Algorithm for Astronomical, Point Source, Signal to Noise Ratio Calculations

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ALGORITHM FOR ASTRONOMICAL POINT SOURCE, SIGNAL TO NOISE RATIO CALCULATIONS

I. GENERAL DESCRIPTION

The starting point for this program is three other programs [1] (OTF, PSF and Camera) developed by Dr. D. J. Schroeder. References 2 through 4 provide the basis for the development work. The programs OTF and PSF compute the monochromatic point spread function for a star of a given visual magnitude as seen through a telescope with a circular obscuration ratio of 0.33, entrance pupil diameter D and system focal ratio F outside the Earth's atmosphere. In the absence of any degradations, the point spread function (PSF) is computed from the modulation transfer function (MTF) of a perfect lens with a central obscuration. Image degradations can be introduced by multiplying the perfect lens MTF by the MTF's associated with the different types of degradations. The above programs include a MTF for image jitter, and MTFs for high, mid and low frequency wave aberrations associated with the optical telescope assembly. The PSF is assumed to be radially symmetric to considerably simplify to the computations. No ray aberrations due to optical path differences, especially those associated with the off axis imaging of the telescope, are considered. Any off axis defects are assumed to be compensated for by the following instrument. The change in the PSF for a selected wavelength can be examined for different combinations and magnitudes of the degradations included in the program.

Program Camera, in conjunction with OTF and PSF, adds a detector pel array, to the focal plane, a uniform cosmic background and the option to add a second star in the vicinity of the target star. For a given wavelength and for each pel in the detector array (maximum array size is 20 x 8) two relative signals are computed. The first is a target star relative signal. The second is the target star plus cosmic background signal. If a second star has been included, three relative signals are computed: the target star relative signal, the second star relative signal, and the target star plus second star plus cosmic background relative signal.

The above program has been modified to also compute the encircled energy for the monochromatic PSFs, the polychromatic PSF, and signal to noise ratios ($S/N$) for a given wavelength interval as a function of observation time and observation times as a function of signal to noise ratios using the combined Space Telescope and Wide Field/Planetary Camera quantum efficiencies [5], filter functions [6], and a detector characterized by an rms readout noise, a mean dark current, and an optional dead space between the detector pels.

II. USER INSTRUCTIONS

The user is first asked to input the visual magnitude of the signal star and then to choose whether or not the spectral distribution of the star is uniform or blackbody. The cosmic background and background star (if requested) will also have the same type of distribution. If a blackbody distribution is chosen, the user is asked to input the effective temperature (degrees Kelvin) of the signal star.
The effective temperatures of the cosmic background and background star (if requested) will be asked for later.

Next the user is asked to input the number of wavelengths to consider and the lower and upper wavelengths (cm). For Space Telescope, the lower and upper wavelengths are 10^{-5} and 11 \times 10^{-5} cm. The maximum number of wavelengths that the program can handle is 21, which makes the wavelength increment 0.5 \times 10^{-5} cm. For this case, the filter functions (0 < \text{transmission} < 1) and quantum efficiencies (electrons/photon) must be specified and read in (all 21 of them) in increments of 0.5 \times 10^{-5} cm. The user is asked if there is a table of quantum efficiencies and filter functions. If the answer is yes to both questions then two files of data must have been previously prepared, which are now read into the program. One file contains the quantum efficiencies and the other contains the filter transmission. Because the program integrates over wavelength, the first and last non-zero filter transmissions should be divided by 2 in order to use the trapezoidal rule for numerical integration. If the answer to both questions is no, the user is assumed to desire a system with 100 percent quantum efficiency and 100 percent transmission.

To obtain a monochromatic star and background, the lower wavelength will be chosen as the desired wavelength, if the number of wavelength calculations is set to 1. The user is then asked to input the quantum efficiency and filter transmission for that wavelength.

Next the user is asked to input the diameter (cm) of the telescope entrance pupil, the focal ratio of the optical system and the central obscuration ratio (ratio of obscured to clear entrance pupil diameter). For Space Telescope the obscuration ratio is 0.33.

The user is then asked to input the image degradation factors of jitter and high, mid and low frequency wave aberrations. For Space Telescope [1] the rms image jitter is anticipated to be 0.007 arc sec or less and the high, mid and low frequency aberrations are reported to be 0.121 \times 10^{-5} cm, 0.1304 \times 10^{-5} cm, and 0.2361 \times 10^{-5} cm, respectively.

At this stage of the program the user may request a printout of the monochromatic point spread function and encircled energy. If this option is requested, the program outputs the Airy radius in arc seconds and cm using the shortest wavelength for which the product of the quantum efficiency and filter function is greater than 10^{-4}. The user is then asked to input the maximum radius (arc seconds) to be considered and the number of radius calculations desired. If the maximum radius to be considered is 0.2 and the number of radius calculations is 51, the point spread function and encircled energy values will be output in the range from 0 to 0.2 arc sec in increments of 0.004 arc sec. The wavelength is selected by entering a wavelength number from 1 to 21 for space telescope. For example, 9 is 5 \times 10^{-5} cm, 10 is 5.5 \times 10^{-5} cm, and 11 is 6 \times 10^{-5} cm. The program then outputs the normalized point spread function [PSF(0)=1], the encircled energy, EE, [EE(\omega)=1], the normalized point spread function scaled in units of energy cm^{-2} sec^{-1} as a function of radius along with the radius in units of cm and arc seconds. After the printout has been completed the user is asked if there is a desire to change any of the PSF...
parameters. A yes answer allows the user to re-input a maximum radius, change the number of radius calculations or select another wavelength. The procedure described in this paragraph is repeated until a no answer is received for changing the PSF parameters.

Next, the user is asked to enter the x,y widths of a pel (arc seconds) and the x,y pel center separation (arc seconds), the number of pels in the x,y directions and the x,y coordinates of the signal star chief ray in the detector plane. To visualize the effects of the inputs, see Figure 1, using the definitions listed below:

\[
\begin{align*}
XPW &= \text{x direction width of the CCD pel in arc seconds} \\
YPW &= \text{y direction width of the CCD pel in arc seconds} \\
XPD &= \text{x distance between pel centers in arc seconds, } XPD \geq XPW \\
YPD &= \text{y distance between pel centers in arc seconds, } YPD \geq YPW \\
NPX &= \text{number of pels in the x direction} \\
PSX &= \text{number of pels in the y direction} \\
PSX &= \text{x coordinate (arc seconds) of signal star chief ray, } 0 \leq \text{PSX} \leq \text{NPX.PD}/2 \\
PSY &= \text{y coordinate (arc seconds) of signal star chief ray, } 0 \leq \text{PSY} \leq \text{NPD}/2
\end{align*}
\]

Dead space between the pels is created whenever \(XPD > XPW\) and/or \(YPD > YPW\), i.e., the pel center separation is greater than the pel width in the same direction. The chief ray coordinates, which are now considered to be the optic axis, should be as close or closer to the origin than to the farthest corner of the pel that is farthest from origin. Otherwise, redundant situations are created that serve no purpose other than to complicate the programming. Smaller array sizes produce more accurate results in the \((S/N)\) calculations because the PSF is computed at 200 different radii that range from zero to a value in arc seconds that is equal to the distance from the signal star chief ray coordinates to the coordinates of the farthest corner of the farthest pel from the origin (i.e., the larger the array size, the larger radius increment). The normalized point spread function is integrated over a pel to determine the fraction of energy, which will be denoted as diffraction efficiency, that falls within the pel. This integration requires radial interpolation of the point spread function, which produces less error when the radial increments are made smaller.

From these inputs just discussed in the paragraph, the program computes and outputs the diffraction efficiency as a function of wavelength for each pel in the array.

Next, the user is asked to enter the visual magnitude of the cosmic background, and if a blackbody distribution was previously selected, an effective background temperature (degrees Kelvin) will be requested. The user is also asked whether or not to include a background star. If the answer is yes the user is asked for the visual magnitude of the background star, and if a blackbody distribution was previously selected, the effective temperature (degrees Kelvin) of the star will be requested. The x,y coordinates (arc seconds) of the background star chief ray are now requested. The diffraction efficiency, as discussed before, is computed for this star as a function of wavelength for each pel. In this case the point spread is also computed at 200 different radii. If the chief ray coordinates lie within the detector array, the radius ranges from a value of zero to a value equal to the distance from the chief ray coordinates to the farthest corner of the pel that is farthest from the
chief ray. If the chief ray coordinates lie outside the detector array, the radius ranges from a value that is equal to the minimum distance from the chief ray to the nearest pel to a value that is equal to the distance from the chief ray to the farthest corner of the farthest pel. From these just discussed inputs, the program computes and outputs the diffraction efficiency of the background star as a function of wavelength for each pel in the array.

To complete the detector characteristics, the user inputs the rms readout noise in electrons per pel and the detector temperature in degrees Kelvin. For the particular CCD array used with the Space Telescope and Wide Field/Planetary Camera, the readout noise has been suggested [5] to range from 13.9 to 17.8 electrons per pel rms. For the above CCD array, the maximum operating temperature [5] is expected to be around 178°K (-95°C), and the mean dark current in electrons per pel per second is computed from this output temperature. The user can input a mean dark current directly by first entering a temperature less than 4°K. A prompt will then appear requesting the desired dark current.

The next set of inputs are concerned with the output data that is desired. The user first specifies the observation start and end times in seconds and the number of time calculations. For example, to compute the (S/N) in hour intervals from 1 to 14 hours, the user would input a start time of 3600 seconds, and an end time of 50400 seconds and 14 is the number of time calculations. To reverse the situation and determine the observation times needed to achieve a desired range of (S/N), the user inputs a start and end (S/N) and the desired number of (S/N) calculations. The program will proceed to output the desired data.

After the output is complete, the user may repeat the calculations for a signal star with a different visual magnitude and/or a cosmic background with a different visual magnitude and/or a background star with a different visual magnitude, without having to re-input all the previous input data. An additional repeat calculation that can be accomplished with or without the magnitude changes, is to change the characteristics of the detector. This option, however, requires that the user re-input all data starting with the detector readout noise and ending with the number of (S/N) calculations. The program terminates when the user answers no to all the repeat calculation options.

II. EQUATIONS USED

The system of units used is the cgs system, except for the image jitter and detector focal plane coordinates and dimensions, which are in arc seconds. The total energy emitted in the eye responsive spectral region from a star of visual magnitude m and received per square meter per second outside the Earth's atmosphere is given by [7]

\[
I_m = 2.54 \times 10^{-6} \times 10^{-0.4m} \text{ lux}. \tag{1}
\]

The above equation is converted from photometric to radiometric units by dividing by
\[ 0.68 \int_{3.8 \times 10^{-5} \text{cm}}^{7.6 \times 10^{-5} \text{cm}} K(\lambda) \, d\lambda \text{ lux per (ergs cm}^{-2} \text{ sec}^{-1} \Delta \lambda^{-1}) \] ,

(2)

where \( \Delta \lambda \) is the wavelength interval (expressed in cm) and \( K(\lambda) \) is the photopic eye response [7] given in Table 1. Dividing equation (1) by equation (2) is tantamount to assuming a uniform spectral distribution for the star and gives

\[ I_m(\lambda) = \frac{3.7353 \times 10^{-6} \times 10^{-0.4m}}{7.6 \times 10^{-5} \text{cm}} \text{ ergs cm}^{-2} \text{ sec}^{-1} \Delta \lambda^{-1} \] .

(3)

If \( B(\lambda,T) \) is the blackbody distribution having an effective temperature \( T \) (degrees Kelvin), then equation (3) can be converted to a blackbody distribution in the following manner [8]:

\[ I_m(\lambda,T) = \frac{3.7353 \times 10^{-6} \times 10^{-0.4m} B(\lambda,T)}{7.6 \times 10^{-5} \text{cm}} \text{ ergs cm}^{-2} \text{ sec}^{-1} \Delta \lambda^{-1} \] .

(4)

Any type distribution can be used for \( B(\lambda,T) \) in equation (4), since multiplying equation (4) by \( 0.68K(\lambda) \) and integrating from \( 3.8 \times 10^{-5} \text{cm} \) to \( 7.6 \times 10^{-5} \text{cm} \) reproduces equation (1). If \( B(\lambda,T) \) were a uniform distribution, then equation (3) would be obtained again. Assume that a uniform distribution has been selected and that the following quantities have been specified:

- \( A = \) area of the telescope entrance pupil (cm\(^2\))
- \( hc/\lambda = \) ergs/photon
- \( Q(\lambda) = \) quantum efficiency in electrons per photon
- \( F(\lambda) = \) optical filter transmission
- \( \text{DE}_s(\lambda) = \) diffraction efficiency

The quantum efficiencies [5] used in the output example are shown in Table 2. This includes the combined effects of the optical telescope assembly and the Wide Field/Planetary Camera. The filter function [6] used in the output example is listed in Table 3. The diffraction efficiency is quite involved and will be discussed in
detail later. However, it is the integral of the normalized point spread function over the area of a pel. For a given wavelength, this integral gives the fraction of energy intercepted by that pel, since the integral over all space of the normalized point spread function is one.

The rate at which electrons are liberated from a particular pel per wavelength interval at a chosen wavelength is given by

\[ S_s(\lambda) = \frac{A}{\hbar c} \int_{\lambda_1}^{\lambda_2} \lambda I_m(\lambda)Q(\lambda)F(\lambda)DE_s(\lambda) d\lambda \text{ electrons sec}^{-1} \Delta \lambda^{-1} \]  

(5)

Equation (5) is called the monochromatic signal current. Integrating equation (5) over the wavelength region influenced by the quantum efficiency and optical filter produces the polychromatic signal current. If there is a dead space between the detector pels, the diffraction efficiency is adjusted to account for the dead space and the location at the chief ray of the diffraction pattern relative to the pel in question. The polychromatic signal current is given by

\[ S_s = \frac{A}{\hbar c} \int_{\lambda_1}^{\lambda_2} \lambda I_m(\lambda)Q(\lambda)F(\lambda)DE_s(\lambda) d\lambda \text{ electrons sec}^{-1} \]  

(6)

By the same analogy the signal current from a second star would be given by

\[ S_u = \frac{A}{\hbar c} \int_{\lambda_1}^{\lambda_2} \lambda I_m'(\lambda)Q(\lambda)F(\lambda)DE_u(\lambda) d\lambda \text{ electrons sec}^{-1} \]  

(7)

where \( m' \) could be a different visual magnitude and \( DE_u(\lambda) \) is the diffraction efficiency of the second star for the same pel. The signal current for the cosmic background is given by

\[ S_b = \frac{A(d\Omega)}{\hbar c} \int_{\lambda_1}^{\lambda_2} I_m''(\lambda)Q(\lambda)F(\lambda) d\lambda \text{ electrons sec}^{-1} \]  

(8)

where \( d\Omega \) is the area of a pel in arc seconds squared (\( \text{\textdegree}^2 \)). The cosmic background is treated as an extended source of visual magnitude \( m'' \) and \( I_m''(\lambda) \) has units of per \( \text{\textdegree}^2 \). The \( d\Omega \) for an extended source replaces \( DE(\lambda) \) for a point source.

To compute a signal to noise ratio, the contributions to the signal term and the noise term must be defined. For the polychromatic case, for example, the signal is equation (6) multiplied by the time in seconds. If the incoming photon flux is
assumed to be Poisson distributed, then the variance of the signal is also equal to
the mean signal. If the signal to noise ratio is defined as the ratio of the mean to
the standard deviation of the mean, then the signal star will contribute to the noise.
Other contributions to the noise include the cosmic background signal, the unwanted
star signal if requested, the rms readout noise in electrons per pel and the mean
dark current in electrons per pel per second. If all of the noise contributors are
assumed to be statistically independent, then the signal to noise ratio is given by

\[
\frac{S_s}{\sqrt{N}} = \frac{S_s \cdot t}{(S_s \cdot t + S_u \cdot t + S_b \cdot t + R^2 + D \cdot t)^{1/2}}, \tag{9}
\]

where \(R\) is the rms readout noise, \(D\) is the mean dark current and \(t\) is the time in
seconds. If the second star option was not requested, \(S_u\) would be set to zero.
Equation (9) is computed for every pel and the only quantity that changes in the
diffraction efficiency in the expressions for \(S_s\) and \(S_u\). As previously mentioned,
the user has the option of inputing the dark current directly or inputing the detector
temperature and having the dark current computed for the CCD array on Space
Telescope for the Wide Field/Planetary Camera. The equations (9) for the dark
current are given by

\[
D = 29.3 \times 10^9 \times T^{3/2} \exp(-5802.1Z) \tag{10},
\]

where

\[
Z = \frac{1.1557}{T} - 7.021 \times 10^{-4} \frac{T}{1108 + T} \tag{11},
\]

and where \(T\) is the detector temperature in degrees Kelvin.

Equation (9) can also be solved for the time required to achieve a particular
\((S/N)\). Solving for the time gives

\[
t = \frac{(S/N)^2}{2S_s^2} \left( S_s + S_u + S_b + D \right) + \left( \frac{(S/N)^4}{4S_s^4} \left( S_s^2 + S_u^2 + S_b^2 + D^2 \right) + (S/N)^2 \left( R/S_s \right)^2 \right)^{1/2} \tag{12} \text{ sec}.
\]

How the diffraction efficiency is calculated will now be discussed. As previously
mentioned, the diffraction efficiency is the integral of the normalized point spread
function over the detector pel area. The analytic expression for the normalized
rotationally symmetric point spread function is given by [2]
\[
\text{PSF}(r, \lambda) = \frac{8}{\tau_p} \int_0^1 \text{MTF}(\nu_n, \lambda) J_0(2\pi r [D_p / \lambda, 648000] \nu_n) \nu_n d\nu_n ,
\]  

(13)

where \( r \) is the radius of the PSF in arc seconds, \( D_p \) is the diameter of the telescope entrance pupil (cm), \( \lambda \) is the wavelength (cm), \( \nu_n \) is the normalized spatial frequency, \( J_0 \) is a zero order Bessel function of the first kind, \( \tau_p \) is the pupil transmittance, 648000 is the number of arc seconds in \( \pi \) radians, and MTF is the modulation transfer function. For a central obstruction, the pupil transmittance is given by \( 1-\varepsilon^2 \), where \( \varepsilon \) is the ratio of the obstruction diameter to the clear diameter. Equation (13) is normalized such that \( \text{PSF}(0, \lambda) = 1 \), however it can be scaled [2] in units of energy \( \text{cm}^{-2} \text{sec}^{-1} \) by multiplying by \( \tau_p \pi^2 D_p^2 / 16\lambda^2 F^2 \), where \( F \) is the system focal ratio.

The MTF is composed of the product of the individual MTF's representing the perfect lens, the image jitter, and the high, mid and low frequency aberrations of the optical telescope assembly. The MTF equation for the perfect lens is given by [2]

\[
\text{TPL}(\nu_n) = (A+B+C)/(1-\varepsilon^2) ,
\]  

(14)

where

\[
A = \frac{2}{\pi} \left[ \arccos(\nu_n) - \nu_n (1-\nu_n^2)^{1/2} \right] ; \quad 0 \leq \nu_n \leq 1.0
\]

(15)

\[
A = 0 ; \quad \nu_n > 1.0
\]

\[
B = \frac{2\varepsilon^2}{\pi} \left\{ \arccos(\nu_n / \varepsilon) - (\nu_n / \varepsilon)[1-(\nu_n / \varepsilon)^2]^{1/2} \right\} ; \quad 0 \leq \nu_n / \varepsilon \leq 1.0
\]

(16)

\[
B = 0 ; \quad \nu_n / \varepsilon > 1.0
\]

\[
C = -2\varepsilon^2 \quad ; \quad 0 \leq \nu_n \leq (1-\varepsilon)/2
\]

\[
C = \frac{2}{\pi} \left\{ \varepsilon \sin \phi + \frac{\phi}{2} \left(1+\varepsilon^2 - (1-\varepsilon^2) \arctan \left(\frac{1+\varepsilon}{1-\varepsilon} \tan \frac{\phi}{2}\right)\right) \right\} - 2\varepsilon^2 ; \quad (1-\varepsilon/2) \leq \nu_n \leq (1+\varepsilon/2)
\]

(17)

\[
C = 0 ; \quad \nu_n \geq (1+\varepsilon/2) ,
\]

and

\[
\phi = \arccos \left[ \frac{1-\varepsilon^2-4\nu_n^2}{2\varepsilon} \right] .
\]
The MTF for image jitter is given by [2]

\[ TJ(\nu_n, \lambda) = \exp \left[ \frac{-2\pi^2 \cdot \sigma_j^2 \cdot D_p^2 \cdot \nu_n^2}{\lambda^2 \cdot 648000^2} \right] \]  

(18)

where \( \sigma_j \) is the rms jitter in arc seconds, \( D_p \) is the telescope entrance pupil diameter (cm), 648000 is the number of arc seconds in pi radians and \( \lambda \) is the wavelength (cm). The MTF for the high frequency wave aberrations is given by [4]

\[ TH(\lambda) = \exp \left[ \frac{-4\pi^2 \cdot \sigma_h^2}{\lambda^2} \right] \]  

(19)

where \( \sigma_h \) is the rms high frequency aberration (cm). The MTF for the mid frequency aberrations are given by [1]

\[ TM(\nu_n, \lambda) = \exp \left[ \left( \frac{-4\pi^2 \cdot \sigma_m^2}{\lambda^2} \right) \left( 1 - [1 - 18 \cdot \nu_n]^{3/2} \right) \right] ; \nu_n < 1/18 \]  

(20)

\[ TM(\nu_n, \lambda) = \exp \left( \frac{-4\pi^2 \cdot \sigma_m^2}{\lambda^2} \right) ; \nu_n \geq 1/18 \]

where \( \sigma_m \) is the rms mid frequency aberration (cm). The MTF for the low frequency aberrations is given by [1]

\[ TL(\nu_n, \lambda) = 1 - \frac{2.815}{\lambda} \sin \left( \frac{5\pi \nu_n}{3} \right) \]  

(21)

\[ TL(\nu_n, \lambda) = 1 - \frac{\sigma_L}{0.0373 \lambda} (-0.9 + \sec[3/4(\nu_n-0.4)]) \]  

(22)

where \( \sigma_L^2 \) is the rms low frequency aberration (cm). If \( TL(\nu_n, \lambda) \) becomes negative, it is set to zero. Thus, the MTF(\( \nu_n, \lambda \)) in equation (13) is given by

\[ \text{MTF}(\nu_n, \lambda) = TPL(\nu_n) \cdot TJ(\nu_n, \lambda) \cdot TH(\lambda) \cdot TM(\nu_n, \lambda) \cdot TL(\nu_n, \lambda) \]  

(22)

For a given wavelength, the encircled energy, \( EE(r_o, \lambda) \), is given by [2]
Since PSF is radially symmetric, equation (23) becomes

\[
EE(r_o, \lambda) = \frac{\pi r_o}{2} \left[ \frac{D_p \cdot \pi}{\lambda \cdot 648000} \right]^2 \int_0^{r_o} \int_0^{2\pi} PSF(r, \lambda, \theta) r \, dr \, d\theta .
\] (24)

The encircled energy is normalized such that \( EE(\infty, \lambda) = 1 \). Substituting (13) into (24) gives

\[
EE(r_o, \lambda) = 4\pi^2 \left[ \frac{D_p \cdot \pi}{\lambda \cdot 648000} \right]^2 \int_0^{1} MTF(\nu_n, \lambda) \left[ \int_0^{r_o} J_0 \left( 2\pi \left[ \frac{D_p \cdot \pi}{\lambda \cdot 648000} \right] \nu_n \right) r \, dr \right] \nu_n \, d\nu_n
\] (25)

or

\[
EE(r_o, \lambda) = 2\pi r_o \left[ \frac{D_p \cdot \pi}{\lambda \cdot 648000} \right] \int_0^{1} MTF(\nu_n, \lambda) J_1 \left( 2\pi \left[ \frac{D_p \cdot \pi}{\lambda \cdot 648000} \right] \nu_n \right) d\nu_n
\]

where \( J_1 \) is a first order Bessel function of the first kind, \( D_p \) is the diameter (cm) of the telescope entrance pupil, \( r_o \) the radius of the encircled energy in arc seconds, \( \lambda \) is the wavelength and 648000 is the number of arc seconds in pi radians. To achieve reasonable accuracy \( \nu_n \) needs to be calculated for at least 400 points in the range \( 0 \leq \nu_n \leq 1 \). Thus, \( \nu_n \) is incremented in increments of 1/400. It is also recommended [1] that the following numerical integration scheme [10] be used:

\[
\int_a^b f(x) \, dx = \frac{h}{4.5} \left[ 1.4y_0 + 6.4y_1 + 2.4y_2 + 6.4y_3 + 2.8y_4 \\
+ 6.4y_5 + 2.4y_6 + 6.4y_7 + 2.8y_8 \\
\cdots \\
+ 6.4y_{4n-3} + 2.4y_{4n-2} + 6.4y_{4n-1} + 1.4y_{4n} \right]
\] (26)

where \( 4nh=b-a \).
The computation of the diffraction efficiency is very similar to computing encircled energy except that the integration is performed over a square or rectangular area instead of a circular area. Thus, for a particular pel, equation (23) becomes

$$DE(\lambda) = \frac{\pi r_p}{4} \left[ \frac{D_p \cdot \pi}{\lambda \cdot 648000} \right]^2 \int_{y_1}^{y_2} \int_{x_1}^{x_2} PSF(x,y,\lambda) \, dx \, dy \quad , \quad (27)$$

where \( x, y \) are expressed in arc seconds. Each pel is broken into a 20 by 20 sub-pixel array. The distance from each subpixel to the chief ray of the diffraction image is computed. Since the PSF is radially symmetric, the value of PSF(\( x, y, \lambda \)) is found by interpolating the value of PS (\( r, \lambda \)) to a value of \( r = (x^2+y^2)^{1/2} \). This procedure is used for the signal star as well as the second star.
REFERENCES


### TABLE 1. PHOTOPIC EYE RESPONSE

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<tr>
<th>$\lambda$ (10$^{-5}$ cm)</th>
<th>$K(\lambda)$</th>
<th>$\lambda$ (10$^{-5}$ cm)</th>
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### TABLE 2. QUANTUM EFFICIENCIES

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<th>$\lambda$ (10$^{-5}$ cm)</th>
<th>$Q(\lambda)$</th>
<th>$\lambda$ (10$^{-5}$ cm)</th>
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*Interpolated Values

### TABLE 3. FILTER FUNCTION

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*The values were divided by 2 for using trapezoidal rule in numerical integration over wavelength.
FIGURE 1. RELATIONSHIP OF POINT SOURCE DIFFRACTION IMAGE TO CCD CONFIGURATION
APPENDIX A

OUTPUT EXAMPLE

ENTER SIGNAL STAR(#1) VISUAL MAGNITUDE
?77.

ENTER CHOICE OF SPECTRAL DISTRIBUTION FOR SOURCE
FLAT DISTRIBUTION=0; BLACKBODY DISTRIBUTION=1
?0.

ENTER NUMBER OF WAVELENGTHS
?21.

ENTER LOWER AND UPPER WAVELENGTHS (CM)
71.E-5, 11.E-5

IS THERE A TABLE OF QUANTUM EFFICIENCIES? (YES=1, NO=0)
?1.

IS THERE A TABLE OF FILTER TRANSMISSIONS? (YES=1, NO=0)
?1.

ENTER TELESCOPE ENTRANCE PUPIL DIAMETER (CM)
?240.

ENTER SYSTEM FOCAL RATIO
?12.9

ENTER OBSCURATION RATIO
?0.33

ENTER RMS IMAGE JITTER (ARC SECONDS)
?0.007

ENTER HIGH FREQUENCY RMS ABERATIONS (CM)
?0.121E-5

ENTER MID FREQUENCY RMS ABERATIONS (CM)
?0.1304E-5

ENTER LOW FREQUENCY RMS ABERATIONS (CM)
?0.2361E-5

DO YOU WISH TO COMPUTE NORMALIZED POINT SPREAD
FUNCTION, PSF(0)-1, AND THE NORMALIZED ENCIRCLED ENERGY,
EE(INF)-1? (YES=1, NO=0)
?1.

MINIMUM AIRY RADIUS= .78690000E-03 CM= .52425539E-01 ARC SECONDS

ENTER MAXIMUM RADIUS TO CONSIDER (ARC SECONDS)
?0.5

ENTER NUMBER OF RADIUS CALCULATIONS
?51.

ENTER DESIRED WAVELENGTH NUMBER (1, 2, ..., 21)

CLEAR SCREEN AND RETURN TO CONTINUE
?
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<th>Pnt Spread Fnl Encircled Energy (Arc Seconds)</th>
<th>Energy/Sec/CM²</th>
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**Do you wish to change any PSF parameters? (Yes=1; No=0)**

Yes

**Clear screen and return to continue**
ENTER X,Y PEL WIDTHS (ARC SECONDS)
?0.1, 0.1

ENTER X,Y PEL CENTER SEPARATIONS (ARC SECONDS)
?0.1, 0.1

ENTER NUMBER OF PELS IN X,Y DIRECTIONS
?2, 2.

ENTER X,Y COORDINATES OF SIGNAL STAR (ARC SECONDS)
?70.05, 0.05

WAV= .50000000E-04 IX=1 IV=1 DFE= .57745045E-08
WAV= .50000000E-04 IX=1 IV=2 DFE= .47940138E-01
WAV= .50000000E-04 IX=2 IV=1 DFE= .10263014E-01
WAV= .50000000E-04 IX=1 IV=1 DFE= .57273858E-00
WAV= .55000000E-04 IX=1 IV=2 DFE= .47984166E-01
WAV= .55000000E-04 IX=2 IV=1 DFE= .78841666E-01
WAV= .55000000E-04 IX=2 IV=2 DFE= .1232071E-01
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ENTER COSMIC BACKGROUND VISUAL MAGNITUDE
?23.

DO YOU WISH TO INCLUDE A BACKGROUND STAR? (YES=1, NO=0)
?0.

ENTER RMS READOUT NOISE (ELECTRONS/PEL)
?18.

ENTER DETECTOR TEMPERATURE (KELVIN)
?178.

ENTER OBSERVATION START AND END TIMES (SECONDS)
?3500, 50400.

ENTER NUMBER (>1) OF TIME CALCULATIONS
?14.

ENTER START AND END SIGNAL TO NOISE RATIOS (S/N)
?1, 10.

ENTER NUMBER (>1) OF (S/N) CALCULATIONS
?10.

CLEAR SCREEN AND RETURN TO CONTINUE ?
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DO YOU WISH TO CHANGE SOURCE MAGNITUDE? (YES=1, NO=0)

DO YOU WISH TO CHANGE COSMIC BACKGROUND MAGNITUDE? (YES=1, NO=0)

DO YOU WISH TO CHANGE BACKGROUND STAR MAGNITUDE? (YES=1, NO=0)

DO YOU WISH TO CHANGE DETECTOR EFFECTS? (YES=1, NO=0)

DID YOU MAKE ANY CHANGES? (YES=1, NO=0)

STOP 0
APPENDIX B

PROGRAM LISTING

07:46 AUG 10, 84 DC/PNTSRC.JAYROE

IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DIMENSION TPL(400), TJ(400), TH(21)
DIMENSION TM(400,21), TL(400,21), FX1(400), DFE(10,10)
DIMENSION SGNL(200), PELS(10,10), PELB(10,10), DIFE(10,10)
COMMON /BESF/FX0(400), PR(200,21), TTOT(400,21), QUEF(21), FF(21)
COMMON /PHOF/XK(40)
WRITE(102, 6000)
6000 FORMAT("ENTER SIGNAL STAR(#1) VISUAL MAGNITUDE")
READ(101, 4000) XS1
4000 FORMAT(20D6)
   XIS1=0.37353*10.0**(-0.4*XSI1=5.)
   HC=1.9865E-16
   HCOK=1.4388
   DLA=1.E-6
   KRESP=0
WRITE(102, 6001)
6001 FORMAT(/,"ENTER CHOICE OF SPECTRAL DISTRIBUTION FOR SOURCE")
WRITE(102, 6002)
6002 FORMAT("FLAT DISTRIBUTION=0; BLACKBODY DISTRIBUTION=1")
READ(101, 4000) KRESP
IF(KRESP.EQ.1) GO TO 90
XISL1=(XSI1*10.0**5)/(1.0682*HC)
GO TO 120
90 WRITE(102, 6004)
6004 FORMAT(/,"ENTER SIGNAL STAR EFFECTIVE TEMPERATURE(KELVIN)")
READ(101, 4000) ST1
DO 110 IK=1,39
READ(20, 5000) XK(IK)
5000 FORMAT(E15.8)
110 CONTINUE
CALL PHO1(ST1, HCOK, DLA, HC, XINT)
   XISL1=XSI1*XINT
WRITE(102, 6008)
6008 FORMAT(/,"ENTER NUMBER OF WAVELENGTHS")
READ(101, 4000) NNAV
DL=0.
   DO 125 I=1,21
   QUEF(I)=1.
120 CONTINUE
   FF(I)=0.5
   FF(21)=0.5
IF(NNAV.EQ.1) GO TO 130
WRITE(102, 6012)
6012 FORMAT(/,"ENTER LOWER AND UPPER WAVELENGTHS(CM)")
READ(101, 4000) WAVL, WAVH
   X=NNAV=1
   DL=(WAVH-WAVL)/X
   GO TO 140
130 WRITE(102, 6016)
6016 FORMAT(//,"ENTER WAVELENGTH(CM)"")
READ(101,4000)WAVL
WRITE(102,6020)

6020 FORMAT(//,"ENTER QUANTUM EFFICIENCY AND FILTER TRANSMISSION")
READ(101,4000)QUEF(1),FF(1)
GO TO 180
WRITE(102,6024)

6024 FORMAT(//,"IS THERE A TABLE OF QUANTUM EFFICIENCIES?(YES=1;NO=0)"")
READ(101,4000)IRESP
IF(IRESP.NE.1)GO TO 160
DO 150 I=1,NWAV
150 READ(22,5000)QUEF(I)
WRITE(102,6028)

6028 FORMAT(//,"IS THERE A TABLE OF FILTER TRANSMISSIONS?(YES=1;NO=0)"")
READ(101,4000)IRESP
IF(IRESP.NE.1)GO TO 180
DO 170 I=1,NWAV
170 READ(24,5000)FF(I)
WRITE(102,6032)

6032 FORMAT(//,"ENTER TELESCOPE ENTRANCE PUPIL DIAMETER(CM)"")
READ(101,4000)DP
WRITE(102,6036)

6036 FORMAT(//,"ENTER SYSTEM FOCAL RATIO")
READ(101,4000)FR
CUTF=1./(FR*WAVL)
WRITE(102,6040)

6040 FORMAT(//,"ENTER OBSCURATION RATIO")
READ(101,4000)EN
EN2=EN*EN
ENPLS=1.+EN2
ENMNS=1.-EN2
ENRAT=(1.+EN)/(1.-EN)
PI=3.1415926
TOPI=2./PI
DM=0.0025063
XARG=-DM
DO 210 I=1,400
DAO
A=0.
B=0.
C=0.
XARG=XARG+DM
IF(XARG.GT.1.)GO TO 210
A=TOPI*(ACOS(XARG)-XARG*SQRT(1.-XARG*XARG))
IF(EN.LT.0.001)GO TO 210
YARG=XARG/EN
IF(YARG.GT.1.)GO TO 195
B=TOPI*EN2*(ACOS(YARG)-YARG*SQRT(1.-YARG*YARG))
195 ZARG=XARG*2./(1.-EN)
IF(ZARG.GT.1.)GO TO 200
C=-2.*EN2
GO TO 210
200 WARG=XARG*2./(1.+EN)
IF(WARG,GT,1.)GO TO 210
PHI=ACOS((1.+EN2-4.*XARG*XARG)/(2.*EN))
C=EN*SIN(PHI)+PHI*ENPLS/2.-ENMNS*ATAN(ENRAT*TAN(PHI/2.))
C=C*TOPI-2.*EN2
210 TPL(I)=(A+B+C)/ENMNS
DN=0.0025063*WAVL
WRITE(102,6044)
6044 FORMAT(/"ENTER RMS IMAGE JITTER(ARC SECONDS)"")
READ(101,4000)SIGJ
WRITE(102,6048)
6048 FORMAT(/"ENTER HIGH FREQUENCY RMS ABERATIONS(CM)"")
READ(101,4000)SIGH
WRITE(102,6052)
6052 FORMAT(/"ENTER MID FREQUENCY RMS ABERATIONS(CM)"")
READ(101,4000)SIGM
WRITE(102,6056)
6056 FORMAT(/"ENTER LOW FREQUENCY RMS ABERATIONS(CM)"")
READ(101,4000)SIGL
WAV=WAVL=DL
FACJ=PI*SIGJ*DP*PI/648000.
FACJ=2.*FACJ*FACJ
FACH=-4.*PI*SIGH*SIGH
FACM=-4.*PI*SIGM*SIGM
DNU=DM
DO 250 NW=1,NWAV
PROD=QUEF(NW)*FF(NW)
WAV=WAV+DL
XNU=DNU
TH(NW)=EXP(FACH/WAV**2)
DO 250 I=1,400
XNU=XNU+DNU
TJ(I)=EXP(FACJ*XNU*XNU/WAV**2)
TTOT(I,NW)=0.
IF(PROD.LT.1.E-4)GO TO 250
XARG=XNU
TM(I,NW)=EXP(FACM/WAV**2)
YARG=1.-18.*XARG
IF(YARG.LT.0.)GO TO 220
ZARG=-FACM*YARG**1.5/WAV**2
TM(I,NW)=TM(I,NW)*EXP(ZARG)
220 IF(XARG,GT,0.355)GO TO 230
YARG=PI*XARG/0.6
TL(I,NW)=1.-2.815*SIGL*SIN(YARG)/WAV
GO TO 240
230 YARG=0.75*(XARG=0.4)
TL(I,NW)=1.-SIGL*(-0.9+1./COS(YARG))/(0.0373*WAV)
240 IF(TL(I,NW),LT,0.)TL(I,NW)=0.
TTOT(I,NW)=TPL(I)*TJ(I)*TM(I,NW)*TL(I,NW)*TH(NW)
250 CONTINUE
WRITE(102,6060)
6060 FORMAT(/"DO YOU WISH TO COMPUTE NORMALIZED POINT SPREAD"")
WRITE(102,6064)
FORMAT("FUNCTION,PSF(0)=1,AND THE NORMALIZED ENCIRCLED ENERGY,")
WRITE(102,6065)
FORMAT("EE(INF)=1?(YES=1;NO=0)")
READ(101,4000)IRES
IF(IRES.NE.1)GO TO 350
WAV=WAVL+DL
DO 255 I=1,NWAV
WAV=WAV+DL
PROD=QUEF(I)*FF(I)
IF(PROD.LT.1,E=4)GO TO 255
WAV=WAV
GO TO 256
255 CONTINUE
256 RAD=1.22*WAVM*FR
ANG=1.22*WAVM*648000./(DP*PI)
WRITE(102,6066)RAD,ANG
FORMAT(/,"MINIMUM AIRY RADIUS=",E15.8," CM=",E15.8,
& " ARC SECONDS")
WRITE(102,6068)
FORMAT(/,"ENTER MAXIMUM RADIUS TO CONSIDER(ARC SECONDS)"
READ(101,4000)RMAX
WRITE(102,6072)
FORMAT(/,"ENTER NUMBER OF RADIUS CALCULATIONS")
READ(101,4000)NR
WRITE(102,6076)
FORMAT(/,"ENTER DESIRED WAVELENGTH NUMBER(1,2,...,21)"
READ(101,4000)JW
WRITE(102,6077)
FORMAT(/,"CLEAR SCREEN AND RETURN TO CONTINUE")
READ(101,4000)DUMMY
X=NR=1
RCM=RMAX*DP*FR*PI/648000.
DR=RCM/X
DRY=RMAX/X
DWAV=JW=1
WAVX=WAVL+DWAV*DL
SCAL=ENMNS*(PI*DP/(4.*FR*WAVX))**2
DRZ=RMAX*DP*PI/(X*WAVX*648000.)
ZR=DRZ
YR=DRY
XR=DR
WRITE(102,7000)RMAX,RCM
7000 FORMAT("MAXIMUM RADIUS=",E15.8," ARC SECONDS=",E15.8," CM")
WRITE(102,7004)NR,WAV
7004 FORMAT("NUMBER OF RADIUS CALCULATIONS",I4," WAVELENGTH=",E15.8,
& " CM")
WRITE(102,7008)
7008 FORMAT(/," RADIUS(CM), PNT,SPREAD FN, ENCIRCLED ENERGY",
& (ARC SECONDS) ENERGY/SEC/CM**2")
DO 340 IR=1,NR
XR=XR+DR
YR=YR+HR
340
ZR=ZR+DRZ
XNU=-DNU
DO 310 I=1,400
XNU=XNU+DNU
X=2.*PI*ZR*XNU
IF(X.GT.0.)GO TO 285
BES0=1.
BES1=0.
GO TO 300
285 IF(X.GT.4.)GO TO 290
X2=X/2.
X22=X2*X2
X24=X22*X22
X26=X24*X22
X28=X24*X24
BES0=1.*X22+X24/4.*X26/36.*X28/576.*X28*X22/14400.
BES0=BES0+X28*X24/518400.*X28*X26/25401600.
X3=X22*X2
X5=X24*X2
X7=X26*X2
X9=X28*X2
BES1=X2-X3/2.*X5/12.*X7/144.*X9/2880.*X9*X22/86400.
BES1=BES1+X9*X22/3628800.*X7*X28/203212800.
GO TO 300
290 X2=X*X
X3=X2*X
X4=X3*X
X5=X4*X
X6=X5*X
P0=1.*0.0703125/X2+0.112152/X4=0.5725/X6
Q0=-0.125/X+0.0732422/X3=0.227108/X5
BES0=0.7978846/SQRT(X)
AQ=X=0.7853982
BES0=BES0*(P0*COS(AQ)-Q0*SIN(AQ))
P1=1.*0.1171875/X2-0.1441956/X4+0.6765926/X6
Q1=0.375/X=0.1025391/X3+0.2775764/X5
BES1=0.7978846/SQRT(X)
BQ=X=2.35619449
BES1=BES1*(P1*COS(BQ)-Q1*SIN(BQ))
300 FX0(I)=TTOT(I,JW)*BES0*XNU
FX1(I)=TTOT(I,JW)*BES1
310 CONTINUE
A2=0.
A3=0.
A4=0.
B2=0.
B3=0.
B4=0.
DO 320 J=2,399,2
A2=A2+FX0(J)
B2=B2+FX1(J)
320 CONTINUE
**DO 330 J=3,398,4**
**A3=A3+FX0(J)**
**A4=A4+FX0(J-2)+FX0(J+2)**
**B3=B3+FX1(J)**
**B4=B4+FX1(J-2)+FX1(J+2)**

**330 CONTINUE**
**SAS=DNU*(1.4*A4+2.4*A3+6.4*A2)/4.5**
**SBS=DNU*(1.4*B4+2.4*B3+6.4*B2)/4.5**
**PSF=8.*SAS/ENMNS**
**SPSF=SCAL*PSF**
**EE=2.*PI*ZR*SBS**
**WRITE(102,7012)XR,PSF,EE,YR,SPSF**

**7012 FORMAT(5(E15.8,1X))**

**340 CONTINUE**
**WRITE(102,6080)**

**6080 FORMAT(/,"DO YOU WISH TO CHANGE ANY PSF PARAMETERS?(YES=1;NO=0)")**
**READ(101,4000)IRESP**
**IF(IRESP.EQ.1)GO TO 260**

**350 WRITE(102,6077)**
**READ(101,4000)DUMMY**
**WRITE(102,6084)**

**6084 FORMAT(/,"ENTER X,Y PEL WIDTHS(ARC SECONDS)")**
**READ(101,4000)XPW,YPW**
**WRITE(102,6088)**

**6088 FORMAT(/,"ENTER X,Y PEL CENTER SEPARATIONS(ARC SECONDS)")**
**READ(101,4000)XPD,YPD**
**WRITE(102,6092)**

**6092 FORMAT(/,"ENTER NUMBER OF PELS IN X,Y DIRECTIONS")**
**READ(101,4000)NX,NY**
**DO 360 IX=1,NX**
**DO 360 IY=1,NY**

**360 PELB(IX,IY)=0.**
**WRITE(102,6096)**

**6096 FORMAT(/,"ENTER X,Y COORDINATES OF SIGNAL STAR(ARC SECONDS)")**
**READ(101,4000)PSX,PSY**
**XN=NX**
**YN=NY**
**XMAX=XN*XPD**
**YMAX=YN*YPD**
**RMAX=SQRT((XMAX-PSX)**2+(YMAX-PSY)**2)**
**DRZ=RMAX*DP/PI/(199.*648000.)**
**DRI=RMAX/199.**
**ZR=DRZ**

**C ***** COMPUTE POINT SPREAD FUNCTION **********************
CALL BES(NWAV,DRZ,ZR,DNU,ENMNS,WAVL,DL)**

**C ***** INTEGRATE PSF OVER WAVELENGTH **********************
IF(NWAV.GT.1)GO TO 440**
**PROD=QUEF(1)*FF(1)*XISL1*PI*DP*DP/4.**
**DO 430 IR=1,200**
**IF(KRESP.EQ.1)GO TO 425**
**SIGNL(IR)=PROD*PR(IR,1)/WAVL**
**GO TO 430**
425 \[ Y = \text{HCOK}/(\text{WAVL}*\text{ST1}) \]
\[ \text{SIGNL}(\text{IR}) = \text{PROD} \times \text{PR}(\text{IR}, 1)/((\exp(Y)-1.)*\text{WAVL}^{*6}) \]
430 CONTINUE
GO TO 480
440 MWAV = NWAV = 1
FAC = XI = 1*PI*DP*DP*DL/4.
DO 470 IR = 1, 1200
WAV = WAVL
PROD = QUEF(1)*FF(1)*PR(\text{IR}, 1)
IF (KRESP EQ. 1) GO TO 441
SUM = PROD/WAV
GO TO 442
441 \[ Y = \text{HCOK}/(\text{WAVL}*\text{ST1}) \]
\[ \text{SUM} = \text{PROD}/((\exp(Y)-1.)*\text{WAVL}^{*6}) \]
442 IF (NWAV EQ. 2) GO TO 460
DO 450 J = 2, MWAV
WAV = WAVL
PROD = QUEF(JW)*FF(JW)
IF (PROD LT. 1, E=4) GO TO 450
IF (KRESP EQ. 1) GO TO 446
SUM = SUM+PROD*PR(\text{IR}, JW)/WAV
GO TO 450
446 \[ Y = \text{HCOK}/(\text{WAVL}*\text{ST1}) \]
\[ \text{SUM} = \text{SUM}+\text{PROD} \times \text{PR}(\text{IR}, JW)/((\exp(Y)-1.)*\text{WAVL}^{*6}) \]
450 CONTINUE
460 WAV = WAVL
PROD = QUEF(NWAV)*FF(NWAV)*PR(\text{IR}, NWAV)
IF (KRESP EQ. 1) GO TO 465
SUM = SUM+PROD/WAV
GO TO 466
465 \[ Y = \text{HCOK}/(\text{WAVL}*\text{ST1}) \]
\[ \text{SUM} = \text{SUM}+\text{PROD}/((\exp(Y)-1.)*\text{WAVL}^{*6}) \]
466 \[ \text{SIGNL}(\text{IR}) = \text{SUM} \times \text{FAC} \]
470 CONTINUE
C ****** INTEGRATE SIGNAL OVER PELS ****************************
480 \[ \text{DPX} = \text{XPW}/20. \]
\[ \text{DPY} = \text{YPW}/20. \]
DO 500 I = 1, NX
DO 500 J = 1, NY
PELS(I, J) = 0.
XI = I - 1
YI = J - 1
XX = DPX/2. * XI * XP
SUM = 0.
DO 490 IDX = 1, 20
YY = DPY/2. * YI * YP
XX = XX + DPX
DO 490 IDY = 1, 20
YY = YY + DPY
DIF = (PSX - XX)**2 + (PSY - YY)**2
DST = SQRT(DIF)
FRAC = DST/DRR
NUM=FRAC
IF(NUM,GE.,199)GO TO 500
XNUM=NUM
FRAC=FRAC=XNUM
FAC=1.
IF((IDY.EQ.,1).OR.(IDY.EQ.,20))FAC=0.5
IF((IDX,EQ,.1).OR.(IDX,EQ.,20))FAC=FAC*0.5
SUM=SUM+FAC*(SINGL(NUM+1)+FRAC*(SINGL(NUM+2)-SINGL(NUM+1)))
490 CONTINUE
PELS(IY,IX)=SUM*ENMNS*PI*DPX*DPY*(PI*DP/1296000.)**2
500 CONTINUE
C ***** CALCULATE DIFFRACTION EFFICIENCY ****************************
WAV=AVL+DL
DO 920 NW=1,NWAV
WAV=WAV+DL
PROD=QUEF(NW)*FF(NW)
IF(PROD,LT.,1.E-4)GO TO 920
DO 920 IX=1,NX
DO 920 IY=1,NY
XI=IX-1
YI=IY-1
XX=DPX/2.+XI*XP
YY=DPY/2.+YI*YP
XX=XX+DPX
DO 920 IDY=1,NW
YY=YY+DPY
DST=SQR((PSX-XX)**2+(PSY-YY)**2)
FRAC=DST/DRR
NUM=FRAC
IF(NUM,GE.,199)GO TO 910
XNUM=NUM
FRAC=FRAC=XNUM
FAC=1.
IF((IDY.EQ.,1).OR.(IDY.EQ.,20))FAC=0.5
IF((IDX.EQ.,1).OR.(IDX,EQ.,20))FAC=FAC*0.5
SUM=SUM+FAC*(PR(NUM+1,NW)+FRAC*(PR(NUM+2,NW)-PR(NUM+1,NW)))
900 CONTINUE
DIFE(IY,IX)=SUM*ENMNS*PI*DPX*DPY*(PI*DP/(WAV*1296000.))**2
WRITE(102,7017)WAV,IX,IY,DIFE(IY,IX)
7017 FORMAT("WAV=',E15.8,' IX=",I1," IY=",I1," DIFE=",E15.8)
920 CONTINUE
C ***** CALCULATE BACKGROUND SIGNAL ****************************
WRITE(102,6100)
6100 FORMAT("ENTER COSMIC BACKGROUND VISUAL MAGNITUDE")
READ(101,4000)XIMB
XIMLB=0.37353*10.0**(-0.4*XIMB-5.)
IF(KRESP.EQ.,1)GO TO 510
XIMLB=(XIMB*10.0**5.)/(-1.0682*HC)
GO TO 515
WRITE(102, 6104)
6104 FORMAT(/ "ENTER BACKGROUND EFFECTIVE TEMPERATURE (KELVIN)"
READ(101, 4000) BEFT
CALL PHQI(BEFT, HCOK, DLX, HC, XINT)
XIMLB=XIMB*XINT
C
***** INTEGRATE OVER WAVELENGTH ****************************
515 IF(NAV, GT, 1) GO TO 520
IF(KRESP, EQ, 1) GO TO 517
SUM=XIMLB*QUEF(1)*FF(1)*WAVL
GO TO 550
517 Y=HCOK/(WAV*BEFT)
SUM=XIMLB*QUEF(1)*FF(1)/((EXP(Y)-1.)*WAVL**4)
GO TO 550
520 WAV=WAVL
IF(KRESP, EQ, 1) GO TO 521
SUM=QUEF(1)*FF(1)*WAVL
GO TO 522
521 Y=HCOK/(WAV*BEFT)
SUM=QUEF(1)*FF(1)/((EXP(Y)-1.)*WAVL**4)
522 IF(NAV, EQ, 2) GO TO 540
MWAV=NAV=1
DO 530 I=2, MWAV
WAV=WAV+DL
PROD=QUEF(1)*FF(I)
IF(PROD, LT, 1.E-4) GO TO 530
IF(KRESP, EQ, 1) GO TO 528
SUM=SUM+PROD*WAV
GO TO 530
528 Y=HCOK/(WAV*BEFT)
SUM=SUM+PROD/((EXP(Y)-1.)*WAVL**4)
530 CONTINUE
540 WAV=WAV+DL
IF(KRESP, EQ, 1) GO TO 545
SUM=SUM+QUEF(NAV)*FF(NAV)*WAV
GO TO 546
545 Y=HCOK/(WAV*BEFT)
SUM=SUM+QUEF(NAV)*FF(NAV)/((EXP(Y)-1.)*WAVL**4)
546 SUM=SUM*DL*XIMLB
550 BGRND=0.7854*DP*DP*SUM*XPW*YPW
IRESP=0
WRITE(102, 6108)
6108 FORMAT(/ "DO YOU WISH TO INCLUDE A BACKGROUND STAR? (YES=1; NO=0)"
READ(101, 4000) IRESP
IF(IRESP, NE, 1) GO TO 700
WRITE(102, 6112)
6112 FORMAT(/ "ENTER VISUAL MAGNITUDE OF BACKGROUND STAR"
READ(101, 4000) XVM
XIMV=0.37353*10.0**(-0.4*XVM-5.)
IF(KRESP, EQ, 1) GO TO 560
XIMLV=(XIMV*10.0**5.)/(1.0682*HC)
GO TO 570
560 WRITE(102, 6116)
6116 FORMAT(/,"ENTER BACKGROUND STAR EFFECTIVE TEMPERATURE(KELVIN)"")
READ(101,4000)ST2
CALL PHOI(ST2,HCOK,DLA,HC,XINT)
XIMLV=XIMV*XINT
570 WRITE(102,6120)
6120 FORMAT(/,"ENTER X,Y COORDINATES OF BACKGROUND STAR(ARC SECONDS)"")
READ(101,4000)BSX,BSY
DX=BSX=XMAX
DY=BSY=YMAX
D00=BSX*BSX+BSY*BSY
DX0=DX*DX+BSY*BSY
DXY=DX*DY
D0Y=BSX*BSX+DY*DY
DMAX=0.
DMIN=10000.
IF(D00.GT.DMAX)DMAX=D00
IF(DX0.GT.DMAX)DMAX=DX0
IF(DXY.GT.DMAX)DMAX=DXY
IF(D0Y.GT.DMIN)DMIN=D0Y
IF(DX0.LT.DMIN)DMIN=DX0
IF(DXY.LT.DMIN)DMIN=DXY
DMAX=SQR(DMAX)
DMIN=SQR(DMIN)
IF(DX.GT.0.) GO TO 580
IF(DY.GT.0.) GO TO 600
DMIN=0.
GO TO 610
580 IF(DY.GT.0.) GO TO 610
DMIN=DX
GO TO 610
600 IF(DX.GT.0.) GO TO 610
DMIN=DY
610 RMAX=DMAX=DMIN
DRZ=RMAX*DP*PI/(199.*648000.)
DRR=RMAX/199.
ZR=DMIN*DP*PI/648000.-DRZ
CALL BES(NWAV,DRZ,ZR,DNU,ENMNS,WAVL,DL)
C
***** INTEGRATE OVER WAVELENGTH ****************************
630 IF(NWAV.GT.1) GO TO 630
PROD=QUEF(1)*FF(1)*XIMLV*PI*DP*DP/4.
DO 620 IR=1,200
620 IF(KRESP.EQ.1) GO TO 615
SIGNL(IR)=PROD*PR(IR,1)/WAVL
GO TO 620
615 Y=HCOK/(WAVL*ST2)
SIGNL(IR)=PROD*PR(IR,1)/((EXP(Y)-1.)*WAVL**6)
620 CONTINUE
GO TO 670
630 MWAV=NWAV-1
FAC=XIMLV*PI*DP*DP*DL/4.
DO 660 IR=1,200
WAV=WAVL
PROD=QUEF(1)*FF(1)*PR(IR,1)
IF(KRESP.EQ.1)GO TO 631
SUM=PROD/WAV
GO TO 632
631 Y=HCOK/(WAV*ST2)
SUM=PROD/((EXP(Y)-1.)*WAV**6)
632 IF(NWAV.EQ.2)GO TO 650
DO 640 JW=2,NWAV
WAV=WAV+DL
PROD=QUEF(JW)*FF(JW)
IF(PROD,LT.1.E-4)GO TO 640
IF(KRESP.EQ.1)GO TO 636
SUM=SUM+PROD*PR(IR,JW)/WAV
GO TO 640
636 Y=HCOK/(WAV*ST2)
SUM=SUM+PROD*PR(IR,JW)/((EXP(Y)-1.)*WAV**6)
640 CONTINUE
650 WAV=WAV+DL
PROD=QUEF(NWAV)*FF(NWAV)*PR(IR,NWAV)
IF(KRESP.EQ.1)GO TO 655
SUM=SUM+PROD/WAV
GO TO 656
655 Y=HCOK/(WAV*ST2)
SUM=SUM+PROD/((EXP(Y)-1.)*WAV**6)
656 SIGNL(IR)=SUM+FAC
660 CONTINUE
C ***** INTEGRATE SIGNAL OVER PEL **********************************************
670 DPX=XPW/20,
DPY=YPW/20,
DO 690 IX=1,NX
DO 690 IY=1,NY
PELB(IX,IY)=0.
XI=IX-1
YI=IY-1
XX=DPX/2.+XI*XPD
SUM=0.
DO 680 IDX=1,20
YY=DPY/2.+YI*YPD
XX=XX+DPX
DO 680 IDY=1,20
YY=YY+DPY
DIF=(BSX-XX)**2+(BSY-YY)**2
DST=SQRT(DIF)-DMIN
FRAC=DST/DRR
NUM=FRAC
IF((NUM,GE,199).OR.(NUM,LT.0))GO TO 690
XNUM=NUM
FRAC=FRAC-XNUM
FAC=1.
IF((IDY,EQ.1).OR.(IDY,EQ.20))FAC=0.5
IF((IDX.EQ.I).OR.(IDX.EQ.20))FAC=FAC*0.5
SUM=SUM+FAC*SIGNL(NUM+1)+FAC*FRAC*(SIGNL(NUM+2)-SIGNL(NUM+1))
680 CONTINUE
PELB(IX,IY)=SUM*ENMNS*PI*DPX*DPY*(PI*DP/FR/1296000.)**2
690 CONTINUE
C  ***** CALCULATE DIFFRACTION EFFICIENCY  ***********************
WAV=WAVL=DL
DO 950 NW=1,NWAV
WAV=WAV+DL
PROD=QUEF(NW)*FF(NW)
IF(PROD.LT.1.E-4)GO TO 950
DO 950 IX=1,NX
DO 950 IY=1,NY
XI=IX-1
YI=IY-1
XX=DPX/2.+XI*XPD
SUM=0.
DFE(IX,IY)=0.
DO 930 IDX=I
DO 930 IDY=I
YY=DPY/2.+YI*YPD
XX=XX+DPX
DO 930 IDY=20
YY=YY+DPY
DST=SQR((BSX-XX)**2+(BSY-YY)**2)-DMIN
FRAC=DST/DRR
NUM=FRAC
IF((NUM.GE.199).OR.(NUM.LT.0))GO TO 940
XNUM=NUM
FRAC=FRAC=XNUM
FAC=1.
IF((IDX.EQ.1).OR.(IDX.EQ.20))FAC=0.5
IF((IDX.EQ.1).OR.(IDX.EQ.20))FAC=FAC*0.5
SUM=SUM+FAC*(PR(NUM+1,NW)+FRAC*(PR(NUM+2,NW)+PR(NUM+1,NW))
930 CONTINUE
DFE(IX,IY)= SUM*ENMNS*PI*DPX*DPY*(PI*DP/(WAV*1296000.))**2
940 WRITE(102,7027)WAV,IX,IY,DFE(IX,IY)
950 CONTINUE
700 WRITE(102,6124)
6124 FORMAT("ENTER RMS READOUT NOISE (ELECTRONS/PEL")
READ(101,4000)RON
RN=RON*RON
WRITE(102,6128)
6128 FORMAT("ENTER DETECTOR TEMPERATURE (KELVIN")
READ(101,4000)DTEM
IF(DTEM.LT.4.)GO TO 710
FAC=(-6705.487/DTEM+4.0737*DTEM/((1108.+DTEM)
FAC=EXP((FAC)*(29.3E+9)
DC=FAC*DTEM**1.5
GO TO 720
710 WRITE(102,6132)
6132 FORMAT("ENTER MEAN DARK CURRENT (ELECTRONS/PEL/SEC")

READ(101,4000)DC
720 WRITE(102,6136)
6136 FORMAT(/'ENTER OBSERVATION START AND END TIMES(SECONDS)'/)
READ(101,4000)STRT,ENDT
WRITE(102,6140)
6140 FORMAT(/'ENTER NUMBER(>1) OF TIME CALCULATIONS'/)
READ(101,4000)NT
X=NT-1
DT=(ENDT-STRT)/X
WRITE(102,6144)
6144 FORMAT(/'ENTER START AND END SIGNAL TO NOISE RATIOS(S/N)'/)
READ(101,4000)SNST,SNND
WRITE(102,6148)
6148 FORMAT(/'ENTER NUMBER(>1) OF (S/N) CALCULATIONS'/)
READ(101,4000)NS
Y=NS-1
DNS=(SNND-SNST)/Y
READ(101,4000)DUMMY
725 WRITE(102,6152)DP
6152 FORMAT(/'TELESCOPE ENTRANCE PUPIL DIAMETER ','F7.3,' CM'/)
WRITE(102,6156)FR
6156 FORMAT(/'OPTICAL SYSTEM FOCAL RATIO ','F5.2'/)
WRITE(102,6160)
6160 FORMAT(/'SOURCE STAR CHARACTERISTICS'/)
WRITE(102,6164)XS1
6164 FORMAT(5X,'SOURCE MAGNITUDE ','F5.2)
IF(KRESP.EQ.0)GO TO 726
WRITE(102,6168)ST1
6166 FORMAT(5X,'SOURCE TEMPERATURE(KELVIN) ','F9.3)
726 WRITE(102,6172)
6172 FORMAT(/'COSMIC BACKGROUND CHARACTERISTICS'/)
WRITE(102,6176)XMB
6176 FORMAT(5X,'COSMIC BACKGROUND MAGNITUDE ','F5.2)
IF(KRESP.EQ.0)GO TO 727
WRITE(102,6180)BEFT
6180 FORMAT(5X,'COSMIC BACKGROUND TEMPERATURE(KELVIN) ','F9.3)
727 IF(IRESP.NE.1)GO TO 730
WRITE(102,6184)
6184 FORMAT(/'BACKGROUND STAR CHARACTERISTICS'/)
WRITE(102,6188)XVM
6188 FORMAT(5X,'BACKGROUND STAR MAGNITUDE ','F5.2)
WRITE(102,6192)ST2
6192 FORMAT(5X,'BACKGROUND STAR TEMPERATURE(KELVIN) ','F9.3)
730 WRITE(102,6196)
6196 FORMAT(/'SYSTEM SPECTRAL CHARACTERISTICS'/)
WRITE(102,6200)
6200 FORMAT(4X,'WAVELENGTH(CM)*,5X,'QUAN,EFFIC.*',3X,'FILTER FUNCTION')
WAV=WAVL
DO 740 I=1,NWAV
PROD=QUEF(I)*FF(I)
740 IF(PROD.LT.1.E-4)GO TO 739
WRITE(102,6204)WAV,QUEF(I),FF(I)
6204 FORMAT(3(6X,E11.4))
739 WAV=WAV+DL
740 CONTINUE
WRITE(102,6208)
6208 FORMAT(/,"DETECTOR CHARACTERISTICS")
WRITE(102,6212)XPW,YPW
6212 FORMAT(5X,"PEL DIMENSIONS(ARC SECONDS): X",F8.4," Y",F8.4)
WRITE(102,6216)XPD,YPD
6216 FORMAT(5X,"PEL CENTER SEPARATION(ARC SECONDS): X",F8.4," Y",F8.4)
WRITE(102,6220)PSX,PSY
6220 FORMAT(5X,"SOURCE STAR COORDINATES(ARC SECONDS): X",F8.4," Y",F8.4)
IF(IRESP.NE.1)GO TO 750
WRITE(102,6224)RSLX,BSY
6224 FORMAT(5X,"BACKGROUND STAR COORDINATES(ARC SECONDS): X",F8.4," Y",F8.4)
750 WRITE(102,6228)NX,NY
6228 FORMAT(5X,"ARRAY SIZE: X",I2," BY Y",I2)
WRITE(102,6232)RNM
6232 FORMAT(5X,"RMS READOUT NOISE(ELECTRONS/PEL):",F5.2)
IF(DET.M.LT.4.)GO TO 760
WRITE(102,6236)TEM
6236 FORMAT(5X,"DETECTOR TEMPERATURE(KELVIN):",F8.4)
760 WRITE(102,6240)DC
6240 FORMAT(5X,"DARK CURRENT(ELECTRONS/PEL/SECOND):",E15.8)
WRITE(102,6246)
6246 FORMAT(/,"SIGNAL TO NOISE RATIO AS A FUNCTION OF OBSERVATION", & " TIME")
WRITE(102,6250)
6250 FORMAT("AND PEL NUMBER")
WRITE(102,6254)
6254 FORMAT(4X,"TIME(SECONDS) ",2X," (S/N) ",2X,"PEL NUMBER")
DO 770 IX=1,NX
770 IY=1,NY
T=STRT+DT
DO 770 IT=1,NT
T=T+DT
XNUM=PELS(IX,IY)*T
XDNOM=XNUM+BGRND*T+PELB(IX,IY)*T+RN+DC*T
SNP=XNUM/SQRT(XDNOM)
WRITE(102,6256)T,SNP,IX,IY
6256 FORMAT(5X,E12.5,6X,E11.4,2X,2I3)
770 CONTINUE
WRITE(102,6260)
6260 FORMAT(/,5X,"(S/N)",4X," SECONDS ",2X,"PEL NUMBER")
DO 780 IX=1,NX
780 IY=1,NY
SN=SNST-DNS
DO 780 IS=1,NS
SN=SN+DNS
A = SN * SN / (PELS(IX, IY) ** 2)
B = A * (PELS(IX, IY) + BGRND + PELB(IX, IY) + DC) * 0.5
C = A * RON
T1 = B + SQRT(B * B + C)
WRITE(102, 6264) SN, T1, IX, IY

6264 FORMAT(5X, F5.2, 5X, E12.5, 2X, 2I3)
780 CONTINUE
WRITE(102, 6268)
6268 FORMAT(//, "DO YOU WISH TO CHANGE SOURCE MAGNITUDE? (YES=1; NO=0)")
READ(101, 4000) JRESP
IF(JRESP.EQ.0) GO TO 800
WRITE(102, 6272)
6272 FORMAT(//, "ENTER NEW SOURCE MAGNITUDE")
READ(101, 4000) XSM
XNEW = 0.37353 * 10.0 ** (-0.4 * XSM)
XNEW = XNEW / XIM
DO 790 IX = 1, NX
DO 790 IY = 1, NY
790 PELS(IX, IY) = PELS(IX, IY) * XNEW
800 WRITE(102, 6276)
6276 FORMAT(//, "DO YOU WISH TO CHANGE COSMIC BACKGROUND MAGNITUDE?", & "(YES=1; NO=0)")
READ(101, 4000) JRESP
IF(JRESP.EQ.0) GO TO 810
WRITE(102, 6280)
6280 FORMAT(//, "ENTER NEW COSMIC BACKGROUND MAGNITUDE")
READ(101, 4000) XMB
XNEW = 0.37353 * 10.0 ** (-0.4 * XMB)
XNEW = XNEW / XIM
BGRND = BGRND * XNEW
810 WRITE(102, 6284)
6284 FORMAT(//, "DO YOU WISH TO CHANGE BACKGROUND STAR MAGNITUDE?", & "(YES=1; NO=0)")
READ(101, 4000) JRESP
IF(JRESP.EQ.0) GO TO 830
WRITE(102, 6288)
6288 FORMAT(//, "ENTER NEW BACKGROUND STAR MAGNITUDE")
READ(101, 4000) XMV
XNEW = 0.37353 * 10.0 ** (-0.4 * XMV)
XNEW = XNEW / XIM
DO 820 IX = 1, NX
DO 820 IY = 1, NY
820 PELB(IX, IY) = PELB(IX, IY) * XNEW
830 WRITE(102, 6292)
6292 FORMAT(//, "DO YOU WISH TO CHANGE DETECTOR EFFECTS? (YES=1; NO=0)")
READ(101, 4000) JRESP
IF(JRESP.EQ.1) GO TO 700
WRITE(102, 6296)
6296 FORMAT(//, "DID YOU MAKE ANY CHANGES? (YES=1; NO=0)")
READ(101, 4000) JRESP
IF(JRESP.EQ.1) GO TO 725
STOP
SUBROUTINE PHOI(TOTAL, HCOK, DLA, HC, XINT)
IMPLICIT DOUBLE PRECISION(A-H, O-Z)
COMMON /PHOF/XK(40)
X=3.8E-5
Y=HCOK/(X*TEMP)
SUM=XK(1)/((EXP(Y)-1.)*X**5)
X=X+DLA
DO 10 IK=2,38
ICHEK=2*(IK/2)
FAC=2.
IF(IK.EQ.ICHEK)FAC=4.
Y=HCOK/(X*TEMP)
SUM=SUM+FAC*XK(IK)/((EXP(Y)-1.)*X**5)
X=X+DLA
10 CONTINUE
Y=HCOK/(X*TEMP)
SUM=SUM+XK(39)/((EXP(Y)-1.)*X**5)
XINT=3./SUM*DLA*HC
RETURN
SUBROUTINE BES(NWAV, DRZ, ZR, DNU, ENMNS, WAVL, DL)
IMPLICIT DOUBLE PRECISION(A-H, O-Z)
COMMON /BESF/X0(400), P(200,21), TTOP(400,21), QUEF(21), FF(21)
TW0PI=6.283185307
WAV=WAVL-DL
DO 60 NW=1,NWAV
WAV=WAV+DL
DR=DRZ/WAV
Z=ZR/WAV
PROD=QUEF(NW)*FF(NW)
DO 60 IR=1,200
PR(IR,NW)=0.
Z=Z+DR
IF(PROD.GT.1.E-4)GO TO 5
GO TO 60
5 XNU=DNU
DO 31 I=1,400
XNU=XNU+DNU
X=TWOPI*Z*XNU
IF(X.GT.0.)GO TO 10
BS=1.
GO TO 30
10 IF(X.GT.4.))GO TO 20
X2=X/2.
X22=X2*X2
X24=X22*X22
X26=X24*X22
X28=X24*X24
BS=BS+X28*X24/518400.-X28*X26/25401600.
GO TO 30
20 X2=X*X
\[ x_3 = x_2 x \]
\[ x_4 = x_3 x \]
\[ x_5 = x_4 x \]
\[ x_6 = x_5 x \]
\[ p = 1.0 \cdot 0.0703125 / x_2 + 0.112152 / x_4 = 0.5725 / x_6 \]
\[ q = -0.125 / x + 0.0732422 / x_3 = 0.227108 / x_5 \]
\[ \text{BS} = 0.797846 / \sqrt{x} \]
\[ \text{AQ} = x - 0.7853982 \]
\[ \text{BS} = \text{BS} \cdot (p \cdot \cos(AQ) - q \cdot \sin(AQ)) \]

30 \text{FX0}(I) = \text{TTOT}(I, NW) \times \text{BS} \times \text{XNU} \]

31 \text{CONTINUE} \]
\[ A_2 = 0. \]
\[ A_3 = 0. \]
\[ A_4 = 0. \]
\[ \text{DO} 40 \ I = 2, 399, 2 \]
\[ A_2 = A_2 + \text{FX0}(I) \]

40 \text{CONTINUE} \]
\[ \text{DO} 50 \ I = 3, 398, 4 \]
\[ A_3 = A_3 + \text{FX0}(I) \]
\[ A_4 = A_4 + \text{FX0}(I-2) + \text{FX0}(I+2) \]

50 \text{CONTINUE} \]
\[ \text{SAS} = \text{DNU} \cdot (1.4 \cdot A_4 + 2.4 \cdot A_3 + 6.4 \cdot A_2) / 4.5 \]
\[ \text{PR}(I, NW) = 8 \cdot \text{SAS} / \text{ENMNS} \]

60 \text{CONTINUE} \]
\[ \text{RETURN} \]
\[ \text{END} \]
Algorithm for Astronomical, Point Source, Signal to Noise Ratio Calculations

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An algorithm was developed to simulate the expected signal to noise ratios as a function of observation time in the charge coupled device detector plane of an optical telescope located outside the Earth's atmosphere for a signal star, and an optional secondary star, embedded in a uniform cosmic background. By choosing the appropriate input values, the expected point source signal to noise ratio can be computed for the Hubble Space Telescope using the Wide Field/Planetary Camera science instrument.