SHEAR-LAYER CORRECTION AFTER AMIET UNDER CONSIDERATION OF ADDITIONAL TEMPERATURE GRADIENT. WORKING DIAGRAMS FOR CORRECTION OF SIGNALS

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Amiet's correction scheme for sound wave transmission through shear-layers is extended to incorporate the additional effects of different temperatures in the flow-field in the surrounding medium at rest. Within a parameter-regime typical for acoustic measurements in wind tunnels amplitude-and angle-correction is calculated and plotted systematically to provide a data base for the test engineer.
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Notation

c \quad \text{m/s} \quad \text{Speed of sound}

h \quad \text{m} \quad \text{Separation of sound source from the shear layer}

M \quad - \quad \text{Mach number}

P_A \quad \text{N/m}^2 \quad \text{Acoustic pressure at the point (A)}

P_M \quad \text{N/m}^2 \quad \text{Acoustic pressure at the measurement point (M)}

P_{c-} \quad \text{N/m}^2 \quad \text{Acoustic pressure immediately ahead of the shear layer (on the source side)}

P_{c+} \quad \text{N/m}^2 \quad \text{Acoustic pressure adjacent to the rear of the shear layer (measurement side)}

t \quad \text{K} \quad \text{Temperature}

U \quad \text{m/s} \quad \text{Flow speed}

x, y, z \quad \text{m} \quad \text{Cartesian coordinates}

x_0 \quad \text{m} \quad \text{Axial separation of the sound source from the intersection point of a sound ray with the shear layer}

Y \quad \text{m} \quad \text{Side separation of the measurement point from the sound source}

\alpha \quad \text{deg.} \quad \text{Angle between wave front and shear layer}

\mu \quad - \quad = \left( \frac{t_i}{t_a} \right)^{1/2}

\psi \quad \text{deg.} \quad \text{Angle between the shear layer and the sound propagation direction in the surrounding medium at rest}

\psi' \quad \text{deg.} \quad \text{Through radiation angle from the sound source in the flow}

\psi_M \quad \text{deg.} \quad \text{Angle between the connection line of sound source-measurement point and the shear layer}

\rho \quad \text{kg/m}^3 \quad \text{Density}

Subscripts

i \quad - \quad \text{In the flow}

a \quad - \quad \text{In the surrounding medium at rest}
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1. **Introduction**

When studying aeroacoustic phenomena using wind tunnel tests (free jet configuration), often one measures the acoustic field outside of the wind tunnel flow which is produced by the noise source inside the flow. The acoustic signal has to pass through the flow shear layer of the wind tunnel free jet on its way to the measurement microphone. Consequently, there is a change in the pressure amplitude and in the original radiation direction.

In order to obtain accurate measurement data, additional corrections are required. From the large number of research papers published on this topic, the method of Amiet [1] published in the last few years and its correction method has proven itself. It has been systematically tested already by experimental investigations [2]. The correction equations given by Amiet, however, are restricted to the case of uniform temperature in the wind tunnel flow and the surrounding medium. The theory, which is the basis of the correction method, however, does not require this restriction, which was already noticed by Amiet.

During investigations, in a free jet wind tunnel, especially in large facilities, it is easy for a temperature difference between the flow and the surroundings to occur, or the temperature of the air flow is used as a parameter in aeroacoustic measurements. Therefore, in the present paper, we will extend the correction equations of Amiet for sound propagation through a wind tunnel shear layer to the influence of different temperatures in the flow and the surroundings. In particular, we will then give a systematic result presentation to the test engineer, which will enable him to perform a fast correction of the measured data.

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* Numbers in margin indicate foreign pagination
2. Expansion of the Correction Method of Amiet to Different Temperatures Inside and Outside of the Flow

The correction method of Amiet is based on a paper by Ribner [3], in which the amplitude changes and angular changes when a plane acoustic wave passes through an infinitesimally thin and plane flow shear layer are given. Amiet, in his paper [1], relates this solution to laws of geometric acoustics. In particular, he allows the shear layer to be located in the near field of the acoustic source. In the following we will give a theoretical derivation of the correction equations for the changes of the acoustic propagation direction and pressure amplitude, with the additional influence of an infinitesimally thin temperature shear layer as well. The theoretical expansion, however, will be restricted to steps in which the temperature influence becomes effective. The complete derivation of the theoretical relationships can be found in the literature [1 and 3].

2.1 Theoretical Analysis

Figure 1 shows the geometric configuration of an acoustic source (Q) in a flow, the position of the velocity and temperature shear layer and the position of the receiver (M) in the surrounding medium at rest. The purpose of the theoretical analysis is to determine the acoustic pressure level at the point A (Figure 1) which is emanated under the true radiation angle, as related to the one measured at (M), as a function of suitable dimensionless influencing parameters. Therefore, we wish to find the following relationships:

\[
(\psi - \psi_M) = \text{function} \left( \psi_M, \frac{h}{\gamma}, M, \frac{t_i}{t_a} \right) \quad (1)
\]

\[
\frac{p_A}{p_M} = \text{function} \left( \psi_M, \frac{h}{\gamma}, M, \frac{t_i}{t_a} \right). \quad (2)
\]
In order to determine the rotation of the wave front when passing through the separation layer between the moving and the rest medium, we use the Mach wave analogy shown in Figure 2, which is based on an analysis by Ribner [3]. The sound waves which impinge on the separation surface and come from the moving medium (i) produce a disturbance (ripple) in it, which propagates with a speed above the speed of sound. In the present case, this perturbation velocity is found as the sum of the velocity component parallel to the separation layer of the velocity triangle formed with the local speed of sound and the flow velocity $U$.

Similar to this representation, according to the discussion by Ribner, the perturbation can be considered to be fixed, and instead the medium can be considered to be moving with the perturbation velocity, but in the opposite direction. The acoustic wave reflected by the separation layer as well as the acoustic waves radiated into the medium at rest (a) can be interpreted as Mach waves produced by the fixed perturbation. From the velocity triangles shown in Figure 2, we then find the following relationship:

$$U_a = U_1 + U$$

in other words

$$\frac{C_a}{\cos \phi} = \frac{C_i}{\sin \alpha} + U$$

In dimensionless notation, equation (4) then becomes

$$\frac{1}{u \cos \phi} = \frac{1}{\sin \alpha} + M$$

with $u = \sqrt{t_i / t_a}$

where $M$ is the Mach number of the flow.

Using some geometrical analysis on the propagation of the wave fronts, in the flow, Amiet then finds the following relationship

$$\frac{1}{u \cos \phi} = \frac{1}{\sin \alpha} + M$$
\[
\frac{\cos a}{\sin a + M} = \frac{h}{x_o} = \tan \varphi'
\]  

(6)

and

\[
Y \cot \varphi'_M = h \cot \varphi' + (Y - h) \cot \varphi
\]  

(7)

or

\[
\tan \varphi'_M = \frac{\tan \varphi' \tan \varphi}{(h/Y) \tan \varphi + [1-(h/Y)] \tan \varphi'}
\]

From equations (5) and (6) one finally obtains the following relationship

\[
\tan \varphi' = \frac{h}{x_o} = \frac{\zeta}{(1-M^2) \mu \cos \varphi + M}
\]

(8)

with

\[
\zeta = \sqrt{(1-M \mu \cos \varphi)^2 - \mu^2 \cos^2 \varphi}
\]

(9)

Based on equations (7) and (8), by using an iteration one can find the desired relationship (equation (1)) for the angle of the source (Q) and receiver (M) (Figure 1) connection line and the true radiation angle. In order to determine the change in the pressure amplitude when a sound wave passes through the separation layer, Ribner [3] gives the transmission loss in the following form

\[
\frac{P_{C-}}{P_{C+}} = \frac{1}{T} = \frac{1}{2} + \frac{0.1 \frac{c^2}{d}}{2 \rho_a c_d M_a (M^2 - 1)^{0.5}} \left( \frac{M^2 - 1}{M_a} \right)^{0.5}
\]

(10)

The pressure \( P_{C-} \) describes the amplitude of the acoustic
wave which impinges onto the separation surface of pressure $P_{c+}$ of the transmitted wave. In the present case, the density difference between the medium at rest and the moving medium is only to be attributed to a temperature change by using

$$\frac{\partial \rho}{\partial \frac{\partial^2 c}{\partial t^2}} = 1$$  \hspace{1cm} (11)

It is found that an amplitude change is only indirectly dependent if there is a temperature shear layer, that is only through an angle change. Using equations (5), (9) and (11), and a few geometric relationships, equation (10) can be written in the form

$$\frac{P_{c-}}{P_{c+}} = \frac{1}{T} = \frac{1}{2\zeta} \left\{ \zeta + \sqrt{1 - u^2 \cos^2 \varphi} \left( 1 - Mu \cos \varphi \right)^2 \right\}$$  \hspace{1cm} (12)

In order to calculate the pressure amplitude at the position (A) in the medium at rest (Figure 1), we must now determine the divergence of the acoustic rays along the propagation path. According to the geometry shown in Figure 3, according to Amiet, we find the following for the propagation in the x-y plane

$$\frac{dl_2}{dl_1} = 1 + (Y-h) \frac{1}{\sin^2 \varphi} \frac{d \varphi}{dx_o}$$  \hspace{1cm} (13)

By substitution

$$\left| \frac{dx_o}{d \varphi} \right| = \left| \frac{dx_o}{\frac{d \alpha}{d \varphi}} \right| = \left| \frac{\alpha}{\cos^2 \varphi} \right| \frac{h(1 + M \sin \alpha)}{(1 - M \cos \varphi)} \frac{\left( -u \right) \sin \varphi}{\zeta (1 - Mu \cos \varphi)} = \left| \frac{-uh \sin \varphi}{\zeta^3} \right|$$  \hspace{1cm} (14)

in equation (13) one finally obtains

$$\frac{dl_2}{dl_1} = 1 + \left( \frac{Y}{h} - 1 \right) \frac{\zeta^3}{\mu \sin^2 \varphi}$$  \hspace{1cm} (15)
For the divergence in the z direction according to Amiet we find

\[ \frac{dz_2}{dz_1} = 1 + (Y-h) \frac{1}{\sin \psi} \frac{d\psi'}{dz_1} \] (16)

the angle \( \psi' \) is the solid angle of the acoustic ray with respect to the x-y plane (Figure 4).

For the boundary condition for equal phase of the incoming and transmitted acoustic wave in the separation layer, Amiet obtains the following relations

\[ \frac{dz}{d\psi}_{\psi=0} = \frac{\cos \psi}{(1-M \cos \psi)} \frac{h}{(1-M^2)} \left( \sqrt{(1-M^2) + \left( \frac{x_0}{h} \right)^2} - M \frac{x_0}{h} \right) \] (17)

If the ratio \( x_0/h \) is eliminated using equation (8), then from this we obtain

\[ \frac{dz}{d\psi}_{\psi=0} = \frac{h \cos \psi}{\zeta} \frac{(1-M \mu \cos \psi)}{(1-M \cos \psi)} \] (18)

For small angles \( \psi \) we have \( \psi' = \psi \cdot \sin \phi \) and \( \phi = \pi/2 - \psi \), that is \( \psi' = \psi \cdot \cos \phi \). Therefore we find

\[ \frac{dz}{d\psi'} = \frac{dz}{d\psi} \frac{d\psi}{d\psi'} = \frac{h}{\zeta} \frac{(1-M \mu \cos \phi)}{(1-M \cos \phi)} \] (19)

and equation (16) is given the form

\[ \frac{dz_2}{dz_1} = 1 + \left( \frac{Y}{h} - 1 \right) \frac{\zeta}{\sin \psi} \frac{(1-M \cos \psi)}{(1-M \mu \cos \psi)} \] (20)

The ratio of the pressure amplitudes between the location of the separation layer and the measurement point can therefore now be determined as a consequence of the divergence of the sound rays, according to
\[ \frac{P_{c+}}{P_{c-}} = \frac{d^2_{z1}}{d^2_{z2}} \]  

where \( P_{c+} \) is the acoustic pressure after passing through the separation layer. The ratio of the pressure amplitudes \( P_{c-}/P_{c+} \), that is the pressure change when passing through the separation layer, was specified by equation (12). Therefore, we have the following for the desired ratio of the pressure amplitude at the point A and the pressure amplitude at the point (M)

\[ \frac{P_A}{P_M} = \frac{P_{c+}}{P_{c-}} \cdot \frac{P_A}{P_{c+}} = \frac{P_{c-}}{P_{c+}} \cdot \frac{h}{Y} \]  

Using equations (12) and (21) we finally obtain the desired correction equation for the acoustic pressure amplitude in the desired dimensionless form

\[ \frac{P_A}{P_M} = \frac{h}{Y} \cdot \frac{2 \zeta \sin^2 \varphi}{\sin \varphi + \left( \frac{Y}{h} - 1 \right) \frac{(1-M \mu \cos \varphi)}{(1-M \mu \cos \varphi)}} \cdot \left[ \sqrt{\sin^2 \varphi + \left( \frac{Y}{h} - 1 \right) \frac{\cos^2 \varphi}{(1-M \mu \cos \varphi)}} \right] \]  

In the case of equal temperature in the flow and in the surrounding medium, the temperature ratio takes on the value of 1, and the correction equations (7), (8), (9) and (23) then are given the form specified by Amiet [1].

2.2 Discussion of Results

Figures 5, 6 and 7 show calculation results of the shear layer correction for selected values of the dimensionless influence parameters. That is, the geometric ratio \( h/Y \), the
temperature ratio, $t_i/t_a$ and the flow Mach number $M$. In order to show the substantial important trends for the deviations referred to the measurement point (M) as well as that of the true radiation angle and the corresponding acoustic pressure level, first we assume two limiting values for the geometric ratio $h/Y$.

Figure 5 shows the correction quantities as a function of the geometric propagation angle $\varphi_M$ to the measurement point for the ratio $h/Y \rightarrow 0$. This is for a measurement location whose distance from the source is substantially greater than the lateral dimensions of the flow field in which the source is located. For the case of equal temperature in the flow and in the surrounding medium at rest, we then obtain the correction values which are well known from the literature: the true radiation angle from the source is always smaller than the geometric angle between the measurement point. The magnitude of the angular difference increases with increasing flow Mach number. In addition, in the range of small angles, we have the phenomena of total reflection at the shear layer, i.e., the sound radiated by the source against the flow direction is "captured" in the flow (for example, at $M = 0.3$ for $\varphi_M < 40^\circ$). The condition for the occurrence of total reflection according to equations (8) and (9) is the following

$$\varphi < \arccos \left( \frac{1}{\mu(1+M)} \right)$$

(24)

because of $\varphi' = 0$ in equation (8).

Regarding the correction of the acoustic pressure level determined at the measurement point (M) it is found that in the considered angular range, when recalculating the level values to the geometric location (A) of the intersection point between a ray with the true radiation angle and a line parallel to the flow direction through the measurement point at angles
of $\gamma M > 90^\circ$, the sound pressure levels are greater. On the other hand, below $90^\circ$ they are smaller. With increasing flow Mach number, the magnitude of the level difference to be corrected increases.

From these results, we find that nothing changes when one assumes a temperature difference between the medium at rest and the medium in motion. It is only due to a corresponding change in the angular correction that there is also a change in the correction of the pressure amplitudes.

For smaller Mach numbers and a relatively larger temperature change in the flow (in Figure 6: $m = 0.05; t_i/t_a = 1.5$) however there is a special feature for large radiation angles: the angular correction here becomes negative and therefore the sign in the level difference is reversed in this angular range, where otherwise it is positive. The reason for this is the fact that for angles $\gamma M > 90^\circ$, the rotation of the wave fronts caused by the temperature jump compared with the rotation caused by the flow separation layer has the opposite sense, and can dominate for correspondingly lower flow Mach numbers.

The correction values shown in figure 6 apply for a ratio of $h/Y = 0.8$, that is for a measurement location, which is close to the shear layer compared to the distance of the sound source. The magnitude of the correction quantities varies as a function of the measured radiation angle caused by the resulting geometric differences. However, from this there is no principal change with regard to the previously described relationships. However, it is remarkable, that at high flow Mach numbers there is a great influence of the temperature ratio on the correction of the pressure amplitude.

Figure 7 finally shows the influence of different temperature in the flow and the surrounding medium at rest for parameter values which can be considered typical for
noise measurements in low speed wind tunnels. It becomes clear that the temperature influence as a rule will only become effective at large measurement angles.

3. Representation of the Correction Parameters in a Nomogram

Figures 8 to 44 show the correction parameters - amplitude correction and angle correction - for selected parameter ranges in the form of nomograms. The temperature ratio $t_i/t_a$ varies between 1.0 and 1.15 and the geometric ratio $h/Y$ varies in the range 0 to 0.8. In each of these correction diagrams, the flow Mach number occurs as a parameter and passes through values between 0.05 and 0.3.

4. Summary

Starting with the basic equations of Ribner and Amiet for the correction of acoustic signals after passing through a flow shear layer, we have determined the additional influence of different temperatures on both sides of the shear layer on the correction parameters. It is found that especially for large radiation angles compared with the flow direction, the influence of different temperatures in the flow and in the surrounding medium at rest must be considered for an accurate amplitude and angle correction.

For the parameter range which can be considered typical for noise measurements in free jet wind tunnels, we have calculated the correction parameters and have systematically compiled the calculated results in the form of nomograms.
5. References


Figure 1. Geometric relationships when sound passes through a shear layer according to Amiet [1].
Figure 2. Sound propagation and reflection through and at a separation layer between a moving and a medium at rest according to Ribner [3].
Figure 3. Sound propagation in the x-y plane.

Figure 4. Angle definition for determining the sound propagation in the z direction.
Figure 5. Influence of the flow Mach number and the temperature ratio on the amplitude and angle corrections for the ratio $h/Y = 0$ (large distance to the shear layer).
Figure 6. Influence of the flow Mach number and temperature ratio on the amplitude and angle corrections for the ratio $h/Y = 0.8$ (small measurement distance to the shear layer).
Figure 7. Determination of an amplitude and angle correction for typical measurement conditions in low speed wind tunnels.
Figure 8. Amplitude and angle correction for different flow Mach numbers for \( t_i / t_a = 1.0 \); \( h/Y \to 0 \).
Figure 9. Amplitude and phase deviation for different flow Mach numbers for $t_i/t_a = 1.0$; $h/Y = 0.1$. 

$$AL = 20 \log \left( \frac{P_a}{P_M} \right)$$

Parameters:
- $t_i/t_a = 1.0$
- $h/Y = 0.1$
- $M = 0.30$
- $0.25$
- $0.20$
- $0.15$
- $0.10$
- $0.05$

Variables:
- $\theta_M$
- $\phi_M$
- $\theta_M' - \phi_M'$
Fig. 10. Curves of ultrasound attenuation for different values of each curve for \( \frac{t_i}{t_a} = 1.0 \) and \( h/Y=0.2 \).
\[ \Delta L = 20 \log \left( \frac{P_A}{P_M} \right) \]

\[ M = 0.30 \]

\[ M = 0.25 \]

\[ M = 0.20 \]

\[ M = 0.15 \]

\[ M = 0.10 \]

\[ M = 0.05 \]

\[ t_i/t_a = 1.0 \]

\[ h/Y = 0.3 \]

\[ \phi_{M} - \phi' \]

\[ 30^\circ \quad 50^\circ \quad 70^\circ \quad 90^\circ \quad 110^\circ \quad 130^\circ \quad 150^\circ \]

Figure 12. Amplitude and phase shift curves for each number t_i/t_a = 1.0; h/Y = 0.4
Figure 13. Amplitude and angle correction for different values of $\theta$ with $t_i/t_a = 1.0$ and $h/Y = 0.5$. 
Figure 14. Amplitude and angle correction for different $M$ values for $t_i/t_a = 1.0$; $h/Y = 0.6$. 

$\Delta L = 20 \log \left( \frac{p_A}{p_M} \right)$
\[ \Delta L = 20 \log \left( \frac{P_A}{P_M} \right) \]

- \( t_i/t_a = 1.0 \)
- \( h/Y = 0.7 \)
- \( M = 0.30 \)
- \( 0.25 \)
- \( 0.20 \)
- \( 0.15 \)
- \( 0.10 \)
- \( 0.05 \)

\[ \Phi_M - \Phi_M' \]

\[ \Phi_M \]

\[ 30^\circ, 50^\circ, 70^\circ, 90^\circ, 110^\circ, 130^\circ, 150^\circ \]
\[ \Delta L = 2 \log \left( \frac{p_A}{p_M} \right) \]

Graphs showing the relationship between \( \phi_M \) and \( \phi' \) for different Mach numbers (M) with \( t_i/t_a = 1.0 \) and \( h/Y = 0.8 \).

Text: 16. 16. Graphs showing the relationship between \( \phi_M \) and \( \phi' \) for different Mach numbers (M) with \( t_i/t_a = 1.0 \) and \( h/Y = 0.8 \).
$\Delta L = 20 \log \left( \frac{P_A}{P_M} \right)$

$t_i/t_a = 1.05$

$h/Y \to 0$

$M = 0.30$
$0.25$
$0.20$
$0.15$
$0.10$
$0.05$

$\phi_M - \phi'$

$\phi_M$

$30^\circ$
$50^\circ$
$70^\circ$
$90^\circ$
$110^\circ$
$130^\circ$
$150^\circ$

$\phi_M$

Diagram 17. Amplitude and angle correction for different flow such as $t_i/t_a = 1.05$; $h/Y \to 0$. 

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Figure 18. Amplitude and angle correction for different flow Mach numbers for $t_i/t_a=1.05$; $h/Y=0.1$. 
Figure 19. Amplitude and phase characteristics for a flow with

\[ t_i/t_a = 1.05 ; \ h/Y = 0.2 \]
Figure 20. Amplitude and angle correction for different flow Mach numbers for $\tau_i/\tau_a = 1.05$ ; $h/Y = 0.3$. 
Figure 21. Amplitude and phase correction for different flow mach numbers for $t_i/t_a = 1.05$; $h/Y = 0.4$. 
Figure 22. Table correction for different flow mach
$\frac{t_1}{t_a} = 1.05$ ; $h/Y = 0.5$. 

$\Delta L = 20 \log \left( \frac{p_A}{p_M} \right)$
Figure 23. Variation of sound level for different flow Mach numbers. $t_i/t_a = 1.05$; $h/Y = 0.6$. 
Figure 24. Amplitude and phase deviation for different flow Mach numbers for $t_i/t_a = 1.05$; $h/Y = 0.7$. 
Figure 25. Amplitude and phase correction for different flow Mach numbers for $\frac{t_i}{t_a} = 1.05$ and $h/Y = 0.8$. 
Figure 26: Amplitude and angle correction for different flow mach
numbers for $\frac{t_i}{t_a} = 1.10; \frac{h}{\gamma} = 0$. 

$M = 0.30, 0.25, 0.20, 0.15, 0.10, 0.05$
\[ \Delta L = 20 \log \left( \frac{P_A}{P_M} \right) \]

- \( t_i / t_a = 1.10 \)
- \( h / Y = 0.1 \)
- \( M = 0.30 \)
- \( 0.25 \)
- \( 0.20 \)
- \( 0.15 \)
- \( 0.10 \)
- \( 0.05 \)

\[ \varphi - \varphi_M \]

\[ \varphi_M \]

\[ \varphi_M \]

**Figure 27.** Latitude and \( \varphi - \varphi_M \) correction for different flow mach numbers \( t_i / t_a = 1.10 \); \( h / Y = 0.1 \).
28. Angle correction for different flow Mach numbers for $t_i/t_a = 1.10$ and $h/Y = 0.2$. 

\[
\Delta L = 20 \log \left( \frac{P_A}{P_M} \right)
\]
Figure 29. Amplitude and angle correction for different flow paths for $t_i/t_a=1.10$; $h/Y=0.3$. 

\[
\Delta L = 20 \log \left( \frac{p_A}{p_M} \right) 
\]

$M = 0.30$

0.25

0.20

0.15

0.10

0.05.
Figure 30. Amplitude and angle correction for different flow mach numbers for \( t_i/t_a = 1.10 \); \( h/Y = 0.4 \).
Figure 31: Amplitude and angle correction for different flow conditions. For $t_i/t_a=1.10$ and $h/Y=0.5$.
\( t_i/t_a = 1.10 \)
\( h/Y = 0.6 \)
\( M = 0.30 \)
\( 0.25 \)
\( 0.20 \)
\( 0.15 \)
\( 0.10 \)
\( 0.05 \)

**Figure 32**: Variation of \( \Delta L = 20 \log \left( \frac{P_{A}}{P_{M}} \right) \) and \( \Phi_M - \Phi' \) with \( \Phi_M \) for each

\( t_i/t_a = 1.10 \); \( h/Y = 0.6 \).
Figure 33. Right side of figure correction for different flow mach numbers for \( t_i/t_a = 1.10 \); \( h/Y = 0.7 \).
Figure 34. Amplitude and phase correction for different flow Mach numbers for \( t_i/t_a = 1.10 \); \( h/Y = 0.8 \).
Figure 35. Amplitude and angle diagrams for a typical case with $t_i/t_a = 1.15$; $h/Y \to 0$. 

$\Delta L = 20 \log \left( \frac{p_A}{p_M} \right)$

$\Phi_M = \Phi' \pm \Phi_M$

$M = 0.30, 0.25, 0.20, 0.15, 0.10, 0.05$
Figure 36. Amplitude and angle of vibration for different flow-ratio curves for $t_i/t_a = 1.15$, $h/\gamma = 0.1$. This graph illustrates the relationship between $\Delta L = 20 \log (p_A/p_M)$ and $\Phi_M - \Phi_M'$ for varying $M$ values.
Figure 37. Amplitude and angle correction for different flow Mach numbers for $\frac{ti}{ta}=1.15$; $h/Y=0.2$. 

$\Delta L = 20 \log \left( \frac{P_A}{P_M} \right)$
Figure 38. Amplitude and angle correction for different flow ratios for 
$\frac{t_i}{t_a} = 1.15$ and $h/Y = 0.3$.
Figure 39. Theoretical prediction of $20 \log \left( \frac{p_A}{p_M} \right)$ and $\Phi_M - \Phi'_M$ for $t_i/t_a = 1.15$; $h/Y = 0.4$.
Figure 40  Amplitude and angle correction for different flow ratios for $t_i/t_a = 1.15$ and $h/Y = 0.5$. 

\[ \Delta L = 20 \log \left( \frac{p_A}{p_M} \right) \]

\[ \phi = M \phi_M \]

1. $t_i/t_a = 1.15$
2. $h/Y = 0.5$
3. $M = 0.30$
4. 0.25
5. 0.20
6. 0.15
7. 0.10
8. 0.05
Figure 41: Amplitude and angle correction for different flow conditions for $t_i/t_a = 1.15$; $h/Y = 0.6$. 

\[ \Delta L = 20 \log \left( \frac{p_A}{p_M} \right) \]
$t_i/t_a = 1.15$

$h/Y = 0.7$

$M = 0.30$
$0.25$
$0.20$
$0.15$
$0.10$
$0.05$

$\Delta L = 20 \log_{10} \left( \frac{P_A}{P_M} \right)$

$\Phi_M - \Phi'$

$\phi_M$

$t_i/t_a = 1.15 ; h/Y = 0.7$
Figure 43. Amplitude and angle correction for different Mach numbers for \( \frac{t_i}{t_a} = 1.15 \); \( h/Y = 0.8 \).