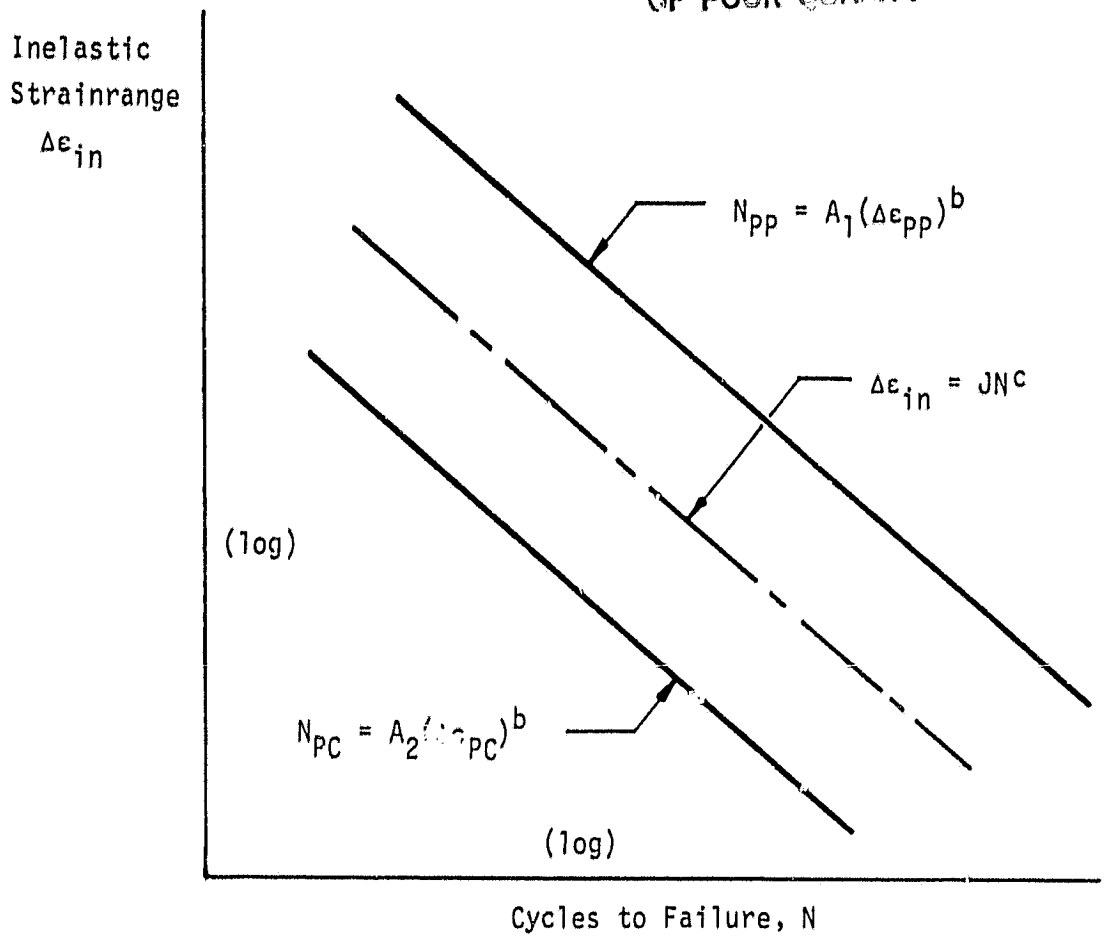

Fig. 8. An Example of an Inelastic Strain Life Curve When Only PP and PC Strain is Present.

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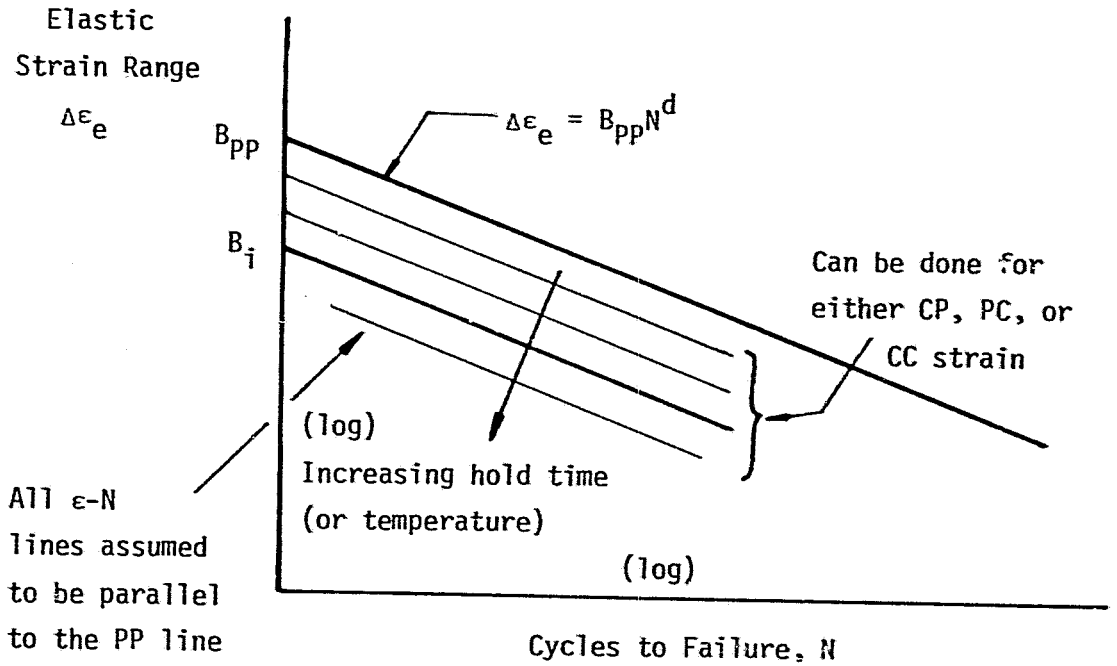

Fig. 9. The Elastic Strain Life Curves.

(Note: It is possible for another line to be above the PP line, e.g., for a material which experiences strain age hardening.)

The subscript i refers to the form of the hysteresis loop, i.e., PC, CP, CC, or a combination of these with PP.

Experimental data has suggested that the assumption of (a) parallel ε-N lines and (b) an empirical form

$$\ln[B_{PP}/B_i] = D_i T^{\delta} \tag{12}$$

are reasonable. An illustration of data which supports Eq. 12 is shown later in Fig. 16. In general, D_i and δ will be functions of hold time, temperature, and type of cycle. Using the general scheme as described in Appendix A, D_i will be a random variable, and δ will be constant. D_i and δ will be established from experimental data so that the distribution of D_i will reflect both material and statistical uncertainty.

In general, the elastic strain-life expression is given by Eq. 11, substituting B_i from Eq. 12,

$$B_i = B_{PP} \exp[-D_i T^{\delta}] \tag{13}$$

Combining the plastic and elastic strain ranges, the total strain life curve is given as

$$\Delta\varepsilon_T = \Delta\varepsilon_e + \Delta\varepsilon_{in} = B_i N^d + JN^c \tag{14}$$

The strain-life curves are illustrated in Fig. 10.

Upon substituting the expressions for J and B_i the total strain life expression becomes

$$\Delta\varepsilon_T = \{B_{PP} \exp[-D_i T^{\delta}]\} N^d + \{\sum_j f_j/A_j\}^c N^c \tag{15}$$

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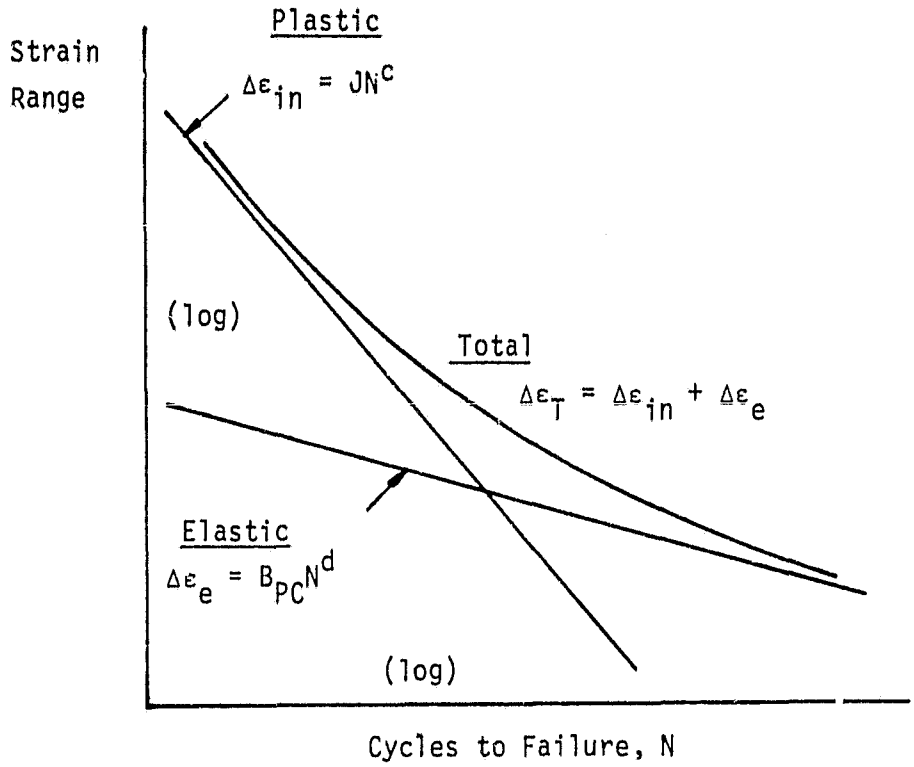

Fig. 10. The Total Strain-Life Curves.

5a

This is the definition of fatigue strength. It is assumed that B_{PP}, D_i, f_i, and A_i are random variables. Therefore, for a given life $\Delta\varepsilon_T$ also will be a random variable.

6. RELIABILITY ANALYSIS

The probability density function (pdf) of the fatigue strength, $\Delta\varepsilon_T$, denoted as f_{ε_T}, will be a function of N; it is illustrated in Fig. 11 at the intended service life N_o. The service strain range is denoted as $\Delta\varepsilon_S$. Also shown on this figure is the pdf of $\Delta\varepsilon_S$, denoted as f_{ε_S}. It is assumed that the strain range will be constant over the life of the component, but the magnitude ($\Delta\varepsilon_S$) is treated as a random variable to reflect uncertainties in the environment as well as the procedures used to compute the strains.

The event of failure is defined as ($\Delta\varepsilon_T < \Delta\varepsilon_S$), and the probability of failure is

$$p_f = P(\Delta\varepsilon_T < \Delta\varepsilon_S) \tag{16}$$

In the language of mechanical reliability, $\Delta\varepsilon_T$ is the "strength," and $\Delta\varepsilon_S$ is "stress."

EXAMPLE 1

Consider a component, subjected to a constant amplitude oscillatory stress, which is expected to experience some inelastic strain, of the PP and PC types only. Thus, the fatigue strength of the material would be defined by a special case of Eq. 15,

$$\Delta\varepsilon_T = \{B_{PP} \exp[-D_{PC}\, T^{\delta}]\}\, N^d + \{\frac{f_{PP}}{A_1} + \frac{f_{PC}}{A_2}\}N^c \tag{17}$$

which includes only PP and PC strain terms.

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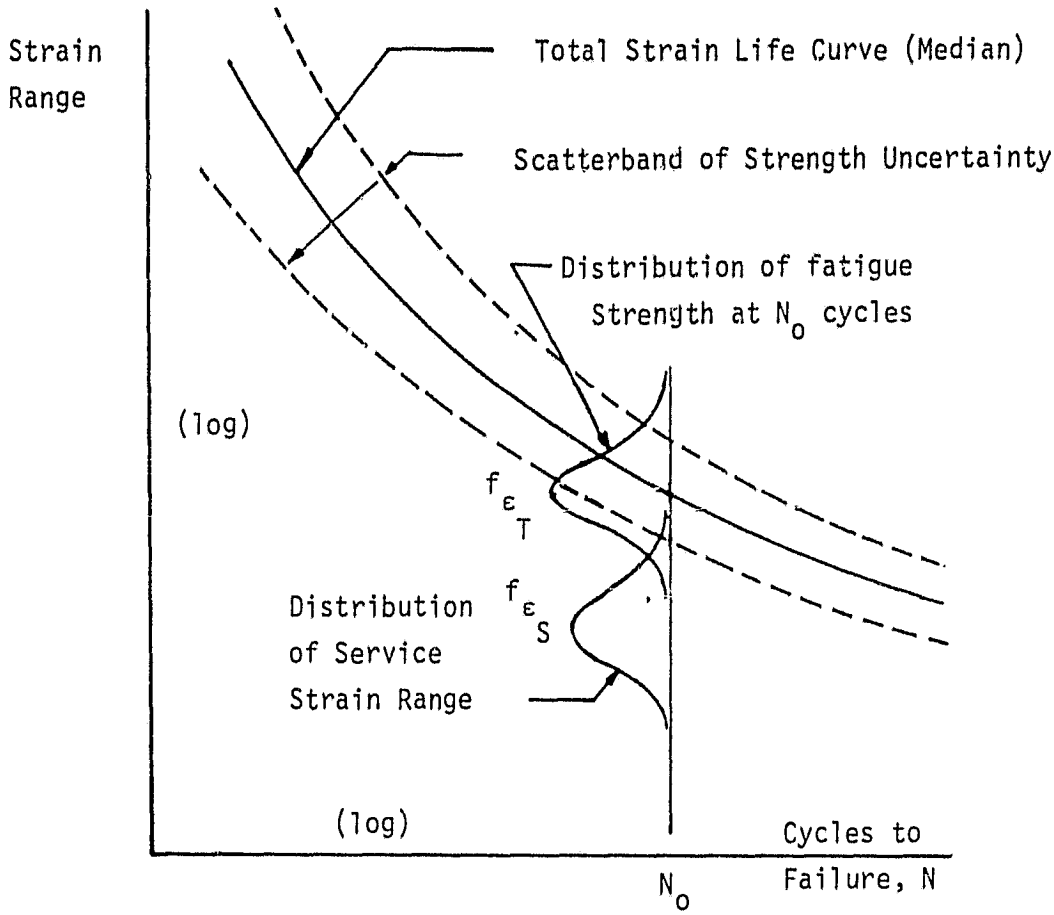

Fig. 11. Relationship Between the Distribution of Service Strain and Fatigue Strength of the Material

10a

The material is to be a nickel base alloy AF2-1DA at 1400 F (760 C). A summary of the mechanical properties is given in Table 1. This example illustrates the use of modern reliability methods to assess the structural performance of the component. For a specified life, and the associated service strainrange, it is required to estimate the safety index and the probability of failure. Data used for all design factors are summarized in Table 2. Commentary on how the parameters are determined from these data is provided in the following.

Hold time, f_{PP}, f_{PC}. Assume that there is uncertainty in the hold time, T. Thus, T can be a random variable, and for this example the coefficient of variatic was assumed to be only 5%. But f_{PP} and f_{PC} will be functions of T, and it is therefore necessary to provide explicit functions. Fig. 12 shows how such a relationship might appear. This is used for the example and is not based on actual data. In fact, such a relationship could be established from a simple test. Scatter in material behavior is not considered here. It should be noted that the f_{PP}-f_{PC}-T relationship will also depend upon temperature Θ and service strain range, $\Delta\varepsilon_S$; the latter is also a function of T and Θ. A simplified physical model is employed herein for demonstration purposes.

In this example, it is assumed that the uncertainty in hold time will be relatively small, and that in the first approximation the relationship between f_{PP} and T is the tangent to the curve at the mean value (in this example, 100 sec.) as shown in Fig. 12.

$$f_{PP} = 1.1 - 0.10 \log_{10} T \tag{18}$$

$$f_{PC} = 1 - f_{PP} \tag{19}$$

Thus, upon substituting Eqs. 18 and 19 into Eq. 17, the plastic strain life curve can be expressed as a function of T.

TABLE 1

Mechanical Properties of AF2-1DA at 1400 F (760C); Ref. 6

Young's Modulus	25×10^3 ksi
Yield Strength	123 ksi
Ultimate Strength	164 ksi
Reduction of Area	22.3%

Stress Rupture Properties

Stress (ksi)	Reduction in Area (%)	Time to Rupture (hrs)
135	15.8	1.1
130	14.6	2.1
125	15.0	196.

TABLE 2

Data for Example 1

Random Variables	Distribution (a)	Mean	COV(%)
$\Delta\varepsilon_S$	EVD	4.45E-3	20
T	N	100	5
B_{PP}	LN	0.0216(b)	9.9
D_{PC}	LN	0.0447(b)	30
f_{PP}	N	0.80	5
A_1	LN	0.0281(b)	47
A_2	LN	0.0156(b)	69

Constants	
δ	0.25
c = 1/b	-0.637
d	-0.117
N_o, service life	1,000 cycles

Notes
(a) Abbreviations

EVD Type I extreme value distribution of maxima

N Normal

LN Lognormal

(b) For lognormal variates, the median is used rather than the mean

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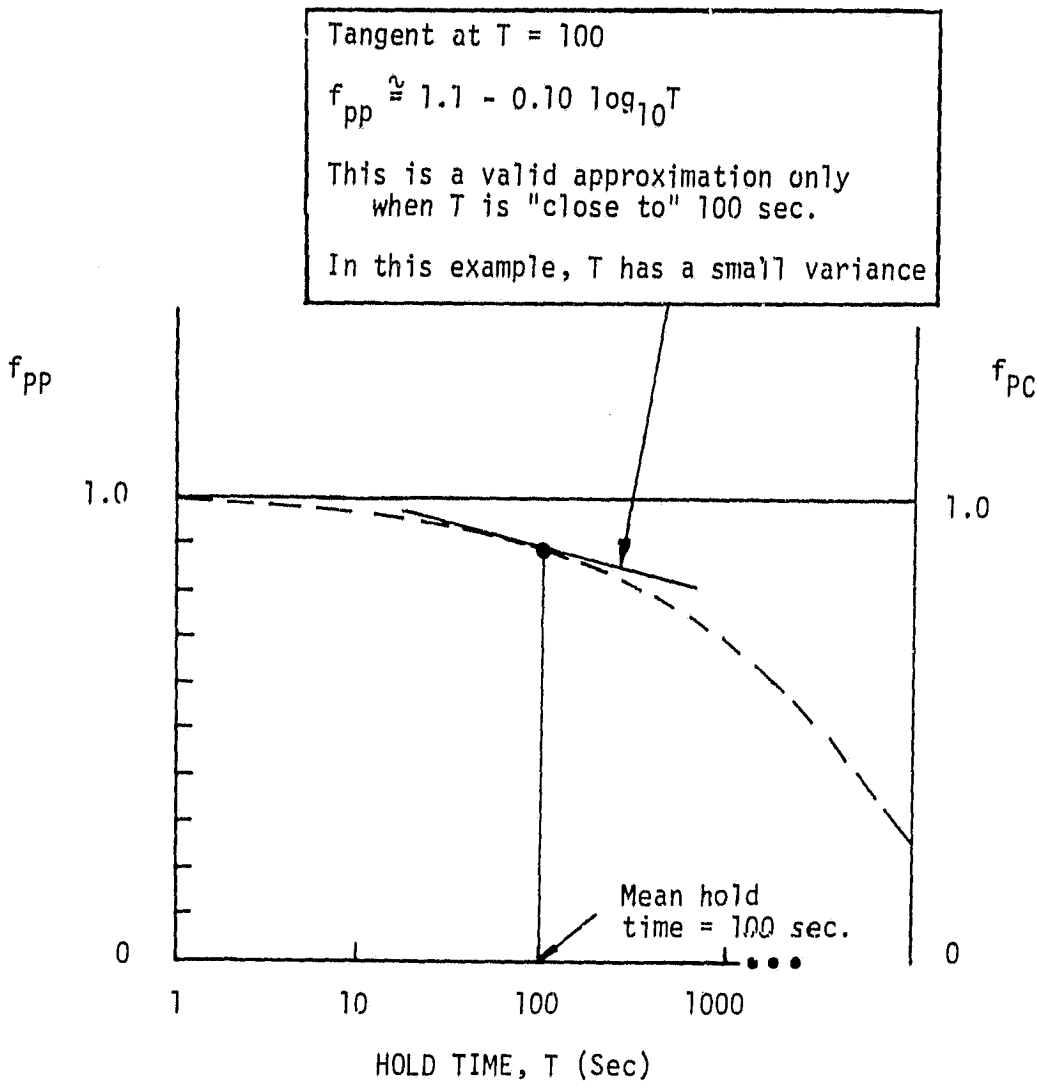

Fig. 12. Relationship Between f_{PP} and f_{PC} as a Function of Hold Time for a Given Strain Range.

Data used for all of the design factors are summarized in Table 1. Commentary on these data is provided in the following.

Stress. It is assumed that the service strain amplitude, $\Delta\varepsilon_S$, has a Type I extreme value distribution of maxima, (EVD). The COV of 20% is fairly typical of loading variables and reflects primarily modelling error resulting from assumptions made in the computational procedures which translate environment into notch strains. A more refined and complete model in which $\Delta\varepsilon_S$ is expressed as a function of temperature and hold time is presented above in Eq. 4, but the simplified approach is used here simply to illustrate the reliability methods.

PP Strain range Data. The PP strain range-life data for AF2-1DA at 1400 F is shown in Fig. 13. Methods of analysis of these data are described in Appendix A, and a summary of the results is given in Table 3. It should be noted that the uncertainty in fatigue strength is described by the random variable A_1, whose COV includes data scatter as well as statistical uncertainti in the estimates of the least squares parameters.

PC Strain range Data. The PC strain range-life data is shown in Fig. 14. To analyze the data, it is first assumed that the slope will be the same as the PP curve, i.e., b = - 1.57. A least squares method, with the exponent known, is employed, and the results are summarized in Table 4. The COV of A_2 reflects both data scatter and statistical uncertainty, the latter which is quantified using the methods described in Appendix B.

Elastic Strainrange Data. The PP elastic strain life data is plotted in Fig. 15. A summary of the statistical analysis of this data is provided in Table 5. Basic analysis methods are summarized in Appendix A. Note that for this data, the least squares analysis is applied to the form

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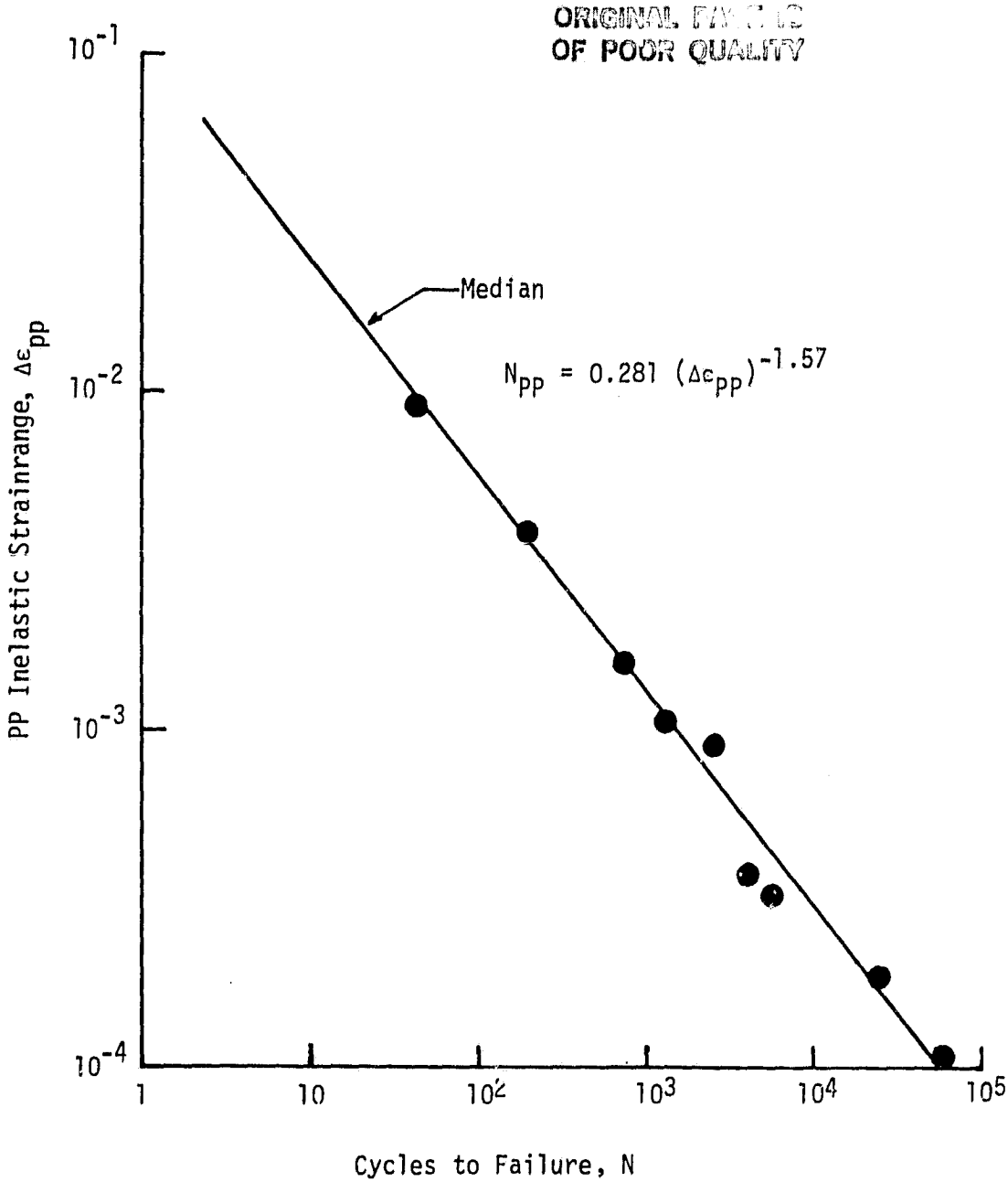

Fig. 13. PP Strain Range-Life Data for AF2-1DA at 1400 F.

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TABLE 3

STATISTICAL ANALYSIS OF PP DATA
(AF2-1DA at 1400F, 760C)

- Transformation

$Y_i = \log_{10} N_i$ $\qquad$ $X_i = \log_{10} \Delta\varepsilon_i$

- Data (See Fig. 13); Sample Size, n = 9

$\Delta\varepsilon_{PP}$(%)	N_f (cycles)	X_i	Y_i
.896	43	-2.047	1.633
.368	200	-2.434	2.301
.154	756	-2.812	2.878
.104	1,322	-2.983	3.121
.089	2,695	-3.0506	3.430
.037	4,205	-3.432	3.624
.032	5,745	-3.495	3.759
.018	25,433	-3.745	4.405
.011	59,121	-3.959	4.772

- Least Squares Analysis [See Apperdix A]

$\hat{Y} = \hat{a} + \hat{b}x$

$\hat{a} = -1.552$ $\qquad$ $\hat{b} = -1.57$

$s = 0.138$

- Statistical Model (See Appendix A for detail of this example)

$N_{PP} = A_1(\Delta\varepsilon_{PP})^b$

$b = \hat{b} = -1.57$

Median of A_1; $\tilde{A}_1 = 0.0281$

COV of A_1; $C_{A_1} = 0.472$

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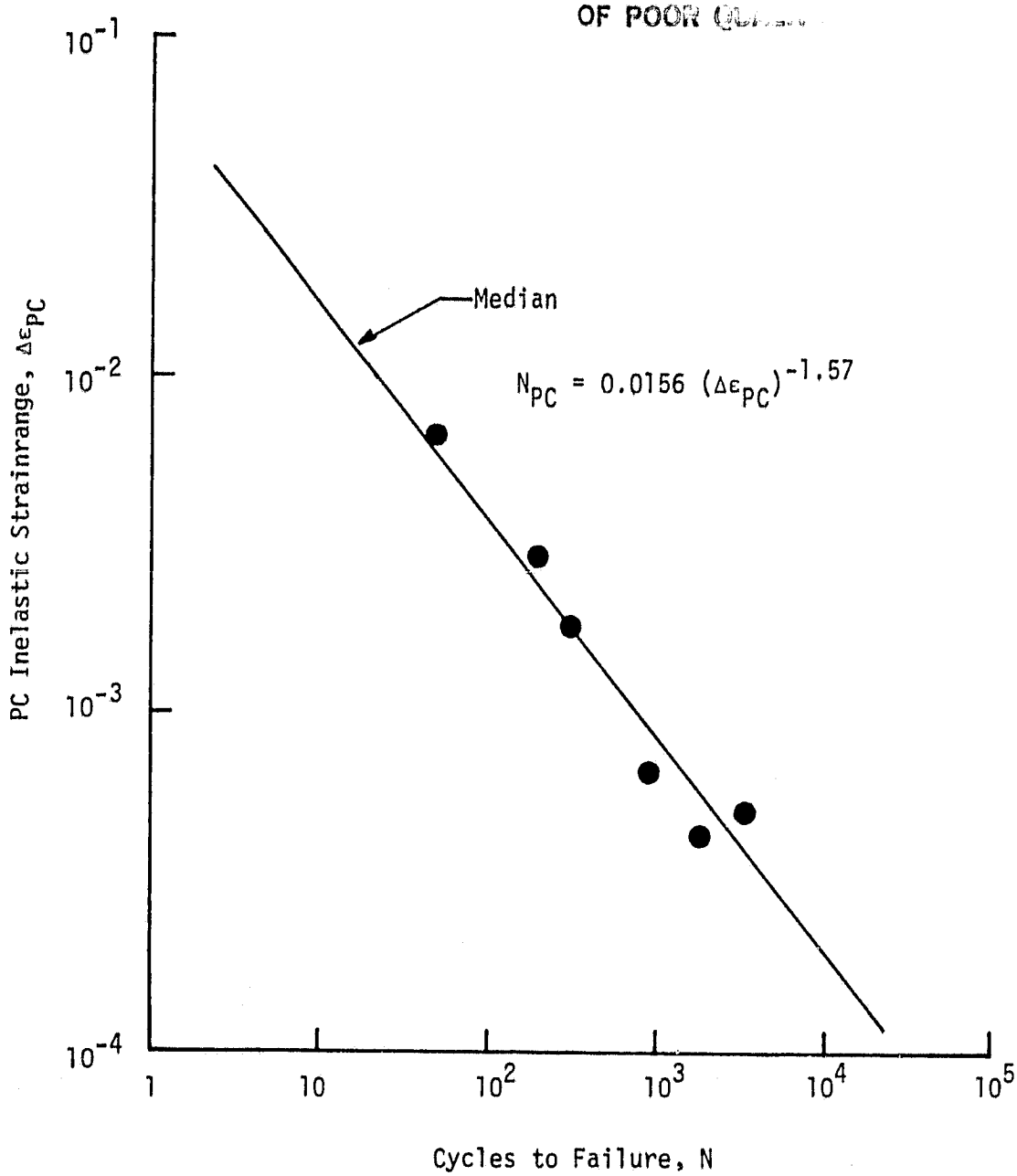

Fig. 14. PC Strain Range-Life Data for AF2-1DA at 1400 F.

TABLE 4

STATISTICAL ANALYSIS OF PC DATA

- Transformation

$$Y_i = \log_{10} N_i \qquad\qquad X_i = \log_{10}\Delta\varepsilon_i$$

- Data (See Fig. 14); Sample Size, n = 6

$\Delta\varepsilon_{PC}$(%)	N_f (cycles)	X_i	Y_i
.671	51	-2.173	1.710
.290	212	-2.537	2.326
.184	300	-2.735	2.478
.069	904	-3.158	2.956
.043	1,807	-3.366	3.257
.051	3,380	-3.291	3.529

- Least Squares Analysis (assume the same slope as the PP data; b = -1.57)

$$\hat{Y} = \hat{a} + bX$$

$$\hat{a} = -1.807$$

$$s = 0.173$$

- Statistical Model (See Appendix B for detail of this example)

$$N_{PC} = A_2(\Delta\varepsilon_{PC})$$

$$b = \hat{b} = -1.57$$

Median of A_2; $\tilde{A}_2 = 0.0156$

COV of A_2; $C_{A_2} = 0.687$

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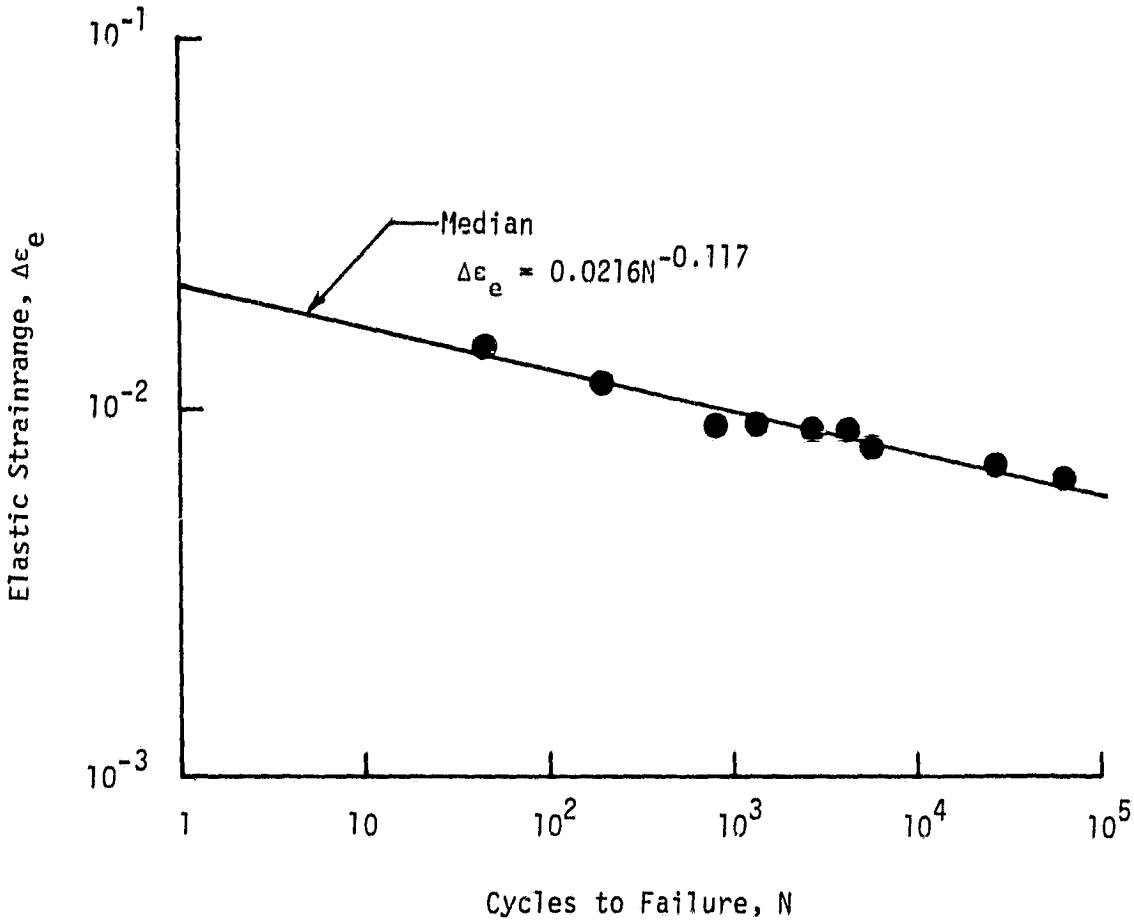

Fig. 15. Elastic Strain Range-Life Data for AF2-1DA at 1400 F.

TABLE 5

Statistical Analysis of Elastic Strain Life Data
(AF2-1DA at 1400F)

- Transformation

$Y_i = \log_{10} N_i$ $\qquad$ $X_i = \log_{10} \Delta\varepsilon_i$

- Data (See Fig. 15); Sample Size, n = 9

$\Delta\varepsilon_e$ (%)	N_f (cycles)	X_i	Y_i
1.492	43	-1.826	1.633
1.155	200	-1.937	2.301
.898	756	-2.047	2.878
.898	1,322	-2.047	3.121
.799	2,695	-2.097	3.430
.800	4,205	-2.097	3.624
.783	5,745	-2.106	3.759
.703	25,433	-2.153	4.405
.652	59,121	-2.186	4.772

- Least Squares Analysis (See Appendix A)

$\hat{Y} = \hat{a} + \hat{b}x$

$\hat{a} = -14.21$ $\qquad$ $\hat{b} = -8.532$

$s = 0.2581$

- Statistical Model

$N = A(\Delta\varepsilon_e)^b$

$b = -8.532$

Median of $A = \tilde{A} = 10^{-14.21}$

COV of A, $C_A = 1.01$ (See Eqs. A.7 and A.10; g(.01,9) = 1.41 used here)

TABLE 5 (continued)

- Alternate Form Used in Analysis

$$\Delta\varepsilon_e = B_{PP}N^d$$

i) $d = 1/b$

ii) $\tilde{B}_{PP} = (1/\tilde{A})^{1/b} = 0.0216$

iii) $C_{B_{PP}} = \left[(1 + C_A^2)^{(1/b)^2} - 1\right]^{1/2} = 0.099$

The relationships of ii) and iii) are valid only when A (and therefore, B) have lognormal distributions. These are basic forms for lognormal variates [e.g., See Ref. 5].

$N = A(\Delta\varepsilon)^b$. For the strength formulation, statistics on the parameters of the equation $\Delta\varepsilon = B\ N^d$ are required. Forms for relating statistics between A and B and c and d are given in Table 5.

Elastic Strain-Life vs. Hold Time Relationship. The relationship between elastic strain-life curves and hold time is established from experimental data as shown in Fig. 16. Data from CC and CP strains, not shown, supported a selection of a slope of 0.25 for the data. Thus, the empirical form, relating B_{PP} and B_{PC} is,

$$\ln(B_{PP}/B_{PC}) = D_{PC}\ T^{0.25} \tag{20}$$

Least squares analysis is performed; the statistics on D_{PC} are presented in Table 6.

Elastic and Inelastic Strainrange-Life Relationships. The strain-life curves, employing the total strainrange version of SRP and the data of Table 2, are presented in Fig. 17 for reference only. These curves suggest that the influence of creep in this example is relatively small.

Reliability Analysis. The fatigue strength, $\Delta\varepsilon_T$, of the material is given by Eq. 17 with substitutions of Eqs. 18 and 19 for f_{PP} and f_{PC}. The event of failure is $(\Delta\varepsilon_S < \Delta\varepsilon_T)$. The following methods will be used to evaluate the probability of failure,

1) Monte Carlo. This method is widely employed for solving complicated probability problems. It is a very useful tool, but it suffers from high computer costs relative to accuracy.

2) Rackwitz-Fiessler (R-F). The R-F scheme is a numerical method for evaluating reliabilities in problems such as this one [5]. It is now widely employed and details of the method are well documented [7 , 8 , 9]. It has been demonstrated by Wu et al. [10]

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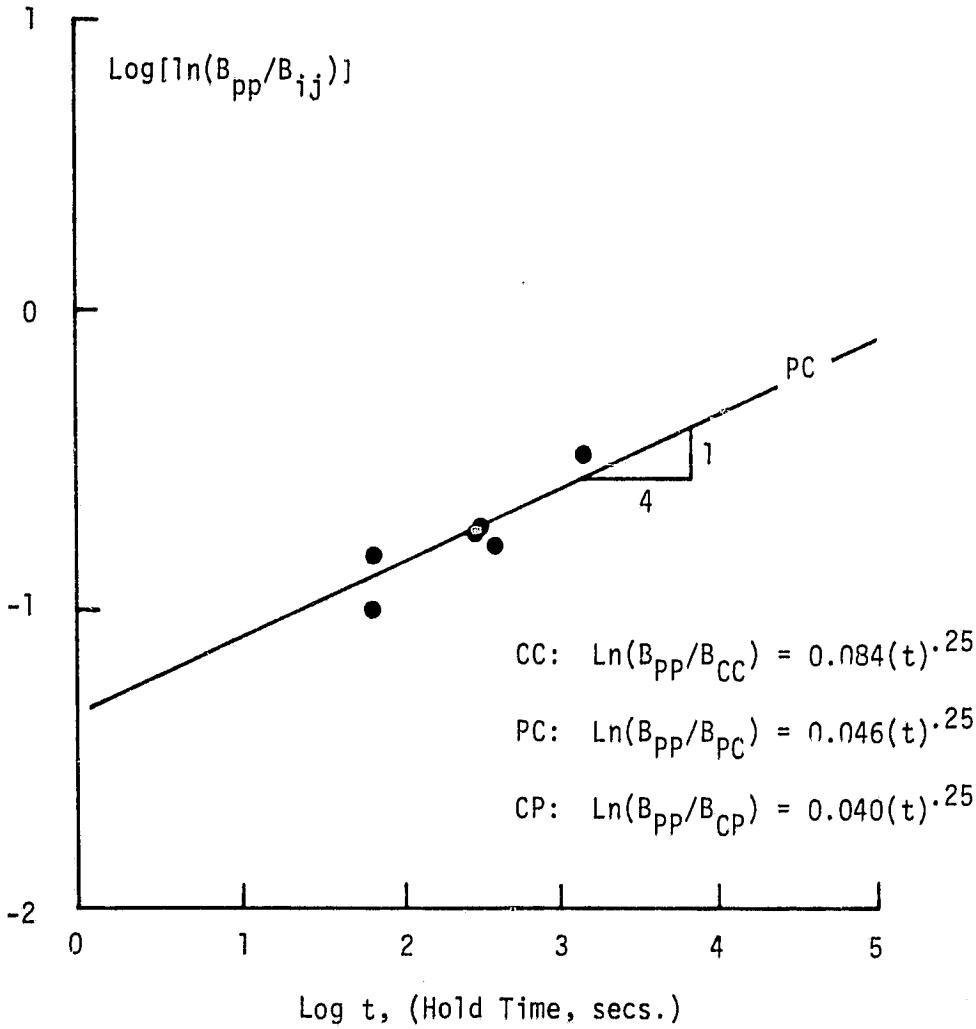

Fig. 16. Time-Dependent Intercepts for Elastic Strain Range Life Relations, AF2-1DA, 760°C, Halford and Nachtigall Data [6].

TABLE 6

STATISTICS ON D_{PC}

- Transformation

$$Y = \log_{10} [\ln(B_{PP}/B_{PC})]$$

$$X = \log_{10} t$$

- Data (See Fig. 16); Sample Size, n = 6

Y_i	X_i
-1.00	1.83
-0.813	1.86
-0.779	2.62
-0.733	2.46
-0.707	2.51
-0.466	3.16

- Least Squares Analysis

$$\hat{Y} = \hat{A} + 0.25\ X$$

Least squares relationship

$$\hat{A} = (\sum y_i - 0.25 \sum X_i)/n = -1.35$$

$$s = \sqrt{\frac{1}{n-1} \sum (Y_i - \hat{Y}_i)^2} = 0.081$$

- Statistics on D_{PC} (See Appendix B for definitions of terms)

Median, $\tilde{D}_{PC} = 10^{\hat{A}} = 10^{-1.35}$

$= 0.0447$

COV $\sigma_o = g(\alpha,n)s$

$g(.01,6) = 1.56$

$\sigma_o = (1.56)(0.081) = 0.126$

Then $C_D = \sqrt{10^{(\cdot 126)^2/.434} - 1} \cong 0.30 \cong 0.30$

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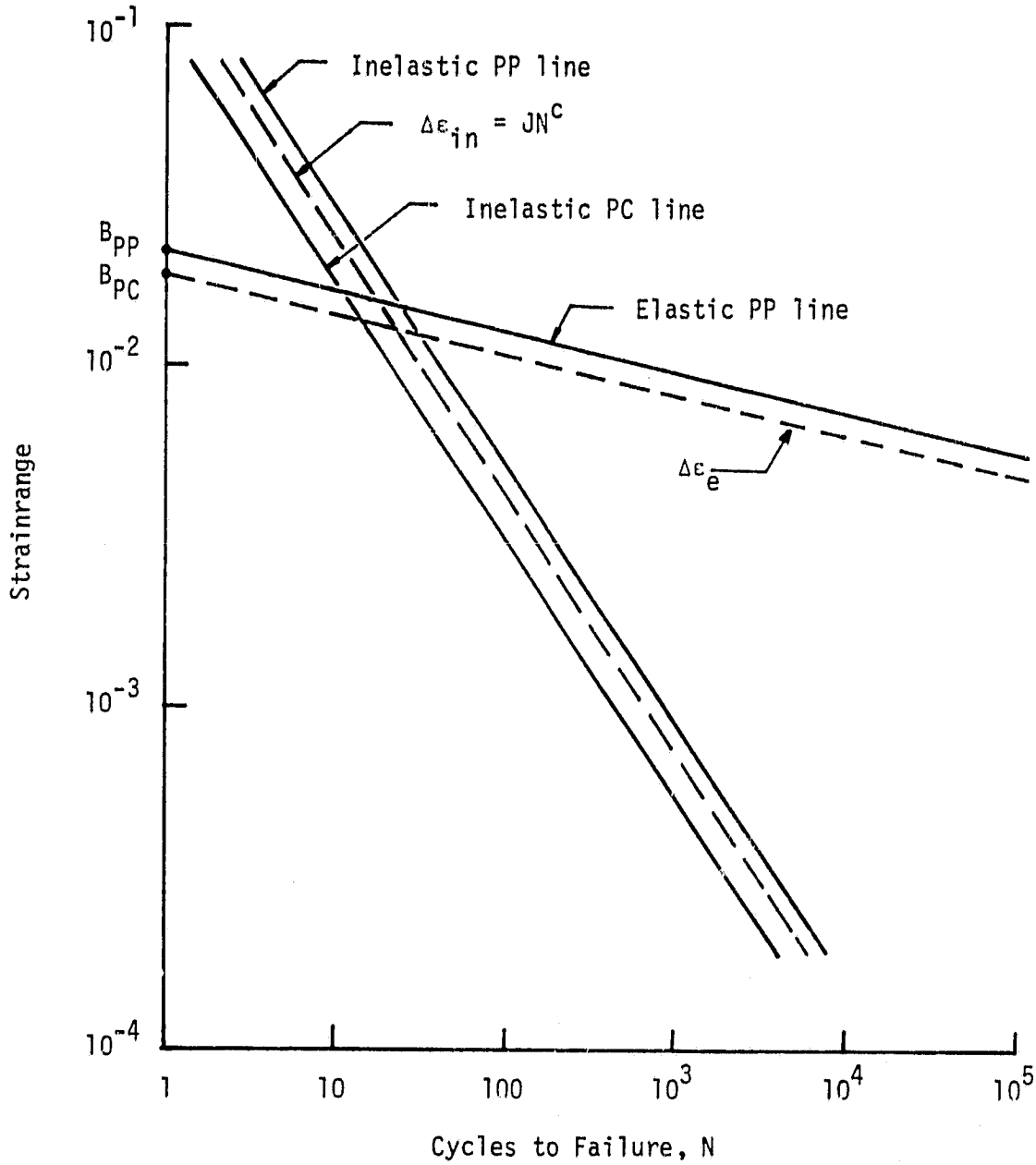

Fig. 17. Median Strain-Life Curves Which Define Fatigue Strength for Example 1.

that the R-F method does an adequate job of estimating reliabilities at computer costs far less than Monte Carlo. A summary of the R-F method is given in Appendix C.

3) The Wu Algorithm. This scheme was developed by Y.-T. Wu on the same NASA/Lewis grant which sponsored this project [10]. This numerical method, summarized in Appendix D, is more complicated than R-F, but increased computer costs, relative to R-F, are insignificant. All evidence seems to indicate that the accuracy in estimating probabilities of failure is substantially better. At this time the Wu algorithm has not been subjected to peer review and has not been published, but its performance has been demonstrated to be of consistently high quality in a large number of examples. This SRP problem is another example.

The Results. The output of the R-F and the Wu programs are provided in Tables 6 and 7 respectively. Results are summarized in Table 8. Agreement of the three methods in this example is better than usual [10]. In this example, three approximations to β and p_f are being compared, although Monte Carlo is exact as $n \to \infty$.

A practical limitation to Monte Carlo for structural risk problems is demonstrated by this example. Note the relatively broad range of the 98% confidence interval for a sample of n = 100,000. This range would be even broader for the "more typical" risk levels of 10^{-3} or lower. To sharpen the limits, a much larger sample would be required. But even for this problem, approximate relative computer costs presented in Table 8 illustrate the inefficiency of Monte Carlo.

In summary, for a single problem, Monte Carlo computer costs may not be excessive. But for a large scale program, the R-F and Wu schemes may be much more efficient. Furthermore, these methods provide a basis for developing safety check expressions for design criteria documents (See Example 3 below.)

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Table 6

Output of Rackwitz-Fiessler Program for Example 1

COMPUTATION OF THE SAFETY INDEX USING R-F ALGORITHM

NUMBER OF DESIGN VARIABLES, N= 6
STOP SENSITIVITY = .00010000
INITIAL GUESS OF REDUCED VARIABLES = 0.0
INITIAL STEP SIZE = 0.1

LIMIT STATE G(R,S)=0. : STRAIN RANGE PARTITIONING MODEL

DESIGN VARIABLES

VARIABLE	TRANSFORMATION	MEAN/MEDIAN	COV
ES	EVD	.53400E-02	.20000E+00
T	NORMAL	.10000E+03	.50000E-01
BPP	LOG	.21600E-01	.99000E-01
DCP	LOG	.44700E-01	.30000E+00
A1	LOG	.28100E-01	.47000E+00
A2	LOG	.15600E-01	.69000E+00

NOTE: THE MEDIAN IS SPECIFIED FOR A LOGNORMAL VARIABLE ONLY.

DESIGN POINT

VARIABLE	REDUCED VALUE		BASIC VALUE	
ES	XR(1)=	2.34338	X(1)=	.87259E-02
T	XR(2)=	.01660	X(2)=	.10008E+03
BPP	XR(3)=	-.78652	X(3)=	.19985E-01
DCP	XR(4)=	.36855	X(4)=	.49814E-01
A1	XR(5)=	-.28086	X(5)=	.24782E-01
A2	XR(6)=	-.07246	X(6)=	.14907E-01

SAFETY INDEX , BETA = 2.5164

PROBABILITY OF FAILURE = .59346E-02*

*$p_f = \Phi(-\beta)$

Table 7

Output of Program Which Uses the Wu Algorithm; Example 1

COMPUTATION OF THE SAFETY INDEX USING THE Y. T. WU ALGORITHM

LIMIT STATE G(R,S)=0. : STRAIN RANGE PARTITIONING MODEL

DESIGN VARIABLES

VARIABLE	TRANSFORMATION	MEAN/MEDIAN	COV
ES	EVD	.53400E-02	.20000E+00
T	NORMAL	.10000E+03	.50000E-01
BPP	LOG	.21600E-01	.99000E-01
DCP	LOG	.44700E-01	.30000E+00
A1	LOG	.28100E-01	.47000E+00
A2	LOG	.15600E-01	.69000E+00

NOTE: THE MEDIAN IS SPECIFIED FOR A LOGNORMAL VARIABLE ONLY.

DESIGN POINT

VARIABLE	REDUCED VALUE		BASIC VALUE	
ES	XR(1)=	6.16752	X(1)=	.86154E-02
T	XR(2)=	.01855	X(2)=	.10009E+03
BPP	XR(3)=	-.85075	X(3)=	.19755E-01
DCP	XR(4)=	.40547	X(4)=	.50354E-01
A1	XR(5)=	-.31582	X(5)=	.24395E-01
A2	XR(6)=	-.07969	X(6)=	.14840E-01

SAFETY INDEX , BETA = 2.5050

PROBABILITY OF FAILURE = .61295E-02*

*$p_f = \Phi(-\beta)$

Table 8

Comparison of Reliability Analyses Between Monte Carlo, Rackwitz-Fiessler, and the Wu Algorithms

	Safety Index, β	Probability of Failure[a], p_f	Relative Cost Factor
Rackwitz-Fiessler[a]	2.516	5.93E-3	1
Wu[a]	2.505	6.13E-3	2
Monte Carlo[b]	2.485	6.48E-3 98% Confidence Limits (5.90, 7.10E-3)	50

[a] β computed first. Then $p_f = \Phi(-\beta)$ where Φ is the standard normal distribution function

[b] p_f computed by counting the number of failures in a sample of n = 100,000. Then $\beta = -\Phi^{-1}(p_f)$.

EXAMPLE 2. Details of constructing the random variable A are described in Appendix A for the two variable case and in Appendix B for a single variable. The function $g(\alpha,n)$ is introduced to quantify the statistical uncertainty component, essentially by enlarging the sample standard deviation, s. It is s which quantifies variability in material properties. A summary is provided in Table 9.

As n becomes larger, this statistical uncertainty becomes smaller, and $g(\alpha, n)$ approaches unity. In this example, it is assumed that the data of Example 1 is now based on large samples so that all statistical uncertainty disappears (i.e., $g(\alpha, n) = 1$). Table 9 summarizes those variables and their COV's in Example 1 for which this error term was included. Also shown is the reduction in COV if the sample size were large and $g = 1$ . . . assuming the same statistics for all variables.

The goal of this exercise is to demonstrate the effect of statistical uncertainty on the design. How important is it to the overall reliability analysis to increase sample sizes to reduce this statistical error? To accomplish this goal (1) the statistical uncertainty component was removed from the COV's of the four variables considered in Table 9, (2) the mean value of service strain, $\Delta\varepsilon_S$ was increased so that the safety index was the same as in Example 1, (for both R-F and Wu).

The results using both schemes were identical, as summarized in Table 10. As the statistical uncertainty is removed, the mean value of $\Delta\varepsilon_S$ can be increased at the same level of risk. But the increase in allowable strain (mean) is only 2.6%. Thus, in this example at least, it seems that while statistical uncertainty may strongly influence the COV of a design

Table 9

A Summary of the Effect of Statistical Uncertainty on the COV's of the Random Design Factors of Example 1

For the random variable, X, the coefficient of variation (COV) is

$$C = \sqrt{10^{\sigma_0^2/.434} - 1}$$

where σ_0 is the equivalent standard deviation

$$\sigma_0 = g\ s$$

s = sample standard deviation

g = factor to account for statistical uncertainty in estimating parameters

Variable	COV, including statistical uncertainty, i.e., g > 1	COV, assume n large enough so that there is no statistical uncertainty; g = 1
B_{PP}	9.9%	7.0%
D_{PC}	30.	19.
A_1	47.	33.
A_2	69.	41.

factor, its impact on the overall reliability seems almost negligible. On the basis of this one example, it would, of course, be dangerous to conclude that statistical uncertainty is unimportant and that small samples are OK. Clearly more studies need to be made on this problem.

Change in the Mean Value of Service Strain, $\Delta\varepsilon_S$ which would be allowed at the same level of risk as Example 1. When the Statistical Uncertainty Component of B_{pp}, D_{PC}, A_1, and A_2 is Removed. (Essentially same results for both Rackwitz-Fiessler and Wu algorithms.)

Including Statistical Uncertainty (Example 1)	5.34E-3
Excluding Statistical Uncertainty	5.48E-2
Percentage Increase: Would be equal to the percent increase in the requirement for the cross sectional area of a tension member if stress and strain were linearly proportional	2.6

EXAMPLE 3. This example demonstrates the use of the R-F and Wu algorithms to derive a safety check expression which could be used for a design criteria document. The problem of Example 1 is used. The general method for deriving partial safety factors, (PSF's) is described in the literature [8, 10, 12]. A simple tutorial in PSF's is provided in Appendix E for readers who are unfamiliar with the concepts.

In conventional design practice, typically a single safety factor is employed to account for all uncertainty. A more refined criterion, could be developed by applying safety factors to each random design factor. In theory, a criterion using these PSF's would produce a more efficient design. Described in Appendix E is how probabilistic design methods, namely the R-F scheme, can be used to derive the PSF's.

It should be noted that a probability based design criterion could require that the designer compute p_f (or β) for the component in question. The component would be safe if $p_f < p_o$ (or $\beta > \beta_o$) where p_o and β_o are the target risk and safety index respectively. To require a designer to exercise skills in probability theory may be impractical. The much more familiar format, a deterministic inequality involving safety factors, is easy to understand and use. In summary, a reliability based safety check expression is derived, having a format, familiar to designers, such that probabilistic and statistical analyses are invisible.

In this example, the problem is defined as follows:

1. The limit state is defined by Eq. 17 (with the substitution of Eqs. 18 and 19).

2. Distributional forms and statistics of the design factors are defined in Table 2.

3. The target safety index, β_o is given as $\beta_o = 3.0$. Just for reference, . . . the notional probability of failure associated with this value is $p_f = \Phi(-3) \cong 0.0013$.

4. The tuning factor A (See Appendix E) is defined by replacing $\Delta\varepsilon_S$ with $A \cdot \Delta\varepsilon_S$. In the R-F algorithm, A is adjusted so that $\beta = 3.0$. As an alternative viewpoint, $\Delta\varepsilon_S$ can be taken as unity, and A then could be thought of as the mean of $\Delta\varepsilon_S$.

5. The nominal values are defined as the median values for variables having lognormal distributions and the mean values for the other variables.

With this information, the PSF program at the University of Arizona was run; the results are presented in Table 11. Input to the program are the variables, their distributions and statistics, the nominal values (at the top of the table), and β_o (at the bottom). The program computes the partial safety factors, γ_i, as listed.

Combining the PSF's with the limit state expression, a safe design results when the following inequality is satisfied.

$$1.84\ (\overline{\Delta\varepsilon_S}) \leq \{0.91\ \tilde{B}_{PP} \exp\ [-1.14\tilde{D}_{PC}\ \bar{T}^{\delta}]\}\ N^d + \left\{\frac{\bar{f}_{PP}}{0.86\tilde{A}_1} + \frac{\bar{f}_{PC}}{0.95\tilde{A}_2}\right\} N^c \quad (21)$$

where,

$$\bar{f}_{PP} = 1.1 - 0.10 \log_{10} \bar{T}$$

$$\bar{f}_{PC} = 1 - \bar{f}_{PP}$$

Note that the relationship for f_{PP} is valid only for a limited range of hold time (See Fig. 12).

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Table 11

Results of PSF Program

FAILURE FUNCTION: SRP MODEL

DESIGN VARIABLES

VARIABLE	DISTRIBUTION	NOMINAL	MEAN/MEDIAN	COV
ES	EVD	.10000E+01	.10000E+01	.20000
T	NORMAL	.10000E+03	.10000E+03	.05000
BPP	LOG	.21600E-01	.21600E-01	.09900
DCP	LOG	.44700E-01	.44700E-01	.30000
A1	LOG	.28100E-01	.28100E-01	.47000
A2	LOG	.15600E-01	.15600E-01	.69000

NOTE: THE MEDIAN IS SPECIFIED FOR A LOGNORMAL VARIABLE ONLY.

DESIGN POINT

VARIABLE	REDUCED VALUE		BASIC VALUE		(γ) PARTIAL SAFETY FACTOR*
ES	XR(1)=	2.80029	X(1)=	.18404E+01	1.8404
T	XR(2)=	.02031	X(2)=	.10010E+03	1.0010
BPP	XR(3)=	-.92031	X(3)=	.19723E-01	.9131
DCP	XR(4)=	.44062	X(4)=	.50873E-01	1.1381
A1	XR(5)=	-.33125	X(5)=	.24235E-01	.8624
A2	XR(6)=	-.08437	X(6)=	.14785E-01	.9478

SAFETY INDEX, BETA= 3.00 $\leftarrow \beta_0$

SCALE FACTOR= .46658E-02 $\leftarrow$ A

*Assumes that nominal values = [median for lognormal variates; mean for $\Delta\varepsilon_S$ and T]

where the bar over a variable denotes mean, and the tilde denotes median (the nominal values by definition).

This example was presented only for demonstration purposes. Limitations of its use in a design criteria document center around the fact that each PSF is a function of all of the statistics and parameters.

1. The expression was derived on the basis of known statistics of the design factors. It applies to a specific case. For application, a range of possible statistics and corresponding PSF's should be studied to construct characteristic PSF's. The problem may require some engineering judgement in smoothing the PSF's.

2. The PSF's were derived for a specific life, N = 1,000 cycles. If the requirement should include other values of N, then the behavior of the PSF's should again be scrutinized.

SUMMARY COMMENTS

Reliability technology has now developed to the point where application to complicated problems is a practical reality in many cases. The Rackwitz-Fiessler and Wu algorithms provide an estimate of reliability of a component experiencing the combined effects of creep and fatigue. The strain range partitioning form of the limit state has a relatively complex and highly non-linear form; yet, as demonstrated herein, these algorithms easily handle this problem with negligible computer costs.

Reliability analysis was used to assess the impact of small sample sizes on component risk. In addition to uncertainty due to inherent data scatter resulting from material behavior, statistical uncertainty, resulting from the fact that parameter estimates are random variables, is present. An example provides an illustration that statistical uncertainty may be relatively insignificant, but it would be dangerous at this time to present this as a general conclusion.

These advanced reliability methods can also be used to derive safety check expressions which employ partial safety factors. A maximum allowable risk is the basic criterion. An example in which PSF's are derived for the SRP problem was presented. As a general comment, caution should be exercised in specifying PSF's for general application simply because they are functions of all of the statistical parameters in a given limit state.

APPENDIX A. ANALYSIS OF STRAIN-LIFE DATA

Methods for analyzing strain-life fatigue data are discussed in this appendix. The goal of such data analysis is to provide a characterization or statistical summary of the ε-N relationship in a form suitable for inclusion in a comprehensive reliability analysis.

Considered will be uncertainty associated with (a) inherent behavior of the material as evidenced by scatter in the data, and (b) statistical behavior of the least squares estimators. This problem was addressed in Ref. 11, and the following summary describes a model for quantifying both uncertainties, thereby producing a model for reliability analysis.

The Least Squares Line

Consider a constant amplitude fatigue test in which pairs of data $(\Delta\varepsilon_i, N_i)$, $i = 1, n$ are collected. N_i is the cycles to failure associated with strain-range (or amplitude, $\Delta\varepsilon_i$) and n is the sample size. $\Delta\varepsilon$ is the independent (or controlled) variable and N is the dependent variable. Hypothetical test data are shown in Fig. A.1 plotted on log-log paper. There data imply a model of the form

$$N = A(\Delta\varepsilon)^b \tag{A.1}$$

where A is a random variable and b is constant. Therefore, N would be a random variable also; its density function $f_N|\Delta\varepsilon$ is shown in Fig. A.1.

Because b and A would be those parameters in a design algorithm which represents the fatigue strength of a material, it is necessary to provide a description of b and A.

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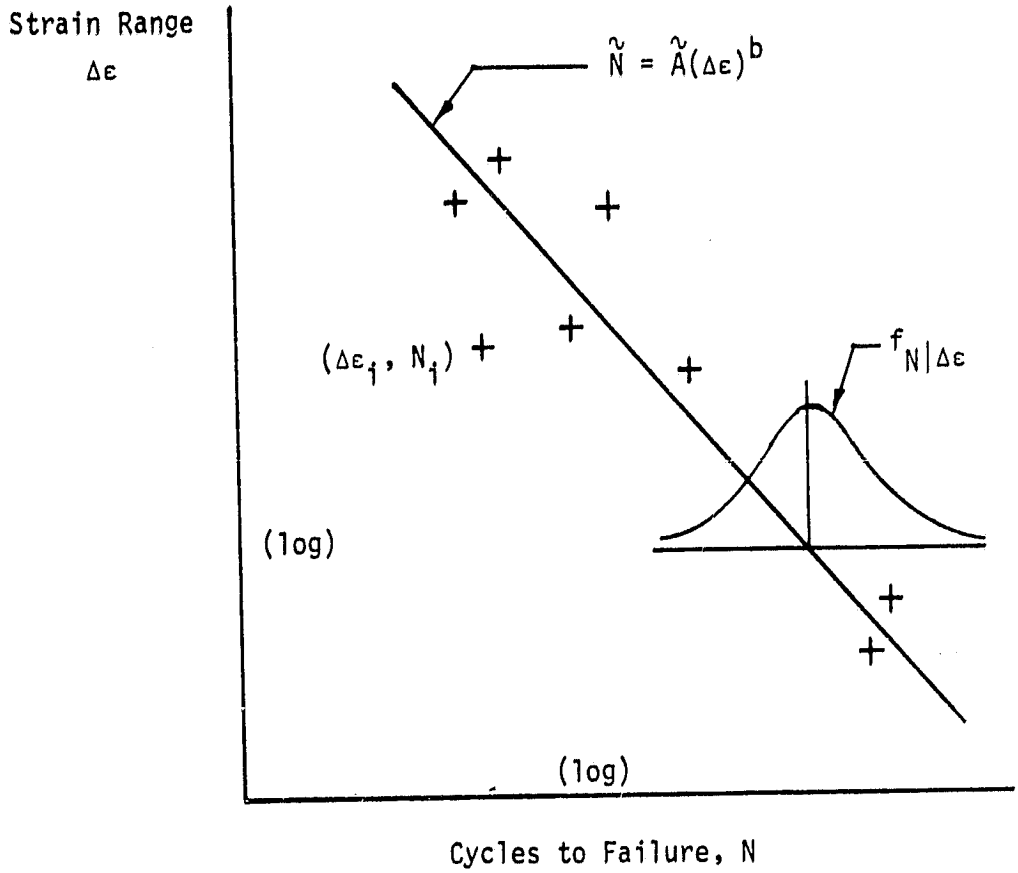

Fig. A.1. Typical Fatigue Data Illustrating the Median Curve and the Distribution of Cycle Life.

Therefore, the problem is to translate the data $(\Delta\varepsilon_i, N_i)$ into the value of b and a distribution of A. In order to do this, first consider the median curve defined by $\tilde{A}$ (the median of A) in Fig. A.1. A linear form of this median curve is

$$Y = a + bX \tag{A.2}$$

where

$$Y = \log N, \quad X = \log \Delta\varepsilon, \quad a = \log \tilde{A} \tag{A.3}$$

Eq. A.3 translates the data $(\Delta\varepsilon_i, N_i)$ into (X_i, Y_i), i = 1, n. Eq. A.2 defines the mean of Y (log N) given X (log $\Delta\varepsilon$). The scatter in the data is defined by the standard deviation of Y given X, denoted as σ and assumed to be constant (not a function of X). Moreover, Y is assumed to have a normal distribution for all X.

Using the method of least squares, a, b, and σ are estimated by $\hat{a}$, $\hat{b}$, and $\hat{s}$ respectively [11],

$$\begin{aligned} \hat{b} &= \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y}) / \sum_{i=1}^{n} (X_i - \bar{X})^2 \\ \hat{a} &= \bar{Y} - b\bar{X} \\ s^2 &= \frac{1}{n-2} \sum_{i=1}^{n} [Y_i - (\hat{a} + \hat{b}X_i)]^2 \end{aligned} \tag{A.4}$$

where $\bar{X}$ and $\bar{Y}$ are the sample means of X and Y respectively. Because each Y_i is a random variable, the estimates $\hat{a}$, $\hat{b}$, and s are also random variables. The "best fit" line

$$\hat{Y} = \hat{a} + \hat{b}X \tag{A.5}$$

is called the least squares line, $\hat{Y}$ is the estimate of the mean of Y given X.

Note that (a) Y given X is normal and (b) the least squares line is the estimate of the mean of $Y|X$. Therefore, it follows that (a) $N|\Delta\varepsilon$ is lognormal and (b) the least squares line, N, is the estimate of the <u>median</u> of $N|\Delta\varepsilon$. As illustrated in Fig. A.2, the least squares line, $\hat{Y}$, is only an estimate of the actual median by virtue of the fact that $\hat{a}$ and $\hat{b}$ are only estimates.

The general goal of this study is to develop an empirical relationship between Y and X which accounts for both the scatter in the data and the distribution of the estimators, but is easy to use in probability-based design formats. A proposed model, suggested by the above discussion, is as follows:

1. Let $b = \hat{b}$ be a constant.
2. Assume that the uncertainty due to both sources is accounted for in $\hat{a}$ (and therefore A) by considering the y intercept as a random variable.
3. Therefore, let the empirical relationship be

$$Y = a_o + \hat{b}x \tag{A.6}$$

where a_o has a normal distribution with mean $\ddot{a}$ and standard deviation σ_o.

The concept of an equivalent prediction interval (EPI) was employed to derive σ_o[11].

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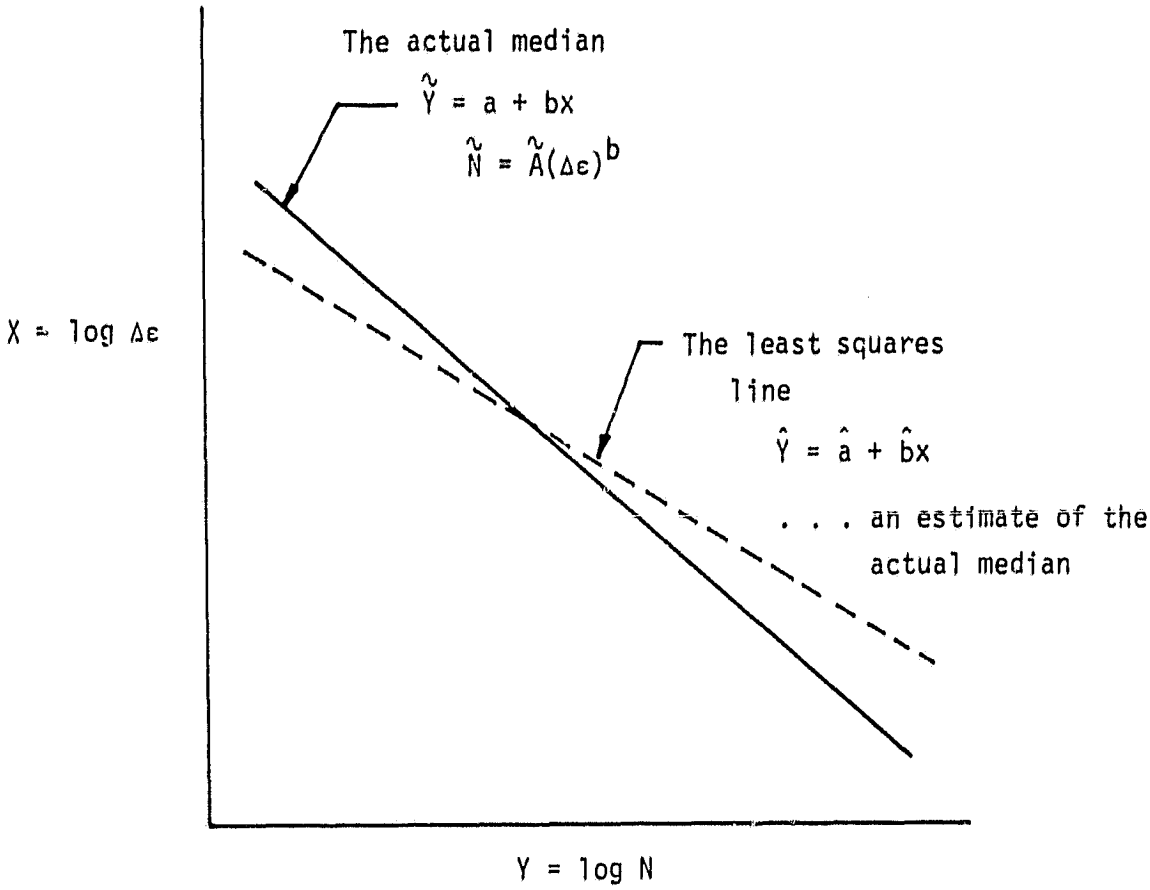


Fig. A.2. An Illustration that the Least Squares Line is Only an Approximation to the Median Curve.

$$\sigma_o = g(n,\alpha)s \tag{A.7}$$

where

$$g(n,\alpha) = \exp[A(\alpha)\{\ln n\}^{-B(\alpha)}]$$

$$A(\alpha) = 1.56[\frac{1}{2}\,\ell n(\frac{2-\alpha}{\alpha})]^{1.12} \tag{A.8}$$

$$B(\alpha) = 3.32 - 1.7\alpha$$

$$6 \le n \le 50;\ 0.01 \le \alpha \le 0.15$$

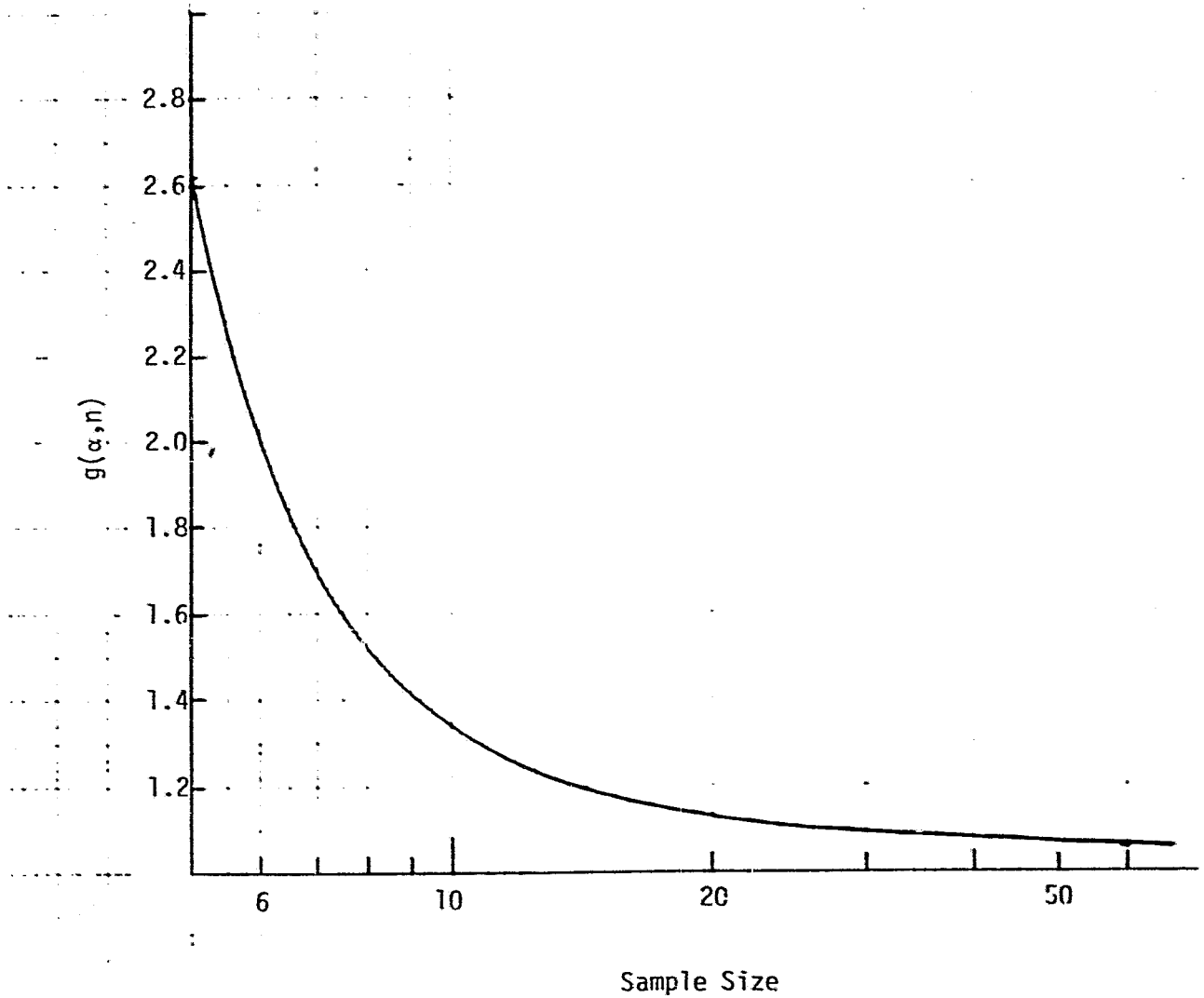
$g(n,\alpha)$ is in essence, an adjustment factor to s to account for the fact that there is uncertainty in the estimates of a and b and s. In turn, s accounts for the scatter in material behavior.

The value of α is arbitrary. It refers to that region of the tail area where it is desired to have a good fit [5, 11]. As a general rule, a value of $\alpha = 0.01$ is reasonable. For reference, $g(\alpha,n)$ is plotted as a function of n for $\alpha = 0.01$ in Fig. A.3. g is the measure of statistical error, and it is interesting to note how quickly it drops as n increases, . . . thus suggesting that statistical uncertainty may be small for $n \geq 10$.

The consequences of the model described above, relative to reliability analyses are:

1. $Y|X$ has a normal distribution. (Thus N given $\Delta\varepsilon$ has a lognormal distribution)
2. The mean value of $Y|X$ is $\hat{a} + \hat{b}x$. (Thus the median of N is $\tilde{N} = 10^{\hat{a}}\ (\Delta\varepsilon)^b$
3. The standard deviation of $Y|X$ is σ_o (and is not a function of X).

Fig. A.3. $g(\alpha,n)$ as a Function of n for $\alpha = 0.01$; Simple Linear Model.



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4. $a_o = \log A$ is normal and A is lognormal. The median $\tilde{A}$ and coefficient of variation C_A of A can be obtained from the lognormal (base 10) forms

$$\tilde{A} = 10^{\hat{a}} \tag{A.9}$$

$$C_A = \sqrt{10^{(\sigma_o^2/.434)} - 1} \tag{A.10}$$

<u>EXAMPLE</u> Given the PP strain-life data (n = 9) as illustrated in Fig. 13 and given in Table 3, it is required to produce to statistics on A. From the least squares analysis,

$$\hat{a} = -1.552 \quad \hat{b} = -1.57 \quad s = 0.138$$

The median of A is computed by Eq. A.9,

$$\tilde{A} = 10^{-1.552} = 0.0281$$

From Fig. A.3, for $\alpha = .01$ and $n = 9$

$$g(.01, 9) = 1.41$$

The equivalent standard deviation is,

$$\sigma_o = g \cdot s$$

$$= (1.41)\ (0.138) = 0.194$$

and the COV of A is computed from Eq. A.10 as,

$$C_A = \sqrt{10^{0.194^2/.434} - 1} = 0.472$$

APPENDIX B. STATISTICAL ANALYSIS OR SINGLE RANDOM VARIABLE

Consider a random sample of size n of a single random variable, X.

$$\underset{\sim}{X} = (X_1, X_2, \ldots X_n) \tag{B.1}$$

It is known that X has a normal distribution. The sample mean, $\overline{X}$, and sample standard deviation, s_x, are computed. To establish a "design value" of X, the notion of an equivalent prediction interval (EPI) can be used [5]. But the EPI can also be used to provide "improved" statistics for probabilistic design.

Define an equivalent standard deviation as,

$$\sigma_o = g_1(\alpha,n)s_x \tag{B.2}$$

where

$$g_1(\alpha,n) = t_{\alpha;n-1}\sqrt{1 + (1/n)} \Big/ z_\alpha \tag{B.3}$$

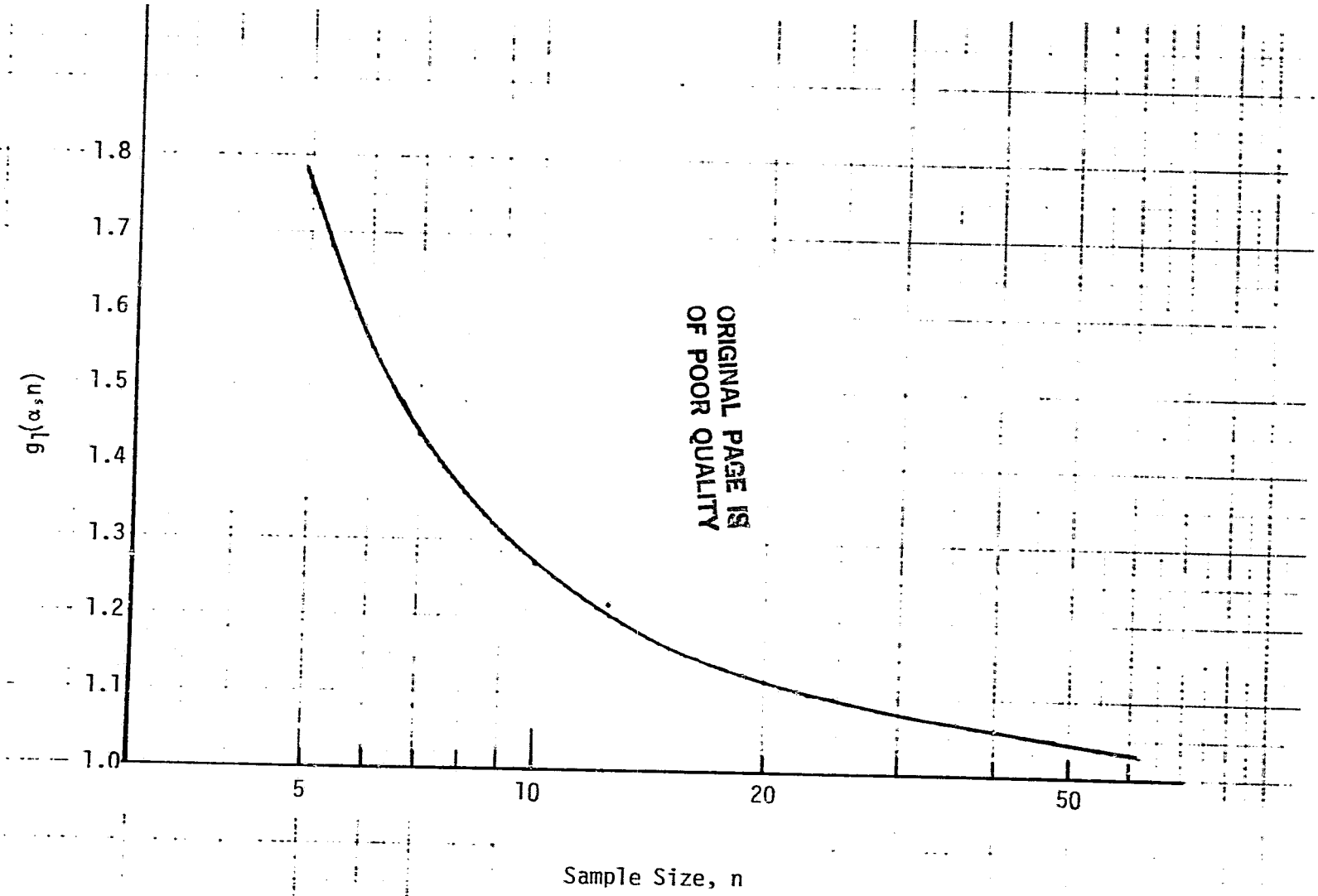
$t_{\alpha;n-1}$ = students t variate

z_α = standard normal variate

α = reference probability level

The choice of α is arbitrary, but for general design, a value of 0.01 is recommended. Reference 5 provides additional discussion. For convenient reference, the value of g_1 for $\alpha = 0.01$ is presented as a function of n in Fig. B.1.

Fig. B.1. $g_1(\alpha,n)$ as a Function of n for $\alpha = 0.01$; Single Random Variable.



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For design purposes, one can state that X has a normal distribution having mean and standard deviation $(\overline{X}, \sigma_o)$. Scatter inherent in the phenomena is described by s_x, and statistical scatter, i.e., the fact that $\overline{X}$ and s_x are random variables is described by g.

Example: Let Z be a lognormally distributed random variable. Let $Y = \log_{10} Z$. Then Y has a normal distribution. A random sample, $\underset{\sim}{Z}$, of size n = 6 is taken. Transforming to Y and computing the statistics, $\overline{Y} = -1.807$ and $s = 0.173$.

The equivalent standard deviation of Y is given by Eq. B.2. From Fig. B.1, $g_1 = 1.56$ for $\alpha = .01$ and $n = 6$. Thus,

$$\sigma_o = (1.56)(0.173)$$

$$= 0.270$$

Invoking basic properties of the lognormal distribution (See Appendix C).

- Median of Z

$$\tilde{Z} = 10^{\overline{Y}} = 0.0156$$

- Coefficient of variation of Z

$$C_Z = \sqrt{10^{\sigma_o^2/.434} - 1} = 0.687$$

APPENDIX C. THE RACKWITZ-FIESSLER (R-F) ALGORITHM

The algorithm proposed by Rackwitz and Fiessler (7) has been extensively described in recent literature (8, 9, 12). The procedure for calculating the R-F algorithm safety index can be summarized as follows:

1. Define each design factor, X_i (i=1,n) and its corresponding probability distribution; F_i and f_i denotes to cdf and pdf of X_i respectively.
2. Define reduced variables

$$u_i = \frac{X_i - \mu_i}{\sigma_i} \qquad i = 1,n \qquad \text{(C.1)}$$

where (μ_i, σ_i) = mean and standard deviation of X_i respectively.

3. Define the limit state in reduced variables

$$g'(\underset{\sim}{u}) = 0 \qquad \text{(C.2)}$$

where $\underset{\sim}{u} = (u_1, u_2, \ldots u_n)$

4. Make an initial estimate of the safety index

$$\beta = \min \sqrt{u_1^2 + u_2^2 + \ldots + u_n^2} \qquad \text{(C.3)}$$

subject to $g'(\underset{\sim}{u}) = 0$. This is the Hasofer-Lind generalized safety index.

5. Calculate the corresponding design point, X^* where

$$X_i^* = u_i^* \sigma_i + \mu_i \qquad i = 1,n \qquad \text{(C.4)}$$

The design point is that point on the failure surface closest to the origin of reduced coordinates.

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6. Calculate the means and standard deviations of the equivalent normal distributions for each non-normal variable

$$\sigma_{Ni} = \frac{\phi[\Phi^{-1}(F_i(X_i^*))]}{f_i(X_i^*)}$$

$$\mu_{Ni} = X_i^* - \Phi^{-1}[F_i(X_i^*)]\sigma_{Ni} \tag{C.5}$$

where ϕ = standard normal pdf and Φ = standard normal cdf for each variable.

7. Define the new reduced variables

$$u_i' = \frac{X_i - \mu_{Ni}}{\sigma_{Ni}} \tag{C.6}$$

8. Calculate a new estimate of the safety index

$$\beta_1 = \min\sqrt{(u_1')^2 + (u_2')^2 + \ldots + (u_N')^2} \tag{C.7}$$

subject to $g'(\underset{\sim}{u}') = 0$.

9. Repeat steps 6 through 10 until the difference

$$|\beta_N - \beta_{N-1}| \leq t \tag{C.8}$$

where t is the "error." In this study, the value t = 0.001 was used.

10. The probability of failure is calculated using $\hat{\beta} = \beta_N$

$$P_f = \Phi(-\hat{\beta}) \tag{C.9}$$

APPENDIX D. A NEW METHOD OF CONSTRUCTING EQUIVALENT NORMAL DISTRIBUTION AND COMPUTING PROBABILITY OF FAILURE

This method constructs, for each non-normal variable, a scaled three parameter equivalent normal distribution function (cdf) employing a least square scheme. The probability of failure p_f is computed using the obtained parameters by assuming that the limit state is linear at the design point.

Consider a limit state function $g(\underset{\sim}{X})$ involving n independent random variables which is linearized at the design point,

$$g(\underset{\sim}{X}) = a_o + \sum_{i=1}^{n} a_i X_i \tag{D.1}$$

Three equivalent normal parameters, (A, μ_N, σ_N), for each X_i are found, one by one, and the probability of failure is estimated as

$$p_f = \int_{\Omega_1} f_{\underset{\sim}{X}}(\underset{\sim}{x}) = \int_{\Omega_2} \phi_{\underset{\sim}{u}}(\underset{\sim}{u}) d\underset{\sim}{u} \left(\prod_{i=1}^{n} A_i\right) \tag{D.2}$$

where A_i are the scale factors, $f_{\underset{\sim}{X}}(\underset{\sim}{x})$ and $\phi_{\underset{\sim}{u}}(\underset{\sim}{u})$ denote the joint probability density function (pdf) of the original and equivalent normal variables, respectively. Ω_1 is the failure region on the original coordinates, $\underset{\sim}{x}$, and Ω_2 is the corresponding failure region on the reduced coordinates, $\underset{\sim}{u}$, in which

$$u_i = \frac{X_i - \mu_{Ni}}{\sigma_{Ni}} \tag{D.3}$$

Using Eq. D.3, the reduced limit state function is also linear. A minimum distance, δ, on the u space can be found, and p_f is estimated as

$$p_f = \Phi(-\delta) \left(\prod_{i=1}^{n} A_i\right) \tag{D.4}$$

The generalized safety index, β, is computed as

$$\beta = -\Phi^{-1}(p_f) \tag{D.5}$$

Consider constructing the equivalent normal cdf for one of the variables, X_i, and let

$$Y = a_1 X_1 + \ldots + a_{i-1} X_{i-1} + a_{i+1} X_{i+1} + \ldots a_n X_n \tag{D.6}$$

Eq. D.1 becomes

$$g(\underset{\sim}{X}) = a_o + a_i X_i + Y \tag{D.7}$$

Thus, the limit state involves only two variables; X_i is the variable to be normalized and Y represents the sum of the other variables. Assume that the Rackwitz-Fiessler (R-F) algorithm has been performed, and $\underset{\sim}{X}$ are replaced by the equivalent normals, then Y is also a normal variable with pdf of

$$\phi'(y) = \frac{1}{\sqrt{2\pi}\,\sigma_Y} e^{-\frac{1}{2}\left(\frac{y-\mu_Y}{\sigma_Y}\right)^2} \tag{D.8}$$

which will be used in the following procedure.

Define the R-F reduced design point as

$$z_i^* = \frac{x_i^* - \mu_i}{\sigma_i} \tag{D.9}$$

where μ_i and σ_i are the R-F equivalent normal mean and standard deviation, respectively. A variable may be defined as a "strength" variable if $z_i^* < 0$, and a "stress" variable if $z_i^* > 0$.

Assume that a non-normal variable, denoted as X (without the subscript i) is a strength variable with cdf of F(x) and pdf of f(x). The three equivalent normal parameters can be found by minimizing the sum of the errors of the squares between two functions, $F(x)\phi'(y)$ and $A\Phi(x)\phi'(y)$, i.e.,

$$\text{Min:} \quad E = \int [A\Phi(x)\phi'(y) - F(x)\phi'(y)]^2 dx \tag{D.10}$$

$$\text{Subject to } g(\underset{\sim}{X}) = a_o + aX + Y = 0$$

where $A\Phi(x)$ is the equivalent normal cdf with mean μ_N and standard deviation σ_N.

The procedure described in the folloiwng imposes two constraints, similar to the R-F algorithm, to Eq. D. 10, i.e., match cdf's and pdf's at the design point,

$$A\Phi(u^*) = F(x^*) \tag{D.11}$$

$$A\frac{\phi(u^*)}{\sigma_N} = f(x^*) \tag{D.12}$$

where $\Phi(u^*)$ and $\phi(u^*)$ are the "standardized" normal cdf and pdf, respectively. Using Eq. D.11 and Eq. D.12, the error sum, E, can be evaluated for a given A value.

The procedure for determining the normal parameters can be summarized as follows:

1. Calculate $\phi'(y)$ as a function of x.

   Define

$$z_1 = \frac{x - \mu_X}{\sigma_X} \tag{D.13}$$

$$z_2 = \frac{y - \mu_Y}{\sigma_Y} \tag{D.14}$$

   The reduced limit state using R-F results can be derived as

$$z_1^* z_1 + z_2^* z_2 = \beta^2 \tag{D.15}$$

   so that

$$z_2 = \frac{\beta^2 - z_1^* z_1}{z_2^*} \tag{D.16}$$

   Given any x value, z_2 can be calculated. Therefore, $\phi'(y)$ can be computed using Eq. D.8. Note that because σ_Y, in Eq. D.8, is a constant, it can be taken out from the E integral without affecting the result of the parameters.

2. Make an initial guess of A (e.g., A = 1) and calculate u^* from Eq. D.11,

$$u^* = \Phi^{-1}[\frac{F(x)}{A}] \tag{D.17}$$

3. Calculate σ_N from Eq. D.12

$$\sigma_N = A \frac{\phi(u^*)}{f(x^*)} \tag{D.18}$$

4. Calculate μ_N using Eq. D.3,

$$\mu_N = x - u\sigma_N \tag{D.19}$$

5. Compute the error sum, E, in which $\Phi(x)$ is evaluated using μ_N and σ_N.

6. Choose other values of A and repeat step 2 through step 5. An "optimum" A, which minimizes E, can be determined using a suitable optimization routine. Three parameters are thereby determined.

7. Repeat the above procedure for other non-normal variables. If the variable is a stress variable, F(x) and $\Phi(x)$ should be replaced by $1 - F(x)$ and $1 - \Phi(x)$, respectively, in all the formulations.

8. Compute p_f and β according to Eq. D.4 and Eq. D.5.

Because, in general, there is no closed form solution for the E integral, a numerical scheme must be used to approximate E by replacing the integral by a summation and replacing dx by Δx, i.e., x values must be discretized. The region of x must also be set. It can be determined such that

$$\frac{F(x)\ \phi'(y)}{F(x^*)\phi'(y^*)} \leq H \qquad \text{for the two limits of } x \tag{D.20}$$

where H is a reasonably small value, say, 0.2. Note that $F(x)\phi'(y)$ relates closely to p_f, therefore small H value implies that a sufficiently wide region of x will be included in the summation of E. However, when a variable has a relatively small z^* (e.g., $z_i^*/\beta < 0.1$) and a large coefficient of variation (e.g., 0.4), the range of x may become very wide (therefore, too many

points of x need to be included in the summation) to satisfy Eq. D.20. In such cases, it is suggested that R-F equivalent normal parameters may be used (i.e., A = 1) directly. Because z_i^* is small, the difference in p_f estimate is usually negligible.

A user-oriented computer program applying the above numerical scheme has been developed in the University of Arizona. The process of choosing x values is automated; only the distributional information and the limit state need be input by the user to generate the probability of failure estimate.

APPENDIX E. A SIMPLE ILLUSTRATION OF THE COMPUTATION OF PARTIAL SAFETY FACTORS

Definition of and derivation of partial safety factors (PSF) is described in the literature [8 , 10, 12]. Because techniques of constructing design criteria using PSF's are relatively new and not widely known in the aerospace and mechanical design community, a simple example is presented which provides a tutorial.

THE EXAMPLE

The limit state is

$$L + D = AR \tag{E.1}$$

where L and D are stress variables and AR is strength variable.

The target safety index is chosen as

$$\beta_o = 3.0 \tag{E.2}$$

The statistical parameters are given as follows: (μ = mean, σ = standard deviation, tilde indicates median, C = COV)

L	Extreme Value Distribution (EVD)	$\mu_L = 10$	$\sigma_L = 2.0$
D	Lognormal (LN)	$\tilde{D} = 20$	$C_D = 0.15$
R	Weibull (WEI) or Lognormal (LN)	$\mu_R = 50$	$\sigma_R = 5.0$

A = constant; here we could assume that it is a geometric variable, e.g., cross sectional area. But in the process of computing the point, A plays the role of an adjustment or "tuning factor." Its role will be described later.

HOW THE PARTIAL SAFETY FACTORS ARE COMPUTED

The Rackwitz-Fiessler (R-F) method for approximating non-normal variates with an equivalent normal in a Hasofer-Lind generalized safety index approach is employed. An R-F program, (named RACA), based on an optimation method for computing the safety index, β, has been developed at the University of Arizona. This program was used for the calculations. The PSF's were computed by the following steps.

First RACA was used to compute the design point so that $\beta = 3.0$, the target safety index. This has to be done (with the present version of the program) by iteration by adjusting the value of A so that $\beta = 3.0$. Thus, A is called the <u>tuning factor</u>. The output of the program is illustrated in Table 2. The results of the program are the <u>design point</u> (L^*, D^*, R^*), . . . and A. Note that the design point is on the failure surface, i.e.,

$$\underbrace{L^* + D^*}_{\text{"stress"}} = \underbrace{AR^*}_{\text{"strength"}} \qquad \text{(E.3)}$$

Define <u>partial safety factors</u>, γ_i

$$L^* = \gamma_L L_n \qquad D^* = \gamma_D D_n \qquad (AR^*) = \gamma_R (AR_n) \qquad \text{(E.4)}$$

where the subscript "n" refers to the nominal value. This value is arbitrarily chosen. It could be chosen as the mean or median, or perhaps a value in right tail for stress variable or in left tail for strength variable. Clearly the partial safety factors depend upon the definition of nominal values and therefore, codified safety check expressions should clearly specify the definition of a nominal value.

Substituting Eq. E.4 into E.3

$$\gamma_L L_n + \gamma_D D_n = \gamma_R (AR_n) \tag{E.5}$$

But if we let

$$\gamma_L L_n + \gamma_D D_n \leq \gamma_R (AR_n) \tag{E.6}$$

we will insure that $\beta \geq \beta_o$. Here we are just saying that it is okay to lower stresses or increase strength.

In this example, A = 1.06, and

$$\text{Design Point} \quad \begin{aligned} L^* &= 11.17 \\ D^* &= 23.53 \\ AR^* &= (1.06)(32.73) \end{aligned} \tag{E.7}$$

Assume that <u>means</u> are nominal values (often the mean value is used for stress variables, but some number in lower tail for strength variable). Then it follows from Eq. E.4 that

$$\gamma_L = \frac{L^*}{\mu_L} = \frac{11.17}{10.0} \cong 1.12$$

$$\gamma_D = \frac{D^*}{\mu_D} = \frac{23.53}{20.0} = 1.18$$

$$\gamma_R = \frac{AR^*}{\mu_R} = \frac{32.73}{50.00} = 0.655$$

Thus, the safety check expression or condition for a safe design Eq. E.5 expressed in terms of the PSF's, becomes,

$$1.12\ L_n + 1.18\ D_n \leq 0/65\ (\ldots_n) \qquad \text{(E.9)}$$

Eq. E.9 could then be employed as a safety check expression in a design criteria document. It is of course understood that the inequality would be valid only for cases when the design factors are assumed to possess the same statistics as the variables used to derive the PSF's.

REFERENCES

1. Manson, S. S., Halford, G. R., and Hirschberg, M. H., "Creep-Fatigue Analysis by Strainrange Partitioning," NASA TM X-67838, 1971.

2. Halford, G. R., and Saltsman, J. F., "Strainrange Partitioning - A Total Strainrange Version," Advances in Life Prediction Methods, ASME, 1983.

3. Hirschberg, M. H., and Halford, G. R., "Use of Strainrange Partitioning to Predict High Temperature Low Cycle Fatigue," NASA TN D-8072, Jan. 1976.

4. Manson, S. S., Halford, G. R., and Nachtigall, A. J., "Separation of Strain Components for Use in Strainrange Partitioning," Advances in Design for Elevated Temperature Environment," ASME, 1975.

5. Wirsching, P. H., "The Application of Probabilistic Design Theory to High Temperature Low Cycle Fatigue," NASA CR-165488, Nov. 1981.

6. Halford, G. R., and Nachtigall, A. J., "The Strainrange Partitioning Behavior of an Advanced Gas Turbine Disk Alloy, AF2-IDA," Paper No. 79-1192 AIAA/SAE/ASME 15th Joint Propulsion Conference, June 18-20, 1979, Las Vegas.

7. Rackwitz, R., and Fiessler, B., "Structural Reliability Under Combined Load Sequences," Computers and Structures, Vol. 8, 1978, pp. 489-494.

8. Thoft-Christensen, P., and Baker, M. J., Structural Reliability Theory and Its Applications, Springer-Verlag, N. Y., 1982.

9. Ang, A. H.-S., and Tang, W., Probability Concepts in Engineering Planning and Design, Vol. II, Wiley, 1984.

10. Wu, Yih-Tsuen, Efficient Algorithm for Performing Fatigue Reliability Analyses, Ph.D. Dissertation, The University of Arizona, 1984.

11. Wirsching, P. H., and Hseih, S., "Linear Model in Probabilistic Fatigue Design," Journal of the Engineering Mechanics Division, ASCE, Vol. 106, No. EM6, Dec. 1980.

12. CIRIA; Rationalization of Safety and Serviceability Factors in Structural Codes, Construction Industry Research and Information Association, Report No. 63, 1977.