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1. CURRENT TECHNICAL OBJECTIVES

1. Optimal Utilization of Laser and VLBI Observations for Reference Frames for Geodynamics

2. Utilization of Range Difference Observations in Geodynamics

3. Estimation Techniques in Crustal Deformation Analysis
2. ACTIVITIES

2.1 Earth Rotation Parameter Determination from Different Space Geodetic Systems

Introduction

Since the last report on this study, much work has been done on developing and/or testing software to allow simulation and adjustment of Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR), and Very Long Baseline Interferometry (VLBI) data. This progress on readying the software for each system is described separately below, along with some comments on the adjustment program to be used and other work to be done.

SLR

The NASA GEODYN program (version 8210.1) has successfully been used to simulate SLR data. Given approximate station coordinates, observational accuracies, observation cutoff elevation angle, time period, a satellite state vector, and earth rotation information, there is no problem in simulating any amount of data. Although the effects due to other systematic error sources existing in SLR could be easily considered, at least initially a simple point-mass earth and satellite model will be used. The operation of this specific option has also been verified.

The problem noted in the last report that this program was failing due to OC4 errors has been avoided by not printing the solar flux tables (with a "FLUX_2" card). This temporary fix to the problem was provided by Dr. Peter Dunn of EG&G and was in turn reported to Ms. Barbara Putney of NASA/GSFC.

LLR

To simulate LLR data, the same basic assumption mentioned above will be made, namely, that the satellite (in this case the moon) will be considered as a simple point mass body revolving around a point mass earth. The situation is then identical to that of SLR, except that an approximation of the moon's orbit will be used rather than that of Lageos. As before, the main reason this simplifying assumption is made is to allow the simulation to be as straightforward as possible, by ignoring the systematic effects which are not considered likely to propagate strongly into the earth rotation parameter (ERP) estimates. In addition, there is no software currently available here to consider some of the systematic effects occurring in LLR, such as the lunar
physical librations. (No further indication has yet been received as to when GEODYN will be changed to allow real LLR observations; however, as mentioned in the last report, this is apparently still a long-term goal of GSFC.)

VLBI

The simulation of VLBI data is being pursued along the following lines:

(1) IRIS (the International Radio Interferometric Surveying Network) VLBI schedules have been obtained from the National Geodetic Survey (NGS) (via Ross McKay on June 18, 1984). These schedules are in the VLBI Mark III System SKED program format as described in [Vandenberg and Schaffer, 1983] (also received from Mr. McKay on August 22, 1984). These schedules are the actual schedules now in use by the five IRIS observatories. Since in the next few years substantial changes in the number of such observatories (permanently dedicated to ERP observations) and in the observing schedules seem unlikely, these schedules will be used to define all the VLBI observations to be simulated. The only change will be to consider various observation session intervals, rather than only every five days. And if substantial changes should occur in the schedule or number of observatories, the new schedules could be easily obtained from NGS and used instead.

(2) Since GEODYN has never been properly set up to simulate VLBI observations (as it has for SLR), an intermediate program is being written ("SKEDVIP") to read the schedules and provide information to another program for data simulation. A modified version (now also being made) of Yehuda Bock's "VIP" program [Bock, 1980] will be used to generate these simulated observations in a GEODYN-compatible format.

(3) As reported in the last semiannual report, the current version of GEODYN (8210.1) will not process VLBI observations since the changed source regarding VLBI which was to have been installed in that version was left out. Discussions with Dr. Dunn and Ms. Putney in August have indicated that these changes will also not now be added in the future but may be incorporated eventually in the new GEODYN II program. (GEODYN itself has been permanently frozen at the 8210.1 level.)

Rather than wait an extremely long time for GEODYN II to become available with those changes, the changed source itself was requested from their author, N. Zelensky of EG&G so they could be incorporated here into a special version of GEODYN. One attempt at sending these changes failed in late September due to a tape mixup at GSFC, and another tape is awaited. When it arrives, the special version of GEODYN will be easily created and tested with a VLBI test run previously sent by Mr. Zelensky.

(4) In any case, once the program is operating here with VLBI data, as with SLR and LLR it can provide individual solutions directly or pass the normal equations to the SOLVE program for individual or combination solutions.
The SOLVE Program

The SOLVE program (version 8212.0), which will be used at least to provide the combination SLR-LLR-VLBI solutions, has now been tested with simulated SLR data. Although apparently working, there are still some problems occurring, which are still being checked to verify the correct operation of the program.

Summary and Future Work

The readying and testing of the software to simulate and adjust the three data types is now well underway and should be completed by the time of the next report.

Final decisions on the stations participating, observational accuracies, and reference frame differences to be used should also be made soon. The main problem to be considered next is the simulation of the ERP information itself, including over what time period they are to be estimated. Software to combine the estimates of the individual systems using a weighted averaging method (à la BIH) also will be obtained or developed.

References


2.2 Utilization of Range-Difference Observations in Geodynamics

Introduction

The research reported in the last semianual report in reference to the utilization of simultaneous laser range-differences for the determination of baseline variations evidently demonstrated that a 30 m disagreement between the GEOSPP81 and GEODYN baseline estimate came from the presence of blunders in the original laser ranging data set. Therefore, at that point no conclusions could be drawn regarding the performance of the simultaneous laser range-difference method as applied in the determination of baseline variations. Naturally, rejection of all the observations containing blunders should follow in this investigation.

Data Processing

The rejection of the observations containing blunders was accomplished through the so-called "data-snooping" procedure. The performance of this procedure for editing laser range observations is well established (see 12th Semianual Status Report) because rejection of less than 10% of the observations out of the original data set first randomizes the residuals and, second, reduces their magnitude between -90 and 90 cm. Before applying this procedure the residuals ranged between -300 and +300 m. Based on the fact that 1% of the residuals had magnitude greater than 75 cm and the remaining ones ranged between -40 and +40 cm (see Fig. 1), it was decided to edit out the observations with residuals greater than 75 cm. A sample plot of the resulting residuals is shown in Fig. 2. This plot confirms that the observations which survived in the above procedure form a "clean" (i.e., no presence of blunders) laser-range data set.

Based on this clean data set, the next step is to create the quasi-simultaneous ranges by first determining which of the two stations--7114 or 7115--has the denser data distribution for each pass. Once this is determined, the data of the station with the most observations are fed into an interpolator, and ranges are obtained for each of the data points in the alternate station's record. These ranges, together with the corresponding ones of the alternate station, are subtracted to produce simultaneous range-difference observables which are part of the input for the final adjustment program (i.e., GEOSPP81). It is clear, therefore, that the choice of the interpolation method is very critical, because choice of the wrong interpolator will annihilate the precision of the third-generation lasers. Among the interpolation methods, for reasons well explained in [Pavlis, 1982], the cubic spline and Chebychev interpolators were chosen. Through these two interpolators, the quasi-observables (simultaneous range-differences) were generated for two sample passes (i.e., pass #11 and pass #12) and together with

--Lageos initial state vectors for the starting epoch at each pass
--short planetary ephemeris file for the period span of the observations
Fig. 1 Residuals of Chebychev interpolation (after data snooping)
Fig. 2 Residuals of Chebychev interpolation (after data snooping and editing out residuals 750 cm)
coordinates of the pole and variations in UT1 at the epochs of the observations were fed into the GEOSPP81 adjustment program. The coordinate system in these solutions was realized through the following standard deviation values.

--station 7114, fixed (i.e., $\sigma_x = \sigma_y = \sigma_z = 0.0001$ m)
--station 7115, $\sigma_x = \sigma_y = \sigma_z = 5.0$ m
--initial state vectors: position, $\sigma_x = \sigma_y = \sigma_z = 10.0$ m
velocity, $\sigma_x = \sigma_y = \sigma_z = 10$ cm/s

The orbit residuals as computed using the quasi-observables (i.e., simultaneous range-differences) which were generated through cubic spline and Chebychev interpolators have an rms of 22 cm and 11 cm respectively, and the plots of these residuals are depicted in Figs. 3 and 4. Inspecting Fig. 2 the existence of a high correlation between orbit residuals (GEOSPP81/CUBIC SPLINE) and gaps in successive data points can be seen. This, of course, is very likely to happen because the cubic spline interpolator is a piece-wise interpolator and therefore very sensitive to the successive data gaps.

To confirm this sensitivity, the differences between cubic spline quasi-observables (simultaneous range differences) and Chebychev quasi-observables were computed and plotted (see Fig. 5). These differences have exactly the same behavior as the orbit residuals (GEOSPP81/CUBIC SPLINE)(compare Figs. 3 and 5), and this assures us that the high correlation of the orbit residuals (GEOSPP81/CUBIC SPLINE) and the successive data gaps comes merely from the cubic spline interpolator in view of the fact that the Chebychev interpolator is a global, gap insensitive interpolator. Hence, cubic spline interpolation should not be used when successive data gaps are greater than 10 s. This is exactly the situation with the data set under question. Therefore, the Chebychev interpolator was deployed to generate the quasi-observables (simultaneous-range differences).

Next, using these observables and in response to D. Christodoulidis' request in July, 1984 (private communication), it was decided to compute through the GEOSPP81 program the 7114-7115 baseline for the periods shown in Table 1. These baseline estimates can then be compared with the corresponding ones as determined by the GEODYN program at GSFC Geodynamics Branch and which were kindly supplied to us by D. Christodoulidis (private communication, August, 1984).

Before estimating the 7114-7115 baseline, its relative orientation with the satellite passes for the periods shown in Table 1 should have been examined (see Fig. 6). Fortunately enough, 95% of the passes for each of these periods are parallel to the estimated baseline, assuring, therefore, its reliable determination. Inspecting Table 1 it is clear that both methods (i.e., simultaneous range-difference mode and range mode) perform about the same with a bias of about 17 cm. It is very interesting to see that the rate of change of the baseline in successive periods is about the same in both methods. But does it really mean anything?
Fig. 3 Orbit residuals after data snooping and editing (750 cm residuals) (GEOSPP81/CUBIC SPLINE)
Fig. 4 Orbit residuals after data snooping and editing (750 residuals)
GEOSPP81/CHEBYCHEV
Fig. 5 Differences between differential ranges as computed from cubic spline and Chebychev interpolations.
<table>
<thead>
<tr>
<th>Time Span of Observations (see Fig. 6)</th>
<th>Groups (see Fig. 6)</th>
<th>No. of Observations</th>
<th>GEOSPRRI Length Change in Successive Periods</th>
<th>GROBY Length Change in Successive Periods</th>
<th>Baseline (m)</th>
<th>Length Change in Successive Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/18/79 - 10/30/79</td>
<td>2 - 6</td>
<td>3279</td>
<td>258 289.891</td>
<td>0.04</td>
<td>258 289.759</td>
<td></td>
</tr>
<tr>
<td>10/31/79 - 11/09/79</td>
<td>7 - 9</td>
<td>2751</td>
<td>258 289.931</td>
<td>-0.051</td>
<td>258 289.693</td>
<td>-0.066</td>
</tr>
<tr>
<td>11/10/79 - 11/14/79</td>
<td>10 - 12</td>
<td>1115</td>
<td>258 289.88</td>
<td>-0.051</td>
<td>258 289.775</td>
<td>-0.082</td>
</tr>
<tr>
<td>11/15/79 - 11/20/79</td>
<td>unacceptable data</td>
<td></td>
<td></td>
<td></td>
<td>258 289.702</td>
<td>-0.055</td>
</tr>
<tr>
<td>11/21/79 - 11/22/79</td>
<td>16 - 17</td>
<td>1855</td>
<td></td>
<td></td>
<td>258 289.690</td>
<td>-0.018</td>
</tr>
<tr>
<td>12/13/79 - 12/14/79</td>
<td>18 - 20</td>
<td>1118</td>
<td></td>
<td></td>
<td>258 289.890</td>
<td></td>
</tr>
<tr>
<td>12/20/79 - 12/22/79</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conclusions

It is evident from the above discussion that the GEOSPP81 baseline estimate is compatible with the GEODYN one, but the question of the bias existence should be answered as well as the performance of the simultaneous range difference method in determining real (due to crustal motions) baseline change, over extended periods of time, for instance one or two years. This is exactly the direction which this investigation will follow.

References

2.3 An Algorithm for Crustal Deformation Analysis

Introduction

The complexity and unknown nature of the laws of crustal deformations have forced geodesists to approach geotectonics problems only by providing the necessary measurements to related disciplines at the beginning. Although the physical interpretation of the underlying mechanism of deformation still comes from other disciplines, a more detailed analysis of deformations becomes a necessity. This is caused mainly by the departures of observed quantities from the expected deformations along plate boundaries predicted by the long-term averages of crustal deformations. This led geodesists to modify their simplistic approach of providing only the changes in repeatedly observed geodetic entities to more complicated ones.

Currently, the problem is to find models of the deformation process which fit well to the measurements and explain the discrepancies between small scale and global components of driving mechanisms. Some of the difficulties in the analysis in both scales may arise from single modeling. Therefore, elucidation of deformation mechanisms is, in nature, inescapably iterative on different models.

Today there exists an inherent propensity among geodesists to employ the sample oriented analysis of deformation measurements. However, increasing measurements indicate a better agreement with theoretically predicted values. It is, therefore, another kind of inference, namely, improving and testing global solutions through sample information rather than inferring them from local or regional studies, should also be kept in mind.

Fig. 1 depicts an algorithm that can be used for the analysis of regional or local deformation measurements. Some new techniques and testing procedures are proposed.

Model Identification and Improved Estimation

Geodetic observations are subject to error. In the first step they must be tested for blunders and outliers. Diagnostic checking is performed on each epoch of observations. Since observations are invariant quantities with respect to datum definitions, a proper minimum constraint solution is possible. Prior information on parameters or on observations are most likely to increase residuals and effect sample statistics. Therefore adjustments at this stage should always be based only on sample information.

The second step in the analysis is the identification of the proper descriptive model of deformation process. The qualitative information provided by other disciplines and previous in situ measurement analysis play an important role at this stage. Nevertheless, postulating a general class of descriptive deformation models is mostly based on the experience and intuition of the analyst. Recently, different global tests have been developed for discriminating different models [Chrzansowsky, 1981]. In this study, a new test based on the measure of entropy is being investigated for discriminating
models from the observations performed at different epochs. Several models are tested until an agreement is reached between model and data.

In the next step quantitative prior information is identified and tested versus the sample information (Appendix B). If prior and sample information is found to be compatible, an improved estimation of parameters is performed. Different estimation techniques are being tested for improved estimation. An efficiency function is set up to measure the gain for different estimators against weighted least squares estimation (Appendix A).

Reference

Fig. 1  Deformation analysis: model identification and improved estimation.
Comparing two estimators (particularly unbiased versus biased), we have to weigh the advantages against the disadvantages of both estimators. Obviously, there exist so many ways to express such preferences. Each can be best for different purposes. Unfortunately, a universal comparison criterion does not exist for such purposes. As a criterion of "betterness" the scalar mean square error criterion is considered. The reason for such a choice is discussed through the definitions. An efficiency function based on the scalar mean square error is set up.

Definition: Mean Square Criterion (MSE). An estimator \( \tilde{\xi}_1 \) of \( \xi \) is said to be strongly better than \( \tilde{\xi}_2 \) if the difference
\[
\Delta(\tilde{\xi}_1, \tilde{\xi}_2) = \text{MSE}(\tilde{\xi}_2) - \text{MSE}(\tilde{\xi}_1)
\]
is non-negative definite.

Definition: Scalar Mean Square Error Criterion (SMSE). \( \tilde{\xi}_1 \) is said to be SMSE-better than \( \tilde{\xi}_2 \) if it has smaller scalar MSE, i.e., if
\[
\text{E}(\tilde{\xi}_2 - \xi)^T(\tilde{\xi}_2 - \xi) - \text{E}(\tilde{\xi}_1 - \xi)^T(\tilde{\xi}_1 - \xi)
= \text{tr}(\text{MSE}(\tilde{\xi}_2) - \text{MSE}(\tilde{\xi}_1)) \geq 0
\]
Clearly, if \( \tilde{\xi}_1 \) is MSE better than \( \tilde{\xi}_2 \), then this holds for the SMSE criterion too.

The problem of optimal estimates can only be attacked after the establishment of a generally acceptable criterion of optimality. Specific practical problems are sometimes best treated with different criteria of optimality.

Let \( L(\xi, \tilde{\xi}) \) be a function which, for each true value of parameter \( \xi \) and its estimated value \( \tilde{\xi} \), assigns a "loss" or "cost." The loss function cannot always be chosen as one which most naturally fits the physical problem. Compromises in the nature of the function are usually necessary, and the selection of a loss function in a specific case is often governed more by manipulative convenience than by physical arguments.

Definition: A loss function \( t \) a member of class \( L, t \in L \) if it has the following properties
\[
L: t(q) = 0, \quad \text{if } q = 0
\]
\[
t(q_2) > t(q_1) \quad \text{if } q_2 > q_1 > 0; \text{ monotonic}
\]
\[
t(q) = t(q) \quad \text{symmetric}
\]
where \( q = \hat{t} - t \). It is generally convenient to choose the loss function as a convex function (for minimization purposes). A natural selection of a loss function from the mathematical and physical point of view is

\[
L(\hat{t}, A) = (\hat{t} - t)^T A (\hat{t} - t)
\]

where \( A \) is a positive definite symmetric matrix of corresponding order.

**Definition: Risk Function.** Given the loss \( L(\hat{t}, A) \), the risk \( R \) is defined as the expected value of the loss function,

\[
R(\hat{t}, A) = E(L(\hat{t}, A)) = E((\hat{t} - t)^T A (\hat{t} - t))
\]

Since a priori restrictions that reduce the dimensionality of the parameter space results in an increase in estimation efficiency, then the question is how important is this increase in practice. That is, we need to investigate the "magnitude" of the efficiency increase that results from a priori restrictions. In order to speak of the amount by which efficiency is increased, we must define some cardinal measure of efficiency.

**Definition: A measure of efficiency of an estimator is defined by the following scalar**

\[
\delta = \frac{(R(\hat{t}_1) - R(\hat{t}_2))}{R(\hat{t}_1)}
\]

which gives the decrease of risk relative to the sample estimation. The nearer \( \delta \) is to one, the higher is the influence of the prior information on the efficiency of the estimation. If \( \delta \) tends to zero, this indicates that the sample information is the dominating one. If the risk is chosen as the SME, the efficiency function is by the following expression

\[
\delta = \frac{(\text{tr MSE}(\hat{t}_1) - \text{tr MSE}(\hat{t}_2))}{\text{tr MSE}(\hat{t}_1)}
\]

or

\[
\delta = \frac{(\text{SMSE}(\hat{t}_1) - \text{SMSE}(\hat{t}_2))}{\text{SMSE}(\hat{t}_1)}
\]
APPENDIX B: TEST FOR COMPATIBILITY OF PRIOR AND SAMPLE INFORMATION

Once the model of deformation is identified (discriminatory analysis), the next step is to check the possibility that prior and sample information are in agreement with each other. This step is one of the deciding factors on whether prior information should be included in the estimation procedure.

Let $\mathbf{x}$ denote the prior information on deformation parameters $\mathbf{T} = \mathbf{x} + \mathbf{e}$ where $\mathbf{e}$ is the error vector. Then the following null hypothesis is tested:

$H_0$: Prior and sample information are in agreement.

Under this hypothesis if prior information and an estimate of parameters of WLS type are in agreement, then their difference is expected to be close to zero,

$$\delta = \mathbf{x} - \mathbf{\bar{x}} = \mathbf{e} - N^{-1} \mathbf{A}^T W^{-1} \mathbf{u}$$

where $N = (\mathbf{A}^T W^{-1} \mathbf{A})$, $W$ the weight matrix, and $\mathbf{u}$ is the error vector of observations. The matrix of second moments of this difference is

$$E(\delta \delta') = \Sigma_{ee} - \sigma^2 N^{-1}$$

if

$$E(\delta \delta') = \Sigma_z z'$$

where $z = \Sigma_{ee} - \sigma^2 B^{-1}$. $\Sigma_{ee}$ is the covariance matrix of a priori information. $\sigma^2$ is the a priori variance of unit weight. Then,

$$\delta^T \Sigma_z^{-1} \delta \sim (0, 1)$$

If the disturbance terms in sample observations and prior information are assumed to be distributed normally, i.e.,

$$\mathbf{u} \sim N(0, \sigma^2 \Sigma_{uu}), \ \bar{\mathbf{e}} \sim N(0, \Sigma_{ee})$$

then

$$\delta^T \Sigma_z^{-1} \delta \sim N(0, 1)$$

It can be shown that the scalar $\tau = \delta^T \Sigma_z^{-1} \delta$ can be used as a test statistic since it follows a central $\chi^2$ distribution. $\tau$ is called "compatibility statistic" [Theil, 1963],

$$\tau = \delta^T \Sigma_z^{-1} \delta = (\mathbf{R} - \mathbf{\bar{x}})^T (\Sigma_{ee} + \sigma^2 N^{-1})(\mathbf{R} - \mathbf{\bar{x}})$$

can be used which has asymptotically the same distribution. Here, $s$ is an estimate of a priori variance of unit weight.
Reference

2.4 Preliminary Results of the Reference Frame Comparisons in Terms of the Pole Coordinates from Different Techniques

The data sets used in the present comparisons are from Project MERIT Circulars issued by the Joint Working Group on the Rotation of the Earth and from the Earth Orientation Bulletins issued by IRIS. The data involves astrometry (BIH, IPMS), Doppler (coded as Dop92(1967-921), Dop67(1970-671) and Dop44(1981-441)), SLR (Lageos, coded as CSR39) and VLBI. Starlette is excluded from comparison because of its sparse amount of data. All the data sets, except Dop92 and VLBI, span the whole comparison period of September, 1983, through June, 1984. The Dop92 data for September, 1983, February, May and June, 1984, are not available. The VLBI data for September, 1983, are also not available.

The data are smoothed. The smoothing models of the coordinates of the pole are as follows:

Model 1:

\[
\begin{align*}
\text{x} &= k_1 + k_2 \cos A + k_3 \sin A + k_4 \cos C + k_5 \sin C \\
\text{y} &= k_6 - k_2 \sin A + k_3 \cos A - k_4 \sin C + k_5 \cos C
\end{align*}
\]

Model 2:

\[
\begin{align*}
\text{x} &= k_1 + k_2 \cos A + k_3 \sin A + k_4 \cos C + k_5 \sin C \\
\text{y} &= k_6 + k_7 \cos A + k_8 \sin A - k_4 \sin C - k_5 \cos C
\end{align*}
\]

where

\[
\begin{align*}
A &= 2\pi (\text{MJD} - 45578) / 365.25 \\
C &= 2\pi (\text{MJD} - 45578) / 435
\end{align*}
\]

in which MJD 45578 corresponds to September 1, 1983.

Table 1 lists these coefficients with their standard deviations and post-fit standard deviations of "observations." The values of coefficients \( k_1 \) and \( k_8 \) representing relative polhode centers are drawn in Fig. 1. It should be noted that the time intervals of tabulated values vary among different techniques. Here the five-day values for BIH and IPMS data, the four-day values for Doppler data, and the roughly five-day values for SLR and VLBI data are used.

Table 1 and Fig. 1 show that no large systematic difference exists in the pole origin. Note that the BIH coordinates and the coefficients derived from them have larger standard deviations. As mentioned earlier, Doppler 92 lacks four-month data, so it has a relatively large standard deviation.

For comparison purposes, the means of pole coordinate differences between various techniques are computed, using the simple formulae:
\[ \Delta x_m = \frac{1}{n} \sum_{i=1}^{n} (x^I - x^{II}) \]
\[ \Delta y_m = \frac{1}{n} \sum_{i=1}^{n} (y^I - y^{II}) \]

Table 2 includes these means of differences along with the differences between \( k_1 \)'s and between \( k_4 \)'s as listed in Table 1. It should be noted that in order to obtain simultaneity among all types of "observations" from different techniques the five-day pole coordinates for CSR39, Doppler 44 and VLBI are interpolated at the time of BIH coordinates, using a fourth-order Lagrangian interpolation method. It is found that \( \Delta x_m \) and \( \Delta k_1 \), and \( \Delta y_m \) and \( \Delta k_4 \) tend to agree in general, but are not in good agreement. This is because the periodic terms (annual and Chandler) in pole coordinates are not averaged out in the comparison duration (ten months).

Finally, the transformation parameters between various reference frames inherent in different techniques are calculated in terms of pole coordinates, using the following models:

\[ \Delta y = -(y^I - y^{II}) = -\beta_1 + \alpha_1 \cos \phi + \alpha_2 \sin \phi \]
\[ \Delta x = -(x^I - x^{II}) = -\beta_2 + \alpha_1 \sin \phi + \alpha_2 \cos \phi \]

where \( \beta_1, \beta_2 \) are the rotation angles about the first and second axes of the terrestrial coordinate system II; \( \alpha_1, \alpha_2 \) of the corresponding inertial coordinate system II; \( \phi \) is the Greenwich sidereal time. The values of \( \beta_1, \beta_2, \alpha_1, \alpha_2 \) angles and their standard deviations with post-fit standard deviations of "observations" are listed in Table 3.

It can be seen from Table 3 that the rotation angles are of the order of 10 milliarcseconds, much smaller than expected. The interesting thing is that post-fit standard deviations (and thus the standard deviations of transformation parameters) for VLBI \( \rightarrow \) CSR39 are the smallest among them, while those associated with the BIH system are much larger. This is possibly an indication that VLBI and CSR39 pole coordinates are more consistent and probably more precise. On the other hand, the BIH coordinates are seemingly questionable. Doppler coordinates are in between.
### Table 1 Coefficients of Annual and Chandler Motions with Their Standard Deviations (units in \( 0.001 \))

<table>
<thead>
<tr>
<th>MODEL 1</th>
<th>( k_1 ) (( \sigma ))</th>
<th>( k_2 ) (( \sigma ))</th>
<th>( k_3 ) (( \sigma ))</th>
<th>( k_4 ) (( \sigma ))</th>
<th>( k_5 ) (( \sigma ))</th>
<th>( k_6 ) (( \sigma ))</th>
<th>( k_7 ) (( \sigma ))</th>
<th>Motion Amplitude</th>
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ORIGINAL PAGE IS OF POOR QUALITY
Model 1

Model 2

Fig. 1  Relative polhode centers with standard deviation
Table 2

Table 2 (units in 0.001)

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Table 3

Table 3 (units in 0.001)

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3. PERSONNEL

Ivan I. Mueller, Project Supervisor, part time
Brent Archinal, Graduate Research Associate, part time
George Dedes, Graduate Research Associate, part time
Huseyin Baki Iz, Graduate Teaching Associate, without compensation
Irene B. Tesfai, Secretary, part time
Ziqing Wei, Visiting Researcher, without compensation

4. TRAVEL

Ivan I. Mueller, Brent A. Archinal, George Dedes
Cincinnati May 14-17, 1984
To attend AGU Annual Spring Meeting. Mueller presented two papers.

Ivan I. Mueller
Graz, Austria June 25 - July 7, 1984
To represent CSTG (IAG Commission VIII) at the 25th COSPAR Plenary.
No project support.

Ivan I. Mueller
Sopron, Hungary July 9-13, 1984
To give the keynote speech at the International Symposium on Space
Techniques for Geodynamics. To attend business meetings of CSTG,
MERIT and Project ADOS. No project support.

Ivan I. Mueller
Herstmonceux, England September 8-14, 1984
To attend 5th International Workshop on Laser Ranging Instrumentation,
Royal Greenwich Observatory. No project support.

Ivan I. Mueller
Magdeburg, GDR September 23-29, 1984
To present the Helmert Commemorative Lecture and attend the 5th
International Symposium on Geodesy and Physics of the Earth. No
project support.
5. REPORTS PUBLISHED TO DATE

OSU Department of Geodetic Science Reports published

262 The Observability of the Celestial Pole and Its Nutations
by Alfred Leick
June, 1978

263 Earth Orientation from Lunar Laser Range-Differencing
by Alfred Leick
June, 1978

284 Estimability and Simple Dynamical Analyses of Range (Range-Rate and
Range-Difference) Observations to Artificial Satellites
by Boudewijn H.W. van Gelder
December, 1978

289 Investigations on the Hierarchy of Reference Frames in Geodesy and
Geodynamics
by Erik W. Grafarend, Ivan I. Mueller, Haim B. Papo, Burghard Richter
August, 1979

290 Error Analysis for a Spaceborne Laser Ranging System
by Erricos C. Pavlis
September, 1979

298 A VLBI Variance-Covariance Analysis Interactive Computer Program
by Yehuda Bock
May, 1980

299 Geodetic Positioning Using a Global Positioning System of Satellites
by Patrick J. Fell
June, 1980

302 Reference Coordinate Systems for Earth Dynamics: A Preview
by Ivan I. Mueller
August, 1980

320 Prediction of Earth Rotation and Polar Motion
by Sheng-Yuan Zhu
September, 1981

329 Reference Frame Requirements and the MERIT Campaign
by Ivan I. Mueller, Sheng-Yuan Zhu and Yehuda Bock
June, 1982

337 The Use of Baseline Measurements and Geophysical Models for the
Estimation of Crustal Deformations and the Terrestrial Reference System
by Yehuda Bock
December, 1982
On the Geodetic Applications of Simultaneous Range-Differencing to Lageos
by Erricos C. Pavlis
December, 1982

A Comparison of Geodetic Doppler Satellite Receivers
by Brent A. Archinal
November, 1982
(partial support)

On the Time Delay Weight Matrix in VLBI Geodetic Parameter Estimation
by Yehuda Bock
July, 1983

Model Choice and Adjustment Techniques in the Presence of Prior Information
by Burkhard Schaffrin
September, 1983
The following papers were presented at various professional meetings and/or published:

"Concept for Reference Frames in Geodesy and Geodynamics"
AGU Spring Meeting, Miami Beach, Florida, April 17-21, 1978
IAU Symposium No. 82, Cadiz, Spain, May 8-12, 1978
7th Symposium on Mathematical Geodesy, Assisi, Italy, June 8-10, 1978

"What Have We Learned from Satellite Geodesy?

"Parameter Estimation from VLBI and Laser Ranging"
IAG Special Study Group 4.45 Meeting on Structure of the Gravity Field Lagonissi, Greece, June 5-6, 1978

"Estimable Parameters from Spaceborne Laser Ranging"
SGRS Workshop, Austin, Texas, July 18-23, 1978

"Defining the Celestial Pole," manuscripts geodaetica, 4 (1979), No. 2 pp. 149-183.

"Three-Dimensional Geodetic Techniques"
Technology Exchange Week, Inter-American Geodetic Survey Fort Clayton, Canal Zone, May 14-19, 1979


"Space Geodesy for Geodynamics, A Research Plan for the Next Decade"
Sonderforschungsbereich - Satellitengodöasie - SFB 78 Colloquium in Viechtach, FRG, October 23-24, 1979

"Concept of Reference Frames for Geodesy and Geophysics"
seminar given at University of Stuttgart, West Germany, June 19, 1980

"Space Geodesy and Geodynamics,"
seminar given at University of Stuttgart, West Germany, June 26, 1980

"Geodetic Applications of the Global Positioning System of Satellites and Radio Interferometry," seminar given at University of Stuttgart, West Germany, July 3, 1980


"Precise Positioning with GPS" seminar given at Deutsche Geodätische Forschungsinstitut, Munich, West Germany, September 18, 1980

"Tecnica Geodesicas Tridimensionales" (translated from English by IAGS), ASIA Journal (Asociacion Salvadorena de Ingenieros y Arquitectos) San Salvador, No. 61, Oct. 80, pp. 40-51; cont'd in No. 62, Dec. 80, pp. 31-39.


"A Comparison of Geodetic Doppler Satellite Receivers" Proc. of 3rd International Geodetic Symp. on Satellite Doppler Positioning, Las Cruces, New Mexico, Feb. 8-12, 1982 (Brent Archinal and Ivan I. Mueller)


"Results of a Comparison of Geodetic Doppler Satellite Receivers," Third International Symp. on the Use of Artificial Satellites for Geodesy and Geodynamics, Porto Hydra, Greece, Sept. 20-25 (Brent A. Archinal and Ivan I. Mueller)


"Baseline Determination from Simultaneous Lageos Ranging," Annual Spring Meeting of the AGU, Cincinnati, May, 1984 (Erricos C. Pavlis)


"Reference Systems, Collocations and Ties," 5th International Workshop on Laser Ranging Instrumentation, Royal Greenwich Observatory, September 8-14, 1984


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