ATMOSPHERIC-PROFILE IMPRINT IN FIREBALL ABLATION-COEFFICIENT

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During the past two decades three different projects for registration of meteoric fireballs were put into operation using multistation photographic technique. They have yielded data on several hundreds of fireball trajectories, some of them with deep atmospheric penetration down to heights of 20 kilometers. The immediate results of multistation photograph of a fireball are the relative distances along the trajectory, $l_{\text{obs}}$, and the heights $h_{\text{obs}}$ measured at each shutter time-mark, $t_r$ (shutter breaks of the image). The precision of one value of $l_{\text{obs}}$ is of the order of several tens of meters. There are usually many tens of independent points (breaks) available for long fireball trajectories with independently measured $l_{\text{obs}}$ and $h_{\text{obs}}$. We need a good theoretical relation for a least-squares solution of $l(t)$ or $h(t)$, where $l$ is the theoretically given distance along the trajectory and $h$ the height, until recently there were no adequate formulae expressing theoretically the distance along the fireball trajectory, $l$, as function of $t$. We have been able to find such formulae and moreover to find their general form for any atmospheric profile used.

The motion and ablation of a single non-fragmenting meteor body can be expressed by the following set of differential equations first presented by Hoppe (1937):

\[\frac{dv}{dt} = -\frac{\Gamma A_0}{d} \rho \, v^{-2/3} \, m^{-1/3} \, v \, \rho \, m^{-1/3} \, v \, \rho \, m^{-1/3} \, v \]

(1)

\[\frac{dn}{dt} = -\frac{\Delta}{2c} \rho \, m^{-2/3} \, v \, \rho \, m^{-2/3} \, v \]

(2)

\[\frac{dh}{dt} = -v \, \cos \, \gamma \]

(3)

where $v$ is the velocity, $m$ the mass, $h$ the height of the meteoroid, $\rho$ the air density at any time instant $t$. The parameters are: $\Gamma$ the drag coefficient, $h$ the heat-transfer coefficient, $A$ the shape factor: $A = \pi \, m /2 \, \rho d$, where $S$ is the head cross-section and $\rho_d$ the density of the meteoroid, $\gamma$ is the angle between the fireball trajectory and the vertical. The ablation coefficient, is defined by

\[v = \frac{A}{2c} \rho_d^{-2/3} \]

and the shape-density coefficient, $K$, by

\[K = \Gamma A \rho_d^{-2/3} \]

(5)

To solve the system of equations (1) to (3), we have to assume a density profile of the atmosphere. Until now everybody working in the field of physical theory of meteors and its application to observations solely used the assumption of exponential decrease of the air density with height corresponding to an isothermal atmosphere:

\[\rho = \rho_c \, \exp(-bh) \]

(6)
where \( b \), the air density gradient, was assumed constant as well as the zero-level air-density, \( \rho_0 \). Moreover, the solution of the system (1) to (3) was known only in the form of \( v = v(t) \):

\[
\frac{E_1 \left( \frac{1}{6} \rho_0 v^2 \right) - E_1 \left( \frac{1}{6} \rho_o v^2 \right)}{b \cos z_R} = \frac{2K \rho_m^{-1/3} \exp \left( \frac{1}{6} \rho_o v^2 \right)}{b \cos z_R}
\]

(7)

where \( v_o \) is the initial velocity (before entering the atmosphere) and \( \rho_m \) is the initial mass and \( E_1(x) = \int_0^x \frac{\exp\left(-\frac{1}{6} z^2\right) dz}{z^2} \) is the exponential integral. Thus numerical differentiation of directly observed distances along the fireball trajectory, \( \text{obs} \), was necessary to get \( v \) for application of formula (7) to observations. Such an indirect method yielded ablation coefficients and initial velocities with standard deviations much larger than corresponding to the accuracy of the measured distance, \( \text{obs} \), so the accuracy of the observed quantities was far from being utilized fully.

Recently we have proposed a solution of the system (1)-(3) in a closed form expressing \( t - t(t) \):

\[
t - t_0 = \frac{2}{b \cos z_R} \int_{v_0}^{v} \frac{\exp \left( \frac{1}{6} \rho_0 z^2 \right) dz}{v^2 \left( E_1 \left( \frac{1}{6} \rho_o v^2 \right) - E_1 \left( \frac{1}{6} \rho_o v^2 \right) \right)}
\]

(8)

\[
1 - 1_0 = \frac{1}{b \cos z_R} \ln \frac{E_1 \left( \frac{1}{6} \rho_o v^2 \right) - E_1 \left( \frac{1}{6} \rho_0 v^2 \right)}{E_1 \left( \frac{1}{6} \rho_o v^2 \right) - E_1 \left( \frac{1}{6} \rho_0 v^2 \right)}
\]

(9)

Here \( v_o \), \( 1_0 \) are the velocity and the distance at the point \( t = 0 \) from where the relative time is counted. The integration variable \( z \) is denoted \( x \). The equations (8) and (9) hold under the assumption (6). The observed values, \( 1_\text{obs} \) and \( h_\text{obs} \) for each independent time instant, \( t \), can be fitted to equations (8) and (9) by the least-square method and the parameters \( 1_0, v_0, \rho_0, v_o, \rho_o \) can be determined.

In applying our formulae (8) and (9) to observations of fireballs we started to suspect much greater importance of the air density profile for the resulting values of the parameters than it was assumed previously, when the simplistic approach to the air density profile with a constant air density gradient was assumed as correct. Our solution of (1) to (3) for a general air density profile has the form:

\[
t - t_0 = \int_{t_0}^{1} v^{-1} \frac{dh}{\rho} \frac{E_1 \left( \frac{1}{6} \rho_0 v^2 \right) - E_1 \left( \frac{1}{6} \rho_0 v^2 \right)}{h \rho_{\text{obs}}} = \int_{h_0}^{h_\text{obs}} \frac{dh}{\rho}
\]

(10)

\[
\frac{E_1 \left( \frac{1}{6} \rho_0 v^2 \right) - E_1 \left( \frac{1}{6} \rho_0 v^2 \right)}{h \rho_{\text{obs}}} = \int_{h_0}^{h_\text{obs}} \frac{dh}{\rho}
\]

(11)

where \( h \) is the height at the time instant \( t_0 \). Among other parameters \( v_0 \) and \( \rho_0 \) can be determined by the least-square method to fit the observations to (10) and (11).

Details of both solutions (8), (9) and (10), (11) can be found in two recent papers (PECMA and CEPLECHA, 1983, 1984) together with outlines of the numerical procedures and of the computer programs used. The ablation
coefficients and initial velocities computed for 10 Prairle Network and one
European Network fireballs for the isothermal atmosphere (1962) and for the
seasonal atmosphere (1966) are compared in Table 1. Graphical comparison of
ablation coefficients computed for the isothermal atmosphere and for the
seasonal atmosphere are plotted in Figure 1. The following results are evident.

a) The computed ablation coefficient is strongly dependent on the atmospheric
model used. Differences by using a simplified isothermal atmosphere are up
to factor of two.
b) The standard deviations when using a seasonal atmosphere are significantly
smaller than for the simple isothermal model.
c) The initial velocity differs also far outside the standard deviations for
the majority of cases and the values from the seasonal atmosphere are
better. This has astronomical significance in computing the orbits.

The main conclusion is evident. The generous assumption of simple
atmospheric model used up to now for theoretical considerations of meteoroid
penetration into the atmosphere and for computational applications to fireballs
yields incorrect results. At least, the density profile of "monthly
atmospheres" should be used (CIRA, 1972) for any future theoretical and
experimental applications to get any reliable data on ablation coefficients and
initial velocities of fireballs with good dynamic data. Analysing ablation
coefficients computed for many fireballs of different structure and composition
of their meteoroids, we could better recognize different types of bodies and
their average characteristics. Then, using the average statistical value of the
ablation coefficient for each separate structural and compositional group of
fireballs, we could determine details of the instant air density profile of the
Middle Atmosphere at the particular moment of any fireball with good dynamic
data and moreover the local disturbance by large meteoric bodies. And in this
direction we want to contribute to studies of the Middle Atmosphere in the near
future.

REFERENCES

CIRA (1972), COSPAR International Reference Atmosphere 1972, Akademie Verlag,
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(for 1983, No. 63).
U.S. Standard Atmosphere (1962), Washington
U.S. Standard Atmosphere Supplements (1966), Washington
Table 1. Comparison of ablation coefficient, \( \gamma \), and initial velocity, \( v_\infty \), computed with constant air density gradient, \( b \), and with seasonal air density profile.

<table>
<thead>
<tr>
<th>Fireball Number</th>
<th>( \gamma ) ( \text{s}^2/\text{km}^2 )</th>
<th>( \gamma ) ( \text{exp}(-bh) ) Seasonal Atmosphere</th>
<th>( \gamma ) ( \text{exp}(-bh) ) Seasonal Atmosphere</th>
<th>( v_\infty ) ( \text{km/s} )</th>
<th>Height Interval km</th>
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<td>FN39057</td>
<td>0.0229</td>
<td>3.0195</td>
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<td>14.350</td>
<td>74</td>
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<td>±0.005</td>
<td>±0.006</td>
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<td>±.004</td>
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</table>

*Different average values of \( b \) were used corresponding to different height intervals for particular fireballs and to the U.S. Standard Atmosphere (1962).

Seasonal atmosphere were taken from U.S. Standard Atmosphere Supplements (1966).
Figure 1. The average ablation coefficient, $\gamma$, computed from the simple isothermal atmospheric model ($\gamma = c \cdot \exp(-bh)$) is plotted against the average ablation coefficient computed from the seasonal atmospheric model. Bars are the standard deviations. The $45^\circ$ line marks equal values of $\gamma$ from both computations.