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Dilution Jet Configurations in a Reverse Flow Combustor

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DILUTION JET CONFIGURATIONS IN A REVERSE FLOW COMBUSTOR

Abstract
by
JAMES ZIZELMAN

Results of measurements of both temperature and velocity fields within a reverse flow combustor are presented. Flow within the combustor is acted upon by perpendicularly injected cooling jets introduced at three different locations along the inner and outer walls of the combustor. The oddity of such combustor configurations is best exemplified by recognizing that flows within them accelerate both transversely and longitudinally.

Each experiment is typified by a group of parameters: density ratio, momentum ratio, spacing ratio, and confinement parameter. Two more quantities, Froude number and Mach number, although calculated were found not to be important in these experiments. Measurements of both temperature and velocity are presented in terms of normalized profiles at azimuthal positions through the turn section of the combustion chamber. A rake of probes that spans the combustor laterally allows for the
presentation of some three dimensional plots giving some indication of the lateral spreading. In addition, jet trajectories defined by minimum temperature and maximum velocity give a qualitative indication of the location of the jet within the cross flow. Results of a model from a previous temperature study are presented in some of the plots of data from this work.

During injection from all three injection locations (inner wall prior to the turn, outer wall prior to the turn, and outer wall into the turn) a migration of the injected fluid toward the inner wall is observed both from temperature and velocity fields.

Penetration into the cross flow is shown to be affected as follows: increasing injection jet momentum increases penetration, increasing the ratio of the jet density to the cross flow density increases penetration, and increasing spacing between jets in multiple jet injection increases penetration.

Lateral spreading seemed to be greater during higher momentum injection and during injection from the outer wall.

The above conclusions appear consistent with
the flow field that sets up in the combustor. The flow is observed as inertially dominated and characteristically irrotational as a pressure gradient develops to support fluid turning.
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<td>A</td>
<td>Area</td>
</tr>
<tr>
<td>b</td>
<td>Jet radius</td>
</tr>
<tr>
<td>b₀</td>
<td>Initial jet radius</td>
</tr>
<tr>
<td>C_D</td>
<td>Coefficient of drag</td>
</tr>
<tr>
<td>Dr</td>
<td>Density ratio, ρ_j₀/ρ</td>
</tr>
<tr>
<td>E₁</td>
<td>Entrainment coefficient</td>
</tr>
<tr>
<td>E₂</td>
<td>Entrainment coefficient</td>
</tr>
<tr>
<td>h_w</td>
<td>Differential pressure in mm water</td>
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<tr>
<td>H₀</td>
<td>Combustor height at entrance</td>
</tr>
<tr>
<td>H₁</td>
<td>Combustor height at exit</td>
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<tr>
<td>J</td>
<td>Momentum Ratio, ρ_j₀ V_j₀²/ρ V²</td>
</tr>
<tr>
<td>l</td>
<td>Perimeter of turbulent jet</td>
</tr>
<tr>
<td>Pₛ</td>
<td>Static pressure</td>
</tr>
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<td>P_T</td>
<td>Total pressure</td>
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<tr>
<td>ΔP</td>
<td>Total pressure - static pressure</td>
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<td>q</td>
<td>Volumetric flux</td>
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<td>Jet volumetric flux</td>
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<td>Initial jet volumetric flux</td>
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<td>R</td>
<td>Gas constant for air</td>
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<td>R_i</td>
<td>Radius of inner wall prior to bend</td>
</tr>
<tr>
<td>R_o</td>
<td>Radius of outer wall prior to bend</td>
</tr>
<tr>
<td>s</td>
<td>Longitudinal jet coordinate</td>
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Sr  
Spacing ratio, length between jet centers

$\frac{2b_o}{\text{spacing ratio, length between jet centers}}$

T  
Cross flow temperature

$T_{\text{jet}}$  
Initial jet temperature

$T_L$ or $T_L$  
Local temperature

V  
Cross flow velocity

$V_{\text{jet}}$  
Initial jet velocity

$V_{\text{jet}}$  
Initial jet velocity

$V_{\text{jet}}$  
X-direction jet velocity

$V_{\text{jet}}$  
Y-direction jet velocity

$V_L$ or $V_L$  
Local velocity

x, X  
x-axis

y, Y  
y-axis

z, Z  
z-axis

GREEK

τ  
Normalized temperature,

\[ \frac{T_L - T}{T_{\text{jet}} - T} \]

γ  
Normalized velocity,

\[ \frac{V_L}{V} - 1 \]

θ  
Angle of jet centerline from initial cross flow streamlines

ξ  
Longitudinal jet coordinate
CHAPTER I
INTRODUCTION

In recent years, many studies have been made that deal directly with the concept of jet injection. Both theoretical and experimental work has been done. Much of this work deals with one of the following configurations: (1) jet injection into a non-moving environment, (2) perpendicular jet injection into a medium with constant velocity, and (3) multiple jet and two-dimensional jet injection into a medium with constant velocity.

Work of the above description can be viewed in most of the references in the bibliography. It must be remembered, however, that these studies are not addressing the problem of turbulent jet injection into an accelerating medium. The studies involving a moving cross flow are set up to provide a constant area for the flow, and hence a constant velocity. In most dilution jet applications in jet engine combustors, this type of investigation is entirely adequate because the chamber involved is really nothing more than a straight-through annulus resulting in cross flow velocities that remain nearly the same from inlet to exit. However, in the geometry of the reverse flow combustor, the cross
flow negotiates a 180° turn while experiencing a
decrease in cross sectional area. This geometric
complexity gives rise to experimental studies such
as this one associated with determining both
velocity and temperature fields throughout the
combustor. From this information, immediate
qualitative conclusions are available as well as more
detailed information describing the flow conditions
for use as experimental input to a semi-empirical
model. As can be seen, then, this investigation has
some peculiarities resulting from the unique
combustor design. By the nature of the geometry,
the flow must accelerate both longitudinally and
transversely as it negotiates the 180° turn.

Figure 1: Simplified Combustor Geometry [18]
When one examines the combustor more closely, it
becomes evident that one can identify two apparent
modes of acceleration. The first to be identified
is the transverse acceleration that must exist
perpendicular to the flow direction due to the fact
that the flow negotiates the turn. The second
identifiable acceleration is that due to the cross
sectional area decrease as one approaches the exit.
This acceleration is in the direction of the flow.
By rotating the schematic in figure 1 about the
centerline, one can see that this combustor is
actually an annulus turned inside of itself. This
gives rise to more acceleration in the direction of
the flow because the inner annulus (the exit) is of
smaller radius, and therefore carves out less area.
The following figures depict the two modes of
acceleration:

\[ \text{FLOW} \]

Longitudinal Acceleration

Figure 2: Longitudinal Acceleration
Due to this unique combustor design, little theoretical, semi-empirical, or purely empirical information is available that describes flow conditions within the combustor. Lipshitz[18], in a recent study using a model reverse flow combustor, generated a large quantity of empirical information regarding temperature fields throughout the turn section of the combustor. However, an attempted model to predict jet trajectories in single jet injection showed poor agreement with trajectories inferred from the temperature profiles apparently due to the lack of information regarding the cross flow velocity field. In the case of Lipshitz's experiment, a thermal jet trajectory location was defined as the minimum temperature observed between
the inner and outer walls of the combustor at a given angular position in the turn section. Individual trajectory locations could then be connected with a smooth line.

To more fully examine the complexities of this combustor with its various cooling jet injection configurations, an experiment was designed to gather both temperature and velocity data from the end of the primary zone (at the entrance to the turn section) to the outlet of the combustor (at the turbine blades). Both radial (from inner wall to outer wall) and azimuthal (angularly through the turn) sampling points were set up so that both the velocity and temperature fields could be well represented.

In an actual turbine-engine situation, it is advantageous to know what effect changes in dilution jet injection would have on temperature and velocity fields at the outlet of the combustor. Since the outlet is the location of the turbine blades, knowing detailed information here gives one the ability to operate the engine at a higher, more efficient temperature with considerably less uncertainty as to when mechanical failure of the blades will occur. Of course, the preferred
temperature profile at the blades is best exemplified by figure 4.

![Diagram showing temperature profile at turbine blades]

Figure 4: Preferred temperature profile (turbine)

Figure 4 shows a temperature profile that increases in going from the root to the tip of the blade. The reasoning here is based upon centrifugal loading. At the root, the blade structure must exert enough inward force to support nearly the entire length of the blade thereby causing this location to experience the maximum stress. Now, choose a position near the tip of the blade. Here, the only centrifugal loading is that caused by that small portion between your chosen position and the blade tip. It follows that the stress in the blade at the tip approaches zero. As one begins to operate a jet engine near its allowable maximum, the limit is usually set by the mechanical capability of the
turbine blades, so the lower the temperature at the maximum stress location, the longer the blade can maintain mechanical integrity.

Additionally, a known velocity profile at the exit would allow a more detailed calculation of convective heat transfer coefficients of the turbine blades, resulting in a more thorough understanding of the heat transfer taking place, again providing the opportunity to operate the engine at a more efficient temperature.

It can be seen, then, that if enough information is gathered, it will be possible to tailor both velocity and temperature profiles at the outlet by merely changing the dilution jet initial conditions. It is this tailoring that makes this work so important.
CHAPTER II

PREVIOUS RELATED STUDIES

Since this research concerns itself with a specific geometric design, there is relatively little information regarding temperature and velocity profiles and trajectories. In fact, one previous study by Lipshitz [18] is all that is known to exist. However, some studies have been made that relate in a generic way because the basic concept of jet injection is investigated. As stated earlier, these studies include: (1) free turbulent jet injected into a non-moving medium, (2) turbulent jet injected perpendicularly into a non-accelerating medium, and (3) multiple jets and two-dimensional jets injected into a non-accelerating medium. Since these specific types of studies will not aid a great deal in the understanding of this one, they will be reviewed here fairly quickly. In a later chapter, a presentation of the temperature study conclusions by Lipshitz [18] will be conducted.

2.1. The Free Jet

As is suggested in any work on the fluid
mechanics of jets, turbulent profiles are generally considered to be self-similar, that is, the velocity profiles remain similar with respect to the space variable. Discussions on similarity solutions for the free jet can be viewed in works by Abramovich\cite{1} and Albertson\cite{2}. Using the idea of the similarity solution, it becomes evident that in the fully developed region of a free jet, the jet boundary layer grows linearly as a function of distance from the origin, i.e.,

\[ b = s \] (II.1)

\[ \text{core transition fully-developed region region region} \]

\[ \text{mean velocity profiles} \]

\[ \text{instantaneous picture} \]

\textbf{Figure 5: A turbulent free jet}\cite{18}
In addition, the similarity solution allows one to recognize that the centerline velocity of the jet decays as the inverse of the distance travelled from the origin:

\[ V_j = \frac{1}{s} \]

(II.2)

Investigating further, one can see from Albertson\textsuperscript{[2]} and Ricou and Spalding\textsuperscript{[25]} that the gross volume flux for a single turbulent free jet was found to be given by:

\[ \frac{q_j}{q_{jo}} = \frac{0.16s}{b_0} \]

(II.3)

Something that goes hand in hand with the idea of gross volume flux is the notion of entrainment. Qualitatively, it is a measure of how much fluid experiences motion in the direction of an injected jet. Rouse, Yih, and Humphreys\textsuperscript{[26]} in an early work investigated injection where the jet and the ambient fluids were of different densities. Their modification to the gross volume flux equation (II.3) manifests itself in a density correction factor as follows:
In the above analyses, the consideration is given to the free turbulent jet pictured in figure 5 above. In such a turbulent jet, the velocity profile at the origin is nearly uniform. From the origin outward, there are three distinct regions of the jet: (1) the potential core, (2) the transition region, and (3) the fully developed region. The center of the potential core region is referred to as the core. At this location, the fluid has the same properties as the fluid issuing from the nozzle of the jet. Still in the potential core region, but moving laterally from the centerline, a free shear layer must be encountered. In this area, the fluid smoothly returns to the ambient conditions. This potential core region has been seen to exist about four to five nozzle diameters downstream of the jet. It is this type of information that becomes useful in the analysis of the data of this investigation as a guide to give some rough idea about what to expect.

The second region is the transition region, immediately followed by the fully developed region. It is typical for the transition region to dissipate
ten nozzle diameters downstream of the injection location. Of course, these estimated distances depend largely on the initial conditions of the jet as it issues from the nozzle. After transition has occurred in the second region, the jet remains turbulent.

2.2 Jet Injection into a Non-Accelerating Medium

In studies involving a single jet introduced into a non-accelerating cross flow, it is typical for a round jet to be issued perpendicularly into the oncoming flow with what is considered a uniform velocity profile. Also, it is generally considered typical for the cross flow duct to be of constant cross sectional area to eliminate acceleration. As the jet moves outward from the nozzle, a complex deflection begins to occur. Figure 6 below illustrates this typical injection configuration.

Figure 6: Injection configuration for a non-accelerating cross flow.
In an attempt to qualitatively describe how the "jet bending" is taking place, a number of different mechanisms have been identified and can be considered. First, as the jet issues from the nozzle, it can be viewed as a slug of fluid, even a rigid cylinder, that has associated with it a coefficient of drag, $C_D$.\cite{1} As the dynamic pressure of the cross flow begins to exert force on the "cylinder" of fluid, it begins to experience a drag in the direction of velocity of the cross flow. Immediately, of course, a "bending over" of the jet begins to appear. As quickly as this occurs, the drag force vector forms two components: one tangentially along the deformed jet and one normal to it.

\textbf{Figure 7: Jet bending due to the drag mechanism.}
Another mechanism which is attributed to the complex bending of a jet injected perpendicularly into a cross flow is jet entrainment. As the jet penetrates outward into the cross flow, it pulls along with it particles of fluid belonging to the cross flow, due to the frictional force apparent there. As these cross flow particles are entrained, so, too, is the momentum associated with them. Since the momentum vector of the cross flow is perpendicular to that of the issuing jet, it follows that this becomes a mechanism responsible for the direction change of the jet. This momentum transfer, in other words, causes the jet to acquire a velocity component in the direction of the cross flow.

The third mechanism is related to the first and second in that it considers the jet to be a cylindrical slug of fluid, and gives some physical meaning to entrainment. In this mechanism, though, one looks more in-depth at what occurs fluid dynamically as fluid passes over a cylinder. Most obviously, there is a high pressure area where the cross flow impinges upon the issuing jet. Secondly, a low pressure wake forms downstream of the cylinder as the flow detaches. The cross flow, then, will
begin to flow in the direction of the favorable pressure gradient, or into the low pressure region behind the jet. This action sets up a pair of vortices, causing the jet to become kidney-like in shape as seen in the figure below.

![Vortex pair formation in a jet](image)

Figure 8: Vortex pair formation in a jet.

This induced vortex action is held partially responsible for the entrainment that occurs in cross flow injection situations. Platten and Keffer\[23\] show that the increased spreading rate associated with this injection configuration is attributable to this vortex pair formation as well.

Just as the free jet was disassembled into its three basic regions, the single jet in a cross flow
can be similarly viewed. This type of jet also has three principal regions, the first of which is referred to as the potential core and is based mainly on the jet to cross flow momentum ratio but is also somewhat dependant on the jet Reynolds number based on initial jet diameter[24].

The next region identifiable is downstream of the potential core and is fully turbulent. Vorticity produced by the cross flow negotiating around the cylindrical jet gives rise to the formation of the vortex pair within the jet, giving it a kidney shape while average jet velocities begin to diminish toward the ambient value rather quickly. After a relatively short distance, the jet trajectory becomes nearly parallel to that of the cross flow. This zone is commonly referred to as the "zone of maximum deflection,"[24] or the "curvilinear zone."[17]

In the third zone, the "far field zone,"[16] the two vortices are literally overrun by the cross flow and subsequently swept downstream at velocities nearly that of the cross flow, although such conditions are reached asymptotically. Additionally, the amount of circulation appears to decrease as one moves downstream. This zone is
known to exist up to 1000 nozzle diameters downstream. Again, these statements serve as crude approximations of the reverse flow combustor because they do not consider acceleration of the cross flow, but a general idea of what to expect is presented.

The most successful modelling of a jet in a cross flow includes some consideration for jet entrainment. In this brief discussion on related works, some of those are presented now. Fan[10] developed an interesting model for the jet entrainment. For this fluid mechanism, he suggested an entrainment coefficient, $E$. Using this coefficient, the development was taken further, and an entrainment velocity was defined as follows:

$$V_e = E \cdot \text{abs}(V_j - V) = E \left( (V_{jx} - V)^2 + V_{jy}^2 \right)^{1/2}$$ (II.5)

In a turbulent jet, as mentioned above, mass flux within the turbulent region increases by entraining the surrounding fluid as progression downstream occurs. In accordance with Kamotsni and Greber[14], and using the coordinates of figure 8, the rate of increase of mass flux is given by:
where $\rho$ is the density of the cross flow, $l$ is the perimeter of the turbulent region, and $V_e$ is the entrainment velocity, or the velocity with which the turbulent front is advancing into the cross flow at a particular $\xi$.

Equation II.5 shows $E$ being used as a correlation coefficient for the entrainment velocity that uses the vector difference between local jet and local cross flow velocities. The value of $E$ was found to fall in the range of 0.4 to 0.5 for $V_j/V$ ratios ranging from five to 20. This correlation, however, is intended for a uniform area cross flow, and begins to break down under acceleration.

Hoult et al. [13] show an asymptotic solution for jet trajectories compared to experimental values at various velocity ratios. From this investigation, yet another description of entrainment velocity was developed. In this correlation, the description of entrainment is dependant upon the difference between local jet velocity and the parallel and normal components of the cross flow velocity. It contains two entrainment coefficients.
Figure 9: Diagram for Hoult’s entrainment equation.

The correlation developed is as follows:

\[ V_e = E_1(V_j - V\cos(\theta)) + E_2(V\sin(\theta)) \]  \hspace{1cm} (II.7)

The values for \( E_1 \) and \( E_2 \) are given as 0.11 and 0.6 respectively for a velocity ratio ranging from one to ten.

In a similar development, Kamotani and Greber [14] constructed a mathematical relation depicting entrainment velocity which is also dependant on cross flow velocities both tangential and normal to the jet trajectory. In this development, however, the value of jet velocity used was taken as the maximum value of fluid speed in a given normal plane instead of the average jet velocity as had been in earlier investigations. Their relation is written:

\[ V_e = E_1(V_{j\text{max}} - V\cos(\theta)) + E_2V\sin(\theta) \]  \hspace{1cm} (II.8)
where $\Theta$ is defined in figure 9. The values of $E_1$ and $E_2$ were seen to vary with changing jet to cross flow momentum ratio, $J$. For instance, with $J = 15.3$, $E_1$ and $E_2$ were found to be 0.07 and 0.32 respectively. As $J$ was increased to 59.6, $E_1$ and $E_2$ were found to be 0.067 and 0.182 respectively. Incidentally, Kamotani and Greber [14] showed that for a single jet, a decrease in the channel height did not have a considerable affect on the trajectory of the jet unless the opposing wall was brought so near that the jet impinged on its surface. They also show that increasing momentum ratio caused increased penetration of the trajectory into the cross flow.

In addition to the above statements concerning perpendicular jet injection, numerous other studies have been done using a wide array of experimental techniques and theoretical approaches. As an overview, some of these will be stated here. Chien and Schetz [7] actually took the time to solve the three-dimensional Navier-Stokes equations and energy equation written in terms of vorticity, velocity, and temperature. Specifically, they solved the single jet perpendicular injection into a cross
flow problem. Comparisons with a laminar cross flow experiment did show good agreement. However, when modifications were made to more closely model a turbulent buoyant jet, less promising results were seen. Analysis showed that trajectory information was quantitatively acceptable, but the three-dimensional information was only qualitatively acceptable. Inaccuracies were blamed on the simple turbulence model. Campbell and Schetz [6] also produced a three-dimensional analysis for the jet in a cross flow. Their scheme for predicting trajectories included buoyancy forces and entrainment terms in the momentum equation perpendicular to the trajectory. Finally, one of the only investigations performed on a non-uniform cross flow was done by Sucec and Bowley [28]. This investigation was not detailed combining drag and entrainment concepts into a single force. This approach uses the Abramovich idea that the jet can be treated as a cylinder in a cross flow.

As can be seen, then, little work pertains directly to flow within the reverse flow combustor by the nature of its odd geometry. However, the work described here does allow one to make some intelligent judgements as to what might be expected.
2.3. **Two-Dimensional Jets and Multiple Jets**

In a jet engine combustion chamber, it is typical for jets to be injected in rows that normally run perpendicular to the length of the combustor. In this work, the study of multiple jet injection is conducted. Therefore, it becomes important to scan quickly some related jet investigations to get a notion of what to expect.

As spacing between a multiple number of jets in a row decreases, the behavior of the row becomes increasingly similar to that of a two-dimensional slot jet. Albertson\(^2\) showed both theoretically and experimentally, that in the case of a free slot jet, the velocity is a function of the reciprocal of the square root of the distance from the jet:

\[
V_j = \frac{1}{s^{1/2}} \quad (II.9)
\]

Also, spreading of the jet is a function of the square root of the distance from the jet injection point:

\[
b = s^{1/2}. \quad (II.10)
\]
This behavior is quite different from that of a single round jet described above. Lastly, Albertson\cite{2} gives an expression for volume flux as:

$$\frac{q_j}{q_{j0}} = 0.62 \left[ \frac{b}{b_0} \right]^{1/2} \quad (II.11)$$

In searching for literature on rows of single jets, it was found that all of the work was experimental in nature. Empirical correlations generally accompanied the experimental results. This type of work is inevitably useful only for similar flow configurations, since the governing correlations do not normally include a provision for different cross flow geometry and injection geometry.

One of the most important results from the multiple jet investigations is the concept of vortex interaction between adjacent jets. Vortex interaction causes a downward movement of the jet trajectory. However, in the limit where the spacing between adjacent jets becomes very small, the vortex formation is limited, hence the downwash is less. Experiment shows the deteriorating trajectory is followed by a higher trajectory as the spacing
continues to decrease. In the following diagram,

![Diagram of vortex interaction](image)

**Figure 10:** Schematic of vortex interaction

Each kidney shape represents a single jet. Each shape contains two arrows indicating the vortex pair associated with each jet. Upon examination of the effects on jet two above, one can see that vortex 1B has a downward effect on jet 2. Vortex 1A, however, has an upward effect, but since the distance from vortex 1A to jet 2 is larger than the distance from vortex 1B to jet 2, the resultant effect is a downward tendency. The effect of the vortices associated with jet 3 on jet 2 are identical to those of jet 1 on jet 2. It must be noted, however, in the limit as one decreases the spacing between the jets, the vortex formation is hindered and the effect described above becomes minimal. During injection into a confined cross flow, as one
decreases the spacing ratio and approaches the geometry of a two-dimensional slot jet, a second mechanism that has a downward effect on the jet trajectory is defined. As the injected flow becomes increasingly two-dimensional, the cross flow must pass over the top of the jet and not between individual jets. The resultant effect is a suppressed trajectory to allow the cross flow sufficient area.

2.4. Reverse Flow Combustor Experiments

One previous study on jet injection in a reverse flow combustor is available. After the presentation of descriptions of the physical facility, experimental procedures, and relevant parameters, conclusions from this study will be summarized. In addition, in the presentation of data from this work, trajectories from the model developed in the previous work will be shown and compared to experimental trajectories.
3.1. **Apparatus**

For this experimental project, data was taken from a 90° subsection model of a reverse flow combustor. This model combustor was designed and built exclusively for the purpose of performing dilution jet injection experiments. Dimensions of the combustor are similar to full-size rigs in operation. Figure 11 shows a schematic of the experimental rig fully dimensioned. From this schematic, the aspect ratio of the apparatus, defined as:

\[
AR = \frac{\text{Mean Combustor Arclength}}{\text{Combustor Channel Height}}
\]

is calculated to be 7.125 for the primary zone (that section downstream of the fuel injection but upstream of the turn) and 7.00 at the exit. With such aspect ratios, it is expected that at the centerline of the combustor, no secondary flows with significance will exist. The primary zone of the device is constructed from Inconel-750X super
alloy, allowing wall temperatures of up to 1200°F (650°C) without significant warpage. The turn section is constructed from split 90° stainless steel elbows. Figure 12 shows a photographic view of the entire set-up including supporting apparatus required for device operation and data acquisition.

The fuel used in this model combustor is natural gas. As is typical in using natural gas for fuel, air is used as the oxidizing agent in the combustion process. In addition, air is used as the primary cooling agent as well as the dilution jet cooling agent. Consequently, all of these air requirements are taken from the same source, a centrifugal blower that develops one psig while delivering two lbs. of air per second. The natural gas is supplied at 0.4 psig.

The injection points of the combustor exist in four distinct rows, three on the outer wall and one on the inner wall. The spacing between the dilution jet ports is different for each row. Each row is placed perpendicular to the length of the combustor on rays extending from the combustor centerline. Each row contains 21 ports each with an initial nozzle diameter of 7.11 mm.

The combustion process occurs 432 mm upstream
Figure 12: Photographic view of experimental apparatus
of the 180° bend in the combustor. At this point, nine burners are situated that deliver a mixture of natural gas and air that is slightly fuel rich. The mixing of air and natural gas for the premixed flame occurs outside the combustor. Although each natural gas burner has an adjustment to vary the quantity of fuel delivered, no such convenience exists for the combustion air delivery. It is assumed that an equal amount of air is delivered to each burner from the manifold-like apparatus that is attached to the end of the combustion air delivery pipe. Within the combustion chamber, the primary cooling air is delivered from a series of small holes that encircle each burner. It is thought that the hot combustion gas mixes with the primary cooling air as it moves down stream. Once again, there is no indication whether or not each primary cooling air delivery station is experiencing the same mass flux from the cooling air manifold. It may be seen later that this lack of information may result in a skewed temperature or velocity profile in the combustor before the turn and without injection.

As was stated earlier, all of these requirements are met by using a centrifugal blower.
This centrifugal blower delivers air to four pipes in a network. Each pipe is equipped with an orifice meter to determine its flow rate. The natural gas delivery is monitored in the same manner. Using an ASME[3] standard on orifice meters, calibration curves previously determined were verified. The pressure taps used in the orifice meters most closely approximate the corner tap type. The orifice plates used are sharp edged. The following table shows pipe and orifice sizes used for the experiment:

<table>
<thead>
<tr>
<th></th>
<th>Pipe (in.)</th>
<th>Orifice (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combustion Air</td>
<td>4.297</td>
<td>1.400</td>
</tr>
<tr>
<td>Cooling Air</td>
<td>4.297</td>
<td>2.418</td>
</tr>
<tr>
<td>Dilution Jet Air</td>
<td>4.297</td>
<td>0.904</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>2.255</td>
<td>0.502</td>
</tr>
</tbody>
</table>

The procedure used to verify the calibration curves uses empirical information. Required information for this particular experiment includes inlet pressure before the orifice plate, specific weight of the flowing gas, orifice to pipe ratio, and a correlation coefficient taken from ASME[3]. Each of the orifice meters were calibrated in this way, and each matched within 0.5% the calibration.
equations found in the previous experiment. Additionally, the following calibration equations represent the resulting curves:

\[
\begin{align*}
\text{Combustion Air (Kg/s)} & = 0.0030(h_w,\text{comb})^{1/2} \\
\text{Cooling Air (Kg/s)} & = 0.0094(h_w,\text{cool})^{1/2} \\
\text{Dilution Air (Kg/s)} & = 0.0012(h_w,\text{dil})^{1/2} \\
\text{Natural Gas (Kg/s)} & = 0.0003(h_w,\text{natgas})^{1/2}
\end{align*}
\]

In the above equations, the values of \( h_w \) are the manometer readings and must be given in \( \text{mm of water differential} \).

In a logical progression, the next topic to be discussed is the data gathering capability of the system. As shown in Figure 13 below, a rake of probes is attached to a rotating and traversing mechanism.

Figure 13[20]: Cutaway sketch of the combustor
Using figure 13 as a guide, a radial movement within the combustor is from wall to wall as shown by the arrow placed beside the five-probe rake. Azimuthal movement is an angular displacement through the turn as shown by the curved arrow.

The rake of probes consists of 10 distinct probes at five locations (spaced at 3.57 initial jet radii apart). As can be seen in figure 14, the probes are located 0.50 inches (12.7 mm) apart giving a total rake width of 2.00 inches (50.8 mm). At each location exists a pitot-static tube, 0.167 inches (4.24 mm) in diameter for the measurement of total and static pressure from which velocity can be calculated. Tack-welded to each pitot-static tube is a chromel-alumel thermocouple measuring 0.01 inches (0.254 mm) in diameter for temperature measurement of the flow.

Total and static pressure values from each tube are routed to five individual pressure transducers. The transducer used is Setra Systems High Accuracy model 239 for pressure ranging from 0 to 0.2 psid. An excitation voltage of 24 volts is required to operate each device. The output of the transducer ranges from zero to five volts and is directly proportional to the input delta pressure.
Appendix 2 gives full details of the device by presenting a factory specification sheet.

Figure 14: Pitot-Static/Thermocouple Rake
Each thermocouple is routed to a switching box and ultimately to a model 400A Doric Trendicator digital temperature measuring device. The Doric Trendicator is equipped with an analog linearizing circuit. This circuit reads the LED display of the device and outputs a voltage in millivolts identical to the number on the display.

All five transducer outputs and the five analog temperature signals are then routed to an IBM personal computer, model XT. Since the signals are analog, it is first necessary to process them through an analog to digital conversion system. The system used in this case is the Labmaster model manufactured by Tecmar, Inc. This particular board allows 12 bit accuracy. In addition, it is possible to channel the input signal through a programmable gain so that it remains as near full scale as possible for the most accurate conversions. The software used to take and process the data as it is converted or typed at the keyboard was written entirely by this author. Additionally, programming to output information as plots or in formatted form was written by this author.

The system also has some supporting apparatus that should be mentioned here. A major component
that was designed and built is a safety mechanism. Its basic purpose is to halt the flow of natural gas by closing an in-line electric gas valve when flame blow-out is detected. The schematic below describes the apparatus:

![Schematic diagram of safety control mechanism](image)

**Figure 15: Safety control mechanism schematic.**

Through the use of an operational amplifier, a comparator circuit was constructed. One input to the op-amp is the voltage generated across the junction of a thermocouple. The other input is an adjustable voltage for comparison. When the thermocouple voltage falls below that set by the adjustable one, the op-amp shuts down. This, in turn, cuts current to the relay that controls the
power to the natural gas valve thereby closing it.

Three DC power supplies are used in this experiment, as well. One is used to supply power to the safety control mechanism. The other two are mounted in the transducer panel supplying the required excitation voltage for them.

3.2. Experimental Procedures

When one wants to run an experiment using the model reverse flow combustor, one simply operates through a computer and a series of computer programs. This series of programs can be viewed in Appendix 4. Found in Appendix 3 is a detailed version of the experimental procedure including crucial start-up and shut-down procedures. The first general purpose of these programs is to create a means by which to acquire and store pressure and temperature data and calculate velocities. Secondly, proper storage gives the ability to later retrieve this information and print it so that each experiment can be properly and easily identified.

With the IBM XT at the experimental site, at the outset of each run, one must input relevant information via the keyboard. This information typically characterizes the experiment in progress.
Specifically, the following must be identified:

1. Filename
2. The row in which jets appear, if any
3. The number of dilution jets, if any
4. The number of unused jets between operating jets
6. Combustor pressure
7. Barometric pressure
8. Combustion air flow rate
9. Primary cooling air flow rate
10. Dilution jet air flow rate
11. Natural gas flow rate
12. Cooling jet temperature
13. Cross flow temperature
14. Six wall temperatures

The software saves these values for future reference. Additionally, the software performs all the necessary conversions to metric units as well as placing the manometer readings into the corresponding calibration equations. After displaying each calculated value, a provision is made for adjustments to the subsequent operating conditions. If any are required, all calculations are re-performed. Last, the software then sets up the data acquisition procedure. Automatically, five delta-pressures are taken from the five pitot-static tubes while temperature data is taken one thermocouple at a time. After the set is taken, velocity is calculated and appears immediately on
the CRT, and the whole group is sent to the storage file.

Since the azimuthal spacing increment through the combustor is chosen at the time of experiment, there is not a fixed number of data point locations. As an example, however, if one chooses 20° as the azimuthal increment (this was by far the most commonly used increment in this investigation), a total number of 76 rake locations results. With five probes on the rake, this generates 380 data points. In the chapter on presentation of data, these locations are identified in detail. In addition, data point averaging and standard deviation calculations will be discussed.

In the above discussion, radial rake movement was defined. Figure 16 points out that this radial movement is not really radial with respect to the center of curvature of the inner wall or the outer wall of the combustor. This is a result of the probes extending into the flow 13 mm.

3.3. **Experimental Preliminaries**

As with any scientific investigation, there are a group of important quantities that tend to characterize the experiment. In this case, of
course, pressure and temperature measurements are taken so that detailed temperature and velocity fields can be examined within the combustor.

![Combustor Coordinates and Measuring Stations](image)

**Figure 16: Combustor Coordinates and Measuring Stations [20]**

However, in addition, there are governing quantities that provide uniqueness for each given procedure. The first of these quantities is the density ratio.
Dr, defined as:

$$Dr = \frac{\rho_{jo}}{\rho}$$

and is usually referred to as the ratio of the density of the injection fluid to that of the cross flow. In this investigation, this non-dimensional parameter typically is about 2.15 or 2.75. These two distinct values of Dr arise from using two distinct cross flow temperatures. Specifically, about 710°F (650 K) and 1020°F (820 K) are the common cross flow temperatures. The dilution jet injection occurs at room temperature, generally somewhere near 70°F (300 K). From this information, densities of both gasses are calculable through the ideal gas equation.

For each density ratio, experiments can also be characterized by momentum ratio, $J$, defined as:

$$J = \frac{\rho_{jo} V_{jo}^2}{\rho V^2}$$

This quantity is typically referred to as the ratio of the momentum associated with the jet initially to that of the cross flow. It typically ranges from zero (no injection) to about 11.7. The
cross flow velocity for this rig ranges from eight to 12 m/s while jet injection velocities are normally between 12 and 22 m/s.

Thirdly, a characterizing parameter, injection jet spacing ratio, $S_r$, is defined as follows:

$$S_r = \frac{\text{Distance between jet centerlines}}{2b_o}$$

This variable in the operation of the combustor allows one to examine the effect of bringing individual jets closer and closer together. In addition to this variable, there is a variable concerning injection jet location. Injection can occur from the inner wall (the wall with the smaller radius of curvature in the turn section) prior to the bend or from the outer wall prior to the bend or in the bend.

With these characterizing parameters, the analysis of the acquired data is more sensibly accomplished since detailed information is available as to how the dilution jets compare with the cross flow.

With the acquisition of temperature, total pressure, and static pressure at five probe locations, local velocity is calculated as follows:
This relation for velocity is found from the Bernoulli equation. Of course, this assumes that the measured flow is incompressible and suffers from no frictional losses. In this investigation, Ma is typically 0.02 for the cross flow and 0.05 for the jet. With this information, it can be said that the compressibility effect on the accuracy of the velocity measuring device is negligible for this purpose. Also, since the Ma is much less than one, it can be said that there is never a significant deviation between impact and static temperatures.

One can now effectively characterize an experiment in the reverse flow combustor and have confidence in the quantities being measured.

Presentation of the data from this experiment is done in a variety of plots. Information plotted on them are generally normalized temperature and velocity profiles. The normalization for temperature is derived from the pattern factor Pf, a parameter used to examine temperature non-uniformity within combustion chambers. The pattern factor is defined as:

\[
V_1 = \left[ \frac{2(P_T - P_s)}{\rho_1} \right]^{1/2}
\]
The normalized temperature relation used here is defined as:

\[ Pf = \frac{(T_{\text{max}} - T_{\text{ave}})}{(T_{\text{ave}} - T_{j0})} \]

The normalized temperature relation used here is defined as:

\[ \tau = \frac{(T_1 - T)}{(T_{j0} - T)} \]

The normalization of velocity is as follows:

\[ \gamma = \frac{V_1}{V} - 1 \]

The normalization of velocity is as follows:

This normalization gives the deviation from the cross flow velocity anywhere in the combustor. Cross flow conditions are the temperature and velocity one finds at the first azimuthal measuring station, midway between the outer and inner walls. Figure 16 shows that the first azimuthal measuring location is the one immediately before the turn section.

For purposes of plotting simplicity, the combustor sketch shown in figure 16 was unwrapped, such that the inner wall became a straight line, and the outer wall became a mathematical curve moving closer to the inner wall depicting the area decrease that occurs in moving through the combustor. Shown in
Figure 17 is a set of radial normalized temperature profiles characteristic of this combustor at the lower Dr with no dilution jet injection taken from the center probe of the rake. The numbers along the bottom curve (the outer wall) are the azimuthal abscissa locations corresponding to those in Figure 16. Looking closely at the first azimuthal test location, one can identify nine test points that move radially from the inner wall to the outer wall. The one nearest the inner wall is radial position 95, the next one toward the outer wall is 85 and so on until one arrives at that point closest to the outer wall which is labeled 15. As one moves azimuthally through the combustor, one can see that the number of radial positions decreases. At the exit, for instance, only five radial positions exist: position 95 at the inner wall through position 55 at the outer wall. The area decrease can be seen by noticing that the outer wall has moved significantly closer to the inner wall. More specifically, there are nine equally spaced (1.56 initial jet radii) radial positions at azimuthal stations one through five. Azimuthal station six has eight radial positions, azimuthal station seven has seven, azimuthal station eight has six, and
azimuthal stations nine and ten have five radial positions.

At the top of the plot is a set of information that characterizes the particular plot. In this case, we see Dr = 2.23 identifying the cross flow temperature, and a zero J due to the fact that there is no injection, and an Sr labeled "no injection" to avoid confusion.

Upon examining the profiles themselves, one can see that at each azimuthal station, the profile can move to the right or the left of the abscissa (lines drawn out by the probe tips during radial movement). Moving to the right indicates a temperature that is greater than the cross flow temperature. Moving to the left of the abscissa indicates that the local temperature has fallen below that of the cross flow (and r has become positive). In the case of jet injection, it is not uncommon to observe profiles to the extreme left of their respective abscissae.

Examining figure 18, once observes a set of normalized velocity profiles for the same flow conditions as those for the temperature profile in figure 17. The plot is identical except for the magnitude of unity for the normalized quantity. In this plot, movement to the right of the abscissa
indicates a velocity that is greater than the cross flow velocity. Conversely, movement to the left of the baseline indicates a velocity magnitude less than that at the cross flow location.

Figures 19 and 20 make use of the five-probe rake. These plots are referred to as lateral representations of the flow conditions. Both figures 19 and 20 indicate normalized temperature profiles, but these can be made for velocity as well. In the case of figure 19, represented is the lateral span of the rake at one particular azimuthal location and one particular radial position. In figure 20, a three-dimensional view is constructed by using all the radial positions at a given azimuthal station. As can be seen on these plots, all the characterizing information is present including data locations.

Each '+' sign shown on the plots is a data point. Each data point, as discussed earlier, is constructed from various temperature and pressure information. In this experiment, every temperature and pressure measurement is actually an average of 25 samples. The analog to digital conversion software is set up so that 25 samples are taken, the average calculated and stored and the standard
MOMENTUM RATIO: 0.00
DENSITY RATIO: 2.23
SPACING RATIO: NO INJECTION
FILENAME: \datfiles\d1000

NORMALIZED VELOCITY = 1

Figure 18: γ, No Injection.
deviation calculated and presented. This method of data acquisition allows the experimenter to obtain a reasonable average from fluctuating conditions. In addition, the calculation of standard deviation is helpful in the error analysis. Also, a growing standard deviation indicates some problem with the data gathering apparatus and allows one to search out the problem before it propagates through all the data.

3.4. Presentation of Expected Error

Using a conservative error analysis clearly documented in detail in Appendix 5, the following errors are expected. The values listed are percentages of the indicated quantity.

Temperature, $T \pm 1.2\%$

Pitot-static pressure, $P_T - P_s \pm 4.2\%$

Velocity, $V \pm 2.7\%$

Normalized temperature, $\tau \pm 6.5\%$

Normalized velocity, $\gamma \pm 6.6\%$

Momentum ratio, $J \pm 18\%$

Density ratio, $Dr \pm 6.4\%$

On the data plots, the error in $\tau$ corresponds to $\pm 3$ data point widths. The error in $\gamma$ corresponds to $\pm 0.25$ data point widths.
Figure 19: Representative Lateral Plot of T, Outer Wall Prior to Bend, 0°.
**Figure 20**: Representative 3-D Lateral Plot of T, Outer Wall into Bend, 60°.
CHAPTER IV

CONCLUSIONS FROM PREVIOUS REVERSE FLOW COMBUSTOR STUDY

Prior to presenting data and conclusions from this investigation, a summary of the conclusions found from the previous temperature study [18] is given below. As a presentation of the model formulated in the previous work, calculated dilution jet trajectories are shown along with the experimental trajectories of this work where possible in the next chapter.

The summary is as follows:

1. Increasing density ratio gives rise to a deeper penetration of the jet into the cross flow.

2. Increasing momentum ratio gives rise to a deeper penetration into the cross flow.

3. Inward drifting phenomenon identified due to the nature of the flow accelerating along the inner wall.

4. Longitudinal acceleration suppresses single jet thermal spreading rate.

5. Confinement effect, measured by the ratio of channel height to initial jet diameter, suppresses trajectory as spacing ratio
6. Smoothing of radial temperature gradients occurs far before the exit.

7. Tightly spaced row of jets injected from the inner wall attaches to that wall, shielding it from the outer hot stream. Injection from the outer wall penetrates deeply into the cross flow and does not attach to the outer wall.

8. Very low momentum flow injected from the outer wall tends to stay there through the turn.

9. Apparently, proper injection can result in a desired temperature profile.
CHAPTER V

SINGLE JET INJECTION DATA AND CONCLUSIONS

In this chapter, a representative group of single jet injection configurations will be examined in detail by looking closely at radial and lateral plots. To obtain a representative group, one must look at injection from the inner wall before the bend (14.67 initial jet radii upstream, point A in figure 16), injection from the outer wall before the bend (14.28 initial jet radii upstream, point B in figure 16), and injection from the outer wall into the bend (5.53 initial jet radii into the bend, point C in figure 16).

Enough information must be examined so that trends associated with changing momentum and density ratios become evident. In this section, spacing ratio is not important.

Below, find observations from individual tests with conclusions drawn from all of them at the close of the chapter.

5.1 No Injection

To properly analyze the effects of dilution jet
injection on flow conditions in the combustor, one must first examine the results associated with no injection. In figure 17, one can see that temperatures seem to be skewed near the inner wall just prior to the bend. Calculating the pattern factor for this position, one finds that the value never exceeds 0.07. Comparing this value to the typical value of 0.2 found in the exit patterns of operational combustors\textsuperscript{[18]}, it can be said the the oncoming cross flow has a relatively flat profile.

With movement downstream, one sees a decrease in normalized temperature, indicating that temperatures are falling below that of the defined cross flow. This decrease in temperature is attributable to heat losses through the combustor walls.

Examining the normalized velocity profiles presented in figure 18, the immediately obvious feature is that the radial profile at the first azimuthal station is quite uniform. As one moves downstream, an increase in velocity is detected from inner wall to outer wall. In addition, an increase in velocity at the inner wall with respect to the outer wall is observed. This phenomenon is attributable to the fact that the inertially
dominated flow attempts to negotiate the turn like an irrotational one. For this to occur, a greater acceleration at the inner wall with respect to the outer wall is necessary. It is also thought, that due to this greater acceleration, a drifting inward of mass is expected to satisfy conservation of mass. Lastly, pressure must increase from inner to outer wall to support the turning fluid.

Generally, to identify a temperature trajectory of cool injected fluid, one connects the minimum temperature at one azimuthal position to the minimum at the next. However, in doing this, one must be careful to consider the profiles with no injection to avoid choosing an inappropriate maximum $r$. In identifying velocity trajectories, one connects the maximum velocities. In this case, one must clearly consider the increasingly skewed profiles that appear through the turn section. Typically, during injection, one sees a velocity decrease downstream of the wake created by the "cylinder" of fluid that obstructs the flow. As one moves from the injection wall, a jump in velocity is detected indicating probe movement out of the wake and into a region where velocities become greater than those expected with no injection.
5.2. **Low Momentum Injection--Outer Wall--Prior to Bend**

Examining figure 21, one can see that at azimuthal station one, all values of $\tau$ are affected except the two nearest the inner wall. These positions are showing a temperature decrease, and hence, a $\tau$ increase. Radial position 65 shows the maximum $\tau$, thereby qualifying for the trajectory location.

At azimuthal station two, a significant temperature decrease occurs at radial positions 45 through 95 with a maximum $\tau$ occurring at 75. This shows movement closer to the inner wall.

At azimuthal station three, significant temperature deterioration occurs at positions 65 through 95 with maximum deviation occurring at position 85. Azimuthal location four shows lower temperatures at the inner wall with maximum $\tau$ occurring again at position 85, showing the thermal trajectory quite close to the inner wall.

Azimuthal stations from this point to the exit show temperature decreases at the inner wall region, but no maxima can be identified. It can be said qualitatively that the injection jet does migrate toward the inner wall even at these locations, but
due to mixing in the radial direction, a trajectory is no longer meaningful.

Figure 22 shows the normalized velocity plot for this case. As indicated earlier, at the injection point, the jet acts as a flow obstruction to the cross flow, causing a low pressure, low velocity wake downstream. As one exits from the wake, one expects to experience a jump in velocity followed by some velocity distribution within the spreading jet. At the first azimuthal measuring station, one observes a significant decrease in velocity at radial positions 15 through 45. The jump is observed at position 55 with the maximum occurring at position 75.

At azimuthal station two, the jump is observed again, with maximum occurring at position 75. This choice for the trajectory location is a little less obvious. Position 75 also shows the maximum positive deviation from that for the no injection situation.

After azimuthal position two, some increased velocities can be detected but trajectories are no longer definable. In comparison to the temperature profiles, these show a much less drastic change due to injection. Trajectories are definable for a
lesser distance downstream as well. When one considers a free jet as a very crude approximation, one sees a deterioration in velocity proportional to the inverse of the distance from the virtual origin, giving velocities at the start of the turn section already much less than that of the cross flow. With this in mind, these velocity deviations seem reasonable.

5.3. High Momentum Injection—Outer Wall—Prior to Bend

Examining the temperature profiles for high momentum injection from the outer wall \((J = 9.74)\), one observes that all the radial positions of the first azimuthal station are showing a temperature decrease. The maximum occurs at radial position 65, qualifying this as the trajectory position.

Radial position 85 shows the maximum temperature decrease at the second azimuthal station, while the brunt of the cooling seems to be at radial positions 45 through 95, near the inner wall. The indication here, again, is that the thermal trajectory is moving toward the inner wall.

At azimuthal station three, radial positions 65 through 95 indicate cooling due to injection. In
this radial set however, the maximum occurs at the radial position nearest the wall (95). This is somewhat suspect, but when comparing this profile to that for no injection, one can see that the temperature deviation is also greatest at this position. This verifies that with the higher momentum ratio, injection from the outer wall does in fact migrate to the inner wall. Downstream of azimuthal position three, one observes that cooling is more evident at the inner wall, but due to mixing, trajectory locations are no longer apparent.

Figure 24 gives the normalized velocity profiles for this injection configuration. Examining the first azimuthal location, the same wake phenomenon is evident. Wake flow conditions occur until a jump in velocity is detected at radial position 65. From 65 to 95, flow velocities are higher than expected for the no injection configuration. Maximum occurs at position 75, the trajectory location.

Azimuthal position two exhibits a small jump in velocity near the inner wall at position 85. Already at this position, a maximum is not really detectable, nor is a maximum positive deviation from the no injection profile. The velocity trajectory.
then is near the inner wall at this point, but is not technically definable.

Azimuthal stations three, four, and continuing downstream have smoothed so that no velocity deviation is detectable.

5.4. Low Momentum Injection--Inner Wall--Prior to Bend

Examining the temperature plots shown in figure 25 for inner wall injection at \( J = 5.80 \), one sees at azimuthal station one, a large temperature decrease in all radial locations but the one nearest the outer wall. Maximum \( v \) occurs at radial position 65, labeling that position as the trajectory location.

Azimuthal station two exhibits a similar variance from the no injection profile with the greatest temperature deviation occurring at radial position 75, near the inner wall. This indicates that after a penetration into the cross flow, the jet begins to migrate back toward the inner wall.

Continuing with this thought, as one examines azimuthal positions three and four, one finds that maximum \( v \) occurs at radial position 85 in both cases. This shows a continued movement of the thermal trajectory back toward the inner wall with movement downstream.
Further downstream, a definite cool region exists at the inner wall for quite a long distance. In fact, more cooling at the inner wall with respect to the outer wall is evident even at the exit. Apparently, mixing at the inner region is less than that at the outer wall.

Examining the plot of $\gamma$ shown in figure 26, one should be aware of the profile for no injection. Upon observance of azimuthal station one, the expected wake is evident from the inner wall out to radial position 55. Recall that for injection from the outer wall, the wake region was located from the outer wall toward the center. The expected velocity jump does occur at radial station 45. This station also serves as the trajectory location since it is also the local maximum $\gamma$.

Azimuthal station two indicates a similar profile as above. Once again, the wake effect is evident at the inner wall from radial positions 55 to 95. At position 45, one sees a peak velocity and one that qualifies for the trajectory location.

After this point, smoothing of the velocity profiles causes difficulty in identifying trajectory locations. A quick observation here is that in the inner wall injection configuration, velocity
trajectories tend to deviate more from the temperature trajectories than in the outer wall injection case. Also, the velocity trajectory does not migrate as perceptably toward the inner wall as the temperature trajectory.

5.5. High Momentum Injection--Inner Wall--Prior to Bend

Figure 27 indicates at azimuthal position one a significant temperature deterioration near the inner wall. In this case, the maximum occurs at radial position 55, the first trajectory location. In this higher momentum case, all of the temperatures along the station have been lowered, giving rise to the idea of greater penetration due to greater momentum ratio.

Temperature profiles at azimuthal station two show a similar behavior but not quite as skewed as station one. Here again, all temperatures seem to be affected, but not to the same degree as in station one. The trajectory location is at radial position 55.

Azimuthal station three begins to flatten somewhat, but a maximum is still observable at radial position 65. Even azimuthal station four shows a maximum, also at position 65. After this
position, considerable cooling is seen throughout the combustor, although cooling at the inner wall is much more pronounced. This set of profiles shows that even at this elevated momentum ratio, a migration toward the inner wall is evident after a deep penetration into the cross flow.

Radial velocity plots shown in figure 28 for this injection configuration show behavior consistent with the temperature plots. Upon close examination of the initial azimuthal station, the lower wake velocity is observed at radial positions 55 through 95. At location 45 a jump in velocity occurs with \( \gamma \) peaking at position 35. Position 35 is marked as the velocity trajectory position.

At azimuthal station two, the expected velocity jump is seen near position 45 with the peak in \( \gamma \) occurring at radial position 35, clearly marking it as the trajectory location.

Downstream of azimuthal station three, the profiles begin to flatten out and take the form of those seen with no injection: a positive velocity gradient sets up from outer to inner wall.

5.6. Low Momentum Injection—Outer Wall—Into the Bend

This injection configuration differs from all
those presented earlier in that it delivers dilution cooling air from the outer wall 5.63 initial jet radii downstream of the start of the turn section.

Upon examining figure 29, the most obvious change is that the first two azimuthal measuring locations are nearly unaffected by the upstream injection.

Dramatically at station three, there exists a highly skewed profile with maximum $\tau$ occurring at radial position 35. Radial positions 55 through 95 show almost no temperature change whatsoever. This shows that penetration here is limited in comparison to the upstream injection locations. Obviously, maximum $\tau$ and the trajectory location are at radial position 35.

Azimuthal location four shows significant temperature reduction at radial position 55, the recognized trajectory position. Note here that temperatures at both walls remain near those observed with no injection. Little spreading of the jet has occurred by this azimuthal location.

Azimuthal position five, however, begins to show significant cooling along the inner wall, while the outer wall remains at its previous temperature. This serves as yet another confirmation of the
migration toward the inner wall, even with a relatively weak initial penetration. At this location, one can observe maximum $\gamma$ and consequently the trajectory location at radial position 75.

Surprisingly, at azimuthal station six, a trajectory location can visually be located at radial position 75. In addition, significant cooling can be observed from radial positions 35 through 95. After this azimuthal location, cooling along the inner wall is clearly more prominent, but trajectory locations are no longer identifiable.

Figure 30 displays the normalized velocity profiles for this low momentum injection from the outer wall into the bend. First, no effect is noticed at the first two azimuthal locations. At location three, one sees the typical velocity decrease at radial position 15 due to a flow disturbance at the injection point. At radial position 25, a marked jump in velocity to above what is expected in the no injection configuration is observed. Although no maximum $\gamma$ is seen, it is likely that the velocity trajectory location is near the inner wall.

Azimuthal position four and those further
downstream show no evidence of velocity trajectories, with the profiles resuming the typically irrotational one of the no injection plot.

5.7. High Momentum Injection--Outer Wall--Into the Bend

The temperature profiles shown in figure 31 present results for \( J = 9.81 \). Just as in the case of low momentum injection into the turn section, no detectable change occurs at the first two azimuthal stations while the third sees a major temperature decrease. Positions 65 through 95 appear unaffected at station three, with skewing beginning at station 55 and the maximum \( T \) occurring at position 35. Note that this trajectory, as well as all the others, is marked on its associated baseline.

Azimuthal station four shows marked skewing as well, with its maximum \( T \) occurring at radial position 75, showing a definite migration of the high momentum injection toward the inner wall.

Azimuthal locations five and six show an increase in \( T \) as one moves toward the inner wall. In both cases, maximums occur at the radial position nearest the inner wall (95) or at most, one position away from the inner wall (85). Again, this is strong evidence suggesting migration toward the
inner wall.

Figure 32 shows the normalized radial velocity profiles for high momentum injection into the bend. As is true with all the velocity profiles, variations are somewhat subtle. At azimuthal station three, the jump in velocity and maximum Y occur at radial position 25. This, then, is the location of the trajectory.

Examining azimuthal station four, one sees a marked increase in velocity near inner the wall with the jump occurring at position 75. The local maximum is observed at position 85, giving it the velocity trajectory identification. Further downstream, some velocity increase is detectable near the inner wall, but not significant enough to choose trajectory positions.

5.8. Conclusion From Normalized Radial Profiles

To this point, various representative flow configurations for single jet injection have been discussed in detail. In each subsection of this chapter, normalized radial temperature and velocity profiles were the basis of these discussions. Thermal and velocity trajectories were defined where possible. At this point, it is necessary to look at
all the profiles, and make conclusions concerning
the general behavior. In summary, the following
conclusions are evident from these profiles:
1. Without injection, temperature profiles show a
pattern factor within reasonable limits set up by
operational combustors. Generally, a low
temperature region exists along the outer wall of
the combustor due to heat transfer through the
walls. Velocity profiles show a fairly uniform
condition at the inlet to the turn section. As the
inertially dominated flow attempts to negotiate the
turn like an irrotational one, particles at the
inner wall must accelerate with respect to those at
the outer wall. This gives rise to an increase in
velocity at the inner wall with respect to the outer
wall. In addition, an area decrease through the turn
section causes an increase in velocity. As the
acceleration occurs along the inner wall,
conservation of mass calls for a movement of fluid
toward this region. Also, pressure increases as one
moves from inner wall to outer wall in the turn
section.
2. The injection of cooling air at the inner wall
before the bend, the outer wall before the bend, and
the outer wall into the bend all cause significant
changes in temperature and velocity downstream of their respective injection locations.

3. The greater the momentum ratio, the deeper the penetration into the cross flow according to both temperature and velocity profiles.

4. The greater the density ratio, the deeper the penetration into the cross flow.

5. Migration of the injection jet toward the inner wall occurs during injection from all locations. This indicates that the centrifugal effect of a heavier fluid naturally moving toward the outer wall of a turning channel is overcome by the pressure and migration effects discussed in (1).

6. Both thermal and velocity trajectories allow one to make the same statements concerning jet movement in the cross flow, although they rarely coincide for all azimuthal stations.

7. Although still qualitatively consistent, velocity and temperature trajectories show a greater deviation from each other during injection from the inner wall. The velocity profiles typically show a deeper penetration and a slower return to the inner wall than the temperature trajectories. This is due to the recirculation zone that is set up downstream of the injection location. During injection from
the inner wall the trajectory moves back through the radial positions associated with the wake region, but further downstream.

8. During injection from the inner wall, cooling and increased velocities are more apparent along the inner wall even at the exit giving rise to the conclusion that a jet injected from this wall experiences less mixing due to its position.

9. Velocity and temperature profiles at azimuthal stations nearest the exit for outer wall injection prior to the turn and in the turn are quite similar for similar injection conditions.

10. Lipshitz[18] developed a model to predict the centerline of an injected jet from conservation equations and empirical relations. Three velocity and three temperature plots from those discussed above have computed model trajectories marked on them for comparison. Agreement is poor for all cases except velocity trajectory from the inner wall. In this case, though, too few velocity trajectory locations are identifiable for a conclusive comparison.

5.9. **Lateral Distributions**

Also plotted for various azimuthal locations in
the combustor are lateral plots that make use of the five-probe rake. With these plots, one can view laterally across the combustor at given radial and azimuthal positions. In addition, one can view laterally all of the radial positions at a given azimuthal station resulting in a three-dimensional plot of combustor conditions. Ten of these plots, each for one of the ten azimuthal stations, gives a three-dimensional plot of combustor conditions for the entire turn section.

Plots of this type were generated for various azimuthal positions downstream of injection to investigate jet spreading behavior.

Upon examining the three-dimensional plots for no injection (figures 33 through 36) plotted at 60° increments from the start of the turn section to the exit, one observes that there is a non-uniformity laterally across the rake width. From prior to the turn continuing completely through the combustor to the exit one sees that this three-dimensionality takes the same form: an increase in when going from -7.14 initial jet radii to 7.14 initial jet radii across the combustor centerline. Each plotted line represents a probe location. Probe one is at the far left and is -7.14 initial jet radii from the
combustor centerline. Probe five is at the far right and 7.14 initial jet radii from the combustor centerline. Probes two, three, and four fall logically between these two.

As indicated in the radial velocity plots, combustor velocity is fairly insensitive to single jet injection more than two or three azimuthal stations (40° to 60°) downstream of the injection point. Therefore, only thermal plots will be used for investigating lateral spreading.

When examining the plots for low momentum injection from the outer wall (figures 37 through 40), the most obvious change occurs in figure 37 (first azimuthal station). Probes two and three show a significant temperature decrease, while the others remain relatively unaffected. This results in a three-dimensional look to the plot, and indicates that jet spreading is still quite limited at this point.

At 60°, the plot shows the flow returning to the condition seen in the no injection plot, where decrease in temperature is detected in going from probe one to five. However, upon careful examination, one can see a definite increase in temperature when going from probe four to five,
giving evidence that lateral spreading is not complete. The plots for 120° and 180° show that the temperature distribution has returned to one similar to that for no injection, indicating that spreading across the rake width is complete.

The set of plots for outer wall injection at a high momentum ratio (figures 41 through 44) shows similar behavior to the low momentum configuration. At the high momentum ratio, though, lateral spreading seems complete at an azimuthal value of 60°. This is faster than the low momentum configuration.

Three-dimensional lateral plots for injection from the inner wall at a low momentum ratio (figures 45 through 48) and at a high momentum ratio (figures 49 through 52) show similar behavior. In both of these cases, lateral spreading does not seem complete at 60°. By 120°, however, profiles look no different than those for no injection, indicating that lateral spreading has surpassed the width of the rake.

Finally, from the plots above, individual radial positions can be identified and plotted. Two plots with low momentum injection from the outer wall and two with high momentum injection from the
inner wall are presented (figures 53 through 56). In figures 53 and 54 one can see that the center probe temperature regains temperature in moving from 0° to 100° azimuthally. In this case, the outer probes do not detect a significant change in temperature. Figures 55 and 56 show similar plots for high momentum injection from the inner wall. In this case, the center probe temperature also shows a rebounding toward the cross flow value. The outermost probes indicate a more significant decrease in temperature this time, giving evidence of wider spreading.

5.10. Additional Conclusions From Lateral Plots
11. Lateral spreading rate increases with increasing momentum ratio. This is in agreement with the experiment by Kamotani and Greber[14].
12. Lateral spreading rate is greater for outer wall injection than for inner wall injection. This observation confirms conclusion eight above, indicating that mixing is greater at the outer wall region than the inner wall region.

In the following chapter, multiple jet injection will be considered. There, lateral plots will become important again in the determination of two-
dimensional behavior of a closely spaced row of jets and how this affects trajectory.
CHAPTER VI
MULTIPLE JET INJECTION DATA AND CONCLUSIONS

All of the introductory information that pertains to single jet injection such as definitions for \( \tau \) and \( \gamma \) as well as numerical values for pattern factors apply here as well. Once again, as the flow irrotationally negotiates the turn section, acceleration of fluid particles along the inner wall with respect to the outer wall occurs. From mass conservation, a migration of fluid inward is evident. Additionally, an increase in pressure is experienced when going from inner wall to outer wall to support the turning fluid.

As was true for single jet injection, experiments were carried out at momentum ratios from about 3.7 to about 11.7. Density ratios ranged from about 2.15 to about 2.75. With multiple jets, though, two more parameters become a part of the investigation: spacing ratio and confinement ratio. Spacing ratio, \( S_r \), defined above, varied from 2.67 to 9.21. Each injection configuration was tested in three spacing ratios: the smallest, followed by double that, followed by triple that. Values for spacing ratio differ from outer wall to inner wall.
since injection ports are positioned on rays extending from the combustor centerline.

Confinement ratio, defined as:

\[
\text{Confinement Ratio} = \frac{H_o}{b_o}
\]

is 16.77 for injection prior to the bend and 16.95 for injection into the bend.

Data acquisition procedures, plotting procedures, and trajectory identification procedures are identical to those used in single jet injection.

Upon examining the data and conclusions from single jet injection, one can see the effects of changing momentum ratio and density ratio. Therefore, it is unnecessary to present data that displays the effects of changing these parameters. Instead, injection location and spacing ratio will be the variables here. On each plot, though, two trajectories are marked. One is that for the row of jets, and the other is for the single jet for comparison.

6.1. High Momentum Injection--Outer Wall--Prior to Bend

This section contains information on three different spacing ratios for this flow
configuration. First examining the normalized temperature profile with the widest spacing ratio shown in figure 57 one discovers a spacing ratio of 9.21. The profiles are inevitably similar to those of a single jet. The trajectory location (maximum $\gamma$ is at radial positions 55, 75, and 85 for azimuthal location one, two, and three respectively. Comparing these locations to those found in single injection, one finds that the obvious difference is a shorter penetration, and a continued "lower" trajectory. Even though some suppression of trajectory is detected, a cooling region along the inner wall is evident even as far downstream as the exit.

The velocity profiles for this configuration (figure 58) are also quite similar to the high momentum single injection profiles. In this case, the velocity jump at azimuthal position one occurs at radial position 55. The maximum $\gamma$ occurs at radial position 65. Looking closely at azimuthal positions two and three, one can see that the radial positions closest to the inner wall actually show no change in velocity. However, at position two, radial location 75 is seen as having the greatest positive deviation from the no injection velocity.
At position three, radial location 75 is indicated as having the largest positive displacement from the no injection velocity. Although these positions do not stand out as definite local maximums, they are likely trajectory locations by comparison to profiles with no injection. Further downstream, one can no longer identify with any certainty any trajectory location, but a noticeable velocity increase along the inner wall is evident. From the velocity trajectory estimates in the previous chapter, one can see an apparent 'lower' penetration amounting to at least one radial position.

Remaining at high momentum injection from the outer wall, the spacing ratio is decreased to 6.14 by adding four jets for a total of 11. The question to ask now is how does this affect the flow within the combustor.

Looking at the temperature profile diagram shown in figure 59, a marked increase in $\tau$ is detected about midway between combustor walls at azimuthal station one. At radial position 55, $\tau$ peaks, giving the thermal trajectory location. Similarly, position 75 at azimuthal station two qualifies as the second trajectory location.
Azimuthal position three becomes a little more difficult to analyze since the maximum local $t$ actually occurs at the endpoint (position 95). By comparison with the no-injection configuration, it can be seen that the trajectory really cannot be pinpointed. However, positions 75 through 95 indicate the most temperature deviation. It is likely, then, that the trajectory falls somewhere between these two radial positions.

At azimuthal measuring station four, a peak is again detectable as a trajectory location at radial station 85. Marked cooling is detected downstream across the combustor with the inner wall region affected the most. In comparison with single jet temperature profiles, this spacing ratio does not produce noticeably different results from the larger Sr. The initial thermal trajectory location shows some suppression. This mild suppression seems to be carried downstream as well.

The velocity profile (figure 60) at azimuthal station one displays the typical jump in velocity at radial point 55. Position 65 is near the peak $\gamma$ and deviates the greatest from the no injection configuration, giving indication as the trajectory location.
At both azimuthal stations two and three, velocity trajectory locations are difficult to detect. However, upon careful examination, it appears that trajectories lie somewhere between radial positions 65 and 95 due to increased velocity in this region. The initial trajectory location shows evidence of some suppression by appearing one radial position lower than in the single jet case. Velocity trajectory locations downstream of the initial one cannot be identified clearly enough, so a statement concerning suppression in this region cannot be made.

Once again, keeping with high momentum injection from the outer wall, the number of jets is increased to 21, producing a spacing ratio of 3.07. This is the smallest value for this parameter for jets injected from the outer wall before the bend. Examining the normalized profiles for this case (figures 61 and 62), trajectory suppression is immediately evident.

The radial temperature profiles show a peak at radial position 25 for the first azimuthal station. The second azimuthal abscissa shows an obvious peak at radial position 45. Radial position 55 is the maximum for azimuthal station three.
These three trajectory locations marked on figure 61 show a significant suppression in comparison with the single jet case. By the third azimuthal station in the single jet case, the trajectory was identified as existing along the inner wall. For 21 jets, though, the trajectory is located only at the midpoint between the two walls.

Upon examining the velocity profiles, one does not find such a dramatic change in trajectory location. At azimuthal position one, the velocity jump occurs at radial location 35. Velocities 55 through 95 appear fairly uniform. In contrast, with no injection, velocity increases in going from 55 to 95. The point with the largest deviation from the no injection profile is position 65, identifying this as the location of the trajectory.

At azimuthal baseline two, one observes the velocity jump at radial position 55. Points 65 through 95 display an increase over the no injection case, with no particular y exhibiting a maximum. It is concluded that the velocity trajectory must fall somewhere between radial points 65 and 95, with the exact location unidentifiable.

Azimuthal station three is similar to two in behavior with the trajectory location falling
between radial positions 75 and 95. As one moves downstream, an increase in velocity is observable at all radial locations. This behavior gives some evidence of suppression, but nowhere near the magnitude of suppression displayed in the thermal profiles. In addition, with this large number of injection jets (21), the increased mass flux is indicated by the noticeably higher velocity as far downstream as the exit.

6.2. High Momentum Injection--Inner Wall--Prior to Bend

Due to a probe problem, only pressures were obtainable from the rake for multiple jet injection from the inner wall. So that some statement could be made concerning the relative behavior of multiple jets from the inner wall, temperature data was taken from Lipshitz [18]. This temperature data was then used to calculate a local density which, in turn, was used in the calculation of local velocity. Due to probe geometry differences between the previous temperature study and this study, it becomes evident that probe tip location for each case differs by two to three millimeters. It is estimated, that as a maximum, this could result in a local temperature error of 100°F near the injection point. This
results in an additional error in local velocity of about 5%, according to the procedure used in estimating error in Appendix S. The error in $\gamma$ then increases to about 19%. However, this still results in an error band on the normalized velocity profiles slightly less than the width of one data point. Therefore, qualitative trends depicted by these profiles remain believable.

Following the same procedure as earlier, using a high momentum ratio (9.68) and the low density ratio (2.18), each of the three possible spacing ratios are investigated. The injection of seven jets results in a spacing ratio of 7.41, a relatively large value. Normalized temperature profiles seen in figure 63 show that at azimuthal position one, the maximum $r$ occurs at radial position 75, indicating a shallow penetration.

Azimuthal position two shows a maximum at the same radial position. Strikingly, from azimuthal abscissa three and continuing downstream, the maximum $r$ occurs at position 95, against the inner wall. It appears as if the thermal jet trajectory, then, is directed along the inner wall as well. In comparison with the single jet thermal trajectory for a similar momentum ratio, considerable
suppression of trajectory is evident, even at this wide spacing ratio.

The velocity profiles, again in this case, do not show such a dramatic suppression (see figure 64). The velocity jump at azimuthal baseline one occurs at radial position 65, with the maximum deviation from the no injection velocity occurring at position 45.

Azimuthal station two shows similar behavior with maximum occurring at radial position 55. Station three also has a trajectory location at position 55. Downstream of position four, increased velocities across the combustor are detectable.

In comparing these velocity profiles to temperature profiles for the same set of flow conditions, one sees that they predict a much deeper penetration than their thermal counterparts. However, in comparing them to the velocity profiles for single jet injection, one sees that a suppressed behavior is evident.

Decreasing the spacing ratio to 4.94 by increasing the number of dilution jets to 11, one would expect further suppression in accordance with what occurred during injection from the outer wall. Examining the first azimuthal station of the
temperature profiles (figure 65), one observes a maximum \( t \) at radial position 75. Position two shows the maximum \( t \) at radial position 85, identifying both of these positions as trajectory locations.

Position three and those downstream show the maximum \( t \) falling against the inner wall, showing a suppressive behavior in comparison with the single jet. However, this thermal trajectory suppression is apparently no greater than the wider spacing ratio configuration.

The velocity profiles shown in figure 66 depict trajectories appearing at radial position 55 for the first three azimuthal baselines. This, again, shows suppression in comparison with the single jet, but no appreciable difference from the 7.41 spacing ratio. Also, as above, the velocity trajectories indicate a deeper penetration into the cross flow.

By using 21 cooling jets, a spacing ratio of 2.47 is achieved. Upon examining the temperature profiles in figure 67, one finds the most striking change in profiles thus far. At all azimuthal baselines, without exception, the maximum \( t \) occurs at radial position 95, indicating quite a significant suppression of thermal trajectory. Single injection of the same momentum ratio normally
produces a trajectory somewhere near the midpoint of the combustor.

Examining the velocity profiles given in figure 68 for this flow configuration, dramatic suppressive behavior is again detected. At the initial azimuthal station, velocity is seen to peak at radial position 55. Azimuthal position two shows maximum velocity occurring at position 65. Position three indicated radial location 75 while positions four and five both show radial position 85 as the location for maximum $\gamma$. As can be seen by the indicated trajectory for the single jet, these profiles, too, show suppression so great that velocity trajectory as well moves very near the inner wall.

### 6.3. High Momentum Injection--Outer Wall--Into the Bend

Again following similar procedures as above, the first configuration to be discussed is the one with the widest spacing ratio. Upon examining the temperature profile diagram in figure 69, it is easily seen that injection occurs just upstream of the third azimuthal station. A huge temperature decrease is detected at this station, with $\tau$ peaking at radial position 35.
Azimuthal station four gives the peak \( r \) at radial position 75, indicating the trajectory is moving toward the inner wall. Azimuthal station four, then, depicts radial position 75 as its trajectory location as well, while station five indicates position 85. In comparison with the single jet injection configuration, there is no perceivable difference in penetration into the cross flow as is obvious during injection prior to the turn section.

The velocity profiles in figure 70 are just as difficult to interpret as those were for single jet injection. Although not clear, radial position 25 is the likely location for the velocity trajectory at azimuthal position three. Even less clear, azimuthal position four indicates a likely trajectory position as 75, the position with the maximum deviation from the no injection configuration. In agreement with the thermal trajectories, these do not indicate any suppressive behavior.

Adding four jets to the earlier seven, one achieves a \( Sr = 6.00 \). At the injection locations prior to the bend, a decrease in spacing ratio generally led to a suppression of the jet trajectory in the cross flow. Upon examining the normalized
temperature profiles in figure 71, one sees the skewed temperatures at azimuthal station three. Incidentally, both temperature and velocity appear unchanged upstream of injection throughout the course of multiple jet injection into the bend.

Azimuthal station three shows radial position 35 serving as the trajectory location by exhibiting the maximum \( T \). Position four also shows quite a skewed profile with its maximum \( T \) appearing at radial position 65. Both azimuthal positions five and six show trajectory locations at radial position 75. With this trajectory, suppressive behavior is becoming evident.

The velocity profiles (figure 72), as in the past, show nowhere near the dramatic change seen in the temperature profile. At azimuthal station three, the maximum local \( v \) occurs at radial position 25. The next azimuthal station shows a clear trajectory location at radial position 75. Downstream of this location, it becomes impossible to define a trajectory. With the trajectory located as it is, however, a slight suppression of approximately one radial position is observed. Although some suppression is evident, migration of the jet toward the inner wall remains intact.
The final spacing ratio at this injection location is 3.00. It is achieved by injecting 21 jets. In figure 73, one sees the first thermal trajectory location at azimuthal station three is found at radial position 25. Azimuthal station four shows the maximum $\gamma$ at radial position 45. Downstream of this location, a trajectory is no longer definable. Although cooling exists across the combustor, one sees easily a marked increase in cooling at the outer wall. The trajectory pointed out by this set of data shows a definite suppression toward the outer wall.

The velocity profile seen in figure 74 is similar to those observed above. Azimuthal station three shows the velocity trajectory at radial position 25. Radial position 65 is identified at azimuthal position four as the trajectory location by exhibiting the maximum $\gamma$. Downstream of this azimuthal station, the trajectory is no longer meaningful. However, with the trajectory shown, it is easy to conclude the some suppression toward the outer wall takes place. This suppression does not stop the migration toward the inner wall.

6.4. Conclusions From Normalized Radial Profiles
To this point, a representative group of tests examining the effects of spacing ratio have been closely examined. Each test was looked at individually. At this time, it is important to make some general statements concerning these tests:

1. As was expected from Lipshitz\textsuperscript{[18]}, trajectories determined from velocities generally show a greater penetration into the cross flow than those determined from temperature. This seems to be a result of the recirculation zone set up downstream of the injection location by the issuing dense jet.

2. This deviation increases as spacing ratio decreases since the low pressure recirculation zone becomes more pronounced as the jet becomes increasingly two-dimensional.

3. Injection from the inner wall shows penetration into the cross flow, followed by a migration back toward the inner wall. As spacing ratio decreases, this penetration becomes suppressed. At minimum spacing ratio, the maximum $\tau$ occurs at inner wall without exception.

4. For injection from the outer wall, one can identify migration toward the inner wall as well. Again, as in the single jet configurations, this is attributed to the drifting and pressure gradient...
effects overcoming the centrifugal effects. As Sr decreases, there is a noticeable suppression toward the outer wall, but migration toward the inner wall remains evident.

5. This suppression is attributed to two phenomena: As spacing ratio decreases, it becomes increasingly difficult for the cross flow to pass between individual jets. Instead of penetrating through the denser fluid, it passes over it, suppressing its trajectory. Also, as spacing ratio decreases, a significant low pressure recirculation zone sets up downstream, since according to experiment, the cross flow tends pass over the injected fluid. This low pressure causes the injected jet to collapse against the wall from which it was injected. In the case of the inner wall, this low pressure adds to an already low pressure region resulting in significant suppression. In the case of the outer wall, this low pressure fights a high pressure region, resulting in some suppression, but a continued migration toward the inner wall as the turn is negotiated.

6.5. Lateral Profiles

To examine the notion of two-dimensionality downstream of the injection point, three-dimensional
lateral plots were produced, making use of the width of the rake. As in the single jet case, since velocity two or more azimuthal stations downstream of the injection point seems somewhat unaffected, these plots make use of temperature distributions only.

Injection from the outer wall at the higher momentum ratio and wide spacing ratio is represented in figures 75 through 78. At the initial azimuthal station (0°), one observes the three-dimensionality of temperature at that point. Since the set of injection jets do not continuously span the combustor width, cross flow has the ability to flow inbetween them, thereby causing little or no suppression, as observed experimentally.

Examining the lateral plots further downstream (60°, 120°, and 180°), a degree of three dimensionality remains, even at the exit. This shows that the spacing is in fact wide enough to allow the cross flow to penetrate between them.

As one examines the tight spacing ratio (J = 3.08) for high momentum injection from the outer wall (figures 79 through 82), one observes immediately at 0° that the flow is nearly two-dimensional. By 60°, the flow is clearly two-
dimensional. The two figures showing downstream locations again show little or no change laterally from probe one to five.

From these lateral plots, the suppression of jet trajectory can again be attributed to the low pressure recirculation zone that forms downstream of the injection location. This region is more pronounced during injection with small spacing ratios because, as shown here, because the row of jets causes the cross flow to pass over the top.

Kamotani and Greber\[14] show a suppression in trajectory with decreasing spacing ratio until spacing ratio falls below about two or three. At this point, the issuing jet acts nearly like a slot jet and the trajectory shows less suppression. In this experiment, spacing ratio evidently cannot be decreased to the point where suppression begins to deteriorate.
CHAPTER VII

GENERAL CONCLUSIONS AND RECOMMENDATIONS

This is the second work using a reverse flow combustor that investigates the perpendicular injection of a cooling jet into a hot cross flow that is accelerating both longitudinally and transversely.

This work shows that there is, in fact, good agreement between trends associated with thermal and velocity trajectories. Both trajectories indicate a migration toward the inner wall independent of injection location, density ratio, momentum ratio, and spacing ratio.

The semi-empirical model developed by Lipshitz[18] for a single jet shows similar agreement as it did in his experiment. It is in need of better entrainment information along with knowledge of the pressure field throughout the combustor. Considering the vortex structure that sets up during perpendicular injection, a model which uses this as its dominant effect is under development.

With the experimental data available from this study, from the previous temperature study, and the
expected empirical information on entrainment, all of the necessary components are available to carry out a careful investigation to produce a semi-empirical model for a single jet as well as a row of jets.

From these experiments, then, it can be seen that injection from the outer wall allows cooling at the inner wall due to the migration effect. This is practically useful since injection from the outer wall is more easily carried out. Since migration toward the inner wall is evident, cooler temperatures and higher velocities can be developed there. If cooling of the outer wall is desired, the leakage of cooling air from the outer wall injection ports at low momentum ratios will result in a cool region along that wall.

Compact reverse flow combustors are a viable alternative. Flow conditions within them are predictable, and desirable modifications can be made through dilution jet injection.
CHAPTER IX

DATA PLOTS

The following pages contain all of the data plots described and referred to above.
Figure 23: T, High J, Outer Wall Prior to Bend.
MOMENTUM RATIO: 9.74  
DENSITY RATIO: 2.20  
SPACING RATIO: SINGLE INJECTION  
FILENAME: \datfiles\d1913

Normalized Velocity = 1

Figure 24: χ, High J, Outer Wall Prior to Bend.
<table>
<thead>
<tr>
<th>MOMENTUM RATIO: 5.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>DENSITY RATIO: 2.17</td>
</tr>
<tr>
<td>SPACING RATIO: SINGLE INJECTION</td>
</tr>
<tr>
<td>FILENAME: \datfiles\d1514</td>
</tr>
</tbody>
</table>

**Normalized Velocity = 1**

Figure 26: γ, Low J, Inner Wall.
MOMENTUM RATIO: 5.83
DENSITY RATIO: 2.17
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1511

NORMALIZED TEMPERATURE = 1

Figure 29: τ, Low J, Outer Wall Into Bend.
MOMENTUM RATIO: 5.83
DENSITY RATIO: 2.17
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1511

Trajectory Centerline
○ Experimental
□ Model

NORMALIZED VELOCITY = 1

Figure 30: γ, Low J, Outer Wall Into Bend.
MOMENTUM RATIO: 9.81
DENSITY RATIO: 2.18
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1911

NORMALIZED TEMPERATURE = 1

Figure 31: τ, High J, Outer Wall Into Bend.
Figure 32: Y, High J. Outer Wall into Bend.
INNER WALL

AZIMUTHAL LOCATION: 0 DEGREES
MOMENTUM RATIO: 0.00
DENSITY RATIO: 2.23
SPACING RATIO: NO INJECTION
FILENAME: \datfiles\d1000

NORMALIZED TEMPERATURE = 1

Figure 33: τ, No Injection, 0°.
INNER WALL

AZIMUTHAL LOCATION: 60 DEGREES
MOMENTUM RATIO: 0.00
DENSITY RATIO: 2.23
SPACING RATIO: NO INJECTION
FILENAME: \datfiles\d1000

NORMALIZED TEMPERATURE = 1

Figure 34: \( \tau \), No Injection, 60°.
Figure 35: I. No Injection, 120°.
Figure 36: T, No Injection, 180°.
INNER WALL

AZIMUTHAL LOCATION: 0 DEGREES
MOMENTUM RATIO: 5.80
DENSITY RATIO: 2.20
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1513

NORMALIZED TEMPERATURE = 1

Figure 37: τ, Low J, Outer Wall Prior to Bend, 0°.
Figure 38: T, Low J, Outer Wall Prior to Bend, 60°.
Figure 39: $\tau$, Low J, Outer Wall Prior to Bend, 120°.
Figure 40: t. Low J, Outer Wall Prior to Bend, 180°.
INNER WALL

AZIMUTHAL LOCATION: 0 DEGREES
MOMENTUM RATIO: 9.74
DENSITY RATIO: 2.20
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1913

NORMALIZED TEMPERATURE = 1

Figure 41: \( \tau \), High J, Outer Wall Prior to Bend, 0°.
INNER WALL

AZIMUTHAL LOCATION: 60 DEGREES
MOMENTUM RATIO: 9.74
DENSITY RATIO: 2.20
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1913

NORMALIZED TEMPERATURE = 1

Figure 42: \( \tau \), High J, Outer Wall Prior to Bend, 60\(^\circ\).
INNER WALL

AZIMUTHAL LOCATION: 120 DEGREES
MOMENTUM RATIO: 9.74
DENSITY RATIO: 2.20
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1913

NORMALIZED TEMPERATURE = 1

Figure 43: \( \tau \), High J, Outer Wall Prior to Bend, 120°.
INNER WALL

AZIMUTHAL LOCATION: 180 DEGREES
MOMENTUM RATIO: 9.74
DENSITY RATIO: 2.20
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1913

NORMALIZED TEMPERATURE = 1

Figure 44: τ, High J, Outer Wall Prior to Bend, 180°.
INNER WALL

AZIMUTHAL LOCATION: 0 DEGREES
MOMENTUM RATIO: 5.80
DENSITY RATIO: 2.17
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1514

NORMALIZED TEMPERATURE = 1

Figure 45: τ, Low J, Inner Wall, 0°.
INNER WALL

AZIMUTHAL LOCATION: 60 DEGREES
MOMENTUM RATIO: 5.80
DENSITY RATIO: 2.17
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1514

NORMALIZED TEMPERATURE = 1

Figure 46: $\tau$, Low J, Inner Wall, 60°.
INNER WALL

AZIMUTHAL LOCATION: 120 DEGREES
MOMENTUM RATIO: 5.80
DENSITY RATIO: 2.17
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1514

NORMALIZED TEMPERATURE = 1

Figure 47: τ, Low J, Inner Wall, 120°.
Figure 48: $\tau$, Low J, Inner Wall, 180°.
Figure 49: t, High J, Inner Wall, 0°.
INNER WALL

AZIMUTHAL LOCATION: 60 DEGREES
MOMENTUM RATIO: 9.70
DENSITY RATIO: 2.19
SPACING RATIO: SINGLE INJECTION
FILENAME: \datfiles\d1914

NORMALIZED TEMPERATURE = 1

Figure 50: $\tau$, High J, Inner Wall, 60°.
AXIAL LOCATION: 1.20 DEGREES
MOMENTUM RATIO: 0.70
SPACING RATIO: 2.19
FILE: .\files\1974

NORMALIZED TEMPERATURE = 1

Figure 51: High J, Inner Wall, 120°
INNER WALL

AZIMUTHAL LOCATION: 180 DEGREES
MOMENTUM RATIO: 9.70
DENSITY RATIO: 2.19
SPACING RATIO: SINGLE INJECTION
FILENAME: datfiles\d1914

NORMALIZED TEMPERATURE = 1

Figure 52: τ, High J, Inner Wall, 180°.
MOMENTUM RATIO: 5.80
DENSITY RATIO: 2.20
SPACING RATIO: SINGLE INJECTION
AZIMUTHAL STATION: 0 DEGREES
RADIAL STATION: 65 UNITS
FILENAME: \datfiles\d1513

NORMAL TEMP. = 0.5

Initial Jet Diameters

Figure 53: \( \tau \), Low J, Outer Wall Prior to Bend, 0°, lateral spreading.
MOMENTUM RATIO: 9.70  
DENSITY RATIO: 2.19  
SPACING RATIO: SINGLE INJECTION  
AZIMUTHAL STATION: 0 DEGREES  
RADIAL STATION: 55 UNITS  
FILENAME: \datfiles\d1914

NORM. TEMP. = 0.5

Figure 55: τ, High J, Inner Wall, 0°, lateral spreading.
MOMENTUM RATIO: 9.70
DENSITY RATIO: 2.19
SPACING RATIO: SINGLE INJECTION

AZIMUTHAL STATION: 100 DEGREES
RADIAL STATION: 75 UNITS
FILENAME: \datfiles\d1914

NORM. TEMP. = 0.5

**Initial Jet Diameters**

Figure 56: τ, High J, Inner Wall, 100°, lateral spreading.
Figure 57: 1, High J, Outer Wall Prior to Bend, 7 Jets.
<table>
<thead>
<tr>
<th>MOMENTUM RATIO: 9.69</th>
<th>SPACING RATIO: 9.21</th>
</tr>
</thead>
<tbody>
<tr>
<td>DENSITY RATIO: 2.18</td>
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<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
</table>

Trajectory Centerline
○ Multiple Jets
□ Single Jet

NORMALIZED VELOCITY = 1

Figure 58: y, High J, Outer Wall Prior to Bend, 7 jets.
Figure 59: \( \tau \), High J, Outer Wall Prior to Bend, 11 jets.
Figure 61: T. High J, Outer Wall Prior to Bend, 21 jets.
Figure 62: γ, High J, Outer Wall Prior to Bend, 21 jets.
Figure 64: Y, High J. Inner Wall, 7 jets.
Figure 67: \( \tau \), High J, Inner Wall, 21 jets.
Figure 69: $\tau$, High J, Outer Wall Into Bend, 7 jets.
MOMENTUM RATIO: 9.70
DENSITY RATIO: 2.18
SPACING RATIO: 9.00
FILENAME: \datfiles\d1971

NORMALIZED VELOCITY = 1

Figure 70: γ, High J, Outer Wall Into Bend, 7 jets.
Figure 71: τ, High J, Outer Wall into Bend, 11 jets.
MOMENTUM RATIO: 9.72
DENSITY RATIO: 2.18
SPACING RATIO: 6.00
FILENAME: \datfiles\d19111

Trajectory Centerline
○ Multiple Jets
□ Single Jet

NORMALIZED VELOCITY = 1

Figure 72: γ, High J, Outer Wall Into Bend, 11 Jets.
<table>
<thead>
<tr>
<th>MOMENTUM RATIO: 9.86</th>
<th>SPACING RATIO: 3.00</th>
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</thead>
<tbody>
<tr>
<td>DENSITY RATIO: 2.19</td>
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</tbody>
</table>

Trajectory Centerline
- Multiple Jets
- Single Jet

NORMALIZED VELOCITY = 1

Figure 74: γ, High J, Outer Wall Into Bend, 21 Jets.
Figure 75: T, High J, Outer Wall Prior to Bend, 7 Jets, 0°.

INNER WALL

AZIMUTHAL LOCATION: 0 DEGREES

MOMENTUM RATIO: 2.18

SPACING RATIO: 9.21

FILENAME: datfilesd1973

NORMALIZED TEMPERATURE = 1
INNER WALL

AZIMUTHAL LOCATION: 60 DEGREES
MOMENTUM RATIO: 9.69
DENSITY RATIO: 2.18
SPACING RATIO: 9.21
FILENAME: \datfiles\d1973

NORMALIZED TEMPERATURE = 1

Figure 76: \( \tau \), High J, Outer Wall Prior to Bend, 7 Jets, 60\(^\circ\).
Azimuthal Location: 120 degrees
Momentum Ratio: 9.69
Density Ratio: 2.18
Spacing Ratio: 9.21

Filename: \data\files\m1973

Normalized Temperature = 1

Figure 77: T, High J, Outer Wall Prior to Bend, 7 Jets, 120°
INNER WALL

AZIMUTHAL LOCATION: 180 DEGREES
MOMENTUM RATIO: 9.69
DENSITY RATIO: 2.18
SPACING RATIO: 9.21
FILENAME: \datfiles\d1973

NORMALIZED TEMPERATURE = 1

Figure 78: \tau, High J, Outer Wall Prior to Bend, 7 Jets, 180°.
Figure 79: 1 High J, Outer Wall Prior to Bend, 21 Jets, 0°.
INNER WALL

AZIMUTHAL LOCATION: 60 DEGREES
MOMENTUM RATIO: 9.81
DENSITY RATIO: 2.18
SPACING RATIO: 3.07
FILENAME: \datfiles\d19213

NORMALIZED TEMPERATURE = 1

Figure 80: τ, High J, Outer Wall Prior to Bend, 21 Jets, 60°.
INNER WALL

AZIMUTHAL LOCATION: 120 DEGREES
MOMENTUM RATIO: 9.81
DENSIY RATIO: 2.18
SPACING RATIO: 3.97
FILENAME: datfiles_d19213

NORMALIZED TEMPERATURE = 1

Figure 81: T. High J, Outer Wall Prior to Bend, 21 Jets, 120°.
APPENDIX 1

PARAMETER CALCULATIONS

1. Velocity is calculated from the Bernoulli equation

\[ V = \left( \frac{2\Delta P}{\rho} \right)^{1/2} \]

where \( \Delta P \) = pilot-static pressure difference, \( \rho \) = density of oncoming flow.

2. Momentum ratio calculated as

\[ J = \frac{\rho_{\text{jet}} V_{\text{jet}}^2}{\rho V^2} \]

where \( \rho, V \) are cross flow conditions.

\[ \therefore V_{\text{jet}} = \frac{\dot{m}_{\text{jet}}}{\text{Area}_{\text{jet}}} \]

\[ \therefore J = \frac{\left( \frac{P}{RT} \right)_{\text{jet}} \left[ \frac{\dot{m}}{\text{Area}} \right]_{\text{jet}}^2}{2\Delta P} \]

3. Density ratio is calculated as:
\[ Dr = \frac{P_{\text{let}}}{P} \]

\[ Dr = \frac{\left(\frac{P}{RT}\right)_{\text{let}}}{\left(\frac{P}{RT}\right)} \]

Other parameter calculations are straightforward, and need no additional explanation.
APPENDIX 2

SPECIFICATION SHEETS
Properties and Characteristics

**MACH NUMBER RANGE**

The lower usable limit for Pitot-Static Probes depends on the sensitivity of the readout device used with the probe. A differential pressure of 1" of water, for example, is about the minimum that can be measured with 1% accuracy with ordinary slant gauges, so the lower limit is approximately at a Mach Number of 0.08 or a velocity of 70 ft/sec for air at standard atmospheric conditions. There is no minimum Mach Number for the tube itself. The upper limit is at about Mach 0.95 for the total pressure reading and 0.70 for the static as shown in Figure 1. The static reading is accurate to 0.5% to a Mach Number of 0.50 and to 1.5% up to Mach 0.70. At this point the calibration becomes erratic due to the formation of local shock waves on and around the tip of the probe and the reading can vary as much as 15% with slight changes in flow conditions or proximity to solid boundaries. Above Mach 1.0 both the total and static readings vary considerably from true stream values but they can be corrected theoretically.

**TURBULENCE ERRORS**

Pitot-Static tubes appear to be insensitive to isotropic turbulence which is the most common type. Under some conditions of high intensity, large scale turbulence which makes the angle of attack at a probe vary over a wide range, the probe would presumably have an error corresponding to the average yaw or pitch angle caused by the turbulence.

**REYNOLDS NUMBER RANGE**

Pitot-Static probes are not directly affected by Reynolds Number except at very low velocities. Therefore, in liquids where Mach Number effects are absent, their calibration is substantially constant at all velocities.

The minimum Reynolds Number for the total pressure measurement is about 30 where the characteristic length is the diameter of the impact hole. Below this value the indicated impact pressure becomes higher than the stream impact pressure due to viscosity effects. This error is only noticeable in air under standard atmospheric conditions for velocities under 12 ft/sec with impact holes 0.015" diameter or less.

**BOUNDARY EFFECTS**

The static pressure indication is sensitive to distance from solid boundaries. Figure 3 shows how this error increases the indicated velocity pressure at a Mach Number of 0.35. The probe and boundary form a Venturi passage which accelerates the flow and decreases the static pressure on one side. The curve shows that static readings should not be taken closer than 5 tube diameters from a boundary for 1% accuracy and 10 tube diameters is safer.

**TIME CONSTANT**

The speed of reading depends on the length and diameter of the pressure passages inside the probe, the size of the pressure tubes to the manometer, and the displacement volume of the manometer. The time constant is very short for any of the standard tubes down to 1/16" diameter. It increases rapidly for smaller diameters, however. For this reason 1/16" O.D. is the smallest recommended size for ordinary use - this will take 15 to 60 seconds to reach equilibrium pressure with ordinary manometer hook-ups. These tubes have been made as small as 1/32" O.D., but their time constant is as long as 15 minutes and they check up very slowly on time dirt in the air stream. If very small tubes are required, it is preferable to use separate total and static tubes rather than the combined total-static type. Where reinforcing stems are specified on small sizes, the inner tubes are enlarged at the same point to ensure minimum time constant.

**INSTALLATION INFORMATION**

Probes are installed in the fluid stream with the impact hole facing upstream, the head parallel to the flow direction and the stem perpendicular. Types PA and PB (Fig. 4) are well suited to mounting on thin-walled ducts where the probe is to be inserted from the outside. Types PC and PD (Fig. 5) are designed with removable pressure take-offs for installation from within the duct, in thick-walled ducts where it is not practical to make an insertion hole of diameter equal to the length of the probe tip. Figure 6 shows limiting lengths and diameters for installation.
The AK line of PC board mounting budget priced (B) and standard (S) includes ranges from 3.6 V to 30 VDC. Critical components in each model are epoxy encapsulated and ground to ensure long serviceability. All units are given a minimum shock of 3 hours under full load and short circuit conditions. This is why a full 2 year for B series and a full 3 year warranty for S and C series is given and made applicable. All products are available and custom input/output terminations and voltages will be made upon request.

### PC BOARD MOUNTING BUDGET PRICED (B) SERIES

<table>
<thead>
<tr>
<th>Model</th>
<th>Voltage (V)</th>
<th>Current (A)</th>
<th>Type</th>
<th>Price</th>
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<tr>
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<td>50.0</td>
<td>S</td>
<td>600</td>
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### PC BOARD MOUNTING STANDARD (S) SERIES

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<th>Current (A)</th>
<th>Type</th>
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<td>50.0</td>
<td>S</td>
<td>600</td>
</tr>
</tbody>
</table>

### Case I

**General Specifications**
- DC input: 10V to 125V
- AC input: 200 to 240V
- Operating Temp: -20°C to 60°C
- Temp Coefficient: ±0.02%/°C
- Input: 25°C, 50Hz
- Isolation: 500V
- Compliance: Etta-A to common telecommunication equipment.

**Power Supply Options**
- Standard: 25°C and 50Hz
- Temperature: -20°C to 60°C
- Input: 25°C, 50Hz
- Voltage: ±5V, ±12V, ±15V
- Current: 5mA, 10mA, 15mA
- Output: ±5V, ±12V, ±15V
- Regulation: ±1%
- Ripple: ±50mV
- Isolation: 500V
- Compliance: Etta-A to common telecommunication equipment.
Features
- Instant warm-up
- High output: 5 volts or 12.5 volts
- ± 0.02% Repeatability
- ± 0.1% Hysteresis
- ± 0.1% Non-linearity
- ± 0.01°F Thermal effects
- ± 0.02% Output noise
- Fast response: less than 1 millisecond
- Will withstand high overpressure
- Many options, including remote control

Description
The Model 239 is a complete pressure transducer system for accurate measurement of low pressures. The unique Setra electronic circuitry is combined with a rugged capacitance-type sensor in the transducer. The high output usually requires no further signal conditioning.

The pressure media may be compatible liquid or gas. The reference pressure media must be clean dry air or non-condensable gas. The reference chamber for absolute pressure units is well-sealed at high vacuum.

The high level output signal, excellent stability, combined with fast dynamic response make this transducer well suited for many industrial laboratory, and aerospace applications requiring the highest accuracy. The unit differs from Model 239E, in that the Model 239 is carefully compensated to minimize both zero and sensitivity shifts due to environmental temperature variations.

Full Scale Ranges

Applications
- For highest accuracy, obtaining very high linearity, low hysteresis, fast dynamic response, very small temperature effects, insensitivity to motion and vibration, and high stability.
- For high accuracy applications where low power consumption, instant warm-up, and small size are needed.
- For accurate low absolute pressure (vacuum-welded reference chamber) and for low differential pressure measurements when the reference pressure is clean dry air or gas.
- Standard units for most applications connect directly into most A/D converters. Can be attenuated to match low level indicators, scanners, and loggers.
- Many options listed on back page for special instrumentation needs, including remote control.
- Replacing low output strain gage type transducers and associated signal conditioning, getting higher accuracy at much lower cost.

Construction
Setra's patented variable capacitance sensors approach the ultimate in design simplicity. A stainless steel diaphragm and an insulated electrode form a variable capacitance. As the pressure increases, the capacitance changes. This capacitance is detected and converted to a linear d.c. electric signal by Setra's unique electronic circuit.

Low Pressure and Absolute Pressure
Low range pressure sensors have a thin preched stainless steel diaphragm positioned close to the electrode. Positive pressure moves the diaphragm toward the electrode, increasing the capacitance. High positive overpressure pushes the diaphragm against the electrode, thereby providing high positive overpressure protection.

Higher Pressure
Higher pressure range sensors have an insulated electrode fastened to the center of the diaphragm, forming a variable capacitance. As the pressure increases, the capacitance will decrease.

46 Nagog Park, Acton, Massachusetts 01720 / Telephone: (617) 263-1420
Model 239 Specifications

<table>
<thead>
<tr>
<th>Pressure Ranges</th>
<th>Unidirectional differential</th>
<th>Bi-directional differential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 to 0.02, 0.02, 1.5, 10, 20, 100 psid</td>
<td>0 to ±0.01, 0.1, 0.5, 2.5, 5.10, 50 psid</td>
</tr>
<tr>
<td>Positive pressure media</td>
<td>Gases or liquids compatible with stainless steel, hard anodized 5061 aluminum, Buna N &quot;O&quot; ring (Stainless steel in place of aluminum on special order)</td>
<td></td>
</tr>
</tbody>
</table>

### Reference pressure media
- Differential pressure
- Absolute pressure
- Measuring pressure
- Excitation Power

<table>
<thead>
<tr>
<th>Zero pressure output</th>
<th>Output impedance</th>
<th>Non-linearity</th>
<th>Thermal effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;10 psid</td>
<td>&lt;20 MΩ</td>
<td>&lt;±1% full range output</td>
<td>Reversal excitation protected</td>
</tr>
</tbody>
</table>

### Natural frequency
2000 Hz nominal

### Output noise
<200 microvolts RMS (in band, 0 Hz to 10 kHz)

### Electrical Connection
- 2 foot multi-conductor cable
- 5' foot multi-conductor cable

### Weight
8 ounces

### Maximum Working Pressure
800 psig

### Operating Range of Transducer
0-01 and 0-04 psi

### Pressure Range of Transducer
0-10, 0-20, 0-100, 0-200 psi and higher

### Electronic Circuit Information
- Four terminal equivalent circuit; either negative excitation or negative output should be connected to case. Unit calibrated at the factory with negative excitation connected to case
- Options:
  - Remote calibration signal (specify percent full scale or adjustable)
  - Remote zero pressure adjustment
  - Remote sensitivity adjustment
  - 32VDC or 28VDC ground referenced excitation (order power output voltage)
  - 10VDC power output (minimum 0.01VDC)
  - Compensated temperature range -60°F to 250°F (typical temperature effects 2% standard)
  - Intrinsic safety design (Class 1, Div. 1, Groups C and D)

### Ordering Information
- Order as Model 239 pressure transducer
- Specify pressure range noting differential pressure or absolute pressure, desired electrical output, and desired options

### Outline Drawing
- All dimensions are in inches
- 100% full range output
- Orders above 1000 psig proof positive on special order
- Orders above 1000 psig factory test
- Orders above 1000 psig on special order
- Orders above 1000 psig for 0.1000VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0500VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0200VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0100VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0020VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0010VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0005VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0002VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0001VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.00005VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.00002VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.00001VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.000005VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.000002VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.000001VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0000005VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0000002VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0000001VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.00000005VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.00000002VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.00000001VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.000000005VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.000000002VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.000000001VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0000000005VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0000000002VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.0000000001VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.00000000005VDC test, minimum 0.02 ohms and not be damaged
- Orders above 1000 psig for 0.00000000002VDC test, minimum 0.02 ohms and not be damaged
APPENDIX 3

OPERATING PROCEDURES

1. START UP & SHUT DOWN

START-UP AND SHUT-DOWN INSTRUCTIONS
for the
REVERSE FLOW COMBUSTOR
Revised July, 1982

START-UP

1. Verify the operation of the hanging electrical outlets.
2. Plug power supply, ignition system, and temperature indicator into the outlets.
3. Move blower main electrical switch to the "on" position.
4. Check to see that the "primary cooling air" valve is closed.
5. Open "dilution jet air" valve to 1 cm differential to prevent the flow of hot air into the blower.
6. Check to see that the main and small natural gas valves are closed.
7. Set automatic gas shut-off system to "off" position.
8. Set 3-way natural gas valve to "upstream to main valve" position (pointer to the left).
9. Set temperature indicator to monitor central probe of the rake.

10. Supply power to the pressure transducers for delta pressure acquisition.

11. Start blower by depressing the "on" button located on the remote control unit.

12. Supply combustion air at 0.018 kg/sec (40 mm $H_2O$).

13. Check to see that the exhaust system is open and the ejectors are operational.

14. Set the automatic gas shut-off system to the "manual" mode of operation.

15. Open the small natural gas valve (allowing flow to the central burner only) and supply gas to the ignitor at 20 CFH (as indicated on the small flow meter).

16. Activate the ignition system by depressing the right switch and then the left switch.

17. Listen for the sound of igniting natural gas; verify the ignition by monitoring the central probe temperature.

18. Increase the flow rate through the small natural gas valve to 25 CFH.

19. Open large natural gas valve, supplying gas to all burners at 0.001 kg/sec (18 mm $H_2O$).

20. Listen for the sound of ignition at the remaining burners.

21. Immediately supply primary cooling air at a rate of at least 0.065 kg/sec (55 mm $H_2O$).
22. Switch the 3-way natural gas valve to the "gas manifold" position (pointer to the right).


24. Take out ignition system plug from the hanging electrical outlet.

25. The combustor is now in the "operational" mode. Keep flow rates within the "operational zone" while adjusting them to produce different temperatures.

   Do not exceed exhaust temperature of 1292°F (700°C).

   Do not exceed bend wall temperatures of 932°F (500°C).

   Do not force the scanning mechanism. If it becomes immovable due to thermal warpage, allow it to cool, thereby relieving the stress.

   Do not over-tighten dilution jet plugs. Close only slightly more than hand tight.

26. Set automatic gas shut-off system to the "automatic" position.

SHUT-DOWN


2. Allow blower to continue running until the wall temperature has dropped to 140°F (60°C).

3. Shut down the blower by depressing the "off" button on the remote control unit. Close all air valves.

4. Move automatic gas shut-off system to "off" position.

5. Move blower main electrical switch of "off" position.
B. OPERATING PROCEDURE

If using a multiple probe, call program sectry 2.bas.

Before beginning, if pressure transducers are to be used, call transres.bas. Upon running it will ask for a channel number. Channel numbers zero through four correspond to transducers one through five. The act of typing a channel number and returning causes 25 pressure measurements to be taken, and the average to be calculated. The last number in the column is the average. Write it down and save it for each transducer.

Load your choice of program for multiple or single probes. It will first ask what the zero values for the transducers are. It will then ask for a jet raw location. Number one is the location furthest into the turn, number two is the second run in the turn, number three is prior to the turn outer wall, and number four is prior to the turn inner wall.

The next question asked is how many operating dilution jets exist. Answer with the appropriate number. Following this, answer how many non-working jets exist between each set of two operating jets. The next question asks which azimuthal increment is desired. Answer with 1 for 10 degrees, 2 for 20 degrees, etc. The following two questions ask you to name an introductory file name and a data file name. Do so.

Following that, manometers must be read for the various flow rates. The program will then ask you to enter values and pressures. The units are usually mm water differential and are mentioned in each question. Required entries are: combustor pres-
sure, combustor air flow rate, natural gas flow rate, dilution jet flow rate, and barometric pressure.

Following pressure input, one must key in cooling jet temperature, cross flow temperature, and six wall temperatures (numbers five through ten) in degrees F.

The computer will give you back all of the input parameters in SI units, and ask you for your first temperature. Locate the probe where it asks (should be radial position 95, and zero azimuthal degrees). Press button 11 on the Doric trendicator and press F5 on the keyboard. On the Doric, keying F5 on the computer after each. After 15, the pressures are all automatically acquisitioned, their values printed along with the calculated local velocities. The computer will then tell you where to locate the probe. Do so. Repeat the above procedure.

To plot out data radially, choose iasplimpt.bas for temperature and iasplimpv.bas for velocity. The program will ask for introductory and data file names. Supply them and the plot will be made.

C. Thermocouple Locations

THERMOCOUPLES

1. Right Burner
2. Middle Burner
3. Left Burner
4. Straight Inner Wall
5. Inner Wall at Middle Bend
6. Outer Wall at Exit
7. Outer Wall at Middle Bend
8. Straight Outer Wall
9. Side Wall at Middle Bend
10. Side Wall at Exit
11. Rake
12. Rake
13. Rake
14. Rake
15. Rake
16. Jet Injection Port
17. Exhaust Port
18. Single Probe
APPENDIX 4

COMBUSTOR SOFTWARE
1. SECTRY2.BAS--Data Acquisition

10 DIM TVS(12)
20 DIM ZER(5)
30 INPUT "zero(0)/1 ZER(0)
40 INPUT "zero(1)/1 ZER(1)
50 INPUT "zero(2)/1 ZER(2)
60 INPUT "zero(3)/1 ZER(3)
70 INPUT "zero(4)/1 ZER(4)
80 DIM T(4,6)
90 DIM F(4,6)
100 T(1,1,0) = 100; T(1,1,1) = 100; T(1,2,0) = 100; T(1,2,1) = 100; T(2,1,0) = 100; T(2,1,1) = 100
110 T(1,2,0) = 100; T(1,2,1) = 100; T(2,1,0) = 100; T(2,1,1) = 100
120 T(4,6,0) = 100
130 F(1,1,0) = 1224.5; F(1,1,1) = 1224.5; F(2,1,0) = 1224.5; F(2,1,1) = 1224.5; F(4,1,0) = 1224.5; F(4,1,1) = 1224.5
140 IF NS = 0
150 INPUT "If applicable, what is the jet row location 1 to 4?" ; JL
160 INPUT "Number of operating dilution jets" ; NO
170 IF JL = 1 THEN SF1 = 2 ELSE IF JL = 2 THEN SF1 = 2.05 ELSE IF JL = 3 THEN SF1 = 2.45
180 PRINT "RVM Row one starts at highest outer wall location with rows two and three progressively at lower positions on the outside wall. Row four includes dilution jets on the inner wall.
190 PRINT "The azimuthal increment goes with 1, 1.5, 2, 2.5, 3, etc.
200 PRINT "Recall that 0 degrees azimuthal position refers to probe position fully in combustor. Also, radial position 95 refers to probe position fully at the outer wall.
210 PRINT "INPUT "What file name and number for intro. information?" ; FI1
220 PRINT "What file name and number for data?" ; FD1
230 OPEN FI1 FOR OUTPUT AS #1
240 OPEN FD1 FOR OUTPUT AS #2
250 INPUT "Combustor Pressure (gage) in mm of water-Mm-kr=", PR
260 INPUT "Combustor Air Flow Rate in m/s water differential-Mgr=", MGR
270 INPUT "Cooling Air Flow Rate in m/s water differential-Mkr=", Mkr
280 INPUT "Natural Gas Flow Rate in m/s water differential-MGR=", MGR
290 INPUT "Multiple dilution jet flow rate in m/s water differential-MJgr=", MJgr
300 IF NJ = -1 THEN INPUT "Single Jet Flow Rate in S.C.F.M. m-Jr=", MJcr
310 PRINT "Barometric Pressure in inches of Hg-Bgr=", Bgr
320 PRINT "Bareometric Pressure in inches of Hg-Bgr=", Bgr
330 PRINT "Parameter values given below in desired units are designated with MC-, MD-, MP-, ETC.
340 PRINT "If desired, variables names as follows: PR becomes P, PR becomes T, TIA. ETC.
350 MC = .00035*BDK(MCR); PRINT "ac = " MC " l/sec";
360 MI = .00044*BDK(MMR); PRINT "m1 = " MI " l/sec"
370 MJ1 = .0002*BDK(MJR); PRINT "m1 = " MJ1 " l/sec"
380 IF NJ = -1 THEN MSD = (.001208)*BDK(MJR)/NO; PRINT "md = " MSD " l/sec"
IF AJ = -1 THEN MSG = MSG + 5.23007E+04: PRINT "m = " MSG " kg/sec"

REM Lines 710 to 730 are orifice meter calibration equations. These and
other equations in this section convert to desired units.
4 M = RC = GC: PRINT "m = " M " kg/sec"
6: B = C = 4.766944E0: PRINT "p = " P " pascals"

72 T = 273.15 + (5/9)*(TR - 32): PRINT "t = " T " degrees kelvin"

76 T = 273.15 + (5/9)*(TR - 32): PRINT "T = " T " degrees kelvin"

78 T6 = 273.15 + (5/9)*(TR - 32): PRINT "t6 = " T6 " degrees kelvin"

82 T10 = 273.15 + (5/9)*(TR - 32): PRINT "t10 = T10 " degrees kelvin"

85 RC = F*(128-71): PRINT "ro = " RC " kg/cubic meter"

86 J = (FG)*A*: J: PRINT "raj = " ROJ " kg/cubic meter"

88 V = M*FG: 0.25: PRINT "v = " V " meter/sec"

87 V = M*FG: 0.25: PRINT "v = " V " meter/sec"

92 FPRINT "The number of jets used for injection is " NOJ

93 DH = NOJ/2: PRINT "Density ratio = " 1

95 PRINT USING "6E.6": DH

j = (FG)*A*: J: PRINT "Momentum ratio = " 1

97 PRINT USING "6E.6": J

1: AI = I = 1

11: PRINT "Sensitivity ratio = 1"

17: PRINT USING "6E.6": J

19: PRINT "Planar increment, AI = " AI

21: PRINT "Input information file is named " Files

23: PRINT "Data file is named " Files

25: INPUT "Check calculated values, if adjustments are required, make them 
GIVING TIME FOR CHANGES TO TAKE PLACE. WERE ADJUSTMENTS NEEDED?"

27: IF F = 4: "yes" THEN 520

29: PRINT USING "6E.6": 1

31: PRINT USING "6E.6": 1

33: PRINT USING "6E.6": 1

35: NEXT AI

37: IF AI = 0 TO 8 STEP AI

39: NEXT AI

41: FOR AI = 0 TO 8 STEP AI

43: FOR RC = 0 TO 1 STEP (-1)

45: FOR RC = 0 TO 1 STEP (-1)

47: GOSUB 6000

49: NEXT RC

51: NEXT RC

53: NEXT RC

55: NEXT RC

57: FOR AI = 0 TO 14 STEP AI

59: FOR RC = 0 TO 4 STEP (-1)

61: GOSUB 6000

63: NEXT RC

65: NEXT RC
This page contains a program for temperature conversion and acquisition. The program is written in a language similar to BASIC. The function of the program is to read temperature data from a source and convert it to a decimal number. The code includes comments explaining the process and the conversion algorithm. The conversion is performed using a series of instructions that read and process the data from memory locations specified in the program.

The program starts by initializing variables and setting up the conversion process. It then enters a loop that iterates through the memory addresses where the temperature data is stored. For each address, it reads the data, converts it to a decimal number, and prints the result. The conversion process involves bitwise operations and arithmetic calculations to transform the binary representation of the temperature data into a decimal value.

The comments throughout the code provide explanations of the different steps and conditions in the conversion process. For example, the program checks for specific conditions such as when the conversion fails or when a particular address is encountered. In such cases, it attempts to convert the data again or proceeds to the next step.

Overall, the program appears to be a utility for data acquisition and conversion, possibly for use in a control system or monitoring application where temperature readings need to be processed and displayed in a more user-friendly format.
```
200: TASC M1 = VOLTAGE*1000/8
210: Y = M = TPSD(M1)
220: NEXT M1
230: BEEP
235: PRINT "SC " "X/Y (MI-1)"
240: PRINT "1/26 M1A = 1 TO 25 STEP 1" ; MIA = 164
245: FSD = FSD + ((TPSD(MIA) - T1(RA,RA2)) * 2/25 - 1)
250: NEXT MIA
255: END = FSD .5
260: PRINT "Channel " "N-1 " std. dev. is 0 SD(1)"
270: T1(RA,RA2) = (S/9)*((3/(M1-1)) + 459.69)
275: X = 0
280: END = 0
290: NEXT I
300: RETURN

1.6.1 REN subroutine to perform A/D conversion of channels 0 through 4 in the proper sequence. The board must be in the 1/0 mapped mode. These conversions are for pressure.

6.20 ADDRESS = 18-7
6.21 OUT ADDRESS = 8, 170
6.22 REN Line B:1 calls gain=4 since input is 0 to 5 volts (ma. 35 m.s)
6.23 FOA CH = 0 TO 4 STEP 1
6.24 OUT ADDRESS = 5, CH
6.25 REN Line B:5 designates channel number
6.26 FOA FI = 1 TO 25 STEP 1
6.27 OUT ADDRESS = 6, CH
6.28 REN Line B:7 starts conversion procedure.
6.29 C = INITIAL ADDRESS - 6
6.30 REN Line B:6 assigns from board the condition of status bit concerning conversion. Below, if an unacceptable conversion occurs, then the channel will attempt the code again.

6.31 IF C = 1 THEN "HEX 6 -"
6.32 CALL A 192
6.33 IF C = 12 THEN: ELSE END 010
6.34 FINISH - "CONVERSION: STOP"
6.35 END 0
6.36 REN below, we convert to decimal values.
6.40 VALUE = 256*HIGH + LOW
6.41 IF VALUE > 32767 THEN VALUE = VALUE - 65535
6.42 VOLTAGE = VALUE/204.6
6.43 REN Calibration is 0.1 psi per 2.5 volts of output.
6.44 CALIBRATION = 1.15
6.45 VOLTAGE = VOLTAGE*CALIBRATION/4 - ZEROMIC"
6.46 Y = Y + FI(RA,RA2)
6.47 NEXT FI
6.48 FI(RA,RA2) = Y/(FI-1)
6.49 Y = 0
6.50 NEXT CH
6.51 RETURN
0
```
2. INFOUT2.BAS--Displays data in tables.

[Program code and comments not transcribed for this text representation.]
3. LASPLMPT.BAS--Plots set of radial temperature profiles.
4. LASPLMPV.BAS--Plots set of radial velocity profiles.
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ORIGINAL PAGE OF POOR QUALITY

1810 FOR AZ = 0 TO 1 STEP (AID/10)
1815 AILOC = AILOC + 1
1820 FOR RA = 4 TO 1 STEP (-1)
1825 GOSUB A10
1830 NEXT RA
1835 NEXT AZ
2810 ALES = A1S - (AID + 2 + 3.141592654/30)
2815 FOR AZ = A12 TO 8 STEP (AID/10)
2820 AILOC = AILOC + 1
2825 GOSUB A10
2830 FOR RA = 4 TO 2 STEP (-1)
2835 GOSUB 12405
2840 NEXT RA
2845 NEXT AZ
2855 FOR AZ = A12 TO 10 STEP (AID/10)
2860 AILOC = AILOC + 1
2865 GOSUB 1101
2870 FOR RA = 4 TO 2 STEP (-1)
2875 GOSUB 12405
2880 NEXT RA
2885 NEXT AZ
2890 FOR AZ = A12 TO 14 STEP (AID/10)
2895 AILOC = AILOC + 1
2900 GOSUB 1101
2905 FOR RA = 4 TO 2 STEP (-1)
2910 GOSUB 12405
2915 NEXT RA
2920 NEXT AZ
2925 FOR AZ = A12 TO 18 STEP (AID/10)
2930 AILOC = AILOC + 1
2935 GOSUB 1101
2940 FOR RA = 4 TO 2 STEP (-1)
2945 GOSUB 12405
2950 NEXT RA
2955 NEXT AZ
3870 FOR THETA = 0 TO .1746 STEP (11/560) + 2.3.141592654/30
3875 XCOORD = (THETA + 357.295779518) + 25
3880 PSET (XCOORD, 166)
3885 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
3890 XCOORD = (THEA + 357.295779518) + 25
3895 PSET (XCOORD, 166)
3900 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
3905 XCOORD = (THETA + 357.295779518) + 25
3910 PSET (XCOORD, 166)
3915 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
3920 XCOORD = (THETA + 357.295779518) + 25
3925 PSET (XCOORD, 166)
3930 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
3935 XCOORD = (THETA + 357.295779518) + 25
3940 PSET (XCOORD, 166)
3945 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
3950 XCOORD = (THETA + 357.295779518) + 25
3955 PSET (XCOORD, 166)
3960 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
3965 XCOORD = (THEA + 357.295779518) + 25
3970 PSET (XCOORD, 166)
3975 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
3980 XCOORD = (THEA + 357.295779518) + 25
3985 PSET (XCOORD, 166)
3990 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
3995 XCOORD = (THEA + 357.295779518) + 25
4000 PSET (XCOORD, 166)
4005 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
4010 XCOORD = (THEA + 357.295779518) + 25
4015 PSET (XCOORD, 166)
4020 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
4025 XCOORD = (THEA + 357.295779518) + 25
4030 PSET (XCOORD, 166)
4035 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
4040 XCOORD = (THEA + 357.295779518) + 25
4045 PSET (XCOORD, 166)
4050 FOR AIS = 0 TO .1746 STEP (AID + 2 + 3.141592654/30)
15 THETA = .2617994
7220 RETURN
8910 FOR THETA = THETA TO 3.141592654# STEP (8\#-3.141592654#)/80)
9010 DELTA = (1/8*(1 + SIN(THETA)))
9110 ALPH = 41*(DELTA.1 = DELTA 2/.5)
9210 B = (1/8*COS(THETA)*COS(THETA) + B)*COS(ALPH) + COS(THETA)
9310 L = SIN(THETA) + B*COS(ALPH) + SIN(THETA) - 2 + (1/8)*COS(THETA) + B)
9410 X = (6 + 15*COS(THETA))/2 + (8 + 15*THETA)/26
9510 Y = 15*ALPHA/2 + 1.75
9520 L = (1/2 + Y/2) .5
9610 XCDOR = (THETA)**57.26277951# + 25
9620 YCDOR = (L - XCDOR**2 + 5)
9710 XCDOR = (XCDOR)**57.26277951# + 25
9720 YCDOR = (L - XCDOR**2 + 5)
16100 IF -- nep, GOSF, FILE
1010 FOR ALR = A15 TO 3.144 STEP (A10**2.1511592654#)/30)
10210 X = XCDOR - 50
1510 IF ALR = A12 THEN DAV AN "yep! get the next": LOCATE VT; X: PRINT A10
16110 THEA = Theta + .06766426# GOTO 10620
10610 NEXT ALR
10620 NEXT THEA
10630 RETURN
13411 Y = 16.16: IF - (X - XCDOR, A12) < 50) THEN: IF X12 = (X, ALR, A12) / (V12 = 1, V12 = 1, XCDOR; YCDOR; PSET; X+1, Y; PSET; X+1, Y+1; PSET; X+1, Y+1; PSET; X, Y)
12 = 1: RETURN.
5. LATPLTES.BAS—Plots all rake temperatures laterally for a given azimuthal and radial position.

10 CLS
20 SCREEN 2
30 DIM PNT(9,16)
40 DIM RAK(9,16)
50 DIM ANG(9,16)
60 DIM T(4,4,16)
70 DIM V(4,9,16)
80 INPUT "STATE INTRODUCTORY FILENAME IN QUOTES"; F18
90 INPUT "STATE DATA FILENAME IN QUOTES"; F94
100 OPEN F18 FOR INPUT AS #1
110 OPEN F94 FOR INPUT AS #2
130 INPUT "WHAT TYPE OF PLOT (lateral velocity or temperature)"; FLYTF18
140 INPUT "WHAT RADIAL POSITION IS DESIRED (9, 8, ..., 1)"; RAF
150 INPUT "WHAT AZIMUTHAL POSITION IS DESIRED (18, 17, ..., 2, 1)"; AZF
160 CLS
170 E=OFF
180 FOR A = 0 TO 6 STEP A1
190 FOR RA = 4 TO 1 STEP (-1)
200 GOSUB 5000
210 NEXT RA
220 NEXT A2
230 FOR A = 2 TO 6 STEP A1
240 FOR RA = 4 TO 1 STEP (-1)
250 GOSUB 5000
260 NEXT RA
270 NEXT A2
280 FOR A = 2 TO 6 STEP A1
290 FOR RA = 4 TO 1 STEP (-1)
300 GOSUB 5000
310 NEXT RA
320 NEXT A2
330 FOR A = 0 TO 5 STEP A1
340 FOR RA = 4 TO 1 STEP (-1)
350 GOSUB 5000
360 NEXT RA
370 NEXT A2
380 FOR A = 0 TO 5 STEP A1
390 FOR RA = 4 TO 1 STEP (-1)
400 GOSUB 5000
410 NEXT RA
420 NEXT A2
430 LOCATE 7, 45; PRINT "FILENAME:"; F18
440 LOCATE 5, 9; PRINT "MOMENTUM RATIO:"; "0.5: J
450 LOCATE 6, 9; PRINT "DENSITY RATIO:"; "1:PRINT USING "0.00": J
460 LOCATE 7, 9; PRINT "SPACING RATIO:"; "1:PRINT "SINGLE INJECTION":
470 LOCATE 8, 45; PRINT "AZIMUTHAL STATION:"; "1:PRINT (AZF+10); PRINT "DEGREES";
480 LOCATE 6, 45; PRINT "RADIAL STATION:"; "1:PRINT (RAF+.5)*101: PRINT "UNITS":
490 PBST(296, 176); DRAW *192 1384 r192
500 PBST(225, 176); DRAW *192 1384 r192
510 PBST(225, 176); DRAW *192 1384 r192
520 PBST(225, 176); DRAW *192 1384 r192
530 PBST(225, 176); DRAW *192 1384 r192
540 LOCATE 24, 11: PRINT "-7.141"
550 LOCATE 24, 12: PRINT "-3.571"
560 LOCATE 24, 13: PRINT "0.00"
570 LOCATE 24, 14: PRINT "-0.5"
580 LOCATE 24, 15: PRINT "-1.1:"
590 LOCATE 9, 1: PRINT "Norm. Temp. = 0.5"
5010 IF RA=RAF AND A2=AZF THEN BODUS 6000
5010 RETURN
5020 XCODR=104: YCODR=(176-192*(T(D, RA, A2)-T)/(T3-T)): PSET(XCODR, YCODR): DRAW
5030 D4 d4 d4 r4 18 r4
5040 XCODR=104: YCODR=(176-192*(T(D, RA, A2)-T)/(T3-T)): PSET(XCODR, YCODR): DRAW
5050 D4 d4 d4 r4 18 r4
5060 XCODR=246: YCODR=(176-192*(T(D, RA, A2)-T)/(T3-T)): PSET(XCODR, YCODR): DRAW
5070 d4 d4 r4 18 r4
5080 XCODR=246: YCODR=(176-192*(T(D, RA, A2)-T)/(T3-T)): PSET(XCODR, YCODR): DRAW
5090 d4 d4 r4 18 r4
5100 RETURN
6. 3DLAT.BAS--Plots laterally all rake temperatures for all radial positions at a given azimuthal location.

10 CLS
20 INPUT "What azimuthal location"; Az
30 DIM \(x(10)\)
40: DIM X(9)
50: FOR RA = 6 TO 1 STEP (-1)
60: GOSUB 500
70: NEXT RA
80: NEXT Az
90 CLS
100 SCREEN 2
101 LINE (50, 190)-(50, 110)
102 LINE (50, 110)-(150, 90)
103 LINE (150, 90)-(150, 110)
104 LINE (50, 110)-(50, 190)
110 LOCATE 4, 48: PRINT "INNER WALL"
111 LOCATE 14, 50: PRINT "FILENAME:"; FS
112 IF Az = 20 THEN GOTO 120
113 LOCATE 10, 50: PRINT "AZIMUTHAL LOCATION:" Az "DEGREES"
114 LOCATE 11, 50: PRINT "MOMENTUM RATIO:" "1: PRINT USING ";0.00"; 1
115 LOCATE 12, 50: PRINT "DENSITY RATIO:" "1: PRINT USING ";0.00"; 1
116 LOCATE 20, 45: PRINT "NORMALIZED TEMPERATURE = 1"
117 IF Az = 1 THEN B600 1425
118 IF Az = 2 THEN B600 1450
119 LOCATE 15, 50: PRINT "SPACING RATIO:" "1: PRINT USING ";0.00"; 15
LOCATE 12,50: PRINT "SPACING RATIO: ":PRINT "NO INJECTION":
GOTO 1510
LOCATE 12,50: PRINT "SPACING RATIO: ":PRINT "SINGLE INJECTION":
GOTO 1510
REM now continue with data plotting
FSET(B,102)
X=5+102
LIM = 1
X = 80
IF A1 = 10 THEN LIM = 2
IF A1 = 12 THEN LIM = 3
IF A1 = 14 THEN LIM = 4
IF A1 = 16 THEN LIM = 5
IF A1 = 18 THEN LIM = 5
FOR LAT = 0 TO 4 STEP 1
FOR RA = 9 TO LIM STEP -1
VALUE = 300*(T(LAT,RA,A1)) -Tj (Tj - T)
IF RA = 9 THEN GOTO 1655
GOTO 1655
Z(RA) = X + VALUE: PSET(Z(RA),Y(RA)): DRAW"L=value"
GOTO 1655
X(RA) = X + VALUE
PSET(Z(RA),Y(RA))
LINE ((Z(RA) + 1), (Y(RA) + 1)) - ((Z(RA)), (Y(RA)))
IF RA = LIM THEN DRAW "L=VALUE"
Y(RA) = Y(RA) + 40
NEXT RA
FSET(X(9),Y(9)): DRAW "BBU 630"
Y(9) = Y(8) - 4
IF LAT = 4 THEN GOTO 1765
FSET(X(9),Y(9)): DRAW "BBU 650"
FSET(X(9),Y(9)): DRAW "BBU 650"
LINE ((Z(RA) + 1), (Y(RA) + 1)) - ((Z(RA)), (Y(RA)))
IF RA = LIM THEN DRAW "L=VALUE"
414 U = U -1
47 END
APPENDIX 5

ERROR ANALYSIS

This error analysis is based on the approximation that error can be estimated using the first term of the Taylor Expansion.

Sources of error in this experiment are from:

- thermocouple
- conduction
- pilot-static tube
- flow fluctuations
- Doric temperature indicator
- Setra Systems transducer
- Tecmar A/D board.

1. Temperature Error:

\[
\frac{\sigma(T)}{T} = \left( \frac{\text{std. dev.}}{\sqrt{25}} \right) + \text{Conduction error} + \text{Thermocouple error}
\]

\[
\frac{\sigma(T)}{T} = 100 \left( \frac{1.2}{600} \right) + 0.18 + 0.75 = 1.13\%
\]

Fluctuation must include Doric flow fluctuations and A/D board.
2. Pressure Error:

\[ \frac{\theta(\Delta P)}{\Delta P} = \left( \frac{\text{std. dev.}}{\sqrt{25}} \right) + \text{pitot-static error} \]

\[ \frac{\theta(\Delta P)}{\Delta P} = 100 \left( \frac{8 \times 10^6}{0.004} \right) + 4 = 4.2\% \]

Fluctuations seen in standard deviation must include Sebra transducer, flow fluctuations, and A/D board.

3. Error in Velocity

Since velocity has multiple components, each having some error associated with it, the Taylor formula will therefore be used:

\[ V_L = \left[ \frac{2 \Delta P}{P_L} \right]^{1/2} \]

where \( V_L, P_L \) are "local" quantities.

\[ P_L = \frac{P}{RT_L} = \frac{P_{\text{atm}} + P_{\text{comb}}}{RT_L} \]

where \( P_{\text{atm}} = \) atmospheric pressure

\( P_{\text{comb}} = \) combustor pressure (gage)

\[ \therefore V_L = \left[ \frac{2 \Delta P}{\left( \frac{P_{\text{atm}} + P_{\text{comb}}}{RT_L} \right)} \right]^{1/2} \]
Using the Taylor expansion formula, and taking the absolute value of each term:

\[
\frac{\partial V}{V} = \frac{1}{V} \left[ \frac{\partial V}{\partial (\Delta P)} \Delta (\Delta P) \right] + \frac{1}{V} \left[ \frac{\partial V}{\partial T_L} \Delta T_L \right]
\]

\[
\quad + \frac{1}{V} \left[ \frac{\partial V}{\partial P_{atm}} \Delta (P_{atm}) \right] + \frac{1}{V} \left[ \frac{\partial V}{\partial P_{comb}} \Delta (P_{comb}) \right]
\]

After carrying out the differentiation in the first term, and multiplying by \(1/V\) and \(\Delta (\Delta P)\), one gets:

\[
\frac{1}{V} \left[ \frac{\partial V}{\partial (\Delta P)} \right] = \frac{1}{2} \left[ \frac{2RT_L}{P_{atm} + P_{comb}} \right] \Delta (\Delta P)
\]

\[
= \frac{1}{2} \frac{\partial (\Delta P)}{\Delta P}
\]

Similarly, term two becomes:

\[
\frac{1}{V} \left[ \frac{\partial V}{\partial T_L} \Delta T_L \right] = \frac{1}{2} \frac{\partial T_L}{T_L}
\]

Similarly, term three becomes:
Similarly, term four becomes:

\[ \frac{1}{V} \left[ \frac{\partial V}{\partial P_{\text{atm}}} \delta(P_{\text{atm}}) \right] = \frac{1}{2} \frac{\delta(P_{\text{atm}})}{P_{\text{atm}} + P_{\text{comb}}} \]

Therefore, uncertainty in velocity is

\[ \frac{\delta V}{V} = \frac{1}{2} \left[ \frac{\delta (\Delta P)}{\Delta P} + \frac{\delta (T_{\text{L}})}{T_{\text{L}}} + \frac{\delta (P_{\text{atm}}) + \delta (P_{\text{comb}})}{P_{\text{atm}} + P_{\text{comb}}} \right] \]

\[ \frac{\delta (\Delta P)}{\Delta P} = 4.2\% \quad \text{from above} \]

\[ \frac{\delta (T_{\text{L}})}{T_{\text{L}}} = 1.13\% \quad \text{from above} \]

Combustor and atmospheric errors are estimated by how well manometers can be read. The barometer is ± 0.01 inches Hg. The combustor pressure manometer is ± 0.05 cm water. This gives

\[ \frac{\delta (P_{\text{atm}}) + \delta (P_{\text{comb}})}{P_{\text{atm}} + P_{\text{comb}}} = \frac{0.01 + 0.0014 \text{ inches Hg}}{28 + 0.06 \text{ inches Hg}} = 0.04\% \]

Therefore

\[ \frac{\delta V}{V} = \frac{1}{2} \left[ 4.2 + 1.13 + 0.04 \right] \]
4. Error in Momentum Ratio

\[ J = \frac{p_j v_j^2}{\rho v^2} \quad \text{where} \quad p, v \quad \text{are cross flow conditions} \]

\[ J = \left( \frac{P_j}{RT_j} \right) v_j^2 \]

where

\[ v_j = \left( \frac{\dot{m}_j}{\rho A_j} \right) \]

\[ v = \left( \frac{\dot{m}}{\rho A} \right) \]

therefore,

\[ J = \left( \frac{\dot{m}_j}{\dot{m}} \right)^2 \frac{P_j A^2 T_j}{\rho^2 A^2 T} \]

Use this expression for \( J \), and write the error equations as follows:

\[ \frac{\delta J}{J} = \frac{1}{J} \left[ \frac{\partial J}{\partial \dot{m}_j} \delta (\dot{m}_j) \right] + \frac{1}{J} \left[ \frac{\partial J}{\partial \dot{m}} \delta (\dot{m}) \right] \]
\[
+ \frac{1}{J} \left[ \frac{\partial J}{\partial P} \delta(P) \right] + \frac{1}{J} \left[ \frac{\partial J}{\partial P_i} \delta(P_i) \right] \\
+ \frac{1}{J} \left[ \frac{\partial J}{\partial T_j} \delta(T_j) \right] + \frac{1}{J} \left[ \frac{\partial J}{\partial T} \delta(T) \right]
\]

Just as in the velocity error calculations:

\[
\frac{1}{J} \left[ \frac{\partial J}{\partial \dot{m}_j} \delta(\dot{m}_j) \right] = \frac{\delta(\dot{m}_j)}{\dot{m}_j}
\]

\[
\frac{1}{J} \left[ \frac{\partial J}{\partial \dot{m}} \delta(\dot{m}) \right] = \frac{\delta(\dot{m})}{\dot{m}}
\]

\[
\frac{1}{J} \left[ \frac{\partial J}{\partial \delta P} \delta(P) \right] = \frac{\delta(P)}{P}
\]

\[
\frac{1}{J} \left[ \frac{\partial J}{\partial \delta P_i} \delta(P_i) \right] = \frac{\delta(P_i)}{P_i}
\]

\[
\frac{1}{J} \left[ \frac{\partial J}{\partial \delta T_j} \delta(T_j) \right] = \frac{\delta(T_j)}{T_j}
\]

\[
\frac{1}{J} \left[ \frac{\partial J}{\partial \delta T} \delta(T) \right] = \frac{\delta(T)}{T}
\]

Estimated uncertainties:

\[
\dot{m}_j = 0.05 \text{CFM} \quad 1.5 \text{CMF} = 0.033
\]

\[
P = \frac{0.5 \text{mm}}{11 \text{mm}} + \frac{1 \text{mm}}{80 \text{mm}} + \frac{1 \text{mm}}{40 \text{mm}} = 0.08295
\]
\[ P = \frac{0.01 \text{ inches Hg}}{29 \text{ inches Hg}} + \frac{1 \text{ mm}}{23 \text{ mm}} = 0.0438 \]

\[ T_0 = \frac{1}{544} + 0.0075 + 0.00025 = 0.0096 \]

(DORIC) (T-C) (TECMAR)

\[ T = \frac{1}{1180} + 0.0075 + 0.00025 + 0.0018 = 0.01040 \]

(DORIC) (T-C) (TECMAR) (CONDUCTION)

Add all of the above according to the Taylor formula, but using the sum of the squares for a less conservative estimate to get

\[ \frac{\delta J}{J} = \pm 18\% \]

5. Error in Density Ratio

\[ \rho_1 = \frac{P_1}{RT_1} \]

\[ \rho = \frac{P}{RT} \]

\[ Dr = \frac{\left( \frac{P_1}{RT_1} \right)}{\left( \frac{P}{RT} \right)} = \frac{P_1T}{P_T} \]

\[ \frac{\partial (Dr)}{Dr} = \frac{1}{Dr} \left[ \frac{\partial Dr}{\partial P_1} \frac{\partial (P_1)}{\partial Dr} + \frac{1}{Dr} \left[ \frac{\partial Dr}{\partial P} \frac{\partial (P)}{\partial Dr} \right] \right] \]
\[ \frac{\delta(Dr)}{Dr} = \frac{\delta P}{P} + \frac{\delta P}{P} + \frac{\delta T}{T} + \frac{\delta T}{T} \]

Using values from the momentum calculation

\[ \frac{\delta(Dr)}{Dr} = \pm 6.4\% \]

6. Error in Tau

\[ \tau = \frac{T_1 - T_3}{T_2 - T_3} \]

where

\[ T_1 = T_k \]
\[ T_2 = T_1 \]
\[ T_3 = T \]

\[ \frac{\delta T}{T} = \frac{1}{T} \left[ \frac{\delta T}{\delta T_1} \delta(T_1) \right] + \frac{1}{T} \left[ \frac{\delta T}{\delta T_2} \delta(T_2) \right] + \frac{1}{T} \left[ \frac{\delta T}{\delta T_3} \delta(T_3) \right] \]

Carrying out the differentiation:

\[ \frac{\delta T}{T} = \frac{\delta(T_1)}{T - T_3} + \frac{\delta(T_2)}{T_2 - T_3} + \delta(T_3) \left[ \frac{1}{T_1 - T_3} + \frac{1}{T_2 - T_3} \right] \]

For conservative values, take
\[ |T_1 - T_3| = 150 \, \text{K} \]
\[ |T_2 - T_3| = 350 \, \text{K} \]
\[ T_1 = 500 \, \text{K} \]
\[ T_2 = 300 \, \text{K} \]
\[ T_3 = 650 \, \text{K} \]

Recall

\[ \frac{\delta(T_1)}{T_1} = \pm 1.13\% \]
\[ \frac{\delta(T_2)}{T_2} = \pm 0.96\% \]
\[ \frac{\delta(T_3)}{T_3} = \pm 1.04\% \]

This gives absolute errors as:

\[ T_1 = \pm 5.65 \, \text{K} \]
\[ T_2 = \pm 2.9 \, \text{K} \]
\[ T_3 = \pm 6.75 \, \text{K} \]

As a less conservative estimate, the square root of the sum of the squares of the terms in the Taylor expansion gives:
\[
\frac{\delta(V)}{V} = \pm 6.5\%
\]

(\pm 3 data point widths)

7. Error in Gamma

\[\gamma = \frac{V_1}{V} - 1\]

\[\frac{\delta\gamma}{\gamma} = \frac{1}{\gamma} \left[ \frac{\partial \gamma}{\partial V_1} \delta(V_1) \right] + \frac{1}{\gamma} \left[ \frac{\partial \gamma}{\partial V} \delta(V) \right]\]

\[\frac{\delta\gamma}{\gamma} = \frac{\delta(V_1)}{V_1 - V} + \frac{\delta(V)}{V_1 - V}\]

Recall

\[\frac{\delta V}{V} = 2.7\%\]

Representative values of \(V, V_1\) are 8.15 m/s

Using a less conservative estimate of the square root of the sum of the squares to get

\[\frac{\delta\gamma}{\gamma} = \pm 6.6\%\]

(\pm 1/4 data point width)
REFERENCES


