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Geometric Error Analysis for Shuttle Imaging Spectrometer Experiment

Shyh Jong Wang
Che-Hang Charles Ih

December 15, 1984

NASA
National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology
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ABSTRACT

The demand of more powerful tools for remote sensing and management of earth resources has been steadily increasing over the last decade. With the recent advancement of area array detectors, high resolution multichannel imaging spectrometers can be realistically constructed.

This report documents the error analysis study for the Shuttle Imaging Spectrometer Experiment system for the purpose of providing information for design, tradeoff, and performance prediction.

Error sources including the Shuttle attitude determination and control system, instrument pointing and misalignment, disturbances, ephemeris, earth rotation, etc., have been investigated. Geometric error mapping functions were developed, characterized, and illustrated extensively with tables and charts. Selected ground patterns and the corresponding image distortions have been generated for direct visual inspection of how the various error sources affect the appearance of the ground object images.
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I. INTRODUCTION

A. BACKGROUND

Earth resource management and utilization have experienced great success over the last decade through the Landsat programs. In more recent years, both NASA and user communities have envisioned the need for development of better and more powerful instruments for surveying and managing earth resources. The Landsat D's new sensor, Thematic Mapper (TM), the proposed utilization of Tracking and Data Relay Satellite System (TDRSS), and a more advanced ground system represent an advancement in the earth resource satellite development [1].

The Thematic Mapper of the Landsat D (launched in 1982) has seven spectral bands, two more than those of the Multispectral Scanner associated with the earlier Landsats. However, study shows that the reflectance spectrum of earth surface materials contains a significant amount of information which can only be identified with spectral resolution much finer than those of the Thematic Mapper [2]. With the advancement of area array detectors, a push broom imaging spectrometer can be realistically constructed for simultaneous imaging and registration of hundreds of spectral bands. For the case of Shuttle Imaging Spectrometer Experiment, 128 spectral channels have been proposed to cover the spectral range of 0.4 to 1.0 µm for VNIR (visible and near infrared) and 1.0 to 2.5 µm for SWIR (short-wavelength infrared) with instantaneous field of view of 30m. Table 1 shows the required sensor performance [2].

The purpose of this report is to document the error analysis study for the imaging spectrometer experiment. Error analysis is an important aspect of the overall remote sensing system, since errors from many sources, including
the spectrometer itself, the spacecraft that carries the instrument, knowledge limitations on the true spacecraft attitude and locations, earth rotation, curvature, and terrain variations, etc., will all contribute to image distortion, shift, rotation, and misregistration. Error correction or compensation are necessary and are an integral part of the image processing. This work covers the analysis of fundamental and geometric errors and error sensitivities, the development of geometric mapping functions, and the computation of ground

Table 1. Sensor Performance Requirements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comments</th>
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<tbody>
<tr>
<td>Spectral Coverage</td>
<td>0.4 to 2.5 µm</td>
<td>Although the entire spectrum is probably not required for any one discipline, in the aggregate of all remote sensing disciplines, the entire region is required</td>
</tr>
<tr>
<td>Spectral Sampling Interval</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VNIR (0.4 to 1.0 µm)</td>
<td>0.01 µm or better</td>
<td>Adequate for most research topics</td>
</tr>
<tr>
<td>SWIR (1.0 to 2.5 µm)</td>
<td>0.02 µm or better</td>
<td></td>
</tr>
<tr>
<td>Instantaneous Field of View</td>
<td>30 m</td>
<td></td>
</tr>
<tr>
<td>Swath Width</td>
<td>at least 10-12 km</td>
<td>This is adequate for research if pointing capability is provided to assure target access</td>
</tr>
<tr>
<td>Pointing Mirror Range</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Along track</td>
<td>at least ± 45 deg</td>
<td>Essential for atmospheric and BRDF (Bidirectional Reflectance Distribution Function) Studies</td>
</tr>
<tr>
<td>C.crosstalk</td>
<td>not more than ± 25 deg</td>
<td></td>
</tr>
<tr>
<td>Radiometric Performance (NEdR)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VNIR</td>
<td>&lt; 0.5%</td>
<td></td>
</tr>
<tr>
<td>SWIR</td>
<td>&lt; 1.0%</td>
<td></td>
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</table>
pattern distortions. The results are translated into many tables, plots, and patterns for visual apprehension. It is believed that the results reported here are important for design, trade-off, and performance prediction.

B. APPROACH

This study has been performed in four progressive stages as shown in block diagram form in Fig. 1. In Stage I, the error sources were identified, dynamic disturbances and the Space Shuttle error dynamics were modeled, and the error power spectral densities for two in-orbit configurations were developed. Stage II of this study concentrated on the development of geometric error mapping functions and error sensitivity analysis. Geometric errors due to ephemeris uncertainties, attitude deviations, earth rotation, etc., were studied. In Stage III, ground pattern image distortions caused by various error effects and forward and side looking angular offsets, altitude change, and effect of earth rotation were generated. Stage IV consists of the analysis of the imaging spectrometer instrument errors. These errors include optical jitter, nonlinearities, processing errors, and repeatability. The study of Stages I, II, and III has been completed and the results are included in this report. The study called for by Stage IV has not been planned. It is emphasized here that the imaging spectrometer error model development is an important step for the overall performance prediction and design of the imaging spectrometer system.

Major findings of this work are summarized in the following subsection. In Section II, the orbital and imaging spectrometer configurations are described. The attitude dynamics and error power spectral densities are documented in Section III; and parametric analysis of geometric errors are treated in Section IV. Section V deals with the ground pattern image distortions. Conclusions are summarized in Section VI.
Figure 1. IS Error Analysis System Block Diagram
C. SUMMARY OF MAJOR FINDINGS

The following is a summary of major findings. The details of these are treated in the sections to follow.

1. The results show that the IS Experiment with image pickup period of 20 seconds at a time is feasible with the shuttle properly phased inside the control deadband. The error PSD (power spectral density) characteristic reveals that the system resonates at very low frequencies (in the $10^{-5}$ to $10^{-2}$ Hz region). Excitations at these frequencies must be avoided through design precautions. The analysis also showed that errors below 0.01 Hz are dominated by the shuttle dynamics reacting to disturbances, whereas those above 0.01 Hz are dominated by the shuttle inertial measurement system uncertainties and its inherent noise. These high frequency errors limit pointing performance and result in a one-sigma ground track error of 54.8 meters per axis. One-sigma rate errors are shown to be less than 4 meters per second per axis in the frequency range of $10^{-5}$ to $4 \times 10^{-2}$ Hz. The image smear is not significant because of the short millisecond-level line time.

2. The effects of earth curvature are very small for the application here (see Fig. 42).

3. Altitude uncertainties cause only moderate geometric errors. The worst 1σ geometric errors are 11.71 m in position and 0.133 m/sec in rate with STDN and large unmodeled perturbations at 200 km orbit. The performance improves with TDRSS. For the 300 km orbit and with small unmodeled perturbations, the 1σ geometric errors will reduce to 0.22m and 0.0032 m/sec (see Table 11).
4. The effects of other navigation errors are significantly greater. The downtrack errors range from 203m (300 km orbit) to 8128m (200 km orbit); and those for the crosstrack are 152m to 508m (see Table 12).

5. The effects of roll and pitch attitude errors are relatively large compared with, for instance, those caused by yaw errors and altitude uncertainties. The error sensitivity is 1.94 m/arc sec or approximately 7000 m/degree. The yaw sensitivity is 0 for 0° view angle and 0.028 m/arc sec for the maximum view angle of ±0.825° (see Table 13).

6. The error sensitivities of attitude errors increase significantly for large attitude offsets. For 20° side looking, the sensitivity increases to 2.28 m/arc sec for roll errors and to 0.75 m/arc sec for yaw errors and the pitch error sensitivity is almost unaffected. For the 45° forward looking case, the sensitivities for the pitch, roll, and yaw errors increase to 2.83, 4.36, and 1.97 m/arc sec, respectively (see Table 13).

7. The performance of the Shuttle Imaging Spectrometer is limited by the Shuttle IMU (Inertial Measuring Unit) accuracy, instrument misalignment, etc., Shuttle RCS (Reaction Control Subsystem) deadband, etc. unless some means of error reduction are employed. For instance, ground control points may be used to reduce navigation prediction errors; and precision point mounts, such as AGS (ASPS* Gimbal System) and IPS (Instrument Pointing System), may be used to reduce the

*Annular Suspension Pointing System
attitude errors.

The single axis geometric errors due to the combined Shuttle/IS misalignment, for instance, for normal nadir pointing, 20° side looking, and 45° forward looking are 169m, 198m, and 379m, respectively (refer to Table 10 for detailed breakdowns).

8. Earth rotation causes shifts of images toward the direction of rotation. The magnitude of this shift depends on the latitude of the object. For instance, at the equator the object moves approximately 462m in one second (for 400 km orbit), and at 60° latitude it moves only 231m in one second.
II. ORBITAL AND IMAGING SPECTROMETER CONFIGURATIONS

A. SHUTTLE NOMINAL FLIGHT CONFIGURATIONS

Two shuttle in-orbit configurations have been considered, the Payload-Bay Nadir and the Nose-Down Nadir, as illustrated in Figures 2 and 3, respectively. It will be shown later that the Nose-Down Configuration is gravity gradient stabilized and the Payload-Bay Nadir Configuration is not. However, the Payload-Bay Nadir Configuration offers simpler instrument mounting and less aerodynamic drag. Besides, for certain experiments that require large forward looking angles, the Nose-Down Configuration will be unsuitable.

A circular orbit of 400 km altitude has been selected for this analysis. This selection is consistent with the SIS-B parameters.*

There are a number of possible instrument mounting options that will affect the pointing and the geometric errors of the instrument. These options include:

a) Direct mount
b) AGS (ASPS Gimbal System) mount
c) IPS (Instrument Pointing System) mount

Since both the AGS and IPS systems are capable of providing precision payload pointing and measurements, the system performance will be improved at the expense of significant extra cost. The direct mount approach is the least complex and most cost effective, provided that the errors are within the tolerance. In this report the direct mount approach is considered. Furthermore, the shuttle IMU

*During the period when this part of the work was performed, SIS-B (Shuttle Imaging Spectrometer-Configuration B) was considered. However, the methods used here are mostly generic, and hence, can be applied to systems of similar configurations.
Figure 2. Payload-Bay Nadir Pointing Configuration (A)

Figure 3. Nose-Down Nadir Pointing Configuration (B)
(Inertial Measurement Unit) and the shuttle state estimator unit are used for obtaining attitude and rate information, without additional instrumentation. Other options may be studied in the future if necessary.

B. IMAGING SPECTROMETER SYSTEM DESCRIPTION

In order to correlate the analysis reported here to the IS applications, it is desirable to understand the basic operating principle of the IS instrument. Figure 4 shows a sketch of the basic elements of the IS, except the memory banks, registers, and the processors. Some of the IS parameters that are relevant to the geometric error analysis, along with definitions of the IS terminologies, are also listed in Fig. 4. It is understood that the sketch and the parameter values used are for illustrative purposes only since the parameters may change as the development of the IS system is finalized.

Referring to Fig. 4, as the reflection from the ground objects passes through a narrow slit, it strikes the incident surface of the prism. The prism separates the incident light into spectral images projected onto an area array of light detectors. The area array consists of 384 linear arrays, corresponding to 384 spectral channels. Each linear array consists of 384 detector elements. Each detector element corresponds to an image area of 30m x 30m on the ground. Hence, each linear array corresponds to an image of a specific spectral channel of the same ground area of 30m (along-track) x 11520m (cross-track). This 11.52 km cross-track measure, referred to as the swath width, defines the IS coverage for each orbit pass. The 30m x 30m area is referred to as the pixel (picture element) which defines the resolution of the image, i.e., within this element no features can be resolved. The IS is operated based on a push broom principle. That is, a specific detector element on the instrument (moving with the spacecraft) collects photons from the 30m x 30m moving window for a specific period
KEY PARAMETERS
• LINE TIME = 4 ms
• SPECTRUM:
  VISIBLE REGION 0.4 – 1.0 μm
  SHORT WAVELENGTH IR 1.0 – 2.5 μm
• FIELD OF VIEW: ± 0.825°
• SWATH WIDTH: 11.52 km
• POINTING AngLES (RANGE):
  ± 45° (PITCH)
  ± 20° (ROLL)
• ORBIT: 400 km CIRCULAR

DEFINITIONS
GIFOV (GROUND INSTANTANEOUS FIELD OF VIEW): PROJECTION ON THE GROUND OF EACH SQUARE DETECTOR
LINE TIME: TIME TO MOVE ALONG THE GROUND A DISTANCE OF 1 GIFOV
DN: DATA NUMBER, THE "BRIGHTNESS" OF THE ASSOCIATED PIXEL
PIXEL: PICTURE ELEMENT (30 m x 30 m HERE, IT DEFINES THE RESOLUTION OF THE IMAGE)
SWIR: SHORT WAVE LENGTH IR
VIS: VISIBLE WAVE LENGTH

Figure 4. Imaging Spectrometer Basic Operating Principles
of time called the line time. The line time in this case is the time required to move 30m along-track, which is determined by the orbit. For a 400 km circular orbit, it is about 4 ms. At the end of each line time, the total number of photons collected by each detector is recorded and processed and the detector/ register is reset and a new 4 ms cycle is repeated. For digital processing, the amount of light collected by a collector is assigned a number called DN (data number), which is proportional to the number of photons accumulated. Therefore, the processor has to record and process 384 x 384, or approximately $1.475 \times 10^5$ DN's every 4 ms.

As the shuttle flies over an area, banks after banks of DN's are collected. By spacing the banks of DN's 30m apart, the features of the ground image emerge. The ratios of the DN's of various channels for the same area are of special interest, as these ratios are closely correlated to geological and ecological states of the earth including mineral deposits, forestry, crops,

Figure 5. Shuttle Imaging Spectrometer
Optical System Configuration
disease and insect infestations, land and soil erosion, precipitation in
the form of snow and ice, air and water quality, etc.

The Imaging Spectrometer optical system configuration and the arrange-
ment of lenses, mirrors, slit, prisms, focal plane detector, etc. are
shown in Fig.5 [2]. Additional information on the instrument design and require-
ments can be found in Refs. 2 and 3.
III. ERROR SOURCES AND SHUTTLE ATTITUDE DYNAMICS

A. ERROR SOURCES

There are two types of errors that are important to the imaging spectrometer experiment — the direct errors and the derived errors. The direct errors are those pertaining to the spacecraft and instrument pointing, ephemeris, and instrument errors as shown in Fig. 6. The derived errors are the geometric errors and pattern distortions of ground objects which result from the direct errors and the effects of earth rotation, curvature, oblateness, and local vertical uncertainties. Fig. 1 shows relationships of the error sources and the system dynamics.

- Attitude Deviations, Rate Errors and Structural Vibration
- Measurement Noise and Drift
- Misalignment of Shuttle IMU and Attitude Reference Frame
- Misalignment Between Shuttle and IS Instrument References
- Ephemeris Prediction Errors
- Earth Rotation, Curvature, and Oblateness
- IS Instrument Errors Including
  - Optical Jitter
  - Nonlinearities
  - Processing
  - Repeatability

Figure 6. Error Sources
In this section, the steady state analysis is performed in the frequency domain. The PSD (power spectral density) for the pointing errors and the rate errors were obtained by considering the dynamics of the Space Shuttle Orbiter, the IMU (Inertial Measurement Unit), the attitude control state estimator, the measurement noise, the misalignment of reference frames, and the disturbances including gravity gradient, gyroscopic torques, and aero-dynamic drag torques.

B. SHUTTLE MASS PROPERTY

The shuttle mass properties employed here were obtained from the Shuttle Operational Data Book [4] for OV-102/STS-3. The mass and the c.m. are, respectively:

Shuttle mass: 102,153.73 kg (224,738.21 lbs)

Shuttle c.m.: \( x_o = 1105.5", y_o = 0", z_o = 374.3" \)

where the coordinates \( x_o, y_o, \) and \( z_o \) are the Orbiter Coordinates [5] defined in Figure 7. The moment of inertia matrix (kg-m\(^2\)) is, in the shuttle body frame (refer to Fig. 8),

\[
I_B = \begin{bmatrix}
1.36 \times 10^6 & -4.69 \times 10^3 & -3.49 \times 10^5 \\
-4.69 \times 10^3 & 1.00 \times 10^7 & -3.32 \times 10^3 \\
-3.49 \times 10^5 & -3.32 \times 10^3 & 1.05 \times 10^7
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1.36 \times 10^6 & 0 & -3.49 \times 10^5 \\
0 & 1.00 \times 10^7 & 0 \\
-3.49 \times 10^5 & 0 & 1.05 \times 10^7
\end{bmatrix} \quad (i)
\]

The magnitude of the off-diagonal terms of the inertia matrix \( I_B \) suggests strong couplings exist especially between the \( x_o \) and the \( z_o \)-axis. To simplify the dynamic equations, it is convenient to consider principal-axis
ORBITER COORDINATES

TYPE: Rotating, Orbiter referenced

ORIGIN: Approximately 200 inches (5.1m) ahead of the nose and approximately 400 inches (10.2m) below the centerline of the cargo bay

ORIENTATION AND LABELING:

The X-axis is parallel to the centerline of the cargo bay, negative in the direction of launch.

The Z-axis is positive upward in landing attitude.

The Y-axis completes the right-handed system.

The standard subscript is 0.

Figure 7. Orbiter Coordinate System
pointing instead of body-axis pointing, e.g., pointing $-Z_p$ rather than $-Z_B$ as illustrated in Figure 8.

(Figure 8. Shuttle Body and Principal Coordinates (Payload-Bay Nadir Pointing Shown))

The orientations of the principal axes can be determined by rotating the body frame an angle $\alpha$ about the $Y_B$-axis, i.e., let $B: X_B \rightarrow X_p$, then

$$B = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix}$$  \hspace{1cm} (2)$$

and

$$I_p = B I_B B^T$$  \hspace{1cm} (3)$$

It can be shown that
\[ \alpha = \frac{1}{2} \tan^{-1} \left( \frac{-2 I_{BXX}}{I_{BZZ} - I_{BYY}} \right) \]

(4)

provided that \( I_{BXY} = I_{BYZ} = 0 \).

For \( I_{BXX} = 1.36 \times 10^6 \text{ kg-m}^2 \), \( I_{BZZ} = 1.05 \times 10^7 \text{ kg-m}^2 \), and \( I_{BZX} = 3.49 \times 10^5 \text{ kg-m}^2 \), the angle \( \alpha = -2.18^\circ \). Therefore, in order to do principal-axis nadir pointing, the shuttle has to rotate \( 2.18^\circ \) about the orbit normal (see Figure 8). The moments of inertia about the principal axes are

\[ I_p = \text{diag}(1.38 \times 10^6, 1.00 \times 10^7, 1.05 \times 10^7), \text{ kg-m}^2. \]

C. ASSESSMENT OF DISTURBANCES

The shuttle motion is characterized in part by the environmental disturbances, the major sources of which are the aerodynamic drag, gravity gradient, gyroscopic effect, solar radiation, and on-board causes such as astronaut activities, equipment vibrations, and venting. On-board activities may be partially eliminated or reduced through mission planning and their effects will be assessed in the future. In this subsection, the gyroscopic torques, the gravity gradient torques, and the aerodynamic drag torques are estimated. The solar pressure is at least one order of magnitude less than the aerodynamic drag forces and, hence, it is not included in this analysis.

C.1 The Aerodynamic Drag Torques

To estimate the aerodynamic drag torques, an approach referred to as the three-plate model [6] has been used. Referring to Figure 9, let \( \mathbf{n}_1, \mathbf{n}_2, \) and \( \mathbf{n}_3 \) be the unit normal vectors for the equivalent plates 1, 2, and 3. Where \( \mathbf{n}_1, \mathbf{n}_2, \) and \( \mathbf{n}_3 \) are in the direction of \( X_B, Y_B, \) and \( Z_B \)-axis, respectively.

To express \( \mathbf{n}_4 \) in body frame,
Let $A_1$, $A_2$, and $A_3$ be the corresponding areas on which the aerodynamic pressure applies, and $\overline{c_{p1}}$, $\overline{c_{p2}}$, and $\overline{c_{p3}}$ the three corresponding centers of pressure, respectively. The model assumes that the aerodynamic force applies to each area in the direction opposite to the vehicle velocity vector and assumes no shading among the plates. It is further assumed that the drag coefficient $C_D$ is constant and the lift coefficient $C_L$ is zero.

Let $\overline{v_B} = (v_{B1}, v_{B2}, v_{B3})^T$ be the inertial velocity vector in the body frame, with magnitude $v$. The force and torque applied on $A_i$ are

$$\overline{F_{Bi}} = - \left( \frac{1}{2} C_D \rho \overline{v^2} A_i \right) |\overline{n_i}, \overline{u_B}| \overline{u_B}$$

$$\overline{T_{Bi}} = \overline{F_{Bi}} \times \overline{F_{Bi}} = - \left( \frac{1}{2} C_D \rho \overline{v^2} A_i \right) |\overline{n_i}, \overline{u_B}| \overline{r_{Bi}} \times \overline{u_B}$$

where $\rho$ is the atmospheric density at the orbital altitude, $\overline{u_B} = \overline{v_B}/v$ the unit velocity vector, and $\overline{r_{Bi}}$ the vectors of the center of pressure of $A_i$ relative to the vehicle center of mass.
The total aerodynamic forces and torques on the vehicle are

\[ \overline{F}_B = \sum_{i=1}^{3} - \left( \frac{1}{2} C_D \rho v^2 A_i \right) \left| \overline{\nu}_i \cdot \overline{u}_B \right| \overline{u}_B \]

(8)

\[ \overline{T}_B = \sum_{i=1}^{3} - \left( \frac{1}{2} C_D \rho v^2 A_i \right) \left| \overline{\nu}_i \cdot \overline{r}_{B1} \times \overline{u}_B \right| \]

(9)

The value of \( C_D \) depends on the shape of the vehicle. In Ref. 7, the values of 2.5 to 3.0 were suggested. The value of \( C_D = 2.0 \) is used here as it was used in Ref. 6 for Space Shuttle Simulation.

The atmospheric mass density \( \rho \) can be found in a JPL internal memorandum. For 1985 mission time, the peak density is expected to occur in April for the 400 km orbit; the densities are

\[ 2.64 \times 10^{-12} \text{ kg/m}^3 \quad \text{(predicted)} \]

and

\[ 3.77 \times 10^{-12} \text{ kg/m}^3 \quad \text{(97.7 percentile)} \]

The orbital velocity \( v \) for a 400 km circular orbit is computed as

\[ v = 7669.60 \text{ m/sec} \quad \text{or} \quad v^2 = 5.882 \times 10^7 \text{ (m/sec)}^2 \]

The plate areas, according to the attachment (SSFS On-Orbit Aero Data, 7/24/80) to Ref. 6, with Cargo Bay doors open, are

\[ A_1 = 119.45 \text{ m}^2 \]
\[ A_2 = 229.92 \text{ m}^2 \]
\[ A_3 = 454.46 \text{ m}^2 \]

(10)

However, a different set of values are given in Ref. 8.

\[ A_1 = 64.1 \text{ m}^2 \]
\[ A_2 = 212.7 \text{ m}^2 \]
\[ A_3 = 367.0 \text{ m}^2 \]

(11)
The values in (10) were used in this work.

The position vectors $\mathbf{r}_{B1}$ may be obtained using data given in Ref. 4 and Ref. 6. The values are,

$$\begin{align*}
\mathbf{r}_{B1} &= \begin{bmatrix} 3.797 \\ 0 \\ -0.231 \end{bmatrix} \text{ m} \\
\mathbf{r}_{B2} &= \begin{bmatrix} 0.927 \\ 0 \\ -0.742 \end{bmatrix} \text{ m} \\
\mathbf{r}_{B3} &= \begin{bmatrix} 1.166 \\ 0 \\ -1.074 \end{bmatrix} \text{ m}
\end{align*}$$

The torques $\mathbf{\tau}_{B1}$ for the predicted density, are

$$\begin{align*}
\mathbf{\tau}_{B1} &= -1.855 \times 10^{-2} |u_{B1}| \begin{bmatrix} -r_{B13} u_{B2} + r_{B12} u_{B3} \\ r_{B13} u_{B1} + r_{B11} u_{B3} \\ -r_{B12} u_{B1} + r_{B11} u_{B2} \end{bmatrix} \\
\mathbf{\tau}_{B2} &= -3.570 \times 10^{-2} |u_{B2}| \begin{bmatrix} -r_{B23} u_{B2} + r_{B22} u_{B3} \\ r_{B23} u_{B1} - r_{B21} u_{B3} \\ -r_{B22} u_{B1} + r_{B21} u_{B2} \end{bmatrix} \\
\mathbf{\tau}_{B3} &= -7.057 \times 10^{-2} |u_{B3}| \begin{bmatrix} -r_{B33} u_{B2} + r_{B32} u_{B3} \\ r_{B33} u_{B1} - r_{B31} u_{B3} \\ -r_{B32} u_{B1} + r_{B31} u_{B2} \end{bmatrix}
\end{align*}$$

$$(12)$$
In Eq. (13), the torques are known once the unit velocity vector \( \overrightarrow{u}_B \) is specified; \( \overrightarrow{u}_B \) varies with the pointing configuration and spacecraft attitude.

C.1.1. Drag Torques for Configuration A

Referring to Fig. 2, since this is "-Zp" pointing, for nominal attitude, the vehicle moves in the \( X_p \) direction. Let \( \overrightarrow{u}_p \) be the unit velocity vector in the \( P \)-frame and let \( A_\alpha \) be the rotation matrix due to \( \alpha \), then

\[
\overrightarrow{u}_B = A_\alpha^T \overrightarrow{u}_p = \begin{bmatrix}
\cos\alpha & 0 & \sin\alpha \\
0 & 1 & 0 \\
-\sin\alpha & 0 & \cos\alpha
\end{bmatrix} \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
\cos\alpha \\
0 \\
-\sin\alpha
\end{bmatrix}
\]  

(14)

Since \( \alpha(= -2.18^\circ = -0.038 \text{ rad}) \) is small,

\[
\overrightarrow{u}_B = \begin{bmatrix}
1 \\
0 \\
-\alpha
\end{bmatrix}
\]  

(15)

Using Eqs. (12), (13), and (15), and

\[
\overrightarrow{T}_p = \sum_{i=1}^{3} A_\alpha \overrightarrow{T}_{B_i}
\]

(16)

the torques in the \( P \)-Frame, for nominal attitude, are

\[
\overrightarrow{T}_p = \begin{bmatrix}
0 \\
9.84 \times 10^{-3} \\
0
\end{bmatrix} \text{ N-m}
\]  

(17)

For small attitude errors, \( \phi \), \( \theta \), and \( \psi \), from the rotating orbital frame,

\[
\overrightarrow{u}_p = A \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]  

(18)
where,

\[ A = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ 0 & -\phi & 1 \end{bmatrix} \]

and

\[ \bar{u}_B = A^T \bar{u}_A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (20) \]

Using Eqs. (12), (13), and (20), the predicted aerodynamic torques become, by retaining only the first order terms, in N\cdot m,

\[ \bar{T}_P = \sum_{i=1}^{3} A_\alpha \bar{T}_{B1} \]

\[ = \begin{bmatrix} 4.29 \times 10^{-3} \psi \\ 4.29 \times 10^{-3} + 7.04 \times 10^{-2} (\theta - \alpha) + 7.58 \times 10^{-2} |\theta - \alpha| + 2.65 \times 10^{-2} |\psi| \\ 7.04 \times 10^{-2} \psi \end{bmatrix} \]

C.1.2. Drag Torques for Configuration B

Referring to Figure 3, under this configuration, \( X_p \) is in the Nadir direction and \( Y_p \) is in the direction of motion for nominal attitude. In this case,

\[ \bar{u}_p = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (22) \]

\[ \bar{u}_B = A^T \bar{u}_p = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (23) \]

and from Eqs. (12), (13), and (23),

\[ \bar{T}_P = \sum_{i=1}^{3} A_\alpha \bar{T}_{B1} \]
\[
\begin{bmatrix}
-3.96 \times 10^{-2} \\
0 \\
-4.53 \times 10^{-2}
\end{bmatrix}
\text{N-m} 
\]

In the presence of small attitude errors, \(\phi\), \(\theta\), and \(\psi\), the aero-
dynamic drag torques are

\[
\mathbf{T}_p = \begin{bmatrix}
-2.45 \times 10^{-2} - 7.58 \times 10^{-2} |\phi| -4.29 \times 10^{-3} |\phi| +3.31 \times 10^{-2} \\
-3.31 \times 10^{-2} \phi + 2.65 \times 10^{-2} \phi \\
-(3.31 \times 10^{-2} + 8.23 \times 10^{-2} |\theta| +7.04 \times 10^{-2} |\theta| +2.65 \times 10^{-2} \phi)
\end{bmatrix}
\]  

C.1.3. Estimation of Disturbance PSD

Before the PSD's are estimated, the uncertainty part of the distur-
bance torques has to be determined. The torques of Eqs. (21) and (25) consist
of static parts and the dynamic parts. The dynamic parts are functions of the
attitude errors \(\phi\), \(\theta\), and \(\psi\). The attitude errors are assumed to be random and time-
varying with standard deviation of 1° (.01745 rad.) per axis. Therefore, the
estimated values of the random disturbance torques are, for Configuration A

\[
\bar{\sigma}_{T_{PA}} = \begin{bmatrix}
7.48 \times 10^{-5} \\
1.86 \times 10^{-3} \\
1.23 \times 10^{-3}
\end{bmatrix}
\text{N-m} 
\]

and that for Configuration B are,

\[
\bar{\sigma}_{T_{PB}} = \begin{bmatrix}
1.33 \times 10^{-3} \\
7.40 \times 10^{-4} \\
1.89 \times 10^{-3}
\end{bmatrix}
\text{N-m} 
\]

and the corresponding PSD's are obtained by the following approximation with
the correlation time of \(T = 180\) seconds,

\[
Q_{T_{PA}} = 2T \bar{\sigma}_{T_{PA}}^2
\]
The Gravity Gradient Torques and Gyroscopic Torques

The gravity gradient torques and the gyroscopic torques can be estimated using the following equations, respectively:

\[
\begin{align*}
\ddot{Q}_{T,\text{PB}} & = 2T \left( \sigma_{T,\text{PB}}^2 \right) \\
& = \begin{bmatrix}
2.01 \times 10^{-6} \\
1.25 \times 10^{-3} \\
5.44 \times 10^{-4}
\end{bmatrix} (N\cdot m)^2 \cdot \sec
\end{align*}
\]

(28)

\[
\begin{align*}
\dot{Q}_{\text{GRP}} & = -O_{\text{PB}} \times \omega_{\text{PB}} \\
& = \begin{bmatrix}
6.32 \times 10^{-6} \\
1.97 \times 10^{-6} \\
1.29 \times 10^{-3}
\end{bmatrix} (N\cdot m)^2 \cdot \sec
\end{align*}
\]

(29)

C.2 The Gravity Gradient Torques and Gyroscopic Torques

The gravity gradient torques and the gyroscopic torques can be estimated using the following equations, respectively:

\[
\ddot{T}_{\text{GRP}} = 3\omega_0^2 u_{\text{RP}}^T I_F u_{\text{RP}}
\]

(30)

and

\[
\ddot{T}_{\text{GRP}} = \omega_{\text{OP}} \times \hat{H}_P = \omega_{\text{OP}}^T I_F \omega_{\text{OP}}
\]

(31)

Where \( u_{\text{RP}} \) and \( \omega_{\text{OP}} \) are the unit earth vector and the spin velocity vector, respectively, in principal frame, and \( u_{\text{RP}}^T \) is the skew symmetric matrix of the vector \( u_{\text{RP}} \). For Configuration A,

\[
\ddot{u}_{\text{RP}} = A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]

(32a)

and for Configuration B,

\[
\ddot{\omega}_{\text{OP}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \omega_0
\]

(32b)
\[ \mathbf{\tau}_{RP} = A \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \]  
(33a)

\[ \mathbf{\omega}_P = A \begin{bmatrix} 0 \\ 0 \\ \mathbf{\omega}_o \end{bmatrix} \]  
(33b)

The corresponding torques, to include only the first order effect, are, for

Configuration A

\[ \mathbf{\tau}_{gP} = 3 \mathbf{\omega}_o^2 \begin{bmatrix} (I_{PZZ} - I_{PYY})^\phi \\ (I_{PZZ} - I_{PXX})^\theta \\ 0 \end{bmatrix} = \begin{bmatrix} 1.92^\phi \\ 33.11^\theta \\ 0 \end{bmatrix} \text{ N-m} \]  
(34a)

\[ \mathbf{\tau}_{GRP} = \mathbf{\omega}_o^2 \begin{bmatrix} - (I_{PZZ} - I_{PYY})^\phi \\ 0 \\ (I_{PYY} - I_{PXX})^\psi \end{bmatrix} = \begin{bmatrix} -.64^\phi \\ 0 \\ 11.04^\psi \end{bmatrix} \]  
(34b)

And for Configuration B

\[ \mathbf{\tau}_{gP} = - 3 \mathbf{\omega}_o^2 \begin{bmatrix} 0 \\ (I_{PZZ} - I_{PXX})^\theta \\ (I_{PXX} - I_{PYY})^\phi \end{bmatrix} = \begin{bmatrix} 0 \\ -35.03^\theta \\ 43.71^\phi \end{bmatrix} \]  
(35a)

\[ \mathbf{\tau}_{GRP} = \mathbf{\omega}_o^2 \begin{bmatrix} - (I_{PZZ} - I_{PYY})^\phi \\ (I_{PZZ} - I_{PXX})^\theta \\ 0 \end{bmatrix} = \begin{bmatrix} -.64^\phi \\ 11.68^\theta \\ 0 \end{bmatrix} \]  
(35b)

Since the gravity gradient torques and gyroscopic torques are proportional to the attitude errors, they may be included in the equations of motion.
as part of the vehicle dynamics rather than disturbances to the plant.

D. THE MISALIGNMENT AND MEASUREMENT NOISE

In this subsection, the errors that will contribute to pointing uncertainties are estimated. These error sources include the misalignment errors of the IMU and the IS instrument, and the sensor noise.

D.1 The Misalignment Errors

Let \( b_{MS} \) be the misalignment between the IMU and the shuttle reference frame; let \( b_{SI} \) be the misalignment between the shuttle and the imaging spectrometer reference frame.

Based on the space shuttle performance requirement [5], the IMU 3\( \sigma \) misalignment uncertainty is \( \pm 0.133^\circ / \text{axis} \), hence,

\[
\sigma_{b_{MS}} = 0.044^\circ / \text{axis} = 159.6 \text{ arc-sec/axis} \quad (36a)
\]

However, based on a JPL internal memorandum, the shuttle flight test performance is much better,

\[
\sigma_{b_{MS}} = 82 \text{ arc-sec/axis} \quad (36b)
\]

The performance data Eq. (36b) was used in this report.

The misalignment data for the IS instrument is not directly available. Based on Ref. 9, the estimated LANDSAT D initial alignment bias between the sensor optical axis and the vehicle pointing vector was \( \pm 200 \) arc-sec. The alignment bias can be measured before launch and can be removed from the image data. The variation part that cannot be removed without ground control points was estimated as \( \pm 30 \) arc-sec. This latter number was used in this work,

\[
\sigma_{b_{SI}} = 30 \text{ arc-sec} \quad (37)
\]

Therefore, the total misalignment error becomes

\[
\sigma_b = (\sigma_{b_{MS}}^2 + \sigma_{b_{SI}}^2)^{1/2} = 87.32 \text{ arc-sec.} \quad (38)
\]
D.2 The IMU Model

The sensor dynamics for the shuttle IMU was modeled approximately by the first order low pass filter as shown in Figure 10.

\[
\frac{1}{1 + \tau s}
\]

Figure 10. Simplified IMU Model

Where \( \tau \) is the time constant and \( v \) the white gaussian noise. The time constant was estimated from a DRIRU II Model,

\[
\tau = \frac{1}{\omega_s} = \frac{1}{14\pi} = .023 \text{ sec}
\]  
(39)

That is, this measurement is assumed to have a 7 Hz bandwidth.

To determine \( v \), from Ref. 5, the 3\( \sigma \) IMU readout error is \( \pm .073^\circ/\text{axis} \).

Assume that the measurement noise is the readout error, then

\[
\sigma_v = .0243^\circ/\text{axis} = 87.6 \text{ arc-sec/axis}
\]
(40)

However, the actual performance of the shuttle IMU was much better; it has a 1\( \sigma \) gyro resolution error (\( \sigma_{\text{RESO}} \)) of 20 arc-sec/axis. If we assume the random noise has the same amplitude as that of the resolution noise, then

\[
\sigma_v = \sqrt{2} \sigma_{\text{RESO}}
\]
(41)

The corresponding measurement noise PSD, \( R \), is estimated as,

\[
R = \frac{1}{14\pi} \sigma_v^2
\]
\[
= 8.55 \times 10^{-10} (\text{rad})^2 \text{ -sec}
\]
(42)
E. SHUTTLE DYNAMICS AND INSTRUMENT POINTING ERRORS

E.1 The Equations of Motion

Consider Configuration A. For simplicity, the subscript P for principal axes is dropped from all equations. Again, let \( \phi \), \( \theta \), and \( \psi \) be the small roll, pitch, and yaw angles, respectively. Let \( \bar{\omega} = (\omega_x, \omega_y, \omega_z)^T \) be the angular velocity vector of the shuttle, then for small attitude errors,

\[
\begin{align*}
\dot{\omega}_x &= \dot{\phi} + \dot{\psi} \omega_0 \\
\dot{\omega}_y &= \dot{\theta} + \omega_0 \\
\dot{\omega}_z &= \dot{\psi} - \dot{\phi} \omega_0
\end{align*}
\]  

and

\[
\begin{align*}
\dot{\omega}_x &= \dot{\phi} + \dot{\psi} \omega_0 \\
\dot{\omega}_y &= \dot{\theta} \\
\dot{\omega}_z &= \dot{\psi} - \dot{\phi} \omega_0
\end{align*}
\]

With this simplified relation between the inertial rates and the attitude error rates, one can show that the equations of motion may be summarized as follows, accounting only for first order effects:

\[
\begin{bmatrix}
I_x \ddot{\phi} + (I_x - I_y + I_z) \omega_0 \dot{\psi} + 4 (I_y - I_z) \omega_0^2 \phi \\
I_y \ddot{\theta} + 3 (I_x - I_z) \omega_0^2 \theta \\
I_z \ddot{\psi} - (I_x - I_y + I_z) \omega_0^2 \phi - (I_x - I_y) \omega_0^2 \psi
\end{bmatrix} = \bar{T}_d + \bar{T}_c
\]

The left-hand side of Eq. (45) accounts for the body dynamics and the gravity gradient and the gyroscopic torques; and on the right-hand side of Eq. (45) \( \bar{T}_d \) and \( \bar{T}_c \) are the disturbance torques and the control torques, respectively.

E.2 The Space Shuttle State Estimator

The Shuttle State Estimator consists of two parallel Kalman-type filters, one for acceleration estimation and one for rate estimation [10]. The attitude estimate is the extrapolation of the measured attitude and the rate estimate may be approximated by using a second order filter with parameters.
determined by the filter gains. With the current baseline filter data, the
equivalent damping coefficient and the corner frequency for the rate filter are
.8 and .04 Hz (for vernier rate filter), respectively [11].

E.3 The System Pointing Errors

E.3.1. The Pitch Loop (Configuration A)

From Eq. (45) and ignoring the initial conditions, one has the

following output function,

$$
\theta(s) = \frac{1}{I_Y S^2 + 3\omega_o^2 (I_X - I_Z)} \left( T_d Y + T_c Y \right)
$$

Figure 11 shows the open-loop block diagram for the dynamics of the instrument
pointing error excited by the random disturbances. Included in the diagram are
the dynamics of the vehicle, the IMU, and the rate filter. Measurement error
and the misalignment error are also included.

E.3.2. The Roll and Yaw Loops (Configuration A)

The output equations for the coupled roll and yaw dynamics are, from

Eq. (45)

$$
\begin{bmatrix}
\phi(s) \\
\psi(s)
\end{bmatrix} = \frac{1}{D(s)} \begin{bmatrix}
I_Z S^2 + \omega_o^2 (I_Y - I_X) & -\omega_o (I_X - I_Y + I_Z) \\
\omega_o (I_X - I_Y + I_Z) & I_X S^2 + 4 \omega_o^2 (I_Y - I_Z)
\end{bmatrix}
\begin{bmatrix}
T_X \\
T_Z
\end{bmatrix}
$$

where

$$
D(s) = I_X I_Z S^4 + \omega_o^2 \left( I_Z I_Y + I_Y^2 + 3 I_Y I_Z - 3 I_Z^2 \right) S^2 + 4 \omega_o^4 (I_Y - I_X)(I_Y - I_Z)
$$

and

$$
T_X = T_{cX} + T_{dX}
$$

$$
T_Z = T_{cZ} + T_{dZ}
$$

Figure 12 shows the block diagram for the roll and yaw instrument error dynamics.
The block diagrams for the pitch and the roll-yaw loops are similar except for
the coupling terms for the roll and yaw axes.

![Pitch-Axis Block Diagram (Configuration A)](image)

Figure 11. Pitch-Axis Block Diagram (Configuration A)

E.4 The Instrument Pointing Error PSD

E.4.1 The Pitch Error PSD (Configuration A)

The PSD's for the pitch-axis instrument pointing error and the rate error are,

\[
P_\theta^p(\omega) = F_\theta^p(\omega) F_\theta(\omega) Q_Y + R + \sigma_{b0}^2 \delta(\omega) \tag{50}
\]

\[
P_\theta^p(\omega) = (F_\theta^p(\omega) F_\theta(\omega) Q_Y + R) H_R^*(\omega) H_R(\omega) \tag{51}
\]

where \(Q_Y = Q_{T PA}\) and \(Q_{T PA}\) is the second component of Eq. (28), and where \(R\), \(\sigma_{b0}\) are defined in preceding sections, and \(\delta(\omega)\) is the Dirac delta function; and where,

\[
F_\theta(\omega) = G_\theta(\omega) H(\omega) \tag{52a}
\]

\[
G_\theta(\omega) = \left[ \frac{1}{I_Y \omega^2 + 3 \omega_0^2 (I_X - I_Z)} \right] \tag{52b}
\]

\[S = j\omega\]
Figure 12. Roll and Yaw Axes Block Diagram. (Configuration A)
The output power spectral density for the system is closely related to the frequency response of the system, which characterizes the steady state dynamics of the system, and, hence, it is meaningful only if the system is stable. Unfortunately, in Eq. (52b), \( I_x < I_z \) which implies that the system is unstable. The destabilizing term comes from the gravity gradient because the nadir pointing axis is not the axis of minimum inertia. To proceed, one has to consider the gravity gradient as external disturbance rather than a part of the dynamics. Figure 13 shows the instrument pointing error PSD in \((\text{rad})^2\)-sec as a function of frequency in Hz. Figure 14 shows the rate error PSD in \((\text{rad/sec})^2\)-sec as a function of frequency.

E.4.2. The Roll and Yaw Error PSD (Configuration A)

The instrument pointing error PSD's for the roll and yaw axes are,

\[
P_\phi (\omega) = F_\phi^*(\omega) F_\phi (\omega) \sigma_x^2 + F_\psi^*(\omega) F_\psi (\omega) Q_x + R + \sigma_{bo}^2 \delta(\omega) \tag{53a}
\]

\[
P_\psi (\omega) = F_\psi^*(\omega) F_\psi (\omega) Q_z + F_\phi^*(\omega) F_\phi (\omega) Q_x + R + \sigma_{bo}^2 \delta(\omega) \tag{53b}
\]

and the instrument pointing rate error PSD's for the roll and yaw axes are,

\[
P^*_\phi (\omega) = [F_\phi^*(\omega) F_\phi (\omega) Q_x + F_\psi^*(\omega) F_\psi (\omega) Q_z + R] H_\phi^*(\omega) H_\phi (\omega) \tag{54a}
\]
\[ P_\phi (\omega) = [F^*_\phi (\omega) F^*_\psi (\omega) Q_x + F^*_\phi (\omega) F_{\phi \psi} (\omega) Q_z + R] H^*_z(\omega) H_z(\omega) \] (54b)

where

\[ F^*_\phi (\omega) = G^*_\phi (\omega) H(\omega) \] (55a)

\[ G^*_\phi (\omega) = \left[ \frac{I_z S^2 + \omega_0^2 (I_y - I_x)}{D(S)} \right] S = j\omega \] (55b)

\[ F_{\phi \psi} (\omega) = G_{\phi \psi} (\omega) H(\omega) \] (56a)

\[ G_{\phi \psi} (\omega) = \left[ \frac{\omega_0 (I_y - I_x + I_z)}{D(S)} \right] S = j\omega \] (56b)

\[ F^*_\psi (\omega) = G^*_\psi (\omega) H(\omega) \] (57a)

\[ G^*_\psi (\omega) = \left[ \frac{I_x S^2 + 4\omega_0^2 (I_y - I_z)}{D(S)} \right] S = j\omega \] (57b)

and \( Q_x \) and \( Q_z \) are the first and the third components of \( \bar{Q}_{T PA} \) in Eq. (28).

For the same reason discussed earlier, the roll and yaw dynamics are also unstable (referring to Eq. (48), \( D(s) \) has roots in the right-half complex plane). The PSD's for this case are obtained again by treating gravity gradient and gyroscopic torques as disturbances. Figures 13 and 14 show the roll and yaw power spectral densities.

E.4.3. The Error PSD's for Configuration B

The stability problem may be resolved by reorienting the shuttle from payload-bay nadir (Configuration A) to nose-down nadir (Configuration B). The advantage of this new configuration is that it is a gravity gradient stabilized system. However, there are drawbacks for this configuration. First, it will require a larger support-tower for the IS instrument, and the second drawback is that the aerodynamic forces and torques have increased significantly over the payload-bay nadir case.
Figure 13. Instrument Pointing Error PSD, (rad)²·sec

Figure 14. Instrument Pointing Rate Error, (rad/sec)²·sec
To obtain the error power spectral density for Configuration B, it is only necessary to replace \( [X] \) and the corresponding notations of the equations in this section by \( [-Z] \). That is, for instance, Eq. (52b) becomes,

\[
G_\theta(\omega) = \left[ \frac{1}{I_Z s^2 + 3 \omega^2 (I_Y - I_X)} \right] S \omega
\]

where \( G_\theta(\cdot) \) here still represents the pitch dynamics. Since \( I_Y > I_X \), Eq. (58) is stable.

The instrument pointing error PSD's are shown in Figure 15 and the rate error PSD's are illustrated in Figure 16.

The computer programs that are used for generating these results are included in Appendices C and D.

F. SUMMARY OF MAJOR FINDINGS

F.1 The Major Environmental Disturbances

The major disturbance sources that are modeled are the aerodynamic drag torques, the gravity gradient torques, and the gyroscopic torques. The unmodeled disturbances are the solar pressure torques, the on-board equipment vibrations, crew motions, and venting. Table 2 shows the static disturbance torques in N-m and the stochastic torque PSD's in \( (N\cdot m)^2 \)-sec for both Configurations A and B with a circular orbit of 400 km altitude.

F.2 The Measurement Uncertainties

The modeled measurement uncertainties are summarized in Table 3.

F.3 Ground Track Errors and Navigation Uncertainties

The power spectral densities for the ground track errors and the rate errors due to instrument pointing uncertainties for Configurations A and B are shown in Figure 17. It is important to note that strong resonances occur within

36
the frequency band of $10^{-5}$ Hz to $10^{-3}$ Hz. The ground track errors at higher frequencies, .01 Hz and above, are dominated by the measurement noise. Recall that the 1σ measurement noise is 28.28 arc-sec/axis. The corresponding ground track error is 54.84 m/axis or 77.56 m/lateral motion.

The 3σ ground track errors due to navigational uncertainties are about one order of magnitude greater than the attitude errors with the aid of TDRSS; and the error is greater with STDN as indicated in Table 4 [12]. Note that the values tabulated in Table 3 are the 3σ rms values.
Figure 15. Instrument Pointing Error PSD, (rad)$^2$-sec

Figure 16. Instrument Pointing Rate Error, (rad/sec)$^2$-sec
**Table 2. Modeled Environmental Disturbances**

<table>
<thead>
<tr>
<th>Sources</th>
<th>Static, N-m</th>
<th>Stochastic -- PSD, (N-m)²/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Y&lt;sub&gt;p&lt;/sub&gt;</td>
</tr>
<tr>
<td>Aerodynamic Drag Torques</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Config. A</td>
<td>0</td>
<td>-0.68x10⁻²</td>
</tr>
<tr>
<td>Config. B</td>
<td>-2.78x10⁻²</td>
<td>0</td>
</tr>
<tr>
<td>Gravity Gradient and Gyroscopic Torques</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 3. Modeled Measurement Uncertainties**

<table>
<thead>
<tr>
<th>Sources</th>
<th>Performance (10)</th>
<th>Requirement (10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMU/Shuttle Misalignment</td>
<td>82 arc-sec</td>
<td>160 arc-sec</td>
</tr>
<tr>
<td>IMU Resolution</td>
<td>20 arc-sec</td>
<td>20 arc-sec</td>
</tr>
<tr>
<td>IMU Noise (20) arc-sec</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IMU Derived Rate (See filter)</td>
<td></td>
<td>dynamic model</td>
</tr>
<tr>
<td>Rate Gyro (60 arc-sec/sec)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shuttle/IS Misalignment (10)</td>
<td>30 arc-sec</td>
<td></td>
</tr>
</tbody>
</table>
Figure 17. IS Ground Track Error and Rate Error PSD
Figure 17. (Continued)
Table 4. Expected On-Orbit Navigation Accuracies (3a)

<table>
<thead>
<tr>
<th>CASE</th>
<th>NAVIGATION TRACKING SYSTEM</th>
<th>UNMODELED PERIGEE PERTURBATION</th>
<th>MINIMUM PERIGEE n.mi.</th>
<th>POSITION, FEET</th>
<th>VELOCITY, FPS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>RADIAL</td>
<td>DOWNTRACK</td>
<td>CROSSTrack</td>
<td>RADIAL</td>
</tr>
<tr>
<td>1</td>
<td>STDN*</td>
<td>105</td>
<td>5,000</td>
<td>34,500</td>
<td>5,000</td>
</tr>
<tr>
<td></td>
<td>TDRSS**</td>
<td>105</td>
<td>3,600</td>
<td>18,500</td>
<td>4,000</td>
</tr>
<tr>
<td>2</td>
<td>STDN</td>
<td>105</td>
<td>3,000</td>
<td>22,000</td>
<td>1,500</td>
</tr>
<tr>
<td></td>
<td>TDRSS</td>
<td>150</td>
<td>1,200</td>
<td>5,500</td>
<td>1,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>105</td>
<td>1,800</td>
<td>10,000</td>
<td>2,000</td>
</tr>
<tr>
<td>3</td>
<td>STDN</td>
<td>150</td>
<td>800</td>
<td>2,000</td>
<td>1,500</td>
</tr>
<tr>
<td></td>
<td>TDRSS</td>
<td>105</td>
<td>8,000</td>
<td>80,000</td>
<td>5,000</td>
</tr>
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<td></td>
<td>105</td>
<td>6,000</td>
<td>50,000</td>
<td>5,000</td>
</tr>
</tbody>
</table>

NOTE:

The correlation between downtrack position and radial velocity is -0.95

The correlation between radial position and downtrack velocity is -0.80

* Spaceflight Tracking and Data Network

** Tracking Data Relay Satellite System
IV. PARAMETRIC ANALYSIS OF GEOMETRIC ERRORS

In section III, the steady state error dynamics of the Shuttle Imaging Spectrometer system has been analyzed with the major error sources and disturbance effects estimated. In this section, the emphasis is on the geometric error analysis. Geometric errors are consequences of the more direct errors including attitude and rate errors, ephemeris uncertainties, misalignment errors, earth rotation and curvature, etc. Figure 1 shows how the various errors propagate and how the geometric errors and the ground pattern distortions can be generated through dynamic analysis and simulation. The block that is considered in this section is II. Due to mounting and other practical considerations, only the payload-bay principal-axis nadir pointing configuration is considered (see Fig. 2).

The geometric error mapping functions due to the individual error sources as well as the aggregated errors are derived. The earth curvature effect is incorporated in all of the results. For the purpose of quick reference, the key mapping functions are tabulated in Appendix A. A list of the source code of the computer program that has been used for generating the geometric error characteristic curves is given in Appendix E.

A. COORDINATE CONFIGURATIONS

In Fig. 18, the coordinate frame \((X_p,Y_p,Z_p)\) on the Shuttle c.m. consists of the principal body axes. For the purpose of geometric analysis, another set of coordinates is used, i.e., the \((X,Y,Z)\) frame centered at the nadir point on the ground. This frame is the nadir projection of the orbital rotating frame centered at the Shuttle c.m. Specifically, the X-axis is in the direction of the projected motion on the ground, or the along-track direction, the Z-axis is in the nadir direction, and the Y-axis is in the cross-track direction, so that a right-hand coordinate system is formed.

For the purpose of this report, it is assumed that the Imaging Spectrometer is attached to the payload bay with its optical axis aligned with the body.
Figure 18. Payload-Bay Nadir Pointing Configuration and Coordinates
Z_p-axis. For normal IS operations, this is the desired configuration. However, for some IS experiments, such as those for assessing the atmospheric effects on the images, the IS instrument is required to point up to ±45° about the pitch axis (Y) from nadir (forward and backward looking) or up to ±20° about the roll axis (X) (side looking). For those cases, it is assumed that the instrument is gimbal mounted. However, since the analysis performed here is not primarily concerned with dynamics, no specific details are made at this time regarding mounting configurations.

B. GEOMETRIC ERRORS INDUCED BY EPHEMERIS UNCERTAINTIES

Ephemeris uncertainties include radial, along-track, and cross-track prediction errors. The geometric errors induced by ephemeris uncertainties are discussed in the following subsections.

B.1 Geometric Errors Due to Altitude Uncertainties

When the altitude of the shuttle varies, the ground point will shift in both the X and Y directions accordingly. This can be investigated in the following two ways.

B.1.1. With Fixed Viewing Angles

Referring to Figure 19, when the shuttle flies at nominal altitude h (position A), a ground point B with coordinates (Xo, Yo) corresponding to a view angle \( \lambda \) from the shuttle IS is located. After the shuttle elevates \( \Delta h \) to position A', point B' with coordinates (Xo', Yo') corresponding to the same view angle \( \lambda \) is located. The problem is to determine the shift of image due to the altitude change. Consider that when the altitude increases, the image of ground objects tends to move toward the nadir point, which causes reduction in image size and increase in field of view. Mathematically, the shift of image may be characterized by computing \( \Delta X \) and \( \Delta Y \), where

\[
\Delta X = X_0 - X_0' \\
\Delta Y = Y - Y_0'
\]
Starting by assuming \((X_0, Y_0)\) known, the task is to determine \((X_0', Y_0')\) by first computing the view angle \(\lambda\). Figure 19b is generated by taking a side view from Fig. 19a in the direction perpendicular to Plane \(A'C\). Apparently,

\[
\sqrt{X_0^2 + Y_0^2} = R \sin \beta
\]

GROUND AREA COVERED WHEN SHUTTLE IS AT NOMINAL ALTITUDE (POSITION A IN FIGURE 4 (b))

GROUND AREA COVERED WHEN SHUTTLE IS AT HIGHER ALTITUDE (POSITION A' IN FIG. 4 (b))

LEGEND:

\(B\) = A GROUND POINT CORRESPONDING TO VIEW ANGLE \(\lambda\) WHEN SHUTTLE IS AT NOMINAL ALTITUDE (POSITION A IN FIG. 4(b))

\(B'\) = THE GROUND POINT CORRESPONDING TO THE SAME VIEW ANGLE AFTER THE SHUTTLE ELEVATED \(\Delta h\) (POSITION A' IN FIG. 4(b))

\((X_0, Y_0)\) = COORDINATES OF POINT B

\((X_0', Y_0')\) = COORDINATES OF POINT B'

\(\Delta X, \Delta Y\) = X AND Y COMPONENTS OF THE POSITION CHANGE

(a) TOP VIEW (LOOKING TOWARD NADIR)

Figure 19: Ground Point Shift in X and Y Direction due to Altitude Error
LEGEND:

- $h = \text{NOMINAL ALTITUDE}$
- $\Delta h = \text{ALTITUDE ERROR}$
- $\lambda = \text{VIEW ANGLE}$
- $A = \text{SHUTTLE'S POSITION AT NOMINAL ALTITUDE}$
- $A' = \text{SHUTTLE'S POSITION AFTER BEING ELEVATED } \Delta h$

Figure 19. (Continued)
or

\[ \beta = \sin^{-1} \left( \frac{\sqrt{X_0^2 + Y_0^2}}{R} \right) \]  

(59)

and

\[ \alpha = 180^\circ - \lambda - \beta \]  

(60)

From triangle ABC, we have

\[ \frac{R}{\sin \lambda} = \frac{R + h}{\sin \alpha} \]  

(61)

Substituting Eqs. (59) and (60) into (61), it becomes

\[ \frac{R}{\sin \lambda} = \frac{R + h}{\sin \left[ 180^\circ - \lambda - \sin^{-1} \left( \frac{\sqrt{X_0^2 + Y_0^2}}{R} \right) \right]} \]

which leads to

\[ \tan \lambda = \frac{\sqrt{X_0^2 + Y_0^2}}{R + h - \sqrt{R^2 - X_0^2 - Y_0^2}} \]

hence,

\[ \lambda = \tan^{-1} \left( \frac{\sqrt{X_0^2 + Y_0^2}}{R + h - \sqrt{R^2 - X_0^2 - Y_0^2}} \right) \]  

(62)

To determine \( \sqrt{X_0'{}^2 + Y_0'{}^2} \), triangle A'B'C is used,

\[ \frac{R}{\sin \lambda} = \frac{R + h + \Delta h}{\sin \alpha'} \]

\[ \therefore \alpha' = \sin^{-1} \left[ \left( 1 + \frac{h + \Delta h}{R} \right) \sin \lambda \right] \]  

(63)

and

\[ \sqrt{X_0'{}^2 + Y_0'{}^2} = R \sin \beta' \]

\[ = R \sin (\lambda + \alpha') \]  

(64)
Substituting Eq. (63) into (64),
\[
\sqrt{X_0^2 + Y_0^2} = R \sin \left\{ \lambda + \sin^{-1} \left[ \left( \frac{1 + h + \Delta h}{R} \right) \sin \lambda \right] \right\}
\]

(65)

Expanding the right hand side of Eq. (65), and adopting the "-" sign for
\[
\cos \left\{ \sin^{-1} \left[ \left( \frac{1 + h + \Delta h}{R} \right) \sin \lambda \right] \right\},
\]
since \(90^\circ < \sin^{-1} \left[ \left( \frac{1 + h + \Delta h}{R} \right) \sin \lambda \right] < 180^\circ\),
one has
\[
\sqrt{X_0^2 + Y_0^2} = R \left[ \left( \frac{1 + h + \Delta h}{R} \right) \cos \lambda - \sqrt{1 - \left( \frac{1 + h + \Delta h}{R} \right)^2 \sin^2 \lambda} \right] \sin \lambda
\]

(66)

Now, referring to Fig.19(a), since \(B'\) is on the line \(OB\), the angle \(\rho\) is
determined by
\[
\rho = \sin^{-1} \frac{X_0}{\sqrt{X_0^2 + Y_0^2}} = \cos^{-1} \frac{Y_0}{\sqrt{X_0^2 + Y_0^2}}
\]

(67)

Hence, from Eqs. (66) and (67),
\[
X_0' = \sqrt{X_0^2 + Y_0^2} \sin \rho
\]
\[
= \frac{RX_0 \sin \lambda}{\sqrt{X_0^2 + Y_0^2}} \left[ \left( \frac{1 + h + \Delta h}{R} \right) \cos \lambda - \sqrt{1 - \left( \frac{1 + h + \Delta h}{R} \right)^2 \sin^2 \lambda} \right]
\]

(68)

\[
Y_0' = \sqrt{X_0^2 + Y_0^2} \cos \rho
\]
\[
= \frac{R Y_0 \sin \lambda}{\sqrt{X_0^2 + Y_0^2}} \left[ \left( \frac{1 + h + \Delta h}{R} \right) \cos \lambda - \sqrt{1 - \left( \frac{1 + h + \Delta h}{R} \right)^2 \sin^2 \lambda} \right]
\]

(69)

where \(\lambda\) is obtained by Eq. (62).

Hence, from Eqs. (68), (69), and (62) the values for \(\Delta X = X_0 - X_0'\) and
\(\Delta Y = Y_0 - Y_0'\) can be obtained.
When point $A'$ is below point $A$, the negative value for $\Delta h$ should be used in the above equations.

However, if one starts with given view angle $\lambda$ and $\rho$, the values of $X_0$, $Y_0$, $X_0'$, and $Y_0'$ can be readily obtained using Eqs. (68) and (69) directly (set $\Delta h$ to 0 for $X_0$ and $Y_0$).

B.1.2 With Fixed Ground Points

Referring to Fig. 20, the view angle for the fixed ground point $B$ will change as the shuttle altitude varies. Follow the same approach of paragraph B.1.1, the view angles $\Omega$ and $\Omega'$ can be found,

$$\Omega = \tan^{-1} \left( \frac{\sqrt{X_0^2 + Y_0^2}}{R + h - \sqrt{R^2 - X_0^2 - Y_0^2}} \right)$$  \hspace{1cm} (70)

and

$$\Omega' = \tan^{-1} \left( \frac{\sqrt{X_0^2 + Y_0^2}}{R + h + \Delta h - \sqrt{R^2 - X_0^2 - Y_0^2}} \right)$$  \hspace{1cm} (71)

Using Eqs. (70) and (71), the value of $\Delta \Omega = \Omega' - \Omega$ can be obtained. Again if point $A'$ is below point $A$, the negative value for $\Delta h$ should be used in the above equations.

B.2. Geometric Errors Due to Intrack and Crosstrack Prediction Errors

The intrack (along X-direction or direction of orbit) and the crosstrack (along Y-direction or "-" orbit normal direction) prediction errors will cause the ground objects to shift along the X and Y directions, respectively, as shown in Fig. 21. To determine the overall geometric errors associated with ephemeris uncertainties, the intrack error $\Delta X^*$ and the crosstrack error $\Delta Y^*$ can be incorporated in $\Delta X$ and $\Delta Y$ found in Section B.1.1, respectively.
Figure 20. Change of View Angle of a Ground Point due to Altitude Error
Figure 21. Geometric Errors Induced by Intrack and Crosstrack Prediction Errors
C. GEOMETRIC ERRORS INDUCED BY ATTITUDE UNCERTAINTIES

Attitude uncertainties refer to the angular errors with respect to the nominal roll (ζ), pitch (θ), and yaw (ψ) axes. The effects of these errors to the images of ground objects are studied in this subsection first on the individual error basis and then on the aggregated basis. For convenience, in the subsequent discussions, the term nominal flight condition will be used to signify the condition of the shuttle flying in a 400 km circular orbit with the attitude of payload-bay nadir pointing.

C.1 Geometric Errors Induced by Roll Error

Referring to Fig. 22, consider the nominal flight condition with φ₀ offset angle about the roll axis. The IS slit in this case is directed at the Y-axis on the ground. For a given view angle λ, the image point (or pixel) on the ground is P₀ with coordinates (0, Y*). For the same roll offset and view angle the ground point P' is located after a roll error φ is introduced. The coordinates of P' are (0, Y*''). Equivalently, by rotating the optical instrument to the "left," the image recorded on the film appears to move to the "right." Therefore, the geometric error may be defined as the displacement of the object image after error is introduced relative to the image before the error is introduced. Mathematically, this is P₀ - P' or P₀φo⁺λ - P₀φo⁻φ⁺λ. Since, in the case being considered, the image is on the Y-axis, one has

\[ \Delta X = 0 \]  \hspace{1cm} (72)

\[ \Delta Y = Y* - Y*' \]  \hspace{1cm} (73)

where Y* and Y*' are,

\[ Y* = -R \sin (\lambda + \phi_0) \left[ \left(1 + \frac{h}{R}\right) \cos (\lambda + \phi_0) - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 (\lambda + \phi_0)} \right] \]  \hspace{1cm} (74)
Legend:

$\lambda$ = View Angle

$\phi_o$ = Roll Offset Angle

$\phi$ = Roll Error

$P_0$ = The ground point corresponding to view angle $\lambda$ and roll offset $\phi_o$

$P_o$ = The ground point corresponding to the same $\lambda$ and $\phi_o$ after roll error $\phi$ is introduced

Figure 22. Shift of Ground Point Induced by Roll Error
C.2 Geometric Errors Induced by Pitch Error

Consider the case of pitch attitude error. Let \( \theta_0 \) be the offset angle about the pitch axis (desired attitude), and \( \theta \) be the pitch error. Referring to Fig. 23, for a given view angle \( \lambda \), projecting the slit onto the XY-plane forms two line segments: \( \overline{BC} \), corresponding to null pitch error (\( \theta = 0 \)), and \( \overline{B'C'} \), corresponding to arbitrary pitch error \( \theta \). Projecting along the line of view \( \overline{AC} \) onto the ground, \( P(X*,Y*) \) is obtained; similarly, \( P'(X*'',Y*'') \) is obtained, corresponding to line of view \( \overline{AC'} \). The geometric error here is defined the same way as that for the roll case, i.e.,

\[
\begin{align*}
\Delta X &= X* - X*' \\
\Delta Y &= Y* - Y*'
\end{align*}
\]

(76)

(77)

The formulae for computing \( X* \), \( Y* \), \( X*' \), and \( Y*' \) are derived as follows. In Fig. 23, assume that \( h \), \( \theta \), \( \theta_0 \), and \( \lambda \) are known. The lengths of the line segments \( \overline{OB} \) and \( \overline{OB'} \) are

\[
\overline{OB} = h \tan \theta_0
\]

(78a)

and

\[
\overline{OB'} = h \tan (\theta_0 + \theta)
\]

(78b)

and that of \( \overline{AB} \) and \( \overline{AB'} \) are

\[
\overline{AB} = \frac{h}{\cos \theta_0}
\]

(79a)

\[
\overline{AB'} = \frac{h}{\cos (\theta_0 + \theta)}
\]

(79b)
LEGEND:

\( \lambda \) = VIEW ANGLE

\( \theta_0 \) = PITCH OFFSET ANGLE

\( \theta \) = PITCH ERROR

\( P \) = THE GROUND POINT CORRESPONDING TO VIEW ANGLE \( \lambda \) AND PITCH OFFSET \( \theta_0 \)

\( P' \) = THE GROUND POINT CORRESPONDING TO THE SAME \( \lambda \) AND \( \theta_0 \) AFTER PITCH ERROR \( \theta \) IS INTRODUCED

Figure 23. Shift of Ground Point Induced by Pitch Error
BC and B'C', with respect to Y-axis, can be obtained as

\[ BC = -AB \tan \lambda = -h \tan \frac{\lambda}{\cos \theta} \]  \hfill (80a)

and

\[ B'C' = -AB' \tan \lambda = -h \tan \frac{\lambda}{\cos(\theta + \theta)} \]  \hfill (80b)

implies that the signs of BC and B'C' are opposite to that of λ. λ and λ' are, referring to Fig. 23,

\[ \lambda = \sqrt{OB^2 + BC^2} = h \sec \theta \sqrt{\sin^2 \theta + \tan^2 \lambda} \]  \hfill (81a)

and

\[ \lambda' = \sqrt{OB'^2 + B'C'^2} = h \sec (\theta + \theta) \sqrt{\sin^2 (\theta + \theta) + \tan^2 \lambda} \]  \hfill (81b)

the angles τ and ξ (see Fig. 23) can then be determined,

\[ \tau = \tan^{-1} \left( \frac{\xi}{h} \right) = \tan^{-1} \left( \frac{\sec \theta \sqrt{\sin^2 \theta + \tan^2 \lambda}}{\sin \theta \tan \lambda} \right) \]  \hfill (82a)

and

\[ \xi = \tan^{-1} \left( \frac{\xi'}{h} \right) = \tan^{-1} \left[ \frac{\sec (\theta + \theta) \sqrt{\sin^2 (\theta + \theta) + \tan^2 \lambda}}{\sin \theta \tan \lambda} \right] \]  \hfill (82b)

Finally,

\[ X^* = \sqrt{X^2 + Y^2 \frac{OB}{R}} \]

\[ = R \left[ \left( 1 + \frac{h}{R} \right) \cos \tau - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \tau} \frac{\sin \tau \sin \theta}{\sqrt{\sin^2 \theta + \tan^2 \lambda}} \right] \]  \hfill (83a)

\[ Y^* = \sqrt{X^2 + Y^2 \frac{BC}{R}} \]

\[ = -R \left[ \left( 1 + \frac{h}{R} \right) \cos \tau - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \tau} \frac{\sin \tau \tan \lambda}{\sqrt{\sin^2 \theta + \tan^2 \lambda}} \right] \]  \hfill (83b)
\[ X^{*'} = \sqrt{X^{*2} + Y^{*2}} \frac{OB'}{x'} \]
\[ = R \left[ (1 + \frac{h}{R}) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi} \right] \frac{\sin \xi \sin (\theta_o + \theta)}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda}} \]
\[ (83c) \]
\[ Y^{*'} = \sqrt{X^{*2} + Y^{*2}} \frac{B'C'}{x'} \]
\[ = -R \left[ (1 + \frac{h}{R}) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi} \right] \frac{\sin \xi \tan \lambda}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda}} \]
\[ (83d) \]

However, if one starts with \((X^*, Y^*)\) given, then \(\lambda\) and \(\theta_o\) would have to be computed. In that case,
\[ \lambda = -\tan^{-1} \left[ \frac{Y^*}{\left( R + h - \sqrt{R^2 - X^*2 - Y^*2} \right) \sqrt{1 + \frac{X^*}{\left( R + h - \sqrt{R^2 - X^*2 - Y^*2} \right)^2}}} \right] \]
\[ (84a) \]
and
\[ \theta_o = \tan^{-1} \left( \frac{X^*}{R + h - \sqrt{R^2 - X^*2 - Y^*2}} \right) \]
\[ (84b) \]

Once \(\lambda\) and \(\theta_o\) are determined, \(\tau\) and \(\xi\) can be computed using (82), and \(X^{*'}\) and \(Y^{*'}\) are determined using (83).

C.3 Geometric Errors Induced by Yaw Error

For a given yaw offset angle, \(\psi_o\), and the view angle, \(\lambda\), the ground point \(P(X^*, Y^*)\) is located. A new point \(P'(X^{*'}, Y^{*'})\) is found as yaw attitude error, \(\psi\), is introduced (see Fig.24). The values for \(X^*, Y^*, X^{*'}\), and \(Y^{*'}\) can be computed as follows:
\[ X^* = \left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \]
\[ \times R \sin \lambda \sin \psi_o \]
\[ (85a) \]
LEGEND:

$\psi_0$ = YAW OFFSET ANGLE
$\psi$ = YAW ERROR
$P$ = THE GROUND POINT CORRESPONDING TO VIEW ANGLE $\lambda$ AND YAW OFFSET $\psi_0$
$P'$ = THE GROUND POINT CORRESPONDING TO THE SAME $\lambda$ AND $\psi_0$ AFTER YAW ERROR $\psi$ IS INTRODUCED

Figure 24. Shift of Ground Point Induced by Yaw Error
y* = \left[ \left(1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \right] R \sin \lambda \cos \psi \tag{85b}

X^* = \left[ \left(1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \right] R \sin \lambda \sin (\psi_0 + \psi) \tag{86a}

and

Y^* = \left[ \left(1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \right] R \sin \lambda \cos (\psi_0 + \psi) \tag{86b}

From Eqs. (85) and (86), we can obtain

\Delta X = X^* - X^*

\Delta Y = Y^* - Y^*

C.4 Geometric Errors Induced by Roll, Pitch, and Yaw Attitude Errors

C.4.1 The Shift of IS Line-of-Sight due to Attitude Error

Referring to Fig. 25, the shuttle is flying at an arbitrary attitude (\phi, \theta, \psi). Consider the 3-2-1 sequence, i.e., the yaw, pitch, and roll rotation sequence. After yaw and pitch rotations, the Imaging Spectrometer line-of-sight will shift to AB, where B(X_1, Y_1) is its intersection with the XY-plane. The coordinates are determined as follows:

\[ X_1 = X_0 + h \tan \phi \cos \psi \tag{87a} \]

\[ Y_1 = Y_0 + h \tan \phi \sin \psi \tag{87b} \]

Through a roll angle rotation, point B will move to C(X_2, Y_2) on the XY-plane, where

\[ X_2 = X_0 + h \tan \phi \cos \psi + \frac{h}{\cos \phi} \tan \phi \sin \psi \tag{88a} \]

\[ Y_2 = Y_0 + h \tan \phi \sin \psi - \frac{h}{\cos \phi} \tan \phi \cos \psi \tag{88b} \]
Figure 25. The Shift of Ground Projection of the IS Line-of-Sight due to Attitude Error
The interest here is to determine the coordinates of O'(X,Y) corresponding to the final line-of-sight projected onto the ground. By taking a view normal to the ACO-plane, Figure 26 is obtained. The quantities \( \varepsilon \), \( \lambda \), and \( \rho \) are obtained as follows,

\[
\varepsilon = \tan^{-1}\left( \sqrt{\frac{x^2 + y^2}{h}} \right)
\]

\[
\lambda = R \sin \varepsilon \left[ (1 + \frac{h}{R}) \cos \varepsilon - \sqrt{1 - (1 + \frac{h}{R})^2 \sin^2 \varepsilon} \right]
\]

and

\[
\rho = \sin^{-1}\left( \frac{y}{\sqrt{x^2 + y^2}} \right) = \cos^{-1}\left( \frac{x}{\sqrt{x^2 + y^2}} \right)
\]

Hence, the coordinates of X and Y are,

\[
X = \lambda \cos \rho = R \sin \varepsilon \left[ (1 + \frac{h}{R}) \cos \varepsilon - \sqrt{1 - (1 + \frac{h}{R})^2 \sin^2 \varepsilon} \right] \frac{x}{\sqrt{x^2 + y^2}}
\]

\[
Y = \lambda \sin \rho = R \sin \varepsilon \left[ (1 + \frac{h}{R}) \cos \varepsilon - \sqrt{1 - (1 + \frac{h}{R})^2 \sin^2 \varepsilon} \right] \frac{y}{\sqrt{x^2 + y^2}}
\]

where \( X_2 \) and \( Y_2 \) are given in Eq. (88).

Note that this derivation is general enough so that the formulae are valid if one replaces the attitude errors with errors plus offsets, \( \psi \), \( \theta \), and \( \phi \) by \( \phi_0 + \phi \), \( \theta_0 + \theta \), and \( \psi_0 + \psi \), respectively.
Figure 26. Side View of Figure 25
C.4.2. The Shift of Ground Image Due to Attitude Error

For simplicity, \( \phi \), \( \theta \), and \( \psi \) are used to represent the attitude angles as sums of attitude offsets and attitude errors. To fix the derivation, the yaw \( (\psi) \), pitch \( (\theta) \), and roll \( (\phi) \) Euler sequence is assumed.

In this subsection, the results of subsection C.4.1 are extended to cover the entire field of view of the IS slit. First consider the case of a nominal flight with zero attitude offset. Referring to Fig. 27, let 0 be the nadir point which is the origin of the XY-Frame and the X'Y'-Frame. The X'Y'-Frame is formed by rotating the XY-Frame through an angle \( \psi \) about the yaw axis. Let P be a point corresponding to a view angle \( \lambda \) (negative value shown in Fig. 27) on the Y-axis before any rotation occurs. After a \( \psi \) rotation, for the same view angle, the IS is sighting point \( P_1 \). Similarly, after a pitch \( (\theta) \) and then roll \( (\phi) \) rotation, the points \( P_2 \) and then \( P_3 \) are sighted, respectively.

The coordinates of \( P_2 \) in the X'Y'-Frame are, from the results of subsections C.1 and C.2 and Fig. 23,

\[
X'_2 = \frac{R \sin \theta \sin \xi_2 \left[ \cos \xi_2 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_2} \right]}{\sqrt{\sin^2 \theta + \tan^2 \lambda}} \quad (91a)
\]

\[
Y'_2 = -\frac{R \tan \theta \sin \xi_2 \left[ \cos \xi_2 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_2} \right]}{\sqrt{\sin^2 \theta + \tan^2 \lambda}} \quad (91b)
\]

where the angle \( \xi_2 \) is the angle \( \xi \) defined in Fig. 23 corresponding to point \( P_2 \) here, and

\[
\xi_2 = \tan^{-1} \left( \sec \theta \sqrt{\sin^2 \theta + \tan^2 \lambda} \right) \quad (91c)
\]
Figure 27. Shift of Ground Point Induced by Attitude Errors.
The coordinates of \( P_3 \) in the \( X'Y' \)-Frame are

\[
X'_3 = \frac{R \sin \theta \sin \xi_3 \left[ \left( 1 + \frac{h}{R} \right) \cos \xi_3 - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}
\]

(92a)

\[
Y'_3 = -\frac{R \tan (\lambda + \phi) \sin \xi_3 \left[ \left( 1 + \frac{h}{R} \right) \cos \xi_3 - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}
\]

(92b)

where \( \xi_3 \) is similarly defined as \( \xi_2 \) and

\[
\xi_3 = \tan^{-1} \left( \frac{\sec \theta \sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}{} \right)
\]

(92c)

or, finally, in the \( XY \)-Frame

\[
X_3 = X'_3 \cos \psi - Y'_3 \sin \psi
\]

(93a)

\[
Y_3 = X'_3 \sin \psi + Y'_3 \cos \psi
\]

(93b)

where \( X'_3 \) and \( Y'_3 \) are given in (92).

The geometric error caused by the attitude errors is therefore,

\[
\Delta X = X - X_3
\]

(94a)

\[
\Delta Y = Y - Y_3
\]

(94b)

where

\[
X = 0
\]

\[
Y = -R \sin \lambda \left[ \left( 1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \right]
\]

Now, consider the case of the shuttle (or the IS) flying at a nominal attitude \((\phi_0, \theta_0, \psi_0)\) with attitude uncertainties of \((\phi, \theta, \psi)\). Let \( P_o(X_o, Y_o) \) be a ground point that is sighted by the IS with a view angle \( \lambda \) and attitude \((\phi_0, \theta_0, \psi_0)\).

After \((\phi, \theta, \psi)\) attitude rotations from the nominal, the ground point \( P_3(X_3, Y_3) \) is sighted with the same view angle. In this case, the geometric error is
where
\[ X_o = X'_o \cos \psi_o - Y'_o \sin \psi_o \]  
(96a)
\[ Y_o = X'_o \sin \psi_o + Y'_o \cos \psi_o \]  
(96b)
\[ X_3 = X'_3 \cos (\psi_o + \psi) - Y'_3 \sin (\psi_o + \psi) \]  
(96c)
\[ Y_3 = X'_3 \sin (\psi_o + \psi) + Y'_3 \cos (\psi_o + \psi) \]  
(96d)
and
\[ X'_o = \frac{R \sin \theta_o \sin \xi_o \left[ (1 + \frac{h}{R}) \cos \xi_o - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_o} \right]}{\sqrt{\sin^2 \theta_o + \tan^2 (\lambda + \phi_o)}} \]  
(97a)
\[ Y'_o = \frac{R \tan (\lambda + \phi_o) \sin \xi_o \left[ (1 + \frac{h}{R}) \cos \xi_o - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_o} \right]}{\sqrt{\sin^2 \theta_o + \tan^2 (\lambda + \phi_o)}} \]  
(97b)
where
\[ \xi_o = \tan^{-1} \left( \sec \theta_o \sqrt{\sin^2 \theta_o + \tan^2 (\lambda + \phi_o)} \right) \]  
(97c)
and
\[ X'_3 = \frac{R \sin (\theta_o + \phi) \sin \xi_3 \left[ (1 + \frac{h}{R}) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 (\theta_o + \phi) + \tan^2 (\lambda + \phi_o + \psi)}} \]  
(98a)
\[ Y'_3 = \frac{R \tan (\lambda + \phi_o + \psi) \sin \xi_3 \left[ (1 + \frac{h}{R}) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 (\theta_o + \phi) + \tan^2 (\lambda + \phi_o + \psi)}} \]  
(98b)
and where
\[ \xi_3 = \tan^{-1} \left[ \sec (\theta_o + \phi) \sqrt{\sin^2 (\theta_o + \phi) + \tan^2 (\lambda + \phi_o + \psi)} \right] \]  
(98c)

\[ \Delta X = X_o - X_3 \]  
(95a)
\[ \Delta Y = Y_o - Y_3 \]  
(95b)
To include errors caused by altitude uncertainties, one can replace $h$ by $h_0$ in Eq. (97), and $h$ by $h_0 + \Delta h$ in Eq. (98), where $h_0$ is the nominal or estimated altitude and $h_0 + \Delta h$ is the actual altitude.

D. GEOMETRIC ERRORS INDUCED BY ATTITUDE RATE ERRORS

Attitude rates cause attitude angle changes which in turn cause geometric errors. To account for attitude rates, one integrates the rates to obtain the attitude angles, and then uses the formulae derived to obtain the corresponding time-varying geometric errors. The instantaneous attitude angles are:

$$
\phi' = \phi_0 + \int_{t_0}^{t} \dot{\phi} dt \quad (99a)
$$

$$
\theta' = \theta_0 + \int_{t_0}^{t} \dot{\theta} dt \quad (99b)
$$

$$
\psi' = \psi_0 + \int_{t_0}^{t} \dot{\psi} dt \quad (99c)
$$

E. GEOMETRIC ERRORS INDUCED BY MISALIGNMENT ERRORS

The misalignment errors can be incorporated in the attitude errors. Let $(\gamma_b, \phi_b, \psi_b)$ be the attitude bias or misalignment, then the effective IS attitude angles will be

$$
\gamma' = \gamma_0 + \gamma_b + \gamma \quad (100a)
$$

$$
\phi' = \phi_0 + \phi_b + \phi \quad (100b)
$$

$$
\psi' = \psi_0 + \psi_b + \psi \quad (100c)
$$
The corresponding geometric errors can be determined using the formulae derived in subsection C.

F. GEOMETRIC ERRORS INDUCED BY EARTH ROTATION

The effect of earth rotation on images varies with the position of the Shuttle relative to the earth and the Shuttle orbital elements. In general, ground images become skewed as a result of earth rotation.

Referring to Fig. 28. Let O be the nadir point at which the latitude is ζ. Let \( \omega_e \) be the spin rate of the earth, where \( \omega_e = \frac{2\pi}{86400} = 7.2722 \times 10^{-5} \) rad/sec.

Let \( V_e \) be the linear velocity of the earth at O,

\[
V_e = \omega_e R \cos \zeta \tag{101}
\]

Let \( \Gamma \) be the orbital inclination at the equator, and let \( \Lambda \) be the orbital inclination to the local meridian at O. From Fig. 29 and Appendix B, one has the following spherical geometrical relation,

\[
\cos \Gamma = \cos \zeta \sin \Lambda
\]

or

\[
\sin \Lambda = \frac{\cos \Gamma}{\cos \zeta} \tag{102a}
\]

and

\[
\cos \Lambda = \sqrt{1 - \sin^2 \Lambda} = \sqrt{1 - \left( \frac{\cos \Gamma}{\cos \zeta} \right)^2} \tag{102b}
\]

From Fig. 29 again, \( \Lambda \) is also the angle between the linear velocity vector \( V_e \) and the Y-axis. Hence, velocity components \( V_{eX} \) and \( V_{eY} \) are, using (102),

\[
V_{eX} = V_e \sin \Lambda = \frac{V_e \cos \Gamma}{\cos \zeta} \tag{103a}
\]

and

\[
V_{eY} = V_e \cos \Lambda = V_e \sqrt{1 - \left( \frac{\cos \Gamma}{\cos \zeta} \right)^2} \tag{103b}
\]
LEGEND:

\[ \omega_e = \text{ANGULAR VELOCITY OF EARTH SPIN} \]
\[ \zeta = \text{LATITUDE AT POINT } O \]
\[ V_e = \text{LINEAR VELOCITY OF THE EARTH AT POINT } O \]

Figure 28. Linear Velocity of the Earth at the Nadir Point
(a) INTRACK AND CROSSTRAK COMPONENTS OF EARTH LINEAR VELOCITY VECTOR
AT THE NADIR POINT

(b) ORBIT INCLINATION TO THE LOCAL MERIDIAN (I), degrees

<table>
<thead>
<tr>
<th>ORBITAL INCLINATION (degrees)</th>
<th>NORTH LATITUDE, degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>18.5</td>
<td>71.5</td>
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<tr>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td>85</td>
<td>5</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 29. Earth Spherical Geometry
During the time interval from $t_1$ to $t_2$, the ground point has moved from $P(X, Y)$ to $P'(X', Y')$, the corresponding position changes are,

$$
\Delta X = \int_{t_1}^{t_2} v_x \cos \xi \, dt = \int_{t_1}^{t_2} \frac{v_e \cos \Gamma}{\cos \xi} \, dt \quad (104a)
$$

$$
\Delta Y = \int_{t_1}^{t_2} v_y \, dt = \int_{t_1}^{t_2} v_e \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \xi}\right)^2} \, dt \quad (104b)
$$

Note that for small time intervals, $\xi$ may be considered constant and $(104)$ may be approximated as

$$
\Delta X \approx \frac{\cos \Gamma}{\cos \xi} v_e (t_2 - t_1) \quad (105a)
$$

$$
\Delta Y \approx v_e \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \xi}\right)^2} (t_2 - t_1) \quad (105b)
$$

and

$$
X' = X + \Delta X \quad (106a)
$$

$$
Y' = Y + \Delta Y \quad (106b)
$$

G. NUMERICAL RESULTS: GEOMETRIC ERRORS AND ERROR SENSITIVITIES

Numerical results of error sensitivities and geometric errors due to various direct errors have been generated. These data are summarized in Tables 5-10 and discussed in the following subsections.

G.1 Geometric Errors Due to Altitude Uncertainties

Table 5 shows the geometric error for a 1 km altitude change and error sensitivity caused by altitude uncertainty. The nominal altitude of 400 km was used to generate these data. This result is also plotted in Fig. 30 as a function of view angles for altitude changes ranging from 1 km to 4 km. As indicated in this figure, the geometric errors are quite linear with the view angles and the altitude errors for the range shown.
For the Imaging Spectrometer illustrated in Fig. 4, the view angles are limited by the field of view of $\pm 0.825^\circ$. Using the various expected on-orbit navigation accuracies shown in Table 4, and the sensitivity data of Table 5, the corresponding geometric error can be determined. For instance, from Table 4, the $3\sigma$ altitude uncertainty, with TDRSS in the tracking system for the 150 nmi orbit, if the small unmodeled perturbation is used, is 800 feet (or $\sim 244$ m) and with the STDN system and large unmodeled perturbation and 105 nmi orbit, the $3\sigma$ altitude uncertainty is 8000 feet (or $\sim 2440$ m). From Table 5, the corresponding $3\sigma$ geometric errors for the largest view angle ($0.825^\circ$) are 3.51 m and 35.1 m, respectively. Note that the nominal pixel size is 30 m.

G.2 Geometric Errors Due to Roll Uncertainties

The geometric errors for view angles ranging from $-9^\circ$ to $+9^\circ$ due to a roll error of $1^\circ$ are tabulated in Table 6. The error sensitivity at the nominal attitude is also tabulated in Table 6. By comparing the errors for $1^\circ$ and the sensitivity for the same view angle, it is found that the differences are $0.2\%$ or less. That is, for small angles, the error sensitivity data can be used to compute errors. Fig. 31 shows the plots of geometric errors for roll errors of up to $5^\circ$. Unlike the errors corresponding to altitude uncertainties, these curves are relatively flat. The geometric error is about 7 km per $1^\circ$ of roll error.

It is noted from Fig. 31, that for small roll errors, the geometric errors are nearly symmetrical with the view angles. For larger roll angles, the geometric errors are greater for view angles that have the same sign of $\phi$. This is more visible from Fig. 32 which shows the plots of geometric errors corresponding to roll errors of $1^\circ$ and $2^\circ$ from a roll offset of $20^\circ$. 

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G.3 Geometric Errors Due to Pitch Uncertainties

Table 7 shows the error sensitivity and the geometric errors corresponding to the 1° pitch error for the view angles of up to ±9°. The geometric errors caused by pitch errors are about the same in amplitude as those caused by roll errors. Fig. 33 shows that pitch curves are rather flat, unlike the corresponding roll curves. Figs. 34 and 35 show the ±1° and ±2° pitch uncertainty induced geometric error curves with 22.5° and 45° pitch offsets, respectively. These curves, especially those in Fig. 35, are more similar to the roll curves in shape.

G.4 Geometric Errors Due to Yaw Uncertainties

Geometric errors due to rotations about the yaw or the local vertical axis (assuming zero offsets) are tabulated in Table 8 for 1° and plotted in Fig. 36 for up to 5°. Linearity applies for both the view angles and the yaw angles for the ranges covered here. Unlike roll and pitch errors, yaw errors induce much smaller geometric errors.

G.5 Geometric Errors Due to Attitude Uncertainties

Table 9 shows the geometric errors due to 1° error in each of the roll, pitch, and yaw axes. The error magnitude for a given view angle is approximately the vector sum of the roll and pitch induced errors.

Fig. 37 shows the plots of geometric errors caused by up to 5° roll, pitch, and yaw errors. Fig. 38 shows two curves, one corresponding to ±1° pitch error and the other, ±1° yaw error, with the roll offset of 20°.

Comparing pitch curve with that of Fig. 33, the 20° roll offset has no noticeable effects on the geometric errors. On the other hand, the effects on the yaw curve are quite pronounced, as a large bias of approximately 2.5 km results due to the 20° roll.

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Fig. 39 shows the 1° roll and 1° yaw curves for a pitch offset of 22.5°. Fig. 40 shows the same curves for 45° pitch offset.

With the large angular pitch offsets, the yaw curves appear quite different from those without pitch offset, as noted in Figs. 36, 39, and 40. With these offsets, yaw-induced error curves look quite similar to those induced by roll errors in shape. By offsetting the pitch angle to 45°, the yaw-induced geometric errors increase to about sevenfold from zero pitch offset, and more than twofold over the 22.5° pitch offset.

The roll-induced geometric errors were less drastically affected by pitch angular offsets. For instance, for 22.5° pitch offset, the error has increased about 7% from that with zero offset, and about 46% for 45° pitch offset.

G. 6 Ground Point Shift Due to Shuttle Motion and Earth Rotation

The earth rotation effect on the shift of the ground image varies with the latitude of the shuttle, and the combined effects of the earth rotation and shuttle motion vary with the orbital inclination in addition to the latitude. Table 10 shows the tabulation with latitude ranging from 0° to 80°. The largest earth rotation effect occurs at the equator, .46 km in one second, and diminishes to .08 km in one second at the latitude of 80° as shown in Fig. 41.

The combined effects are dominated by the motion of the shuttle since at 400 km altitude the projected ground speed is much greater than the earth rotation speed. In a one-second period, the image will shift more than 7 km for any latitude.
Table 5. Geometric Errors and Error Sensitivity Due to Altitude Uncertainties

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Error Induced by 1 km Altitude Error, km</td>
<td>0.01746</td>
<td>0.03492</td>
<td>0.05242</td>
<td>0.06995</td>
<td>0.08753</td>
<td>0.1052</td>
<td>0.1229</td>
<td>0.1407</td>
<td>0.1586</td>
<td></td>
</tr>
<tr>
<td>Geometric Error Sensitivity, km/km</td>
<td>0.01746</td>
<td>0.03492</td>
<td>0.05242</td>
<td>0.06995</td>
<td>0.08753</td>
<td>0.1052</td>
<td>0.1229</td>
<td>0.1407</td>
<td>0.1586</td>
<td></td>
</tr>
</tbody>
</table>
Figure 30. Geometric Error due to Altitude Uncertainties ($\Delta h$)
Table 6. Geometric Errors and Error Sensitivity Due to Roll Attitude Error

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>-9</th>
<th>-8</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Error Induced by +1° Roll Error, km</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometric Error Sensitivity, km/deg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Error Induced by +1° Roll Error, km</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometric Error Sensitivity, km/deg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 31. Geometric Error due to Roll Attitude Error ($\phi$)
Figure 32. Geometric Error due to Roll Uncertainty (φ) About 20° Roll Offset
Table 7. Geometric Errors and Error Sensitivity Due to Pitch Attitude Error

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>-9</th>
<th>-8</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
</table>
Figure 33. Geometric Error due to Pitch Attitude Error (θ)
Figure 34. Geometric Error due to Pitch Uncertainty ($\theta$) About 22.5° Pitch Offset
Figure 35. Geometric Error due to Pitch Uncertainty ($\theta$) About 45° Pitch Offset
Table 8. Geometric Errors and Error Sensitivity Due to Yaw Attitude Error

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Error Induced by 1 Yaw Error, km</td>
<td>0</td>
<td>0.1219</td>
<td>0.2438</td>
<td>0.3659</td>
<td>0.4883</td>
<td>0.6109</td>
<td>0.7340</td>
<td>0.8576</td>
<td>0.9818</td>
<td>1.107</td>
</tr>
<tr>
<td>Geometric Error Sensitivity, km/deg.</td>
<td>0</td>
<td>0.1219</td>
<td>0.2438</td>
<td>0.3659</td>
<td>0.4883</td>
<td>0.6109</td>
<td>0.7340</td>
<td>0.8576</td>
<td>0.9818</td>
<td>1.107</td>
</tr>
</tbody>
</table>
Figure 36. Geometric Error due to Yaw Attitude Error ($\psi$)
Table 9. Geometric Errors Induced by Yaw, Pitch, and Roll Errors

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km
- Yaw Error = +1°
- Pitch Error = +1°
- Roll Error = +1°

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>-9</th>
<th>-8</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>View Angle, deg.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Error, km</td>
<td>9.966</td>
<td>10.06</td>
<td>10.16</td>
<td>10.27</td>
<td>10.37</td>
<td>10.48</td>
<td>10.60</td>
<td>10.72</td>
<td>10.85</td>
<td></td>
</tr>
</tbody>
</table>
Figure 37. Geometric Error due to Roll ($\phi$), Pitch ($\theta$), and Yaw ($\psi$) Errors
Figure 38. Geometric Error due to Pitch ($\theta$) and Yaw ($\psi$) Uncertainties About 20° Roll Offset
Figure 39. Geometric Error due to Roll ($\phi$) and Yaw ($\psi$) Uncertainties About 22.5° Pitch Offset
Figure 40. Geometric Error due to Roll ($\phi$) and Yaw ($\psi$) Uncertainties About 45° Pitch Offset
Table 10. Ground Point Shift Induced by Shuttle Motion and Earth Rotation

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Orbit Inclination = 85°
- Nominal Shuttle Ground Speed = 7.202 km/sec

<table>
<thead>
<tr>
<th>Latitude, deg.</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground Point Shift Velocity due to Earth Rotation, km/sec</td>
<td>0.4623</td>
<td>0.4344</td>
<td>0.3541</td>
<td>0.2311</td>
<td>0.08027</td>
</tr>
<tr>
<td>Ground Point Shift Velocity Relative to Nadir Point, km/sec</td>
<td>7.177</td>
<td>7.175</td>
<td>7.171</td>
<td>7.166</td>
<td>7.162</td>
</tr>
<tr>
<td>Angle of Velocity Vector with Y-axis, deg.</td>
<td>-86.32</td>
<td>-86.54</td>
<td>-87.19</td>
<td>-88.18</td>
<td>-89.44</td>
</tr>
<tr>
<td>Shift in 0.1 sec, km</td>
<td>0.7177</td>
<td>0.7175</td>
<td>0.7171</td>
<td>0.7166</td>
<td>0.7162</td>
</tr>
</tbody>
</table>
Figure 41. Geometric Error Caused by Earth Rotation for the Time Period of 1 Second
H. SUMMARY OF MAJOR FINDINGS

In this section geometric errors caused by the "direct error sources" are analyzed. Equations that map the direct errors onto the geometric errors are derived in Subsections B through G. Extensive illustrations of geometry are used to assist the derivations. Direct error effects included here are those of pointing errors and angular rates, ephemeris prediction errors including altitude, intrack, and crosstrack uncertainties, earth rotation and curvature, and shuttle and instrument misalignment. For quick reference the key error mapping functions are listed in Appendix A. A program list is given in Appendix E.

The numerical results for both the geometric errors and error sensitivities are tabulated in Tables 5 through 10 and also plotted in Figs. 30 through 41. To illustrate the effects of view angles and linearity, an extended range of $\pm 90^\circ$ is used. For the push broom imaging spectrometer studied here, the field of view is limited by $\pm 825^\circ$ (see Fig. 4). The following are the specific findings resulting from further analysis:

1. The effects of earth curvature are very small for the applications here. For instance, the linear displacement of a 10 km arc is $9.9999\ldots$ km. Fig. 42 illustrates several cases of earth curvature effects.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure42.png}
\caption{Effects of Earth Curvature}
\end{figure}
2. Altitude uncertainties cause only moderate geometric errors. Table 11 shows that the worst lo geometric error is 11.71 m and the corresponding rate is .133 m/sec. The worst case associates with the STDN system and large unmodeled perturbation and 200 km orbit. The least lo errors are .22 m and .0032 m/sec corresponding to the TDRSS system and small unmodeled perturbation and 300 km orbit. The effects of other navigation uncertainties such as downtrack and crosstrack position errors are not small, however. Table 12 shows the 3σ uncertainties in feet. Downtrack and crosstrack position errors are mapped directly to the geometric errors.

3. Geometric errors caused by altitude uncertainties are, within the range of interest here, proportional to the view angle and altitude error (see Table 5 and Fig. 30).

4. The effects of roll and pitch attitude errors are relatively large compared with those of altitude errors and yaw attitude errors. Table 13 shows that, for nominal flight conditions, the error sensitivity is approximately 1.94 m/arc sec, or ~7000 m/degree. The yaw sensitivity is very small, from 0 for 0° view angle to .0279 m/arc sec for maximum IS view angle ±825°.

The effects of attitude errors increase significantly for large attitude offsets. For instance, for 20° roll offset (side looking), the roll sensitivity increases to 2.28 m/arc sec from 1.94 and yaw sensitivity increases to .75 m/arc sec from .0279. The pitch sensitivity, in this case, is nearly unaffected. By offsetting the pitch angle to 45° (forward looking), the roll sensitivity increases to 2.83 m/arc sec, the pitch to 4.36 m/arc sec, and the yaw to 1.97 m/arc sec. Table 13 shows the error sensitivities and 1σ geometric errors for many combinations of interest.
5. Without attitude offsets and within the IS field of view, the geometric errors are almost proportional to roll errors (Fig. 31), pitch errors (Fig. 33), and yaw errors (Fig. 36), and are independent of the view angle except for the yaw cases (for which the error is proportional to the view angle). For the cases of large roll or pitch angular offset, most of the properties change only slightly (see Figs. 32, 34, 35, 38, 39, and 40).

6. The performance of the Imaging Spectrometer is limited by the shuttle Inertial Measuring Unit accuracy, the shuttle reference misalignment, and the misalignment between the shuttle and the Imaging Spectrometer, unless means of error reduction, such as using ground control points and a precision point mount between the shuttle and the IS instruments, are employed.

On top of these uncertainties, the shuttle deadband is another source of error that can cause gross geometric errors. This problem, of course, can be resolved by using a precision point mount.

Table 14 shows geometric errors caused by the IMU/shuttle misalignment, IMU resolution, IMU noise, rate gyro drift, and combined shuttle/IS misalignment, for nominal attitude and attitude offsets. Single axis geometric errors corresponding to the combined shuttle/IS misalignment are 169 m, 198 m, and 379 m, respectively, for nominal, 20° roll offset, and 45° pitch offset, for instance. Other cases are shown in detail in Table 14.

7. Earth rotation causes images to shift toward the direction of rotation. The magnitude of this shift depends on the latitude of the object (see Fig. 41). For instance, at the equator, the object moves approximately 462 m in 1 second, while at 60° latitude, it moves only 231 m in one second.
Table 11. Geometric Errors Due to Expected On-orbit Navigation Uncertainty

<table>
<thead>
<tr>
<th>Navigation Tracking System</th>
<th>Unmodeled Perturbation</th>
<th>Minimum Altitude Error, km</th>
<th>1σ Altitude Error, m</th>
<th>Max. Geom. Error in FOV, m</th>
<th>1σ Radial Velocity, m/s</th>
<th>Max. Geom. Error in FOV, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>STDN</td>
<td>Nominal</td>
<td>200</td>
<td>508</td>
<td>7.32</td>
<td>3.87</td>
<td>.056</td>
</tr>
<tr>
<td>TDRSS</td>
<td></td>
<td>200</td>
<td>366</td>
<td>5.27</td>
<td>2.25</td>
<td>.032</td>
</tr>
<tr>
<td>STDN</td>
<td>Small</td>
<td>200</td>
<td>305</td>
<td>4.39</td>
<td>2.50</td>
<td>.036</td>
</tr>
<tr>
<td>TDRSS</td>
<td></td>
<td>300</td>
<td>122</td>
<td>1.76</td>
<td>.59</td>
<td>.0085</td>
</tr>
<tr>
<td>STDN</td>
<td>Large</td>
<td>300</td>
<td>81</td>
<td>2.63</td>
<td>1.12</td>
<td>.016</td>
</tr>
<tr>
<td>TDRSS</td>
<td></td>
<td>200</td>
<td>813</td>
<td>11.71</td>
<td>9.27</td>
<td>.133</td>
</tr>
</tbody>
</table>

STDN — Spaceflight Tracking and Data Network
TDRSS — Tracking Data Relay Satellite System
Table 12. Expected On-Orbit Navigation Accuracies (Ref. 12)

<table>
<thead>
<tr>
<th>Navigation Tracking System</th>
<th>Unmodeled Perturbation</th>
<th>Minimum Perigee, n.mi.</th>
<th>3σ Position, Feet</th>
<th>3σ Velocity, FIS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Radial</td>
<td>Down Track</td>
<td>Cross Track</td>
</tr>
<tr>
<td>STDN</td>
<td>Nominal</td>
<td>105</td>
<td>5,000</td>
<td>34,500</td>
</tr>
<tr>
<td>TDRSS</td>
<td></td>
<td>105</td>
<td>3,600</td>
<td>18,500</td>
</tr>
<tr>
<td>STDN</td>
<td>Small</td>
<td>105</td>
<td>3,000</td>
<td>22,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>1,200</td>
<td>5,500</td>
</tr>
<tr>
<td>TDRSS</td>
<td>Small</td>
<td>105</td>
<td>1,800</td>
<td>10,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>800</td>
<td>2,000</td>
</tr>
<tr>
<td>STDN</td>
<td>Large</td>
<td>105</td>
<td>8,000</td>
<td>80,000</td>
</tr>
<tr>
<td>TDRSS</td>
<td>Large</td>
<td>105</td>
<td>6,000</td>
<td>50,000</td>
</tr>
</tbody>
</table>

NOTE:

The correlation between downtrack position and radial velocity is -0.95.
The correlation between radial position and downtrack velocity is -0.80.

STDN = Spaceflight Tracking and Data Network
TDRSS = Tracking Data Relay Satellite System
Table 13. Geometric Error Sensitivity
Nominal Orbit: 400 km Circular

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Attitude Offset</th>
<th>Error Sensitivity, m/arc sec</th>
<th>Geometric Error for 1° Angular Error, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>For View Angle = 0°</td>
<td>For View Angle = ±.825°</td>
</tr>
<tr>
<td>Roll angle</td>
<td>No offset</td>
<td>1.939</td>
<td>1.940</td>
</tr>
<tr>
<td>Pitch angle</td>
<td></td>
<td>1.939</td>
<td>1.939</td>
</tr>
<tr>
<td>Yaw angle</td>
<td></td>
<td>0</td>
<td>.0279</td>
</tr>
<tr>
<td>Roll angle</td>
<td>20° roll offset</td>
<td>2.25</td>
<td>2.22</td>
</tr>
<tr>
<td>Pitch angle</td>
<td></td>
<td>1.94</td>
<td>1.94</td>
</tr>
<tr>
<td>Yaw angle</td>
<td></td>
<td>.72</td>
<td>.75</td>
</tr>
<tr>
<td>Roll angle</td>
<td>22.5° pitch offset</td>
<td>2.11</td>
<td>2.11</td>
</tr>
<tr>
<td>Pitch angle</td>
<td></td>
<td>2.31</td>
<td>2.31</td>
</tr>
<tr>
<td>Yaw angle</td>
<td></td>
<td>.83</td>
<td>.83</td>
</tr>
<tr>
<td>Roll angle</td>
<td>45° pitch offset</td>
<td>2.82</td>
<td>2.93</td>
</tr>
<tr>
<td>Pitch angle</td>
<td></td>
<td>4.33</td>
<td>4.36</td>
</tr>
<tr>
<td>Yaw angle</td>
<td></td>
<td>1.94</td>
<td>1.97</td>
</tr>
<tr>
<td>Altitude uncertainty</td>
<td>0</td>
<td>.0144 m/m</td>
<td>14.4 m per 1 km altitude change max in FOV</td>
</tr>
</tbody>
</table>
### Table 14. Geometric Error Induced by Shuttle and Imaging Spectrometer Measurement Uncertainties

Nominal Orbit: 400 km Circular

<table>
<thead>
<tr>
<th>Source</th>
<th>Attitude Offset</th>
<th>Per Axis, arc * sec</th>
<th>Per Axis Geometric Error, m</th>
<th>2 Axes, geometric error, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMU/Shuttle Misalignment</td>
<td>No Offset</td>
<td>82</td>
<td>159</td>
<td>225</td>
</tr>
<tr>
<td>IMU Resolution</td>
<td></td>
<td>20</td>
<td>39</td>
<td>55</td>
</tr>
<tr>
<td>IMU Noise</td>
<td></td>
<td>20</td>
<td>39</td>
<td>55</td>
</tr>
<tr>
<td>Rate Gyro</td>
<td></td>
<td>60 arc sec/sec</td>
<td>116 m/s</td>
<td>165 m/s</td>
</tr>
<tr>
<td>Combined Shuttle/IS Misalignment</td>
<td></td>
<td>87</td>
<td>169</td>
<td>239</td>
</tr>
<tr>
<td>IMU/Shuttle Misalignment</td>
<td>20° Roll Offset</td>
<td>82</td>
<td>187</td>
<td>245</td>
</tr>
<tr>
<td>IMU Resolution</td>
<td></td>
<td>20</td>
<td>46</td>
<td>60</td>
</tr>
<tr>
<td>IMU Noise</td>
<td></td>
<td>20</td>
<td>46</td>
<td>60</td>
</tr>
<tr>
<td>Rate Gyro</td>
<td></td>
<td>60 arc sec/sec</td>
<td>137 m/s</td>
<td>180 m/s</td>
</tr>
<tr>
<td>Combined Shuttle/IS Misalignment</td>
<td></td>
<td>87</td>
<td>198</td>
<td>260</td>
</tr>
<tr>
<td>IMU/Shuttle Misalignment</td>
<td>45° Pitch Offset</td>
<td>82</td>
<td>358</td>
<td>426</td>
</tr>
<tr>
<td>IMU Resolution</td>
<td></td>
<td>20</td>
<td>87</td>
<td>104</td>
</tr>
<tr>
<td>IMU Noise</td>
<td></td>
<td>20</td>
<td>87</td>
<td>104</td>
</tr>
<tr>
<td>Rate Gyro</td>
<td></td>
<td>60 arc sec/sec</td>
<td>262 m/s</td>
<td>312 m/s</td>
</tr>
<tr>
<td>Combined Shuttle/IS Misalignment</td>
<td></td>
<td>87</td>
<td>379</td>
<td>452</td>
</tr>
</tbody>
</table>

\[\dagger\] Combined roll and pitch axes.
V. GROUND PATTERNS AND IMAGE DISTORTIONS

The extent that direct errors and earth geometric properties affect the performance of the Shuttle Imaging Spectrometer are analyzed and extensively illustrated with charts and tables in section IV. The advantages of tables and charts are that they carry precise and specific information which is invaluable for design and performance prediction. However, it is difficult to relate this information directly to images of ground objects by human brains. To compensate for this and to make direct observation of how errors in the system affect ground images, selected ground patterns as seen through this optical system are studied here.

The patterns that are readily generated and, most importantly, suitable for exhibiting imaging distortions are square grid patterns.

To cover a large ground that is comparable to the field of view of the imaging spectrometer, a 10 km field is selected and the field is evenly divided into 10 segments of 1 km each. Since the imaging spectrometer employs a push broom principle, each field is then 10 km wide (cross track) and 30 m "long" (along track). However, to see a large ground area, a 10 km x 10 km field is employed with the along track grids tagged with time. Considering a perfectly spherical earth, an imaginary 10 x 10 grid is painted on the ground. As the satellite flies through this region, in the direction from the bottom to the top of the paper, a push broom camera should see a pattern just like that of Fig. 43(a). This pattern is referred to as the nominal ground pattern.

Fig. 43 shows how the ground pattern changes when simple attitude error occurs without considering the effect of Earth rotation. To show the effect of errors, the solid pattern (actual) is overlayed with the "+" pattern (the undistorted nominal pattern). Fig. 43(b) shows that when a 0.1° roll attitude error occurs, the IS instrument points 0.1° left of the object field and the image on...
Fig. 43. Ground Pattern Distortions With No Earth Rotation Effect

(a) NOMINAL GROUND PATTERN

(b) WITH 0.1° ROLL ERROR

(c) WITH 5° YAW ERROR

(d) WITH 0.1° PITCH ERROR
the "film" has shifted to the right; therefore, the solid pattern has shifted to the right of the "+" pattern. The same explanation applies also to the 0.1° pitch error as shown in Fig. 43(d), except that, for pitch, the pattern image has shifted to the "-along track" direction. A 5° yaw error (clockwise rotation) will make the pattern appear skewed as shown in Fig. 43(c).

Fig. 44 shows the effects of simple attitude errors with 20° roll offset. With the large roll angle, the nominal pattern itself has suffered cross track distortions, i.e., the pattern image appears shrunk cross-trackwise as shown in Fig. 44(a). In Fig. 44(b), (c), and (d), the "+" patterns represent the distorted nominal patterns due to roll offset alone and the solid patterns represent those due to attitude errors on top of roll offset. The distortions are similar to those found in Fig. 43 except that with the 20° roll offset the images appear narrower.

The effects of simple attitude errors with 45° pitch offset are shown in Fig. 45. The effect of pitch offset is similar to roll offset, i.e., the nominal pattern image has shrunk cross-trackwise. One might expect also along-track shrinkage. The reason that there is no along-track shrinkage is because of the push-broom effect. Shrinkage will occur for a frame camera, however. It is noted that the pattern images associated with the 45° pitch offset are narrower than those of the 20° roll offset. This is due to the fact that 20° is a lot less than 45°.

The effects of constant attitude rate errors are shown in Fig. 46. Rate errors cause accumulation of attitude errors, i.e., the attitude errors grow with time t. Take the 0.1 deg/sec rate error for the roll axis for instance, the roll error is 0.1 t; whereas, the 0.1° roll error (Fig. 43(b)) at t = 0 is 0; therefore, the "+" pattern image coincides with the solid one at a very
Fig. 44. Ground Pattern Distortions With 20° Roll Offset and No Earth Rotation Effect
Fig. 45. Ground Pattern Distortions With 45° Pitch Offset and No Earth Rotation Effect
Fig. 46. Ground Pattern Distortions With Constant Rotation Rate and No Earth Rotation Effect.
small shift. Shown in Fig. 46(b) is the solid pattern shift to the right at a constant slope with time, which is quite different from Fig. 43(b). Similar comment applies to Fig. 46(c) and (d).

The effects of sinusoidal attitude rates on the pattern images are shown in Fig. 47. For instance, the sinusoidal rate causing a 0.2 sin (4.51378t) degree roll error will make the image appear as the shape "S", as shown in Fig. 47(b). The same function applied to the pitch axis will cause along-track distortions with densely packed grid lines followed by loosely packed ones, etc., as illustrated in Fig. 47(d).

Altitude change causes images to vary in size. Again, it affects the cross-track much more than the along-track due to the push-broom principle. As shown in Fig. 48, the solid image has shrunk cross-trackwise but expanded along-trackwise. The former is due to the altitude increase of 40 km which has no amplification effect on push-broom camera; the latter is due to the fact that, at higher altitude, the orbital rate is decreased, i.e., it will take a longer period of time to fly through the same ground area.

Finally, the effects of earth rotation are shown in Fig. 49. Earth rotation will cause the image to skew toward the direction of earth rotation. As discussed in section IV, the effect of the image shift is most pronounced when the satellite flies through the equator, and reduces as the latitude increases. Earth rotation will not cause along-track distortions.

Computer programs that are used to generate these patterns and distortions are listed in Appendix F.
Fig. 47. Ground Pattern Distortions With Sinusoidal Rotation Rate and No Earth Rotation Effect

(a) NOMINAL GROUND PATTERN

(b) WITH 0.2° SIN (4.51378t) ROLL ERROR

(c) WITH 5° SIN (4.51378t) YAW ERROR

(d) WITH 0.1° SIN (4.51378t) PITCH ERROR
Fig. 48. Ground Pattern Distortions With Effect of Altitude Change (increase) of 40 km and No Earth Rotation Effect
Fig. 49. Ground Pattern Distortions With Effect of Earth Rotation for 40° Latitude and Orbit Inclination of 85°
VI. CONCLUSIONS

1. The effects of earth curvature are very small for the application here (see Fig. 42).

2. Altitude uncertainties cause only moderate geometric errors. The worst 1σ geometric errors are 11.71 m in position and 0.133 m/sec in rate with STDN and large unmodeled perturbations at 200 km orbit. The performance improves with TDRSS. For the 300 km orbit and with small unmodeled perturbations, the 1σ geometric errors will reduce to 0.22 m and 0.0032 m/sec (see Table 11).

3. The effects of other navigation errors are significantly greater. The 1σ downtrack errors range from 203 m (300 km orbit) to 8128 m (200 km orbit); and those for the cross track are 152 m to 508 m (see Table 12).

4. The effects of roll and pitch attitude errors are relatively large compared with, for instance, those caused by yaw errors and altitude uncertainties. The error sensitivity is 1.94 m/arc sec or approximately 7000 m/degree. The yaw sensitivity is 0 for a 0° view angle and 0.028 m/arc sec for the maximum view angle of ± 0.825° (see Table 12).

5. The error sensitivities of attitude errors increase significantly for large attitude offsets. For 20° side looking, the sensitivity increases to 2.28 m/arc sec for roll errors and to 0.75 m/arc sec for yaw errors, and the pitch error sensitivity is almost unaffected. For the 45° forward looking case, the sensitivities for the pitch, roll, and yaw errors increase to 2.83, 4.36, and 1.97 m/arc sec, respectively (see Table 13).

6. The performance of the Shuttle Imaging Spectrometer is limited by the Shuttle IMU (Inertial Measuring Unit) accuracy, instrument misalignment, Shuttle RCS (Reaction Control Subsystem) deadband, etc. unless some means of error reduc-
tion are employed. For instance, ground control points may be used to reduce navigation prediction errors; and precision point mounts, such as AGS (ASPS) and IPS (Instrument Pointing System), may be used to reduce the attitude errors. The single axis geometric errors due to the combined Shuttle/IS misalignment, for instance, for normal nadir pointing, 20° side looking, and 45° forward looking are 169 m, 198 m, and 379 m, respectively (refer to Table 14).

7. Earth rotation causes shifts of images toward the direction of rotation. The magnitude of these shifts depends on the latitude of the object. For instance, at the equator the object moves approximately 462 m in one second (for 400 km orbit), and at 60° latitude it moves only 231 m in one second.

8. Distorted images for selected ground patterns provide revealing information of the effects of system errors to the images of ground objects.

+ Annular Suspension Pointing System
REFERENCES


**APPENDIX A**

**GEOMETRIC ERROR MAPPING FUNCTION TABLE**

Part A: Given the attitude offsets and a view angle, find the coordinates of the corresponding ground points before and after the errors are introduced.

<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Altitude Error, Δh</strong></td>
<td>at nominal altitude ( h ), a view angle ( λ ) is given which sights a ground point ( P(0, Y_0) ). After elevating ( Δh ), the same view angle will sight another ground point ( P'(0, Y'_0) ). The following equations are for obtaining ( Y_0 ) and ( Y'_0 ):</td>
<td>( R = \text{Earth Radius} )</td>
</tr>
<tr>
<td></td>
<td>( Y_0 = -R \sin λ \left[ \left( 1 + \frac{h}{R} \right) \cos λ - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 λ} \right] )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( Y'_0 = -R \sin λ \left[ \left( 1 + \frac{h + Δh}{R} \right) \cos λ - \sqrt{1 - \left( 1 + \frac{h + Δh}{R} \right)^2 \sin^2 λ} \right] )</td>
<td></td>
</tr>
<tr>
<td>**Yaw Error, ( ψ )</td>
<td>given a yaw offset ( ψ ) and a view angle ( λ ) which sights a ground point ( P(X, Y) ). The same view angle will sight another ground point ( P'(X', Y') ) after yaw error ( ψ ) is introduced. The following equations are for obtaining ( X, Y, X' ), and ( Y' ):</td>
<td>( R = \text{Earth Radius} ) ( h = \text{Nominal Altitude} )</td>
</tr>
<tr>
<td></td>
<td>( X = \left[ \left( 1 + \frac{h}{R} \right) \cos λ - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 λ} \right] R \sin λ \sin ψ )</td>
<td></td>
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<tr>
<td></td>
<td>( Y = -\left[ \left( 1 + \frac{h}{R} \right) \cos λ - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 λ} \right] R \sin λ \cos ψ )</td>
<td></td>
</tr>
<tr>
<td>Error Sources</td>
<td>Geometric Error Mapping Function</td>
<td>Comments</td>
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<tr>
<td>---------------</td>
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</table>
| Roll Error, \( \phi \) | \[ \begin{align*} 
X^* &= \left( 1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \] \[ \times \] \[ R \sin \lambda \sin (\psi_o + \phi) \] 
Y^* &= \left( 1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \] \[ \times \] \[ R \sin \lambda \cos (\psi_o + \phi) \] 
| Given: a roll offset \( \phi_o \) and a view angle \( \lambda \) which sights a ground point \( P(0, Y^*) \). The same view angle will sight another ground point \( P'(0, Y'^*) \) after roll error \( \phi \) is introduced. The following equations are for obtaining \( Y^* \) and \( Y'^* \): | \( R = \) Earth Radius \( h = \) Nominal Altitude |
| Pitch Error, \( \theta \) | \[ \begin{align*} 
Y^* &= -R \sin (\lambda + \phi_o) \left[ \left( 1 + \frac{h}{R} \right) \cos (\lambda + \phi_o) - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 (\lambda + \phi_o)} \right] 
Y'^* &= -R \sin (\lambda + \phi_o + \phi) \left[ \left( 1 + \frac{h}{R} \right) \cos (\lambda + \phi_o + \phi) \right. 
\left. - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 (\lambda + \phi_o + \phi)} \right] 
\end{align*} \] 
| Given: a pitch offset \( \phi_o \) and a view angle \( \lambda \) which sights a ground point \( P(X^*, Y^*) \). The same view angle will sight another ground point \( P'(X'^*, Y'^*) \) after pitch error \( \theta \) is introduced. The following equations are for obtaining \( X^*, Y^*, X'^*, \) and \( Y'^* \): | \( R = \) Earth Radius \( h = \) Nominal Altitude For \( \tau \) and \( \xi \), refer to Fig. 23. |
### Error Sources

<table>
<thead>
<tr>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X' = R \left[ (1 + \frac{h}{R}) \cos \xi - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi} \right] \sin \xi \sin (\theta_o + \theta) \frac{\sin \xi \sin (\theta_o + \theta)}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda}} )</td>
<td></td>
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<tr>
<td>( Y' = -R \left[ (1 + \frac{h}{R}) \cos \xi - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi} \right] \sin \xi \tan \lambda \frac{\sin \xi \tan \lambda}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda}} )</td>
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<tr>
<td>( \tau = \tan^{-1} \left( \sec \theta_o \sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda} \right) )</td>
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<tr>
<td>( \xi = \tan^{-1} \left[ \sec (\theta_o + \theta) \sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda} \right] )</td>
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</table>

#### Yaw Error Source

Given a yaw offset \( \psi_o \), followed by a pitch offset \( \theta_o \), then a roll offset \( \phi_o \).

A view angle \( \lambda \) is also given which sights a ground point \( P_0(X_o, Y_o) \). After the errors \( \psi, \theta, \) and \( \phi \) are introduced, the same view angle will sight another ground point \( P_3(X_3, Y_3) \). The following equations are for obtaining \( X'_o, Y'_o, X'_3, \) and \( Y'_3 \):

\[
X'_o = \frac{R \sin \theta_o \sin \xi_o \left[ (1 + \frac{h}{R}) \cos \xi_o - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi_o} \right]}{\sqrt{\sin^2 \theta_o + \tan^2 (\lambda + \phi_o)}}
\]

\[
Y'_o = -\frac{R \tan (\lambda + \phi_o) \sin \xi_o \left[ (1 + \frac{h}{R}) \cos \xi_o - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi_o} \right]}{\sqrt{\sin^2 \theta_o + \tan^2 (\lambda + \phi_o)}}
\]

<table>
<thead>
<tr>
<th>Comments</th>
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<tbody>
<tr>
<td>R = Earth Radius</td>
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<tr>
<td>h = Nominal Altitude</td>
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<tr>
<td>For ( \xi_o ) and ( \xi'_3 ), refer to Fig. 23.</td>
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<tr>
<td>Error Sources</td>
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</tbody>
</table>
Part B: Given the coordinates of a ground point, find the attitude offsets, the corresponding view angle and the coordinates of the point with the same view angle after the errors are introduced.

<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude Error, ( \Delta h )</td>
<td>At nominal altitude ( h ), a ground point ( P(X_o, Y_o) ) corresponding to view angle ( \lambda ) is given. After elevating ( \Delta h ), the same view angle ( \lambda ) will aim at another ground point ( P'(X'_o, Y'_o) ). The following equations are for obtaining ( X'_o ) and ( Y'_o ):</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \lambda = \tan^{-1} \left( \frac{\sqrt{X_o^2 + Y_o^2}}{R + h - \sqrt{R^2 - X_o^2 - Y_o^2}} \right) ]</td>
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<tr>
<td></td>
<td>[ X'_o = \frac{RX_o \sin \lambda}{\sqrt{X_o^2 + Y_o^2}} \left[ (1 + \frac{h + \Delta h}{R}) \cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R}\right)^2 \sin^2 \lambda} \right] ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ Y'_o = \frac{RY_o \sin \lambda}{\sqrt{X_o^2 + Y_o^2}} \left[ (1 + \frac{h + \Delta h}{R}) \cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R}\right)^2 \sin^2 \lambda} \right] ]</td>
<td></td>
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</tbody>
</table>

R = Earth Radius
A ground point \( P(0, y^*) \) corresponding to roll offset \( \phi_o \) and view angle \( \lambda \) is given. After roll error \( \phi \) is introduced, the same view angle \( \lambda \) will aim at another ground point \( P'(0, y^{*'}) \). The following equations are for obtaining \( y^{*'} \):

\[
\lambda + \phi_o = -\text{sgn}(y^*) \tan^{-1}\left( \frac{|y^*|}{R + h - \sqrt{R^2 - y^*2}} \right)
\]

\[
y^{*'} = -R \sin' (\lambda + \phi_o + \phi) \left[ (1 + \frac{h}{R}) \cos (\lambda + \phi_o + \phi) \right.
\]

\[
\left. -\sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 (\lambda + \phi_o + \phi)} \right]
\]

<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll Error, ( \phi )</td>
<td></td>
<td>( R = ) Earth Radius ( h = ) Nominal Altitude</td>
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</tbody>
</table>
### Geometric Error Mapping Function

<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
</table>
| Pitch Error, $\theta$ | A ground point $P(X^*, Y^*)$ corresponding to pitch offset $\theta_o$ and view angle $\lambda$ is given. After pitch error $\theta$ is introduced, the same view angle $\lambda$ will aim at another ground point $P'(X'^*, Y'^*)$. The following equations are for obtaining $X'^*$ and $Y'^*$: | R = Earth Radius  
h = Nominal Altitude  
For $\xi$, refer to Fig.23. |

\[
\lambda = \tan^{-1}\left[\frac{Y^*}{\left(R + h - \sqrt{R^2 - X^*^2 - Y^*^2}\right)\left(1 + \left(\frac{X^*}{R + h - \sqrt{R^2 - X^*^2 - Y^*^2}}\right)^2\right)}\right]
\]

\[
\theta_o = \tan^{-1}\left[\frac{X^*}{R + h - \sqrt{R^2 - X^*^2 - Y^*^2}}\right]
\]

\[
\xi = \tan^{-1}\left[\sec(\theta_o + \theta)\sqrt{\sin^2(\theta_o + \theta) + \tan^2\lambda}\right]
\]

\[
X'^* = \frac{R \sin(\theta_o + \theta) \sin\xi \left(1 + \frac{h}{R}\right) \cos\xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2\xi}}{\sqrt{\sin^2(\theta_o + \theta) + \tan^2\lambda}}
\]

\[
Y'^* = -\frac{R \tan\lambda \sin\xi \left(1 + \frac{h}{R}\right) \cos\xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2\xi}}{\sqrt{\sin^2(\theta_o + \theta) + \tan^2\lambda}}
\]
<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw Error, $\psi$</td>
<td>A ground point $P(X^<em>, Y^</em>)$ corresponding to yaw offset $\psi_o$ and a certain view angle is given. After yaw error $\psi$ is introduced, the same view angle will aim at another ground point $P'(X'^<em>, Y'^</em>)$. The following equations are for obtaining $X'^<em>$ and $Y'^</em>$:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\psi_o = -\tan^{-1}\left(\frac{X^<em>}{Y^</em>}\right)$</td>
<td>$R = $ Earth Radius</td>
</tr>
<tr>
<td></td>
<td>$\lambda = -\text{sgn}(Y^<em>) \tan^{-1}\left(\frac{\sqrt{X^</em>^2 + Y^<em>^2}}{R + h - \sqrt{R^2 - X^</em>^2 - Y^*^2}}\right)$</td>
<td>$h = $ Nominal Altitude</td>
</tr>
<tr>
<td></td>
<td>$X'^* = \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda}\right] R \sin \lambda \sin (\psi_o + \psi)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Y'^* = -\left[\left(1 + \frac{3}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda}\right] R \sin \lambda \cos (\psi_o + \psi)$</td>
<td></td>
</tr>
<tr>
<td>Earth Rotation</td>
<td>Let $\Gamma$ be the shuttle orbital inclination at the equator, and $\zeta$ be the latitude of the nadir point. A given point $P(X, Y)$, after time period $\Delta t$, will move to $P'(X', Y')$. The following equations are for obtaining $X'$ and $Y'$:</td>
<td>$V_e = $ Linear Velocity of Ground Point at Latitude $\zeta$</td>
</tr>
<tr>
<td></td>
<td>$V_e = \omega R \cos \zeta$</td>
<td>$\omega = $ Angular Velocity of Earth Rotation</td>
</tr>
<tr>
<td></td>
<td>$\omega = \frac{2\pi}{86,400}$ radians/sec</td>
<td>$\Delta t$ is assumed small.</td>
</tr>
<tr>
<td></td>
<td>$X' = X + \frac{\cos \Gamma}{\cos \zeta} V_e \Delta t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Y' = Y + \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \zeta}\right)^2} V_e \Delta t$</td>
<td></td>
</tr>
<tr>
<td>Error Sources</td>
<td>Geometric Error Mapping Function</td>
<td>Comments</td>
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</tbody>
</table>
| Yaw Error $\phi$ followed by Pitch Error $\theta$ then Roll Error $\phi$ | A ground point P(0,Y*) corresponding to view angle $\lambda$ (with no attitude offset) is given. A yaw error $\phi$ is introduced first, then a pitch error $\theta$, finally a roll error $\phi$. The same view angle $\lambda$ will then aim at another ground point P'(X*,Y*'). The following equations are for obtaining X* and Y*: | R = Earth Radius  
$\ h = $ Nominal Altitude  
For $\xi$, refer to Fig. 23. |
| $\lambda = - \text{sgn}(Y*) \tan^{-1} \left( \frac{|Y*|}{R + h - \sqrt{R^2 - Y*^2}} \right)$ | |
| $\xi = \tan^{-1} \left[ \frac{\text{sec} \theta \sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}{R \sin \theta \sin \xi} \right]$ | |
| $X = \frac{R \sin \theta \sin \xi \left[ (1 + \frac{h}{R}) \cos \xi - \sqrt{1 - (1 + \frac{h}{R})^2 \sin^2 \xi} \right]}{\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}$ | |
| $Y = \frac{-R \tan (\lambda + \theta) \sin \xi \left[ (1 + \frac{h}{R}) \cos \xi - \sqrt{1 - (1 + \frac{h}{R})^2 \sin^2 \xi} \right]}{\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}$ | |
| $X^* = X \cos \phi - Y \sin \phi$ | |
| $Y^* = X \sin \phi + Y \cos \phi$ | |
APPENDIX B

NAPIER'S RULES FOR RIGHT SPHERICAL TRIANGLES*

A right spherical triangle has five variable parts. If these components and their complements (complement of $\Gamma = 90^\circ - \Gamma$) are arranged in a circle, as illustrated below

then, the following relationships hold between the five components in the circle:

The sine of any component equals the product of either:

1. The tangents of the adjacent components, or
2. The cosines of the opposite components

For our case, from 2:

$$\sin (90^\circ - \Gamma) = \cos \zeta \cos (90^\circ - \Lambda)$$

i.e.,

$$\cos \Gamma = \cos \zeta \sin \Lambda$$

*See Ref. 13
APPENDIX C

COMPUTER PROGRAM FOR SHUTTLE IMAGING SPECTROMETER POINTING ERROR

POWER SPECTRAL DENSITIES FOR CONFIGURATION A -- PAYLOAD-BAY NADIR POINTING

ELT Wed-10/22/82-10:22:28-152,
12537*IS:(1).SAMP-PROG152
1:PROGRAM IMAGING SPECTROMETER SHUTTLE RIGID MOUNT - A
2:INITIAL
3:VARIABLE T=0.0
4: INTEGER FMUM,NPOINT
5:
6:
7:COMMENT INITIALIZE SOME PARAMETERS
8: CONSTANT NPOINT=641
9: CONSTANT FNMR=39
10: CALL FOPEN(FUMM)
11: TFNRM=1.0*NPOINT-1.0
12: CONSTANT FACTR=0.0
13: CONSTANT SCAFAC=1.0E+5
14: SCAFAC2=SCAFAC*SCAFAC
15: SCAFAC21=10.0*LOG10(SCAFAC2)
16:
17:COMMENT SET VARIOUS MATHEMATICAL CONSTANTS
18: PI=3.14159265
19: RSRC2=PI-RS2*3600.0
20: RSRC2=PI/2.0
21:
22:COMMENT DEFINE MOMENTS OF INERTIA
23: CONSTANT IX=1.3BE+6
24: CONSTANT IY=1.00E+7
25: CONSTANT IZ=1.05E+7
26: IXZ=IX*IX
27: IY=1.0Y
28: IZZ=IZ*IZ
29: IXY=IX*Y+IY
30: IXY=IXY
31:
32:COMMENT DEFINE W0
33: CONSTANT W0=0.0011315
34: W0=W0+W0
35: W0=W0+W0
36:
37:COMMENT DEFINE INITIAL ANGLES AND RATES
38: CONSTANT TH0=0.0
39: CONSTANT PHI0=0.0
40: CONSTANT PSI0=0.0
41: CONSTANT TH0D=0.0
42: CONSTANT PHI0D=0.0
43: CONSTANT PSI0D=0.0
44: TH02=TH0*TH0
45: TH0D=TH0D
46: PHI0D=PHI0D
47: PHI0D=PHI0D
48: PSI0D=PSI0D
49: PSI0D=PSI0D
50:
51:COMMENT DEFINE O'S
52: CONSTANT OX=2.014E-6
53: CONSTANT OY=1.251E-3
54: CONSTANT OZ=5.34BE-4
55:
56:COMMENT DEFINE RATE FILTER PARAMETERS
57: CONSTANT WD=0.2513
58: CONSTANT ZETR=0.0
59: W0=W0+W0
60: W0=W0+W0
61: ZETR=ZETR
62:
63:COMMENT DEFINE VARIOUS CONSTANTS
64: CONSTANT R=8.55E-10

125
65: CONSTANT SIGBO=87.32
66: SIGBO=SIGBO*MSCC
67: SIGBO2=SIGBO*SIGBO
68: CONSTANT Tau=0.023
69: Tau=TAU#TRU
70: DELW=1.0
71: 72:COMMENT DEFINE W CONSTANTS AND INITIALIZE W
73: CONSTANT WL=6.26318531E-7
74: CONSTANT WFAC=k.029200527
75: W+WL=WFAC
76: 77:END
78:DYNAMIC
79:CINTERVAL CI+1.0
80:
81:COMMENT COMPUTE W
82: W+WFAC
83: PROCEDURE(DELW=W)
84: DELW=0.0
85: IF(W.LE.O.) DELW=1.0
86: END
87: W=W+H*RSHZCC
88: FREQ=RLG10(WZ)
89: W2=W2+W
90: W=W2+W2
91:
92:COMMENT COMPUTE M42 AND DM2
93: M42=1.0/(TRU=$#1+1.0)
94: DM2=4.0*W2/(W2+W2)#W2*0.0*W2+W2
95: DM2=1.0*W2=W2+(W2+1.0)*(W2+1.0)
96:COMMENT DM2=M42#4.0*W2*(1+1.0-W2*(1+1.0))
97: DM2=DM2/SCFRIC
98: DM2=DM2/DM2
99: 100:COMMENT COMPUTE F’S
101:COMMENT FH2=H2*SCFRIC/(W2+1.0+W2+1.0)
102:COMMENT FH2=H2*W2/(1+1.0+W2+1.0)
103:COMMENT FY2=PHI2/(1+W2+1.0+W2+1.0)
104: FH2=H2*W2*SCFRIC/(W2+1.0+W2)
105: FH2=H2*W2/(1+W2+1.0+W2)
106: FH2=H2*W2/(1+W2+1.0+W2)
107: FH2=H2*W2/(1+W2+1.0+W2)
108: 109:COMMENT COMPUTE PSD’S
110:
111:COMMENT PITCH PSD COMPUTATIONS
112:PPH1=FTH2*Y2*TH2D2
113:PPH2=FTH2*Y2*TH2W2
114:PPH3=FTH2*GY
115:PPH4=SIGBO2*DELW*(PPH1+PPH2+PPH3)/SCFRIC
116:PPH4=W*HD4
117: 118:COMMENT ROLL PSD COMPUTATIONS
119:PRH1=FHR2*Y2*PHI2O2
120:PRH2=FHR2*Y2*PHI2O2
121:PRH3=FHR2*Y2*PHI2O2
122:PRH4=FHR2*O2
123:PRH5=FHR2*Z2*PSI02
124:PRH6=FHR2*Z2*PSI02
125:PRH7=FHR2*O2
126:PRH8=FHR2*O2
127:RS=SIGBO2*DELW
128:PRH4=PRH4+(PRH1+PRH2+PRH3+PRH5+PRH6+PRH7+PRH8)/SCFRIC
129:PRH4=PRH4+HD4

131: COMMENT YFW PSD COMPUTATIONS
132:  PYW1+PYW2+PS102
133:  PYW3+PYW4+PS102+PS202
134:  PYW5+PYW6+PS102
135:  PYW7+PYW8+PS102
136:  PYW9+PYW10+PS102
137:  PYW11+PYW12+PS102
138:  PYW13+PYW14+PS102
139:  PYW15+PYW16+PS102
140:  PYW17+PYW18+PS102
141:  PYW19+PYW20+PS102
142:  
143:  
144: COMMENT PREPARE VARIABLES FOR OUTPUT
145:  RSIG+PS102+PS202
146:  RSIGDB+10.0*ALOG10(RSIG)+FACTOR
147:  PPNDDB+10.0*ALOG10(PPW)+SCF2DB+FACTOR
148:  PPNDWB+10.0*ALOG10(PPW)+FACTOR
149:  PRWDB+10.0*ALOG10(PRWB)+SCF2DB+FACTOR
150:  PRWDB+10.0*ALOG10(PRWB)+SCF2DB+FACTOR
151:  PRWDB+10.0*ALOG10(PRWB)+FACTOR
152:  PRWDB+10.0*ALOG10(PRWB)+SCF2DB+FACTOR
153:  PRWDB+10.0*ALOG10(PRWB)+SCF2DB+FACTOR
154:  PRWDB+10.0*ALOG10(PRWB)+FACTOR
155:  
156: PPNDDB+10.0*ALOG10(PPW)+FACTOR
157:  PRWDB+10.0*ALOG10(PRWB)+FACTOR
158:  PRWDB+10.0*ALOG10(PRWB)+FACTOR
159:  
160: COMMENT SAVE NUMBERS IN FILE
161:  CALL FSAVE(FREG,PPNDWB,PPWDB,PRWDB,FNUM)
162:  
163:  
164:  TERMT(TE,TFINAL)
165:  DERIVATIVE
166:  ALGORITHM ALG 3
167:  GO=INTEG(1.0,0.0)
168:  END
169:  END
170: TERMINAL
171:  
172: COMMENT CLOSE FILE
173:  CALL FCLOSE(FNUM)
174:  END
175: END

EOT: 175
FOR,IS FF.FOPEN
2:    SUBROUTINE FOPEN(N)
3:      REWIND N
4:      RETURN
5:      CIC
6:    FOR,IS FF.FSAVE
7:    SUBROUTINE FSAVE(N,P1,P2,P3,N)
8:      WRITE(N,100)P1,P2,P3
9:      100     FORMAT(4G14.8)
10:     RETURN
11:    END
12:    FOR,IS FF.FCLOSE
13:    SUBROUTINE FCLOSE(N)
14:      ENDFILE N
15:      RETURN
16:     END
END
APPENDIX D

COMPUTER PROGRAM FOR SHUTTLE IMAGING SPECTROMETER PointING ERROR

POWER SPECTRAL DENSITIES FOR CONFIGURATION B — NOSE-DOWN NADIR POINTING

```plaintext
67 WED-10/20/82-10:36:41-14,
123715(111.4RHB/PRG(14)
1:PROGRAM IMAGING SPECTROMETER SHUTTLE RIGID MOUNT - B
2:INITIAL
3:VARIABLE T=0.0
4: INTEGER FIRM,NPOINT
5:
6:
7:COMMENT INITIALIZE SOME PARAMETERS
8: CONSTANT NPOINT=641
9: CONSTANT I/NM=39
10: CALL FORNT(FNLM)
11: TFNLM+1.0*NPOINT=1.0
12: CONSTANT FACTOR=0.0
13: CONSTANT SCFAC=1.0E+10
14: SCFAC2+SCFAC+SCFAC
15: SCF280=10.0*ALOG10(SCFAC2)
16:
17:COMMENT SET VARIOUS MATHEMATICAL CONSTANTS
18: PI=3.14159265
19: RSACC=PI/(180.0*3600.0)
20: ASRZCC=PI/(2.0481)
21:
22:COMMENT DEFINE MOMENTS OF INERTIA
23: CONSTANT I1=1.0E+7
24: CONSTANT I2=1.0E+7
25: CONSTANT I3=1.3E+8
26: IXZ=IX*IX
27: IYZ=IY*IY
28: IZZ=IZ*IZ
29: IXYZ=IXYZ+IXYZ
30: IXYZ=IXYZ+IXYZ
31:
32:COMMENT DEFINE NO
33: CONSTANT NO=0.0011315
34: WO2+WO+WO
35: WO4+WO2+WO2
36:
37:COMMENT DEFINE INITIAL ANGLES AND RATES
38: CONSTANT THO=0.0
39: CONSTANT PHI0=0.0
40: CONSTANT PSI0=0.0
41: CONSTANT THAO=0.0
42: CONSTANT PHIO=0.0
43: CONSTANT PISO=0.0
44: THO2+THO+THO
45: THO2+THO+THO
46: PHI2+PHI+PHI
47: PHI2+PHI+PHI
48: PSI02+PSI0+PSI0
49: PSI02+PSI0+PSI0
50:
51:COMMENT DEFINE O'S
52: CONSTANT G2=1.28E-4
53: CONSTANT G2=1.28E-4
54: CONSTANT G2=1.28E-4
55:
56:COMMENT DEFINE RATE FILTER PARAMETERS
57: CONSTANT W0=0.2513
58: CONSTANT ZETAO=0.8
59: W02+WO2+WO2
60: W04+WO2+WO2
61: ZO2+ZETAO+ZETAO
62:
63:COMMENT DEFINE VARIOUS CONSTANTS
64: CONSTANT R=8.55E-10
```
CONSTANT SIGBO=87.32
SIGBO=SIGBO+PSRCG
SIGBO2=SIGBO+SIGBO
CONSTANT TRU=0.023
TRUE=TRUE TRU
DELW=1.0

COMMENT DEFINE W CONSTANTS AND INITIALIZE W
CONSTANT MLO=6.28318531E-7
CON MFACT=1.02920527
W=MLO*MFACT

END

DYNAMIC
CINTERVAL CI=1.0

COMMENT COMPUTE W
W=W*MFACT
W=WFACT
DELW=W

PROCEDURE DELW=W)
DELW=0.0
IF(W.LE.0.) DELW=0.0
ELSE
WHZ=W+PSRCGC
FREQ=LOG10(WHZ)
HZ=HZM
W=WZ
HZ=HZ+PSRCZ
HZ=HZ+PSRCZ
HZ=HZ+PSRCZ
HZ=HZ+PSRCZ
HZ=HZ+PSRCZ
HZ=HZ+PSRCZ
HZ=HZ+PSRCZ

COMPUTE "5
THWZ=HWZ.SC'ACZ'CfIY.WZ+3.0.WOZ.fIZ-IX» •• Z)
FRWZ=IWZ-WZ-WOZ.IY-IXZ-2.HWZ/DWZ
YWZ. (IX-WZ+4. O.WOZ.fIZ-IY» •• 2.HWZ/DWZ
FRYWZ=WCZ.IXYZ2-ZWZ*MWZ'DWZ

COMMENT
ROLL "5 COMPUTATIONS
PRW1=FRWZ.IXZ2.PSIODZ
PRW2=FRWZ.IXZ2.PHI02Z
PRW3=FRWZ.IXZ2.PHI02Z
PRW4=FRWZ.IXZ2.PSI02Z
PRW5=FRWZ.IXZ2.PSI02Z
PRW6=FRWZ.IXZ2.PSI02Z
PRW7=FRWZ.IXZ2.PSI02Z
PRW8=FRWZ.IXZ2.PSI02Z
RS=SIGBO2+DELW
PRW4=PRW1+PRW2+PRW3+PRW5+PRW6+PRW7+PRW8/SCFACZ
PRW3=PRW3+DELW

COMMENT YAW "5 COMPUTATIONS
PYW1=FYWZ.IXZ2.PSI02Z
PYW2=FYWZ.IXZ2.PSI02Z
YH1 = FYH2 + WY2 * IXY2 * PHI02
YH3 = FYH2 + WY2 * IXY2 * PHI02
YH4 = FYH2 + WY2 * IXY2 * PHI02
YH5 = FYH2 + WY2 * IXY2 * PHI02
YH6 = FYH2 + WY2 * IXY2 * PHI02
YH7 = FYH2 + WY2 * IXY2 * PHI02
YH8 = FYH2 + WY2 * IXY2 * PHI02

PYH = RS * (FYH1 + FYH2 + FYH3 + FYH4 + FYH5 + FYH6 + FYH7 + FYH8) / SCFACZ
PYH = FYH + WY + PHI

140: COMMENT PREPARE VARIABLES FOR OUTPUT
141: RSIG = RSIG02 + DELW
142: RSIG = 10.0 * ALOG10(RSIG) + FACTOR
143: PPW3DB = 10.0 * ALOG10(PPW3) - SCFZDB + FACTOR
144: PPWDB = 10.0 * ALOG10(PPW) + FACTOR
145: PRWDB = 10.0 * ALOG10(PRW) - SCF2DB + FACTOR
146: PRWDB = 10.0 * ALOG10(PRW) + FACTOR
147: PYWDB = 10.0 * ALOG10(PYW) - SCF2DB + FACTOR
148: PYWDB = 10.0 * ALOG10(PYW) + FACTOR
149: PYWDB = 10.0 * ALOG10(PYW) + FACTOR
150: PYWDB = 10.0 * ALOG10(PYW) - SCFZDB + FACTOR
151: PYWDB = 10.0 * ALOG10(PYW) + FACTOR
152: PPWDB = 10.0 * ALOG10(PPW) + FACTOR
153: PPWDB = 10.0 * ALOG10(PPW) + FACTOR
154: PPWDB = 10.0 * ALOG10(PPW) + FACTOR
155: PPWDB = 10.0 * ALOG10(PPW) + FACTOR
156: PPWDB = 10.0 * ALOG10(PPW) + FACTOR

157: COMMENT SAVE NUMBERS IN FILE
158: CALL FSAVE(FREQ, PPWDB, PRWDB, PYWDB, FNUM)
159: TERT = (T.GE.TRINTM)
160: TERMINAL
161: COMMENT CLOSE FILE
162: CALL FCLOSE(FNUM)
ELT WED-10/20-82-10:30:42-(G,)
12437-IS(1),SRMB/FSUB(S)
1:FOR IS FF:OPEN
2: SUBROUTINE FOPEN(N)
3: REWIND N
4: RETURN
5: END
6:FOR IS FF:SAVE
7: SUBROUTINE FSAVE(w,p1,p2,p3,n)
8: WRITE(N,100),p1,p2,p3
9: 100 FORMAT(4G14.8)
10: RETURN
11: END
12:FOR IS FF:FCLOSE
13: SUBROUTINE FCLOSE(N)
14: ENDFILE N
15: RETURN
16: END
EOF:16
APPENDIX E

GEOMETRIC ERROR ANALYSIS PROGRAM LISTINGS

100 D 004552 DOLTA 0000 R 000283 DIPITCH 0000 R 000159 DPSA 0000 R 000667
100 R 000046 DPS12 0000 R 000263 DROLL 0000 D 000262 DQYX 0000 D 000179
100 D 000125 DTETA1 0000 R 000130 DTETA2 0000 R 000213 DMTETA 0000 D 003217
100 D 001200 DY 0000 R 000225 DYN 0000 D 002676 ERR 0000 D 002557
100 D 013625 FI 0000 D 000352 FI1 0000 D 000360 FI2 0000 D 000200
100 I 030711 J 0000 D 030706 J 0000 D 005302 LAMDA 0000 D 000626
100 D 001535 PDY 0000 D 013540 PDY 0000 R 030710 PI 0000 D 017161
100 D 004257 PSI 0000 D 001776 PSY 0000 R 003264 PXY 0000 D 002742
100 D 004286 RA 0000 D 005520 Rm 0000 D 012554 RM1 0000 D 010334
100 D 002652 RD1 0000 D 002522 R 0000 R 000847 BDH 0000 D 013350
100 D 013166 SFFT 0000 D 022332 YUANDA 0000 D 015574 SQURT 0000 D 016201
100 D 027614 SOUT1 0000 D 002235 SB 0000 R 002240 Y 0000 D 022552
100 R 000952 TDH2 0000 D 007314 TET1A1 0000 D 007322 TET2A1 0000 D 017372
100 D 007525 T12 0000 D 007535 T11 0000 D 007536 T12 0000 D 031418
100 D 015552 T1 0000 D 025372 T11 0000 R 005130 T2 0000 D 023266
100 D 022759 VP 0000 D 022759 VPX 0000 D 022759 V8 0000 D 025556
100 D 014756 XI 0000 D 000757 XG 0000 D 000322 XY 0000 D 000333
100 D 012874 X2 0000 D 011530 X2 0000 D 017224 X3 0000 D 006748
100 D 014136 YI 0000 D 005602 Y0 0000 D 023140 Y0 0000 D 011364
100 D 011172 V22 0000 D 026316 Z3

10 C THIS PROGRAM COMPUTES THE ERROR SENSITIVITIES AND GEOMETRIC
20 C ERRORS OF GROUND POINTS INDUCED BY ALTITUDE, ROLL, YAW AND
30 C PITCH ERRORS AS WELL AS EARTH ROTATION AND SHUTTLE MOTION

50

60 PARAMETER NLAMDA, NLOMDA, NODHE, NPSI, NPS1, NTETA, NTETA1
70 B NDTA, NDETA, NDETA1, NDETA2, NDETA3, NDETA4, NDETA5, NDETA6, NDETA7
80 ! REAL NLAMDA(NDHE), NLOMDA(NDHE), NAMDA(NDHE), NLMDA(NDHE), NAMDA(NDHE)
90 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
100 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
110 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
120 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
130 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
140 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
150 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
160 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
170 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
180 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
190 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
200 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
210 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
220 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
230 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
240 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
250 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
260 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
270 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
280 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
290 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
300 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
310 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
320 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)
330 NTETA(NTETA), NTETA1(NTETA), NTETA2(NTETA), NTETA3(NTETA), NTETA4(NTETA), NTETA5(NTETA), NTETA6(NTETA)
340 NTETA7(NTETA), NTETA8(NTETA), NTETA9(NTETA)

133
DO 100 K=1,NOM
DO 90 IE=1,NLAMDA
LAMDA(IE)=DLAMDA(IE)*PI/180.

1Y(I)=RE*(1.0/M)*DCOS(LAMDA(I))+DSRT((1.0/M)*(1.0/M)*)

8DIM(LAMDA(I))=DSIN(LAMDA(I))

FX(K,I)=F(X(K)+0.1)/DCOS(LAMDA(I))+DSRT((1.0/MONH(k)

8J(I)=K*(1.0/MONH(K))+DSIN(LAMDA(I))+DSMT(LAMDA(I)))*8IN

8(LAMDA(I))

DXY(K,I)=DXY(K,I)*XY(I)

IF(DXY(K,I).EQ.0.) GO TO 80

80 PXY(K,I)=P8XY(K,I)+100./P88(XY(I))

GO TO 90

80 PXY(K,I)=100.

90 CONTINUE

100 CONTINUE

DO 102 JK=1,NOM

102 CONTINUE
DEL2(K,J)=DSQRT(TIL2(K)*TII2(K)+TII2(K,J)*TII2(K,J))
RM0L(K,J)=DATAM(DEL2(K,J)/H)
RM02(K,J)=DATAM(DEL2(K,J)/H)
SOCY(K,J)=RSQRT(RM01(K,J)+RM0L(K,J))=DSQRT(1+(100/R)*S)
SOLM(RM01(K,J))=DSQRT(RM02(K,J)-RM01(K,J))=DSQRT(RM02(K,J))
Y11(K,J)=SQRRT(K,J)*TI1(K,J)/DEL2(K,J)
Y22(K,J)=SQRRT(K,J)*TI2(K,J)/DEL2(K,J)
SHFT(K,J)=DSQRT(X11(K,J)*X22(K,J)+X11(K,J)*X11(K,J)-X22(K,J)*X22(K,J))
SHFT(K,J)=MYHFT(K,J)/(DTETA1(K)*DTETA2(K))

207 CONTINUE
208 CONTINUE
209 CONTINUE
210 CONTINUE
211 CONTINUE
212 CONTINUE
213 CONTINUE
214 CONTINUE
215 CONTINUE
216 CONTINUE
BEGIN 200 CONTINUE
DO 200 IK=1,NOM

200 FORMAT (18E12.4)

END 200 CONTINUE

BEGIN 202 CONTINUE
DO 202 K=1,NOM

202 FORMAT (18E12.4)

END 202 CONTINUE
240 FORMAT(1H1,10HGEOMETRIC ERROR SENSITIVITY,1H1)
241 $1 WITH RESPECT TO ALTITUDE ERROR
242 $1/
243 $1 ALTITUDE M = $1#F8.8,9KM,1/1 ALTIUDE ERROR 1 TOMA = $1#F5.8,2KM,
244 $1/
245 $1 VIEW = $1#F5X.8,9 DEGREE,1/1 DIFFERENCE (KM),
246 $1/
247 $1 Altitude Error 2 TOMA = $1#F5.X,2KM,
248 $1/
249 $1 VIEW = $1#F5X.8,9 DEGREE,1/1 DIFFERENCE (KM),
250 $1/
251 $1$10D11.4,1/1$1ERROR1,1/1 SENSITIVITY (KM/DEG),
252 $1/
253 $10D11.4,

240 FORMAT(1H1,10HGEOMETRIC ERROR INDUCED BY1)
241 $1 ROLL ERROR
242 $1/
243 $1 EARTH RADIUS R = $1#F9.8,1KM,1/1 NOMINAL ALTITUDE M = $1#F8.8,9KM,
244 $1/
245 $1 VIEW = $1#F5X.8,9 DEGREE,1/1 DIFFERENCE (KM),
246 $1/
247 $1 NOMINAL LOCATION (KM) = $1#F8.0,9KM,
248 $1/
249 $1 VIEW = $1#F5X.8,9 DEGREE,1/1 DIFFERENCE (KM),
250 $1/
251 $1 LOCATION (KM) = $1#F8.0,9KM,
252 $1/
253 $1 VIEW = $1#F5X.8,9 DEGREE,1/1 DIFFERENCE (KM),
254 $1/
255 $1 LOCATION (KM) = $1#F8.0,9KM,
256 $1/
257 $1 VIEW = $1#F5X.8,9 DEGREE,1/1 DIFFERENCE (KM),
258 $1/
259 $1 LOCATION (KM) = $1#F8.0,9KM,
260 $1/
259 FORMAT(1H1,10HGEOMETRIC ERROR SENSITIVITY,1H1)
261 $1 WITH RESPECT TO ROLL ERROR 1 (KM),
262 $1/
263 $1 ROLL ERROR 1 (KM) = $1#F8.0,9KM,
264 $1/
265 $1 ROLL ERROR 2 (KM) = $1#F8.0,9KM,
266 $1/
267 $1 DIFFERENCE (KM) = $1#F8.0,9KM,
268 $1/
269 $1 SENSITIVITY (KM/DEG) = $1#F8.0,9KM,
270 $1/
271 $1 SENSITIVITY (KM/DEG) = $1#F8.0,9KM,
272 $1/
273 $1 SENSITIVITY (KM/DEG) = $1#F8.0,9KM,
274 $1/
275 $1 SENSITIVITY (KM/DEG) = $1#F8.0,9KM,
276 $1/
277 $1 SENSITIVITY (KM/DEG) = $1#F8.0,9KM,
278 $1/
279 $1 SENSITIVITY (KM/DEG) = $1#F8.0,9KM,
280 $1/
281 $1 SENSITIVITY (KM/DEG) = $1#F8.0,9KM,
ORIGIN OF POOR QUALITY

440 FORMAT (1H1, 6H GEOMETRIC ERROR SENSITIVITY)
441 $TY WITH RESPECT TO PITCH ERROR
442 $EARTH RADIUS = 6,371,000 KM, $ NOMINAL ALTITUDE = 0.
443 $FACTOR = 0.1 KM
444 $ PITCH ERROR 1 D3TA = 115.2 DEGREE
445 $ PITCH ERROR 2 D3TA = 115.3 DEGREE
446 $ ANGLE = 12X + F3, 0.9 (X + F3, 0)
447 $ SHIFTED X COORD. = X + F3, 0.9 (X + F3, 0)
448 $ SHIFTED Y COORD. = Y + F3, 0.9 (Y + F3, 0)
449 $ SHIFTED X COORD. = X + F3, 0.9 (X + F3, 0)
450 $ SHIFTED Y COORD. = Y + F3, 0.9 (Y + F3, 0)
451 $ DIFFERENCE (KM) = 5011, 00
452 $ SENSITIVITY = 1.0011, 00
500 CONTINUE

DO 500 KM = 1, NCOM
500 WRITE (600) R = DYAW (X) + DPITCH (X) + DROLL (X) + (OLMDA (J) * JM1 (10) + X0 (J) * JM1 (10) + Y3 (K) * JM1 (10) + Y3 (K) * JM1 (10) + ODIFF (K) * JM1 (10) + (OLMDA (J) * JM1 (10) + X0 (J) * JM1 (10) + Y3 (K) * JM1 (10) + Y3 (K) * JM1 (10)
510 $ SHIFTED X COORD. = X + F3, 0.9 (X + F3, 0)
511 $ SHIFTED Y COORD. = Y + F3, 0.9 (Y + F3, 0)
512 $ SHIFTED X COORD. = X + F3, 0.9 (X + F3, 0)
515 $ DIFFERENCE (KM) = 5011, 00
520 SENSITIVITY = 1.0011, 00
550 CONTINUE

DO 700 K = 1, NDATA
700 WRITE (600) WRITE (600) R = DTOL (J) * JMJ1 (10) * (DEX (K) + JMJ1 (10) * (DEY (K) + JMJ1 (10) * (DER (K) + JMJ1 (10) + GROUND POINT SHIFT VELOCITY DUE TO EARTH ROTATION =
700 $ DEGREE = 3F15.2
700 $ TIME INTERVAL (SEC) = 13015.4
700 $ X DIRECTION (KM) = 13015.4
700 $ SHIFT IN /
700 $ 141
4900       $1 Y DIRECTION (KM)1.3X.3015.4:////// TOTAL SHIFT
4910       $1 (KM)1.3X.3015.4
4920       708 CONTINUE
4930
4940
4950      DO 707 KM1+NDTA
4960    WRITE(6,706) R+DTAU+DETA(K)+V(K)+V9+VP(K)+DANGL(K)+DT(J)
4970    $J=1+NDT)+DELX(K+J)+J61+NDT)+DELV(K+J)+J61+NDT)+ERRD(K+J)
4980    $6=1+NDT)
5000      $1 BY SHUTTLE MOTION AND EARTH ROTATION EEEEEEEEEEEEEEEEEEEEEEEEEEEEEEE
5010      $1 EARTH RADIUS m=F9.611KM1+/// ORBIT
5020      $1 INCLINATION m=F9.01DEGREE1/// LATITUDE m=F9.0
5030      $1 DEGREE1/// GROUND POINT SHIFT VELOCITY DUE TO EARTH1
5040      $1 ROTATION m=F10.611KM/SEC1/// SHUTTLE GROUND SPEED m1
5050      $10.611KM/SEC1/// GROUND POINT SHIFT VELOCITY RELATIVE
5060      $1 TO NADIR POINT a=D10.6
5070      $1 KM/SEC1/// ANGLE OF VELOCITY VECTOR WITH YAXIS a1
5080      $10.611DEGREE1/// TIME INTERVAL (SEC)1.3F15.21///
5090      $1 SHIFT IN1/// X DIRECTION (KM)1.3X.3015.4://////
5100      $1 SHIFT IN1/// Y DIRECTION (KM)1.3X.3015.4://////
5110      $1 TOTAL SHIFT (KM)1.3X.3015.4
5120      707 CONTINUE
5130
5140
5150      STOP
5160
END

41 CTPE 1.158 SUP 11.10.769
* A BMBTEXF. IMAGE/ABB
283-JB 04/27/83 10131121
2 SUP=73.6 CPU=.000 IO=.302 CC=ERR=433

142
OF POOR QUALITY.

------------------------ GEOMETRIC ERROR INDUCED BY ALTITUDE ERROR ------------------------

EARTH RADIUS R = 6356.7550KM

NOMINAL ALTITUDE HEIGHT, KM

ALTITUDE ERROR ON = 1.0 KM

VIEW ANGLE Lambda (DEGREES) 0° 1° 2° 3° 4° 5° 6° 7° 8°

NOMINAL LOCATION (km) 0.0000 0.492+001 0.137+002 0.264+002 0.276+002 0.270+002 0.269+002 0.260+002 0.250+002

SHIFTED LOCATION (km) 0.0000 0.700+001 0.199+002 0.291+002 0.284+002 0.278+002 0.272+002 0.266+002 0.262+002

ERROR (km) 0.0000 0.177+001 0.149+001 0.526+001 0.875+001 0.873+001 0.871+001 0.869+001 0.336+002

ABS ERROR (km) 0.0000 0.250 0.250 0.250 0.250 0.250 0.250 0.250 0.250

------------------------ GEOMETRIC ERROR SENSITIVITY WITH RESPECT TO ALTITUDE ERROR ------------------------

EARTH RADIUS R = 6356.7550KM

NOMINAL ALTITUDE HEIGHT, KM

ALTITUDE ERROR 1 TON = 0.617 KM

ALTITUDE ERROR 2 TON = 0.617 KM

VIEW ANGLE Lambda (DEGREES) 0° 1° 2° 3° 4° 5° 6° 7° 8°

SHIFTED LOCATION DUE TO ERROR 1 (KM) 0.0000 0.492+001 0.137+002 0.264+002 0.276+002 0.270+002 0.269+002 0.260+002 0.250+002

SHIFTED LOCATION DUE TO ERROR 2 (KM) 0.0000 0.700+001 0.199+002 0.291+002 0.284+002 0.278+002 0.272+002 0.266+002 0.262+002

DIFFERENCE (KM) 0.0000 0.259+001 0.062+001 0.027+002 0.005+002 0.002+002 0.001+002 0.000+002 0.000+002

LAMBD SENSITIVITY (KM/KM) 0.0000 1.000 0.062 0.027 0.005 0.002 0.001 0.000 0.000

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### Geometric Error Induced by Roll Error

**Earth Radius** = 6356.7556 km

**Nominal Altitude** = 8000 km

**Roll Error of 0.1 Degree**

<table>
<thead>
<tr>
<th>View Angle (Degrees)</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
</tr>
</thead>
</table>

**Nominal Location (km)**
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000

**Shifted Location (km)**
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000

**Error (km)**
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000

**Absolute Error (km)**
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000

**Error Sensitivity (km/deg)**
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000

**Roll Error of 0.1 Degree**

<table>
<thead>
<tr>
<th>View Angle (Degrees)</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
</tr>
</thead>
</table>

**Nominal Location (km)**
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000

**Shifted Location due to Error 1 (km)**
- 0.0335000
- 0.0335000
- 0.0335000
- 0.0335000
- 0.0335000
- 0.0335000
- 0.0335000

**Shifted Location due to Error 2 (km)**
- 0.0335000
- 0.0335000
- 0.0335000
- 0.0335000
- 0.0335000
- 0.0335000
- 0.0335000

**Difference (km)**
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000

**Error Sensitivity (km/deg)**
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000

**Shifted Location due to Error 1 (km)**
- 0.0012000
- 0.0012000
- 0.0012000
- 0.0012000
- 0.0012000
- 0.0012000
- 0.0012000

**Shifted Location due to Error 2 (km)**
- 0.0012000
- 0.0012000
- 0.0012000
- 0.0012000
- 0.0012000
- 0.0012000
- 0.0012000

**Difference (km)**
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000

**Error Sensitivity (km/deg)**
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
- 0.0000000
### Geometric Error Sensitivity with Respect to Pitch Error

<table>
<thead>
<tr>
<th>Angle (Degrees)</th>
<th>Error (m)</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
<th>6°</th>
<th>7°</th>
<th>8°</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Original Value</strong></td>
<td>0.005545</td>
<td>0.0011</td>
<td>0.0022</td>
<td>0.0033</td>
<td>0.0044</td>
<td>0.0055</td>
<td>0.0066</td>
<td>0.0077</td>
<td>0.0088</td>
</tr>
<tr>
<td><strong>Error</strong></td>
<td><strong>0.00666</strong></td>
<td><strong>0.0012</strong></td>
<td><strong>0.0024</strong></td>
<td><strong>0.0036</strong></td>
<td><strong>0.0048</strong></td>
<td><strong>0.0060</strong></td>
<td><strong>0.0072</strong></td>
<td><strong>0.0084</strong></td>
<td><strong>0.0096</strong></td>
</tr>
</tbody>
</table>

**Error Calculation**

1. **Original Value** = 0.005545 m
2. **Error** = 0.00666 m
3. **Difference** = 0.0011 m

**Pitch Error Sensitivity (m/degree)**

<table>
<thead>
<tr>
<th>Angle (Degrees)</th>
<th>Error (m)</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
<th>6°</th>
<th>7°</th>
<th>8°</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Original Value</strong></td>
<td>0.00503</td>
<td>0.0010</td>
<td>0.0020</td>
<td>0.0030</td>
<td>0.0040</td>
<td>0.0050</td>
<td>0.0060</td>
<td>0.0070</td>
<td>0.0080</td>
</tr>
<tr>
<td><strong>Error</strong></td>
<td><strong>0.00615</strong></td>
<td><strong>0.0012</strong></td>
<td><strong>0.0024</strong></td>
<td><strong>0.0036</strong></td>
<td><strong>0.0048</strong></td>
<td><strong>0.0060</strong></td>
<td><strong>0.0072</strong></td>
<td><strong>0.0084</strong></td>
<td><strong>0.0096</strong></td>
</tr>
</tbody>
</table>

**Error Calculation**

1. **Original Value** = 0.00503 m
2. **Error** = 0.00615 m
3. **Difference** = 0.0011 m

**Pitch Error Sensitivity (m/degree)**

- **Original Value**: 0.005545 m
- **Error**: 0.00666 m
- **Difference**: 0.0011 m

**Pitch Error Sensitivity (m/degree)**

- **Original Value**: 0.00503 m
- **Error**: 0.00615 m
- **Difference**: 0.0011 m
### Geometric Error Induced by Van Error

**Earth Radius:** 6955.785 km

**Nominal Altitude:** 4088 km

**Van Error DFRA 01 Degree**

<table>
<thead>
<tr>
<th>View Angle DFRA (Degrees)</th>
<th>0°</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
<th>6°</th>
<th>7°</th>
<th>8°</th>
<th>9°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error (km)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1210000</td>
<td>0.2420000</td>
<td>0.3630000</td>
<td>0.4840000</td>
<td>0.6050000</td>
<td>0.7260000</td>
<td>0.8470000</td>
<td>0.9680000</td>
<td>1.0890000</td>
<td></td>
</tr>
</tbody>
</table>

### Geometric Error Sensitivity with Respect to Van Error

**Earth Radius:** 6955.785 km

**Nominal Altitude:** 4088 km

**Van Error 1 DFRA = 0°.5 Degree**

**Van Error 2 DFRA = 0°.5 Degree**

<table>
<thead>
<tr>
<th>View Angle DFRA (Degrees)</th>
<th>0°</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
<th>6°</th>
<th>7°</th>
<th>8°</th>
<th>9°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Shift (km)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Error Sensitivity (km/deg)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>VISTA ANGLE (DEGREES)</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------------</td>
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<td>----</td>
<td>----</td>
<td>----</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y-COORD (H)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X-COORD (H)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VISTA Y-COORD (H)</td>
<td>0.138°</td>
<td>0.25°</td>
<td>0.25°</td>
<td>0.25°</td>
<td>0.25°</td>
<td>0.25°</td>
<td>0.25°</td>
<td>0.25°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VISTA X-COORD (H)</td>
<td>0.48°</td>
<td>0.48°</td>
<td>0.48°</td>
<td>0.48°</td>
<td>0.48°</td>
<td>0.48°</td>
<td>0.48°</td>
<td>0.48°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VISTA Z-COORD (H)</td>
<td>0.138°</td>
<td>0.25°</td>
<td>0.25°</td>
<td>0.25°</td>
<td>0.25°</td>
<td>0.25°</td>
<td>0.25°</td>
<td>0.25°</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EARTH RADIUS [m]</th>
<th>6.378 x 10^6</th>
<th>6.378 x 10^6</th>
<th>6.378 x 10^6</th>
<th>6.378 x 10^6</th>
<th>6.378 x 10^6</th>
<th>6.378 x 10^6</th>
<th>6.378 x 10^6</th>
<th>6.378 x 10^6</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORTH INCLINATION [DEGREE]</td>
<td>90°</td>
<td>90°</td>
<td>90°</td>
<td>90°</td>
<td>90°</td>
<td>90°</td>
<td>90°</td>
<td>90°</td>
</tr>
<tr>
<td>LATITUDE [DEGREE]</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GROUND POINT SHIFT VELOCITY DUE TO EARTH ROTATION [m/s]</th>
<th>9.836 x 10^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHUTTLE GROUND SPEED [m/s]</td>
<td>0.762 x 10^3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GROUND POINT SHIFT VELOCITY RELATIVE TO VEHICLE [m/s]</th>
<th>0.717 x 10^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANGLE OF VELOCITY VECTOR WITH VEHICLE [DEG]</td>
<td>0.032</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIME INTERVAL [SEC]</th>
<th>1.00</th>
<th>.01</th>
<th>.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHIFT IN X DIRECTION</td>
<td>-.0714 x 10^3</td>
<td>.714 x 10^3</td>
<td>.714 x 10^3</td>
</tr>
<tr>
<td>SHIFT IN Y DIRECTION</td>
<td>.0714 x 10^3</td>
<td>.0714 x 10^3</td>
<td>.0714 x 10^3</td>
</tr>
<tr>
<td>TOTAL SHIFT</td>
<td>.0714 x 10^3</td>
<td>.714 x 10^3</td>
<td>.714 x 10^3</td>
</tr>
</tbody>
</table>

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### Ground Point Shift Induced by Shuttle Motion and Earth Rotation

**Earth Radius:** 6356.785 Km

**Orbit Inclination:** 89.0 Degree

**Latitude:** 89.0 Degree

**Ground Point Shift Velocity Due to Earth Rotation:** 0.954+0.001 m/sec

**Shuttle Ground Speed:** .7292+0.001 m/sec

**Ground Point Shift Velocity Relative to Nadir Point:** .7175+0.001 m/sec

**Angle of Velocity Vector with T-axis:** 0.871 Degree

<table>
<thead>
<tr>
<th>Time Interval (sec)</th>
<th>1.00</th>
<th>.10</th>
<th>.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift in X Direction (m)</td>
<td>.7142+0.001</td>
<td>.7142+0.001</td>
<td>.7142+0.001</td>
</tr>
<tr>
<td>Shift in Y Direction (m)</td>
<td>.5514+0.002</td>
<td>.5514+0.002</td>
<td>.5514+0.002</td>
</tr>
<tr>
<td>Total Shift (m)</td>
<td>.7174+0.001</td>
<td>.7174+0.001</td>
<td>.7174+0.001</td>
</tr>
</tbody>
</table>

---

### Ground Point Shift Induced by Shuttle Motion and Earth Rotation

**Earth Radius:** 6356.785 Km

**Orbit Inclination:** 89.0 Degree

**Latitude:** 89.0 Degree

**Ground Point Shift Velocity Due to Earth Rotation:** 0.954+0.001 m/sec

**Shuttle Ground Speed:** .7292+0.001 m/sec

**Ground Point Shift Velocity Relative to Nadir Point:** .7175+0.001 m/sec

**Angle of Velocity Vector with T-axis:** 0.871 Degree

<table>
<thead>
<tr>
<th>Time Interval (sec)</th>
<th>1.00</th>
<th>.10</th>
<th>.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift in X Direction (m)</td>
<td>.7142+0.001</td>
<td>.7142+0.001</td>
<td>.7142+0.001</td>
</tr>
<tr>
<td>Shift in Y Direction (m)</td>
<td>.5514+0.002</td>
<td>.5514+0.002</td>
<td>.5514+0.002</td>
</tr>
<tr>
<td>Total Shift (m)</td>
<td>.7174+0.001</td>
<td>.7174+0.001</td>
<td>.7174+0.001</td>
</tr>
<tr>
<td>TIME INTERVAL (SEC)</td>
<td>1.00</td>
<td>.10</td>
<td>.01</td>
</tr>
<tr>
<td>---------------------</td>
<td>------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>SHIFT IN X DIRECTION (NM)</td>
<td>.7162+001</td>
<td>.7162+000</td>
<td>.7162+001</td>
</tr>
<tr>
<td>SHIFT IN Y DIRECTION (NM)</td>
<td>.2270-002</td>
<td>.2270-000</td>
<td>.2270-002</td>
</tr>
<tr>
<td>TOTAL SHIFT (NM)</td>
<td>.7162+002</td>
<td>.7162+000</td>
<td>.7162+002</td>
</tr>
</tbody>
</table>

---

**Ground Point Shift Induced by Shuttle Motion and Earth Rotation**

**Earth Radius**: 6356,780 NM

**Orbit Inclination**: 85,0 DEGREE

**Latitude**: 66,0 DEGREE

**Ground Point Shift Velocity Due to Earth Rotation**: .7162+002 NM/SEC

**Shuttle Ground Speed**: .7282-001 NM/SEC

**Ground Point Shift Velocity Relative to Nadir Point**: .7162+001 NM/SEC

**Angle of Velocity Vector with Y Axis**: 66,0 DEGREE

<table>
<thead>
<tr>
<th>TIME INTERVAL (SEC)</th>
<th>1.00</th>
<th>.10</th>
<th>.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHIFT IN X DIRECTION (NM)</td>
<td>.7162+001</td>
<td>.7162+000</td>
<td>.7162+001</td>
</tr>
<tr>
<td>SHIFT IN Y DIRECTION (NM)</td>
<td>.2270-002</td>
<td>.2270-000</td>
<td>.2270-002</td>
</tr>
<tr>
<td>TOTAL SHIFT (NM)</td>
<td>.7162+002</td>
<td>.7162+000</td>
<td>.7162+002</td>
</tr>
</tbody>
</table>
APPENDIX F
GROUND PATTERN AND IMAGING

DISTORTIONS GENERATION PROGRAM LISTINGS

```
c000 1 006618 FM 0000 1 006621 JM 0000 1 005706 JPT 0000 1 0
0061 1 006712 J 0000 1 006805 JD 0000 1 005742 J6 0000 1 0
0067 1 006611 JF 0000 1 006822 JH 0000 1 005723 J8 0000 1 0
0061 1 005714 JL 0000 R 000600 LAMDA 0000 1 005303 LFLAG 0000 0 0
0066 1 000003 LMAND 0000 R 000802 LYRDA 0000 1 001374 LFLAG 0000 X 0
0064 1 005673 NJV 0000 1 005674 NGV 0000 1 006590 MILN 0000 X 0
0065 1 006605 NLIN J3 0000 1 006670 NLINN 0000 1 006673 MILNS 0000 I 0
0063 1 006241 MITC 0000 1 006853 MICTI 0000 1 006685 MICTJ 0000 1 0
0063 1 006572 NJTC 0000 1 006806 NJTIC 0000 1 005675 MICTC 0000 I 0
0063 1 006024 MEGA 0000 R 005546 PD 0000 R 005631 PI 0000 R 0
0068 1 005630 R 0000 R 005767 RA 0000 R 005603 RAA 0000 R 0 0
0065 1 005751 RHG1 0000 R 006800 ROLL 0000 R 005656 R5 0000 R 0 0
0065 1 006034 TTY 0000 R 005743 TENTA 0000 R 006633 TIC3 0000 R 0 0
0068 1 005740 YB 0000 R 006040 YBU 0000 R 005756 YB1 0000 R 0 0
0067 1 005669 XCENT 0000 R 005757 XCl 0000 R 005220 XDP 0000 R 2 0
0066 1 003262 XF 0000 R 006612 XG 0000 R 004610 XG1 0000 R 0 0
0065 1 005759 XII 0000 R 005770 XJ 0000 R 005605 XJ 0000 R 0 0
0065 1 005675 XLEFT 0000 R 005667 XLEN 0000 R 006017 XLI 0000 R 0 0
0068 1 005767 XMYC 0000 R 002020 XP 0000 R 002140 XP1 0000 R 0 0
0064 1 003221 XPP 0000 R 003015 XP5 0000 R 005172 XP6 0000 R 0 0
0064 1 005661 YSTA 0000 R 006021 YSTA0 0000 R 005774 XT 0000 R 0 0
0065 1 005720 Y7 0000 H 000421 Y 0000 R 005725 YA 0000 R 0 0
0066 1 005651 YAND 0000 R 005652 YAWS 0000 R 005745 YAI 0000 R 0 0
0066 1 005700 YNOT 0000 R 005755 YG1 0000 R 005703 YG 0000 R 0 0
0066 1 005760 YCH 0000 R 005764 YD 0000 R 005411 YD0 0000 R 0 0
0065 1 003453 YF 0000 R 001003 YG 0000 R 005001 YG1 0000 R 0 0
0065 1 005753 Y11 0000 R 005771 YJ 0000 R 006002 YK 0000 R 0 0
0065 1 005670 YLEN 0000 R 006020 YL1 0000 R 001365 YN 0000 R 0 0
0064 1 005215 YP 0000 R 002153 YP1 0000 R 002201 YP2 0000 R 0 0
0065 1 006324 YP5 0000 R 005205 YP6 0000 R 005615 YP7 0000 R 0 0
0065 1 005644 YSTAP 0000 R 005645 YSTAP1 0000 R 005634 YSTAR 0000 R 0 0
0065 1 005701 YTOP 0000 H 005571 YW 0000 R 000200 YX 0000 R 0 0
```

100 1 0  C DECLARE PARAMETERS
101 2 0  PARAMETER 'NLINES=11' 'NDT=400' 'NLAMDA=460' 'NLMDA=318'
102 3 0  SNLYDA=50 'NLAMDA=480'
103 4 0  REAL LAMDA=LFDA=LYRDA=LMND
104 5 0  INT NLDA=LFDA=LFDA=LMND
105 6 0  INTEGER FLAG=AFLAG
106 7 0  DIMENSION OF VARIABLES
107 8 9  C
109 11 9  S[0],[1],...[2],...[3]=Y[0],...[1],...[2],...[3]=Z[0],...[1],...[2],...[3]
110 12 9  SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]
111 13 9  SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]
112 14 9  SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]
113 15 9  SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]
114 16 9  SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]
115 17 9  SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]=SNF[0],...[3]

150
C0105  10  C  $ALINES$  H LINES$  Y6( H LINES$  X6( H LINES$  
C0105  19  C  SYPLINES$  X90(H LINES$  Y90(H LINES$  
C0105  20  C  SAFLAG(H LINES$  H LINES$  SYPT(H LINES$  
C0106  21  C  DATA R, F, F, YST/PROSVS=0D=05/635672653.1415926540. 
C0106  22  DATA 1000008.000000S=0047.3199.4400000039.099. 
C0106  23  DATA EPS1, EPS2, YSTAS/STAPP=0PS5 £FLAGQ0231=0. 
C0112  25  S-5.1=434.7=00050.6527. 
C0131  26  DATA YSTADYS/NSF=-5.00037.8257. 
C0135  27  DATA DROLL$YSTAD$NSF/05=053.9=00421.0. 
C0142  28  DATA DRAU=DATAYSTAD$NSF/65=053.9=057.2027. 
C0150  29  C  INITIALIZE THE PLOTTER 
C0150  30  C  CALL EGPLT 
C0015  C  CENTER PLCT 
C0151  35  C  XCENT=5.0 
C0152  S5  YCENT=1.0 
C0153  36  CALL ORIGINCENT\YCENT) 
C0154  37  C  DEFINE THE FORM OF THE PLOT 
C0154  38  C  PTYPE=L.LINLE 
C0155  39  SLENF=0. 
C0156  40  CALL PLFORM(PTYPE\XLEN\YLEN) 
C0157  41  C  SCALE THE PLOT 
C0160  44  C  AX119=8 
C0160  45  AX29=8 
C0160  46  AX29=8 
C0163  47  Y411=8 
C0163  48  Y411=8 
C0163  49  MX=2 
C0163  50  MY=2 
C0166  51  NGX=1 
C0166  52  NGY=1 
C0167  53  CALL PLSCFL\XY4, NXY4, NGX, MY, NGY) 
C0171  54  C  TO START PLOTTING 
C0171  55  C  CALL PLGRAF 
C0171  56  C  DEFINE GRID LIMITS 
C0172  57  C  XLEFT=5 
C0172  58  XRIGT=5 
C0173  59  CALLXRIGHT-XLEFT)(H LINES-1) 
C0174  60  YTOP=5 
C0174  61  YTOP=5 
C0174  62  YT=5 
C0176  63  YF=4.(TOP-YTOP)/(H LINES-1) 
C0200  64  C  CONSTRUCT DATA 
C0200  65  C  YC=YTOP 
C0200  66  DO 150 X50=1.H LINES 
C0200  67  DO XLEFT 
C0200  68  DO 100 IPT=1.H LINES 
C0200  69  XNOM=+5IN(YC/N) 
C0200  70  XNOM=+5IN(YC/N) 
C0210  71  C  AND=+5INCMS=5 
C0210  72  C  AND=+5INC=5 
C0210  73  C  AND=+5INC=5 
C0211  74  C  YNOM=YC
I
DO 222 I=1,NLINES
DO 221 J=1,NLINES
IF(EQ(I,J)) GO TO 225
YCI=BI+Y(J)
IG1=I
JG1=J
IFABS(X(J)) .LE. EPS1 .AND. ABS(Y(J)) .LE. EPS1 GO TO 223
221 CONTINUE
DO 222 CONTINUE
DO 225 CONTINUE
YCI=IG1+JG1
YCI=TSTAP1+2(L-1)+0288+Y11
RITE(6,224) YBI+Y(J)+DLPMA+XG1+Y(J)+G1
222 CONTINUE
DLPMA=DLPMA-PO
226 CONTINUE
Y11=YSTAP1*L+0298
Y21=YSTAP1*L+0299
227 CONTINUE
GO TO 605
231 CONTINUE
DO 4 I=1,NLINES
DO 3 J=1,NLINES
KFLAG(I,J)=0
3 CONTINUE
4 CONTINUE
DO 226 L=1,NDT
YD=YSTAY
227 CONTINUE
GO TO 226
Y11=YSTAP1*L+0298
Y21=YSTAP1*L+0299
DLPMA=DLPMA-PO
YSTAP1=L
Y +TSTAP1*L+0298
YSTAP1=L+0299
232 CONTINUE
KFLAG(IN,JM)=1
XH1=JM+2048-1-TAN(LYMDA)
YH1=J+2048-TAN(LYMDA)
RITE(10,230) YH1=J+2048-TAN(LYMDA)
RITE(12,230) YH1=J+2048-TAN(LYMDA)
RITE(10,230) YH1=J+2048-TAN(LYMDA)
RITE(12,230) YH1=J+2048-TAN(LYMDA)
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RITE(10,230) YH1=J+2048-TAN(LYMDA)
RITE(12,230) YH1=J+2048-TAN(LYMDA)
RITE(10,230) YH1=J+2048-TAN(LYMDA)
0633 1450 238 CONTINUE
0633 248* YD=YSTAY+L*L+0288
0635 246* DYA=5.+SIN((L*PI)/174.)
0636 247* 248 CONTINUE
0637 248* GO TO 299
0641 249*

0642 250*
0642 251* 241 CONTINUE
0643 252* DO 6 J=1ALINES
0646 253* DO 5 J=1ALINES
0649 254* LFLAG(1+J)=0
0652 255* 5 CONTINUE
0654 256* 6 CONTINUE
0656 257* ROLL=ROLL*PI/160.
0657 258* XSTAO=R*(1+D/R)*COS(ROLL)-SRT(1-(1+D/R)*(1+D/R))*SIN(ROLL)
0657 259* S=SIN(ROLL)+SIV(ROLL)
0660 260* YK=YSTAO
0661 261* DO 290 L=1,NDT
0664 262* B LAMDA=RS
0665 263* DO 245 K=1,NLAMDA
0670 264* LAMDA=DLAMDA*PI/150.
0671 265* DIS=R*(1+D/R)*COS(LAMDA/ROLL)-SRT(1-(1+D/R)*(1+D/R))*SIN(LAMDA/ROLL)
0671 266* S=LAMDA/ROLL+3IN(LAMDA/ROLL)+SIN(LAMDA/ROLL)
0672 267* KE=DIS**XSTAO
0673 268* DO 66 J=1ALINES
0676 269* DO 55 J=1ALINES
1701 270* XL=XK+YF+J
1702 271* YL=YK+YF+I+J
1703 272* IRA=I
1704 273* JMCJ
1705 274* IF(4+BSV*(1+LE+EPSI+4D+4S(YL)+LE+EPSI))=0 GO TO 242
1707 275* 55 CONTINUE
1711 276* 66 CONTINUE
1713 277* GO TO 24A
1714 278* 242 LFLAG(1+J)=1
1715 279* XMEJ,JM1=DIS*(TAN(LAMDA))
1716 280* YMEJ,JM1=STAO+L*L+0288
1717 281* XMEJ,JM1=DIS*(TAN(LAMDA))
1719 282* FORMAT('X=(',XMEJ,JM1)',Y=(',YMEJ,JM1)')
1720 283* 243 DLAMDA=DLAMDA+RD
1722 284* 245 CONTINUE
1724 285* YK=YSTAO+L*L+0288
1725 286* 250 CONTINUE
1727 287* 251 CONTINUE
1735 288* 250 CONTINUE
1737 289* DO 5 J=1ALINES
1743 290* DO 7 J=1ALINES
1746 291* LFLAG(1+J)=0
1747 292* 7 CONTINUE
1751 293* 8 CONTINUE
1753 294* ROLL=ROLL*PI/180.
1754 295* DER=0.
1755 296* XSTAO=R*(1+D/R)*COS(ROLL)-SRT(1-(1+D/R)*(1+D/R))*SIN(ROLL)
1755 297* S=SIN(ROLL)+SIV(ROLL)
1756 298* YK=YSTAO
1757 299* DO 260 L=1,NDT
1758 300* ERR=ERR+PI/180.
0763 3010  DLMDA=RS
0764 3020  DO 255 K=1,NLMDA
0765 3030  LRMDA=DLMDA*PI/180
0770 3050  S=SIN(LRMDA=ROLL+ER)=SQR(T1-(1+D/R)+(1+D/R))
0771 3060  UK1=DI51*XTAO
0772 3070  DO 90 J=1,NLINES
0775 3090  00 07  J=1,NLINES
1000 3090  0L1=YK1-Y(1,J)
1001 3100  YL1=YK1-Y(1,J)
1002 3110  J=1
1003 3120  J=1
1004 3130  IF(ABS(Y(1,J))=.EPSI.AND.AUS(Y(1,J))=.EPSI) GO TO 252
1006 3140  77 CONTINUE
1010 3150  89 CONTINUE
1012 3160  49 CONTINUE
1013 3170  252 LFLAG1(IP1,JM1)=
1014 3180  (MK1(IP1,JM1)=0)-TAN(LRMDA)
1015 3190  (MK1(IP1,JM1)=0)-TAN(LRMDA)
1016 3200  dRITE(6,255) (MK1(IP1,JM1)=0)
1016 3210  dRITE(6,255) (MK1(IP1,JM1)=0)
1027 3220  255 FORMAT(12F10.5,14E3F10.5)
1030 3230  256 DLMDA=DLMDA+D
1031 3240  255 CONTINUE
1033 3250  YK1=YSTAC*(L+230)
1034 3260  DER=0.2*SIN(L+PI/174.3)
1035 3270  260 CONTINUE
1037 3280  GO TO 299
1040 3290  261 CONTINUE
1041 3300  DO 263 I=1,NLINES
1042 3310  DO 262 J=1,NLINES
1047 3330  AFLAG1(IP1,JM1)=
1050 3340  262 CONTINUE
1052 3350  263 CONTINUE
1054 3360  TAU=DTAU+FX1/100.
1055 3370  OMEGA=2.*X1/06400.
1056 3380  ETA=DETA+PI/150.
1057 3390  V=OMEGA+P*DETA
1060 3400  SLUMDA=COS(ETACOS)/COS(ETACOS)
1061 3410  CLUMDA=SRT(1-SLUMDA*SLUMDA)
1062 3420  VX=V+CLUMDA
1063 3430  VY=V+SLUMDA
1064 3440  VT=V
1065 3450  VTF=VES4-VY
1066 3460  VXX=VXX
1067 3470  VYY=VYY
1070 3480  DO 270 L=1,NL
1073 3490  OLYMDA=VY4S
1074 3500  DO 269 J=1,NLMDA
1077 3510  LMDA=OLYMDA+PI/180.
1100 3520  (F=O(1+D/R)+CO3(1MDA)+SQR(T1-(1+D/R)+(1+D/R)))
1100 3530  WLYMDA=SIN(LMDA)+SIN(LMDA)
1101 3540  (WLYMDA)=SIN(LMDA)
1102 3550  (WLYMDA)=SIN(LMDA)
1103 3560  00 265 I=1,NLINES
1106 3570  00 264 J=1,NLINES

156
ORIGINAl PAl\: OF POOR QUAL\:\.

1111 1112 1113 1114 1115 1116 1117 1118 1119 1120 1121 1122 1123 1124 1125 1126 1127 1128 1129 1130 1131 1132 1133 1134 1135 1136 1137 1138 1139 1140 1141 1142 1143 1144 1145 1146 1147 1148 1149 1150 1151 1152 1153 1154 1155 1156 1157 1158 1159 1160 1161 1162 1163 1164 1165 1166 1167 1168 1169 1170 1171 1172 1173 1174 1175 1176 1177 1178 1179 1180 1181 1182 1183 1184 1185 1186 1187 1188 1189 1190 1191 1192 1193 1194 1195 1196 1197 1198 1199 1200 1201 1202 1203 1204 1205 1206 1207 1208 1209 1210 1211 1212 1213 1214 1215 1216 1217 1218 1219 1220 1221 1222 1223 1224 1225 1226 1227

1111 355* XECYUB=V(I,J)
1112 359* YECYUB=V(I,J)
1113 360* IDD=I
1114 361* JDD=J
1115 362* IF(AMS(VCC).LE.EPSI. AND.ANS(VCC).LE.EPSI) GO TO 266
1116 363* 264 CONTINUE
1117 364* 265 CONTINUE
1118 365* GO TO 268
1119 366* 266 AFLAG(I,J)=1
1120 367* XD(I,DD,JDD)=B=(TAN(LYMDA))
1121 368* YD(I,DD,JDD)=YSTA+(L-1)*.02E
1122 369* WRITE(6,567) XBB,YBB,JD,J0D,LYMDA,KBBD(IDD,J0D),
1123 370* SYD(I,DD,J0D)
1124 371* SYD(IDD,J0D)
1125 372* 267 FORMAT(F210.5,214,3F10.5)
1126 373* 268 DLYMDA=DLYMDA-20
1127 374* 269 CONTINUE
1128 375* 44AXYSTA=L+DT+VT
1129 376* 270 CONTINUE
1130 377* 271 C PLOT HORIZONTAL LINES
1131 378* 272 CONTINUE
1132 379* 273 CONTINUE
1133 380* TICX=1
1134 381* NTIC=1
1135 382* NLIN=NLIINES
1136 383* 00 400 IHOR=1,NLINES
1137 384* 00 300 IPT=1,NLINES
1138 385* XP(IPT)=X(IHOR,IPT)
1139 386* YP(IPT)=Y(IHOR,IPT)
1140 387* 300 CONTINUE
1141 388* 301 CALL PLCUR(XP,YP,NLINES,MTIC,TIC)
1142 389* 400 CONTINUE
1143 390* 401 C PLOT VERTICAL LINES
1144 391* 402 DO 500 IVH=1,NLINES
1145 392* CALL PLCUR(YV(IHOR),YV(IHOR),NLINES,MTIC,TIC)
1146 393* 500 CONTINUE
1147 394* 501 IF(IFLAG.EQ.1) GO TO 655
1148 395* IF(IFLAG.EQ.2) GO TO 805
1149 396* IF(IFLAG.EQ.3) GO TO 501
1150 397* IF(IFLAG.EQ.4) GO TO 609
1151 398* 655 CONTINUE
1152 399* 765 TIC=0
1153 400* NLIN=NLIINES
1154 401* 00 750 IHOR=1,NLINES
1155 402* DO 700 IPT=1,NLINES
1156 403* YP(IPT)=Y(Y(IHOR,IPT))
1157 404* 750 CONTINUE
1158 405* 760 CALL PLCUR(YP1,YP2,NLINES,MTIC,TIC)
1159 406* 750 CONTINUE
1160 407* 750 CONTINUE

157
C PLOT VERTICAL LINES
DO 300 IVERT=1,NLINES
CALL PLCURV(AF1+INVERT+YF1+INVERT+NLIN1+NTIC1+TIC1)
300 CONTINUE
GO TO 9999

C PLOT HORIZONTAL LINES
C PLOT VERTICAL LINES
DO 905 CONTINUE
TIC2=0
NTIC2=0
NLIN2=NLINES
DO 920 IPT=1,NLINES
XP2(IPT)=YG(INOR+IPT)
YP2(IPT)=YG(INOR+IPT)
920 CONTINUE
CALL PLCURV(YP2+YP2+NLIN2+NTIC2+TIC2)

C PLOT VERTICAL LINES
DO 951 IVERT=1,NLINES
CALL PLCURV(YG1+INVERT1+YG1+INVERT1+NLIN2+NTIC2+TIC2)
951 CONTINUE
GO TO 9999

C PLOT HORIZONTAL LINES
TIC6=0
NTIC6=0
NLIN6=NLINES
DO 965 INCR=1,NLINES
XP6(INCR)=YG(INOR+INCR)
YP6(INCR)=YG(INOR+INCR)
965 CONTINUE
GO TO 9999

C PLOT VERTICAL LINES
DO 995 INCR=1,NLINES
CALL PLCURV(YG1+INCR+YG1+INCR+NLIN6+NTIC6+TIC6)
995 CONTINUE

C PLOT HORIZONTAL LINES
TIC3=3
NTIC3=0
NLIN3=NLINES
DO 1005 INCR=1,NLINES
XP3(INCR)=YG(INOR+INCR)
YP3(INCR)=YG(INOR+INCR)
1005 CONTINUE
CALL PLCURV(YP3+YP3+NLIN3+NTIC3+TIC3)
1005 CONTINUE

158
1340 471* C   PLOT VERTICAL LINES
1340 472* DO 907 IVERT=1,NLINES
1343 474* CALL PLOUREXY(IVERT),YM1(IVERT),NLIV5,NLIES,TIC5,TIC3)
1344 475* 907 CONTINUE
1346 476* GO TO 9899
1347 477* C   PLOT HORIZONTAL LINES
1347 479* 911 CONTINUE
1350 480* TIC4=**
1351 481* NLN4=1
1352 482* LIN4=NLINES
1353 483* DO 915 INOR=1,NLINES
1356 484* DO 913 IPT=1,NLINES
1358 485* (PS(IPT)=YM1(INOR+IPT)
1362 486* YPS(IPT)=YM1(INOR+IPT)
1363 487* 913 CONTINUE
1365 489* CALL PLOUREXY(PS,YPS,NLIV4,NLIES,TIC4,TIC4)
1366 489* 915 CONTINUE
1370 490* C   PLOT VERTICAL LINES
1370 492* DO 917 IVERT=1,NLINES
1373 493* CALL PLOUREXY(M1,IVERT),YM1(IVERT),NLIV4,NLIES,TIC4,TIC4)
1374 494* 917 CONTINUE
1376 495* C   PLOT HORIZONTAL LINES
1376 497* 909 CONTINUE
1377 498* TIC5=*
1400 499* NLN5=0
1402 500* LIN5=NLINES
1405 503* DO 918 INOR=1,NLINES
1410 503* (PS(IPT)=YM1(INOR+IPT)
1411 504* YPS(IPT)=YM1(INOR+IPT)
1412 505* 919 CONTINUE
1414 506* CALL PLOUREXY(PS,YPS,NLIV5,NLIES,TIC5,TIC5)
1415 507* 919 CONTINUE
1417 509* C   PLOT VERTICAL LINES
1417 510* DO 922 IVERT=1,NLINES
1422 511* CALL PLOUREXY(M1,IVERT),YM1(IVERT),NLIV5,NLIES,TIC5,TIC5)
1423 512* 920 CONTINUE
1425 513* GO TO 9999
1426 514* C   PLOT HORIZONTAL LINES
1426 516* TIC7=0
1430 517* NLN7=0
1432 518* LIN7=NLINES
1435 519* DO 923 INOR=1,NLINES
1440 522* (PS(IPT)=YPS(INOR+IPT)
1441 523* YPS(IPT)=YPS(INOR+IPT)
1442 524* 922 CONTINUE
1444 525* CALL PLOUREXY(YPS,YPS,NLINES,TIC5,TIC7)
1445 526* 923 CONTINUE

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1447  527+  C PLOT VERTICAL LINES
1447  528+  DO 924  IVERT=1+MLINES
1452  530+  CALL PCLUREDC(YDD1+IVERT)+YDD1+IVERT)+MLINES+TIC7+TIC7)
1453  531+  924 CONTINUE
1455  532+  1455  533+  C TO FINISH PLOTTING
1455  534+  4999 CALL ENDVLT
1456  535+  1456  536+  ** STOP
1457  537+  END

WFL:
PU11=399  CTP=#115  SUP$=9=131
DELETE+2 SHUT+P_IMAGE=1/ABS
UPFUB 2-3-J8  05/19/63  11156:41
TP=.333  SUP=.737  CPU=.000  IO=.382  CC-ER=.433
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