

N85-25868

LAYERED COMPOSITE ANALYSIS CAPABILITY

FOR NASTRAN

Dr. R. (Swami) Narayanaswami and Mr. J. G. Cole
Computerized Structural Analysis and Research Corporation

SUMMARY

Laminated composite material construction is gaining popularity within industry as an attractive alternative to metallic designs where high strength at reduced weights is of prime consideration. This has necessitated the development of an effective analysis capability for the static, dynamic and buckling analyses of structural components constructed of layered composites. Theoretical and user aspects of layered composite analysis and its incorporation into C.S.A.R. Corporation's proprietary version of the NASTRAN® program, CSA/NASTRAN*, are discussed. The availability of stress based and strain based failure criteria is described which aids the user in reviewing the voluminous output normally produced in such analyses. Simple strategies to obtain minimum weight designs of composite structures are discussed. Several example problems are presented to demonstrate the accuracy and user convenient features of the capability.

INTRODUCTION

As structural designers turn more often to high strength light weight composite materials to solve critical design problems, the internal loads and stress analysis tasks become more complex. The finite element models generated to describe composite laminates using existing NASTRAN capabilities either require a considerable number of membrane elements "stacked" on top of one another to represent the plies or require some form of lumping/delumping procedure when using a single element to represent the plies. Both methods of modeling are used to overcome the substantial amount of work involved in determining the material properties to be referenced by an element. Neither method is without its drawbacks. The "stacked" membrane model requires a considerable number of elements and neglects bending effects while the single element representation requires pre-processing functions to generate an appropriate set of element and material properties and also requires post-processing to obtain the individual ply stresses and strains. The

*CSA/NASTRAN is an advanced proprietary version of the NASTRAN® general purpose structural analysis program. NASTRAN is a registered trademark of NASA. CSA/NASTRAN is developed, maintained and marketed by the C.S.A.R. Corporation, Northridge, California.

method of composite material construction analysis discussed in this work and incorporated into CSA/NASTRAN employs the single element technique. The user describes the laminate by means of new PCOMP, PCOMP1, PCOMP2 and MAT8 bulk data cards.

The finite elements used to model the composite structure must be capable of representing coupling between the membrane and bending actions to handle the general case of unsymmetrical ply layups and/or element plane offset from the plane of the grid points. Since none of the existing NASTRAN combination membrane/bending plate elements consider coupling, some work on the finite elements available was also required to implement this new layered composite analysis capability. After some review of the current element formulations and implementations, a decision was made to incorporate new general purpose combined membrane/bending plate elements. The two new elements added to the CSA/NASTRAN library are the CQUAD4 and CTRIA3. The CQUAD4 is a 4-node bilinear isoparametric general quadrilateral element. It is capable of membrane/bending coupling, variable thickness over the element surface and considers the effects of transverse shear flexibility. The CTRIA3 is the 3-node triangular shaped companion to the CQUAD4 element.

User convenient features to scan the voluminous output normally produced in such analyses are provided. The evaluation of a "failure index" for each element based on the commonly used failure theories (maximum stress, maximum strain, Hill, Hoffman and Tsai-Wu) allows the user to review at a glance whether any laminate is stressed (or strained) beyond allowable limits. Some simple techniques to obtain minimum weight designs of layered composite structures are also discussed.

THEORETICAL DISCUSSION

Before discussing the theoretical details of the implementation of the layered composite analysis capability, the theory of the CQUAD4 and CTRIA3 elements is discussed briefly.

The CQUAD4 Element

The CQUAD4 element is a four-node bilinear isoparametric element capable of representing membrane, bending (with transverse shear effects) and membrane-bending coupling behavior. Geometric and kinematic data for the element are shown in figure 1.

Shape of the Element

Standard isoparametric theory is used to represent the shape of the element. A set of element parametric co-ordinates (ξ, η) have been selected which vary linearly between zero and one with the extreme values occurring on the sides of the quadrilateral. Lines of constant ξ and lines of constant η are indicated on figure 1.

Element Co-ordinate System

The x-axis is along the line connecting the first two grid points; the y-axis is perpendicular to the x-axis and lies in the "plane" of the element. If all four grid points do not lie in a plane, a mean plane is defined as discussed below. Finally, the z-axis is normal to the plane of the element and forms a right-handed coordinate system with the x- and y-axes.

Mean-Plane

If the four grid points of the CQUAD4 element do not lie in a plane, a mean-plane containing the projections of the four points is defined such that the four points are alternately H units above and H units below the mean-plane as shown in figure 2 and reference 1.

Membrane Behavior

An enhanced formulation of the four noded isoparametric quadrilateral membrane element available in the NASTRAN® program, the CQDMEM1 element, is used to represent the membrane behavior of the CQUAD4 element. The enhancement consists of using reduced integration to the in-plane shear representation (single point integration, at the center of the element, instead of 2x2 integration). All other details of the element formulation are the same as that discussed in reference 2.

If the element is non-planar, mean-plane transformations that produce only forces and not moments at grid points are used to expand the 8x8 stiffness matrix to 12x12 to allow for three displacements per grid point.

Bending Behavior

A simple, inexpensive to formulate and accurate bending behavior is a necessary prerequisite for plate elements that are to be used in a layered composite analysis. The four-node bilinear isoparametric element discussed by Hughes and Tezduyar (ref. 3) possesses these qualities and is therefore used to represent the bending behavior of the CQUAD4 element. Detailed derivation of the element stiffness, load vector calculations and the stress resultants are provided in reference 3; salient points from the derivations are discussed below.

The concept is to have the transverse displacement interpolated via nine-node Lagrange shape functions and the rotations via four-node bilinear shape functions. The transverse shear strains are calculated in a special way independent of the mid side and center node displacement degrees of freedom; hence four-node bilinear shape functions may be used for transverse displacement also. In other words, the use of special calculations for transverse shear strains allows the use of bilinear shape functions for transverse displacement, rotations and transverse shear strains together with the benefit to the element of quadratic accuracy with respect to Kirchoff modes.

The implementation of the element follows standard isoparametric theory. Interested readers may consult references 3 and 4 for additional details.

If the element is non-planar, mean plane transformations are derived to ensure that the element is in equilibrium when the stiffness matrix is transformed from grid points on the mean-plane to the actual grid point locations.

Membrane/Bending Coupling Effects

The membrane and bending actions are decoupled for plate theory. However, practical situations may necessitate use of CQUAD4 elements to model bending of plates about an axis offset from the geometrical neutral axis. Membrane-bending coupling effects have to be included to analyze such models; these effects also occur in unsymmetric laminates. The CQUAD4 element is therefore designed to include membrane-bending coupling behavior.

The CTRIA3 Element

The CTRIA3 element is a three-node linear element capable of representing membrane, bending (with transverse shear effects) and membrane-bending coupling behavior. The element and the element co-ordinate system are shown in figure 3. The membrane behavior is modeled using the TRMEM element formulation (reference 2), the bending and membrane-bending coupling behavior are modeled using a procedure analogous to that used for the CQUAD4. Being a linear element, the CTRIA3 element is not as accurate as the bilinear CQUAD4 element. It must, therefore, be mentioned in this context that the CQUAD4 element is to be used for all plate modeling requirements; the CTRIA3 element is to be used only where geometrical considerations preclude the use of the CQUAD4 element.

The PSHELL Bulk Data Card

The properties for both the CQUAD4 and the CTRIA3 elements are specified using the new PSHELL element property bulk data card. The PSHELL card data input format provides for specification of element thickness, moment of inertia parameter, transverse shear thickness parameter, stress recovery coefficients and material property references. Provisions are made for specifying up to four different material property identification numbers to separately represent membrane, bending, transverse shear and coupled membrane/bending behaviors.

Force-Displacement Relationship

The relationship between forces and strains used for the CQUAD4 and CTRIA3 elements is

$$\begin{Bmatrix} F \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} tG_1 & t^2G_4 & 0 \\ t^2G_4 & IG_2 & 0 \\ 0 & 0 & t_sG_3 \end{bmatrix} \begin{Bmatrix} \epsilon_m - \epsilon^T \\ \chi - \chi^T \\ \gamma \end{Bmatrix}$$

where

$$\begin{Bmatrix} F \end{Bmatrix} = \begin{Bmatrix} F_x \\ F_y \\ F_{xy} \end{Bmatrix},$$

membrane forces per unit length

$$\begin{Bmatrix} M \end{Bmatrix} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix},$$

bending moments per unit length

$$\begin{Bmatrix} Q \end{Bmatrix} = \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix},$$

transverse shear force per unit length

$$\begin{Bmatrix} \epsilon_m \end{Bmatrix} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix},$$

membrane strains in mean plane

$$\begin{Bmatrix} \chi \end{Bmatrix} = \begin{Bmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \end{Bmatrix},$$

curvatures

$$\begin{Bmatrix} \gamma \end{Bmatrix} = \begin{Bmatrix} \gamma_x \\ \gamma_y \end{Bmatrix},$$

transverse shear strains

and

G_1 is the 3x3 elasticity matrix for membrane action
 G_2 is the 3x3 elasticity matrix for bending action
 G_3 is the 2x2 elasticity matrix for transverse shear action.
 G_4 is the 3x3 elasticity matrix for membrane-bending coupling action.
 t is the element thickness
 I is the element moment of inertia
 t_s is the effective thickness for transverse shear

LAYERED COMPOSITE ANALYSIS

Composite laminates have a number of laminae stacked at various orientations (see figure 4 and reference 5). Ideally, a full 3-dimensional analysis using anisotropic material properties is to be performed. However, lamination theory is a good starting point to perform 2-dimensional analysis that gives satisfactory results at a much reduced cost.

Assumptions Used in Lamination Theory

The following assumptions are used in lamination theory:

- (i) Lamina is in a state of plane stress.
- (ii) Perfect bonding exists between layers so that no slippage of one lamina relative to another occurs.
- (iii) There is no z-variation of the transverse displacement; in other words, thin plate theory can be used.

All of the above assumptions are found to be reasonable in practice (especially in cases where the thickness of the laminate is small in comparison with the length and width).

For unidirectional composites, two orthogonal planes of symmetry exist. One plane is parallel to the fibers and the other is transverse to the fibers. Knowing the material properties in this system, the 3x3 elasticity matrix $[G_M]$ can be evaluated for each lamina. Knowing the orientation of each lamina, the elasticity matrix $[G_M]$ can be transformed to $[G_E]$ in a common element system. Knowing the elasticity matrix $[G_E]$ for all laminae in the common element coordinate system, the membrane, bending membrane-bending and transverse shear elasticity matrices for the laminate are calculated as follows:

$$[G_i] = \frac{1}{t} \int [G_E] dz$$

$$[G_2] = \frac{1}{I} \int (-z)^2 [G_E] dz$$

$$[G_4] = \frac{1}{t^2} \int (-z) [G_E] dz$$

The $[G_3]$ matrix is calculated by assuming that the equations of equilibrium similar to the simple beam theory can be developed independently for the X- and Y- directions. It is to be noted that this is an approximation and that the inter-laminar stresses evaluated by lamination theory are only approximate. However, it is felt that this approximate analysis is better than an analysis neglecting the effects of transverse shear matrix $[G_3]$.

USER INPUT

Composite laminate analysis requires input of laminate data and the orthotropic material property information. This is accomplished by designing the following new bulk data cards shown in the Appendix:

MAT8: two-dimensional orthotropic material data

PCOMP, PCOMP1, PCOMP2: Property cards for composites

User specifies the property identification number of a PCOMP (or PCOMP1 or PCOMP2) property card instead of PSHELL property card for the CQUAD4 and CTRIA3 elements for use as a layered composite element. PCOMP, PCOMP1 and PCOMP2 cards refer to the material properties of each lamina using MAT1, MAT2 or MAT8 cards.

OUTPUT FROM LAYERED COMPOSITE ANALYSIS

The stresses and strains output for each lamina for all the elements yield voluminous output. Some of these output items are shown in figure 5(a) and 5(b) (stresses and strains in each lamina). The concept of failure index for each laminate is introduced to review the output easily. Five commonly used failure theories are provided for this purpose. These are (i) maximum stress theory, (ii) maximum strain theory, (iii) Hill's theory, (iv) Hoffman's theory, and (v) Tsai-Wu theory. Based on the user specified failure theory, the state of stress (or strain) in each lamina is used to evaluate a failure index for the

lamina. The highest failure index value among all laminae of the laminate and the interlaminar stress to allowable bonding stress ratio is defined as the failure index of the laminate. By examining the failure index table, the results of the analysis can be easily reviewed to see whether any lamina has failed according to the specified criterion. A sample output of the failure index table is shown in figure 5(c).

NASTRAN MODIFICATIONS

The design of the layered composite analysis capability that has been incorporated into CSA/NASTRAN was driven by two important requirements. The first was that the capability has maximum versatility and convenience in describing the composite laminate with a minimum of user action necessary. The second was that modifications to existing NASTRAN functional capabilities be kept to a minimum. To achieve the first requirement, seven new bulk data card types shown in the Appendix, and a new functional module were designed and added to the program. The second requirement was satisfied without considerable effort when the decision was made to implement new finite element technology, rather than to try and adapt the existing membrane/bending plate elements to the layered composite environment. This decision also reduced the risk of inadvertently disturbing some aspect of the existing finite element implementations. Thus, the modifications made to the NASTRAN program for the layered composite analysis capability can be divided into two parts: (1) those required for the addition of the new general purpose QUAD4 and TRIA3 finite elements, and (2) those required for the specification and data recovery of the composite laminate itself.

The incorporation of the new 4-node quadrilateral shaped plate and 3-node triangular shaped plate finite elements into NASTRAN has been achieved with the addition of only three new bulk data cards. These new cards are the finite element connection cards, CQUAD4 and CTRIA3, and the element property card, PSHELL, which is referenced by both elements. This new general purpose capability required the following modifications to existing areas of the NASTRAN program for Statics and Normal Modes solution sequences:

- ° Preface IFSiP and IFXiBD routines to process the CQUAD4, CTRIA3 and PSHELL bulk data cards
- ° GPTABD block data routine to add the internal descriptions of the new finite elements to the NASTRAN element library and establish various element-dependent pointer data
- ° TAI table processor module routines to process the material property orientation angle definition options available on the CQUAD4 and CTRIA3 element connection cards
- ° EMG module EMGPRO routine to call the new element matrix generation subroutines

- ° SSG1 module TEMPL routine to call the new element thermal load vector generation subroutines
- ° SDR2 module routines to call the new element stress data recovery subroutines
- ° OFP module routines to provide additional output file page headings for the CQUAD4 and CTRIA3 element forces and stresses

Modifications to the DSMG1 module and RANDOM modules are presently underway to extend this capability to differential stiffness/buckling and the dynamics solution sequences.

The incorporation of the composite laminate specification capability has been achieved with the addition of only four new bulk data cards. These new cards are the MAT8 card which describes the material properties of ply layers, and the PCOMP, PCOMP1 and PCOMP2 cards which describe the laminate ply layup with varying degrees of generality and convenience. One new functional module was also required which performs composite element ply stress data recovery operations. A preface processor converts the supplied PCOMP and MAT8 data into equivalent PSHELL and MAT2 data, updates element connection data references to property data and adds the PSHELL data to the element property table (EPT) and the MAT2 data to the material property table (MPT). The preface then also sets a DMAP parameter to control data recovery operations based upon whether composite element properties are referenced. The layered composite analysis capability required the following modifications to the NASTRAN code:

- ° Preface IFSIP and IFXIBD routines to process the PCOMP, PCOMP1, PCOMP2 and MAT8 bulk data cards
- ° A new preface processor, IFP6, which generates equivalent PSHELL and MAT2 bulk data from the PCOMP and MAT8 data supplied; IFP6 also updates the GEOM2 data block to reflect the element references to the newly generated property data and adds the data to the EPT and MPT
- ° XMPLOD routine to establish module properties for the new composite element stress data recovery processor, SDRCOMP
- ° PREMAT/MAT material property processing routine to handle the MAT8 data
- ° SDRCOMP, a new functional module added to generate individual ply stresses and strains for the layered composite elements and to generate a failure index table of ply data based upon several available failure theories
- ° OFP module to process the new composite element Failure Index Table, ply stress data block and ply strain data block

The action required on the part of the user to access this new layered composite analysis capability is considered to be very minimal. The user must insert the

appropriate PCOMP and MAT8 data which describe the composite laminate into the bulk data deck. Then if the user wishes to have ply data recovery operations performed, a STRAIN= case control request must be present and the SDRCOMP and OFP modules must be altered into the rigid format.

Once the initial capability verification testing process was satisfactorily completed, modifications were made to the NASTRAN rigid format data base to provide the necessary CHKPNT/RESTART functions associated with the addition of the new bulk data cards. Quality assurance testing procedures were then completed satisfactorily demonstrating the incorporation of the layered composite analysis capability as a general purpose feature of CSA/NASTRAN.

CAPABILITY VERIFICATION TESTING

When any new capability is added to NASTRAN, a series of tests is performed to ensure that the capability has been incorporated properly. These tests are designed to ensure that the new capability performs according to design specifications and that existing functional capability has not been adversely affected by the new features. The capability verification tests performed to ensure satisfactory implementation of the layered composite analysis feature were divided into two categories. The first series of tests were designed to validate the proper installation of the new CQUAD4 and CTRIA3 finite elements. They demonstrated the accuracy of the elements under various geometry, material property and loading configurations. Several of the test cases were taken from those proposed in reference 6. The remainder of the tests were designed to demonstrate implementation correctness and not necessarily theoretical accuracy. The test problems considered to be most important are now briefly described:

- ° Patch test - measures the ability of the element to represent constant strain states of deformation. The model geometry and results are shown in figure 6.
- ° Twisted beam - measures the effect of warping on the element. The model geometry and results are shown in figure 7.
- ° Equilibrium tests - each column/row of the elemental stiffness matrix was summed about some point (say, node one) of the element to ensure that the matrix represented a set of forces that were in equilibrium. This test was performed on several configurations which included non-rectangular shapes and element warping. In all instances, the resulting summations were computational zeroes. An additional set of tests were also run which extracted the free-free mode shapes of the plate using unit masses at each degree of freedom. This test resulted in extraction of six distinct rigid body modes which further guarantees equilibrium of the stiffness matrix.

Several other elemental tests were performed to ensure that the elements performed according to specifications. These tests are the same as those described in reference 6 and the twisted ribbon problem described in reference 3. The results from these tests are summarized in table 1.

in addition to the tests performed to validate the addition of new finite elements, a series of tests were also performed to validate the installation of the layered composite analysis capability into CSA/NASTRAN. Once again, many sample problems were run to demonstrate that the capability performed up to design specifications. These tests included exercising all of the various options available on the PCOMP, PCOMP1, and PCOMP2 bulk data cards. Two example problems were run to demonstrate the theoretical correctness of the implementation. These problems were taken from reference 7. The geometry for the first problem, a static analysis of a simply supported square plate subjected to sinusoidally varying pressure load, is presented in figure 8. The geometry for the second problem, a modal analysis of a simply supported cylinder, is presented in figure 9. The modal analysis was accomplished using the cyclic symmetry analysis capability within NASTRAN. The results of these example problems are presented in table 2.

EXAMPLE APPLICATION

To illustrate the application of the layered composite capability in CSA/NASTRAN for design of practical composite structures, the example of an optimum design aluminum rectangular plate (ref. 8, 9 and 10) is analyzed using 8-ply laminated composite made of graphite/epoxy (commercially identified by Type T 300/5208). The geometry, loading and finite element modeling is shown in figure 10(a) and 10(b). The material properties and allowable stresses used in the example are those specified in reference 5. Starting from the optimally designed aluminum plate thickness for the various element groupings, the thickness for the composite model is obtained in 2 or 3 iterations using the enhanced fully stressed design algorithm implemented in CSA/NASTRAN. The thickness for the layered composite plate and the aluminum plate are shown in figure 10(c). The mass for the composite model is 0.40275 kg (0.8879 lbs) versus the mass of 1.034 kg (2.28 lbs) reported in reference 8 and 1.0796 kg (2.38 lbs) reported in reference 9 and 0.9487 kg (2.0915 lbs) reported in reference 10. It is to be noted that it is possible to obtain other optimum designs for the layered composite model. The design shown in figure 10(c) is a feasible design (design that satisfies all specified constraints). The objective here was to find the weight savings possible between layered composite construction and aluminum on a practical example problem subjected to stress and displacement constraints. The capabilities provided in CSA/NASTRAN allows the engineers the option of reviewing such "optimum" designs to select the most economical designs for their needs.

WORK IN PROGRESS

Several additions and extensions to the layered composite analysis capability are currently underway. For versatility, two additional plate elements are being formulated and added to the CSA/NASTRAN finite element library. These elements are the 8-node isoparametric general quadrilateral element CQUAD8 and its 6-node triangular companion, the CTRIA6. Both elements will be able to reference com-

posite material properties. An option to model the composite elements for membrane behavior only (without any reference to bending, transverse shear or membrane-bending coupling) to facilitate modeling the upper and lower covers of a wingbox, for example, is now available. This option will be extended to include property optimization methods to consider the thickness or orientation of each individual ply as a design variable. The OPTPR1 and OPTPR2 modules are being enhanced to incorporate evaluation of design sensitivity coefficients so that structural optimization using mathematical programming techniques can be performed in addition to fully stress design resizing.

CONCLUSION

The theoretical formulations of layered composite analysis capability for addition to the NASTRAN program are presented in this paper. The need to have simple, accurate plate elements for layered composite elements is discussed. The addition of two such elements, the CQUAD4 and CTRIA3 elements, and the layered composite capability into the CSA/NASTRAN program, an enhanced proprietary version of the April 1984 release of NASTRAN, developed and maintained by C.S.A.R. Corporation, is described. Verification problems and example application problems to illustrate the useful features available to engineers to analyze and obtain optimum designs of layered composite structures are also presented.

REFERENCES

1. Cook, R. D. Concepts and Applications of Finite Element Analysis. John Wiley and Sons, New York, 1981.
2. The NASTRAN Theoretical Manual. NASA SP-221 (06) January 1981.
3. Hughes, T. J. R. and Tezduyar, T. E.: Finite Elements Based Upon Mindlin Plate Theory with Particular Reference to the Four-Node Bilinear Isoparametric Element. Journal of Applied Mechanics, September 1981.
4. Zienkiewicz, O. C.: The Finite Element Method. McGraw-Hill Book Company (UK) Limited, 1977.
5. Tsai, Stephen W.; and Hahn, H. Thomas: Introduction to Composite Materials. Technomic Publishing Company, Westport, Connecticut, 1980.
6. MSC/NASTRAN Application Manual. March 1984 Application Note; Joseph, J., Editor; The MacNeal-Schwendler Corporation, Los Angeles, CA.
7. MSC/NASTRAN Application Manual. Section 2.12; Joseph, J., Editor; The MacNeal-Schwendler Corporation, Los Angeles, CA.
8. Prasad, B.; and Haftka, R.: Optimal Structural Design with Plane Finite Elements. Proceedings of the ASCE, Journal of the Structural Div., ST 11, Nov. 1979.
9. Fluery, C.; Ramanathan, R. K.; Salema, M.; and Schmit, L. A.: Access Computer Program for the Synthesis of Large Structural Systems. NASA Langley Research Center Optimization Conference, April 1984.
10. CSAR/OPTIM User's Manual. Computerized Structural Analysis and Research Corporation, Northridge, California, July 1984.

Table 1. Summary of Test Results for CSA/NASTRAN QUAD4 Plate Element.

No.	Test	Element Loading		Element Shape*	Displacement		GRADE
		In-Plane	Out-Of-Plane		Theory	QUAD4 Model	
1.	Patch Test	X		Irregular	Figure 6	Figure 6	A
2.	Patch Test		X	Irregular	Figure 6	Figure 6	A
3a.	Straight Beam - Extension	X		Rectangle	3.000-5	2.985-5	
3b.	Straight Beam - Extension	X		Trapezoid	3.000-5	2.976-5	A
3c.	Straight Beam - Extension	X		Parallelogram	3.000-5	2.997-5	
4.	Straight Beam - Bending	X		Rectangle	.10810	.097632	B
5a.	Straight Beam - Bending	X		Trapezoid	.10810	.007668	
5b.	Straight Beam - Bending	X		Parallelogram	.10810	.008734	F
6.	Straight Beam - Bending		X	Rectangle	.43210	.42389	A
7a.	Straight Beam - Bending		X	Trapezoid	.43210	.41611	
7b.	Straight Beam - Bending		X	Parallelogram	.43210	.42216	B
8a.	Straight Beam - Twist			Rectangle	.03208	.03017	
8b.	Straight Beam - Twist			Trapezoid	.03208	.03433	C
8c.	Straight Beam - Twist			Parallelogram	.03208	.02534	
9.	Curved Beam	X		Rectangle	.58734	.04441	D
10.	Curved Beam		X	Rectangle	.50220	.47305	B
11a.	Twisted Beam	X		Regular	.005424	.005373	
11b.	Twisted Beam		X	Regular	.001754	.001721	A
12a.	Rect. Plate (ss-u,N=4,b/a=5)		X	Regular	12.97	12.6976	
12b.	Rect. Plate (ss-c,N=4,b/a=5)		X	Regular	16.96	15.1614	
12c.	Rect. Plate (cc-u,N=4,b/a=5)		X	Regular	2.56	2.4411	B
12d.	Rect. Plate (cc-c,N=4,b/a=5)		X	Regular	7.23	5.6828	
13.	Scordelis-Lo Roof (N=4)		X	Regular	.3024	.31623	B
14.	Spherical Shell (N=8)		X	Regular	.0940	.09288	A
15a.	Thick-walled Cylinder v=.43	X		Regular	5.0399-3	4.2523-3	
15b.	Thick-walled Cylinder v=.499	X		Regular	5.0602-3	1.803-3	F
15c.	Thick-walled Cylinder v=.4999	X		Regular	5.0623-3	2.649E-4	
16a.	Twisted Ribbon (Case A, L=10)		X	Regular	0.0291+	0.02774	
16b.	Twisted Ribbon (Case B, L=10)		X	Regular	0.0291+	0.02774	B
Number of Failed Tests (D's and F's)							3

*Regular means that element shape has not been intentionally distorted

† Theoretical results are taken from a fine mesh finite element solution

Remarks: 1. Letter grades are used to indicate the following error percentages:

A: Error < 2%; B: 2% < Error < 10% C: 10% < Error < 20%
D: 20% < Error < 50%; F: Error > 50%

2. Letter grades show for problem set numbers 3, 5, 7, 8, 11, 12 and 15 have been assigned by averaging the absolute error for each problem in the set.

3. For details of test problem descriptions 1-15, consult the March 1984 MSC/NASTRAN Application Note in the MSC/NASTRAN Application Manual of the MacNeal-Schwendler Corporation, Los Angeles, CA; for problem 16, refer to "Finite Elements Based Upon Mindlin Plate Theory with particular reference to the Four-node Bilinear Isoparametric Element" by T.J.R. Hughes and T.E. Tezduyar, Journal of Applied Mechanics, September 1981.

TABLE 2. Composite Element Accuracy Test Results

SQUARE PLATE ~ Statics

Center point deflection of a simply supported square plate due to a sinusoidally varying pressure load

Number of Plies	Type of Laminate	Center Point Deflection	
		Theory	NASTRAN
2	Unsymmetric	0.010029	0.010056
3	Symmetric	0.00035347	0.00036245
4	Unsymmetric	0.00042175	0.00042622

CYLINDER ~ Cyclic Symmetry Normal Modes

Harmonic Number	Longitudinal Halfwave	FREQUENCY (rad/sec)	
		Theory	NASTRAN
1	1	18.608	18.576
	3	46.605	46.426
	5	61.435	60.810
2	1	9.391	9.377
	3	27.219	27.143
	5	41.842	41.535
3	1	6.274	6.230
	3	18.895	18.864
	5	30.804	30.644
4	1	4.934	4.893
	3	14.578	14.568
	5	24.494	24.404

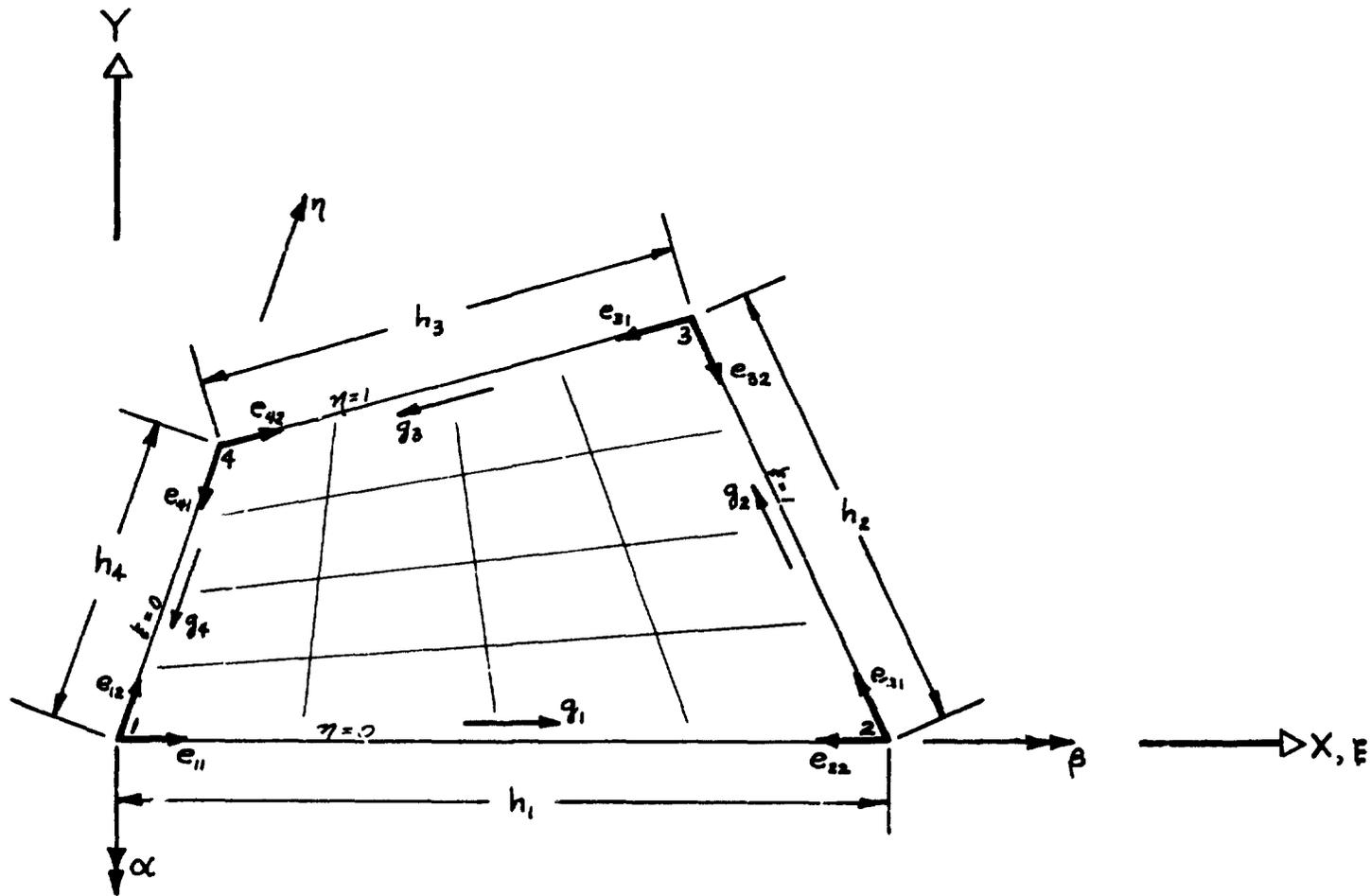
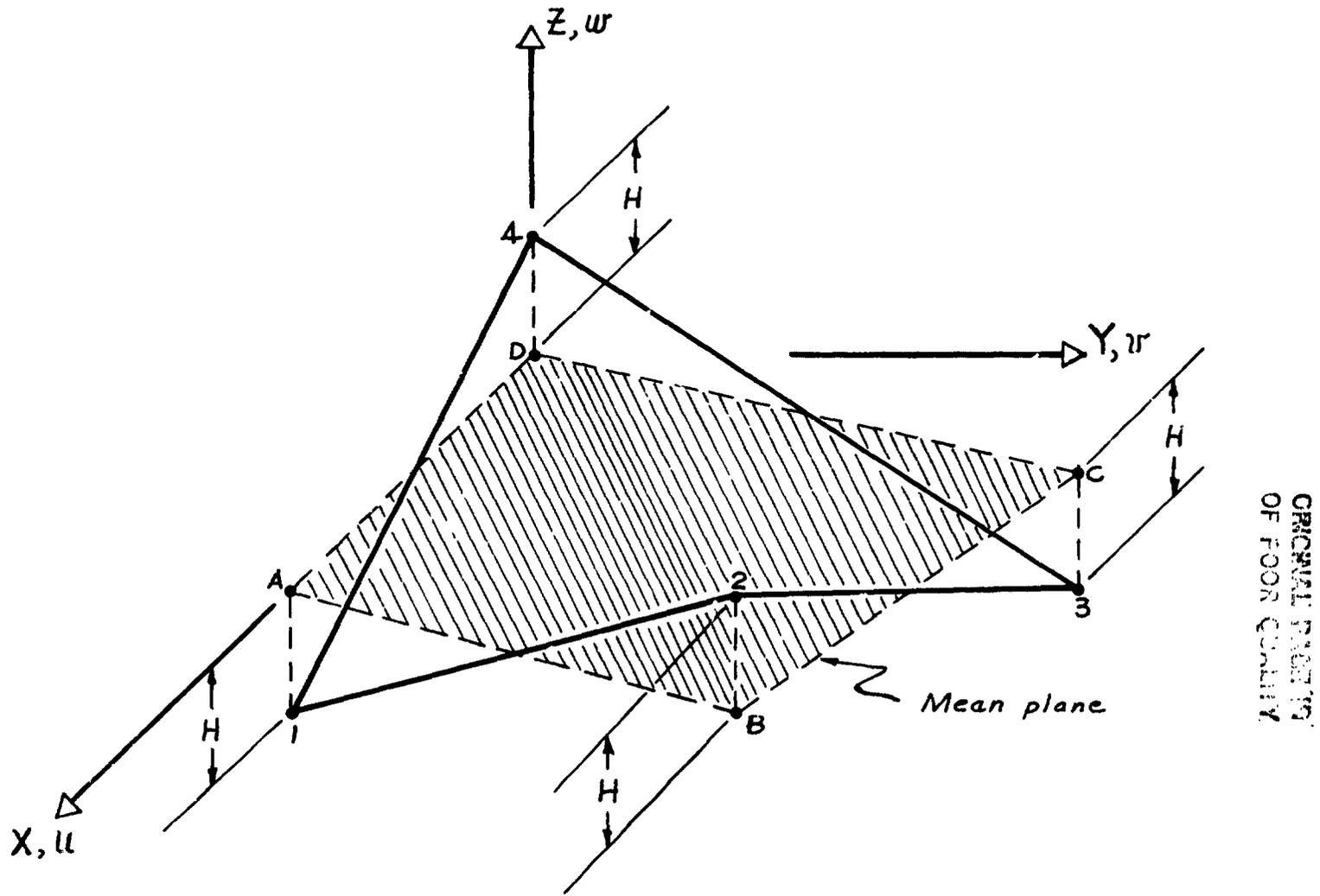


Figure 1. Geometric and Kinematic Data for CQUAD4 Element



ORIGINAL POSITION
OF POOR QUALITY

Figure 2. Warped QUAD4 1-2-3-4 in Local coordinates x, y, z

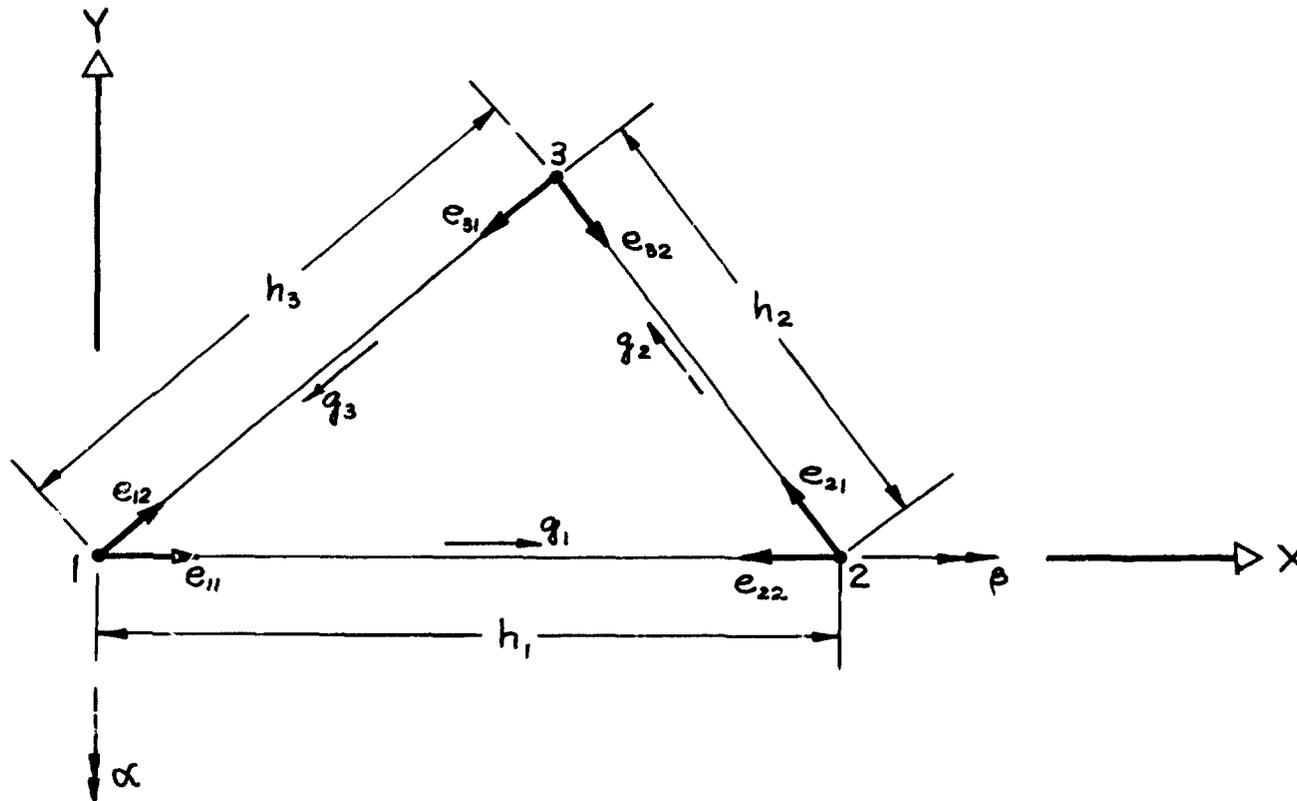


Figure 3. Geometric and Kinematic Data for CTRIA3 Element

ORIGINAL PAGE
OF POOR QUALITY

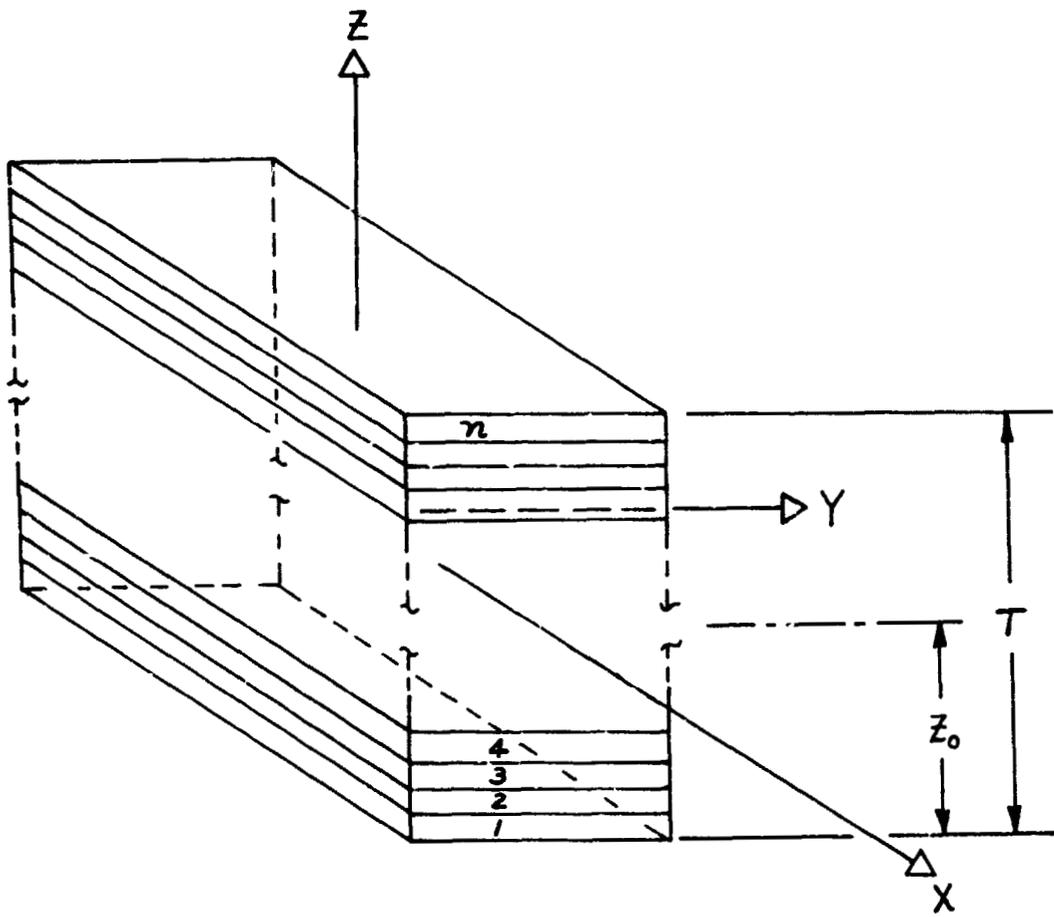


Figure 4. Laminae Identification for n -ply Laminate

USE COMPOSITES INSTEAD OF ALUMINUM

ELEMENT ID	PLY ID	STRESSES IN LAYERED COMPOSITE ELEMENTS (CGUAD4)			INTER-LAMINAR STRESSES		PRINCIPAL STRESSES (ZERO SHEAR) MAJOR	MINOR	MAX SHEAR	
		STRESSES IN FIBRE AND MATRIX DIRECTIONS NORMAL-1	NORMAL-2	SHEAR-12	SHEAR-1Z	SHEAR-2Z				
101	1	-8.94110E+02	-9.83429E+02	1.60752E+02	1.65201E+01	5.08403E+00	37.24	-7.71930E+02	-1.10561E+03	1.66840E+02
101	2	-5.00005E+03	-5.20859E+02	-1.64373E+02	2.01973E+01	2.49705E+01	-84.14	-4.73222E+02	-5.04769E+03	2.28723E+03
101	3	-4.71923E+03	-2.40935E+02	2.78624E+02	2.24036E+01	3.69024E+01	86.45	-2.23667E+02	-4.73650E+03	2.25641E+03
101	4	-2.44536E+03	4.39937E+01	-2.29645E+01	2.25379E+01	4.96653E+01	-89.45	-4.37740E+01	-2.44558E+03	1.20090E+03
101	5	2.44536E+03	4.39937E+01	2.29645E+01	2.24036E+01	3.69024E+01	0.55	2.44558E+03	4.37740E+01	1.20090E+03
101	6	4.71923E+03	2.40935E+02	-2.78624E+02	2.01973E+01	2.49705E+01	-3.55	4.73650E+03	2.23667E+02	2.25641E+03
101	7	5.00006E+03	5.20859E+02	4.64373E+02	1.65201E+01	5.08402E+00	5.86	5.04769E+03	4.73222E+02	2.28723E+03
101	8	8.94110E+02	9.83429E+02	-1.60752E+02	0.0	-3.78227E-06	-52.76	1.10561E+03	7.71930E+02	1.66840E+02
102	1	1.83268E+03	-9.15424E+02	1.35344E+02	7.16597E+00	3.23905E+00	2.81	1.83933E+03	-9.22074E+02	1.38070E+03
102	2	-4.08235E+03	-4.29399E+02	-5.28776E+02	8.76106E+00	1.59088E+01	-81.93	-3.54397E+02	-4.15735E+03	1.90148E+03
102	3	-3.89688E+03	-1.97373E+02	3.17265E+02	9.71811E+00	2.35106E+01	85.13	-1.70362E+02	-3.92369E+03	1.87676E+03
102	4	-2.37724E+03	-2.08959E+01	-1.93348E+01	9.77636E+00	3.16419E+01	-89.53	-2.37371E+01	-2.37740E+03	1.17833E+03
102	5	2.37724E+03	2.08959E+01	1.93348E+01	9.71811E+00	2.35106E+01	0.47	2.37740E+03	2.07371E+01	1.17833E+03
102	6	3.89688E+03	1.97373E+02	-3.17265E+02	8.76106E+00	1.59088E+01	-4.87	3.72389E+03	1.70362E+02	1.87676E+03
102	7	4.08235E+03	4.29399E+02	5.28776E+02	7.16597E+00	3.23905E+00	8.07	4.15735E+03	3.54397E+02	1.90148E+03
102	8	-1.83268E+03	9.15424E+02	-1.35344E+02	0.0	-2.40970E-06	-67.19	9.22074E+02	-1.83933E+03	1.38070E+03

Figure 5(a): Stress Output for Layered Composite Elements

USE COMPOSITES INSTEAD OF ALUMINUM

ELEMENT ID	PLY ID	STRAINS IN LAYERED COMPOSITE			ELEMENTS (CGUAD4)			MAX SHEAR
		STRAINS IN FIBRE AND MATRIX DIRECTIONS			PRINCIPAL	STRAINS (ZERO SHEAR)		
		NORMAL-1	NORMAL-2	SHEAR-12	ANGLE	MAJOR	MINOR	
101	1	-2.357592E-05	-6.489339E-04	1.546283E-04	13.1568	1.256893E-05	-6.850787E-04	3.488238E-04
101	2	-1.849577E-04	-2.954064E-04	-4.466843E-04	-41.4761	2.099030E-04	-6.902672E-04	4.500851E-04
101	3	-1.772439E-04	-1.109746E-04	2.670106E-04	48.5239	1.259418E-04	-4.141603E-04	2.700510E-04
101	4	-9.270482E-05	-3.367991E-06	-2.208976E-05	-76.8432	1.795561E-06	-9.786837E-05	4.983197E-05
101	5	9.270487E-05	3.367984E-06	2.208973E-05	13.1568	9.786840E-05	-1.795554E-06	4.903198E-05
101	6	1.772439E-04	1.109746E-04	-2.670106E-04	-41.4761	4.141603E-04	-1.259418E-04	2.700510E-04
101	7	1.849577E-04	2.954064E-04	4.466843E-04	48.5239	6.902671E-04	-2.099030E-04	4.500851E-04
101	8	2.357592E-05	6.489340E-04	-1.546283E-04	-76.8432	6.850788E-04	-1.256893E-05	3.488239E-04
102	1	7.959603E-05	-6.324911E-04	1.301883E-04	10.0425	1.026514E-04	-6.555465E-04	3.790970E-04
102	2	-1.509667E-04	-2.439583E-04	-5.086337E-04	-42.3885	3.132919E-04	-7.082169E-04	5.107544E-04
102	3	-1.463750E-04	-9.058001E-05	3.051802E-04	47.6115	1.879752E-04	-4.249302E-04	3.064527E-04
102	4	-9.033585E-05	1.137086E-05	-1.859834E-05	-79.9575	1.466449E-05	-9.364948E-05	5.415697E-05
102	5	9.033590E-05	-1.137087E-05	1.859830E-05	10.0425	9.364951E-05	-1.466448E-05	5.415700E-05
102	6	1.463750E-04	9.058004E-05	-3.051802E-04	-42.3885	4.249302E-04	-1.879752E-04	3.064527E-04
102	7	1.509667E-04	2.439583E-04	5.086337E-04	47.6115	7.082170E-04	-3.132919E-04	5.107544E-04
102	8	-7.959604E-05	6.324912E-04	-1.301883E-04	-79.9575	6.555466E-04	-1.026514E-04	3.790970E-04

CHANGES FROM POOR QUALITY

Figure 5(b): Strain Output for Layered Composite Elements

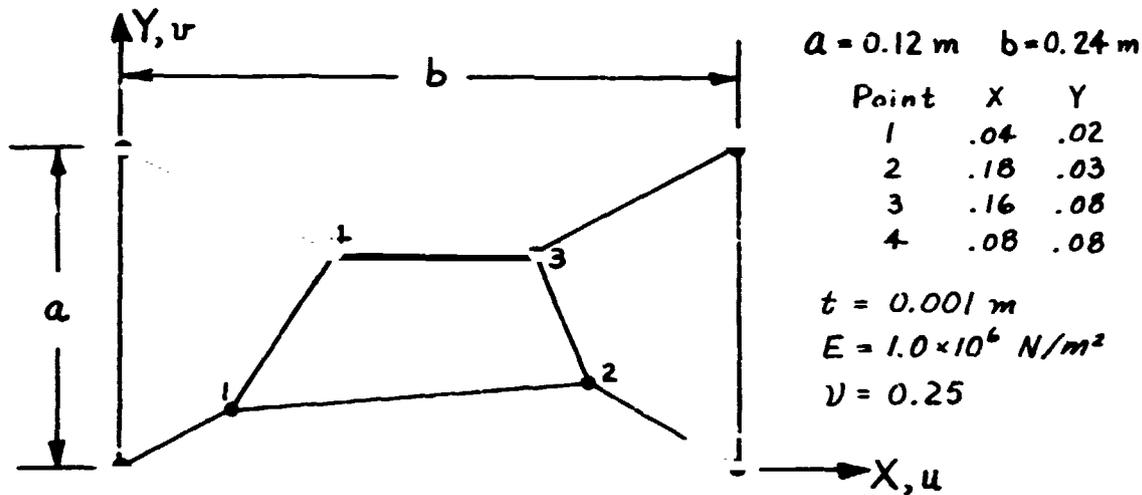
USE COMPOSITES INSTEAD OF ALUMINUM

ELEMENT ID	FAILURE THEORY	PLY ID	FP=FAILURE INDEX FOR PLY (DIRECT STRESSES)	FB=FAILURE INDEX FOR BONDING (INTER-LAMINAR STRESSES)	FAILURE INDEX FOR ELEMENT MAX OF FP,FB FOR ALL PLYS	FLAG
101	HILL	1	0.0013			
		2	0.0025	0.0014		
		3	0.0013	0.0021		
		4	0.0002	0.0031		
		5	0.0002	0.0041		
		6	0.0021	0.0031		
		7	0.0064	0.0021		
		8	0.0153	0.0014		
					0.0153	

88

Figure 5(c): Failure Index Table for Layered Composite Elements

ORIGINAL PAGE IS
OF POOR QUALITY



Membrane Boundary Conditions:

$$u = (x + \frac{1}{2}y) \times 10^{-3} \quad v = (\frac{1}{2}x + y) \times 10^{-3}$$

Bending Boundary Conditions:

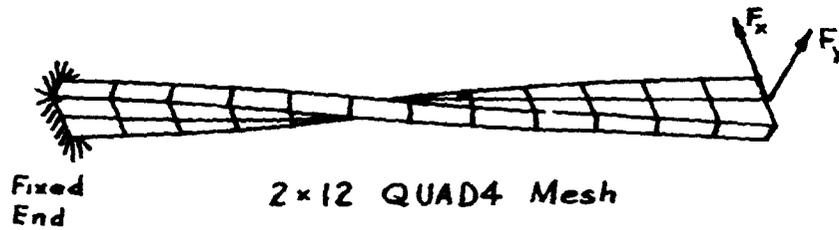
$$w = \frac{1}{2}(x^2 + xy + y^2) \times 10^{-3}$$

$$\theta_x = \frac{\partial w}{\partial y} = (\frac{1}{2}x + y) \times 10^{-3} \quad \theta_y = \frac{\partial w}{\partial x} = (x + \frac{1}{2}y) \times 10^{-3}$$

Test Results:

Element Output Quantity	Theory	NASTRAN
In-Plane Strains $\epsilon_x, \epsilon_y, \gamma$	1.0×10^{-3}	1.0×10^{-3}
Stresses σ_x, σ_y	1333.	1333.
Shear Stress τ_{xy}	400.	400.
Bending Moment per unit length $M_x = M_y$	1.111×10^{-7}	1.111×10^{-7}
Bending Moment per unit length M_{xy}	0.333×10^{-7}	0.333×10^{-7}
Surface Stresses σ_x, σ_y	± 0.667	± 0.667
Surface Shear Stress τ_{xy}	± 0.200	± 0.200

Figure 6. Element Patch Test Results - CQUAD4



Length = 12.0 m

Width = 1.1 m

Depth = 0.32 m

Twist = 90° (root to tip)

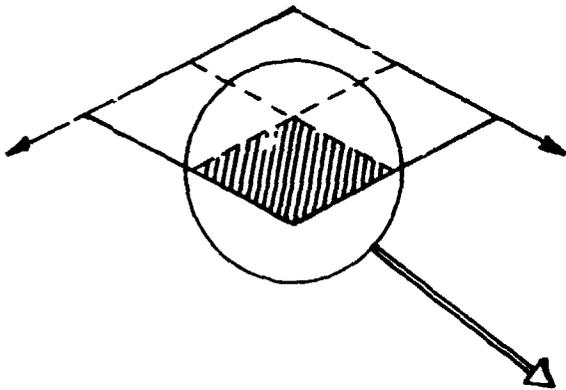
$E = 29.0 \times 10^6 \text{ N/m}^2$ $\nu = 0.22$

Applied Loads: Unit forces at tip

Tip Direction Load	Displacements in Loaded Direction	
	Theoretical	NASTRAN
In-plane Shear	0.005424	0.005373
Out-of-plane Shear	0.001754	0.001721

Figure 7. CQUAD4 Twisted Beam Test Results

ORIGINAL PAGE IS
OF POOR QUALITY



Simply Supported Composite
Square Plate Subjected to
Sinusoidally Varying Pressure
Loading of the form

$$p(x,y) = \frac{1}{2} \sin\left(\frac{\pi x}{l}\right) \sin\left(\frac{\pi y}{l}\right)$$

Finite Element Model

3x3 $\frac{1}{4}$ Symmetric QUAD4 Mesh:

$$E_1 = 20. \times 10^6$$

$$E_2 = .5 \times 10^6$$

$$G_{12} = .25 \times 10^6$$

$$\nu_1 = .25$$

$$\nu_2 = .00625$$

$$t = 0.2$$

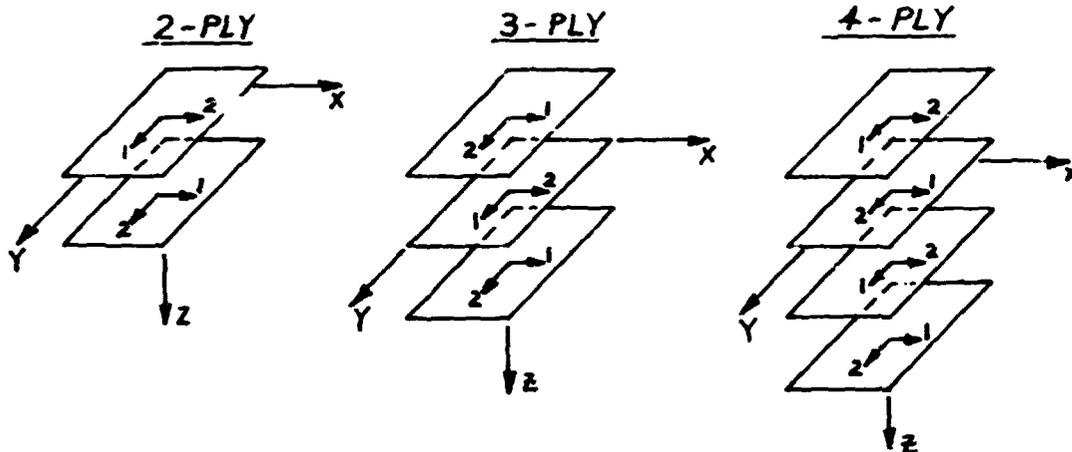
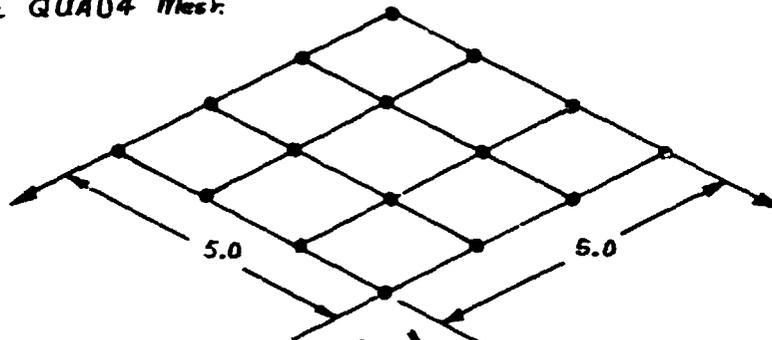
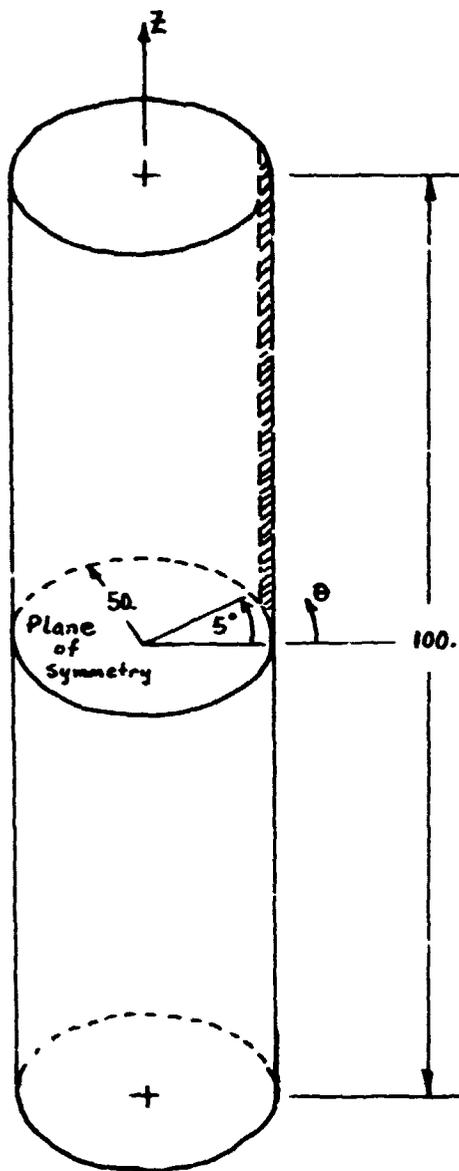
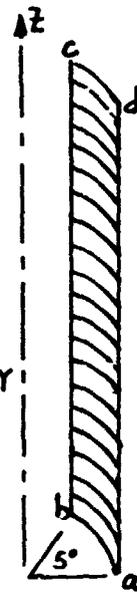


Figure 8. Simply Supported Composite Plate Example

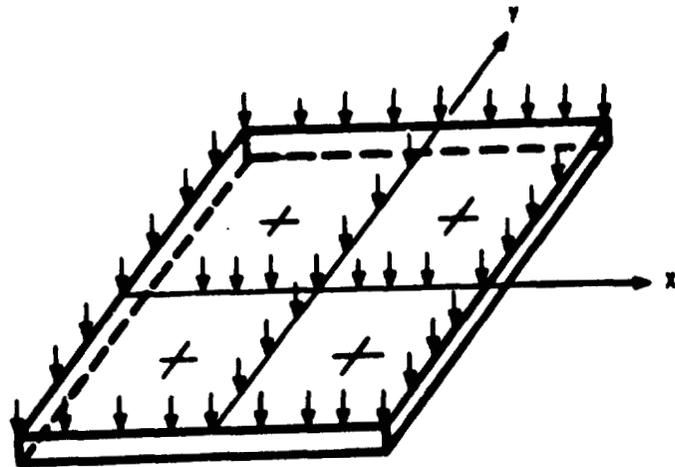


Finite
Element
Model
via
CYCLIC
SYMMETRY



- 20 QUAD4 Elements
Bending, Membrane and
Transverse Shear Effects
- Boundary Conditions
Edge ab:
 $U_z = 0$
 $\theta_r = \theta_z = 0$
 Edge cd: Simply Supported
 $U_r = U_\theta = 0$
 $\theta_z = 0$
- Wall thickness = 0.5
- 12 Segments

Figure 9. Simply Supported Composite Cylinder Example



Composite laminate details:

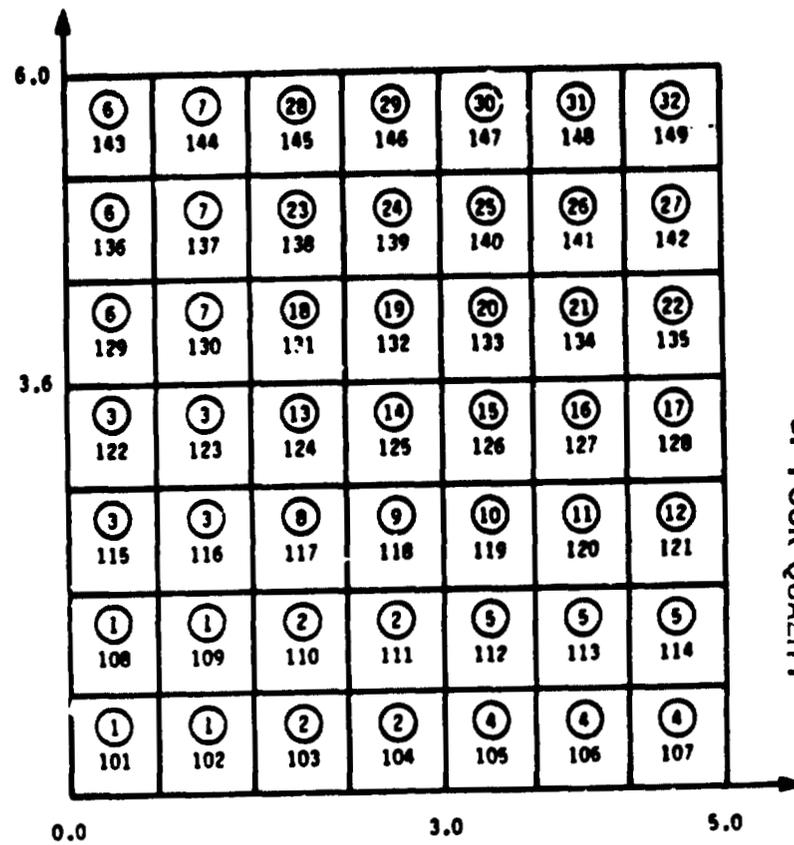
B-ply Graphite/Epoxy (T 300/5208)

(0/+45/-45/90) Symmetric

Constraints:

Element Stress: Hill's Criteria

Displacements: $+.02$ at center and corners
of plate



ORIGINAL PAGE IS
OF POOR QUALITY

(Numbers denote element IDs. Numbers in circles denote element groups by design variable linking.)

(a) Rectangular Plate: Loading and Supports.

(b) Finite Element Model.

Figure 10. Rectangular Plate for Optimum Design

0.199 (0.062)	0.428 (0.268)	0.514 (0.247)	0.251 (0.026)	0.353 (0.181)	0.416 (0.259)	0.246 (0.138)
		0.815 (0.503)	0.847 (0.480)	0.714 (0.365)	0.843 (0.559)	0.317 (0.255)
		0.957 (0.525)	1.005 (0.632)	1.401 (0.829)	0.585 (0.220)	0.235 (0.055)
0.168 (0.028)		0.597 (0.241)	1.260 (0.737)	0.977 (0.576)	1.009 (0.663)	0.302 (0.052)
		0.636 (0.356)	0.659 (0.365)	0.762 (0.347)	0.690 (0.403)	0.524 (0.351)
0.669 (0.406)	0.203 (0.048)	0.143 (0.038)				
		0.624 (0.334)				

(Values denote total thickness of 8-ply graphite/epoxy laminate (mass = 0.8879 lbs). Values in parentheses denote thickness for aluminum (mass = 2.0135 lbs.))

(c) Plate Thickness for Optimum Design.

Figure 10. Rectangular Plate for Optimum Design with Stress and Displacement Constraints. (Cont.)

APPENDIX

CSA/NASTRAN Bulk Data Card Descriptions

CSA/NASTRAN BULK DATA DECK

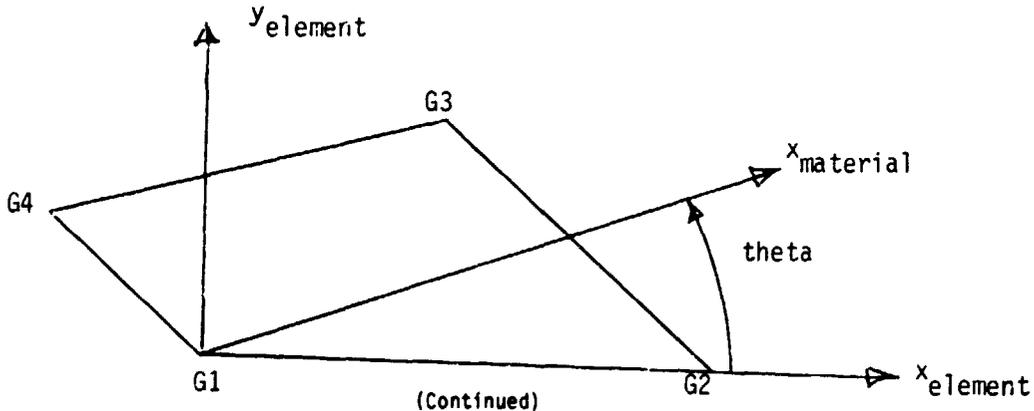
Input Data Card CQUAD4 Quadrilateral Element Connection

Description: Defines a quadrilateral plate element (QUAD4) of the structural model. This is an isoparametric membrane-bending element.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CQUAD4	EID	PID	G1	G2	G3	G4	θ		
CQUAD4	1213	745	92	103	76	1001	32.4		CQ4
			T1	T2	T3	T4			
+Q4			.78	.63	.60	.70			

- | <u>Field</u> | <u>Contents</u> |
|--------------|---|
| EID | Element identification number (unique integer > 0) |
| PID | Identification number of a PSHELL or PCOMP property card (Integer > 0 or blank, default is EID) |
| G1,G2,G3,G4 | Grid point identification numbers of connection points (Integers > 0, all unique) |
| θ | Material property orientation specification (Real or blank; or 0 < Integer < 1,000,000). If Real or blank, specifies the material property orientation angle in degrees. If Integer, the orientation of the material x-axis is along the projection onto the plane of the element of the x-axis of the coordinate system specified by the integer value. The sketch below gives the sign convention for . |
| T1,T2,T3,T4 | Membrane thickness of element at grid points G1 through G4. (Real ≥ 0, or blank, not all zero. See Remark 4 for default.) |



CSA/NASTRAN BULK DATA DECK

CQUAD4 (Continued)

- Remarks:
1. Element identification numbers must be unique with respect to all other element identification numbers.
 2. Grid points G1 through G4 must be ordered consecutively around the perimeter of the element.
 3. All the interior angles must be less than 180° .
 4. The continuation card is optional. If it is not supplied, then T1 through T4 will be set equal to the value of T on the PSHELL data card.
 5. Stresses are output in the element coordinate system.

CSA/NASTRAN BULK DATA DECK

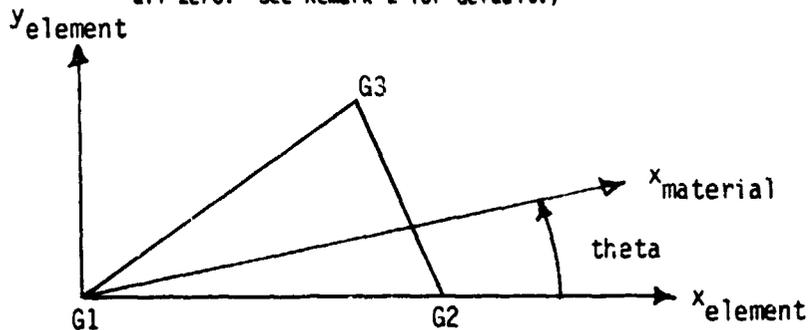
Input Data Card CTRIA3 Triangular Element Connection

Description: Defines a triangular plate element (TRIA3) of the structural model. This is an isoparametric membrane-bending element

Format and Example:

1	2	3	4	5	6	7	8	9	10
CTRIA3	EID	PID	G1	G2	G3	θ			
CTRIA3	401	101	1001	1201	1401	7.5			CTR3
			T1	T2	T3				
+TR3			.83	.81	.74				

<u>Field</u>	<u>Contents</u>
EID	Element identification number (Unique integer > 0)
PID	Identification number of a PSHELL property card (Integer > 0 or blank, default is EID)
G1,G2,G3	Grid point identification numbers of connection points (Integers > 0, all unique)
θ	Material property orientation specification (Real or blank; or 0 < Integer < 1,000,000). If Real or blank, specifies the material property orientation angle in degrees. If Integer, the orientation of the material x-axis is along the projection on to the plane of the element of the x-axis of the coordinate system specified by the integer value. The sketch below gives the sign convention for θ .
T1,T2,T3	Membrane thickness of element at grid points G1, G2, and G3 (Real \geq 0. or blank, not all zero. See Remark 2 for default.)



(Continued)

CSA/NASTRAN BULK DATA DECK

CTRIA3 (Continued)

- Remarks:
1. Element identification numbers must be unique with respect to all other element identification numbers.
 2. The continuation card is optional. If it is not supplied, then T1 through T3 will be set equal to the value of T on the PSHELL data card.

CSA/NASTRAN BULK DATA DECK

Input Data Card MAT8 Material Property Definition, Form 8

Description: Defines the material property for an orthotropic material

Format and Example:

1	2	3	4	5	6	7	8	9	10
MAT8	EID	E1	E2	ν_{12}	G12	G1,Z	G2,Z	RHO	
MAT8	101	28.+6	.5+6	0.25	1.+5	2.+6	1.5+6	0.62	+ABC
	A1	A2	TREF	α_t	α_c	γ_t	γ_c	S	
+ABC	24.-6	1.4-6	70.	1.2+4	1.4+4	1.3+3	1.1+3	1.+3	+DEF
	G_t	F12							
+DEF	1.3-4								

<u>Field</u>	<u>Contents</u>
MID	Material ID (1,000,000 > Integer > 0)
E1	Modulus of elasticity in longitudinal direction (also defined as fibre direction or 1-direction) (Real ≠ 0.0)
E2	Modulus of elasticity in transverse direction (also defined as matrix direction or 2-direction) (Real ≠ 0.0)
ν_{12}	Poisson's ratio (ϵ_2/ϵ_1 for uniaxial loading in 1 - direction). Note that $\nu_{21} = \epsilon_1/\epsilon_2$ uniaxial loading in 2 - direction is related to ν_{12} , E1, E2 by the relation $\nu_{12}E_2 = \nu_{21}E_1$. Real.
G12	Inplane shear modulus (Real > 0.0)
G1,Z	Transverse shear modulus for shear in 1-Z plane (Real > 0.0 or blank)
G2,Z	Transverse shear modulus for shear in 2-Z plane (Real > 0.0 or blank)
RHO	Mass density (Real)
A1	Thermal expansion coefficient in 1-direction (Real)
A2	Thermal expansion coefficient in 2-direction (Real)

(Continued)

CSA/NASTRAN BULK DATA DECK

MAT8 (Continued)

TREF Thermal expansion reference temperature (Real)

X_t, X_c Allowable stresses in tension and compression, respectively, in the longitudinal direction. Required if failure index is desired. (Real > 0.0) (Default value for X_c is X_t)

Y_t, Y_c Allowable stresses in tension and compression, respectively, in the transverse direction. Required if failure index is desired. (Real > 0.0) (Default value for Y_c is Y_t)

S Allowable stress for inplane shear (Real > 0.0)

G_E Structural damping coefficient (Real)

F12 Interaction term in the tensor polynomial theory of Tsai-Mu (Real). Required if failure index by Tsai-Mu theory is desired and if value of F12 is different from 0.0.

Remarks: 1. If $G_{1,z}$ and $G_{2,z}$ values are not supplied, the in-plane shear modulus will be used.

CSA/NASTRAN BULK DATA DECK

Input Data Card PCOMP Layered Composite Element Property

Description: Defines the properties of an n-ply composite material laminate

Format and Example:

1	2	3	4	5	6	7	8	9	10
PCOMP	PID	Z ₀	NSM	S _b	F.T.	TREF	GE	OPT	
PCOMP	991	-1.0	18.7	4.+3	HOFF				+ABC
	MID1	T1	θ ₁	SOUT1	MID2	T2	θ ₂	SOUT2	
+ABC	10	0.25	0.	YES			45.		+DEF
	MID3	T3	θ ₃	SOUT3	MID4	T4	θ ₄	SOUT4	
+DEF			-45.				90.		

<u>Field</u>	<u>Contents</u>
PID	Property ID (1,000,000 > Integer > 0)
Z ₀	Distance from the reference plane to the bottom surface (Real) (Default = -1/2 the thickness of the element)
NSM	Non-structural mass per unit area (Real)
S _b	Allowable shear stress of the bonding material (allowable inter-laminar shear stress). Required if failure index is desired. (Real > 0.0)
F.T.	Failure theory to be used to test whether the element fails or not (BCD). The following theories are presently allowed: HILL for the Hill theory HOFF for the Hoffman theory TSAI for the Tsai-Wu theory STRESS for maximum stress theory STRAIN for maximum strain theory
TREF	Reference temperature (Real)
GE	Damping coefficient (Real)

(Continued)

CSA/NASTRAN BULK DATA DECK

PCOMP (Continued)

OPT Lamination generation option codes (integer ≥ 0)

Code

0 all plies are specified

1 ply layup specified is symmetrical; only the plies on one side of the centerline are specified

2 a "stack" of membrane-only elements will be generated for each element that references this PCOMP card

options may be combined by adding the specific code values

MIDi Material ID of the various plies. MID1 > 0, MID2, ..., MIDn > 0, or blank. The plies are identified by serially numbering them from 1 at the bottom layer. The MIDi must refer to MAT1, MAT2, or MAT8 bulk data cards. (Integer > 0)

Ti Thicknesses of the various plies (Real; T1 > 0.0, T2, T3, ... > 0.0 or blank)

θ_i Orientation angle of the longitudinal direction of each ply with the material axis of the element. (If the material angle on the element connection card is 0.0, the material axis and side 1-2 of the element coincide). The plies are to be numbered serially starting with 1 at the bottom layer (the bottom layer is defined as the surface with the largest -Z value in the element coordinate system). (Real)

SOUTi Stress output required (YES) or not (NO) for the various plies (BCD) (Default = NO)

- Remarks:
1. The default under MID2, MID3, ..., is the last defined MAT card. in this case MID1; so for T2, T3, ..., all these thicknesses will be equal to T1.
 2. At least one of the four values (MIDi, Ti, θ_i , SOUTi) must be present for a ply to exist.
 3. TREF given on the PCOMP card will be used for all plies of the element; it will override values supplied on material cards for individual plies.
 4. CE given on the PCOMP card will be used for the element; it will override values supplied on material cards for individual plies.

CSA/NASTRAN BULK DATA DECK

Input Data Card PCOMP1 Layered Composite Element Property - Alternate Form 1

Description: Defines the properties of an n-ply composite material laminate, all plies being of equal thickness.

Format and Example:

1	2	3	4	5	6	7	8	9	10
PCOMP	PID	Z ₀	NSM	S _b	F.T.	MID	T	OPT	
PCOMP	891	-1.0	18.7	4.+3	STRAIN	10	0.25		+ABC
	θ ₁	θ ₂	θ ₃	θ ₄	θ ₅	θ ₆	θ ₇	θ ₈	
+ABC	0.0	+45.	-45.	+90.	+90.	-45.	+45.	0.0	

Field	Contents
PID	Property ID (1,000,000 > Integer > 0)
Z ₀	Distance from the reference plane to the bottom surface (Real) (Default = -1/2 the thickness of the element)
NSM	Non-structural mass per unit area (Real)
S _b	Allowable shear stress of the bonding material (allowable inter-laminar shear stress). Required if failure index is desired. (Real > 0.0)
F.T.	Failure theory to be used to test whether the element fails or not (BCD). The following theories are presently allowed: HILL for the Hill theory HOFF for the Hoffman theory TSAI for the Tsai-Wu theory STRESS for maximum stress theory STRAIN for maximum strain theory
MID	Material ID of each ply. The plies are identified by serially numbering them from 1 at the bottom layer. The MID must refer to MAT1, MAT2, or MAT8 bulk data cards. (Integer > 0)
T	Thickness of each ply. All plies have the same thickness. (Real > 0.0)

(Continued)

CSA/NASTRAN BULK DATA DECK

PCOMP1 (Continued)

OPT Lamination generation option codes (Integer ≥ 0)

Code

- 0 all plies are specified
- 1 ply layup specified is symmetrical; only the plies on one side of the centerline are specified.
- 2 a "stack" of membrane only elements will be generated for each element that references this PCOMP card

Options may be combined by adding the specific code values

θ_i

Orientation angle of the longitudinal direction of each ply with the material axis of the element. (If the material angle on the element connection card is 0.0, the material axis and side 1-2 of the element coincide). The plies are to be numbered serially starting with 1 at the bottom layer (the bottom layer is defined as the surface with the largest -Z value in the element coordinate system). (Real)

Remarks:

1. Stress and strain output will be generated for all plies if a case control STRESS request is present. See the PCOMP card description for a method of reducing the amount of output generated.

CSA/NASTRAN BULK DATA DECK

Input Data Card PCOMP2 Layered Composite Element Property - Alternate Form 2

Description: Defines the properties of an n-ply composite material laminate

Format and Example:

1	2	3	4	5	6	7	8	9	10
PCOMP2	PID	Z ₀	NSM	S _b	F.T.	MID		OPT	
PCOMP2	891	-1.0	18.7	4.+3	HÖFF	10			+ABC
	T1	θ ₁	T2	θ ₂	T3	θ ₃	T4	θ ₄	
+ABC	.25	0.0	.25	45.		-45.		+90.	+DEF
	T5	θ ₅	T6	θ ₆	T7	θ ₇	T8	θ ₈	
+DEF		+90.		-45.		45.		0.0	

<u>Field</u>	<u>Contents</u>
PID	Property ID (1,000,000 > Integer > 0)
Z ₀	Distance from the reference plane to the bottom surface (Real) (Default = -1/2 the thickness of the element)
NSM	Non-structural mass per unit area (Real)
S _b	Allowable shear stress of the bonding material (allowable inter-laminar shear stress). Required if failure index is desired. (Real > 0.0)
F.T.	Failure theory to be used to test whether the element fails or not (BCD). The following theories are presently allowed: HILL for the Hill theory HÖFF for the Hoffman theory TSAI for the Tsai-Wu theory STRESS for maximum stress theory STRAIN for maximum strain theory
MID	Material ID of the various plies. The plies are identified by serially numbering them from 1 at the bottom layer. The MID must refer to MAT1, MAT2, or MAT8 bulk data cards. (Integer > 0)

(Continued)

CSA/NASTRAN BULK DATA DECK

PCOMP2 (Continued)

OPT Lamination generation option codes (Integer > 0)

Code

- 0 All plies are specified
- 1 ply layup specified is symmetrical; only the plies on one side of the centerline are specified
- 2 a "stack" of membrane only elements will be generated for each element that references this PCOMP card

Options may be combined by adding the specific code values

Ti Thickness of the various plies (Real; T1 > 0.0, T2, T3...> .0 or blank)

θ_i Orientation angle of the longitudinal direction of each ply with the material axis of the element. (If the material angle on the element connection card is 0.0, the material axis and side 1-2 of the element coincide). The plies are to be numbered serially starting with 1 at the bottom layer (the bottom layer is defined as the surface with the largest -Z value in the element coordinate system). (Real)

- Remarks:
1. At least one of the two values, (Ti, θ_i) must be present for a ply to exist.
 2. Stress and strain output will be generated for all plies if a case control STRESS request is present. See the PCOMP card description for a method of reducing the amount of stress output generated.

CSA/NASTRAN BULK DATA DECK

Input Data Card PSHELL Shell Element Property

Description: Defines the membrane, bending, transverse shear, and coupling properties of thin shell elements

Format and Example:

1	2	3	4	5	6	7	8	9	10
PSHELL	PID	MID1	T	MID2	12I/T ³	MID3	TS/T	NSM	
PSHELL	101	700	0.25	701	1.1	703	.5	7.88	PSHL
	Z1	Z2	MID4						
+SHL	.125	-.125							

<u>Field</u>	<u>Contents</u>
PID	Property identification number (Integer > 0)
MID1	Material identification number for the membrane (Integer > 0 or blank)
T	Default value for the membrane thickness (Real)
MID2	Material identification number for bending (Integer > 0 or blank)
12I/T ³	Bending stiffness parameter (Real or blank, default = 1.0)
MID3	Material identification number for transverse shear (Integer > 0 or blank), must be blank unless MID2 > 0) default = MID2.
TS/T	Transverse shear thickness divided by the membrane thickness (Real or blank, default = .833333)
NSM	Nonstructural mass per unit area (Real)
Z1,Z2	Fiber distances for stress computation. The positive direction is determined by the righthand rule and the order in which the grid points are listed on the connection card. (Real or blank, see Remark 11 for defaults).
MID4	Material identification number for membrane-bending coupling (Integer > 0 or blank, must be blank unless MID1 > 0 and MID2 > 0, may not equal MID1 or MID2)

- Remarks:
1. All PSHELL property cards must have unique identification numbers.
 2. The structural mass is computed from the density using the membrane thickness and membrane material properties.

(Continued)

CSA/NASTRAN BULK DATA DECK

PSHELL (Continued)

3. The results of leaving an MID field blank are:
 - MID1 No membrane or coupling stiffness
 - MID2 No bending, coupling, or transverse shear stiffness
 - MID3 MID2 will be used as default for MID3
 - MID4 No bending-membrane coupling
4. The continuation card is not required.
5. The structural damping (for dynamics rigid formats) uses the values defined for the MID1 material.
6. The MID4 field should be left blank if the material properties are symmetric with respect to the middle surface of the shell. If the element centerline is offset from the plane of the grid points, the MID4 field may be used for modeling the offset but it involves laborious calculations that produce physically unrealistic stiffness matrices ("negative terms on factor diagonal") if done incorrectly.
7. This card is used in connection with the CTRIA3 and CQUAD4 cards.
8. For structural problems, PSHELL cards may reference MAT1, MAT2 or MAT8 material property cards.
9. If the transverse shear material, MID3, references a MAT2 data card, the G1 G12 and G22 values will be used to obtain the symmetric 2x2 $[G_3]$ matrix for the element.
10. For heat transfer problems, PSHELL cards may reference MAT4 or MAT5 material property cards.
11. The default for Z1 is $-T/2$, and for Z2 is $+T/2$, where T is the local plate thickness.