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**TRANSIENT ANALYSIS OF A MAGNETIC HEAT PUMP**

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**SUMMARY**

An experimental heat pump that uses a rare earth element as the refrigerant is modeled using NASTRAN. The refrigerant is a ferromagnetic metal whose temperature rises when a magnetic field is applied and falls when the magnetic field is removed. The heat pump is used as a refrigerator to remove heat from a reservoir and discharge it through a heat exchanger. In the NASTRAN model the components modeled are represented by one-dimensional ROD elements. The analysis accounts for heat flow in the solids, heat transport in the fluid, and heat transfer between the solids and fluid. The problem is mildly nonlinear since the heat capacity of the refrigerant is temperature-dependent. One simulation run consists of a series of transient analyses, each representing one stroke of the heat pump. An auxiliary program was written that uses the results of one NASTRAN analysis to generate data for the next NASTRAN analysis. Computed results are compared with experimental data.

**INTRODUCTION**

An experimental heat pump being developed at the David W. Taylor Naval Ship R&D Center (DTNSRDC) uses one or more rare earth elements as the working substance. These elements are ferromagnetic metals whose temperature increases when a magnetic field is applied and decreases when the field is removed. This thermodynamic property is used in a two-stroke cycle to pump heat from a reservoir and discharge it into the surrounding air. Thus, a refrigeration cycle is implied and the heat pump will be referred to as a refrigerator. Using rare earth elements as the refrigerant makes possible magnetic refrigerators that discharge heat at room temperature and refrigerate to very low temperatures. Brown (Reference 1) gives considerations that are required to make magnetic refrigeration practical for such applications and (Reference 2) several configurations that have been proposed for magnetic heat pumps. In the apparatus developed at DTNSRDC, a regenerator serves as the reservoir from which heat is removed and discharged through a heat exchanger. Figure 1 is a schematic description of this test apparatus. Heat is carried by a fluid, compressed nitrogen, that moves through the refrigerator. The refrigerant is formed into thin bands that are wrapped around mandrels to form coils. These coils are formed with gaps between the layers through which fluid can

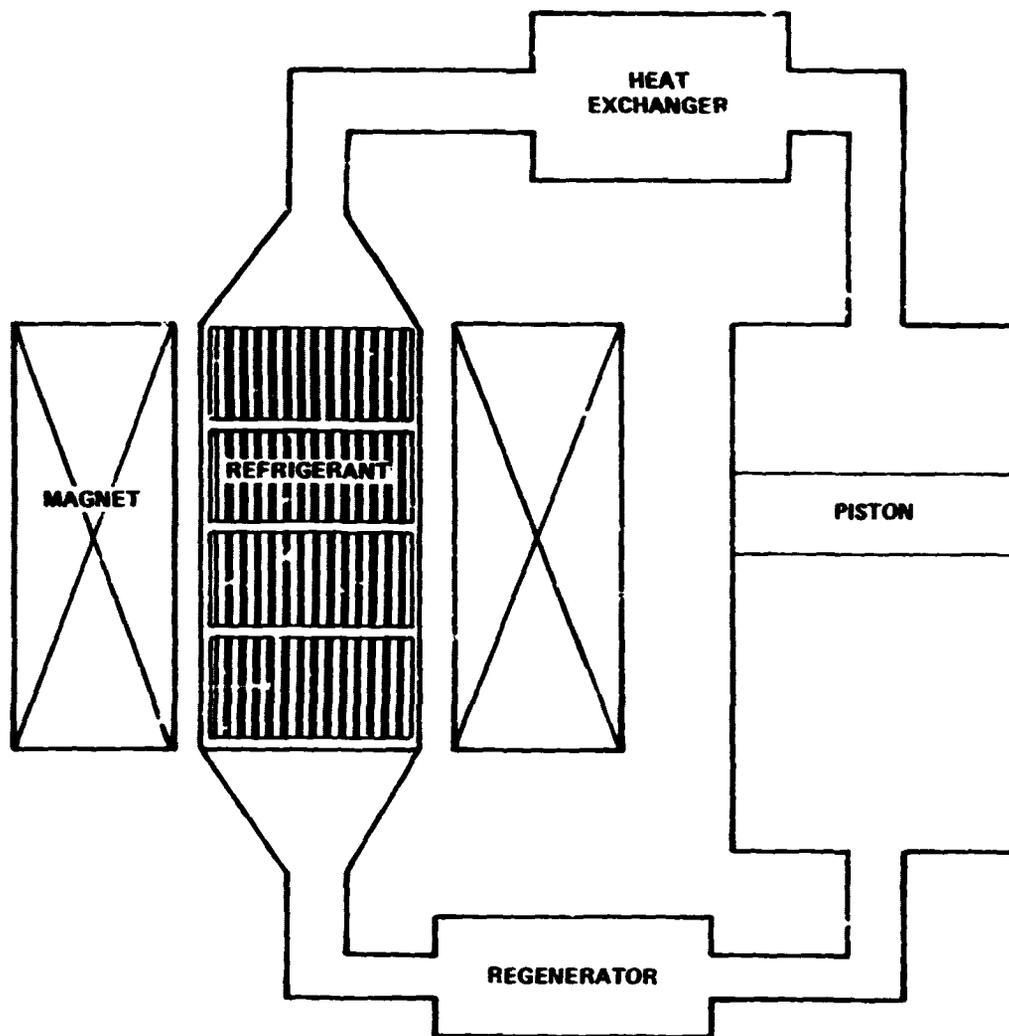


Figure 1. The Magnetic Refrigerator

flow. Several coils are stacked in a stainless steel chamber. The chamber is connected on the cool side to the regenerator and on the warm side to the heat exchanger. The fluid circuit is completed by a cylinder containing a piston that moves the fluid back and forth through the chamber, regenerator, and heat exchanger. The large magnetic field required through the refrigerant is provided

by a superconducting magnet.

The cooling cycle consists of two strokes which will be called the exhaust and intake strokes. During the exhaust stroke the magnetic field is applied to the refrigerant, raising its temperature, and fluid drawn through the cooled regenerator passes over and cools the refrigerant. The heat acquired by the fluid during the exhaust stroke is discharged by a heat exchanger. During the intake stroke, the magnetic field is removed; the temperature of the refrigerant falls, and fluid drawn through the heat exchanger and passing over the refrigerant in the opposite direction is cooled and in turn cools the regenerator.

To achieve very low temperatures, a refrigerator with several stages operating at progressively lower temperatures will be needed. Several rare earth elements or alloys of these elements will be used as refrigerants; each has a temperature range at which it works most effectively. One rare earth element, gadolinium, has its largest temperature change with a change in magnetic field at about 300 K which is near room temperature. Other rare earth metals that may be used are terbium and dysprosium which have largest temperature changes near 225 K and 180 K, respectively.

#### REQUIREMENTS FOR THE MATHEMATICAL ANALYSIS

Different designs for the refrigerator, using various materials must be tested to determine which configurations and material properties hold the most promise for effective and efficient operation. It is more efficient to model the refrigerator analytically and simulate its operation with different designs and materials than to make coils of various materials and then build and test experimental refrigerators.

Since DTNSRDC has considerable experience with the NASTRAN program, it was decided to use its heat transfer capability for this problem. However, several aspects of the problem make it a non-routine application. The material properties of the refrigerant are sufficiently temperature-dependent in the temperature range used to make the problem nonlinear. Also, each test run of the refrigerator consists of many strokes. Each stroke corresponds to a time segment in the solution with initial conditions different from, but determined, by the final conditions of the previous segment. NASTRAN provides for neither nonlinear material properties nor changing temperature values during a transient analysis. To accommodate these requirements, a sequence of NASTRAN analyses is used, one for each stroke. An auxiliary program HEATSTEP was developed that uses results from the NASTRAN analysis of the previous stroke, computes the initial conditions for the next stroke, and prepares data for the next NASTRAN analysis.

## ASSUMPTIONS

A number of simplifying assumptions for the mathematical model keep the analysis manageable. Only the refrigerant, the chamber, and the fluid passing through the chamber are modeled, and a simple approximation is made of the heat capacity of the tubing leading from the chamber to the regenerator. Since the layers of refrigerant and fluid and the stainless steel chamber wall are thin, it is assumed that there is no significant temperature variation through these layers. It is also assumed that there is no significant temperature variation across the chamber. Therefore, the temperature of each type of material is assumed to vary only along the length of the refrigerant chamber, that is, the problem is one-dimensional. Since several seconds are required to apply and remove the magnetic field, it is assumed that at the beginning of each stroke the temperature of the fluid equals the temperature of the refrigerant at the same position. The temperature distribution in the chamber wall is assumed not to change while the magnetic field is changing. The refrigerant coils are stacked one on top of another, so there is a relatively small thermal connection between the coils, and it is approximated by a conductor with a relatively small thermal conductivity.

During the intake stroke the heat exchanger, and during the exhaust stroke the regenerator, are assumed to provide sources of fluid with constant temperatures. Therefore, during the intake stroke, the temperature of the fluid flowing into the refrigerant chamber equals the ambient temperature. Determining the temperature of the fluid entering the chamber during the exhaust stroke is not so straightforward. It is assumed that the temperatures of the fluid and the tubing on the cold side are equal and constant during the exhaust stroke, and at the beginning of the intake stroke the temperature of the cold side is the same as it was during the last exhaust stroke. Therefore, the temperature of the fluid entering the refrigerant chamber during the present exhaust stroke is computed by taking a weighted average of the average temperature of fluid entering the regenerator during the preceding intake stroke and the temperature of fluid leaving the regenerator during the preceding exhaust stroke. In computing the average, these temperatures are weighted by the values of the heat capacity of the fluid moved during one stroke and the heat capacity of the cool side of the apparatus, respectively. The temperature change during one stroke is at most a few degrees, so the temperature-dependent material properties are determined for the initial temperatures for each stroke and remain constant during the stroke.

## THE EQUATIONS THAT DESCRIBE THE REFRIGERATOR MODEL

The partial differential equations that describe the temperature distribution in the refrigerant and the chamber wall are one-dimensional heat equations. A loading term on the right hand side represents film convection of heat to the fluid. These equations are of the form

$$\rho_m c_m \frac{\partial T_m}{\partial t} - k_m \nabla^2 T_m = \frac{A h}{V_m} (T_f - T_m)$$

where  $m = r, s$  or  $f$  for refrigerant, stainless steel or fluid

$\rho_m$  = density of material

$c_m$  = heat capacity of material

$k_m$  = conductivity of material

$A_m$  = area of the surface between the material and fluid

$V_m$  = volume of material

$T_m$  = temperature of material

$h$  = convection coefficient

The partial differential equation that describes the temperature distribution in the fluid is

$$\rho_f c_p \frac{\partial T_f}{\partial t} - k_f \nabla^2 T_f = -\rho_f c_p v \frac{\partial T_f}{\partial x} + \frac{A_r h}{V_f} (T_r - T_f) + \frac{A_s h}{V_f} (T_s - T_f)$$

The term containing  $\partial T_f / \partial x$  represents transfer of heat due to the flow of the fluid. The two terms on the right hand side represent film convection to the refrigerant and stainless steel chamber wall. Boundary conditions for the one-dimensional heat equation consist of the initial temperature distribution in the chamber and specified temperatures or derivatives of the temperature at the ends of the chamber. The boundary conditions are provided by the initial temperatures and constraints contained in the NASTRAN input data.

#### THE FINITE ELEMENT MODEL

The problem is solved using the NASTRAN transient thermal analysis rigid format. The refrigerant, stainless steel, and fluid are modeled by NASTRAN one-dimensional ROD elements. The thermal contacts between the coils of refrigerant are represented by NASTRAN ELAS1 elements. Transfer of heat to and from the fluid from the refrigerant material and the stainless steel chamber is modeled using HBDY elements of type LINE. Figure 2 is a schematic representation of the finite element model.

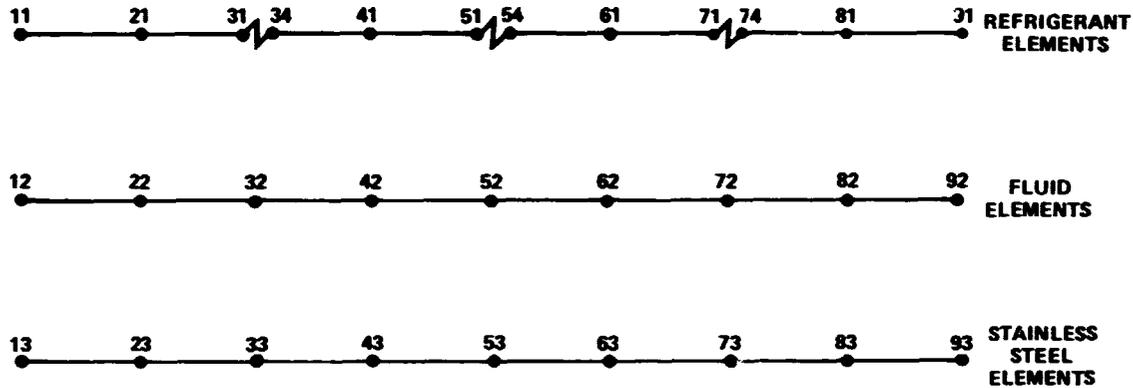


Figure 2. The Finite Element Model

The term in the fluid differential equation containing the spatial derivative of the fluid temperature is obtained by a finite difference approximation using a NASTRAN transfer function. This term is included in the differential equation by applying a load to the appropriate node using the capability for applying nonlinear loads. For example, to compute the finite difference approximation for the fluid node number 32, a value,  $u_{732}$  is assigned to an extra point, number 732, by the equation

$$Bu_{732} + A_1 u_{42} - A_2 u_{32} = 0$$

for  $u_{732} = (u_{42} - u_{32}) / \Delta x$ , where  $\Delta x$  is the distance between adjacent nodes, and  $u_{32}$  is the temperature at node number 32, so  $B = \Delta x$ ,  $A_1 = -1$ , and  $A_2 = 1$ . The preceding approximation is for fluid flowing in the direction of increasing nodal identification numbers; for fluid flowing in the other direction the approximate value is

$$u_{732} = (u_{32} - u_{22}) / \Delta x$$

The term with  $\partial T / \partial x$  is added in the form of a load using a NASTRAN NOLIN card where  $u_{732} = \partial T / \partial x$  is the load and  $\rho_f c_p v W$  is a scale factor.

The temperature of the fluid entering the chamber is maintained by a large source specified by a TLOAD1 card. The magnitude of this force multiplied by a large scale factor  $M$  is given on a DAREA card and applied to a scalar point. A large conductor from the scalar point to ground is specified by an ELAS4 element

that has conductivity equal to the scale factor M. Thus, the value assigned to the scalar point is constrained to equal the input temperature, and then the appropriate fluid grid point temperature is constrained by a multipoint constraint to equal this value. The temperature of the fluid entering the refrigerant chamber is maintained by attaching to the fluid node located where the fluid enters the chamber, a large linear conductor represented by a NASTRAN ELAS4 element, and applying to this node a source of heat with a magnitude that will enforce the required temperature.

#### THE PROGRAM HEATSTEP

The program HEATSTEP, developed at DTNSRDC, is run after each NASTRAN analysis to prepare data for the following NASTRAN run. To determine the temperatures at the end of the stroke and the average temperature of the fluid leaving the refrigerant chamber during an intake stroke, HEATSTEP reads the file of nodal temperatures produced by NASTRAN. Using this information it computes the temperature drop or rise at each refrigerant node depending on whether the preceding stroke was an intake or exhaust stroke. The program then determines the temperature of each refrigerant and stainless steel node for the beginning of the following stroke. The program also computes heat capacities for each refrigerant element.

HEATSTEP determines the temperature of the fluid that will be passing into the chamber on the following stroke, and selects the fluid grid point to which the incoming fluid temperature constraint is to be applied. This is accomplished by selecting the one of two MPCs which connects the appropriate fluid grid point to the scalar point whose temperature has been specified. For an intake stroke, the temperature of the fluid grid point nearest the heat exchanger is constrained to equal the ambient temperature. The rationale for this average is given in the section on Assumptions. The input temperature  $T_{IN}$  is given by the equation

$$T_{IN} = (C_F T_{OUT} + C_T T_{LAST}) / (C_F + C_T)$$

where  $C_F$  is the heat capacity of the volume of fluid moved during one stroke and  $C_T$  is the heat capacity of the tubing on the cold side;  $T_{OUT}$  is the average temperature of fluid leaving the chamber during the preceding intake stroke, and  $T_{LAST}$  is the temperature  $T_{IN}$  for the last exhaust stroke.

Finally, HEATSTEP prepares an input file for the next NASTRAN run that incorporates the new data it has computed.

## RESULTS AND DISCUSSION

Several analyses were made using this method. Some of these analyses were made to get relative indications of the performance of proposed configurations. One analysis modeled the experimental device; results from this analysis are plotted in Figure 3.

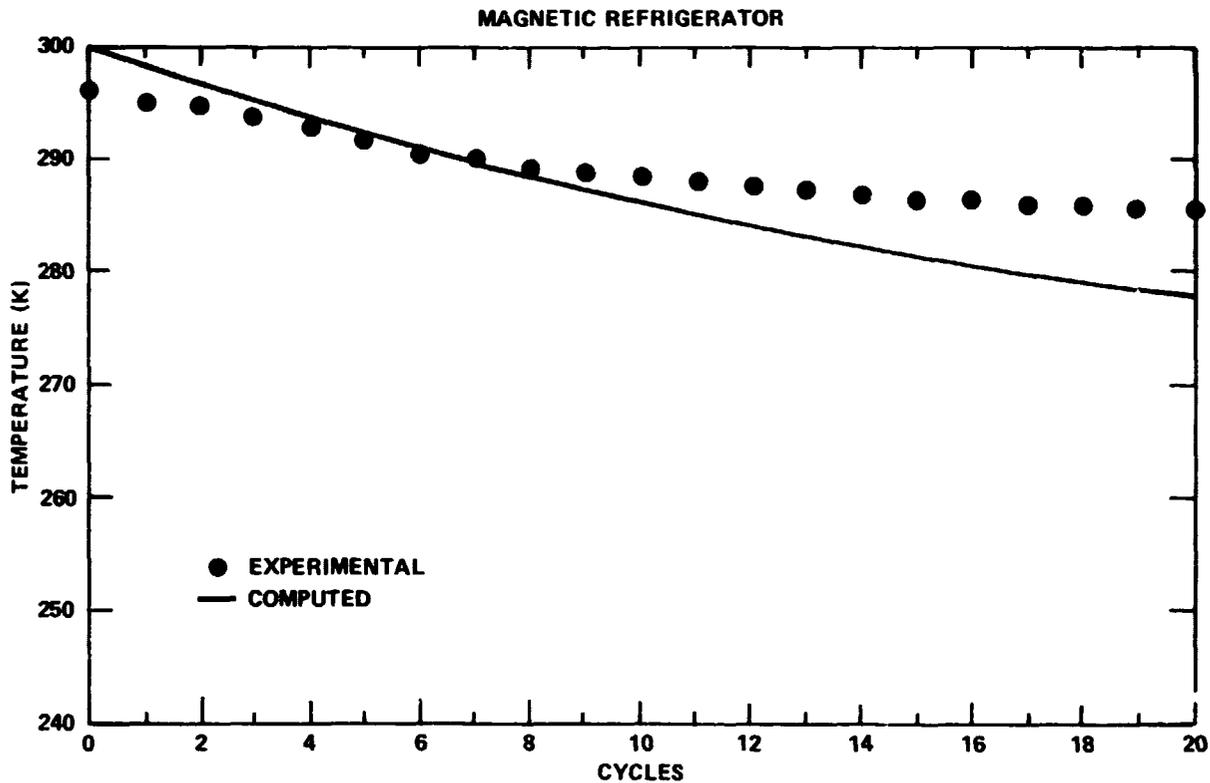


Figure 3. Comparison of Computed and Experimental Temperatures

Several factors may contribute to improving agreement between the computed and experimental results. To avoid penetrating the pressurized chamber, thermocouples were mounted on the outside surface of the stainless steel refrigerant chamber. Steps are being taken to move the thermocouples inside the chamber to more accurately record the temperature of the refrigerant and fluid. The finite element model uses a simple representation of the regenerator and probably its heat capacity is under represented; a larger heat capacity would likely slow the temperature

drop, producing better agreement in the results. It is also likely that better values may be obtained for the film convection coefficients and thermal conductivity of the contact between refrigerant coils. These improvements will be made during future work.

#### REFERENCES

1. Brown, G. V., "Magnetic Heat Pumping Near Room Temperatures," J. Appl. Phys., Vol. 47, pp. 3673-3680 (1976).
2. Brown, G. V., "Basic Principles and Possible Configurations of Magnetic Heat Pumps," ASHRAE Transactions, Vol. 87, Part 2, pp. 783-793 (1981).

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16. Abstract  This publication is the proceedings of a colloquium, and it contains technical papers contributed during the Thirteenth NASTRAN® Users' Colloquium held in Boston, Massachusetts on May 6-10, 1985. The authors review general application of finite element methodology and the specific application of the NASA Structural Analysis System, NASTRAN, to a variety of static and dynamic structural problems.			
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