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Geometric Error Analysis for Shuttle Imaging Spectrometer Experiment

Shyh Jong Wang
Che-Hang Charles lh

December 15, 1984

NASA
National Aeronautics and
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Jet Propulsion Laboratory
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ABSTRACT

The demand of more powerful tools for remote sensing and management of earth resources has been steadily increasing over the last decade. With the recent advancement of area array detectors, high resolution multichannel imaging spectrometers can be realistically constructed.

This report documents the error analysis study for the Shuttle Imaging Spectrometer Experiment system for the purpose of providing information for design, tradeoff, and performance prediction.

Error sources including the Shuttle attitude determination and control system, instrument pointing and misalignment, disturbances, ephemeris, earth rotation, etc., have been investigated. Geometric error mapping functions were developed, characterized, and illustrated extensively with tables and charts. Selected ground patterns and the corresponding image distortions have been generated for direct visual inspection of how the various error sources affect the appearance of the ground object images.
# TABLE OF CONTENTS

List of Illustrations ........................................ viii  
List of Tables .................................................. xi  
I. INTRODUCTION ............................................... 1  
   A. BACKGROUND .............................................. 1  
   B. APPROACH ................................................ 3  
   C. SUMMARY OF MAJOR FINDINGS .............................. 5  
II. ORBITAL AND IMAGING SPECTROMETER CONFIGURATIONS ............. 8  
   A. SHUTTLE NOMINAL FLIGHT CONFIGURATIONS ................ 8  
   B. IMAGING SPECTROMETER SYSTEM DESCRIPTION ................ 10  
III. ERROR SOURCES AND SHUTTLE ATTITUDE DYNAMICS .................. 14  
   A. ERROR SOURCES .......................................... 14  
   B. SHUTTLE MASS PROPERTY .................................. 15  
   C. ASSESSMENT OF DISTURBANCES .............................. 18  
      C.1 Aerodynamic Drag Torque .............................. 18  
      C.2 Gravity Gradient Torques and Gyroscopic Torques ....... 25  
   D. MISALIGNMENT AND MEASUREMENT NOISE ...................... 27  
      D.1 Misalignment Errors .................................. 27  
      D.2 IMU Model ........................................... 28  
   E. SHUTTLE DYNAMICS AND INSTRUMENT POINTING ERRORS .......... 29  
      E.1 The Equations of Motion .............................. 29  
      E.2 The Space Shuttle State Estimator ........................ 29  
      E.3 The System Pointing Errors ............................ 30  
      E.4 The Instrument Pointing Error PSD ..................... 31  
   F. SUMMARY OF MAJOR FINDINGS .............................. 36  
      F.1 The Major Environmental Disturbances .................. 36
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>F.2</td>
<td>The Measurement Uncertainties</td>
<td>36</td>
</tr>
<tr>
<td>F.3</td>
<td>Ground Track Errors and Navigation Uncertainties</td>
<td>36</td>
</tr>
<tr>
<td>IV.</td>
<td>PARAMETRIC ANALYSIS OF GEOMETRIC ERRORS</td>
<td>43</td>
</tr>
<tr>
<td>A.</td>
<td>COORDINATE CONFIGURATIONS</td>
<td>43</td>
</tr>
<tr>
<td>B.</td>
<td>GEOMETRIC ERRORS INDUCED BY EPHEMERIS UNCERTAINTIES</td>
<td>45</td>
</tr>
<tr>
<td>B.1</td>
<td>Geometric Errors Due to Altitude Uncertainties</td>
<td>45</td>
</tr>
<tr>
<td>B.2</td>
<td>Geometric Errors Due to Intrack and Crosstrack Prediction Errors</td>
<td>50</td>
</tr>
<tr>
<td>C.</td>
<td>GEOMETRIC ERRORS INDUCED BY ATTITUDE UNCERTAINTIES</td>
<td>53</td>
</tr>
<tr>
<td>C.1</td>
<td>Geometric Errors Induced by Roll Error</td>
<td>53</td>
</tr>
<tr>
<td>C.2</td>
<td>Geometric Errors Induced by Pitch Error</td>
<td>55</td>
</tr>
<tr>
<td>C.3</td>
<td>Geometric Errors Induced by Yaw Error</td>
<td>58</td>
</tr>
<tr>
<td>C.4</td>
<td>Geometric Errors Induced by Roll, Pitch, and Yaw Attitude Errors</td>
<td>60</td>
</tr>
<tr>
<td>D.</td>
<td>GEOMETRIC ERRORS INDUCED BY ATTITUDE RATE ERRORS</td>
<td>68</td>
</tr>
<tr>
<td>E.</td>
<td>GEOMETRIC ERRORS INDUCED BY MISALIGNMENT ERRORS</td>
<td>68</td>
</tr>
<tr>
<td>F.</td>
<td>GEOMETRIC ERRORS INDUCED BY EARTH ROTATION</td>
<td>69</td>
</tr>
<tr>
<td>G.</td>
<td>NUMERICAL RESULTS: GEOMETRIC ERRORS AND ERROR SENSITIVITIES</td>
<td>72</td>
</tr>
<tr>
<td>G.1</td>
<td>Geometric Errors Due to Altitude Uncertainties</td>
<td>72</td>
</tr>
<tr>
<td>G.2</td>
<td>Geometric Errors Due to Roll Uncertainties</td>
<td>73</td>
</tr>
<tr>
<td>G.3</td>
<td>Geometric Errors Due to Pitch Uncertainties</td>
<td>74</td>
</tr>
<tr>
<td>G.4</td>
<td>Geometric Errors Due to Yaw Uncertainties</td>
<td>74</td>
</tr>
<tr>
<td>G.5</td>
<td>Geometric Errors Due to Attitude Uncertainties</td>
<td>74</td>
</tr>
<tr>
<td>G.6</td>
<td>Ground Point Shift Due to Shuttle Motion and Earth Rotation</td>
<td>75</td>
</tr>
<tr>
<td>H.</td>
<td>SUMMARY OF MAJOR FINDINGS</td>
<td>94</td>
</tr>
<tr>
<td>V.</td>
<td>GROUND PATTERNS AND IMAGE DISTORTIONS</td>
<td>101</td>
</tr>
</tbody>
</table>
VI. CONCLUSIONS. .......................................................... 111
REFERENCES. .............................................................. 113
APPENDIX A - GEOMETRIC ERROR MAPPING FUNCTION TABLE ....... 115
APPENDIX B - NAPIER'S RULES FOR RIGHT SPHERICAL TRIANGLES . 124
APPENDIX C - COMPUTER PROGRAM FOR SHUTTLE IMAGING SPECTROMETER
POINTING ERROR POWER SPECTRAL DENSITIES FOR CONFIGURATION
A -- PAYLOAD-BAY NADIR POINTING ................................ 125
APPENDIX D - COMPUTER PROGRAM FOR SHUTTLE IMAGING SPECTROMETER
POINTING ERROR POWER SPECTRAL DENSITIES FOR CONFIGURATION
B -- NOSE-DOWN NADIR POINTING ..................................... 129
APPENDIX E - GEOMETRIC ERROR ANALYSIS PROGRAM LISTINGS ........ 133
APPENDIX F - GROUND PATTERN AND IMAGING DISTORTIONS GENERATION
PROGRAM LISTINGS .......................................................... 150
List of Illustrations

<table>
<thead>
<tr>
<th>Illustration Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS Error Analysis System Block Diagram</td>
<td>4</td>
</tr>
<tr>
<td>Payload-Bay Nadir Pointing Configuration (A)</td>
<td>9</td>
</tr>
<tr>
<td>Nose-Down Nadir Pointing Configuration (B)</td>
<td>9</td>
</tr>
<tr>
<td>Imaging Spectrometer Basic Operating Principles</td>
<td>11</td>
</tr>
<tr>
<td>Shuttle Imaging Spectrometer Optical System Configuration</td>
<td>12</td>
</tr>
<tr>
<td>Error Sources</td>
<td>14</td>
</tr>
<tr>
<td>Orbiter Coordinate System</td>
<td>16</td>
</tr>
<tr>
<td>Shuttle Body and Principal Coordinates (Payload-Bay Nadir Pointing Shown)</td>
<td>17</td>
</tr>
<tr>
<td>The Three-Plate Aerodynamic Model in Body Frame</td>
<td>19</td>
</tr>
<tr>
<td>Simplified IMU Model</td>
<td>28</td>
</tr>
<tr>
<td>Pitch-Axis Block Diagram (Configuration A)</td>
<td>31</td>
</tr>
<tr>
<td>Roll and Yaw Axes Block Diagram (Configuration A)</td>
<td>32</td>
</tr>
<tr>
<td>Instrument Pointing Error PSD, ((rad)^2)-sec</td>
<td>35</td>
</tr>
<tr>
<td>Instrument Pointing Rate Error, ((rad/sec)^2)-sec</td>
<td>35</td>
</tr>
<tr>
<td>Instrument Pointing Error PSD, ((rad)^2)-sec</td>
<td>38</td>
</tr>
<tr>
<td>Instrument Pointing Rate Error, ((rad/sec)^2)-sec</td>
<td>38</td>
</tr>
<tr>
<td>IS Ground Track Error and Rate Error PSD</td>
<td>40</td>
</tr>
<tr>
<td>Payload-Bay Nadir Pointing Configuration and Coordinates</td>
<td>44</td>
</tr>
<tr>
<td>Ground Point Shift in X and Y Direction Due to Altitude Error</td>
<td>46</td>
</tr>
<tr>
<td>Change of View Angle of a Ground Point Due to Altitude Error</td>
<td>51</td>
</tr>
<tr>
<td>Geometric Errors Induced by Intrack and Crosstrack Prediction Errors</td>
<td>52</td>
</tr>
<tr>
<td>Shift of Ground Point Induced by Roll Error</td>
<td>54</td>
</tr>
<tr>
<td>Shift of Ground Point Induced by Pitch Error</td>
<td>56</td>
</tr>
<tr>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td></td>
</tr>
<tr>
<td>24. Shift of Ground Point Induced by Yaw Error</td>
<td>59</td>
</tr>
<tr>
<td>25. The Shift of Ground Projection of IS Line-of-Sight Due to Attitude Error</td>
<td>61</td>
</tr>
<tr>
<td>26. Side View of Fig. 25</td>
<td>63</td>
</tr>
<tr>
<td>27. Shift of Ground Point Induced by Attitude Errors</td>
<td>65</td>
</tr>
<tr>
<td>28. Linear Velocity of the Earth at the Nadir Point</td>
<td>70</td>
</tr>
<tr>
<td>29. Earth Spherical Geometry</td>
<td>71</td>
</tr>
<tr>
<td>30. Geometric Error Due to Altitude Uncertainties (Δh)</td>
<td>77</td>
</tr>
<tr>
<td>31. Geometric Error Due to Roll Attitude Error (ϕ)</td>
<td>79</td>
</tr>
<tr>
<td>32. Geometric Error Due to Roll Uncertainty (ϕ) About 20° Roll Offset</td>
<td>80</td>
</tr>
<tr>
<td>33. Geometric Error Due to Pitch Attitude Error (θ)</td>
<td>82</td>
</tr>
<tr>
<td>34. Geometric Error Due to Pitch Uncertainty (θ) About 22.5° Pitch Offset</td>
<td>83</td>
</tr>
<tr>
<td>35. Geometric Error Due to Pitch Uncertainty (θ) About 45° Pitch Offset</td>
<td>84</td>
</tr>
<tr>
<td>36. Geometric Error Due to Yaw Attitude Error (ψ)</td>
<td>86</td>
</tr>
<tr>
<td>37. Geometric Error Due to Roll (ψ), Pitch (θ), and Yaw (ϕ) Errors</td>
<td>88</td>
</tr>
<tr>
<td>38. Geometric Error Due to Pitch (θ) and Yaw (ψ) Uncertainties About 20° Roll Offset</td>
<td>89</td>
</tr>
<tr>
<td>39. Geometric Error Due to Roll (ψ) and Yaw (ϕ) Uncertainties About 22.5° Pitch Offset</td>
<td>90</td>
</tr>
<tr>
<td>40. Geometric Error Due to Roll (ψ) and Yaw (ψ) Uncertainties About 45° Pitch Offset</td>
<td>91</td>
</tr>
<tr>
<td>41. Geometric Error Caused by Earth Rotation for the Time Period of 1 Second</td>
<td>93</td>
</tr>
<tr>
<td>42. Effects of Earth Curvature</td>
<td>94</td>
</tr>
<tr>
<td>43. Ground Pattern Distortions With No Earth Rotation Effect</td>
<td>102</td>
</tr>
<tr>
<td>44. Ground Pattern Distortions With 20° Roll Offset and No Earth Rotation Effect</td>
<td>104</td>
</tr>
</tbody>
</table>
45. Ground Pattern Distortions with $45^\circ$ Pitch Offset and No Earth Rotation Effect ........................................... 105

46. Ground Pattern Distortions with Constant Rotation Rate and No Earth Rotation Effect ........................................... 106

47. Ground Pattern Distortions with Sinusoidal Rotation Rate and No Earth Rotation Effect ........................................... 108

48. Ground Pattern Distortions with Effect of Altitude Change (Increase) of 40 km and No Earth Rotation Effect ........................................... 109

49. Ground Pattern Distortions with Effect of Earth Rotation for $40^\circ$ Latitude and Orbit Inclination of $85^\circ$ ........................................... 110
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Sensor Performance Requirements</td>
<td>2</td>
</tr>
<tr>
<td>2.</td>
<td>Modeled Environmental Disturbances</td>
<td>39</td>
</tr>
<tr>
<td>3.</td>
<td>Modeled Measurement Uncertainties</td>
<td>39</td>
</tr>
<tr>
<td>4.</td>
<td>Expected On-Orbit Navigation Accuracies (3o)</td>
<td>42</td>
</tr>
<tr>
<td>5.</td>
<td>Geometric Errors and Error Sensitivity Due to Altitude Uncertainties</td>
<td>76</td>
</tr>
<tr>
<td>6.</td>
<td>Geometric Errors and Error Sensitivity Due to Roll Attitude Error</td>
<td>78</td>
</tr>
<tr>
<td>7.</td>
<td>Geometric Errors and Error Sensitivity Due to Pitch Attitude Error</td>
<td>81</td>
</tr>
<tr>
<td>8.</td>
<td>Geometric Errors and Error Sensitivity Due to Yaw Attitude Error</td>
<td>85</td>
</tr>
<tr>
<td>9.</td>
<td>Geometric Errors Induced by Yaw, Pitch, and Roll Errors</td>
<td>87</td>
</tr>
<tr>
<td>10.</td>
<td>Ground Point Shift Induced by Shuttle Motion and Earth Rotation</td>
<td>92</td>
</tr>
<tr>
<td>11.</td>
<td>Geometric Errors Due to Expected On-Orbit Navigation Uncertainty</td>
<td>97</td>
</tr>
<tr>
<td>12.</td>
<td>Expected On-Orbit Navigation Accuracies (Ref. 12)</td>
<td>98</td>
</tr>
<tr>
<td>14.</td>
<td>Geometric Error Induced by Shuttle and Imaging Spectrometer Measurement Uncertainties</td>
<td>100</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

A. BACKGROUND

Earth resource management and utilization have experienced great success over the last decade through the Landsat programs. In more recent years, both NASA and user communities have envisioned the need for development of better and more powerful instruments for surveying and managing earth resources. The Landsat D's new sensor, Thematic Mapper (TM), the proposed utilization of Tracking and Data Relay Satellite System (TDRSS), and a more advanced ground system represent an advancement in the earth resource satellite development [1].

The Thematic Mapper of the Landsat D (launched in 1982) has seven spectral bands, two more than those of the Multispectral Scanner associated with the earlier Landsats. However, study shows that the reflectance spectrum of earth surface materials contains a significant amount of information which can only be identified with spectral resolution much finer than those of the Thematic Mapper [2]. With the advancement of area array detectors, a push broom imaging spectrometer can be realistically constructed for simultaneous imaging and registration of hundreds of spectral bands. For the case of Shuttle Imaging Spectrometer Experiment, 128 spectral channels have been proposed to cover the spectral range of 0.4 to 1.0 μm for VNIR (visible and near infrared) and 1.0 to 2.5 μm for SWIR (short-wavelength infrared) with instantaneous field of view of 30m. Table 1 shows the required sensor performance [2].

The purpose of this report is to document the error analysis study for the imaging spectrometer experiment. Error analysis is an important aspect of the overall remote sensing system, since errors from many sources, including
the spectrometer itself, the spacecraft that carries the instrument, knowledge limitations on the true spacecraft attitude and locations, earth rotation, curvature, and terrain variations, etc., will all contribute to image distortion, shift, rotation, and misregistration. Error correction or compensation are necessary and are an integral part of the image processing. This work covers the analysis of fundamental and geometric errors and error sensitivities, the development of geometric mapping functions, and the computation of ground

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectral Coverage</td>
<td>0.4 to 2.5 μm</td>
<td>Although the entire spectrum is probably not required for any one discipline, in the aggregate of all remote sensing disciplines, the entire region is required</td>
</tr>
<tr>
<td>Spectral Sampling Interval</td>
<td></td>
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</tr>
<tr>
<td>VNIR (0.4 to 1.0 μm)</td>
<td>0.01 μm or better</td>
<td>Adequate for most research topics</td>
</tr>
<tr>
<td>SWIR (1.0 to 2.5 μm)</td>
<td>0.02 μm or better</td>
<td>This is adequate for research if pointing capability is provided to assure target access</td>
</tr>
<tr>
<td>Instantaneous Field of View</td>
<td>30 m</td>
<td>Adequate for most research topics</td>
</tr>
<tr>
<td>Swath Width</td>
<td>at least 10-12 km</td>
<td>This is adequate for research if pointing capability is provided to assure target access</td>
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<tr>
<td>Pointing Mirror Range</td>
<td></td>
<td></td>
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<tr>
<td>Along track</td>
<td>at least ± 45 deg</td>
<td>Essential for atmospheric and BRDF (Bidirectional Reflectance Distribution Function) Studies</td>
</tr>
<tr>
<td>Cross track</td>
<td>not more than ± 25 deg</td>
<td></td>
</tr>
<tr>
<td>Radiometric Performance (NEdR)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VNIR</td>
<td>≤ 0.5%</td>
<td></td>
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<tr>
<td>SWIR</td>
<td>≤ 1.0%</td>
<td></td>
</tr>
</tbody>
</table>
pattern distortions. The results are translated into many tables, plots, and patterns for visual apprehension. It is believed that the results reported here are important for design, trade-off, and performance prediction.

B. APPROACH

This study has been performed in four progressive stages as shown in block diagram form in Fig. 1. In Stage I, the error sources were identified, dynamic disturbances and the Space Shuttle error dynamics were modeled, and the error power spectral densities for two in-orbit configurations were developed. Stage II of this study concentrated on the development of geometric error mapping functions and error sensitivity analysis. Geometric errors due to ephemeris uncertainties, attitude deviations, earth rotation, etc., were studied. In Stage III, ground pattern image distortions caused by various error effects and forward and side looking angular offsets, altitude change, and effect of earth rotation were generated. Stage IV consists of the analysis of the imaging spectrometer instrument errors. These errors include optical jitter, nonlinearities, processing errors, and repeatability. The study of Stages I, II, and III has been completed and the results are included in this report. The study called for by Stage IV has not been planned. It is emphasized here that the imaging spectrometer error model development is an important step for the overall performance prediction and design of the imaging spectrometer system.

Major findings of this work are summarized in the following subsection. In Section II, the orbital and imaging spectrometer configurations are described. The attitude dynamics and error power spectral densities are documented in Section III; and parametric analysis of geometric errors are treated in Section IV. Section V deals with the ground pattern image distortions. Conclusions are summarized in Section VI.
Figure 1. IS Error Analysis System Block Diagram
C. SUMMARY OF MAJOR FINDINGS

The following is a summary of major findings. The details of these are treated in the sections to follow.

1. The results show that the IS Experiment with image pickup period of 20 seconds at a time is feasible with the shuttle properly phased inside the control deadband. The error PSD (power spectral density) characteristic reveals that the system resonates at very low frequencies (in the $10^{-5}$ to $10^{-2}$ Hz region). Excitations at these frequencies must be avoided through design precautions. The analysis also showed that errors below 0.01 Hz are dominated by the shuttle dynamics reacting to disturbances, whereas those above 0.01 Hz are dominated by the shuttle inertial measurement system uncertainties and its inherent noise. These high frequency errors limit pointing performance and result in a one-sigma ground track error of 54.8 meters per axis. One-sigma rate errors are shown to be less than 4 meters per second per axis in the frequency range of $10^{-5}$ to $4 \times 10^{-2}$ Hz. The image smear is not significant because of the short millisecond-level line time.

2. The effects of earth curvature are very small for the application here (see Fig. 42).

3. Altitude uncertainties cause only moderate geometric errors. The worst 1σ geometric errors are 11.71 m in position and 0.133 m/sec in rate with STDN and large unmodeled perturbations at 200 km orbit. The performance improves with TDRSS. For the 300 km orbit and with small unmodeled perturbations, the 1σ geometric errors will reduce to 0.22 m and 0.0032 m/sec (see Table 11).
4. The effects of other navigation errors are significantly greater. The lo downtrack errors range from 203m (300 km orbit) to 8128m (200 km orbit); and those for the crosstrack are 152m to 588m (see Table 12).

5. The effects of roll and pitch attitude errors are relatively large compared with, for instance, those caused by yaw errors and altitude uncertainties. The error sensitivity is 1.94 m/arc sec or approximately 7000 m/degree. The yaw sensitivity is 0 for 0° view angle and 0.028 m/arc sec for the maximum view angle of ±0.825° (see Table 13).

6. The error sensitivities of attitude errors increase significantly for large attitude offsets. For 20° side looking, the sensitivity increases to 2.28 m/arc sec for roll errors and to 0.75 m/arc sec for yaw errors and the pitch error sensitivity is almost unaffected. For the 45° forward looking case, the sensitivities for the pitch, roll, and yaw errors increase to 2.83, 4.36, and 1.97 m/arc sec, respectively (see Table 13).

7. The performance of the Shuttle Imaging Spectrometer is limited by the Shuttle IMU (Inertial Measuring Unit) accuracy, instrument misalignment, etc., Shuttle RCS (Reaction Control Subsystem) deadband, etc. unless some means of error reduction are employed. For instance, ground control points may be used to reduce navigation prediction errors; and precision point mounts, such as AGS (ASPS* Gimbal System) and IPS (Instrument Pointing System), may be used to reduce the

*Annular Suspension Pointing System
attitude errors.

The single axis geometric errors due to the combined Shuttle/IS misalignment, for instance, for normal nadir pointing, 20° side looking, and 45° forward looking are 169m, 198m, and 379m, respectively (refer to Table 10 for detailed breakdowns).

Earth rotation causes shifts of images toward the direction of rotation. The magnitude of this shift depends on the latitude of the object. For instance, at the equator the object moves approximately 462m in one second (for 400 km orbit), and at 60° latitude it moves only 231m in one second.
II. ORBITAL AND IMAGING SPECTROMETER CONFIGURATIONS

A. SHUTTLE NOMINAL FLIGHT CONFIGURATIONS

Two shuttle in-orbit configurations have been considered, the Payload-Bay Nadir and the Nose-Down Nadir, as illustrated in Figures 2 and 3, respectively. It will be shown later that the Nose-Down Configuration is gravity gradient stabilized and the Payload-Bay Nadir Configuration is not. However, the Payload-Bay Nadir Configuration offers simpler instrument mounting and less aerodynamic drag. Besides, for certain experiments that require large forward looking angles, the Nose-Down Configuration will be unsuitable.

A circular orbit of 400 km altitude has been selected for this analysis. This selection is consistent with the SIS-B parameters.*

There are a number of possible instrument mounting options that will affect the pointing and the geometric errors of the instrument. These options include:

a) Direct mount
b) AGS (ASPS Gimbal System) mount
c) IPS (Instrument Pointing System) mount

Since both the AGS and IPS systems are capable of providing precision payload pointing and measurements, the system performance will be improved at the expense of significant extra cost. The direct mount approach is the least complex and most cost effective, provided that the errors are within the tolerance. In this report the direct mount approach is considered. Furthermore, the shuttle IMU

*During the period when this part of the work was performed, SIS-B (Shuttle Imaging Spectrometer-Configuration B) was considered. However, the methods used here are mostly generic, and hence, can be applied to systems of similar configurations.
Figure 2. Payload-Bay Nadir Pointing Configuration (A)

Figure 3. Nose-Down Nadir Pointing Configuration (B)
(Inertial Measurement Unit) and the shuttle state estimator unit are used for obtaining attitude and rate information, without additional instrumentation. Other options may be studied in the future if necessary.

B. IMAGING SPECTROMETER SYSTEM DESCRIPTION

In order to correlate the analysis reported here to the IS applications, it is desirable to understand the basic operating principle of the IS instrument. Figure 4 shows a sketch of the basic elements of the IS, except the memory banks, registers, and the processors. Some of the IS parameters that are relevant to the geometric error analysis, along with definitions of the IS terminologies, are also listed in Fig. 4. It is understood that the sketch and the parameter values used are for illustrative purposes only since the parameters may change as the development of the IS system is finalized.

Referring to Fig. 4, as the reflection from the ground objects passes through a narrow slit, it strikes the incident surface of the prism. The prism separates the incident light into spectral images projected onto an area array of light detectors. The area array consists of 384 linear arrays, corresponding to 384 spectral channels. Each linear array consists of 384 detector elements. Each detector element corresponds to an image area of 30m x 30m on the ground. Hence, each linear array corresponds to an image of a specific spectral channel of the same ground area of 30m (along-track) x 11520m (cross-track). This 11.52 km cross-track measure, referred to as the swath width, defines the IS coverage for each orbit pass. The 30m x 30m area is referred to as the pixel (picture element) which defines the resolution of the image, i.e., within this element no features can be resolved. The IS is operated based on a push broom principle. That is, a specific detector element on the instrument (moving with the spacecraft) collects photons from the 30m x 30m moving window for a specific period.
KEY PARAMETERS
- LINE TIME = 4 ms
- SPECTRUM:
  - VISIBLE REGION: 0.4 - 1.0 μm
  - SHORT WAVELENGTH IR: 1.0 - 2.5 μm
- FIELD OF VIEW: ± 0.825°
- SWATH WIDTH: 11.52 km
- POINTING ANGLES (RANGE):
  - ± 45° (PITCH)
  - ± 20° (ROLL)
- ORBIT: 400 km CIRCULAR

DEFINITIONS
- GIFOV (GROUND INSTANTANEOUS FIELD OF VIEW): PROJECTION ON THE GROUND OF EACH SQUARE DETECTOR
- LINE TIME: TIME TO MOVE ALONG THE GROUND A DISTANCE OF 1 GIFOV
- DN: DATA NUMBER, THE "BRIGHTNESS" OF THE ASSOCIATED PIXEL
- PIXEL: PICTURE ELEMENT (30 m x 30 m) HERE, IT DEFINES THE RESOLUTION OF THE IMAGE
- SWIR: SHORT WAVE LENGTH IR
- VIS: VISIBLE WAVE LENGTH

Figure 4. Imaging Spectrometer Basic Operating Principles
of time called the line time. The line time in this case is the time required to move 30m along-track, which is determined by the orbit. For a 400 km circular orbit, it is about 4 ms. At the end of each line time, the total number of photons collected by each detector is recorded and processed and the detector/register is reset and a new 4 ms cycle is repeated. For digital processing, the amount of light collected by a collector is assigned a number called DN (data number), which is proportional to the number of photons accumulated. Therefore, the processor has to record and process 384 x 384, or approximately 1.475 x 10^5 DN's every 4 ms.

As the shuttle flies over an area, banks after banks of DN's are collected. By spacing the banks of DN's 30m apart, the features of the ground image emerge. The ratios of the DN's of various channels for the same area are of special interest, as these ratios are closely correlated to geological and ecological states of the earth including mineral deposits, forestry, crops,

![Shuttle Imaging Spectrometer Optical System Configuration](image)

**Figure 5. Shuttle Imaging Spectrometer Optical System Configuration**

12
disease and insect infestations, land and soil erosion, precipitation in the form of snow and ice, air and water quality, etc.

The Imaging Spectrometer optical system configuration and the arrangement of lenses, mirrors, slit, prisms, focal plane detector, etc. are shown in Fig. 5 [2]. Additional information on the instrument design and requirements can be found in Refs. 2 and 3.
III. ERROR SOURCES AND SHUTTLE ATTITUDE DYNAMICS

A. ERROR SOURCES

There are two types of errors that are important to the imaging spectrometer experiment — the direct errors and the derived errors. The direct errors are those pertaining to the spacecraft and instrument pointing, ephemeris, and instrument errors as shown in Fig. 6. The derived errors are the geometric errors and pattern distortions of ground objects which result from the direct errors and the effects of earth rotation, curvature, oblateness, and local vertical uncertainties. Fig. 1 shows relationships of the error sources and the system dynamics.

- Attitude deviations, rate errors and structural vibration
- Measurement noise and drift
- Misalignment of shuttle IMU and attitude reference frame
- Misalignment between shuttle and IS instrument references
- Ephemeris prediction errors
- Earth rotation, curvature, and oblateness
- IS instrument errors including
  - Optical jitter
  - Nonlinearities
  - Processing
  - Repeatability

Figure 6. Error Sources
In this section, the steady state analysis is performed in the frequency domain. The PSD (power spectral density) for the pointing errors and the rate errors were obtained by considering the dynamics of the Space Shuttle Orbiter, the IMU (Inertial Measurement Unit), the attitude control state estimator, the measurement noise, the misalignment of reference frames, and the disturbances including gravity gradient, gyroscopic torques, and aerodynamic drag torques.

B. SHUTTLE MASS PROPERTY

The shuttle mass properties employed here were obtained from the Shuttle Operational Data Book [4] for OV-102/STS-3. The mass and the c.m. are, respectively:

Shuttle mass: 102,153.73 kg (224,738.21 lbs)

Shuttle c.m.: \( \begin{align*} X_o &= 1105.5", \\ Y_o &= 0", \\ Z_o &= 374.3" \end{align*} \)

where the coordinates \( X_o \), \( Y_o \), and \( Z_o \) are the Orbiter Coordinates [5] defined in Figure 7. The moment of inertia matrix (kg-m^2) is, in the shuttle body frame (refer to Fig. 8),

\[
I_B = \begin{bmatrix}
1.36 \times 10^6 & -4.69 \times 10^3 & -3.49 \times 10^5 \\
-4.69 \times 10^3 & 1.00 \times 10^7 & -3.32 \times 10^3 \\
-3.49 \times 10^5 & -3.32 \times 10^3 & 1.05 \times 10^7
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1.36 \times 10^6 & 0 & -3.49 \times 10^5 \\
0 & 1.00 \times 10^7 & 0 \\
-3.49 \times 10^5 & 0 & 1.05 \times 10^7
\end{bmatrix}
\] (i)

The magnitude of the off-diagonal terms of the inertia matrix \( I_B \) suggests strong couplings exist especially between the \( X_a \) and the \( Z_B \)-axis. To simplify the dynamic equations, it is convenient to consider principal-axis
ORBITER COORDINATES

TYPE: Rotating, Orbiter referenced

ORIGIN: Approximately 200 inches (5.1m) ahead of the nose and approximately 400 inches (10.2m) below the centerline of the cargo bay

ORIENTATION AND LABELING:
   The X-axis is parallel to the centerline of the cargo bay, negative in the direction of launch
   The Z-axis is positive upward in landing attitude
   The Y-axis completes the right-handed system
   The standard subscript is 0

Figure 7. Orbiter Coordinate System
pointing instead of body-axis pointing, e.g., pointing $-Z_p$ rather than $-Z_B$ as illustrated in Figure 8.

The orientations of the principal axes can be determined by rotating the body frame an angle $\alpha$ about the $Y_B$-axis, i.e., let $B: X_B \rightarrow X_p$, then

$$B = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix} \quad (2)$$

and

$$I_p = B I_B B^T \quad (3)$$

It can be shown that
\[ \alpha = \frac{1}{2} \tan^{-1}\left( \frac{-2 I_{BZX}}{I_{BZZ} - I_{BXX}} \right) \]  

(4)

provided that \( I_{BXY} = I_{BYZ} = 0 \).

For \( I_{BXX} = 1.36 \times 10^6 \text{ kg-m}^2 \), \( I_{BZZ} = 1.05 \times 10^7 \text{ kg-m}^2 \), and \( I_{BZX} = 3.49 \times 10^5 \text{ kg-m}^2 \), the angle \( \alpha = -2.18^\circ \). Therefore, in order to do principal-axis nadir pointing, the shuttle has to rotate 2.18° about the orbit normal (see Figure 8). The moments of inertia about the principal axes are

\[ I_p = \text{diag} (1.38 \times 10^6, 1.00 \times 10^7, 1.05 \times 10^7), \text{ kg-m}^2. \]

C. ASSESSMENT OF DISTURBANCES

The shuttle motion is characterized in part by the environmental disturbances, the major sources of which are the aerodynamic drag, gravity gradient, gyroscopic effect, solar radiation, and on-board causes such as astronaut activities, equipment vibrations, and venting. On-board activities may be partially eliminated or reduced through mission planning and their effects will be assessed in the future. In this subsection, the gyroscopic torques, the gravity gradient torques, and the aerodynamic drag torques are estimated. The solar pressure is at least one order of magnitude less than the aerodynamic drag forces and, hence, it is not included in this analysis.

C.1 The Aerodynamic Drag Torques

To estimate the aerodynamic drag torques, an approach referred to as the three-plate model [6] has been used. Referring to Figure 9, let \( \vec{n}_1, \vec{n}_2, \) and \( \vec{n}_3 \) be the unit normal vectors for the equivalent plates 1, 2, and 3, where \( \vec{n}_1, \vec{n}_2, \) and \( \vec{n}_3 \) are in the direction of \( X_B, Y_B, \) and \( Z_B \)-axis, respectively. To express \( \vec{n}_4 \) in body frame,
Let $A_1$, $A_2$, and $A_3$ be the corresponding areas on which the aerodynamic pressure applies, and $c_{p1}$, $c_{p2}$, and $c_{p3}$ the three corresponding centers of pressure, respectively. The model assumes that the aerodynamic force applies to each area in the direction opposite to the vehicle velocity vector and assumes no shading among the plates. It is further assumed that the drag coefficient $C_D$ is constant and the lift coefficient $C_L$ is zero.

Let $\vec{v}_B = (v_{B1}, v_{B2}, v_{B3})^T$ be the inertial velocity vector in the body frame, with magnitude $v$. The force and torque applied on $A_1$ are

\[
\vec{F}_{B1} = -\left(\frac{1}{2} C_D \rho v^2 A_1 \right) (\vec{n}_1, \vec{u}_B) \cdot \vec{u}_B
\]

\[
\vec{r}_{B1} = \vec{r}_{B1} \times \vec{F}_{B1} = -\left(\frac{1}{2} C_D \rho v^2 A_1 \right) (\vec{n}_1, \vec{u}_B) \cdot \vec{r}_{B1} \times \vec{u}_B
\]

where $\rho$ is the atmospheric density at the orbital altitude, $\vec{u}_B = \vec{v}_B / v$ the unit velocity vector, and $r_{B1}$ the vectors of the center of pressure of $A_1$ relative to the vehicle center of mass.
The total aerodynamic forces and torques on the vehicle are

\[
\overline{F}_B = \sum_{i=1}^{3} - \left( \frac{1}{2} C_D \rho v^2 A_i \right) \left( \overline{u}_i, \overline{u}_B \right) \overline{u}_B
\]

(8)

\[
\overline{T}_B = \sum_{i=1}^{3} - \left( \frac{1}{2} C_D \rho v^2 A_i \right) \left( \overline{u}_i, \overline{u}_B \right) \overline{r}_{B_i} \times \overline{u}_B
\]

(9)

The value of \( C_D \) depends on the shape of the vehicle. In Ref. 7, the values of 2.5 to 3.0 were suggested. The value of \( C_D = 2.0 \) is used here as it was used in Ref. 6 for Space Shuttle Simulation.

The atmospheric mass density \( \rho \) can be found in a JPL internal memorandum. For 1985 mission time, the peak density is expected to occur in April for the 400 km orbit; the densities are

\[2.64 \times 10^{-12} \text{ kg/m}^3\] (predicted)

and

\[3.77 \times 10^{-12} \text{ kg/m}^3\] (97.7 percentile)

The orbital velocity \( v \) for a 400 km circular orbit is computed as

\[v = 7669.60 \text{ m/sec or } v^2 = 5.882 \times 10^7 (\text{m/sec})^2\]

The plate areas, according to the attachment (SSFS On-Orbit Aero Data, 7/24/80) to Ref. 6, with Cargo Bay doors open, are

\[A_1 = 119.45 \text{ m}^2\]

\[A_2 = 229.92 \text{ m}^2\]

\[A_3 = 454.46 \text{ m}^2\]

(10)

However, a different set of values are given in Ref. 8.

\[A_1 = 64.1 \text{ m}^2\]

\[A_2 = 212.7 \text{ m}^2\]

\[A_3 = 367.0 \text{ m}^2\]

(11)
The values in (10) were used in this work.

The position vectors $\mathbf{r}_{B1}$ may be obtained using data given in Ref. 4 and Ref. 6. The values are,

$$
\mathbf{r}_{B1} = \begin{bmatrix} 3.797 \\ 0 \\ -0.231 \end{bmatrix} \text{ m}
$$

$$
\mathbf{r}_{B2} = \begin{bmatrix} 0.927 \\ 0 \\ -0.742 \end{bmatrix} \text{ m}
$$

$$
\mathbf{r}_{B3} = \begin{bmatrix} 1.166 \\ 0 \\ -1.074 \end{bmatrix} \text{ m}
$$

The torques $\mathbf{\tau}_{B1}$ for the predicted density, are

$$
\mathbf{\tau}_{B1} = -1.855 \times 10^{-2} |u_{B1}| \begin{bmatrix} -r_{B13}u_{B2} + r_{B12}u_{B3} \\ r_{B13}u_{B1} + r_{B11}u_{B3} \\ -r_{B12}u_{B1} + r_{B11}u_{B2} \end{bmatrix}
$$

$$
\mathbf{\tau}_{B2} = -3.570 \times 10^{-2} |u_{B2}| \begin{bmatrix} -r_{B23}u_{B2} + r_{B22}u_{B3} \\ r_{B23}u_{B1} - r_{B21}u_{B3} \\ -r_{B22}u_{B1} + r_{B21}u_{B2} \end{bmatrix}
$$

$$
\mathbf{\tau}_{B3} = -7.057 \times 10^{-2} |u_{B3}| \begin{bmatrix} -r_{B33}u_{B2} + r_{B32}u_{B3} \\ r_{B33}u_{B1} - r_{B31}u_{B3} \\ -r_{B32}u_{B1} + r_{B31}u_{B2} \end{bmatrix}
$$
In Eq. (13), the torques are known once the unit velocity vector \( \vec{u}_B \) is specified; \( \vec{u}_B \) varies with the pointing configuration and spacecraft attitude.

### C.1.1. Drag Torques for Configuration A

Referring to Fig. 2, since this is "-Z" pointing, for nominal attitude, the vehicle moves in the \( X_p \) direction. Let \( \vec{u}_p \) be the unit velocity vector in the \( P \)-frame and let \( A_\alpha \) be the rotation matrix due to \( \alpha \), then

\[
\vec{u}_B = A_\alpha^T \vec{u}_p = \begin{bmatrix}
cosa & 0 & sina \\
0 & 1 & 0 \\
-sina & 0 & cosa
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
cosa \\
0 \\
-sina
\end{bmatrix}
\]

(14)

Since \( \alpha (= -2.18^\circ = -.038 \text{ rad}) \) is small,

\[
\vec{u}_B = \begin{bmatrix}
1 \\
0 \\
-a
\end{bmatrix}
\]

(15)

Using Eqs. (12), (13), and (15), and

\[
\vec{T}_p = \sum_{i=1}^{3} A_\alpha \vec{T}_B i
\]

(16)

the torques in the \( P \)-Frame, for nominal attitude, are

\[
\vec{T}_p = \begin{bmatrix}
0 \\
9.84 \times 10^{-3} \\
0
\end{bmatrix} \text{ N-m}
\]

(17)

For small attitude errors, \( \phi \), \( \theta \), and \( \psi \), from the rotating orbital frame,

\[
\vec{u}_p = A \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

(18)
where,
\[
    A = \begin{bmatrix}
        1 & \psi & -\theta \\
        -\psi & 1 & \phi \\
        \theta & -\phi & 1
    \end{bmatrix}
\]

and
\[
    \bar{u}_B = A^T_a A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\]  \hspace{1cm} (20)

Using Eqs. (12), (13), and (20), the predicted aerodynamic torques become, by retaining only the first order terms, in N-m,
\[
    \bar{T}_p = \sum_{i=1}^{3} A_\alpha \bar{T}_{B_i}
\]  \hspace{1cm} (21)

\[
    = \begin{bmatrix}
        4.29 \times 10^{-3} \psi \\
        4.29 \times 10^{-3} + 7.04 \times 10^{-2} (\theta - \alpha) + 7.58 \times 10^{-2} |\theta - \alpha| + 2.65 \times 10^{-2} |\psi| \\
        7.04 \times 10^{-2} \psi
    \end{bmatrix}
\]

C.1.2. Drag Torques for Configuration B

Referring to Figure 3, under this configuration, \(X_p\) is in the Nadir direction and \(Y_p\) is in the direction of motion for nominal attitude. In this case,
\[
    \bar{u}_p = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}
\]  \hspace{1cm} (22)

\[
    \bar{u}_B = A^T_a \bar{u}_p = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}
\]  \hspace{1cm} (23)

and from Eqs. (12), (13), and (23),
\[
    \bar{T}_p = \sum_{i=1}^{3} A_\alpha \bar{T}_{B_i}
\]
In the presence of small attitude errors, $\phi$, $\theta$, and $\psi$, the aerodynamic drag torques are

\[
\mathbf{T}_p = \begin{bmatrix}
-3.96 \times 10^{-2} \\
0 \\
-4.53 \times 10^{-2}
\end{bmatrix} \text{ N-m}
\]  \hspace{1cm} (24)

\[
\mathbf{T}_p = \begin{bmatrix}
-2.45 \times 10^{-2} - 7.58 \times 10^{-2} |\phi| - 4.29 \times 10^{-3} |\phi| + 3.31 \times 10^{-2} a \\
-3.31 \times 10^{-2} \phi + 2.65 \times 10^{-2} \psi \\
-(3.31 \times 10^{-2} + 8.23 \times 10^{-2} |\phi| + 7.04 \times 10^{-2} |\psi| + 2.65 \times 10^{-2} a)
\end{bmatrix} \hspace{1cm} (25)
\]

C.1.3. Estimation of Disturbance PSD

Before the PSD's are estimated, the uncertainty part of the disturbance torques has to be determined. The torques of Eqs. (21) and (25) consist of static parts and the dynamic parts. The dynamic parts are functions of the attitude errors $\phi$, $\theta$, and $\psi$. The attitude errors are assumed to be random and time-varying with standard deviation of $1^\circ$ (.01745 rad.) per axis. Therefore, the estimated values of the random disturbance torques are, for Configuration A

\[
\mathbf{\sigma}_{\mathbf{T}_A} = \begin{bmatrix}
7.48 \times 10^{-5} \\
1.86 \times 10^{-3} \\
1.23 \times 10^{-3}
\end{bmatrix} \text{ N-m} \hspace{1cm} (26)
\]

and that for Configuration B are,

\[
\mathbf{\sigma}_{\mathbf{T}_B} = \begin{bmatrix}
1.33 \times 10^{-3} \\
7.40 \times 10^{-4} \\
1.89 \times 10^{-3}
\end{bmatrix} \text{ N-m} \hspace{1cm} (27)
\]

and the corresponding PSD's are obtained by the following approximation with the correlation time of $T = 180$ seconds,

\[
Q_{\mathbf{T}_A} = 2T \mathbf{\sigma}_{\mathbf{T}_A}^2
\]

\[
Q_{\mathbf{T}_B} = 2T \mathbf{\sigma}_{\mathbf{T}_B}^2
\]
C.2 The Gravity Gradient Torques and Gyroscopic Torques

The gravity gradient torques and the gyroscopic torques can be estimated using the following equations, respectively:

$$\vec{T}_{GRP} = 2T \left( \vec{\gamma}_{\vec{T}_{PB}} \right)$$

$$\vec{T}_{PB} = \begin{bmatrix} 2.01 \times 10^{-6} \\ 1.25 \times 10^{-3} \\ 5.44 \times 10^{-4} \end{bmatrix} \text{ (N-m)}^2 \text{ - sec} \quad (28)$$

$$\vec{Q}_{T_{PB}} = \begin{bmatrix} 6.32 \times 10^{-6} \\ 1.97 \times 10^{-6} \\ 1.29 \times 10^{-3} \end{bmatrix} \text{ (N-m)}^2 \text{ - sec} \quad (29)$$

$$\vec{T}_{GP} = 3\vec{\omega} \cdot \vec{u}_{RP} I_F \vec{u}_{RP}$$

$$(30)$$

and

$$\vec{T}_{GRP} = \vec{\omega}_{OP} \times \vec{H}_P = \vec{\omega}_{OP} I_P \vec{\omega}_{OP}$$

$$(31)$$

Where $\vec{\omega}_{OP}$ and $\vec{\omega}_{OP}$ are the unit earth vector and the spin velocity vector, respectively, in principal frame, and $\vec{u}_{RP}$ is the skew symmetric matrix of the vector $\vec{u}_{RP}$. For Configuration A,

$$\vec{u}_{RP} = A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$(32a)$$

$$\vec{\omega}_{OP} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \vec{\omega}_o$$

$$(32b)$$

and for Configuration B,
the corresponding torques, to include only the first order effect, are, for Configuration A

\[
\overline{T}_{\text{gP}} = 3 \omega_o^2 \begin{bmatrix}
(I_{PZZ} - I_{PYY})\phi \\
(I_{PZZ} - I_{PXX})\theta \\
0
\end{bmatrix} = \begin{bmatrix}
1.92 \phi \\
33.11 \theta \\
0
\end{bmatrix} \text{ N-m} \tag{34a}
\]

\[
\overline{T}_{\text{GRP}} = \omega_o^2 \begin{bmatrix}
0 \\
-(I_{PZZ} - I_{PYY})\phi \\
(I_{PYY} - I_{PXX})\psi
\end{bmatrix} = \begin{bmatrix}
-0.64 \phi \\
0 \\
11.04 \psi
\end{bmatrix} \tag{34b}
\]

and for Configuration B

\[
\overline{T}_{\text{gP}} = -3 \omega_o^2 \begin{bmatrix}
(I_{PZZ} - I_{PXX})\theta \\
(I_{PXX} - I_{PYY})\psi
\end{bmatrix} = \begin{bmatrix}
-35.03 \theta \\
43.71 \psi
\end{bmatrix} \tag{35a}
\]

\[
\overline{T}_{\text{GRP}} = \omega_o^2 \begin{bmatrix}
-(I_{PZZ} - I_{PYY})\phi \\
(I_{PZZ} - I_{PXX})\theta \\
0
\end{bmatrix} = \begin{bmatrix}
-0.64 \phi \\
11.68 \theta \\
0
\end{bmatrix} \tag{35b}
\]

Since the gravity gradient torques and gyroscopic torques are proportional to the attitude errors, they may be included in the equations of motion.
as part of the vehicle dynamics rather than disturbances to the plant.

D. THE MISALIGNMENT AND MEASUREMENT NOISE

In this subsection, the errors that will contribute to pointing uncertainties are estimated. These error sources include the misalignment errors of the IMU and the IS instrument, and the sensor noise.

D.1 The Misalignment Errors

Let \( b_{MS} \) be the misalignment between the IMU and the shuttle reference frame; let \( b_{SI} \) be the misalignment between the shuttle and the imaging spectrometer reference frame.

Based on the space shuttle performance requirement [5], the IMU 3σ misalignment uncertainty is \( \pm 0.133^\circ/\text{axis} \), hence,

\[
\sigma_{b_{MS}} = 0.044^\circ/\text{axis} = 159.6 \text{ arc-sec/axis} \quad (36a)
\]

However, based on a JPL internal memorandum, the shuttle flight test performance is much better,

\[
\sigma_{b_{MS}} = 82 \text{ arc-sec/axis} \quad (36b)
\]

The performance data Eq. (36b) was used in this report.

The misalignment data for the IS instrument is not directly available. Based on Ref. 9, the estimated LANDSAT D initial alignment bias between the sensor optical axis and the vehicle pointing vector was \( \pm 200 \text{ arc-sec} \). The alignment bias can be measured before launch and can be removed from the image data. The variation part that cannot be removed without ground control points was estimated as \( \pm 30 \text{ arc-sec} \). This latter number was used in this work,

\[
\sigma_{b_{SI}} = 30 \text{ arc-sec} \quad (37)
\]

Therefore, the total misalignment error becomes

\[
\sigma_b = (\sigma_{b_{MS}}^2 + \sigma_{b_{SI}}^2)^{1/2} = 87.32 \text{ arc-sec}. \quad (38)
\]
D.2 The IMU Model

The sensor dynamics for the shuttle IMU was modeled approximately by the first order low pass filter as shown in Figure 10.

\[
\frac{1}{1 + \tau s} + \Sigma
\]

Figure 10. Simplified IMU Model

Where \( \tau \) is the time constant and \( v \) the white gaussian noise. The time constant was estimated from a DRIRU II Model,

\[
\tau = \frac{1}{\omega_s} = \frac{1}{14\pi} = 0.023 \text{ sec}
\]

(39)

That is, this measurement is assumed to have a 7 Hz bandwidth.

To determine \( v \), from Ref. 5, the 3σ IMU readout error is \( \pm 0.073^\circ / \text{axis} \).

Assume that the measurement noise is the readout error, then

\[
\sigma_v = 0.0243^\circ / \text{axis} = 87.6 \text{ arc-sec/axis}
\]

(40)

However, the actual performance of the shuttle IMU was much better; it has a 1σ gyro resolution error (\( \sigma_{\text{RESO}} \)) of 20 arc-sec/axis. If we assume the random noise has the same amplitude as that of the resolution noise, then

\[
\sigma_v = \sqrt{2} \sigma_{\text{RESO}}
\]

(41)

The corresponding measurement noise PSD, \( R \), is estimated as,

\[
R = \frac{1}{(14\pi)^2} \sigma_v^2 = 8.55 \times 10^{-10} \text{ (rad)}^2 \text{ -sec}
\]

(42)
E. SHUTTLE DYNAMICS AND INSTRUMENT POINTING ERRORS

E.1 The Equations of Motion

Consider Configuration A. For simplicity, the subscript P for principal axes is dropped from all equations. Again, let \( \phi, \theta, \) and \( \psi \) be the small roll, pitch, and yaw angles, respectively. Let \( \bar{\omega} = (\omega_x, \omega_y, \omega_z)^T \) be the angular velocity vector of the shuttle, then for small attitude errors,

\[
\begin{align*}
\dot{\omega}_x &= \dot{\phi} + \dot{\psi} \omega_z \\
\dot{\omega}_y &= \dot{\theta} + \omega_z \\
\dot{\omega}_z &= \dot{\psi} - \dot{\phi} \omega_z
\end{align*}
\]  

(43)

and

\[
\begin{align*}
\dot{\omega}_x &= \dot{\phi} + \dot{\psi} \omega_z \\
\dot{\omega}_y &= \dot{\theta} \\
\dot{\omega}_z &= \dot{\psi} - \dot{\phi} \omega_z
\end{align*}
\]  

(44)

With this simplified relation between the inertial rates and the attitude error rates, one can show that the equations of motion may be summarized as follows, accounting only for first order effects:

\[
\begin{bmatrix}
L_x \ddot{\phi} + (I_x - I_y + I_z) \omega_z \dot{\psi} + 4 (I_y - I_z) \omega_z^2 \dot{\phi} \\
L_y \dot{\theta} + 3 (I_x - I_z) \omega_z^2 \theta \\
L_z \ddot{\psi} - (I_x - I_y + I_z) \omega_z \dot{\phi} - (I_x - I_y) \omega_z^2 \psi
\end{bmatrix} = \bar{T}_d + \bar{T}_c
\]  

(45)

The left-hand side of Eq. (45) accounts for the body dynamics and the gravity gradient and the gyroscopic torques; and on the right-hand side of Eq. (45) \( \bar{T}_d \) and \( \bar{T}_c \) are the disturbance torques and the control torques, respectively.

E.2 The Space Shuttle State Estimator

The Shuttle State Estimator consists of two parallel Kalman-type filters, one for acceleration estimation and one for rate estimation [10]. The attitude estimate is the extrapolation of the measured attitude and the rate estimate may be approximated by using a second order filter with parameters.
determined by the filter gains. With the current baseline filter data, the equivalent damping coefficient and the corner frequency for the rate filter are .8 and .04 Hz (for vernier rate filter), respectively [11].

E.3 The System Pointing Errors

E.3.1. The Pitch Loop (Configuration A)

From Eq. (45) and ignoring the initial conditions, one has the following output function,

\[ G(s) = \frac{1}{I_Z s^2 + 3 \omega_o^2 (I_X - I_Z)} (T_{dY} + T_{cY}) \]  

(46)

Figure 11 shows the open-loop block diagram for the dynamics of the instrument pointing error excited by the random disturbances. Included in the diagram are the dynamics of the vehicle, the IMU, and the rate filter. Measurement error and the misalignment error are also included.

E.3.7. The Roll and Yaw Loops (Configuration A)

The output equations for the coupled roll and yaw dynamics are, from Eq. (45)

\[
\begin{bmatrix}
\phi(s) \\
\psi(s)
\end{bmatrix} = \frac{1}{D(s)} \begin{bmatrix}
(I_Z s^2 + \omega_o^2 (I_Y - I_X) & -\omega_o (I_X - I_y + I_z) s \\
\omega_o (I_X - I_y + I_z) s & I_X s^2 + 4 \omega_o^2 (I_Y - I_Z)
\end{bmatrix} \begin{bmatrix}
T_X \\
T_Z
\end{bmatrix}
\]  

(47)

where

\[
D(s) = I_X s^4 + \omega_o^2 (I_Y + I_z + 3 I_Y + 3 I_Z) s^2 + 4 \omega_o^4 (I_Y - I_X) (I_Y - I_Z)
\]  

(48)

and

\[
T_X = T_{cX} + T_{dX}
\]  

(49a)

\[
T_Z = T_{cZ} + T_{dZ}
\]  

(49b)

Figure 12 shows the block diagram for the roll and yaw instrument error dynamics. The block diagrams for the pitch and the roll-yaw loops are similar except for
the coupling terms for the roll and yaw axes.

\[ T_{dy} \]

\[ \begin{align*}
T_{dy} & = \frac{1}{L_y s^2 + 3 \omega_o^2 (I_x - I_z)} \\
T_{dy} & = \frac{1}{\tau s + 1} \\
T_{dy} & = \frac{S}{s^2 + 2 \tau_s + s + \omega_s^2}
\end{align*} \]

Figure 11. Pitch-Axis Block Diagram (Configuration A)

E.4 The Instrument Pointing Error PSD

E.4.1 The Pitch Error PSD (Configuration A)

The PSD's for the pitch-axis instrument pointing error and the rate error are,

\[ P_\theta^0(\omega) = F_\theta^0(\omega) F_\theta(\omega) Q_Y + R + \sigma_{bo}^2 \delta(\omega) \]  
(50)

\[ P_\theta^0(\omega) = (F_\theta^0(\omega) F_\theta(\omega) Q_Y + R) H_\theta^\ast(\omega) H_\theta^*(\omega) \]  
(51)

where \( Q_Y = Q_{PA, Y} \) and \( Q_{PA, Y} \) is the second component of Eq. (28), and where \( R \), and \( \sigma_{bo} \) are defined in preceding sections, and \( \delta(\omega) \) is the Dirac delta function; and where,

\[ F_\theta(\omega) = G_\theta(\omega) H(\omega) \]  
(52a)

\[ G_\theta(\omega) = \left[ \frac{1}{L_y s^2 + 3 \omega_o^2 (I_x - I_z)} \right]_{s=j\omega} \]  
(52b)
Figure 12. Roll and Yaw Axes Block Diagram. (Configuration A)
The output power spectral density for the system is closely related to the frequency response of the system, which characterizes the steady state dynamics of the system, and, hence, it is meaningful only if the system is stable. Unfortunately, in Eq. (52b), $I_x < I_z$ which implies that the system is unstable. The destabilizing term comes from the gravity gradient because the nadir pointing axis is not the axis of minimum inertia. To proceed, one has to consider the gravity gradient as external disturbance rather than a part of the dynamics. Figure 13 shows the instrument pointing error PSD in (rad)$^2$-sec as a function of frequency in Hz. Figure 14 shows the rate error PSD in (rad/sec)$^2$-sec as a function of frequency.

E.4.2. The Roll and Yaw Error PSD (Configuration A)

The instrument pointing error PSD's for the roll and yaw axes are,

$$P_\phi (\omega) = F_\phi^*(\omega) F_\phi (\omega) Q_x + F_\phi^*(\omega) F_\psi (\omega) Q_z + R + \sigma_{bo}^2 \delta(\omega)$$

$$P_\psi (\omega) = F_\psi^*(\omega) F_\psi (\omega) Q_x + F_\phi^*(\omega) F_\psi (\omega) Q_z + R + \sigma_{bo}^2 \delta(\omega)$$

and the instrument pointing rate error PSD's for the roll and yaw axes are,

$$P_\phi^* (\omega) = [F_\phi^*(\omega) F_\phi (\omega) Q_x + F_\phi^*(\omega) F_\psi (\omega) Q_z + R] H_\phi^* (\omega) H_\phi (\omega)$$

$$P_\psi^* (\omega) = [F_\psi^*(\omega) F_\psi (\omega) Q_x + F_\phi^*(\omega) F_\psi (\omega) Q_z + R] H_\psi^* (\omega) H_\psi (\omega)$$
\[ F'_{\psi}(\omega) = [F_{\psi}(\omega) F_{\psi}(\omega) Q_{x} + F_{\psi}(\omega) F_{\psi}(\omega) Q_{z} + R] H_{x}(\omega) H_{\psi}(\omega) \quad (54b) \]

where

\[ F_{\psi}(\omega) = G_{\psi}(\omega) H(\omega) \quad (55a) \]
\[ G_{\psi}(\omega) = \left[ \frac{I_{y} S^{2} + \omega_{0}^{2}(I_{y} - I_{x})}{D(S)} \right] S = j\omega \quad (55b) \]
\[ F_{\psi}(\omega) = G_{\psi}(\omega) H(\omega) \quad (56a) \]
\[ G_{\psi}(\omega) = \left[ \frac{\omega_{0}(I_{x} - I_{y} + I_{z})}{D(S)} \right] S = j\omega \quad (56b) \]
\[ F_{\psi}(\omega) = G_{\psi}(\omega) H(\omega) \quad (57a) \]
\[ G_{\psi}(\omega) = \left[ \frac{I_{x} S^{2} + 4\omega_{0}^{2}(I_{y} - I_{z})}{D(S)} \right] S = j\omega \quad (57b) \]

and \( Q_{x} \) and \( Q_{z} \) are the first and the third components of \( Q_{T,PA} \) in Eq. (28).

For the same reason discussed earlier, the roll and yaw dynamics are also unstable (referring to Eq. (48), \( D(s) \) has roots in the right-half complex plane). The PSD's for this case are obtained again by treating gravity gradient and gyroscopic torques as disturbances. Figures 13 and 14 show the roll and yaw power spectral densities.

E.4.3. The Error PSD's for Configuration B

The stability problem may be resolved by reorienting the shuttle from payload-bay nadir (Configuration A) to nose-down nadir (Configuration B). The advantage of this new configuration is that it is a gravity gradient stabilized system. However, there are drawbacks for this configuration. First, it will require a larger support-tower for the IS instrument, and the second drawback is that the aerodynamic forces and torques have increased significantly over the payload-bay nadir case.
Figure 13. Instrument Pointing Error PSD, (rad)^2/sec

Figure 14. Instrument Pointing Rate Error, (rad/sec)^2/sec
To obtain the error power spectral density for Configuration B, it is only necessary to replace \[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]
and the corresponding notations of the equations in this section by \[
\begin{bmatrix}
Y \\
Z \\
-\dot{X}
\end{bmatrix}_B.
\] That is, for instance, Eq. (52b) becomes

\[
G_\theta(\omega) = \frac{1}{I_Z S^2 + 3 \omega_0^2 (I_Y - I_X)} \text{S} \text{radian} \text{sec} \]

where \(G_\theta(\cdot)\) here still represents the pitch dynamics. Since \(I_Y \geq I_X\), Eq. (58) is stable.

The instrument pointing error PSD's are shown in Figure 15 and the rate error PSD's are illustrated in Figure 16.

The computer programs that are used for generating these results are included in Appendices C and D.

F. SUMMARY OF MAJOR FINDINGS

F.1 The Major Environmental Disturbances

The major disturbance sources that are modeled are the aerodynamic drag torques, the gravity gradient torques, and the gyroscopic torques. The unmodeled disturbances are the solar pressure torques, the on-board equipment vibrations, crew motions, and venting. Table 2 shows the static disturbance torques in N-m and the stochastic torque PSD's in (N-m)^2/sec for both Configurations A and B with a circular orbit of 400 km altitude.

F.2 The Measurement Uncertainties

The modeled measurement uncertainties are summarized in Table 3.

F.3 Ground Track Errors and Navigation Uncertainties

The power spectral densities for the ground track errors and the rate errors due to instrument pointing uncertainties for Configurations A and B are shown in Figure 17. It is important to note that strong resonances occur within
the frequency band of $10^{-5}$ Hz to $10^{-3}$ Hz. The ground track errors at higher frequencies, .01 Hz and above, are dominated by the measurement noise. Recall that the 1σ measurement noise is 28.28 arc-sec/axis. The corresponding ground track error is 54.84 m/axis or 77.56 m/lateral motion.

The 3σ ground track errors due to navigational uncertainties are about one order of magnitude greater than the attitude errors with the aid of TDRSS; and the error is greater with STDN as indicated in Table 4 [12]. Note that the values tabulated in Table 3 are the 3σ rms values.
Figure 15. Instrument Pointing Error PSD, \((\text{rad})^2\text{-sec}\)

Figure 16. Instrument Pointing Rate Error, \((\text{rad/sec})^2\text{-sec}\)
## Table 2. Modeled Environmental Disturbances

<table>
<thead>
<tr>
<th>Sources</th>
<th>Static, N-m</th>
<th>Stochastic -- PSD, (N-m)²·sec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X_p</td>
<td>Y_p</td>
</tr>
<tr>
<td>Aerodynamic Drag Drag Torques</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Config. A</td>
<td>0</td>
<td>-0.68x10⁻²</td>
</tr>
<tr>
<td>Config. B</td>
<td>-2.78x10⁻²</td>
<td>0</td>
</tr>
<tr>
<td>Gravity Gradient and Gyroscopic Torques</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

## Table 3. Modeled Measurement Uncertainties

<table>
<thead>
<tr>
<th>Sources</th>
<th>Performance (10)</th>
<th>Requirement (10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMU/Shuttle Misalignment</td>
<td>82 arc-sec</td>
<td>160 arc-sec</td>
</tr>
<tr>
<td>IMU Resolution</td>
<td>20 arc-sec</td>
<td>20 arc-sec</td>
</tr>
<tr>
<td>IMU Noise</td>
<td>(20) arc-sec</td>
<td>-</td>
</tr>
<tr>
<td>IMU Derived Rate</td>
<td>See filter</td>
<td>dynamic model</td>
</tr>
<tr>
<td>Rate Gyro</td>
<td>-</td>
<td>60 arc-sec/sec</td>
</tr>
<tr>
<td>Shuttle/IS Misalignment (10)</td>
<td>30 arc-sec</td>
<td></td>
</tr>
</tbody>
</table>
Figure 17. IS Ground Track Error and Rate Error PSD
(c) IS Ground Track Error PSD, m²·sec⁻¹

(d) IS Ground Track Rate Error PSD, (m/sec)²·sec⁻¹

Figure 17. (Continued)
Table 4. Expected On-Orbit Navigation Accuracies (30)

<table>
<thead>
<tr>
<th>CASE</th>
<th>NAVIGATION TRACKING SYSTEM</th>
<th>MINIMUM UNMODELED PERIGEE n.mi.</th>
<th>POSITION, FEET</th>
<th>VELOCITY, FPS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>RADIAL</td>
<td>DOWNTACK</td>
<td>CROSSTrack</td>
</tr>
<tr>
<td>1</td>
<td>STDN* NOMINAL</td>
<td>105</td>
<td>5,000</td>
<td>34,500</td>
</tr>
<tr>
<td></td>
<td>TDRSS**</td>
<td>105</td>
<td>3,600</td>
<td>18,500</td>
</tr>
<tr>
<td>2</td>
<td>STDN SMALL</td>
<td>105</td>
<td>3,000</td>
<td>22,000</td>
</tr>
<tr>
<td></td>
<td>TDRSS</td>
<td>150</td>
<td>1,200</td>
<td>5,500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>105</td>
<td>1,800</td>
<td>10,000</td>
</tr>
<tr>
<td>3</td>
<td>STDN LARGE</td>
<td>150</td>
<td>800</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td>TDRSS</td>
<td>105</td>
<td>8,000</td>
<td>80,000</td>
</tr>
</tbody>
</table>

NOTE:

The correlation between downtrack position and radial velocity is -0.95.

The correlation between radial position and downtrack velocity is -0.80.

* Spaceflight Tracking and Data Network

** Tracking Data Relay Satellite System
IV. PARAMETRIC ANALYSIS OF GEOMETRIC ERRORS

In section III, the steady state error dynamics of the Shuttle Imaging Spectrometer system has been analyzed with the major error sources and disturbance effects estimated. In this section, the emphasis is on the geometric error analysis. Geometric errors are consequences of the more direct errors including attitude and rate errors, ephemeris uncertainties, misalignment errors, earth rotation and curvature, etc. Figure 1 shows how the various errors propagate and how the geometric errors and the ground pattern distortions can be generated through dynamic analysis and simulation. The block that is considered in this section is II. Due to mounting and other practical considerations, only the payload-bay principal-axis nadir pointing configuration is considered (see Fig. 2).

The geometric error mapping functions due to the individual error sources as well as the aggregated errors are derived. The earth curvature effect is incorporated in all of the results. For the purpose of quick reference, the key mapping functions are tabulated in Appendix A. A list of the source code of the computer program that has been used for generating the geometric error characteristic curves is given in Appendix E.

A. COORDINATE CONFIGURATIONS

In Fig. 18, the coordinate frame $(X_p,Y_p,Z_p)$ on the Shuttle c.m. consists of the principal body axes. For the purpose of geometric analysis, another set of coordinates is used, i.e., the $(X,Y,Z)$ frame centered at the nadir point on the ground. This frame is the nadir projection of the orbital rotating frame centered at the Shuttle c.m. Specifically, the $X$-axis is in the direction of the projected motion on the ground, or the along-track direction, the $Z$-axis is in the nadir direction, and the $Y$-axis is in the cross-track direction, so that a right-hand coordinate system is formed.

For the purpose of this report, it is assumed that the Imaging Spectrometer is attached to the payload bay with its optical axis aligned with the body.
Figure 18. Payload-Bay Nadir Pointing Configuration and Coordinates
Z_p-axis. For normal IS operations, this is the desired configuration. However, for some IS experiments, such as those for assessing the atmospheric effects on the images, the IS instrument is required to point up to ±45° about the pitch axis (Y) from nadir (forward and backward looking) or up to ±20° about the roll axis (X) (side looking). For those cases, it is assumed that the instrument is gimbal mounted. However, since the analysis performed here is not primarily concerned with dynamics, no specific details are made at this time regarding mounting configurations.

B. GEOMETRIC ERRORS INDUCED BY EPHEMERIS UNCERTAINTIES

Ephemeris uncertainties include radial, along-track, and cross-track prediction errors. The geometric errors induced by ephemeris uncertainties are discussed in the following subsections.

B.1 Geometric Errors Due to Altitude Uncertainties

When the altitude of the shuttle varies, the ground point will shift in both the X and Y directions accordingly. This can be investigated in the following two ways.

B.1.1. With Fixed Viewing Angles

Referring to Figure 19, when the shuttle flies at nominal altitude h (position A), a ground point B with coordinates (Xo, Yo) corresponding to a view angle λ from the shuttle IS is located. After the shuttle elevates Δh to position A', point B' with coordinates (Xo', Yo') corresponding to the same view angle λ is located. The problem is to determine the shift of image due to the altitude change. Consider that when the altitude increases, the image of ground objects tends to move toward the nadir point, which causes reduction in image size and increase in field of view. Mathematically, the shift of image may be characterized by computing ΔX and ΔY, where

\[ \Delta X = Xo - Xo' \]
\[ \Delta Y = Yo - Yo' \]
Starting by assuming \((X_0, Y_0)\) known, the task is to determine \((X_0', Y_0')\) by first computing the view angle \(\lambda\). Figure 19b is generated by taking a side view from Fig. 19a in the direction perpendicular to Plane \(A'AO_{e}B'\). Apparently,

\[
\sqrt{X_0^2 + Y_0^2} = R \sin\theta
\]

---

**Legend:**

- \(B\) = A GROUND POINT CORRESPONDING TO VIEW ANGLE \(\lambda\) WHEN SHUTTLE IS AT NOMINAL ALTITUDE (POSITION A IN FIG. 4(b))
- \(B'\) = THE GROUND POINT CORRESPONDING TO THE SAME VIEW ANGLE AFTER THE SHUTTLE ELEVATED \(\Delta h\) (POSITION A' IN FIG. 4(b))
- \((X_0, Y_0)\) = COORDINATES OF POINT B
- \((X_0', Y_0')\) = COORDINATES OF POINT B'
- \(\Delta X, \Delta Y\) = X AND Y COMPONENTS OF THE POSITION CHANGE

(a) TOP VIEW (LOOKING TOWARD NADIR)

---

Figure 19: Ground Point Shift in X and Y Direction due to Altitude Error
LEGEND:

- \( h \) = NOMINAL ALTITUDE
- \( \Delta h \) = ALTITUDE ERROR
- \( \lambda \) = VIEW ANGLE
- \( A \) = SHUTTLE'S POSITION AT NOMINAL ALTITUDE
- \( A' \) = SHUTTLE'S POSITION AFTER BEING ELEVATED \( \Delta h \)

(b) SIDE VIEW

Figure 19. (Continued)
or

\[ \beta = \sin^{-1} \left( \frac{\sqrt{X_0^2 + Y_0^2}}{R} \right) \]  \hspace{2cm} (59)

and

\[ \alpha = 180^\circ - \lambda - \beta \]  \hspace{2cm} (60)

From triangle ABC, we have

\[ \frac{R}{\sin \lambda} = \frac{R + h}{\sin \alpha} \]  \hspace{2cm} (61)

Substituting Eqs. (59) and (60) into (61), it becomes

\[ \frac{R}{\sin \lambda} = \frac{R + h}{\sin \left[ 180^\circ - \lambda - \sin^{-1} \left( \frac{\sqrt{X_0^2 + Y_0^2}}{R} \right) \right]} \]

which leads to

\[ \tan \lambda = \frac{\sqrt{X_0^2 + Y_0^2}}{R + h - \sqrt{R^2 - X_0^2 - Y_0^2}} \]

hence,

\[ \lambda = \tan^{-1} \left( \frac{\sqrt{X_0^2 + Y_0^2}}{R + h - \sqrt{R^2 - X_0^2 - Y_0^2}} \right) \]  \hspace{2cm} (62)

To determine \( \sqrt{X_0'^2 + Y_0'^2} \), triangle A'B'C is used,

\[ \frac{R}{\sin \lambda} = \frac{R + h + \Delta h}{\sin \alpha'} \]

\[ \therefore \alpha' = \sin^{-1} \left( \frac{1 + \frac{h + \Delta h}{R}}{\left[ \left( \frac{1 + \frac{h + \Delta h}{R} \right) \sin \lambda \right]} \right) \]  \hspace{2cm} (63)

and

\[ \sqrt{X_0'^2 + Y_0'^2} = R \sin \beta' \]

\[ = R \sin (\lambda + \alpha') \]  \hspace{2cm} (64)
Substituting Eq. (63) into (64),

\[
\sqrt{X_0'^2 + Y_0'^2} = R \sin \left\{ \lambda + \sin^{-1} \left[ \left(1 + \frac{h + \Delta h}{R} \right) \sin \lambda \right] \right\} \tag{65}
\]

Expanding the right hand side of Eq. (65), and adopting the "-" sign for \(\cos \left\{ \sin^{-1} \left[ \left(1 + \frac{h + \Delta h}{R} \right) \sin \lambda \right] \right\}\), since \(90^\circ < \sin^{-1} \left[ \left(1 + \frac{h + \Delta h}{R} \right) \sin \lambda \right] < 180^\circ\), one has

\[
\sqrt{X_0'^2 + Y_0'^2} = R \left[ \left(1 + \frac{h + \Delta h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R} \right)^2 \sin^2 \lambda} \right] \sin \lambda \tag{66}
\]

Now, referring to Fig. 19(a), since \(B'\) is on the line \(OB\), the angle \(\rho\) is determined by

\[
\rho = \sin^{-1} \frac{X_o}{\sqrt{X_0^2 + Y_0^2}}
\]

\[
= \cos^{-1} \frac{Y_o}{\sqrt{X_0^2 + Y_0^2}} \tag{67}
\]

Hence, from Eqs. (66) and (67),

\[
X_0' = \sqrt{X_0'^2 + Y_0'^2} \sin \rho \]

\[
= R X_0 \sin \lambda \frac{\cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R} \right)^2 \sin^2 \lambda}}{\sqrt{X_0^2 + Y_0^2}} \tag{68}
\]

\[
Y_0' = \sqrt{X_0'^2 + Y_0'^2} \cos \rho \]

\[
= R Y_0 \sin \lambda \frac{\cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R} \right)^2 \sin^2 \lambda}}{\sqrt{X_0^2 + Y_0^2}} \tag{69}
\]

where \(\lambda\) is obtained by Eq. (62).

Hence, from Eqs. (68), (69), and (62) the values for \(\Delta X = X_0 - X_0'\) and \(\Delta Y = Y_0 - Y_0'\) can be obtained.
When point A' is below point A, the negative value for Δh should be used in the above equations.

However, if one starts with given view angle λ and ρ, the values of X₀, Y₀, X₀', and Y₀' can be readily obtained using Eqs. (68) and (69) directly (set Δh to 0 for X₀ and Y₀).

B.1.2 With Fixed Ground Points

Referring to Fig. 20, the view angle for the fixed ground point B will change as the shuttle altitude varies. Follow the same approach of paragraph B.1.1, the view angles Ω and Ω' can be found,

$$\Omega = \tan^{-1} \left( \frac{\sqrt{X_0^2 + Y_0^2}}{R + h - \sqrt{R^2 - X_0^2 - Y_0^2}} \right)$$  \hspace{1cm} (70)

and

$$\Omega' = \tan^{-1} \left( \frac{\sqrt{X_0^2 + Y_0^2}}{R + h + \Delta h - \sqrt{R^2 - X_0^2 - Y_0^2}} \right)$$  \hspace{1cm} (71)

Using Eqs. (70) and (71), the value of ΔΩ = Ω' - Ω can be obtained. Again if point A' is below point A, the negative value for Δh should be used in the above equations.

B.2. Geometric Errors Due to Intrack and Crosstrack Prediction Errors

The intrack (along X-direction or direction of orbit) and the crosstrack (along Y-direction or "z" orbit normal direction) prediction errors will cause the ground objects to shift along the X and Y directions, respectively, as shown in Fig. 21. To determine the overall geometric errors associated with ephemeris uncertainties, the intrack error ΔX* and the crosstrack error ΔY* can be incorporated in ΔX and ΔY found in Section B.1.1, respectively.
LEGEND:

\[ \Omega = \text{VIEW ANGLE FROM SHUTTLE TO POINT B WHEN SHUTTLE IS AT NOMINAL ALTITUDE } h \]

\[ \Omega' = \text{VIEW ANGLE FROM SHUTTLE TO THE SAME POINT B AFTER SHUTTLE ELEVATES } \Delta h \]

Figure 20. Change of View Angle of a Ground Point due to Altitude Error
Figure 21. Geometric Errors Induced by Intrack and Crosstrack Prediction Errors
C. GEOMETRIC ERRORS INDUCED BY ATTITUDE UNCERTAINTIES

Attitude uncertainties refer to the angular errors with respect to the nominal roll (\(\phi\)), pitch (\(\theta\)), and yaw (\(\psi\)) axes. The effects of these errors to the images of ground objects are studied in this subsection first on the individual error basis and then on the aggregated basis. For convenience, in the subsequent discussions, the term nominal flight condition will be used to signify the condition of the shuttle flying in a 400 km circular orbit with the attitude of payload-bay nadir pointing.

C.1 Geometric Errors Induced by Roll Error

Referring to Fig. 22, consider the nominal flight condition with \(\phi_0\) offset angle about the roll axis. The IS slit in this case is directed at the Y-axis on the ground. For a given view angle \(\lambda\), the image point (or pixel) on the ground is \(P_0\) with coordinates (0,\(Y^*\)). For the same roll offset and view angle the ground point \(P'_0\) is located after a roll error \(\phi\) is introduced. The coordinates of \(P'_0\) are (0,\(Y'^*\)). Equivalently, by rotating the optical instrument to the "left," the image recorded on the film appears to move to the "right." Therefore, the geometric error may be defined as the displacement of the object image after error is introduced relative to the image before the error is introduced. Mathematically, this is \(P_0 - P'_0 = \overline{P_0},_{\phi_0},\lambda - \overline{P_0},_{\phi_0},\phi,\lambda\). Since, in the case being considered, the image is on the Y-axis, one has

\[
\Delta X = 0
\]
\[
\Delta Y = Y^* - Y'^*
\]

where \(Y^*\) and \(Y'^*\) are,

\[
Y^* = -R \sin (\lambda + \phi_0) \left[ \left(1 + \frac{h}{R}\right) \cos (\lambda + \phi_0) - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 (\lambda + \phi_0)} \right]
\]

\(\Delta X = 0\) \hfill (72)
\(\Delta Y = Y^* - Y'^*\) \hfill (73)

where \(Y^*\) and \(Y'^*\) are,

\[
Y^* = -R \sin (\lambda + \phi_0) \left[ \left(1 + \frac{h}{R}\right) \cos (\lambda + \phi_0) - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 (\lambda + \phi_0)} \right]
\]

(74)
LEGEND:

- $\lambda$ = VIEW ANGLE
- $\phi_0$ = ROLL OFFSET ANGLE
- $\phi$ = ROLL ERROR
- $P_o$ = THE GROUND POINT CORRESPONDING TO VIEW ANGLE $\lambda$ AND ROLL OFFSET $\phi_0$
- $P^*$ = THE GROUND POINT CORRESPONDING TO THE SAME $\lambda$ AND $\phi_0$ AFTER ROLL ERROR $\phi$ IS INTRODUCED

Figure 22. Shift of Ground Point Induced by Roll Error
Y*′ = -R sin (λ + φ + θ) \left[ (1 + \frac{h}{R}) \cos (λ + φ + θ) \right. \\
- \left. \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 (λ + φ + θ) \right] \right] \quad (75)

C.2 Geometric Errors Induced by Pitch Error

Consider the case of pitch attitude error. Let \( \theta_o \) be the offset angle about the pitch axis (desired attitude), and \( \theta \) be the pitch error. Referring to Fig. 23, for a given view angle \( \lambda \), projecting the slit onto the XY-plane forms two line segments: \( BC \), corresponding to null pitch error \( (\theta = 0) \), and \( B'C' \), corresponding to arbitrary pitch error \( \theta \). Projecting along the line of view \( AC \) onto the ground, \( P(X*, Y*) \) is obtained; similarly, \( P'(X*', Y'^{*}) \) is obtained, corresponding to line of view \( AC' \). The geometric error here is defined the same way as that for the roll case, i.e., \( \frac{P}{P} = P(\theta_o + \theta, \lambda', \lambda) \), or

\[
\Delta X = X* - X*' \\
\Delta Y = Y* - Y*'
\]

(76)

(77)

The formulae for computing \( X*, Y*, X*' \), and \( Y*' \) are derived as follows. In Fig. 23, assume that \( h, \theta, \theta_o, \) and \( \lambda \) are known. The lengths of the line segments \( OB \) and \( OB' \) are

\[
OB = h \tan \theta_o \quad (78a)
\]

and

\[
OB' = h \tan (\theta_o + \theta) \quad (78b)
\]

and that of \( AB \) and \( AB' \) are

\[
AB = \frac{h}{\cos \theta_o} \quad (79a)
\]

\[
AB' = \frac{h}{\cos (\theta_o + \theta)} \quad (79b)
\]
LEGEND:

\( \lambda \) = VIEW ANGLE

\( \theta_0 \) = PITCH OFFSET ANGLE

\( \theta \) = PITCH ERROR

\( P \) = THE GROUND POINT CORRESPONDING TO VIEW ANGLE \( \lambda \) AND PITCH OFFSET \( \theta_0 \)

\( P' \) = THE GROUND POINT CORRESPONDING TO THE SAME \( \lambda \) AND \( \theta_0 \) AFTER PITCH ERROR \( \theta \) IS INTRODUCED

**Figure 23. Shift of Ground Point Induced by Pitch Error**
\( \overline{BC} \) and \( \overline{B'C'} \), with respect to \( Y \)-axis, can be obtained as

\[
\overline{BC} = -AB \tan \lambda = \frac{-h \tan \lambda}{\cos \theta_0} \tag{80a}
\]

and

\[
\overline{B'C'} = -AB' \tan \lambda = \frac{-h \tan \lambda}{\cos (\theta_0 + \theta)} \tag{80b}
\]

implies that the signs of \( \overline{BC} \) and \( \overline{B'C'} \) are opposite to that of \( \lambda \). \( l \) and \( l' \) are, referring to Fig. 23,

\[
l = \sqrt{OB^2 + BC^2} = h \sec \theta_0 \sqrt{\sin^2 \theta_0 + \tan^2 \lambda} \tag{81a}
\]

and

\[
l' = \sqrt{OB'^2 + B'C'^2} = h \sec (\theta_0 + \theta) \sqrt{\sin^2 (\theta_0 + \theta) + \tan^2 \lambda} \tag{81b}
\]

the angles \( \tau \) and \( \xi \) (see Fig. 23) can then be determined,

\[
\tau = \tan^{-1} \left( \frac{l}{h} \right) = \tan^{-1} \left( \sec \theta_0 \sqrt{\sin^2 \theta_0 + \tan^2 \lambda} \right) \tag{82a}
\]

and

\[
\xi = \tan^{-1} \left( \frac{l'}{h} \right) = \tan^{-1} \left[ \sec (\theta_0 + \theta) \sqrt{\sin^2 (\theta_0 + \theta) + \tan^2 \lambda} \right] \tag{82b}
\]

Finally,

\[
X^* = \sqrt{X^2 + Y^2} \frac{OB}{l} = R \left[ (1 + \frac{h}{R}) \cos \tau - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \tau} \right] \frac{\sin \tau \sin \theta_0}{\sqrt{\sin^2 \theta_0 + \tan^2 \lambda}} \tag{83a}
\]

\[
Y^* = \sqrt{X^2 + Y^2} \frac{BC}{l} = -R \left[ (1 + \frac{h}{R}) \cos \tau - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \tau} \right] \frac{\sin \tau \tan \lambda}{\sqrt{\sin^2 \theta_0 + \tan^2 \lambda}} \tag{83b}
\]
\[ X^{*'} = \sqrt{X^{*2} + Y^{*2}} \frac{OB'}{R} \]

\[ = R \left[ \left(1 + \frac{h}{R}\right) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi} \right] \frac{\sin \xi \sin (\theta_o + \theta)}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda}} \]

\[ Y^{*'} = \sqrt{X^{*2} + Y^{*2}} \frac{B'C'}{R} \]

\[ = -R \left[ \left(1 + \frac{h}{R}\right) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi} \right] \frac{\sin \xi \tan \lambda}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda}} \]

However, if one starts with \((X^*, Y^*)\) given, then \(\lambda\) and \(\theta_o\) would have to be computed. In that case,

\[ \lambda = -\tan^{-1} \left( \frac{Y^*}{\left[ R + h - \sqrt{R^2 - X^2 - Y^2} \right] \sqrt{1 + \left( \frac{X^*}{R + h - \sqrt{R^2 - X^2 - Y^2}} \right)^2}} \right) \]  

(84a)

and

\[ \theta_o = \tan^{-1} \left( \frac{X^*}{R + h - \sqrt{R^2 - X^2 - Y^2}} \right) \]  

(84b)

Once \(\lambda\) and \(\theta_o\) are determined, \(\tau\) and \(\xi\) can be computed using (82), and \(X^{*'}\) and \(Y^{*'}\) are determined using (83).

C.3 Geometric Errors Induced by Yaw Error

For a given yaw offset angle, \(\psi_o\), and the view angle, \(\lambda\), the ground point \(P(X^*, Y^*)\) is located. A new point \(P'(X^{*'}, Y^{*'})\) is found as yaw attitude error, \(\psi\), is introduced (see Fig. 24). The values for \(X^*, Y^*, X^{*'}\), and \(Y^{*'}\) can be computed as follows:

\[ X^* = \left[ \left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \sin \psi_o \]  

(85a)
LEGEND:

$\psi_0 =$ YAW OFFSET ANGLE

$\Psi =$ YAW ERROR

P = THE GROUND POINT CORRESPONDING TO VIEW ANGLE $\lambda$ AND YAW OFFSET $\psi_0$

$P' =$ THE GROUND POINT CORRESPONDING TO THE SAME $\lambda$ AND $\psi_0$ AFTER YAW ERROR $\Psi$ IS INTRODUCED

Figure 24. Shift of Ground Point Induced by Yaw Error
\[ y^* = - \left[ \left(1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \right] R \sin \lambda \cos \psi \]  
(85b)

\[ X^* = \left[ \left(1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \right] R \sin \lambda \sin \left(\psi_0 + \psi\right) \]
(86a)

and

\[ y^* = - \left[ \left(1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \right] R \sin \lambda \cos \left(\psi_0 + \psi\right) \]
(86b)

From Eqs. (85) and (86), we can obtain

\[ \Delta X = X^* - X^* \]

\[ \Delta Y = Y^* - Y^* \]

C.4 Geometric Errors Induced by Roll, Pitch, and Yaw Attitude Errors

C.4.1. The Shift of IS Line-of-Sight due to Attitude Error

Referring to Fig. 25, the shuttle is flying at an arbitrary attitude \((\phi, \theta, \psi)\). Consider the 3-2-1 sequence, i.e., the yaw, pitch, and roll rotation sequence. After yaw and pitch rotations, the Imaging Spectrometer line-of-sight will shift to \(AB\), where \(B(X_1, Y_1)\) is its intersection with the XY-plane. The \((x_1, y_1)\) coordinates are determined as follows:

\[ X_1 = X_o + h \tan \phi \cos \psi \]  
(87a)

\[ Y_1 = Y_o + h \tan \phi \sin \psi \]  
(87b)

Through a roll angle rotation, point \(B\) will move to \(C(X_2, Y_2)\) on the XY-plane, where

\[ X_2 = X_o + h \tan \phi \cos \psi + \frac{h}{\cos \theta \cos \phi} \tan \phi \sin \psi \]  
(88a)

\[ Y_2 = Y_o + h \tan \phi \sin \psi - \frac{h}{\cos \theta \cos \phi} \tan \phi \cos \psi \]  
(88b)
Figure 25. The Shift of Ground Projection of the IS Line-of-Sight due to Attitude Error
The interest here is to determine the coordinates of O*(X,Y) corresponding to the final line-of-sight projected onto the ground. By taking a view normal to the ACO-plane, Figure 26 is obtained. The quantities ε, λ, and ρ are obtained as follows,

\[ ε = \tan^{-1}\left(\frac{\sqrt{x_2^2 + y_2^2}}{h}\right) \quad (89a) \]

\[ λ = R \sin ε \left[ (1 + \frac{h}{R}) \cos ε - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 ε} \right] \quad (89b) \]

and

\[ ρ = \sin^{-1}\left(\frac{y_2}{\sqrt{x_2^2 + y_2^2}}\right) = \cos^{-1}\left(\frac{x_2}{\sqrt{x_2^2 + y_2^2}}\right) \quad (89c) \]

Hence, the coordinates of X and Y are,

\[ X = \lambda \cos ρ = R \sin ε \left[ (1 + \frac{h}{R}) \cos ε - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 ε} \right] \frac{x_2}{\sqrt{x_2^2 + y_2^2}} \quad (90a) \]

\[ Y = \lambda \sin ρ = R \sin ε \left[ (1 + \frac{h}{R}) \cos ε - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 ε} \right] \frac{y_2}{\sqrt{x_2^2 + y_2^2}} \quad (90b) \]

where \(x_2\) and \(y_2\) are given in Eq. (88).

Note that this derivation is general enough so that the formulae are valid if one replaces the attitude errors with errors plus offsets, i.e., replacing \(ψ, θ, \) and \(ψ\) by \(ψ + θ, \) \(θ + θ, \) and \(ψ + θ, \) respectively.
Figure 26. Side View of Figure 25
C.4.2. The Shift of Ground Image Due to Attitude Error

For simplicity, $\phi$, $\theta$, and $\psi$ are used to represent the attitude angles as sums of attitude offsets and attitude errors. To fix the derivation, the yaw ($\psi$), pitch ($\theta$), and roll ($\phi$) Euler sequence is assumed.

In this subsection, the results of subsection C.4.1 are extended to cover the entire field of view of the IS slit. First consider the case of a nominal flight with zero attitude offset. Referring to Fig. 27, let $O$ be the nadir point which is the origin of the XY-Frame and the X'Y'-Frame. The X'Y'-Frame is formed by rotating the XY-Frame through an angle $\psi$ about the yaw axis. Let $P$ be a point corresponding to a view angle $\lambda$ (negative value shown in Fig. 27) on the Y-axis before any rotation occurs. After a $\psi$ rotation, for the same view angle, the IS is sighting point $P_1$. Similarly, after a pitch ($\theta$) and then roll ($\phi$) rotation, the points $P_2$ and then $P_3$ are sighted, respectively.

The coordinates of $P_2$ in the X'Y'-Frame are, from the results of subsections C.1 and C.2 and Fig. 23,

\[
X'_2 = \frac{R \sin \theta \sin \xi_2 \left[ \left( 1 + \frac{h}{R} \right) \cos \xi_2 - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi_2} \right]}{\sqrt{\sin^2 \theta + \tan^2 \lambda}} \tag{91a}
\]

\[
Y'_2 = -\frac{R \tan \theta \sin \xi_2 \left[ \left( 1 + \frac{h}{R} \right) \cos \xi_2 - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi_2} \right]}{\sqrt{\sin^2 \theta + \tan^2 \lambda}} \tag{91b}
\]

where the angle $\xi_2$ is the angle $\xi$ defined in Fig. 23 corresponding to point $P_2$ here, and

\[
\xi_2 = \tan^{-1} \left( \frac{\sec \theta \sqrt{\sin^2 \theta + \tan^2 \lambda}}{\sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi_2}} \right) \tag{91c}
\]
Figure 27. Shift of Ground Point Induced by Attitude Errors.
The coordinates of \( P_3 \) in the \( X'Y' \)-Frame are

\[
X'_3 = \frac{R \sin \theta \sin \xi_3 \left( 1 + \frac{h}{R} \right) \cos \xi_3 - \sqrt{1 - \left( \frac{h}{R} \right)^2 \sin^2 \xi_3}}{\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}
\]

\[
Y'_3 = -\frac{R \tan (\lambda + \phi) \sin \xi_3 \left( 1 + \frac{h}{R} \right) \cos \xi_3 - \sqrt{1 - \left( \frac{h}{R} \right)^2 \sin^2 \xi_3}}{\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}
\]

where \( \xi_3 \) is similarly defined as \( \xi_2 \) and

\[
\xi_3 = \tan^{-1} \left( \sec \theta \sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)} \right)
\]

or, finally, in the \( XY \)-Frame

\[
X_3 = X'_3 \cos \psi - Y'_3 \sin \psi
\]

\[
Y_3 = X'_3 \sin \psi + Y'_3 \cos \psi
\]

where \( X'_3 \) and \( Y'_3 \) are given in (92).

The geometric error caused by the attitude errors is therefore,

\[
\Delta X = X - X_3
\]

\[
\Delta Y = Y - Y_3
\]

where

\[
X = 0
\]

\[
Y = -R \sin \lambda \left( 1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left( \frac{h}{R} \right)^2 \sin^2 \lambda}
\]

Now, consider the case of the shuttle (or the IS) flying at a nominal attitude \((\phi_0, \theta_0, \psi_0)\) with attitude uncertainties of \((\dot{\phi}, \dot{\theta}, \dot{\psi})\). Let \( P_0(X_0, Y_0) \) be a ground point that is sighted by the IS with a view angle \( \lambda \) and attitude \((\phi_0, \theta_0, \psi_0)\).

After \((\dot{\phi}, \dot{\theta}, \dot{\psi})\) attitude rotations from the nominal, the ground point \( P_3(X_3, Y_3) \) is sighted with the same view angle. In this case, the geometric error is
\[ \Delta X = X_0 - X_3 \]  
\[ \Delta Y = Y_0 - Y_3 \]  

where

\[ X_0 = X'_0 \cos \psi_0 - Y'_0 \sin \psi_0 \]  
\[ Y_0 = X'_0 \sin \psi_0 + Y'_0 \cos \psi_0 \]  
\[ X_3 = X'_3 \cos (\psi_0 + \phi) - Y'_3 \sin (\psi_0 + \phi) \]  
\[ Y_3 = X'_3 \sin (\psi_0 + \phi) + Y'_3 \cos (\psi_0 + \phi) \]

and

\[ X'_0 = \frac{R \sin \theta_o \sin \xi_o \left[ (1 + \frac{h}{R}) \cos \xi_o - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_o} \right]}{\sqrt{\sin^2 \theta_o + \tan^2 (\lambda + \phi_o)}} \]  
\[ Y'_0 = \frac{R \tan (\lambda + \phi_o) \sin \xi_o \left[ (1 + \frac{h}{R}) \cos \xi_o - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_o} \right]}{\sqrt{\sin^2 \xi_o + \tan^2 (\lambda + \phi_o)}} \]

where

\[ \xi_o = \tan^{-1} \left( \frac{\sec \theta_o \sin^2 \theta_o + \tan^2 (\lambda + \phi_o)}{\sec \xi_o \sin^2 \xi_o + \tan^2 (\lambda + \phi_o)} \right) \]  

and

\[ X'_3 = \frac{R \sin (\theta_3 + \theta) \sin \xi_3 \left[ (1 + \frac{h}{R}) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 (\theta_3 + \theta) + \tan^2 (\lambda + \phi_3 + \phi)}} \]  
\[ Y'_3 = \frac{R \tan (\lambda + \phi_3 + \phi) \sin \xi_3 \left[ (1 + \frac{h}{R}) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 (\theta_3 + \theta) + \tan^2 (\lambda + \phi_3 + \phi)}} \]

and where

\[ \xi_3 = \tan^{-1} \left[ \frac{\sec (\theta_3 + \theta) \sin^2 (\theta_3 + \theta) + \tan^2 (\lambda + \phi_3 + \phi)}{\sec \xi_3 \sin^2 \xi_3 + \tan^2 (\lambda + \phi_3 + \phi)} \right] \]
To include errors caused by altitude uncertainties, one can replace
$h$ by $h_o$ in Eq. (97), and $h$ by $h_o + \Delta h$ in Eq. (98), where $h_o$ is the nominal or
estimated altitude and $h_o + \Delta h$ is the actual altitude.

D. GEOMETRIC ERRORS INDUCED BY ATTITUDE RATE ERRORS

Attitude rates cause attitude angle changes which in turn cause geometric
errors. To account for attitude rates, one integrates the rates to obtain the
attitude angles, and then uses the formulae derived to obtain the corresponding
time-varying geometric errors. The instantaneous attitude angles are:

\[ \phi' = \phi_o + \int_{t_0}^{t} \dot{\phi} \, dt \quad (99a) \]
\[ \theta' = \theta_o + \int_{t_0}^{t} \dot{\theta} \, dt \quad (99b) \]
\[ \psi' = \psi_o + \int_{t_0}^{t} \dot{\psi} \, dt \quad (99c) \]

E. GEOMETRIC ERRORS INDUCED BY MISALIGNMENT ERRORS

The misalignment errors can be incorporated in the attitude errors. Let
$(\gamma_b, \delta_b, \zeta_b)$ be the attitude bias or misalignment, then the effective IS attitude
angles will be

\[ \gamma' = \gamma_o + \gamma_b + \gamma \quad (100a) \]
\[ \delta' = \delta_o + \delta_b + \delta \quad (100b) \]
\[ \zeta' = \zeta_o + \zeta_b + \zeta \quad (100c) \]
The corresponding geometric errors can be determined using the formulae derived in subsection C.

F. GEOMETRIC ERRORS INDUCED BY EARTH ROTATION

The effect of earth rotation on images varies with the position of the Shuttle relative to the earth and the Shuttle orbital elements. In general, ground images become skewed as a result of earth rotation.

Referring to Fig. 28. Let \( \theta \) be the nadir point at which the latitude is \( \zeta \).

Let \( \omega_e \) be the spin rate of the earth, where \( \omega_e = \frac{2\pi}{86400} = 7.2722 \times 10^{-5} \text{ rad/sec.} \)

Let \( V_e \) be the linear velocity of the earth at \( \theta \),

\[
V_e = \omega_e R \cos \zeta
\]  

(101)

Let \( \Gamma \) be the orbital inclination at the equator, and let \( \Lambda \) be the orbital inclination to the local meridian at \( \theta \). From Fig. 29 and Appendix B, one has the following spherical geometrical relation,

\[
\cos \Gamma = \cos \zeta \sin \Lambda
\]

or

\[
\sin \Lambda = \frac{\cos \Gamma}{\cos \zeta}
\]  

(102a)

and

\[
\cos \Lambda = \sqrt{1 - \sin^2 \Lambda} = \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \zeta}\right)^2}
\]  

(102b)

From Fig. 29 again, \( \Lambda \) is also the angle between the linear velocity vector \( V_e \) and the \( Y \)-axis. Hence, velocity components \( V_{eX} \) and \( V_{eY} \) are, using (102),

\[
V_{eX} = V_e \sin \Lambda = \frac{V_e \cos \Gamma}{\cos \zeta}
\]  

(103a)

and

\[
V_{eY} = V_e \cos \Lambda = V_e \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \zeta}\right)^2}
\]  

(103b)
LEGEND:
\[ \omega_e \] = Angular Velocity of Earth spin
\[ \varsigma \] = Latitude at point 0
\[ V_e \] = Linear Velocity of the Earth at point 0

Figure 28. Linear Velocity of the Earth at the Nadir Point
(a) INTRACK AND CROSSTrack COMPONENTS OF EARTH LINEAR VELOCITY VECTOR AT THE NADIR POINT

(b) ORBIT INCLINATION TO THE LOCAL MERIDIAN ($\Delta$), degrees

<table>
<thead>
<tr>
<th>ORBITAL INCLINATION (degrees)</th>
<th>NORTH LATITUDE, degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 20 40 60 80</td>
</tr>
<tr>
<td>18.5</td>
<td>90 - - - -</td>
</tr>
<tr>
<td>40</td>
<td>71.5 - - - -</td>
</tr>
<tr>
<td>60</td>
<td>50 54.61 90 - - - -</td>
</tr>
<tr>
<td>85</td>
<td>30 32.15 40.75 90 - - -</td>
</tr>
<tr>
<td>90</td>
<td>5 5.32 6.53 10.04 30.13</td>
</tr>
</tbody>
</table>

Figure 29. Earth Spherical Geometry
During the time interval from $t_1$ to $t_2$, the ground point has moved from $P(X, Y)$ to $P'(X', Y')$, the corresponding position changes are,

$$
\Delta X = \int_{t_1}^{t_2} V_e \cos \zeta \, dt = \int_{t_1}^{t_2} \frac{V_e \cos \Gamma}{\cos \zeta} \, dt \tag{104a}
$$

$$
\Delta Y = \int_{t_1}^{t_2} V_e \sin \zeta \, dt = \int_{t_1}^{t_2} \frac{V_e}{\sqrt{1 - \left(\frac{\cos \Gamma}{\cos \zeta}\right)^2}} \, dt \tag{104b}
$$

Note that for small time intervals, $\zeta$ may be considered constant and (104) may be approximated as

$$
\Delta X = \frac{\cos \Gamma}{\cos \zeta} V_e (t_2 - t_1) \tag{105a}
$$

$$
\Delta Y = \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \zeta}\right)^2} V_e (t_2 - t_1) \tag{105b}
$$

and

$$
X' = X + \Delta X \tag{106a}
$$

$$
Y' = Y + \Delta Y \tag{106b}
$$

G. NUMERICAL RESULTS: GEOMETRIC ERRORS AND ERROR SENSITIVITIES

Numerical results of error sensitivities and geometric errors due to various direct errors have been generated. These data are summarized in Tables 5-10 and discussed in the following subsections.

G.1 Geometric Errors Due to Altitude Uncertainties

Table 5 shows the geometric error for a 1 km altitude change and error sensitivity caused by altitude uncertainty. The nominal altitude of 400 km was used to generate these data. This result is also plotted in Fig. 30 as a function of view angles for altitude changes ranging from 1 km to 4 km. As indicated in this figure, the geometric errors are quite linear with the view angles and the altitude errors for the range shown.
For the Imaging Spectrometer illustrated in Fig. 4, the view angles are limited by the field of view of \( \pm 0.825^\circ \). Using the various expected on-orbit navigation accuracies shown in Table 4, and the sensitivity data of Table 5, the corresponding geometric error can be determined. For instance, from Table 4, the 3\( \sigma \) altitude uncertainty, with TDRSS in the tracking system for the 150 nmi orbit, if the small unmodeled perturbation is used, is 800 feet (or \(~244\) m) and with the STDN system and large unmodeled perturbation and 105 nmi orbit, the 3\( \sigma \) altitude uncertainty is 8000 feet (or \(~2440\) m). From Table 5, the corresponding 3\( \sigma \) geometric errors for the largest view angle \( (.825^\circ) \) are 3.51 m and 35.1 m, respectively. Note that the nominal pixel size is 30 m.

G.2 Geometric Errors Due to Roll Uncertainties

The geometric errors for view angles ranging from \(-9^\circ\) to \(+9^\circ\) due to a roll error of \(1^\circ\) are tabulated in Table 6. The error sensitivity at the nominal attitude is also tabulated in Table 6. By comparing the errors for \(1^\circ\) and the sensitivity for the same view angle, it is found that the differences are \(0.2\%\) or less. That is, for small angles, the error sensitivity data can be used to compute errors. Fig. 31 shows the plots of geometric errors for roll errors of up to \(5^\circ\). Unlike the errors corresponding to altitude uncertainties, these curves are relatively flat. The geometric error is about 7 km per \(1^\circ\) of roll error.

It is noted from Fig. 31 that for small roll errors, the geometric errors are nearly symmetrical with the view angles. For larger roll angles, the geometric errors are greater for view angles that have the same sign of \(\phi\). This is more visible from Fig. 32 which shows the plots of geometric errors corresponding to roll errors of \(1^\circ\) and \(2^\circ\) from a roll offset of \(20^\circ\).
Geometric Errors Due to Pitch Uncertainties

Table 7 shows the error sensitivity and the geometric errors corresponding to the 1° pitch error for the view angles of up to +9°. The geometric errors caused by pitch errors are about the same in amplitude as those caused by roll errors. Fig. 33 shows that pitch curves are rather flat, unlike the corresponding roll curves. Figs. 34 and 35 show the 1° and 2° pitch uncertainty induced geometric error curves with 22.5° and 45° pitch offsets, respectively. These curves, especially those in Fig. 35, are more similar to the roll curves in shape.

Geometric Errors Due to Yaw Uncertainties

Geometric errors due to rotations about the yaw or the local vertical axis (assuming zero offsets) are tabulated in Table 8 for 1° and plotted in Fig. 36 for up to 5°. Linearity applies for both the view angles and the yaw angles for the ranges covered here. Unlike roll and pitch errors, yaw errors induce much smaller geometric errors.

Geometric Errors Due to Attitude Uncertainties

Table 9 shows the geometric errors due to 1° error in each of the roll, pitch, and yaw axes. The error magnitude for a given view angle is approximately the vector sum of the roll and pitch induced errors.

Fig. 37 shows the plots of geometric errors caused by up to 5° roll, pitch, and yaw errors. Fig. 38 shows two curves, one corresponding to 1° pitch error and the other, 1° yaw error, with the roll offset of 20°.

Comparing pitch curve with that of Fig. 33, the 20° roll offset has no noticeable effects on the geometric errors. On the other hand, the effects on the yaw curve are quite pronounced, as a large bias of approximately 2.5 km results due to the 20° roll.
Fig. 39 shows the $1^\circ$ roll and $1^\circ$ yaw curves for a pitch offset of $22.5^\circ$. Fig. 40 shows the same curves for $45^\circ$ pitch offset.

With the large angular pitch offsets, the yaw curves appear quite different from those without pitch offset, as noted in Figs. 36, 39, and 40. With these offsets, yaw-induced error curves look quite similar to those induced by roll errors in shape. By offsetting the pitch angle to $45^\circ$, the yaw-induced geometric errors increase to about sevenfold from zero pitch offset, and more than twofold over the $22.5^\circ$ pitch offset.

The roll-induced geometric errors were less drastically affected by pitch angular offsets. For instance, for $22.5^\circ$ pitch offset, the error has increased about 7% from that with zero offset, and about 46% for $45^\circ$ pitch offset.

G. 6 Ground Point Shift Due to Shuttle Motion and Earth Rotation

The earth rotation effect on the shift of the ground image varies with the latitude of the shuttle, and the combined effects of the earth rotation and shuttle motion vary with the orbital inclination in addition to the latitude. Table 10 shows the tabulation with latitude ranging from $0^\circ$ to $80^\circ$. The largest earth rotation effect occurs at the equator, $0.46$ km in one second, and diminishes to $0.08$ km in one second at the latitude of $80^\circ$ as shown in Fig. 41.

The combined effects are dominated by the motion of the shuttle since at $400$ km altitude the projected ground speed is much greater than the earth rotation speed. In a one-second period, the image will shift more than $7$ km for any latitude.
Table 5. Geometric Errors and Error Sensitivity Due to Altitude Uncertainties

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Error Induced by 1 km Altitude Error, km</td>
<td>0.01746</td>
<td>0.03492</td>
<td>0.05242</td>
<td>0.06995</td>
<td>0.08753</td>
<td>0.1052</td>
<td>0.1229</td>
<td>0.1407</td>
<td>0.1586</td>
<td></td>
</tr>
<tr>
<td>Geometric Error Sensitivity, km/km</td>
<td>0.01746</td>
<td>0.03492</td>
<td>0.05242</td>
<td>0.06995</td>
<td>0.08753</td>
<td>0.1052</td>
<td>0.1229</td>
<td>0.1407</td>
<td>0.1586</td>
<td></td>
</tr>
</tbody>
</table>
Figure 30. Geometric Error due to Altitude Uncertainties ($\Delta h$)
Table 6. Geometric Errors and Error Sensitivity
Due to Roll Attitude Error

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>-9</th>
<th>-8</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Error Induced by +1° Roll Error, km</td>
<td>6.987</td>
<td>6.996</td>
<td>7.010</td>
<td>7.029</td>
<td>7.052</td>
<td>7.081</td>
<td>7.114</td>
<td>7.153</td>
<td>7.196</td>
</tr>
</tbody>
</table>
Figure 31. Geometric Error due to Roll Attitude Error (\( \phi \))
Figure 32. Geometric Error due to Roll Uncertainty ($\phi$) About 20° Roll Offset
Table 7. Geometric Errors and Error Sensitivity Due to Pitch Attitude Error

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>-9</th>
<th>-8</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
</table>
Figure 33. Geometric Error due to Pitch Attitude Error ($\theta$)
Figure 34. Geometric Error due to Pitch Uncertainty ($\theta$) About 22.5° Pitch Offset
Figure 35. Geometric Error due to Pitch Uncertainty (θ) About 45° Pitch Offset
Table 8. Geometric Errors and Error Sensitivity Due to Yaw Attitude Error

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Error Induced by 1 Yaw Error, km</td>
<td>0.1219</td>
<td>0.2438</td>
<td>0.3659</td>
<td>0.4883</td>
<td>0.6109</td>
<td>0.7340</td>
<td>0.8576</td>
<td>0.9818</td>
<td>1.107</td>
<td></td>
</tr>
<tr>
<td>Geometric Error Sensitivity, km/deg.</td>
<td>0.1219</td>
<td>0.2438</td>
<td>0.3659</td>
<td>0.4883</td>
<td>0.6109</td>
<td>0.7340</td>
<td>0.8576</td>
<td>0.9818</td>
<td>1.107</td>
<td></td>
</tr>
</tbody>
</table>
Figure 36. Geometric Error due to Yaw Attitude Error (ψ)
Table 9. Geometric Errors Induced by Yaw, Pitch, and Roll Errors

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km
- Yaw Error = +1°
- Pitch Error = +1°
- Roll Error = +1°

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>-9</th>
<th>-8</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>View Angle, deg.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error, km</td>
<td>9.966</td>
<td>10.06</td>
<td>10.16</td>
<td>10.27</td>
<td>10.37</td>
<td>10.48</td>
<td>10.60</td>
<td>10.72</td>
<td>10.85</td>
</tr>
</tbody>
</table>
Figure 37. Geometric Error due to Roll ($\phi$), Pitch ($\theta$), and Yaw ($\psi$) Errors
Figure 38. Geometric Error due to Pitch ($\theta$) and Yaw ($\psi$) Uncertainties About 20° Roll Offset
Figure 39. Geometric Error due to Roll ($\phi$) and Yaw ($\psi$) Uncertainties About 22.5° Pitch Offset
Figure 40. Geometric Error due to Roll (φ) and Yaw (ψ) Uncertainties About 45° Pitch Offset
Table 10. Ground Point Shift Induced by Shuttle Motion and Earth Rotation

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Orbit Inclination = 85°
- Nominal Shuttle Ground Speed = 7.202 km/sec

<table>
<thead>
<tr>
<th>Latitude, deg.</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground Point Shift Velocity due to Earth Rotation, km/sec</td>
<td>0.4623</td>
<td>0.4344</td>
<td>0.3541</td>
<td>0.2311</td>
<td>0.08027</td>
</tr>
<tr>
<td>Ground Point Shift Velocity Relative to Nadir Point, km/sec</td>
<td>7.177</td>
<td>7.175</td>
<td>7.171</td>
<td>7.166</td>
<td>7.162</td>
</tr>
<tr>
<td>Angle of Velocity Vector with Y-axis, deg.</td>
<td>-86.32</td>
<td>-86.54</td>
<td>-87.19</td>
<td>-88.18</td>
<td>-89.44</td>
</tr>
<tr>
<td>Shift in 0.1 sec, km</td>
<td>0.7177</td>
<td>0.7175</td>
<td>0.7171</td>
<td>0.7166</td>
<td>0.7162</td>
</tr>
</tbody>
</table>
Figure 41. Geometric Error Caused by Earth Rotation for the Time Period of 1 Second
H. SUMMARY OF MAJOR FINDINGS

In this section geometric errors caused by the "direct error sources" are analyzed. Equations that map the direct errors onto the geometric errors are derived in Subsections B through G. Extensive illustrations of geometry are used to assist the derivations. Direct error effects included here are those of pointing errors and angular rates, ephemeris prediction errors including altitude, intrack, and crosstrack uncertainties, earth rotation and curvature, and shuttle and instrument misalignment. For quick reference the key error mapping functions are listed in Appendix A. A program list is given in Appendix E.

The numerical results for both the geometric errors and error sensitivities are tabulated in Tables 5 through 10 and also plotted in Figs. 30 through 41. To illustrate the effects of view angles and linearity, an extended range of \( \pm 9^\circ \) is used. For the push broom imaging spectrometer studied here, the field of view is limited by \( \pm 8.25^\circ \) (see Fig. 4). The following are the specific findings resulting from further analysis:

1. The effects of earth curvature are very small for the applications here. For instance, the linear displacement of a 10 km arc is 9.9999... km. Fig. 42 illustrates several cases of earth curvature effects.

\[ \theta = 10 \text{ km} \quad \theta = 413.752 \text{... km} \]
\[ a = 9.99999 \text{... km} \quad c = 413.460 \text{... km} \]
\[ b = 9.99994 \text{... km} \quad a = 400 \text{ km} \]

Figure 42. Effects of Earth Curvature
2. Altitude uncertainties cause only moderate geometric errors. Table 11 shows that the worst geometric error is 11.71 m and the corresponding rate is .133 m/sec. The worst case associates with the STDN system and large unmodeled perturbation and 200 km orbit. The least errors are .22 m and .0032 m/sec corresponding to the TDRSS system and small unmodeled perturbation and 300 km orbit. The effects of other navigation uncertainties such as downtrack and crosstrack position errors are not small, however. Table 12 shows the \(\sigma\) uncertainties in feet. Downtrack and crosstrack position errors are mapped directly to the geometric errors.

3. Geometric errors caused by altitude uncertainties are, within the range of interest here, proportional to the view angle and altitude error (see Table 5 and Fig. 30).

4. The effects of roll and pitch attitude errors are relatively large compared with those of altitude errors and yaw attitude errors. Table 13 shows that, for nominal flight conditions, the error sensitivity is approximately 1.94 m/arc sec, or \(\approx 7000\) m/degree. The yaw sensitivity is very small, from 0 for 0° view angle to .0279 m/arc sec for maximum IS view angle ±825°.

The effects of attitude errors increase significantly for large attitude offsets. For instance, for 20° roll offset (side looking), the roll sensitivity increases to 2.28 m/arc sec from 1.94 and yaw sensitivity increases to .75 m/arc sec from .0279. The pitch sensitivity, in this case, is nearly unaffected. By offsetting the pitch angle to 45° (forward looking), the roll sensitivity increases to 2.83 m/arc sec, the pitch to 4.36 m/arc sec, and the yaw to 1.97 m/arc sec. Table 13 shows the error sensitivities and 1° geometric errors for many combinations of interest.
5. Without attitude offsets and within the IS field of view, the geometric errors are almost proportional to roll errors (Fig. 31), pitch errors (Fig. 33), and yaw errors (Fig. 36), and are independent of the view angle except for the yaw cases (for which the error is proportional to the view angle). For the cases of large roll or pitch angular offset, most of the properties change only slightly (see Figs. 32, 34, 35, 38, 39, and 40).

6. The performance of the Imaging Spectrometer is limited by the shuttle Inertial Measuring Unit accuracy, the shuttle reference misalignment, and the misalignment between the shuttle and the Imaging Spectrometer, unless means of error reduction, such as using ground control points and a precision point mount between the shuttle and the IS instruments, are employed.

On top of these uncertainties, the shuttle deadband is another source of error that can cause gross geometric errors. This problem, of course, can be resolved by using a precision point mount.

Table 14 shows geometric errors caused by the IMU/shuttle misalignment, IMU resolution, IMU noise, rate gyro drift, and combined shuttle/IS misalignment, for nominal attitude and attitude offsets. Single axis geometric errors corresponding to the combined shuttle/IS misalignment are 169 m, 198 m, and 379 m, respectively, for nominal, 20° roll offset, and 45° pitch offset, for instance. Other cases are shown in detail in Table 14.

7. Earth rotation causes images to shift toward the direction of rotation. The magnitude of this shift depends on the latitude of the object (see Fig. 41). For instance, at the equator, the object moves approximately 462 m in 1 second, while at 60° latitude, it moves only 231 m in one second.
Table 11. Geometric Errors Due to Expected On-orbit Navigation Uncertainty

<table>
<thead>
<tr>
<th>Navigation Tracking System</th>
<th>Unmodeled Perturbation</th>
<th>Minimum Perigee, km</th>
<th>1σ Altitude Error, m</th>
<th>Max. Geom. Error in FOV, m</th>
<th>1σ Radial Velocity, m/s</th>
<th>Max. Geom. Error in FOV, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>STDN</td>
<td>Nominal</td>
<td>200</td>
<td>508</td>
<td>7.32</td>
<td>3.87</td>
<td>.056</td>
</tr>
<tr>
<td>TDRSS</td>
<td></td>
<td>200</td>
<td>366</td>
<td>5.27</td>
<td>2.25</td>
<td>.032</td>
</tr>
<tr>
<td>STDN</td>
<td>Small</td>
<td>200</td>
<td>305</td>
<td>4.39</td>
<td>2.50</td>
<td>.036</td>
</tr>
<tr>
<td></td>
<td></td>
<td>300</td>
<td>122</td>
<td>1.76</td>
<td>.59</td>
<td>.0085</td>
</tr>
<tr>
<td>TDRSS</td>
<td></td>
<td>200</td>
<td>183</td>
<td>2.63</td>
<td>1.12</td>
<td>.016</td>
</tr>
<tr>
<td></td>
<td></td>
<td>300</td>
<td>81</td>
<td>1.17</td>
<td>.22</td>
<td>.0032</td>
</tr>
<tr>
<td>STDN</td>
<td>Large</td>
<td>200</td>
<td>813</td>
<td>11.71</td>
<td>9.27</td>
<td>.133</td>
</tr>
<tr>
<td>TDRSS</td>
<td></td>
<td>200</td>
<td>610</td>
<td>8.78</td>
<td>5.69</td>
<td>.082</td>
</tr>
</tbody>
</table>

STDN – Spaceflight Tracking and Data Network  
TDRSS – Tracking Data Relay Satellite System
Table 12. Expected On-Orbit Navigation Accuracies (Ref. 12)

<table>
<thead>
<tr>
<th>Navigation Tracking System</th>
<th>Unmodeled Perturbation</th>
<th>Minimum Perigee, n.mi.</th>
<th>3σ Position, Feet</th>
<th>3σ Velocity, FPS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Radial</td>
<td>Down Track</td>
</tr>
<tr>
<td>STDN</td>
<td>Nominal</td>
<td>105</td>
<td>5,000</td>
<td>34,500</td>
</tr>
<tr>
<td>TDRSS</td>
<td></td>
<td>105</td>
<td>3,600</td>
<td>18,500</td>
</tr>
<tr>
<td>STDN</td>
<td>Small</td>
<td>105</td>
<td>3,000</td>
<td>22,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>1,200</td>
<td>5,500</td>
</tr>
<tr>
<td>TDRSS</td>
<td></td>
<td>105</td>
<td>1,800</td>
<td>10,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>800</td>
<td>2,000</td>
</tr>
<tr>
<td>STDN</td>
<td>Large</td>
<td>105</td>
<td>8,000</td>
<td>80,000</td>
</tr>
<tr>
<td>TDRSS</td>
<td></td>
<td>105</td>
<td>6,000</td>
<td>50,000</td>
</tr>
</tbody>
</table>

NOTE:
The correlation between downtrack position and radial velocity is -0.95.
The correlation between radial position and downtrack velocity is -0.80.

STDN - Spaceflight Tracking and Data Network
TDRSS - Tracking Data Relay Satellite System
Table 13. Geometric Error Sensitivity
Nominal Orbit: 400 km Circular

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Attitude Offset</th>
<th>Error Sensitivity, m/arc sec</th>
<th>Geometric Error for 1° Angular Error, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>For View Angle = 0°</td>
<td>For View Angle = ±0.825°</td>
</tr>
<tr>
<td>Roll angle</td>
<td>No offset</td>
<td>1.939</td>
<td>1.940</td>
</tr>
<tr>
<td>Pitch angle</td>
<td></td>
<td>1.939</td>
<td>1.939</td>
</tr>
<tr>
<td>Yaw angle</td>
<td></td>
<td>0</td>
<td>.0279</td>
</tr>
<tr>
<td>Roll angle</td>
<td>20° roll offset</td>
<td>2.25</td>
<td>2.28</td>
</tr>
<tr>
<td>Pitch angle</td>
<td></td>
<td>1.94</td>
<td>1.94</td>
</tr>
<tr>
<td>Yaw angle</td>
<td></td>
<td>.72</td>
<td>.75</td>
</tr>
<tr>
<td>Roll angle</td>
<td>22.5° pitch offset</td>
<td>2.11</td>
<td>2.11</td>
</tr>
<tr>
<td>Pitch angle</td>
<td></td>
<td>2.31</td>
<td>2.31</td>
</tr>
<tr>
<td>Yaw angle</td>
<td></td>
<td>.83</td>
<td>.83</td>
</tr>
<tr>
<td>Roll angle</td>
<td>45° pitch offset</td>
<td>2.82</td>
<td>2.93</td>
</tr>
<tr>
<td>Pitch angle</td>
<td></td>
<td>4.33</td>
<td>4.36</td>
</tr>
<tr>
<td>Yaw angle</td>
<td></td>
<td>1.94</td>
<td>1.97</td>
</tr>
<tr>
<td>Altitude uncertainty</td>
<td></td>
<td>0</td>
<td>.0144 m/m</td>
</tr>
</tbody>
</table>
Table 14. Geometric Error Induced by Shuttle and Imaging Spectrometer Measurement Uncertainties

Nominal Orbit: 400 km Circular

<table>
<thead>
<tr>
<th>Source</th>
<th>Attitude Offset</th>
<th>Per Axis, arc sec</th>
<th>Per Axis Geometric Error, m</th>
<th>2 Axes, Geometric Error, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMU/Shuttle Misalignment</td>
<td>No Offset</td>
<td>82</td>
<td>159</td>
<td>225</td>
</tr>
<tr>
<td>IMU Resolution*</td>
<td></td>
<td>20</td>
<td>39</td>
<td>55</td>
</tr>
<tr>
<td>IMU Noise</td>
<td></td>
<td>20</td>
<td>39</td>
<td>55</td>
</tr>
<tr>
<td>Rate Gyro</td>
<td></td>
<td>60 arc sec/sec</td>
<td>116 m/s</td>
<td>165 m/s</td>
</tr>
<tr>
<td>Combined Shuttle/IS Misalignment</td>
<td></td>
<td>87</td>
<td>169</td>
<td>239</td>
</tr>
<tr>
<td>IMU/Shuttle Misalignment</td>
<td>20° Roll Offset</td>
<td>82</td>
<td>187</td>
<td>245</td>
</tr>
<tr>
<td>IMU Resolution</td>
<td></td>
<td>20</td>
<td>46</td>
<td>60</td>
</tr>
<tr>
<td>IMU Noise</td>
<td></td>
<td>20</td>
<td>46</td>
<td>60</td>
</tr>
<tr>
<td>Rate Gyro</td>
<td></td>
<td>60 arc sec/sec</td>
<td>137 m/s</td>
<td>180 m/s</td>
</tr>
<tr>
<td>Combined Shuttle/IS Misalignment</td>
<td></td>
<td>87</td>
<td>198</td>
<td>260</td>
</tr>
<tr>
<td>IMU/Shuttle Misalignment</td>
<td>45° Pitch Offset</td>
<td>82</td>
<td>358</td>
<td>426</td>
</tr>
<tr>
<td>IMU Resolution</td>
<td></td>
<td>20</td>
<td>87</td>
<td>104</td>
</tr>
<tr>
<td>IMU Noise</td>
<td></td>
<td>20</td>
<td>87</td>
<td>104</td>
</tr>
<tr>
<td>Rate Gyro</td>
<td></td>
<td>60 arc sec/sec</td>
<td>262 m/s</td>
<td>312 m/s</td>
</tr>
<tr>
<td>Combined Shuttle/IS Misalignment</td>
<td></td>
<td>87</td>
<td>379</td>
<td>452</td>
</tr>
</tbody>
</table>

†Combined roll and pitch axes.
V. GROUND PATTERNS AND IMAGE DISTORTIONS

The extent that direct errors and earth geometric properties affect the performance of the Shuttle Imaging Spectrometer are analyzed and extensively illustrated with charts and tables in section IV. The advantages of tables and charts are that they carry precise and specific information which is invaluable for design and performance prediction. However, it is difficult to relate this information directly to images of ground objects by human brains. To compensate for this and to make direct observation of how errors in the system affect ground images, selected ground patterns as seen through this optical system are studied here.

The patterns that are readily generated and, most importantly, suitable for exhibiting imaging distortions are square grid patterns.

To cover a large ground that is comparable to the field of view of the imaging spectrometer, a 10 km field is selected and the field is evenly divided into 10 segments of 1 km each. Since the imaging spectrometer employs a push broom principle, each field is then 10 km wide (cross track) and 30 m "long" (along track). However, to see a large ground area, a 10 km x 10 km field is employed with the along track grids tagged with time. Considering a perfectly spherical earth, an imaginary 10 x 10 grid is painted on the ground. As the satellite flies through this region, in the direction from the bottom to the top of the paper, a push broom camera should see a pattern just like that of Fig. 43(a). This pattern is referred to as the nominal ground pattern.

Fig. 43 shows how the ground pattern changes when simple attitude error occurs without considering the effect of Earth rotation. To show the effect of errors, the solid pattern (actual) is overlayed with the "tie" pattern (the undistorted nominal pattern). Fig. 43(b) shows that when a 0.1° roll attitude error occurs, the IS instrument points 0.1° left of the object field and the image on
Fig. 43. Ground Pattern Distortions With No Earth Rotation Effect
the "film" has shifted to the right; therefore, the solid pattern has shifted to the right of the "+" pattern. The same explanation applies also to the 0.1° pitch error as shown in Fig. 43(d), except that, for pitch, the pattern image has shifted to the "-along track" direction. A 5° yaw error (clockwise rotation) will make the pattern appear skewed as shown in Fig. 43(c).

Fig. 44 shows the effects of simple attitude errors with 20° roll offset. With the large roll angle, the nominal pattern itself has suffered cross-track distortions, i.e., the pattern image appears shrunk cross-trackwise as shown in Fig. 44(a). In Fig. 44(b), (c), and (d), the "+" patterns represent the distorted nominal patterns due to roll offset alone and the solid patterns represent those due to attitude errors on top of roll offset. The distortions are similar to those found in Fig. 43 except that with the 20° roll offset the images appear narrower.

The effects of simple attitude errors with 45° pitch offset are shown in Fig. 45. The effect of pitch offset is similar to roll offset, i.e., the nominal pattern image has shrunk cross-trackwise. One might expect also along-track shrinkage. The reason that there is no along-track shrinkage is because of the push-broom effect. Shrinkage will occur for a frame camera, however. It is noted that the pattern images associated with the 45° pitch offset are narrower than those of the 20° roll offset. This is due to the fact that 20° is a lot less than 45°.

The effects of constant attitude rate errors are shown in Fig. 46. Rate errors cause accumulation of attitude errors, i.e., the attitude errors grow with time t. Take the 0.1 deg/sec rate error for the roll axis for instance, the roll error is 0.1 t; whereas, the 0.1° roll error (Fig. 43(b)) at t = 0 is 0; therefore, the "+" pattern image coincides with the solid one at a very
Fig. 44. Ground Pattern Distortions With 20° Roll Offset and No Earth Rotation Effect
Fig. 45. Ground Pattern Distortions With 45° Pitch Offset and No Earth Rotation Effect

(a) NOMINAL GROUND PATTERN

(b) WITH 0.1° ROLL ERROR

(c) WITH 5° YAW ERROR

(d) WITH 1° PITCH ERROR
Fig. 46. Ground Pattern Distortions With Constant Rotation Rate and No Earth Rotation Effect
small t. Shown in Fig. 46(b) is the solid pattern shift to the right at a constant slope with time, which is quite different from Fig. 43(b). Similar comment applies to Fig. 46(c) and (d).

The effects of sinusoidal attitude rates on the pattern images are shown in Fig. 47. For instance, the sinusoidal rate causing a 0.2 sin (4.51378t) degree roll error will make the image appear as the shape "S", as shown in Fig. 47(b). The same function applied to the pitch axis will cause along-track distortions with densely packed grid lines followed by loosely packed ones, etc., as illustrated in Fig. 47(d).

Altitude change causes images to vary in size. Again, it affects the cross-track much more than the along-track due to the push-broom principle. As shown in Fig. 48, the solid image has shrunk cross-trackwise but expanded along-trackwise. The former is due to the altitude increase of 40 km which has no amplification effect on push-broom camera; the latter is due to the fact that, at higher altitude, the orbital rate is decreased, i.e., it will take a longer period of time to fly through the same ground area.

Finally, the effects of earth rotation are shown in Fig. 49. Earth rotation will cause the image to skew toward the direction of earth rotation. As discussed in section IV, the effect of the image shift is most pronounced when the satellite flies through the equator, and reduces as the latitude increases. Earth rotation will not cause along-track distortions.

Computer programs that are used to generate these patterns and distortions are listed in Appendix F.
Fig. 47. Ground Pattern Distortions With Sinusoidal Rotation Rate and No Earth Rotation Effect
Fig. 48. Ground Pattern Distortions With Effect of Altitude Change (increase) of 40 km and No Earth Rotation Effect.
Fig. 49. Ground Pattern Distortions With Effect of Earth Rotation for 40° Latitude and Orbit Inclination of 85°
VI. CONCLUSIONS

1. The effects of earth curvature are very small for the application here (see Fig. 42).

2. Altitude uncertainties cause only moderate geometric errors. The worst 1σ geometric errors are 11.71 m in position and 0.133 m/sec in rate with STDN and large unmodeled perturbations at 200 km orbit. The performance improves with TDRSS. For the 300 km orbit and with small unmodeled perturbations, the 1σ geometric errors will reduce to 0.22 m and 0.0032 m/sec (see Table 11).

3. The effects of other navigation errors are significantly greater. The 1σ downtrack errors range from 203 m (300 km orbit) to 8128 m (200 km orbit); and those for the cross track are 152 m to 508 m (see Table 12).

4. The effects of roll and pitch attitude errors are relatively large compared with, for instance, those caused by yaw errors and altitude uncertainties. The error sensitivity is 1.94 m/arc sec or approximately 7000 m/degree. The yaw sensitivity is 0 for a 0° view angle and 0.028 m/arc sec for the maximum view angle of ± 0.825° (see Table 12).

5. The error sensitivities of attitude errors increase significantly for large attitude offsets. For 20° side looking, the sensitivity increases to 2.28 m/arc sec for roll errors and to 0.75 m/arc sec for yaw errors, and the pitch error sensitivity is almost unaffected. For the 45° forward looking case, the sensitivities for the pitch, roll, and yaw errors increase to 2.83, 4.36, and 1.97 m/arc sec, respectively (see Table 13).

6. The performance of the Shuttle Imaging Spectrometer is limited by the Shuttle IMU (Inertial Measuring Unit) accuracy, instrument misalignment, Shuttle RCS (Reaction Control Subsystem) deadband, etc. unless some means of error reduc-
tion are employed. For instance, ground control points may be used to reduce navigation prediction errors; and precision point mounts, such as AGS (ASPS+) Gimbal System) and IPS (Instrument Pointing System), may be used to reduce the attitude errors. The single axis geometric errors due to the combined Shuttle/IS misalignment, for instance, for normal nadir pointing, $20^\circ$ side looking, and $45^\circ$ forward looking are 169 m, 198 m, and 379 m, respectively (refer to Table 14).

7. Earth rotation causes shifts of images toward the direction of rotation. The magnitude of these shifts depends on the latitude of the object. For instance, at the equator the object moves approximately $462$ m in one second (for 400 km orbit), and at $60^\circ$ latitude it moves only $231$ m in one second.

8. Distorted images for selected ground patterns provide revealing information of the effects of system errors to the images of ground objects.

+ Annular Suspension Pointing System
REFERENCES


### APPENDIX A

#### GEOMETRIC ERROR MAPPING FUNCTION TABLE

Part A: Given the attitude offsets and a view angle, find the coordinates of the corresponding ground points before and after the errors are introduced.

<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude Error, $\Delta h$</td>
<td>At nominal altitude $h$, a view angle $\lambda$ is given which sights a ground point $P(0, Y_o)$. After elevating $\Delta h$, the same view angle will sight another ground point $P'(0, Y'_o)$. The following equations are for obtaining $Y_o$ and $Y'_o$:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Y_o = -R \sin \lambda \left[ \left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right]$</td>
<td>$R = \text{Earth Radius}$</td>
</tr>
<tr>
<td></td>
<td>$Y'_o = -R \sin \lambda \left[ \left(1 + \frac{h + \Delta h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R}\right)^2 \sin^2 \lambda} \right]$</td>
<td></td>
</tr>
<tr>
<td>Yaw Error, $\psi$</td>
<td>Given a yaw offset $\psi$ and a view angle $\lambda$ which sights a ground point $P(X^<em>, Y^</em>)$. The same view angle will sight another ground point $P'(X'^<em>, Y'^</em>)$ after yaw error $\psi$ is introduced. The following equations are for obtaining $X^<em>$, $Y^</em>$, $X'^<em>$, and $Y'^</em>$:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X^* = \left[ \left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \sin \psi$</td>
<td>$R = \text{Earth Radius}$</td>
</tr>
<tr>
<td></td>
<td>$Y^* = -\left[ \left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \cos \psi$</td>
<td>$h = \text{Nominal Altitude}$</td>
</tr>
<tr>
<td>Error Sources</td>
<td>Geometric Error Mapping Function</td>
<td>Comments</td>
</tr>
<tr>
<td>---------------</td>
<td>---------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>Roll Error, $\phi$</td>
<td>$X^* = \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda}\right] R \sin \lambda \sin (\psi_o + \psi)$ $\quad Y^* = -\left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda}\right] R \sin \lambda \cos (\psi_o + \psi)$</td>
<td>$R = \text{Earth Radius}$ $h = \text{Nominal Altitude}$ $\lambda$ $\psi_o$ $\phi$</td>
</tr>
<tr>
<td>Pitch Error, $\theta$</td>
<td>$X^* = R \left[\left(1 + \frac{h}{R}\right) \cos \tau - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \tau}\right] \frac{\sin \tau \sin \theta_o}{\sqrt{\sin^2 \theta_o + \tan^2 \lambda}}$ $\quad Y^* = -R \left[\left(1 + \frac{h}{R}\right) \cos \tau - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \tau}\right] \frac{\sin \tau \tan \lambda}{\sqrt{\sin^2 \theta_o + \tan^2 \lambda}}$</td>
<td>$R = \text{Earth Radius}$ $h = \text{Nominal Altitude}$ $\lambda$ $\theta_o$ $\theta$ For $\tau$ and $\xi$, refer to Fig. 23.</td>
</tr>
</tbody>
</table>
### Geometric Error Mapping Function

<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X' = R \left[ \left( 1 + \frac{h}{R} \right) \cos \xi - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi} \right] \sin \xi \sin (\theta + \theta) \over \sqrt{\sin^2 (\theta + \theta) + \tan^2 \lambda}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y' = -R \left[ \left( 1 + \frac{h}{R} \right) \cos \xi - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi} \right] \sin \xi \tan \lambda \over \sqrt{\sin^2 (\theta + \theta) + \tan^2 \lambda}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau = \tan^{-1} \left( \sec \theta \sqrt{\sin^2 \theta + \tan^2 \lambda} \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi = \tan^{-1} \left[ \sec (\theta + \theta) \sqrt{\sin^2 (\theta + \theta) + \tan^2 \lambda} \right]$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Given a yaw offset $\psi_o$, followed by a pitch offset $\theta_o$, then a roll offset $\phi_o$. A view angle $\lambda$ is also given which sights a ground point $P_o(X_o, Y_o)$. After the errors $\psi$, $\theta$, and $\phi$ are introduced, the same view angle will sight another ground point $P_3(X_3, Y_3)$. The following equations are for obtaining $X_o$, $Y_o$, $X_3$, and $Y_3$.*

| $X_o' = R \sin \theta \sin \xi_o \left[ \left( 1 + \frac{h}{R} \right) \cos \xi_o - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi_o} \right] \over \sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi_o)}$ | | |
| $Y_o' = -R \tan (\lambda + \phi_o) \sin \xi_o \left[ \left( 1 + \frac{h}{R} \right) \cos \xi_o - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi_o} \right] \over \sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi_o)}$ | | |

- $R$ = Earth Radius
- $h$ = Nominal Altitude
- For $\xi_o$ and $\xi_3$, refer to Fig. 23.
<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_3'$</td>
<td>$x_3' = \frac{R \sin (\theta_o + \theta) \sin \xi_3 \left[ (1 + \frac{h}{R}) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 (\lambda + \phi_o + \phi)}}$</td>
<td></td>
</tr>
<tr>
<td>$y_3'$</td>
<td>$y_3' = -\frac{R \tan (\lambda + \phi_o + \psi) \sin \xi_3 \left[ (1 + \frac{h}{R}) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 (\lambda + \phi_o + \psi)}}$</td>
<td></td>
</tr>
<tr>
<td>$\xi_o$</td>
<td>$\xi_o = \tan^{-1} \left( \sec \theta_o \sqrt{\sin^2 \theta_o + \tan^2 (\lambda + \phi_o)} \right)$</td>
<td></td>
</tr>
<tr>
<td>$\xi_3$</td>
<td>$\xi_3 = \tan^{-1} \left[ \sec (\theta_o + \theta) \sqrt{\sin^2 (\theta_o + \theta) + \tan^2 (\lambda + \phi_o + \phi)} \right]$</td>
<td></td>
</tr>
<tr>
<td>$x_o$</td>
<td>$x_o = x_3' \cos \psi_o - y_3' \sin \psi_o$</td>
<td></td>
</tr>
<tr>
<td>$y_o$</td>
<td>$y_o = x_3' \sin \psi_o + y_3' \cos \psi_o$</td>
<td></td>
</tr>
<tr>
<td>$X_3$</td>
<td>$X_3 = x_3' \cos (\psi_o + \psi) - y_3' \sin (\psi_o + \psi)$</td>
<td></td>
</tr>
<tr>
<td>$Y_3$</td>
<td>$Y_3 = x_3' \sin (\psi_o + \psi) + y_3' \cos (\psi_o + \psi)$</td>
<td></td>
</tr>
</tbody>
</table>
Part B: Given the coordinates of a ground point, find the attitude offsets, the corresponding view angle and the coordinates of the point with the same view angle after the errors are introduced.

<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude Error, Δh</td>
<td>At nominal altitude h, a ground point P(X₀, Y₀) corresponding to view angle λ is given. After elevating Δh, the same view angle λ will aim at another ground point P'(X', Y'). The following equations are for obtaining X' and Y':</td>
<td>R = Earth Radius</td>
</tr>
<tr>
<td></td>
<td>λ = \tan^{-1}\left(\frac{\sqrt{X_0^2 + Y_0^2}}{R + h - \sqrt{R^2 - X_0^2 - Y_0^2}}\right)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>X' = \frac{RX_0 \sin \lambda}{\sqrt{X_0^2 + Y_0^2}} \left[1 + \frac{h + Δh}{R}\right] \cos \lambda - \sqrt{1 - \left(1 + \frac{h + Δh}{R}\right)^2 \sin^2 \lambda}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Y' = \frac{RY_0 \sin \lambda}{\sqrt{X_0^2 + Y_0^2}} \left[1 + \frac{h + Δh}{R}\right] \cos \lambda - \sqrt{1 - \left(1 + \frac{h + Δh}{R}\right)^2 \sin^2 \lambda}</td>
<td></td>
</tr>
</tbody>
</table>
A ground point \( P(0, y^*) \) corresponding to roll offset \( \phi_0 \) and view angle \( \lambda \) is given. After roll error \( \phi \) is introduced, the same view angle \( \lambda \) will aim at another ground point \( P'(0, y'^*) \). The following equations are for obtaining \( y'^* \):

\[
\lambda + \phi_0 = -\text{sgn}(y^*) \tan^{-1} \left( \frac{|y^*|}{R + h - \sqrt{R^2 - y^*^2}} \right)
\]

\[
y'^* = -R \sin'(\lambda + \phi_0 + \phi) \left[ \left( 1 + \frac{h}{R} \right) \cos (\lambda + \phi_0 + \phi) \right.
\]
\[
- \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 (\lambda + \phi_0 + \phi)} \right]
\]

<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
</table>
| Roll Error, \( \phi \) | \[
\lambda + \phi_0 = -\text{sgn}(y^*) \tan^{-1} \left( \frac{|y^*|}{R + h - \sqrt{R^2 - y^*^2}} \right)
\]
\[
y'^* = -R \sin'(\lambda + \phi_0 + \phi) \left[ \left( 1 + \frac{h}{R} \right) \cos (\lambda + \phi_0 + \phi) \right.
\]
\[
- \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 (\lambda + \phi_0 + \phi)} \right]
\] | \( R = \) Earth Radius \hskip0.2cm \( h = \) Nominal Altitude |
<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch Error, $\theta$</td>
<td>A ground point $P(X^<em>, Y^</em>)$ corresponding to pitch offset $\theta_o$ and view angle $\lambda$ is given. After pitch error $\theta$ is introduced, the same view angle $\lambda$ will aim at another ground point $P'(X'^<em>, Y'^</em>)$. The following equations are for obtaining $X'^<em>$ and $Y'^</em>$:</td>
<td>$R = $ Earth Radius&lt;br&gt;$h =$ Nominal Altitude&lt;br&gt;For $\xi$, refer to Fig.23.</td>
</tr>
</tbody>
</table>

\[
\lambda = \tan^{-1}\left[\frac{Y^*}{\left(R + h - \sqrt{R^2 - X^*^2 - Y^*^2}\right)}\left(1 + \left(\frac{X^*}{R + h - \sqrt{R^2 - X^*^2 - Y^*^2}}\right)^2\right)\right]
\]

\[
\theta_o = \tan^{-1}\left[\frac{X^*}{R + h - \sqrt{R^2 - X^*^2 - Y^*^2}}\right]
\]

\[
\xi = \tan^{-1}\left[\sec(\theta_o + \theta)\sqrt{\sin^2(\theta_o + \theta) + \tan^2\lambda}\right]
\]

\[
X'^* = \frac{R \sin(\theta_o + \theta) \sin\xi \left(1 + \frac{h}{R}\right) \cos\xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2\xi}}{\sqrt{\sin^2(\theta_o + \theta) + \tan^2\lambda}}
\]

\[
Y'^* = -\frac{R \tan\lambda \sin\xi \left(1 + \frac{h}{R}\right) \cos\xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2\xi}}{\sqrt{\sin^2(\theta_o + \theta) + \tan^2\lambda}}
\]
<table>
<thead>
<tr>
<th>Error Sources</th>
<th>Geometric Error Mapping Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw Error, $\psi$</td>
<td>A ground point $P(X^<em>,Y^</em>)$ corresponding to yaw offset $\psi_0$ and a certain view angle is given. After yaw error $\psi$ is introduced, the same view angle will aim at another ground point $P'(X'^<em>,Y'^</em>)$. The following equations are for obtaining $X'^<em>$ and $Y'^</em>$:</td>
<td>$R = $ Earth Radius</td>
</tr>
<tr>
<td></td>
<td>$\psi_0 = -\tan^{-1}\left(\frac{X^<em>}{Y^</em>}\right)$</td>
<td>$h = $ Nominal Altitude</td>
</tr>
<tr>
<td></td>
<td>$\lambda = -\text{sgn}(Y^<em>)\tan^{-1}\left(\frac{\sqrt{X^</em>^2 + Y^<em>^2}}{R + h - \sqrt{R^2 - X^</em>^2 - Y^*^2}}\right)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X'^* = \left[(1 + \frac{h}{R})\cos\lambda - \sqrt{1 - (1 + \frac{h}{R})^2\sin^2\lambda}\right] R \sin\lambda \sin(\psi_0 + \psi)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Y'^* = -\left[(1 + \frac{h}{R})\cos\lambda - \sqrt{1 - (1 + \frac{h}{R})^2\sin^2\lambda}\right] R \sin\lambda \cos(\psi_0 + \psi)$</td>
<td></td>
</tr>
<tr>
<td>Earth Rotation</td>
<td>Let $\Gamma$ be the shuttle orbital inclination at the equator, and $\zeta$ be the latitude of the nadir point. A given point $P(X,Y)$, after time period $\Delta t$, will move to $P'(X',Y')$. The following equations are for obtaining $X'$ and $Y'$:</td>
<td>$V_e = $ Linear Velocity of Ground Point at Latitude $\zeta$</td>
</tr>
<tr>
<td></td>
<td>$V_e = \omega R \cos\zeta$</td>
<td>$\omega = $ Angular Velocity of Earth Rotation</td>
</tr>
<tr>
<td></td>
<td>$\omega = \frac{2\pi}{86,400}$ radians/sec</td>
<td>$\Delta t$ is assumed small.</td>
</tr>
<tr>
<td></td>
<td>$X' = X + \cos\Gamma \cos\zeta V_e \Delta t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Y' = Y - \left(\frac{\cos\Gamma}{\cos\zeta}\right)^2 V_e \Delta t$</td>
<td></td>
</tr>
<tr>
<td>Error Sources</td>
<td>Geometric Error Mapping Function</td>
<td>Comments</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>Yaw Error $\psi$ followed by Pitch Error $\theta$ then Roll Error $\phi$</td>
<td>A ground point $P(O,Y^<em>)$ corresponding to view angle $\lambda$ (with no attitude offset) is given. A yaw error $\psi$ is introduced first, then a pitch error $\theta$, finally a roll error $\phi$. The same view angle $\lambda$ will then aim at another ground point $P'(X'^</em>,Y'^<em>)$. The following equations are for obtaining $X'^</em>$ and $Y'^*$:</td>
<td></td>
</tr>
<tr>
<td>$\lambda = - \text{sgn}(Y^*) \tan^{-1} \left( \frac{</td>
<td>Y^*</td>
<td>}{R + h - \sqrt{R^2 - Y^*^2}} \right)$</td>
</tr>
<tr>
<td>$\xi = \tan^{-1} \left[ \sec \theta \sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)} \right]$</td>
<td>$h$ = Nominal Altitude</td>
<td></td>
</tr>
<tr>
<td>$X = \frac{R \sin \theta \sin \xi \left( 1 + \frac{h}{R} \right) \cos \xi - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi}} {\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}$</td>
<td>For $\xi$, refer to Fig. 23.</td>
<td></td>
</tr>
<tr>
<td>$Y = \frac{-R \tan (\lambda + \theta) \sin \xi \left( 1 + \frac{h}{R} \right) \cos \xi - \sqrt{1 - \left( 1 + \frac{h}{R} \right)^2 \sin^2 \xi}} {\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X'^* = X \cos \psi - Y \sin \psi$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y'^* = X \sin \psi + Y \cos \psi$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX B

NAPIER'S RULES FOR RIGHT SPHERICAL TRIANGLES*

A right spherical triangle has five variable parts. If these components and their complements (complement of $\Gamma = 90^\circ - \Gamma$) are arranged in a circle, as illustrated below:

then, the following relationships hold between the five components in the circle:

The sine of any component equals the product of either:

1. The tangents of the adjacent components, or
2. The cosines of the opposite components

For our case, from 2:

$$\sin (90^\circ - \Gamma) \cos \zeta \cos (90^\circ - A)$$

i.e.,

$$\cos \Gamma = \cos \zeta \sin A$$

*See Ref. 13
APPENDIX C

COMPUTER PROGRAM FOR SHUTTLE IMAGING SPECTROMETER POINTING ERROR

POWER SPECTRAL DENSITIES FOR CONFIGURATION A — PAYLOAD-BAY NADIR POINTING

ELT Wed-10/20/82-10:22:28-(52,)
12537-1S(1),SRM+/PROG152)
1:PROGRAM IMAGING SPECTROMETER SHUTTLE RIGID MOUNT - A
2:INITIAL
3:VARIABLE T=0.0
4: INTEGER FLM, NPOINT
5:
6:
7:COMMENT INITIALIZE SOME PARAMETERS
8: CONSTANT NPOINT=641
9: CONSTANT FLM=39
10: CALL FOPEN(FLM)
11: TFINAL=1.0*NPOINT-1.0
12: CONSTANT FACTOR=0.0
13: CONSTANT SCFAC=1.0E+5
14: SCFAC2=SCFAC*SCFAC
15: SCF2DB=10.0*LOG10(SCFAC2)
16:
17:COMMENT SET VARIOUS MATHEMATICAL CONSTANTS
18: PI=3.14159265
19: RSRC=PI/180.0*3600.0
20: RSRC2=2.0*PI
21:
22:COMMENT DEFINE MOMENTS OF INERTIA
23: CONSTANT IX=1.3BE+6
24: CONSTANT IY=1.00E+7
25: CONSTANT IZ=1.55E+7
26: IXZ=IX*IY
27: IYZ=IX*IY
28: IZZ=IX*IY
29: IXYZ=IXZ+IYZ
30: IXYZ2=IXYZ.IXYZ
31:
32:COMMENT DEFINE W0
33: CONSTANT W0=0.0011315
34: W02=W0*W0
35: W04=W0*W0
36:
37:COMMENT DEFINE INITIAL ANGLES AND RATES
38: CONSTANT TH0=0.0
39: CONSTANT PHI0=0.0
40: CONSTANT PSI0=0.0
41: CONSTANT THD0=0.0
42: CONSTANT PHI0=0.0
43: CONSTANT PSI0=0.0
44: THD2=THD2+THD2
45: PHI02=PHI0*PHI0
46: PHI02=PHI0*PHI0
47: PSI02=PSI0*PSI0
48: PSI02=PSI0*PSI0
49: PSI02=PSI0*PSI0
50:
51:COMMENT DEFINE O'S
52: CONSTANT OX=2.014E-6
53: CONSTANT OY=1.25E-3
54: CONSTANT O2=5.43E-4
55:
56:COMMENT DEFINE RATE FILTER PARAMETERS
57: CONSTANT WI=0.2513
58: CONSTANT ZERO=0.8
59: W02=W0*W0
60: W02=W0*W0
61: ZD2=ZETRA+ZETRA
62:
63:COMMENT DEFINE VARIOUS CONSTANTS
64: CONSTANT R=8.55E-10

125
65:  CONSTANT SIGBO=87.32
66:  SIGBO=SIGBO*MRCC
67:  SIGBO2=SIGBO*SIGBO
68:  CONSTANT TRA=0.023
69:  TRA2=TRA*TRA
70:  DELW=1.0
71:
72: COMMENT DEFINE W CONSTANTS AND INITIALIZE W
73:  CONSTANT ML=6.26318531E-7
74:  CONSTANT WFACT=1.029200527
75:  W=ML*WFACT
76:
77: END
78: DYNAMIC
79: INTERVAL CI=1.0
80:
81: COMMENT COMPUTE W
82:  W=WFACT
83:  PROCEDURAL(DWL=W)
84:  DELW=0.0
85:  IF(W.LE.0.) DELW=1.0
86:  END
87:  W=W*WFRCT
88:  FEO=ERG10(WHZ)
89:  WZ=W4
90:  W=W2*WZ
91:
92: COMMENT COMPUTE M42 AND DM2
93:  M42=1.0/(TRA2*WZ-1.0)
95:  DM2=W2*WFRCT/(WZ**2+1.0)**2
96: COMMENT DM2=DM2*0.4*W04*(IY-IX)*(IY-IX)
97:  DM2=DM2*SCFRC
98:  DM2=DM2*SCFRC
99:  END
100: COMMENT COMPUTE F'S
101: COMMENT FTH2=W4*SCFRC/(W2+1.0)**2
102: COMMENT FTH2=(IY*W2-IY)**2*W2/DW2
103: COMMENT FWT=W12+W24*SCFRC/(IY*W2)**2
104: FTH2=W4*SCFRC/(1.0+W2)**2
105: FTH2=(IY*W2)**2*W2/DW2
106: FTH2=W4*SCFRC/(1.0+W2)**2
107: END
108: COMMENT COMPUTE PSD'S
109: COMMENT PITCH PSD COMPUTATIONS
110: PPW1=FTH2*Y2*THID2
111: PPW2=FTH2*Y2*THID2
112: PPW3=FTH2*Y2*THID2
113: PPW4=FTH2*Y2*THID2
114: PPW5=FTH2*Y2*THID2
115: PPW6+FTH2+DELW=(PPW1+PPW2+PPW3+SCFRC
116: PPW7=PPW4*PHI2
117: END
118: COMMENT ROLL PSD COMPUTATIONS
119: PPW1=FHY2+X*PHI2
120: PPW2=FHY2+X*PHI2
121: PPW3=FHY2+X*PHI2
122: PPW4=FHY2+X*PHI2
123: PPW5=FHY2+X*PHI2
124: PPW6=FHY2+X*PHI2
125: PPW7=FHY2+X*PHI2
126: PPW8=FHY2+X*PHI2
127: RS=R*SIGBO2+DELW
128: PRW=RS*(PPW1+PPW2+PPW3+PPW5+PPW6+PPW7+PPW8)/SCFRC
129: END
130:
131: COMMENT YAW PSD COMPUTATIONS
132: PYW1+FYM2+IZZ+PS10D2
133: PYW3+FYM2+IZZ+PS102+M2
134: PYW3+FYM2+IZZ+PS102+M2
135: PYW4+FYM2+OZ
136: PYW5+FYM2+IX+PS10D2
137: PYW6+FYM2+IX+PS10D2
138: PYW7+FYM2+IX+PS10D2
139: PYW8+FYM2+OX
140: PYW8+PYW2+PYW3+PYW4+PYW5+PYW6+PYW7+PYW8)/SCFAC2
141: PYWD=PYW+HDW2
142:
143:
144: COMMENT PREPARE VARIABLES FOR OUTPUT
145: RSIG+P+SIG102+DELW
146: RSIDB=10.0*ALOG10(RSIG)+FACTOR
147: PPW3DB=10.0*ALOG10(PPW3)-SCF2DB+FACTOR
148: PPW4DB=10.0*ALOG10(PPW4)-SCF2DB+FACTOR
149: PRW4DB=10.0*ALOG10(PRW4)-SCF2DB+FACTOR
150: PRW4DB=10.0*ALOG10(PRW4)-SCF2DB+FACTOR
151: PRW4DB=10.0*ALOG10(PRW4)-SCF2DB+FACTOR
152: PRW4DB=10.0*ALOG10(PRW4)-SCF2DB+FACTOR
153: PRW4DB=10.0*ALOG10(PRW4)-SCF2DB+FACTOR
154: PRW4DB=10.0*ALOG10(PRW4)-SCF2DB+FACTOR
155: PRW4DB=10.0*ALOG10(PRW4)-SCF2DB+FACTOR
156: PPW3DB=10.0*ALOG10(PPW3)+FACTOR
157: PRW4DB=10.0*ALOG10(PRW4)+FACTOR
158: PPW3DB=10.0*ALOG10(PPW3)+FACTOR
159: COMMENT SAVE NUMBERS IN FILE
160: CALL FSAVE(FREQ,PPWDB,PRWDB,PYWDB,FNUM)
161: TERMT(T.E.TFINAL)
162: DERIVATIVE
163: ALGORITHM 1 Alg-3
164: GO=INTEG(1.0,0.0)
165: END
166: TERMINAL
167: END
168: END
169: END
170: END
171: COMMENT CLOSE FILE
172: CALL FCLOSE(FNUM)
173: END
174: END
175: END

EOF: 175
O:ILT  WED-10/20/82-10:30:02-(3,)
12537-IS(1).$RMA/FSUBS(3)
1: $FOR,IS  FF.FOPEN
2:  SUBROUTINE FOPEN(IN)
3:  REWIND N
4:  RETURN
5:  CIC
6: $FOR,IS  FF.FSAVE
7:  SUBROUTINE FSAVE(IN,P1,P2,P3,N)
8:  WRITE(N,100)(P1,P2,P3)
9:  100  FORMAT(4G14.8)
10:  RETURN
11:  END
12: $FOR,IS  FF.FCLOSE
13:  SUBROUTINE FCLOSE(IN)
14:  ENDFILE N
15:  RETURN
16:  END

EDF:16
APPENDIX D

COMPUTER PROGRAM FOR SHUTTLE IMAGING SPECTROMETER POINTING ERROR

POWER SPECTRAL DENSITIES FOR CONFIGURATION B — NOSE-DOWN NADIR POINTING

```
ELT WED-10/20/82-10:36:41-11
12S3715111.ERRE/PROG(14)
1:PROGRAM IMAGING SPECTROMETER SHUTTLE RIGID MOUNT - B
2:INITIAL
3:VARIBLE T=0.0
4:   INTEGER FROM,NPOINT
5:
6:7:COMMENT INITIALIZE SOME PARAMETERS
8:   CONSTANT NPOINT=641
9:   CONSTANT FM#39
10:  CALL FOR(NFM#)
11:  TFMIN=1.0/NPOINT-1.0
12:  CONSTANT FACTOR=0.0
13:  CONSTANT SCFRAC=1.0E+10
14:  SCFRAC2=SCFRAC*SCFRAC
15:  SCF2DB=10.0*LOG10(SCFRAC)
16:
17:COMMENT SET VARIOUS MATHEMATICAL CONSTANTS
18:  PI=3.14159265
19:  ASRCC=PI/(180.0*3600.0)
20:  ASH2CC=1.0/(2.04P1)
21:
22:COMMENT DEFINE MOMENTS OF INERTIA
23:   CONSTANT IX=1.00E+7
24:   CONSTANT IY=1.05E+7
25:   CONSTANT IZ=1.38E+6
26:   IIX=IX*IX
27:   IYI=YI*YI
28:   IIZ=IZ*IZ
29:   IXYI=YX*IX
30:   IXYI=YX*IX
31:
32:COMMENT DEFINE WO
33:   CONSTANT WO=0.0011315
34:   W02=W0*W0
35:   W04=W0*W0
36:
37:COMMENT DEFINE INITIAL ANGLES AND RATES
38:   CONSTANT THO=0.0
39:   CONSTANT PHI0=0.0
40:   CONSTANT PSI0=0.0
41:   CONSTANT THIO=0.0
42:   CONSTANT PHI0D=0.0
43:   CONSTANT PSI0D=0.0
44:   THOZ=THO*THO
45:   THOZ=THO*THO
46:   PHI0Z=PHI0*PHI0
47:   PHI0Z=PHI0*PHI0
48:   PSI0Z=PSI0*PSI0
49:   PSI0Z=PSI0*PSI0
50:
51:COMMENT DEFINE O'S
52:   CONSTANT OX=1.5708E-4
53:   CONSTANT OY=1.28E-3
54:   CONSTANT OZ=6.32E-4
55:
56:COMMENT DEFINE RATE FILTER PARAMETERS
57:   CONSTANT W0=0.2513
58:   CONSTANT ZETAD=0.8
59:   W02=W0*W0
60:   W04=W0*W0
61:   Z02=ZETAD*ZETAD
62:
63:COMMENT DEFINE VARIOUS CONSTANTS
64:   CONSTANT R=8.55E-10
```
65: CONSTANT SIGBO=87.32
66: SIGBO=SIGBO+SIGBO
67: SIGBO=SIGBO+SIGBO
68: CONSTANT TAU=0.023
69: TAU=TAU+TAU
70: DELM=1.0
71: 72:COMMENT DEFINE W CONSTANTS AND INITIALIZE W
73: CONSTANT MLO=6.28318531E-7
74: CONSTANT NFACT=1.028200527
75: W=MLO*NFACT
76: 77:END
78:DYNAMIC
79:CINTERVAL CI=1.0
80: 81:COMMENT COMPUTE W
82: W=W*NFACT
83: PROCEDURE(DELM+W)
84: DELM=0.0
85: IF(W.LEO.) DELM=1.0
86: END
87: W=W*WSLZCC
88: FREQ=RLOG(SW/1000)
89: W2+WH
90: W=WZ+WX
91: 92:COMMENT COMPUTE WZ AND DM2
93: DM2=1.0/(TRUZ+WX+1.0)
94: DM2+DM2*(WZ-WZ)+2.0*(WZ+WZ)WZ
95: DM2+DM2*(WZ-WZ)+2.0*(WZ+WZ)WZ
96: DM2=DM2+3.0*ZH2Z
97: DM2=DM2+DM2
98: 99:
100:COMMENT COMPUTE F'S
101: FTHWZ=DM2+SCFACZ/((WZ+WZ)+3.0*WZ+2.0*DM2)
102: FRWZ=2.0*(WZ+WZ+1.0)*WZ/WZ
103: FW2Z=FW2Z+2.0*Y
104: FRW1=FWW2Z+2.0/SCFACZ
105: PPW1=FW1Z+2.0/PH1DZ
106: PPW2=FW2Z+2.0/PH1DZ
107: PPW3=FW1Z+2.0/PH1DZ
108: PPW4=SIGBO+DELM+(PPW1+PPW2+PPW3)/SCFACZ
109: PPW5=PPW1+PH1DZ
110: 111:COMMENT COMPUTE PSD'S
112: 113: 114:COMMENT COMPUTE PSD COMPUTATIONS
115: PRW1=FW1Z+2.0/PH1DZ
116: PRW2=FW2Z+2.0/PH1DZ
117: PRW3=FW1Z+2.0/PH1DZ
118: PRW4=FW2Z+2.0/PH1DZ
119: PRW5=FW1Z+2.0/PH1DZ
120: PRW6=FW2Z+2.0/PH1DZ
121: PRW7=FW1Z+2.0/PH1DZ
122: PRW8=FW2Z+2.0/PH1DZ
123: PRW9=FW1Z+2.0/PH1DZ
124: PRW10=FW2Z+2.0/PH1DZ
125: PRW11=FW1Z+2.0/PH1DZ
126: PRW12=FW2Z+2.0/PH1DZ
127: 128:COMMENT YAW PSD COMPUTATIONS
129: PYW1=PYW1+2.0/PS10DZ
130: PYW2=PYW2+2.0/PS10DZ
131: PYW1+PYW2+PYW3+PYW4+PYW5+PYW6+PYW7+PYW8/SCFAC2
132: PYWO+PYW1+PYW2
133: COMMENT PREPARE VARIABLES FOR OUTPUT
134: RSIGR+SIGBO2*DELW
135: PPW3D=10.0*ALOG10(PW3)-SCF2DB+FACTOR
136: PRWDD=10.0*ALOG10(PRW)+FACTOR
137: PYWDD=10.0*ALOG10(PYW)-SCF2DB+FACTOR
138: COMMENT SAVE NUMBERS IN FILE
139: CALL FSAVE(FREQ,PPWDB,PRWDB,PYWDB,FNUM)
140: TERMINAL
141: TERMINAL
142: TERMINAL
143: TERMINAL
144: TERMINAL
145: END
146: END
147: END
148: END
149: END
150: END
151: END
152: END
153: END
154: END
155: END
156: END
157: END
158: END
159: END
160: END
161: TERMINAL
162: DERIVATIVE
163: ALGORITHM IALG=3
164: QQ=INTEG(1.0,0.0)
165: END
166: END
167: TERMINAL
168: COMMENT CLOSE FILE
169: CALL FCLOSE(FNUM)
170: END
171: END
172: END
EOF: 172
1: IFOR IS FF.FOPEN
2:   SUBROUTINE FOPEN(N)
3:       REWIND N
4:     RETURN
5:   END
6: IFOR IS FF.FSAVE
7:   SUBROUTINE FSAVE(w,p1,p2,p3,n)
8:     WRITE(N,100),p1,p2,p3
9:   100 FORMAT(4(14.8))
10:   RETURN
11: END
12: IFOR IS FF.FCLOSE
13:   SUBROUTINE FCLOSE(N)
14:   ENDFILE N
15:   RETURN
16: END

EOF: 16
APPENDIX E

GEOMETRIC ERROR ANALYSIS PROGRAM LISTINGS

100 D 004552 DOLTA 0000 R 000263 DPTHCH 0000 R 000159 DPBA 0000 R 000067
100 R 0000b4 DB12 0000 R 000249 DROLL 0000 R 000242 DXY 0000 R 000179
100 D 000125 DTETA1 0000 R 000150 DTETA2 0000 R 000213 DTMETA 0000 D 00321a
100 D 001204 DV 0000 R 000225 DYAM 0000 R 022676 ERR 0000 D 023102
100 D 013629 FI 0000 D 003552 FI1 0000 D 003360 FI2 0000 R 000000
100 D 030711 J 0000 I 03076b K 0000 D 003652 LAMCA 0000 D 000626
100 D 001553 PXY 0000 D 013540 PDV 0000 R 030710 PI 0000 D 01715b
100 D 004257 PSJ 0000 D 017176 PSY 0000 D 003244 PXY 0000 D 000742
100 D 004259 RA 0000 D 005520 RHY 0000 R 012547 RHO1 0000 D 01043b
100 D 026529 R01 0000 D 005222 R 0000 R 000407 RDM 0000 D 013350
100 D 013166 SHFT 0000 D 025352 BLUNDA 0000 D 015576 SQURT 0000 D 016260
100 D 027614 SOUT1 0000 D 002235 SXY 0000 D 003240 Y 0000 D 022521
100 R 008592 TDCH2 0000 D 007314 TETA1 0000 D 007322 TETA2 0000 D 017372
100 D 00752a T112 0000 D 007335 T11 0000 D 007336 T12 0000 D 00318a
100 D 015552 T1 0000 D 025372 T11 0000 D 005130 Y2 0000 D 02326a
100 D 011579 VP 0000 D 02273a VPX 0000 D 022734 V8 0000 D 022556
100 D 011756 XI 0000 D 000075 XO 0000 D 000326 XY 0000 R 000133
100 D 012874 X2 0000 D 011300 XV 0000 D 011720 X3 0000 D 006870
100 D 014163 YI 0000 D 005062 YO 0000 D 023140 Y0 0000 D 01134b
100 D 011712 Y22 0000 D 026316 Y3

10 C
20 C THIS PROGRAM COMPUTES THE ERROR SENSITIVITIES AND GEOMETRIC
30 C ERRORS OF GROUND POINTS INDUCED BY ALTITUDE, ROLL, YAW AND
40 C PITCH ERRORS AS WELL AS EARTH ROTATION AND SMUTTLE MOTION.

50 C
60 C
70 C PARAMETER NLAMDA=10, WLOMDA=19, NDH=8, NPI=3, NPI=3, NTEA=3
80 C
90 C
100 C
110 C
120 C
130 C
140 C
150 C
160 C
170 C
180 C
190 C
200 C
210 C
220 C
230 C
240 C
250 C
260 C
270 C
280 C
290 C
300 C
310 C
320 C
330 C

133
DO 132 KM1=HPSA
   ALFA(K)=ALFA(K)/PI/180.
   DO 133 J=1,HOMDA
   LAMDA(J)=LAMDA(J)/PI/180.
   RA(J)=P(C1+H/R)*DCOS(LAMDA(J))=DBSRT(1+(H/R)*C1+H/R)
   DO 134(J,J=1,HOMDA)
   OTM(K,J)=DBSRT(1+(H/R)*C1+H/R)
   DELTA(K,J)=(RA(J)*DBSRT(PBI(K)))/DBSRT(ALFA(K))
   CONTINUE
   131 CONTINUE
   CONTINUE

DO 136 KM1=HPSI
   DPSI(K)=DPSI(K)/PI/180.
   DBETA(K)=DBETA(K)/PI/180.
   DO 137(J,J=1,HOMDA)
   LAMDA(J)=LAMDA(J)/PI/180.
   RA(J)=P(C1+H/R)*DCOS(LAMDA(J))=DBSRT(1+(H/R)*C1+H/R)
   DO 138(J,J=1,HOMDA)
   OTM(K,J)=DBSRT(1+(H/R)*C1+H/R)
   DELTA(K,J)=(RA(J)*DBSRT(PBI(K)))/DBSRT(ALFA(K))
   CONTINUE
   135 CONTINUE
   CONTINUE

DO 206 KM1=HOTHE
   TTRIA(K)=TTRIA(K)/PI/180.
   DO 207(J,J=1,HOMDA)
   LOMDA(J)=LOMDA(J)/PI/180.
   Y0(J,J)=P(C1+H/R)*DCOS(LOMDA(J))=DBSRT(1+(H/R)*C1+H/R)
   DO 208(J,J=1,HOMDA)
   O0(J,J)=DBSRT(1+(H/R)*C1+H/R)
   DELT(J,J)=(Y0(J,J)*Y(J,J))
   XII(J,J)=DBSRT(1+(H/R)*C1+H/R)
   Y(J,J)=O0(J,J)
   DELT+K,J=O0(J,J)*O(J,J)
   XII+(K,J)=XII(K,J)+Y(J,J)
   CONTINUE
   205 CONTINUE
   CONTINUE

DO 206 KM1=HOTHE
   TTRIA(K)=TTRIA(K)/PI/180.
   DO 207(J,J=1,HOMDA)
   LOMDA(J)=LOMDA(J)/PI/180.
   Y0(J,J)=P(C1+H/R)*DCOS(LOMDA(J))=DBSRT(1+(H/R)*C1+H/R)
   DO 208(J,J=1,HOMDA)
   O0(J,J)=DBSRT(1+(H/R)*C1+H/R)
   DELT(J,J)=(Y0(J,J)*Y(J,J))
   XII(J,J)=DBSRT(1+(H/R)*C1+H/R)
   Y(J,J)=O0(J,J)
   DELT+K,J=O0(J,J)*O(J,J)
   XII+(K,J)=XII(K,J)+Y(J,J)
   CONTINUE
   205 CONTINUE
   CONTINUE

136
DEL2*(X,J)*DSQRT(T12(K)+T12(K)*T12(K))*T12(K)*T12(K,J)

AM1(K,J) = DATAM(DEL1(K,J)/H)

AM2(K,J) = DATAM(DEL2(K,J)/H)

SQRT(K,J)*R(((1+H/R)*DCOS(K,J)) = DSQRT(1*(1+H/R))

R(K,J) = DSQRT(R11(K,J)*T11(K,J))

X1 = X2(K,J)*SQRT(K,J)*T11(K,J)/DE1(K,J)

Y1 = Y2(K,J)*SQRT(K,J)*T11(K,J)/DE1(K,J)

X2(K,J) = X2(K,J) + SQRT(T11(K,J)) + DE1(K,J)

VX = VX(K,J) + SQRT(1*1) + DE1(VX)

VY = VY(K,J) + SQRT(1*1) + DE1(VY)

DO 210 K=1,CON

DO 220 J=1,CON

210 CONTINUE

220 CONTINUE
2620 V8 = 2**PI*ER/5545.484
2630 TAUNDA = USPI/180.
2640 OMEGA = 2**PI*USPI/8400.
2650 DO 222 KM = 120.
2660 ETA = KMOD(KM)*USPI/180.
2670 V(K) = OMEGA * COS(ETA(K)).
2680 SLUNDA(K) = OMEGA * COS(ETA(K)).
2690 CLUNDA(K) = IDGRT(V + SLUNDA(K) + SLUNDA(K)).
2700 VX(K) = V(K) * SLUNDA(K).
2710 VX(K) = V(K) * SLUNDA(K).
2720 VX(K) = V(K) * SLUNDA(K).
2730 VX(K) = V(K) * SLUNDA(K).
2740 VX(K) = V(K) * SLUNDA(K).
2750 VX(K) = V(K) * SLUNDA(K).
2760 VX(K) = V(K) * SLUNDA(K).
2770 VX(K) = V(K) * SLUNDA(K).
2780 VX(K) = V(K) * SLUNDA(K).
2790 VX(K) = V(K) * SLUNDA(K).
2800 VX(K) = V(K) * SLUNDA(K).
2810 VX(K) = V(K) * SLUNDA(K).
2820 CONTINUE
222
2830 CONTINUE
2840 DO 237 J = 1, NDH.
2850 WRITE(230) RE = DM(K) + DALAMDA(I) + IML(ALAMDA) + (XK(I) + IML(NALAN) + S1).EPS(K) + IML(ALAMDA).
2860 \( \sin \eta \)
2870 DO 237 CONTINUE
238
2880 CONTINUE
2890 DO 239 J = 1, NDH.
2900 WRITE(230) RM = SDM(J) + (DALAMDA(I) + IML(ALAMDA) + (XK(I) + IML(NALAN) + S1).EPS(K) + IML(ALAMDA).
2910 \( \sin \eta \)
2920 DO 239 CONTINUE
2930 CONTINUE
2940 \( \sin \eta \)
2950 DO 239 CONTINUE
2960 \( \sin \eta \)
2970 DO 239 CONTINUE
3010 \( \sin \eta \)
3020 DO 239 CONTINUE
3030 \( \sin \eta \)
3040 DO 239 CONTINUE
3050 \( \sin \eta \)
3060 DO 239 CONTINUE
3070 \( \sin \eta \)
3080 DO 239 CONTINUE
3090 \( \sin \eta \)
3100 DO 239 CONTINUE
3110 \( \sin \eta \)
3120 \( \sin \eta \)
3130 \( \sin \eta \)
3140 \( \sin \eta \)
3150 \( \sin \eta \)
3160 \( \sin \eta \)
3170 \( \sin \eta \)
3180 \( \sin \eta \)
ORIGINAL PAGE 3
OF PHOTO QUALITY

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31G -LMOMAD)

260 FORMAT(1H1,**GEOEMIC ERROR SENSITIVITY**)

$1 WITH RESPECT TO ALTITUDE ERROR **********

$1 EARTH RADIUS R = F4.0*1KM */1 NOMINAL

$1 ALTITUDE H = F4.0*1KM */1 ALTITUDE ERROR 1 TMI = F5.2*1KM

$1 ALIITUDE ERROR 2 TMI = F5.2*1KM

$1 VIEW = 5X*ANGLE*/1 DLNGETA(00DEG) = 12X

$1 F2.O*Q X = F2.O*1KM */1 SHIFTED LOCATION*/1 DUE TO ERROR

$1 X1 (KM) = 10011.4 */1 SHIFTED LOCATION*/1 DUE TO

$1 ERROR 2 (KM) = 10011.4 */1 DIFFERENCE (KM)

$1 0011.4 */1 ERROR*/1 SENSITIVITY (KM/KM) = 10

DO 266 KM=OF

350 CONTINUE

360 FORMAT(1H1,**GEOEMIC ERROR SENSITIVITY**)

$1 ROLL ERROR **********

$1 EARTH RADIUS R = F4.0*1KM */1 NOMINAL ALTITUDE H = F4.0*1KM

$1 VIEW = 5X*ANGLE*/1 DLNGETA(00DEG) = 12X

$1 NOMINAL LOCATION (KM) = 10011.4 */1 SHIFTED

$1 LOCATION (KM) = 10011.4 */1 ERROR*/1 YX1 (KM) = 10011.4

$1 VIEW = 5X*ANGLE*/1 DLNGETA(00DEG) = 12X

$1 LOCATION (KM) = 9011.4 */1 ERROR*/1 YX1 (KM) = 9011.4

$1 LOCATION (KM) = 9011.4 */1 ERROR*/1 YX1 (KM) = 9011.4

DO 300 KM=OF

350 CONTINUE

360 FORMAT(1H1,**GEOEMIC ERROR SENSITIVITY**)

$1 ROLL ERROR **********

$1 EARTH RADIUS R = F4.0*1KM */1 NOMINAL ALTITUDE H = F4.0*1KM

$1 VIEW = 5X*ANGLE*/1 DLNGETA(00DEG) = 12X

$1 SHIFTED LOCATION*/1 DUE TO ERROR 1 (KM) = 10011.4

$1 SHIFTED LOCATION*/1 DUE TO ERROR 2 (KM)

$1 10011.4 */1 DIFFERENCE (KM) = 10011.4 */1

$1 VIEW = 5X*ANGLE*/1 DLNGETA(00DEG) = 12X

$1 10011.4 */1 DIFFERENCE (KM) = 10011.4 */1

$1 VIEW = 5X*ANGLE*/1 DLNGETA(00DEG) = 12X

$1 10011.4 */1 DIFFERENCE (KM) = 10011.4 */1

$1 ERROR*/1 SENSITIVITY (KM/DEG) = 10011.4 */1

300 CONTINUE
374 DO 321 K=1,NPSA
375 WRITE(*,320) RHYDPSA(K),(PLANDA(I)),L=1,MLANDA
376 320 FORMAT(15,1X,10F4.0)
377 321 CONTINUE
378
379 DO 400 K=1,NPSB
380 WRITE(*,310) RHYDPSB(K),(PLANDA(I)),L=1,MLANDA
381 310 FORMAT(15,1X,10F4.0)
382 311 CONTINUE
383
384 DO 490 K=1,NTHETA
385 WRITE(*,480) RHYDTHETA(K),(PLOMO(J)),J=1,MLANDA
386 480 FORMAT(15,1X,10F4.0)
387 490 CONTINUE
388
389 DO 500 K=1,NPSB
400 WRITE(*,300) RHYDPSB(K),(PLOMO(J)),J=1,MLANDA
401 300 FORMAT(15,1X,10F4.0)
402 301 CONTINUE
403
404 DO 500 K=1,NPSA
405 WRITE(*,320) RHYDPSA(K),(PLOMO(J)),J=1,MLANDA
406 320 FORMAT(15,1X,10F4.0)
407 321 CONTINUE
408
409 DO 500 K=1,NPSB
410 WRITE(*,480) RHYDTHETA(K),(PLOMO(J)),J=1,MLANDA
411 480 FORMAT(15,1X,10F4.0)
412 490 CONTINUE
413
414 DO 500 K=1,NPSA
415 WRITE(*,320) RHYDTHETA(K),(PLOMO(J)),J=1,MLANDA
416 320 FORMAT(15,1X,10F4.0)
417 321 CONTINUE
4900  $1  Y DIRECTION  (KM)$  1  $3X.3015.4///V$  TOTAL SHIFT$  
4910  $2  (KM)$  1  $3X.3015.4///V$  
4920  709 CONTINUE  
4930  4940  DO 707  KM1+NDelta  
4950  WRITE(6,706) R+Delta(K), Y+Delta(K), V+Delta(K), X+Delta(K), 
4960  $S+Delta(K), Y+Delta(K), V+Delta(K), X+Delta(K), 
4970  $S+Delta(K), Y+Delta(K), V+Delta(K), X+Delta(K), 
4980  706 FORMAT(1X,10F12.8)  
4990  5000  $5  BY SHUTTLE MOTION AND EARTH ROTATION$  
5010  $6  EARTH RADIUS = $7  KM$  ///  ORBIT$  
5020  $8  INCLINATION  = $9  DEGREES///  LATITUDE  = $A  DEGREES$  
5030  $B  DEGREES///  GROUND POINT SHIFT VELOCITY DUE TO EARTH$  
5040  $C  ROTATION  = $D  KM/SEC///  GROUND POINT SHIFT VELOCITY RELATIVE$  
5050  $E  TO NADIR POINT  = $F  KM$  
5060  $G  KM/SEC///  ANGLE OF VELOCITY VECTOR WITH Z AXIS///$  
5070  $H  DEGREES///  TIME INTERVAL (SEC) = $I  SEC$  
5080  $J  SHIFT IN///  X DIRECTION (KM)  = $K  KM$  
5090  $L  SHIFT IN///  Y DIRECTION (KM)  = $M  KM$  
5100  $N  TOTAL SHIFT (KM)  = $O  KM$  
5110  707 CONTINUE  
5120  5130  5140  STOP  
5150  END  

41 CPF=130  SUP=110,769  
* A SMUTEX; IMAGE/ABB  
235-JB  04/27/83  10131121  
2 SUP=736 CPU=.000  ID=302 CC=ERR=.333  

142
### Geometric Error Induced by Altitude Error

**Arith Radius: 6356.755km**

**Nominal Altitude: 1.0km**

<table>
<thead>
<tr>
<th>View Angle</th>
<th>0°</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>5°</th>
<th>7°</th>
<th>10°</th>
<th>20°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Location (km)</td>
<td>0.0000</td>
<td>0.0082×10^-3</td>
<td>0.1917×10^-3</td>
<td>0.2864×10^-3</td>
<td>0.3909×10^-3</td>
<td>0.4964×10^-3</td>
<td>0.5929×10^-3</td>
<td>0.6901×10^-3</td>
</tr>
<tr>
<td>Shifted Location (km)</td>
<td>0.0000</td>
<td>0.0073×10^-3</td>
<td>0.1682×10^-3</td>
<td>0.2651×10^-3</td>
<td>0.3621×10^-3</td>
<td>0.4591×10^-3</td>
<td>0.5561×10^-3</td>
<td>0.6531×10^-3</td>
</tr>
<tr>
<td>Error (km)</td>
<td>0.0000</td>
<td>0.0009×10^-3</td>
<td>0.0199×10^-3</td>
<td>0.0399×10^-3</td>
<td>0.0599×10^-3</td>
<td>0.0799×10^-3</td>
<td>0.0999×10^-3</td>
<td>0.1199×10^-3</td>
</tr>
<tr>
<td>Angle Error (°)</td>
<td>0.0000</td>
<td>0.0025</td>
<td>0.0050</td>
<td>0.0075</td>
<td>0.0250</td>
<td>0.0250</td>
<td>0.0250</td>
<td>0.0250</td>
</tr>
</tbody>
</table>

### Geometric Error Sensitivity with Respect to Altitude Error

**Earth Radius: 6356.755km**

**Nominal Altitude: 6483.4km**

**Altitude Error 1 (°): 0.05°**

**Altitude Error 2 (°): 0.05°**

<table>
<thead>
<tr>
<th>View Angle</th>
<th>0°</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>5°</th>
<th>7°</th>
<th>10°</th>
<th>20°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shifted Location Due to Error 1 (km)</td>
<td>0.0000</td>
<td>0.0094×10^-3</td>
<td>0.1597×10^-3</td>
<td>0.2190×10^-3</td>
<td>0.2703×10^-3</td>
<td>0.3216×10^-3</td>
<td>0.3729×10^-3</td>
<td>0.4242×10^-3</td>
</tr>
<tr>
<td>Shifted Location Due to Error 2 (km)</td>
<td>0.0000</td>
<td>0.0082×10^-3</td>
<td>0.1397×10^-3</td>
<td>0.2081×10^-3</td>
<td>0.2765×10^-3</td>
<td>0.3449×10^-3</td>
<td>0.4133×10^-3</td>
<td>0.4817×10^-3</td>
</tr>
<tr>
<td>Difference (km)</td>
<td>0.0000</td>
<td>0.0002×10^-3</td>
<td>0.0000×10^-3</td>
<td>0.0000×10^-3</td>
<td>0.0000×10^-3</td>
<td>0.0000×10^-3</td>
<td>0.0000×10^-3</td>
<td>0.0000×10^-3</td>
</tr>
<tr>
<td>Error Sensitivity (km/km)</td>
<td>0.0000</td>
<td>0.0043×10^-3</td>
<td>0.0065×10^-3</td>
<td>0.0087×10^-3</td>
<td>0.0109×10^-3</td>
<td>0.0130×10^-3</td>
<td>0.0151×10^-3</td>
<td>0.0172×10^-3</td>
</tr>
</tbody>
</table>

143
*************** GEOMETRIC ERROR INDUCED BY FULL ENUX ***************

EARTH RADIUS A = 6356.755KM

NOMINAL ALTITUDE = 0.000KM

FULL ENUX OF .1 DEGREE

VIEW ANGLE OLOMOA(DEGREES) = 0°  90°  180°  270°


ERROR (KM) = +7153.001  +7153.001  +7153.001  +7153.001  +7153.001  +7153.001  +7153.001  +7153.001


VIEW ANGLE OLOMOA(DEGREES) = 90°  180°  270°


ERROR (KM) = +4914.002  +5625.002  +7153.001  +7153.001  +7153.001  +7153.001  +7153.001  +7153.001


*************** GEOMETRIC ERROR SENSITIVITY WITH RESPECT TO FULL ENUX ***************

EARTH RADIUS A = 6356.755KM

NOMINAL ALTITUDE = 0.000KM

FULL ENUX ± 0.1 DEGREE

VIEW ANGLE OLOMOA(DEGREES) = 0°  90°  180°  270°


SHIFTED LOCATION DUE TO ERROR 2 (KM) = +563.002  +563.002  +482.002  +482.002  +482.002  +482.002  +482.002  +482.002

DIFFERENCE (KM) = 10.0400  +10.0400  +10.0400  +10.0400  +10.0400  +10.0400  +10.0400  +10.0400

ERROR SENSITIVITY(°/DEG) = 1.08  0.54  0.54  0.54  0.54  0.54  0.54  0.54

VIEW ANGLE OLOMOA(DEGREES) = 90°  180°  270°

SHIFTED LOCATION DUE TO ERROR 1 (KM) = +7052.002  +7052.002  +7052.002  +7052.002  +7052.002  +7052.002  +7052.002  +7052.002

SHIFTED LOCATION DUE TO ERROR 2 (KM) = +5625.002  +5625.002  +5625.002  +5625.002  +5625.002  +5625.002  +5625.002  +5625.002

DIFFERENCE (KM) = -1397.000  +1397.000  +1397.000  +1397.000  +1397.000  +1397.000  +1397.000  +1397.000

ERROR SENSITIVITY(°/DEG) = -0.092  +0.092  +0.092  +0.092  +0.092  +0.092  +0.092  +0.092

144
### Geometric Error Sensitivity with Respect to Tan Error

**Earth Radius:** 6356.756 km

**Nominal Altitude:** 400 km

**Tan Error 1 (DPS1) = 0.1° (Degree)**

<table>
<thead>
<tr>
<th>View Angle (Degree)</th>
<th>0°</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
<th>6°</th>
<th>7°</th>
<th>8°</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Error (km) 1</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Tan Error 2 (DPS2) = 0.1° (Degree)**

<table>
<thead>
<tr>
<th>View Angle (Degree)</th>
<th>0°</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
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<th>6°</th>
<th>7°</th>
<th>8°</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Shift (km)</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Error Sensitivity (km/°) 2</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**146**
### Earth Radii
- X Coordinate: 0.00
- Y Coordinate: 0.00
- Z Coordinate: 0.00

### Angular Errors
- Pitch: 0.00 Degree
- Roll: 0.00 Degree
- Yaw: 0.00 Degree

### Coordinate System
- X: 0.00 x 0.00 x 0.00
- Y: 0.00 x 0.00 x 0.00
- Z: 0.00 x 0.00 x 0.00

### Ground Point Shift

#### Earth Radius: 2,667,470

#### Orbit Inclination: 0.00 Degree

#### Latitude: 0.00 Degree

#### Ground Point Shift Velocity Due to Earth Rotation: 0.0002 m/sec

#### Shuttle Ground Speed: 0.7024 m/sec

#### Ground Point Shift Velocity Relative to Vehicle (North): 0.0004 m/sec

#### Angle of Velocity Vector with Vehicle: 0.00032 Degree

### Time Interval (Sec)
- 0.00

#### Shift in X Direction
- 0.00

#### Shift in Y Direction
- 0.00

#### Total Shift
- 0.00

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**ORIGINAL PAGE 13**

**OF POOR QUALITY**
### Ground Point Shift Induced by Shuttle Motion and Earth Rotation

**Earth Radius**: 6356.710km

**Orbit Inclination**: 0.00 DEGREE

**Latitude**: 0.00 DEGREE

**Ground Point Shift Velocity Due to Earth Rotation**: 0.000000 km/sec

**Shuttle Ground Speed**: 0.7202801 km/sec

**Ground Point Shift Velocity Relative to Radio Point**: 0.7195401 km/sec

**Angle of Velocity Vector with T-axis**: 0.0000000 DEGREE

<table>
<thead>
<tr>
<th>Time Interval (Sec)</th>
<th>1.00</th>
<th>0.10</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift in Z direction (km)</td>
<td>0.7192001</td>
<td>0.7192000</td>
<td>0.7192000</td>
</tr>
<tr>
<td>Shift in Y direction (km)</td>
<td>0.7192000</td>
<td>0.7192000</td>
<td>0.7192000</td>
</tr>
<tr>
<td>Total Shift (km)</td>
<td>0.7195401</td>
<td>0.7195400</td>
<td>0.7195400</td>
</tr>
</tbody>
</table>

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### Ground Point Shift Induced by Shuttle Motion and Earth Rotation

**Earth Radius**: 6356.710km

**Orbit Inclination**: 0.00 DEGREE

**Latitude**: 0.00 DEGREE

**Ground Point Shift Velocity Due to Earth Rotation**: 0.000000 km/sec

**Shuttle Ground Speed**: 0.7202801 km/sec

**Ground Point Shift Velocity Relative to Radio Point**: 0.7195401 km/sec

**Angle of Velocity Vector with T-axis**: 0.0000000 DEGREE

<table>
<thead>
<tr>
<th>Time Interval (Sec)</th>
<th>1.00</th>
<th>0.10</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift in Z direction (km)</td>
<td>0.7192001</td>
<td>0.7192000</td>
<td>0.7192000</td>
</tr>
<tr>
<td>Shift in Y direction (km)</td>
<td>0.7192000</td>
<td>0.7192000</td>
<td>0.7192000</td>
</tr>
<tr>
<td>Total Shift (km)</td>
<td>0.7195401</td>
<td>0.7195400</td>
<td>0.7195400</td>
</tr>
<tr>
<td>Time Interval (sec)</td>
<td>1.00</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>---------------------</td>
<td>------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td><strong>Shift in X direction (m)</strong></td>
<td>0.71614989</td>
<td>0.71426680</td>
<td>0.71303508</td>
</tr>
<tr>
<td><strong>Shift in Y direction (m)</strong></td>
<td>0.2276665</td>
<td>0.2276665</td>
<td>0.2276665</td>
</tr>
<tr>
<td><strong>Total Shift (m)</strong></td>
<td>0.71641639</td>
<td>0.71453340</td>
<td>0.71330068</td>
</tr>
</tbody>
</table>

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**ORIGINAL PAGE #: OF POOR QUALITY**
APPENDIX F

GROUND PATTERN AND IMAGING

DISTORTIONS GENERATION PROGRAM LISTINGS

0000 I 006618 IM 0000 I 006621 IM1 0008 I 006754 JPT 0010 I 0

0000 I 007512 J 0000 I 000604 J0 0000 I 005742 J6 0000 I 0

0002 I 005714 JF 0000 R 000000 LMDA 0000 I 005753 LFLAG 0000 I 0

0000 K 000003 LMDA 0000 R 000002 LMDA 0000 I 001747 HFLAG 0000 X 0

0000 I 005673 NGY 0000 I 005674 NGY 0000 R 000000 MLN 0000 I 0

0000 I 006665 NLIN3 0000 I 006670 NLIN4 0000 I 006703 NLIN5 0000 I 0

0000 I 006647 NTIC 0000 I 006653 NTIC1 0000 I 006656 NTIC2 0000 I 0

0000 I 006672 NTIC5 0000 I 006681 NTIC6 0000 I 006775 NTIC7 0000 I 0

0000 I 006026 MEGA 0000 R 005646 PD 0000 R 005654 P1 0000 R 0

0000 R 005630 R 0000 R 005767 RA 0000 R 005637 RAA 0000 R 0

0000 R 005751 RM01 0000 R 006000 ROLL 0000 R 005656 RS 0000 R 0

0000 R 005632 RM1 0000 R 005633 RM2 0000 R 005634 RM3 0000 R 0

0000 R 006652 TIC1 0000 R 006655 TIC2 0000 R 006663 TIC3 0000 R 0

0000 R 006656 TIC6 0000 R 006674 TIC7 0000 R 005730 TI 0000 R 0

0000 R 005747 T21 0000 R 006026 V 0000 R 005636 VS 0000 R 0

0000 R 006034 VTY 0000 R 005631 VA 0000 R 006032 VV 0000 R 0

0000 R 005740 VB 0000 R 006040 VB 0000 R 005756 VB 0000 R 0

0000 R 005664 XCENT 0000 R 005757 XC1 0000 R 005220 XP 0000 R 0

0000 R 003262 X 0000 R 005612 XG 0000 R 004610 XG1 0000 R 0

0000 R 005754 YII 0000 R 005770 YI 0000 R 006003 YX 0000 R 0

0000 R 005675 XLEFT 0000 R 005627 XLEN 0000 R 006017 XL 0000 R 0

0000 R 005767 XCM 0000 R 002020 XP 0000 R 002020 XP 0000 R 0

0000 R 006662 XP4 0000 R 006015 XP5 0000 R 005712 XP6 0000 R 0

0000 R 005752 SYST 0000 R 005631 SYST 0000 R 005774 XT 0000 R 0

0000 R 005720 Y7 0000 R 005042 Y 0000 R 005725 YA 0000 R 0

0000 R 005651 YAVD 0000 R 005652 YAV 0000 R 005745 YAV 0000 R 0

0000 R 005700 YD8 0000 R 005755 YD 0000 R 005703 YC 0000 R 0

0000 R 005760 YC1 0000 R 005764 YD 0000 R 05411 YD 0000 R 0

0000 R 003453 YF 0000 R 001003 YG 0000 R 005001 YG 0000 R 0

0000 R 004753 Y11 0000 R 005771 YJ 0000 R 006002 YK 0000 R 0

0000 R 005670 YLEN 0000 R 006020 YL1 0000 R 001365 YN 0000 R 0

0000 R 006025 YP 0000 R 006025 YP6 0000 R 002201 YP 0000 R 0

0000 R 003247 YP5 0000 R 005645 YSTP 0000 R 005634 YSTAR 0000 R 0

0000 R 005701 YTOP 0000 K 005713 YW 0000 R 000000 YV 0000 R 0

100 1 10 C DECLARE PARAMETERS
101 20 PARAMETER YLINES=11+NDT=+400+YLM=460+YLM=330+
101 30 SYLM=150+YLM=460+YLM=440+
103 40 REAL LMDA+LMDA+LMDA+LMDA+
104 50 INTEGER FLA+FLA+
105 60 C DIMENSION OF VARIABLES
125 90 DIMENSION XN=23+Y1=2+XYP=1+XYP=1+XYP=1+XYP=1+XYP=1+
105 100 SYL IN=1+SYL IN=1+SYL IN=1+SYL IN=1+SYL IN=1+
105 110 SYL INPUT=1+SYL INPUT=1+SYL INPUT=1+SYL INPUT=1+
105 120 SYL INPUT=1+SYL INPUT=1+SYL INPUT=1+SYL INPUT=1+
105 130 XYP=1+XYP=1+XYP=1+XYP=1+XYP=1+XYP=1+XYP=1+
105 140 XYP=1+XYP=1+XYP=1+XYP=1+XYP=1+XYP=1+XYP=1+
105 150 XYP=1+XYP=1+XYP=1+XYP=1+XYP=1+XYP=1+
150 170 SYL INPUT=1+SYL INPUT=1+SYL INPUT=1+SYL INPUT=1+

150
C0105  18*  $\{G\{NLINES\},NLINES\},YG\{NLINES\},NLINES\},XP6\{NLINES\},0
C0105  19*  SYPLN\{NLINES\},XOR\{NLINES\},NLINES\},YDB\{NLINES\},NLINES\}
C0105  20*  SAFLAG\{NLINES\},NLINES\},SYPLN\{NLINES\},XP7\{NLINES\},0
C0106  21*  DATA R=LPI,PI,TYD,YSKVS=605,1567356,7853,1415926540
C0106  22*  $1000000,0000,0000,0000,0039,007/
C0106  23*  DATA EPS=REAL,STAP,TSTAP,PS,PS,FLAG,00210
C0106  24*  S=5,,434,7,, 0055,665,72
C0106  25*  DATA YSTAY=YSK/YSK/YSK/5,,0007,022/
C0106  26*  DATA DROLL=YSTD/95/95/95/0042,1/
C0107  26*  DATA STAU=DATA*DATA*DATA/85,,80,6,,5,,7,,022/
C0108  29*  C  INITIALIZE THE PLOTT
C0108  30*  CALL EPSPLT
C0108  31*  CALL EPSPLT
C0108  32*  C  CENTER PLOT
C0108  33*  XCENT=50
C0108  34*  YCENT=10
C0108  35*  CALL ORIGIN\[XCENT,YCENT]
C0108  36*  C  DEFINE THE FORM OF THE PLOT
C0108  37*  PTYPE=1,LIN
C0108  38*  XLEN=X0
C0108  39*  YLEN=Y0
C0108  40*  CALL PLOM(PTYPE,XLEN,YLEN)
C0108  41*  C  SCALE THE PLOT
C0108  42*  AX(0)=2
C0108  43*  AY(2)=8
C0108  44*  AY(3)=8
C0108  45*  CALL PLOM(AX,AY)
C0108  46*  C  TO START PLOTTING
C0108  47*  CALL EPSGRAF
C0108  48*  C DEFINE GRIND LIMITS
C0108  49*  XLEFT=5
C0108  50*  XRIGHT=5
C0108  51*  CALL EPSGRAF
C0108  52*  YBOT=5
C0108  53*  YTOP=5
C0108  54*  CALL EPSGRAF
C0108  55*  YTOP=4
C0108  56*  YTOP
C0108  57*  C  CONSTRUCT DATA
C0108  58*  YC=YTOP
C0108  59*  DO 150 I=1,NLINES
C0108  60*  XCL=LEFT
C0108  61*  DO 100 IPT=1,NLINES
C0108  62*  XNOM=X+XNOM
C0108  63*  YNOM=YC
C0108  64*  DO 100 I=1,NLINES
C0108  65*  XNOM=X+XNOM
C0108  66*  YNOM=YC

151
0212 75* \( X(\text{INOR},IPT)=X_{\text{DOM}} \)
0213 76* \( Y(\text{INOR},IPT)=Y_{\text{DOM}} \)
0214 77* \( X(\text{COR},YDEL) \)
0215 78* 100 CONTINUE
0217 79* \( Y(\text{YC}-YDEL) \)
0220 80* 150 CONTINUE
0222 81* IF(FLAG..EQ.,1) GO TO 200
0224 82* IF(FLAG..EQ.,2) GO TO 208
0226 83* IF(FLAG..EQ.,3) GO TO 231
0228 84* IF(FLAG..EQ.,4) GO TO 251
0232 85* GO TO 261
0233 86* 230 CONTINUE
0234 87* DO 202 I=1,MINES
0242 93* MFLAG(I,J).=0
0243 94* 201 CONTINUE
0245 95* 202 CONTINUE
0247 96* \( \text{Y}X\text{YSTAR} \)
0250 97* DO 206 L=1,NLT
0253 98* BLAMDA=DL
0254 99* DO 205 K=1,NLAMBA
0260 93* X=W+(1+H/R)*COS(LAMBA)-SORT(1-E2+H/R)*E2+H/R*
0260 94* \( \sin(LAMBA)+2\sin(LAMBA)) \sin(LAMBA) \)
0261 100* DO 201 I=1,MINES
0264 101* DO 70 J=1,MINES
0267 102* \( Z=W-(1+J) \)
0270 103* Y2=V-(1+J)
0271 104* JR=1
0272 105* RX=1
0273 106* IFABS(Y2).LE.EPS1 \&\&ABS(Y3).LE.EPSI GO TO 204
0275 107* 70 CONTINUE
0277 108* 80 CONTINUE
0301 109* GO TO 99
0302 110* 204 MFLAG(I,JR)=1
0303 111* \( \text{XFIR}+(J+J)=(-\text{TAM}(	ext{LAMBA})) \)
0304 112* \( \text{YFIR}+3=\text{YSTAR}+(L+L)-1-1 \)-O228
0306 113* WRITE(6,93) \( \text{XFIR}, \text{J}, \text{K}, \text{L}, \text{M}, \text{N}, \text{O} \)
0316 114* \( \text{98 FOR}^4(1,2\text{F})=(2+1.2+3,2+1,3) \)
0317 115* \( \text{99 BLAMDA}=\text{LAMBA}-99 \)
0320 116* 205 CONTINUE
0322 117* \( \text{Y}X\text{YSTAR}+L=DT+45 \)
0323 118* 206 CONTINUE
0325 119* \( \text{LAMDA}=\text{LAMDA}-99 \)
0326 120* 208 CONTINUE
0327 121* DO 2 I=1,MINES
0332 122* DO 1 J=1,MINES
0335 124* MFLAG(I,J).=0
0336 125* 1 CONTINUE
0346 126* 2 CONTINUE
0342 127* \( \text{DTHETA}=\text{C} \)
0343 128* YASTAP
0344 129* \( \text{DO} \text{217 L}=1,99 \)
0347 130* \( \text{THETA}=\text{DTHETA}=\text{PI}/180 \)
0350 131* \( \text{BLAMDA}=\text{PS} \)
ORIGINAL PAGE 19
OF POOR QUALITY

0351 132* DO 216 K=1,NLPMDA
10354 133* LPMDA=BLPMDA+PI+10.*
*0355 134* T1=EXP(TAN(THETA))
0356 135* T2=ABS(COS(THETA))*TAN(LPMDA)
0357 136* DEL=SQRT(T1*T1+T2*T2)
10358 137* RHO=EXP(DEL/PO)
0359 138* SQRT=EXP(E1+9/R3)*COS(RHO)*SQRT(1-(1+9/R3)*(1+9/R3)+SIN
0360 139* RHO)*SIN(RHO1)+SIN(THETA))
0361 139* 210
0362 140* YI=SQURT+T1/DEL
0463 141* XI=SQURT+T2/DEL
0364 142* YG=1.1*YA
0365 143* KG=XI
0366 144* DO 22 I=1,NLINES
10367 145* DO 11 J=1,NLINES
0374 146* CXRB=XX1+J
0375 147* YC=YA-Y(I+J)
10376 148* IG=I
0377 149* JG=J
0400 150* IFABS(YC).LE.EPSI.AND.ABS(YC).LE.EPSI3 GO TO 210
11 CONTINUE
0404 151* 22 CONTINUE
10406 152* GO TO 215
10407 152* 215
0410 155* NFLAG(IG,JG)=1
0411 156* YG(IG,JG)=YBOU*E(L-1)*0.088
10412 158* CTLE=1235*XR*YD*ID*JG*BLPMDA+XG(IG,JG)*YD(IG,JG)
0425 158* 216
0426 159* DLPMDA=BLPMDA-PO
0425 160* 216 CONTINUE
10427 161* YA=YSTAP+L*0.088
0430 162* DTHETA=RPC/5000.
0431 163* 217 CONTINUE
0433 164* 217 CONTINUE
0434 165* DO 220 I=1,NLINES
0436 166* DO 219 J=1,NLINES
10442 166* NFLAG(I,J)=0
0443 169* 219 CONTINUE
0445 170* 220
10447 171* TENT=DFTHETA+PI/10.
0449 172* ET=DET+PI/10.
0451 173* Y1=YSTAP1
0452 174* DO 227 L=1,NOT
0455 175* DLPMDA=PS
10456 176* DO 226 K=1,NLPMDA
0461 177* LPMDA=DLPMDA+PI/10.
0462 178* T11=EXP(TAN(THETA))
0463 179* T21=ABS(COSRHO)*ET)*TAN(LPMDA)
0464 180* DEL=SQRT(T11*T11+T21*T21)
0468 181* RHO=EXP(DEL/PO)
0466 182* SQURT=EXP(E1+9/R3)*COS(RHO)*SQRT(1-(1+9/R3)*(1+9/R3)+SIN
0466 183* RHO)*SIN(RHO1)+SIN(THETA))
0467 184* Y11=SQURT+T11/DEL
0470 145* XI1=SQURT+T21/DEL
10471 156* YD1+Y11=YA1
0472 187* }01=11

153
0473 193+  DO 222 I=1, NLMINES
0476 196+  DO 221 J=1, NLMINES
0501 190+  I(TAB(I+1)+F(J+1))
0502 190+  YG(I+1)+F(J+1)
0503 190+  IGI=I
0504 193+  JGI=J
0505 190+  IF ABS(YC1)
0506 195+  LE.EPSI. AND. ABS(YC1)
0507 195+  LE.EPSI) GO TO 223
0511 196+  222 CONTINUE
0513 197+  223 CONTINUE
0514 190+  223 VFLAG(I+1)+JG1)+1
0515 193+  223 YGI(I+1)+JG1)+YTAG+l=l+2)
0516 190+  223 YGI(I+1)+JG1)+YSTAP1+3(L-1)+0288+Y11
0517 201+  223 WRITE(6,224) YB1, YB1, XG1, YGI(I+1)+JG1)+XG1(I+1)+JG1)+
0518 202+  223 YG(I+1)+I.G1)
0520 203+  224 FORMAT(6,2F10.5,2E+3F10.5)
0531 204+  225 DLPMDA=DLPMDA-PD
0532 205+  226 CONTINUE
0534 206+  226 YAI=YSTAP1*L+0298
0535 207+  227 CONTINUE
0537 208+  227 GO TO 605
0540 209+  231 CONTINUE
0541 210+  231 DO 4 I=1, NLMINES
0544 212+  231 DO 3 J=1, NLMINES
0547 213+  232 KFLAG(I+J)=0
0550 214+  3 CONTINUE
0552 215+  4 CONTINUE
0556 216+  4 DTA=0.
0559 217+  4 YD=YSTAY
0563 218+  5 DO 190 L=1, NDLT
0566 219+  5 YD=YTAB+PI/180
0567 220+  5 DLMDA=DLMDA+DAS
0570 221+  5 DO 230 E=1, NLMRN
0576 222+  5 DLYMDA=DLYMDA+PI/180.
0577 223+  5 RA=R+(1+O/R)+COS(LYMDA)-SRT4I-(1+O/R)+(1+O/R)+SIN(LYMDA)
0578 224+  5 +SIN(LYMDA)+SIN(LYMDA)
0580 225+  5 E=RA+G3413A4)
0581 226+  5 J=RA+G3I3A(YA6)
0582 227+  5 E=J
0583 228+  5 YD=YA+Y0
0584 229+  5 D0 44 I=1, NLMINES
0587 230+  5 DO 33 J=1, NLMINES
0602 231+  5 X=YE-YJ+J)
0603 232+  5 YE=YE-YJ+J)
0604 233+  5 I=H
0605 234+  5 I=H
0606 235+  5 IFABS(YT)+LE.EPSI. AND. ABS(YT)+LE.EPSI) GO TO 233
0610 236+  33 CONTINUE
0612 237+  33 CONTINUE
0614 238+  44 CONTINUE
0615 239+  44 GO TO 235
0620 242+  233 VFLAG(IN+JMJ2)
0621 241+  233 YHEJ(IN+JMJ2)+D=0.TAN(LYMDA))
0627 241+  233 YHEJ(IN+JMJ2)+YROT+3(L-1)+0288
0620 242+  233 WRITE(6,234) YE=YE-IN+JMJ2+OLYMDA+KIN+JMJ2+WH(IN+JMJ)
0631 243+  234 FORMAT(6,2F10.5,3F10.5)
0632 244+  235 DLYMDA=DLYMDA+YA+9
238 CONTINUE
        YD=YSTAY+L*0.089
239        DY=5.*SIN((L+PI)/174.)
240 CONTINUE
        GO TO 249

241 CONTINUE
        DO 6 1=1,NLINES
243 4 6 CONTINUE
        DO 5 1=1,NLINES
245 5 5 CONTINUE
        LFLAG(1,1:J)=0
247 6 CONTINUE
        5 CONTINUE
249 CONTINUE
        ROLL=ROLL+PI/16.
250       XSTAO=R*(1+D/R)*COS(ROLL)-SQR((1-(1+D/R)*(1+D/R)*SIN(ROLL))
251       5+Sin(ROLL)))+SIN(ROLL)
252       VY=YSTAO
253       DO 250 L=1,N
254 250       DLAMDA=LS
256       DO 256 K=1,NL
258       LR=DLAMDA+PI/16.
260       DIS=R*(1+D/R)*COS(LR)+ROLL)-SQR((1-(1+D/R)*(1+D/R)*SIN(LR)+ROLL))
262       5+Sin(LR+ROLL)))+SIN(LR)+ROLL)
264       X=DIS*XSTAO
265       DO 265 K=1,NL
267 265       YK=YSTAO
269       DO 269 L=1,N
271       XL=YY-XK-LJ
273       YL=YY-YK-LJ
275       IF(E4B.5(LJ+LE_EPSI.AYD.AUS(YL)+LE_EPSI.50 TO 242
277       55 CONTINUE
279       66 CONTINUE
281       GO TO 244
283       56 CONTINUE
285       GO TO 246
287       57 CONTINUE
289       DO 6 1=1,NLINES
291 6 CONTINUE
        DO 7 1=1,NLINES
293 7 CONTINUE
        LFLAG(1,1:J)=0
295 8 CONTINUE
        ROLL=ROLL+PI/189.
297 9 CONTINUE
        DERR=0.
299       XSTAO=R*(1+D/R)*COS(ROLL)-SQR((1-(1+D/R)*(1+D/R)*SIN(ROLL))
301       5+Sin(ROLL)))+SIN(ROLL)
303       VY=YSTAO
305       DO 260 L=1,N
307       ER=ER+PI/189.
309 300 CONTINUE
ORGINAL PAGE IS OF POOR QUALITY.

1111 355  XE$YB=V(1+J)
1112 359  YC$YB=V(1+J)
1113 363  ID$=I
1114 367  JOD$J
1115 361  IF(ANS(YCC)=.LE.EPSI.AND.ANS(YCC)=.LE.EPSI.) GO TO 266
1117 363  CONTINUE
1121 364  263 CONTINUE
1123 365  GO TO 268
1124 366  266 AFLAG=I+J+L
1125 367  XOD$=I+J+L+10*ITOM$=I+J+L+20*ITOM$=10*ITOM$=I+J+L
1126 366  YOD$=I+J+L
1127 370  WRITE(6,627) XOD$=I+J+L+10*ITOM$=I+J+L+20*ITOM$=10*ITOM$=I+J+L
1128 372  ND$=I+J+L+10*ITOM$=I+J+L+20*ITOM$=10*ITOM$=I+J+L
1130 373  267 FORMAT(F7.3,F4.1,F4.1,F4.1)
1141 372  268 OLYMA=OLYMA-FF
1142 373  269 CONTINUE
1144 374  4AAXYSTA=L+DT+VT
1145 375  YAAYSTA=L+DT+VT
1146 376  270 CONTINUE
1150 377  C PLOT HORIZONTAL LINES
1155 380  299 CONTINUE
1159 383  390
1161 394  IC=TIC=1
1162 395  392 MLINE=MLINES
1164 397  393 DO 400 IMOR=1+MLINES
1169 400  395 IPL$=I+IMOR
1171 401  396 XPF(IPL$)=Y(IPL$)
1175 405  397 CONTINUE
1177 407  399 IF(IFLAG.EQ.1) GO TO 655
1201 397  399 IF(IFLAG.EQ.2) GO TO 805
1203 399  400 IF(IFLAG.EQ.3) GO TO 501
1207 400  401 IF(IFLAG.EQ.4) GO TO 609
1210 401  402 DO 500 JTCP=1+MLINES
1214 500  404 CONTINUE
1215 405  404 \hline 500 \hline
1217 405  405 \hline 500 \hline
1221 405  405 \hline 500 \hline
1225 405  405 \hline 500 \hline
1227 405  405 \hline 500 \hline
1210 C PLOT VERTICAL LINES
1213 404 \hline 500 \hline
1215 405 \hline 500 \hline
1221 405 \hline 500 \hline
1225 405 \hline 500 \hline
1227 405 \hline 500 \hline
1231 415- C PLOT VERTICAL LINES
1231 416- DO 300 IVERT=1:NLINES
1231 417- CALL PLCURV(AF1,IVERT),YF(I,VERT),NLIN1,NLIC1,TIC1
1232 418- 800 CONTINUE
1233 419- GO TO 9999
1240 420- C PLOT HORIZONTAL LINES
1240 421- 205 CONTINUE
1241 422- TIC2=0
1242 423- NTIC2=0
1243 424- NLIN2=NLINES
1244 425- DO 250 IHP=1:NLINES
1245 426- XP2(IHP)=YG(IHP,YOR,IP1)
1246 427- YP2(IHP)=YG(IHP,YOR,IP1)
1247 428- 200 CONTINUE
1248 429- CALL PLCURV(YP2,YP2,NLIN2,NTIC2,TIC2)
1249 430- 950 CONTINUE
1261 431- C PLOT VERTICAL LINES
1261 432- DO 951 IVERT=1:NLINES
1262 433- CALL PLCURV(YG1,IVERT),YG1,IVERT),NLIN2,NTIC2,TIC2)
1263 434- 651 CONTINUE
1267 435- GO TO 9999
1270 440- C PLOT HORIZONTAL LINES
1270 441- TIC6=0
1271 442- NTIC6=0
1272 443- NLIN6=NLINES
1273 444- DO 255 ICR=1:NLINES
1274 445- DO 453 IPT=1:NLINES
1275 446- <P6(IPT)=YG(IPT,YOR,IP1)
1276 447- YP6(IPT)=YG(IPT,YOR,IP1)
1277 448- 953 CONTINUE
1278 449- CALL PLCURV(XP6,YP6,NLIN6,NTIC6,TIC6)
1280 450- 554 CONTINUE
1302 460- C PLOT VERTICAL LINES
1302 461- DO 955 IVERT=1:NLINES
1303 462- CALL PLCURV(YG11,IVERT),YG11,IVERT),NLIN6,NTIC6,TIC6)
1304 463- 655 CONTINUE
1307 464- GO TO 9999
1317 465- C PLOT HORIZONTAL LINES
1317 466- 901 CONTINUE
1322 467- TIC3=3
1323 468- NTIC3=0
1324 469- NLIN3=NLINES
1325 470- DO 905 INCR=1:NLIN3
1326 471- CALL PLCURV(YP3,YP3,NLIN3,NTIC3,TIC3)
1332 472- 905 CONTINUE
C    PLOT VERTICAL LINES
01340 471* C    DO 907 IVERT=1,NLINES
01341 472*    CALL PLCURVE(X1,IVERT),YM1(IVERT),NLINES,NTIC3,TIC3)
01342 473* 907 CONTINUE
01343 474*    GO TO 949
01344 475* C    PLOT HORIZONTAL LINES
01345 476*    911 CONTINUE
01346 477*    TIC4=**
01347 478*    NTIC4=1
01348 479*    NLIN4=NLINES
01350 480*    DO 915 IHLOR=1,NLINES
01351 481*    DO 913 IPT=1,NLINES
01352 482*    (P4(IPT)=YM1(IHLOR+IPT)
01353 483*    YP4(IPT)=YM1(IHLOR+IPT)
01355 484* 915 CONTINUE
01356 485*    CALL PLCURVEXP4,YP4,NLINES,NTIC4,TIC4
01357 486* 919 CONTINUE
01358 487* C    PLOT VERTICAL LINES
01359 488*    DO 917 IVERT=1,NLINES
01360 489*    CALL PLCURVE(X1,IVERT),YM1(IVERT),NLINES,NTIC4,TIC4)
01361 490*    917 CONTINUE
01362 491*    TIC5=**
01363 492*    NTIC5=0
01364 493*    NLIN5=NLINES
01366 494*    DO 925 IHLOR=1,NLINES
01367 495*    DO 923 IPT=1,NLINES
01368 496*    (P5(IPT)=YM1(IHLOR+IPT)
01369 497*    YP5(IPT)=YM1(IHLOR+IPT)
01371 498* 919 CONTINUE
01372 499*    CALL PLCURVEXP5,YP5,NLINES,NTIC5,TIC5)
01373 500*    919 CONTINUE
01374 501* C    PLOT VERTICAL LINES
01375 502*    DO 92C IVERT=1,NLINES
01376 503*    CALL PLCURVE(X1,IVERT),YM1(IVERT),NLINES,NTIC5,TIC5)
01377 504* 920 CONTINUE
01378 505*    TIC7=0
01379 506*    NTIC7=0
01380 507*    NLIN7=NLINES
01381 508*    DO 923 IHLOR=1,NLINES
01382 509*    DO 922 IPT=1,NLINES
01383 510*    (P7(IPT)=YD1(IHLOR+IPT)
01384 511*    YP7(IPT)=YD1(IHLOR+IPT)
01385 512* 922 CONTINUE
01386 513*    CALL PLCURVEYP7,YP7,NLINES,NTIC7,TIC7)
01387 514* 923 CONTINUE

159
1447 527*   C   PLOT VERTICAL LINES
1447 529*     DO 924 IVENT=1,NLINES
1452 530*     CALL PLOT(V5D(I+IVENT),YDD(I+IVENT),NLINES,NLINES)
1453 531*     924 CONTINUE
1455 532*     C   TO FINISH PLOTTING
1455 533*     999 CALL LNDVT
1456 534*     **   STOP
1457 537*     END

ND FLR:
PUU1=399 CJP=115 SUPS=9=131
DELETE=4 SHUT=YP,IMAGE=1/4BS
UPFUR 2=3-JB 05/19/63 11:56:41
TP= .333 SUP=.737 CPU=.000 IO=.363 CC-ERR=.433
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