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(NASA-C3-171862) SPACE SHUTTLE PROGRAM
COMMUNICATION AND TRACKING SYSTEMS INTERFAC
ANALYSIS Final Report (Axiomatix, Los Angeles, Calif.) 365 p HC A16/MF A1
Unclass
CSCL 22B G3/16 21158
SPACE SHUTTLE PROGRAM

COMMUNICATION AND TRACKING SYSTEMS

INTERFACE ANALYSIS

Final Report
Contract NAS9-16853

Prepared for
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Houston, Texas 77058

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Axiomatix Report No. 8409-2
September 30, 1984
ADDENDUM

1. Performance of Solar Max Data and Tape Recorder Dumps Using the Ku-Band Channel 2 Through TDRSS

2. Analysis of the Deployed Electronic Assembly Exciter Loop Drop Lock Problem

3. RF Coverage Analysis

4. A Toroidal Antenna Pattern for Centaur ISPM

5. BER Degradation Due to Centaur Transmitted Bit Timing Jitter Noise Through the Shuttle Payload Communication System

6. Performance Analyses of Using Concatenated-Coding for Centaur Signaling Through TDRSS

7. Investigation of Potential Interface Between the Shuttle and the Space Station
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 EXECUTIVE SUMMARY</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Contract Tasks</td>
<td>3</td>
</tr>
<tr>
<td>1.2 Performance of the Contract Tasks</td>
<td>6</td>
</tr>
<tr>
<td>2.0 KU-BAND SYSTEM IMPLEMENTATION AND PERFORMANCE</td>
<td>8</td>
</tr>
<tr>
<td>2.1 Analysis of Solar Max and Tape-Recorder Signals Through the Ku-band System</td>
<td>8</td>
</tr>
<tr>
<td>2.2 Analysis of the Deployed Electronic Assembly Exciter Loop Lock Problem</td>
<td>16</td>
</tr>
<tr>
<td>3.0 S-BAND SYSTEM ANALYSIS</td>
<td>11</td>
</tr>
<tr>
<td>3.1 Shuttle Orbiter S-band Antenna Activities</td>
<td>11</td>
</tr>
<tr>
<td>3.2 RF Coverage Analyses</td>
<td>12</td>
</tr>
<tr>
<td>4.0 CENTAUR INTERFACE COMPATIBILITY EVALUATION</td>
<td>14</td>
</tr>
<tr>
<td>4.1 Summary of Axiomatix Centaur Activities</td>
<td>14</td>
</tr>
<tr>
<td>4.2 Antenna Pattern Analysis</td>
<td>15</td>
</tr>
<tr>
<td>4.3 BER Degradation Due to Centaur Transmitted Bit Timing Jitter</td>
<td>15</td>
</tr>
<tr>
<td>4.4 Performance Analyses of Using Concatenated Coding For Centaur Signaling Through TDRSS</td>
<td>16</td>
</tr>
<tr>
<td>5.0 SHUTTLE COMMUNICATION AND TRACKING SYSTEM COMPATIBILITY ANALYSIS</td>
<td>18</td>
</tr>
</tbody>
</table>
1.0 EXECUTIVE SUMMARY

The Space Shuttle Program Communications and Tracking Systems Interface Analysis of Contract NAS 9-16893 began April 18, 1983. During this time, the Shuttle communication and tracking systems began flight testing. Two areas of analysis documented in this report were a result of observations made during flight tests. These analyses involved the Ku-band communication system. First, there was a detailed analysis of the interface between the solar max data format and the Ku-band communication system including the TDRSS ground station. The second analysis involving the Ku-band communication system was an analysis of the frequency-lock loop of the Gunn oscillator used to generate the transmit frequency. The stability of the frequency-lock loop was investigated and changes to the design were reviewed to alleviate the potential loss of data due to the loop losing lock and entering the reacquisition mode. These analyses are discussed in Section 2 of this report.

Other areas of investigation were the S-band antenna analysis and RF coverage analysis. The S-band antenna system analysis involved the monitoring of the production and deliveries of the flight S-band antenna systems for the Orbiter. This area of analysis is reported in Section 3.1 and discusses the failure analyses primarily concerned with the RF switch box failures due to the lack of hermetic seals. The RF coverage analyses are presented in Section 3.2 and provide assessments of the flight tests of the Shuttle communication systems.

Two additional areas of analysis are documented in this report. The first area is the interface of the Centaur with the Shuttle. Axiomatix performed several interface compatibility analyses. First, Axiomatix proposed an biconical array antenna to meet the requirements of the Solar Polar mission. This antenna concept is reported in Section 4.2. Axiomatix analyzed the BER degradation due to transmitted bit timing jitter noise including continuous or discrete components from the Centaur Digital Computer Unit. The BER degradation was found to be only 0.04 dB. This analysis is reported in Section 4.3. Finally, since the Centaur link through the TDRSS is extremely weak, Axiomatix proposed to use the NASA standard Reed-Solomon code concatenated with currently used convolutional code. To make certain that this concatenating was practical for the Centaur environment, Axiomatix investigated the BER performance degradation due to multiple encryptors and differential coding. The results of these analyses are presented in Section 4.4.
The final area of analysis was the Shuttle communication and tracking system compatibility analysis reported in Section 5. With the advent of the Space Station, the Shuttle will become part of a large number of space vehicles and spacecraft that will be linked together in a communication network. As part of the Shuttle communication and tracking system compatibility analysis, Axiomatix made a preliminary investigation into the potential interface between the Shuttle and the Space Station. In particular, an investigation was made into how to accommodate a large number of users in a multiple access communication network. The results of this investigation is presented in Section 5.
1.1 **Contract Tasks**

There are five major contract tasks which are related to the following Communication and Tracking (C&T) elements of the Space Shuttle Program (SSP). These SSP C&T elements are (1) the Ku-band radar/communication system, (2) the S-band network communication system, (3) the S-band antenna system, and (4) the S-band payload communication system. The detailed tasks follow.

**Task 1: Ku-Band Radar/Communication Systems Analysis**

**Subtask 1—Ku-band Radar System Implementation and Performance Evaluation**

The contractor shall provide the technical support to evaluate the Ku-band rendezvous radar implementation based on his own analytical performance predictions and test data obtained from the hardware contractor (Hughes Aircraft Company), the Orbiter contractor (Rockwell International) and NASA. The contractor shall provide NASA with the performance expected for a given rendezvous profile so that NASA may determine the best operation plan to carry out the mission. As design modifications are proposed to meet mission performance requirements, the contractor shall provide updated analytical performance predictions and recommend tests that are needed to provide an evaluation of the design modification feasibility to meet the SSP operational requirements. The contractor shall provide a performance evaluation of the test data obtained by on-orbit tests and recommend any design modifications required to meet the SSP operational requirements and to determine the operation limitations that must be imposed with the Ku-band radar system, as designed.

**Subtask 2—Ku-band Communication System Implementation and Performance Evaluation**

The contractor shall provide the technical support to evaluate the Ku-band communication system implementation based on his own analytical performance predictions and test data obtained from the hardware contractor (Hughes Aircraft Company), Rockwell and NASA. The contractor shall provide NASA with the acquisition and tracking performance expected of the TDRSS by the Ku-band system. As part of this task, the contractor shall provide an analysis of the latest TDRSS pointing budget and the Ku-band system sidelobe rejection capability. The contractor shall provide an evaluation of overall communication system performance from the payload and Orbiter information sources through both the Ku-band system and the TDRSS. This evaluation shall be based on the contractor's analytical predictions and test data from Hughes, Rockwell and NASA, including on-orbit tests. The contractor shall identify marginal system performance parameters, recommend design changes to the Ku-band system and to the interfaces with the payload and Orbiter information sources, and determine any operational limitations that must be imposed with the Ku-band communication system, as designed.

**Task 2: S-Band Network Communication System Analysis**

The contractor shall provide the technical support to evaluate the S-band network communication system performance through the TDRSS. This evaluation shall be based on the contractor's analytical performance
predictions and test data obtained from the hardware contractor (TRW), the Orbiter contractor (Rockwell International), and NASA. Particular attention shall be given to the spread-spectrum processor of the Orbiter network transponder to establish the performance of its current implementation. An important part of this evaluation is the on-orbit tests with the TDRSS. The contractor shall identify marginal system performance parameters, recommend design changes to the S-band system required to meet the SSP operational requirements and to determine the operational limitations that must be imposed with the S-band network communication system, as designed.

**Task 3: S-band Antenna System Analysis**

The contractor shall review and evaluate the S-band antenna performance and make recommendations for improvements to meet the SSP operational requirements. During this evaluation effort, the contractor shall monitor the antenna pattern measurement testing at JSC and identify areas of concern for achieving system requirements. If the measured performance is found to be marginal or inadequate, the contractor shall evaluate the design of the present Orbiter flight antennas and recommend modifications which will correct deficiencies. New techniques will be investigated to simplify the present antenna concepts which may be considered for future improved space antenna systems.

**Task 4: S-band Payload Communication System Analysis**

**Subtask 1—S-band Payload Communication System Implementation and Performance Evaluation**

The contractor shall provide technical support to evaluate the S-band payload communication system implementation based on his based own analytical performance predictions and test data obtained from the hardware contractor (TRW), the Orbiter contractor (Rockwell International) and NASA. The contractor shall quantify nominal system operations and deviations due to variations in system performance parameters. The contractor shall evaluate the system operational scenarios expected and recommend design changes to the S-band payload system to meet the SSP operational requirements and to determine any operational limitations that must be imposed with the S-band payload communication limitations that must be imposed with the S-band payload communication system, as designed.

**Subtask 2—Payload Interface Compatibility Evaluation and Problem Area Resolution**

The contractor shall provide analytical performance predictions of possible interface compatibility problems associated with interfacing payload elements such as the IUS, TDRS, PAM, Centaur, and other spacecraft command and telemetry systems with the Orbiter communications and data-handling subsystems (or elements) including the Ku-band Signal Processor, Payload Signal Processor, Payload Data Interleaver, Payload Interrogator and Payload Recorder. The contractor shall identify marginal system performance parameters that would result in the payload or Orbiter failing to meet agreed-upon interface requirements contained in the Orbiter/cargo standard interface specification (NASA JSC ICD 2-19001) and payload-unique ICD's. The contractor shall determine any design changes to the Orbiter subsystems or payload systems that must be made in order to meet the mission performance requirements and to determine the operational limitations of the Orbiter subsystems and each of the payload systems, as designed.
Task 5: Shuttle Communication and Tracking System Compatibility  
Analysis and Problem-Area Resolution

The contractor shall analyze the overall Shuttle C&T systems and identify potential performance inadequacies in order to meet updated SSP operational requirements. These areas of updated SSP operational requirements include, but are not limited to, improvements to navigation and encryption of payload data and television. The contractor shall recommend design changes to the Shuttle C&T systems to meet these updated SSP operational requirements and provide analytical performance predictions for any changes proposed to the C&T systems.
1.2 **Performance of the Contract Tasks**

During the contract period from April 18, 1983 to September 30, 1984, the Shuttle communication and tracking systems began on-orbit flight testing. As a result of the on-orbit tests two distinct problems developed concerning the Ku-band communication system. First, there was a detailed analysis of the interface between the solar max data format and the Ku-band communication system including the TDRSS ground station. A similar problem to that predicted for the solar max mission was also observed during tests of dumps of the tape recorders through the Ku-band system. To analyze this problem, the complete payload communication system through the Ku-band system including the TDRSS ground system had to modeled as was proposed as part of Task 1. The second problem that developed concerning the Ku-band communication system was the discovery that the deployed electronic assembly (DEA) exciter would drop lock and transmit random data. These two problems are presented in Section 2 of this report.

Task 3, the S-band antenna system analysis, involved the monitoring of the production and deliveries of the flight S-band antenna systems for the Orbiter. This area of analysis is reported in Section 3.1 and discusses the failure analyses primarily concerned with the RF switch box failures due to lack of hermetic seals. Task 2 was primarily concerned with the RF coverage analyses presented in Section 3.2 which provide assessments of the flight tests of the Shuttle communication systems.

The primary area of investigation of Task 4 was the Centaur interface compatibility evaluation presented in Section 4. Besides the participation in the Centaur Avionics Integration Panel, the Centaur Communications and Data System Subpanel, and the Centaur Telemetry System Interim Critical Design Review, Axiomatix performed several interface compatibility analyses. Axiomatix proposed a biconical array antenna to meet the requirements of the Solar Polar mission. This antenna concept is reported in Section 4.2. Axiomatix analyzed the BER degradation due to transmitted bit timing jitter noise including continuous or discrete components from the Centaur Digital Computer Unit. The BER degradation was found to be only 0.04 dB. This analysis is reported in Section 4.3. Finally, since the Centaur link through the TDRSS is extremely weak, Axiomatix proposed to use the NASA standard Reed-Solomon code concatenated with currently used convolutional code. To make certain that this concatenating was practical for the Centaur environment,
Axiomatix investigated the BER performance degradation due to multiple encryptors and differential coding. The results of these analyses are presented in Section 4.4.

The final area of analysis was Task 5, the Shuttle communication and tracking system compatibility analysis, reported in Section 5. With the advent of the Space Station, the Shuttle will become part of a large number of space vehicles and spacecraft that will be linked together in a communication network. As part of the Shuttle communication and tracking system compatibility analysis, Axiomatix made a preliminary investigation into the potential interface between the Shuttle and the Space Station. In particular, an investigation was made into how to accommodate a large number of users in a multiple access communication network. The results of this investigation is presented in Section 5.
2.0 KU-BAND SYSTEM IMPLEMENTATION AND PERFORMANCE EVALUATION

There were two specific problems that were analyzed as part of the Ku-band System Implementation and Performance Evaluation. First, there was a detailed analysis of the interface between the solar max data format and the Ku-band communication system including the TDRSS ground station. A similar problem to that predicted for the solar max mission was also observed during tests of dumps of the tape recorders through the Ku-band system. The second problem that was analyzed was the tendency of the exciter in the deployed electronic assembly (DEA) of the Ku-band system to drop lock and transmit random data.

2.1 Analysis of Solar Max and Tape-Recorder Signals Through the Ku-band System

The analyses of the solar max data and the tape recorder dumps using the channel 2 of the Ku-band communication system are presented in Addendum 1. Channels 1 and 2 of the Ku-band communication system are in-phase quadrature on a 8.5 MHz square-wave subcarrier. That is, channel 2 is on the inphase (I) component of the QPSK signal and channel 1 is on the quadrature phase (Q) component of the QPSK signal. For the Solar Max Retrieval, computer dump data was to be transmitted to the Shuttle Orbiter at 32 kbps directly modulating the S-band carrier. The first question to be asked was whether the Interrogator (PI) would be able to demodulate the signal without false locking. The second question to be asked was whether the demodulated signal would be completely suppressed by the hard limiter in the channel 2 modulator of the Ku-band communication system. These two questions are analyzed and favorably answered in the Technical Memorandum No. M8310-1, dated October 7, 1983 and entitled "Solar Max Retrieval Signal Structure Before and After Limiting Using the Ku-Band Channel 2 Input", which is included in Addendum 1.

Once it was determined that the solar max signal could be demodulated by the PI and would not be totally suppressed by the hard-limiter in the channel 2 modulator, it was necessary to determine the performance of the TDRSS ground station demodulation. After testing the Ku-band link including the demodulation by the TDRSS ground station in the Electronic Systems Test Laboratory (ESTL), it was found that the I and Q signals reversed at the output of the Medium-Range Demodulation (MRD). An initial evaluation of the test data and the signals transmitted through channel 2 provided some understanding of
the problem. In Addendum 1, the Technical Memorandum No. M8401-1, dated January 10, 1984 and entitled "A Possible Explanation of the Solar Max Retrieval I-Q Swapping due to Noise Filtering in the 8.5 MHz Subcarrier Demodulator" presents an analysis that shows that the power ratio of channel 2 to channel 1 out of the lowpass filters in the TDRSS ground station MRD can be as low as 1:1 when the input signal is lower than -113 dbm. Although approximations were necessary, it is shown that the noise power present in the 5-MHz PI shares the total power in channel 2 with the 32-kbps signal after hardlimiting by the channel 2 modulator but, depending on the SNR in the 5-MHz bandwidth, a good portion of the noise is filtered out in the TDRSS QPSK subcarrier demodulator arm filters. Thus, the QPSK demodulator has great difficulty in deciding whether the normally strong channel 2 or the normally weak channel 1 is in the inphase strong channel position. This could account for the tendency to lock in the wrong I-Q relationship and not be detected.

Following further testing of the Ku-band link including the demodulation by the TDRSS ground station in the ESTL, it was found that the I-Q reversal occurred during track changes when the tape recorder signal was transmitted through the channel 2 of the Ku-band communication system similar to the effect that occurred when the solar max signal was transmitted through channel 2. This result could not be explained by excess noise sharing power with the signal in the channel 2 hardlimiter. Therefore, a more detailed analysis was performed and documented in Axiomatix Report No. R8406-1, dated June 1, 1984 and entitled "Analysis of the Ku-Band Shuttle/MRD I-Q Reversal Phenomena." In this report included in Addendum 1, the I-Q flipping problem of the MRD was studied by analysis and simulation to discover why the flipping occurred. In the case of the tape-recorder dump during start-up or wraparound, it was found that the composite tape-recorder signal contained two tones increasing with frequency, plus a spectrally flat noise term. Depending on the prelimiter-filter bandwidth into the channel 2 subcarrier modulator, this composite signal determined the conditions for I-Q flipping. A 2.5 MHz, six-pole prelimiter-filter stopped the I-Q flipping completely. In the case of the solar max I-Q flipping, the small SNR at the PI caused mainly noise to be transmitted on channel 2. At the receiver, most of the noise was filtered out, greatly reducing the effective I-Q power ratio at the MRD subcarrier loop. It should be noted that observing the existence and modeling of the two tones, along with the noise output from the tape recorder start-up or wraparound, allowed Axiomatix to make very accurate performance predictions. Specifically, the
comparisons between the NASA measurements and the Axiomatix predictions based on simulations or calculations are very good for a variety of cases.

2.2 Analysis of the Deployed Electronic Assembly Exciter Loop Lock Problem

The other area of analysis of the Ku-band system is the analysis of the DEA exciter loop drop lock problem. This analysis is documented in the Axiomatix Report No. R8410-2, dated May 15, 1984 and entitled "Deployed Electronic Assembly Exciter Loop Lock Problem." This report is included in Addendum 2. The DEA provides clean signals for the transmitter TWTA in each of the five Radar frequency channels, and for the single channel in the Communication mode of operation. In the TWTA bypass mode of the Radar operation, the exciter output is used as the transmitter signal. In addition, the exciter provides the first and second LO signals for use in the Receiver mixer downconverters. The problem that was addressed in the report was the intermittent loss of phase lock in the exciter with the relock requiring an additional 20 μs (which is the time for a single sweep). The occurrence of this loss of phase lock appeared to be random with a mean time between losses of about one hour. This report presents the investigation and analysis of the loss of lock problem and describes the meetings held between Hughes and Axiomatix. By changing the gains and hysteresis in the loop, the drop out and the resulting resweep were eliminated. The details of the changes are presented in the report.
3.0 S-BAND ANTENNA SYSTEM ANALYSIS

3.1 Shuttle Orbiter S-Band Antenna Activities

The primary task was to monitor the production and deliveries of the flight S-band antenna systems for the Shuttle. The major problem that arose during the contract period was the failures attributed to the RF switch, which were not hermetically sealed and therefore were vulnerable to multipaction effects characterized by high power arcing during a brief period of operation when the internal pressure within the switch reached a critical value during the slow leakage of ambient nitrogen into the space environment. Since the switches were not designed to maintain the internal nitrogen atmosphere, this partial vacuum condition conducive to multipaction was inevitable, resulting in arcing and subsequent destruction of the RF switch.

The failure analysis conducted by Rockwell International were observed during disassembly of many of the failed units. The evidence indicated that once multipaction was initiated, the resultant plasma created an electrical short which dissipated the incident RF power as thermal energy in the latching reed blade area of the switch creating charring of organic insulating materials and warping the reed blades such that electrical open circuits occurred. The extent of the damage depended on the duration of multipaction and were somewhat minimized by disrupting antenna use during this multipaction period once the cause of the failures was recognized.

The obvious cause of the multipaction was the absence of hermetic sealing of the RF switch box. Due to the extremely high cost of achieving adequate hermetic seals, an interim solution using RTV seals was devised, tested, and implemented to correct this problem, although there were no assurances of a hundred mission capability for this fix. Also, it was suggested that a more heat resistant reed blade material be substituted for the existing phosphor bronze which warped into a permanent bend under high temperatures. A high strength, high temperature metal such as stainless steel (gold plated for electrical conductivity), rather than ductile copper alloys, for example, could withstand the arcing if it occurred and yet maintain its straight shape, thereby retaining the operational capabilities of the RF switch during the remainder of the flight and subsequent flights, but it is believed no such corrective action was considered by the vendor.

Besides this RTV seal interim solution, there were other quality control fabrication problems that had to be corrected. Poor epoxy staking of
fastening screws and wiring were prominent items. It was observed that minute metallic particles resulting from machining were still present in the latching reed blade area and recommendations to fine hone the holes for insulating teflon standoffs were made. Broken connector leads resulting from loosened epoxy-staked attachment screws was another problem. The suggestion to double-nut the fastening screws was not deemed necessary; instead, more epoxy is to be used.

The quad S-band antenna patterns continue to have submarginal performance, but given the existing design, little improvement can be expected. The five horn cross configuration intended to be a possible quad upgrade has been fabricated and will be tested in the near future. Delays were encountered due to the lack of available RF switches. Other schemes to incorporate simultaneous multibeam capabilities and new low level beam switching techniques are being investigated to circumvent this dependence on electromechanical RF switches.

3.2 RF Coverage Analyses

Addendum 3 includes several technical memoranda that provide the assessment of the flight tests of the communication systems. The Technical Memorandum entitled "STS-9 Ascent RF Coverage Communication Support Formulation" provides the recommended STS-9 ascent-phase STDN support plan and the corresponding predicted Orbiter S-band PM automatic antenna switching up to and including TDRS ascent support, as derived from analysis of the STS-9 Cycle 1 reference trajectory.

STS-8 was the first Shuttle mission operation of communications via a TDRS satellite for both S-band and K-band communications. DTO 702 was an STS-8 communications flight test to exercise different operational configurations of the Orbiter's S-band communication system towards establishing flight readiness of the TDRS-mode operation. DTO 702 required exercising S-band communications through TDRS using High Data Rate, Low Data Rate, with and without spreading, with and without encryption, High Frequency and Low Frequency operation, acquisition without Doppler compensation, plus a test part to verify that antenna and beam switching occurs at the projected boundaries. The Technical Memorandum entitled "Shuttle STS-8 Mission DTO 702 Report" provides the assessment.

DTO 710 was an STS-8 communication flight test to exercise different operational configurations of the Orbiter's Ku-band communication system.
towards establishing flight readiness of the Ku-band operations. The intent of DTO 710 was that Ku-band communications be exercised and verified in each of various combinations of Forward and Return modes, with and without spreading, and with and without encryption. Additionally, that K-band acquisition be demonstrated in both the automatic and the manual modes, that Ku-band failover to S-band be demonstrated and finally, a demonstration of maintaining tracking and communications during an OMS burn. The Technical Memorandum entitled "Shuttle STS-8 Mission DTO 710 Report" presents the assessment.

All parts of DTO 711 were previously verified in STS-8 DTO 710 except for exercising and verifying the Ku-band Return link PM mode channel 3 High Data Rate at 48 Mbps. The 48 Mbps High Data Rate was exercised numerous times in STS-9, per mission reports. An assessment of the performance of the 48 Mbps High Data Rate is presented in a one page report entitled "Report on STS-9 DTO 711".

DTO's (i.e., flight tests) have been a key element of the Shuttle program since it began. The Shuttle Program Office nor anyone else has ever formulated the "life cycle" of a DTO, so the tremendous effort involved in the genesis, formulations and reformulations, reviews, scrubs, and re-reviews, and all the multiple coordinations and approvals required, have never been set down. A thorough and comprehensive description of the phases and stages of a DTO is included in Addendum 3. Also included in Addendum 3 is a note on how to analyze the STS-11 Delayed Acquisition Problem.
4.0 CENTAUR INTERFACE COMPATIBILITY EVALUATION

4.1 Summary of Axiomatix Centaur Activities

Meeting Participation

Axiomatix participated in the Centaur Avionics Integration Panel meetings and the Communications and Data System Subpanel meetings. This participation included resolution of technical problems in the areas of Centaur antenna design and performance, DCU data bit jitter, and antenna switching transients and hardline interface specification definition. These meetings were attended on the following dates:

- October 20, 1983 General Dynamics
- February 9-10, 1984 NASA Lewis
- March 7-8, 1984 General Dynamics
- June 5-6, 1984 General Dynamics
- June 22, 1984 General Dynamics

Design Review Participation

Axiomatix actively participated in the Centaur Telemetry System Interim Critical Design Review on November 2, 1983. This participation entailed detailed review of all Design Review documentation and attendance at the Formal Design Review meeting. Several RIDs were generated by Axiomatix.

Analysis of Centaur Telemetry Subsystem

Axiomatix analyzed the degradation due to the data bit jitter and found that the Bit Error Rate (BER) degradation was 0.04 dB. This estimate was based on the data bit jitter plotted in the report by H. J. Choi, et al., listed below. The result was obtained assuming a 50% transition density and neglecting bit duration jitter. It has been determined that the data bit duration jitter introduces a negligible addition BER degradation.

Axiomatix initiated the bit slip rate degradation analysis and performed a comprehensive review of the literature on the subject.
**Review of Reports and Papers**

Axiometix reviewed and studied the following reports and papers that pertain to the Centaur/Shuttle bit jitter issue:


4. J. K. Holmes, "Optimum Bandwidth and Damping for the MA Bit Synchronizer Based on Linearized Tracking Error", TRW


**4.2 Antenna Pattern Analysis**

The present General Dynamics Centaur antenna design is essentially omnidirectional and therefore is not suitable for the Solar Polar mission which requires a moderate gain toroidal pattern along the Centaur vehicle axis. Axiomatix investigated a solution to this incompatibility and proposed a modified biconical array as a candidate configuration for achieving this toroidal pattern since it is readily adapted to the existing Centaur deployable antenna concept. The proposed biconical array is presented in Addendum 4.

**4.3 BER Degradation Due to Centaur Transmitted Bit Timing Jitter**

Axiomatix analyzed the BER degradation due to transmitted bit timing jitter noise including continuous or discrete components. The source of the
bit timing jitter originates in the Digital Computer Unit (DCU), which modulates the Centaur subcarrier and in turn, phase modulates the carrier. After S-band transmission to the Shuttle Orbiter, the PI demodulates the carrier, and the data bits are reconstructed in the PSP bit synchronizer. Frame synchronization is also done in the PSP. The detailed performance analysis is presented in the Axiomatix Report No. R8409-1, dated September 28, 1984 and entitled "BER Degradation Due to Bit Timing Jitter Noise" which is included in Addendum 5. It was determined in this report that the BER degradation due to DCU bit timing jitter, is approximately 0.04 dB. It is concluded that this is a negligible amount of degradation. Neither thermal noise, nor the discrete nature of the loop has been included in this analysis. It is felt that a full bore simulation would be needed to estimate the effects of the discrete loop. However, it is doubtful whether the discrete nature of the loop would add more than 0.1 dB. It is concluded that, due to the very involved nature of the loop, it would not be reasonable to construct such a simulation.

4.4 Performance Analyses of Using Concatenated Coding For Centaur Signaling Through TDRSS

The Centaur link through the TDRSS is extremely weak even using a R=1/2, K=7 convolutional encoding with Viterbi decoding at the TDRSS ground station. In order to improve the BER performance, Axiomatix began an investigation of concatenating a Reed-Solomon code with the convolutional code. By using a RS (255,223) code concatenated with the convolutional code, the required E_b/N_0 can be decreased by 2 dB at Probability of Bit Error (P_b) of 10^-5 and by 2.9 dB at P_b=10^-7. The disadvantages of concatenating the convolutional code with a RS code are: added complexity, increased decoding delay, and more difficult synchronization. In Addendum 6, a viewgraph presentation is included with describes the concatenated coding scheme and presents the BER performance gain that can be obtained.

For the Centaur communication system, there are other concerns that need to be addressed in order to predict the BER performance of a concatenated coding scheme. First, the BER performance of a single-channel concatenated-coding system with multiple encryptors needed to be analyzed. This BER performance was analyzed in Technical Memorandum No. M8404-1, dated April 6, 1984 and entitled "Bit-Error Rate of the Concatenated-Coding System With Multiple"
Encryptors" which is included in Addendum 6. It was found that at $P_b = 10^{-3}$, that for three encryptors in series, decryptor #1 causes a degradation of 0.14 dB, decryptor #2 causes an additional degradation of 0.023 dB and decryptor #3 causes an additional degradation of 0.01 dB. It is also worth noting that due to the steepness of the BER performance curves, a small amount of degradation in SNR still implies a large difference in BER.

A second concern is the need for differential coding to resolve the inherent ambiguity of carrier phase lock. This concern is addressed in the Technical Memorandum No. M8404-2, dated April 10, 1984 and entitled "Bit-Error Rate of the Concatenated-Coding System With Differential Encoding" which is included in Addendum 6. It was found that the best place to put the differential encoder was before the convolutional encoder but after RS encoder and the RS symbol interleaver. It is shown that the use of differential encoding when placed in its best position will cause a BER degradation of 0.02 dB over a range of BER ($10^{-3}$ to $10^{-7}$).
5.0 SHUTTLE COMMUNICATION AND TRACKING SYSTEM COMPATIBILITY ANALYSIS

With the advent of the Space Station, the Shuttle will become part of a large number of space vehicles and spacecraft that will be linked together in a communication network. As part of the Shuttle communication and tracking system compatibility analysis, Axiomatix made a preliminary investigation into the potential interface between the Shuttle and the Space Station. The results of this investigation is presented in Addendum 7. In order to accommodate a large number of users, multiple access (MA) communication was studied. Two methods of channelization were examined: frequency-division MA (FDMA) and time-division MA (TDMA). A third, code-division MA (CDMA), was not considered practical. Principal trade-offs between FDMA and TDMA are complexity versus timing requirements. FDMA has the following advantages relative to TDMA:

(1) lower burst rate and lower user EIRP
(2) greater MA network capacity

TDMA has the following advantages relative to FDMA:

(1) less complex receiver hardware
(2) less complex antenna: single-beam versus multiple simultaneous beams
(3) does not require user power control for simultaneous near/far operations

The performance of phase-shift and frequency-shift-keying (FSK) modulation was examined. Quadrature-phase-shift-keying (QPSK) modulation is practical for the longer symbol times used in FDMA. Methods of coherent and noncoherent demodulation were discussed, including a new technique for acquiring rapid phase reference. Multiple-frequency-shift-keying (MFSK) was examined; it was shown that 4-ary MFSK is superior to higher order modulations for conserving the data bandwidth.

It was found that the FDMA system with QPSK and coherent demodulation has the performance edge over TDMA; however, the complexity of the FDMA system, particularly the antenna requirements may be the deciding factor. At this point, a clear-cut choice of multiple-access techniques (i.e., TDMA or FDMA) cannot be made. Further analyses and hardware trade-offs must be made.
ADDENDUM 1

PERFORMANCE OF SOLAR MAX DATA AND TAPE RECORDER DUMPS USING THE Ku-BAND CHANNEL 2 THROUGH TDRSS
TECHNICAL MEMORANDUM NO. MB310-1

TO: P. Nilsen  DATE: October 7, 1983
FROM: J. K. Holmes
SUBJECT: Solar Max Retrieval Signal Structure Before and After Limiting Using the Ku-Band Channel 2 Input

1.0 SUMMARY

In this memorandum, it is shown that the limiter inherent in the channel 2 input of the Ku-band QPSK subcarrier modulator does not suppress the 32-kbps baseline solar max retrieval signal at the 8.5-MHz QPSK subcarrier channel 2 output located in the TDRSS ground terminal. In fact, with the current phase-modulation indices specified for solar max, the subcarrier (16 kbps) signal is suppressed by at least 80 dB, while the baseband signal is at full power! However, since the subcarrier data is stripped off at the Shuttle payload signal processor (PSP), subcarrier suppression is not a problem. By changing the phase-modulation indices to $\theta_1 = 0.8$ radians and $\theta_2 = 0.6$ radians, it would be possible to give each signal equal power at a level of about -5 dB (0 dB is the total limiter power) if that were required.
In Figure 1, the model for the solar max retrieval data dump is shown. The solar max 32-kbps biphase-M data is modulated onto the carrier along with the modulated 1.024-MHz subcarrier. The subcarrier has 16-kbps of biphase-M BPSK modulation on it. In the Shuttle Payload Interrogator (PI), the carrier phase-locked-loop demodulates the signal, producing the baseband 32-kbps solar max signal and the 16-kbps modulated subcarrier plus spectral components at a frequency of $n_{\omega_{sc}}$, with $n=2,3,4,...$

The lowpass filter (LPF) limits the spectral extent into the Shuttle Ku-band channel 2 input*. This input acts as a limiter. After the 192-kbps NSP data (channel 1) is placed on the QPSK subcarrier, it is bandpass filtered to the fundamental and added to wideband channel 3 (mode 2) data and frequency modulated onto the Ku-band carrier to the TDRSS. After being relayed from the TDRSS to the TDRSS ground terminal, the signal is FM demodulated in order to send the baseband plus modulated subcarrier signal to the medium-rate bit synchronizer (MRBS) for data tracking and detection.

* However, we neglect this filtering in the following analysis.
Figure 1. Model for the Solar Max Retrieval Data Dump
3.0 MODULATED SIGNAL COMPONENTS

First we consider the spectral composition of the modulated solar max retrieval signal. The waveform of the signal is given by

\[ y(t) = \sqrt{2P} \sin (\omega_c t + \theta_1 d_1(t)) \sin \omega_{sc} t + \theta_2 d_2(t) \]  

where:

- \( P \) = signal power
- \( \omega_c = 2\pi f_c \), angular carrier frequency
- \( \theta_1 = 16\text{-}kbps \) phase-modulation angle (0.8\% rad)
- \( \theta_2 = 32\text{-}kbps \) phase-modulation angle (1.0\% rad)
- \( d_1(t) = 16\text{-}kbps \) biphase-M data
- \( d_2(t) = 32\text{-}kbps \) biphase-M data

Expanding, we obtain:

\[ y(t) = \sqrt{2P} \cos(\theta_1 \sin \omega_{sc} t) \left[ \cos \theta_2 \sin \omega_c t + d_2(t) \sin \theta_2 \cos \omega_c t \right] 
\]

\[ + \sqrt{2P} d_1(t) \sin(\theta_1 \sin \omega_{sc} t) \left[ \cos \theta_2 \cos \omega_c t - d_2(t) \sin \theta_2 \sin \omega_c t \right] \]  

We have

\[ \cos(\theta \sin x) = \sum_{n=0}^{\infty} 2J_n(\theta) \cos nx \quad ; \quad \sin(\theta \sin x) = \sum_{n=1}^{\infty} 2J_n(\theta) \sin nx \quad \]  

where \( \sum \) denotes a summation over even integers and \( \sum \) denotes a summation over odd integers. Using (3) in (2), we have the result:

\[ y(t) = \sqrt{2P} \left\{ J_0(\theta_1) \sum_{n=2}^{\infty} 2J_n(\theta_1) \cos n\omega_{sc} t \right\} \left[ \cos \theta_2 \sin \omega_c t + d_2(t) \sin \theta_2 \cos \omega_c t \right] 
\]

\[ + \sqrt{2P} d_1(t) \left\{ \sum_{n=1}^{\infty} 2J_n(\theta_1) \sin n\omega_{sc} t \right\} \left[ \cos \theta_2 \cos \omega_c t - d_2(t) \sin \theta_2 \sin \omega_c t \right] \]
Now, expanding (4) for the first few terms yields

\[
y(t) = \sqrt{P} J_0(\theta_1) \cos_2 \sin \omega_c t + \sqrt{P} d_2(t) \sin \theta_2 J_0(\theta_1) \cos \omega_c t
\]

\[
+ \sqrt{P} d_1(t) \cos_2 \left\{ J_1(\theta_1) \sin \left[ \left( \omega_{\text{sc}} - \omega \right) t \right] + J_{-1}(\theta_1) \sin \left[ \left( \omega_{\text{sc}} + \omega \right) t \right] \right\}
\]

\[
- \sqrt{P} d_1(t) d_2(t) \sin \theta_2 J_1(\theta_1) \cos \omega_{\text{sc}} t + J_{-1}(\theta_1) \cos \left[ \left( \omega_{\text{sc}} + \omega \right) t \right]
\]

+ terms at \( \omega_c \pm n \omega_{\text{sc}} \) \( n \geq 2 \) (5)

Hence, the power distribution is given by:

(a) Carrier power

\[
\frac{P_c}{P} = J_0^2(\theta_1) \cos^2(\theta_2)
\]

(b) Baseband data

\[
\frac{P_{d_2}}{P} = J_0^2(\theta_1) \sin^2 \theta_2
\]

(c) Subcarrier data

\[
\frac{P_{d_1}}{P} = 2 \cos^2 \theta_2 J_1^2(\theta_1)
\]

(d) Cross modulation

\[
\frac{P_{d_1 d_2}}{P} = 2 \sin^2 \theta_2 J_1^2(\theta_1)
\]

Note that the cross-modulation term is in phase quadrature to the subcarrier data term. The power-level change caused by adding the baseband data to the existing subcarrier-modulation process is depicted in Table 1.
Table 1. Nominal Values of Power Levels With and Without The Baseband Modulation Application ($\theta_1 = 0.8$ and $\theta_2 = 1.0$)

<table>
<thead>
<tr>
<th>Power Ratios</th>
<th>Without Baseband Signal (dB)</th>
<th>With Baseband Signal (dB)</th>
<th>Change (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_C/P$</td>
<td>-1.45</td>
<td>-6.80</td>
<td>-5.35</td>
</tr>
<tr>
<td>$P_{d_1}/P$</td>
<td>-5.65</td>
<td>-11.0</td>
<td>-5.35</td>
</tr>
<tr>
<td>$P_{d_2}/P$</td>
<td>-</td>
<td>-2.96</td>
<td>-</td>
</tr>
<tr>
<td>$P_{d_1d_2}/P$</td>
<td>-</td>
<td>-7.15</td>
<td>-</td>
</tr>
</tbody>
</table>
4.0 DEMODULATED SPECTRAL COMPONENTS

In this section, we evaluate the PI carrier demodulated signal, the system model of which is shown in Figure 2.

We determine the power distribution in the wideband output, denoted by $e_{WB}(t)$. To obtain the signal output, we assume that the carrier reference has an error of $\phi$ radians, so that the demodulated signal is of the form $y(t) = \sqrt{E} \cos(\omega_c t - \phi)$, or

$$e_{WB}(t) = \left[ \sqrt{E} \left\{ J_0(\theta_1) + \sum_{n=2}^{\infty} 2J_n(\theta_1) \cos n\omega_{sc} t \right\} \cos \phi + \sin \phi \ d_2(t) \cos \phi \right]$$

$$+ \sqrt{E} \ d_1(t) \left\{ \sum_{n=1}^{\infty} 2J_n(\theta_1) \sin n\omega_{sc} t \right\} \cos \phi - \sin \phi \ d_2(t) \sin \phi \right\}$$

$$+ o(2\omega_c)$$ (10)

Expanding once again, we have

$$e_{WB}(t) = \sqrt{E} \cos \phi \left\{ J_0(\theta_1) + \sum_{n=2}^{\infty} 2J_n(\theta_1) \cos n\omega_{sc} t \right\}$$

$$+ \sqrt{E} \ d_2(t) \sin \phi \left\{ J_0(\theta_1) + \sum_{n=2}^{\infty} 2J_n(\theta_1) \cos n\omega_{sc} t \right\}$$

$$+ \sqrt{E} \ d_1(t) \cos \phi \left\{ \sum_{n=1}^{\infty} 2J_n(\theta_1) \sin n\omega_{sc} t \right\}$$

$$+ \sqrt{E} \ d_1(t) \ d_2(t) \sin \phi \left\{ \sum_{n=1}^{\infty} 2J_n(\theta_1) \sin n\omega_{sc} t \right\}$$ (11)

Hence, the terms at baseband and $\omega_{sc}$ are given by
\[ \sqrt{2} \cos(\omega_c t - \phi) \]

Figure 2. Shuttle PI Carrier-Demodulation Model
Notice from (12) that cross modulation in the desired band (baseband to above $\omega_{sc}$) does not exist if the reference carrier is in perfect quadrature, i.e., ideal carrier tracking with $\phi = 0$. Since the solar max will be on the end of the boom in the Shuttle, the SNR should be very high and the phase error $\phi$, very small; thus, cross modulation should be negligible (since it depends on $\sin \phi$, with $\phi$ the phase error.)
5.0 DEMODULATED AND LIMITED SPECTRAL COMPONENTS

Now we consider the limited version of \( e_{MB}(t) \) since that will be representative of what is demodulated out of channel 2 at the TDRSS Earth terminal QPSK subcarrier-tracking loop.

Using (1) with the reference signal we obtain, as an input to the lowpass filter (in closed form),

\[
e_{MB}(t) = \sqrt{2} \cos(\omega_c t - \phi) \sqrt{P} \sin(\omega_c t + \theta_1 d_1(t) \sin \omega_{sc} t + \theta_2 d_2(t))
\]

or

\[
e_{MB}(t) = \sqrt{P} \sin(\theta_1 d_1(t) \sin \omega_{sc} t + \theta_2 d_2(t) + \phi) + O(2\omega_c)
\]

After filtering the signal, the terms at \( 2\omega_c \) are effectively removed so that we have, to a good approximation,

\[
\tilde{e}_{MB}(t) = \sqrt{P} \sin(\theta_1 d_1(t) \sin \omega_{sc} t + \theta_2 d_2(t) + \phi)
\]

After going through the limiter, we obtain

\[
e_{L}(t) = \text{sgn}(\tilde{e}_{MB}(t)) = \sqrt{P_L} \text{sgn}[\tilde{e}_{MB}(t)] = \sqrt{P_L} \text{sgn}[\theta_1 d_1(t) \sin \omega_{sc} t + \theta_2 d_2(t) + \phi]
\]

where \( \text{sgn} \) is given by \( \text{sgn}(\sin(x)) \) and \( \sqrt{P_L} \) is the voltage output of the limiter.

Denote \( a_1(t) \) as the binary-valued input of channel 1 to the QPSK (8.5-MHz) modulator and let \( a_2(t) = e_{L}(t) \) the channel 2 input to the QPSK modulator. We have

\[
a_1(t) = d_3(t)
\]

and

\[
a_2(t) = \text{sgn}[\theta_1 d_1(t) \sin \omega_{sc} t + \theta_2 d_2(t) + \phi]
\]

\( (1.024 \text{ MHz}) \)
Now the 8.5-MHz subcarrier is modulated in the form

$$s(t) = \sin[\omega_{sc2} t + \phi(t)]$$  \hspace{1cm} (19)

where

$$\phi(t) = \theta a_1(t) a_2(t) - \frac{\pi}{2} (a_1(t) - 1)$$  \hspace{1cm} (20)

The value of $\theta$ determines the power relationship between channels 1 and 2. A value of 26.6° provides a 4:1 power ratio (Ch2:Ch1) after lowpass filtering the 8.50-MHz subcarrier to the fundamental. Now,

$$s(t) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin[n(\omega_{sc2} t) + \phi(t)]$$  \hspace{1cm} (21)

or

$$s(t) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \cos(n\theta) \sin(n\omega_{sc2} t) + \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin(n\theta) \cos(n\omega_{sc2} t)$$  \hspace{1cm} (22)

Now,

$$\cos(n\theta) = \cos\left(\theta a_2 \frac{d_1 - \pi (a_1 - 1)}{2} n\right)$$  \hspace{1cm} (23)

$$\cos(n\theta) = \cos(na_1 a_2 \theta) \cos\left(n \frac{\pi}{2} (a_1 - 1)\right) - \sin(na_1 a_2 \theta) \sin\left(n \frac{\pi}{2} (a_1 - 1)\right)$$  \hspace{1cm} (24)

$$\sin(n\theta) = \sin\left(na_1 a_2 \theta\right) \cos\left(n (a_1 - 1) \frac{\pi}{2}\right) + \cos(na_1 a_2 \theta) \sin(n (a_1 - 1) \frac{\pi}{2})$$  \hspace{1cm} (25)

Also,

$$\cos(n\theta) = \cos(n\theta) \cdot a_1(t)$$  \hspace{1cm} (26)
and

\[ \sin(n\theta) = a_2(t) \cdot \sin\theta \]  \hspace{1cm} (27)

Hence,

\[ s(t) = a_1(t) \sum_{n=1}^{\infty} \frac{4}{n\pi} \cos(n\theta) \sin(n\omega_{sc2} t) + a_2(t) \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin(n\omega_{sc2} t) \]  \hspace{1cm} (28)

Now \( s(t) \) is bandpass filtered (see Figure 1) so that its fundamental is passed; therefore, we obtain out of the BPF

\[ s(t)|_{BPF} = a_1(t) \frac{4}{\pi} \cos \theta \sin \omega_{sc2} t + a_2(t) \frac{4}{\pi} \sin \cos \omega_{sc2} t \]  \hspace{1cm} (29)

This signal is added to channel 3 and frequency modulated onto the carrier which is sent to the TDRS, then bent piped to the TDRS ground terminal. After FM demodulation, the signal is demodulated by the 8.5-MHz QPSK subcarrier demodulator. This produces a channel 2 output at the ground terminal which is given by

\[ s_2(t) = \sqrt{P_2} \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin[n\omega_{sc1} t + \theta_1 d_1(t)] + \frac{4}{n\pi} \sin[n\omega_{sc1} t + \theta_2 d_2(t) + \phi] \]  \hspace{1cm} (30)

where \( P_2 \) is the power in the demodulated channel 2. Note that

\[ \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin[nx] = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin[nx] \ (x \text{ in radians}) \]  \hspace{1cm} (31)

which is a Fourier series expansion of \( \sum_{n=1}^{\infty} \sin[nx] \). Using (31) in (30) produces

\[ s_2(t) = \sqrt{P_2} \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin(n\theta_1 d_1(t) \sin \omega_{sc1} t) \cos(n\theta_2 d_2(t) + n\theta) \]

\[ + \sqrt{P_2} \sum_{n=1}^{\infty} \frac{4}{n\pi} \cos(n\theta_1 d_1(t) \sin \omega_{sc1} t) \sin(n\theta_2 d_2(t) + n\theta) \]  \hspace{1cm} (32)
where \( \sum \) again denotes summation over odd integer values. Using the relationships in (3) produces

\[
e_L(t) = \sqrt{P_2} \sum_{n=1}^{\infty} \frac{4}{n \pi} \left( \cos n\theta_2 \cos \phi - d_2(t) \sin n\theta_2 \sin \phi \right) d_1(t)
\]

\[
\exp\{-\tau / \pi \} \propto 2J_m(n\phi_1) \sin(m\omega_{sc} t) + \sqrt{P_2} \sum_{n=1}^{\infty} \frac{4}{n \pi} \times \left( d_2(t) \sin n\theta_2 \cos \phi + \cos n\theta_2 \sin \phi \right) \left[ J_0(n\phi_1) + \sum_{m=2}^{\infty} 2J_m(n\phi_1) \cos(m\omega_{sc} t) \right]
\]

Now we determine the terms out of the limiter at baseband out to a frequency extent of slightly more than \( \omega_{sc} \). We have

\[
E_L(t)|_{LP} = \frac{8}{\pi} \sqrt{P_2} d_1(t) \left[ \sum_{n=1}^{\infty} \frac{1}{n} \cos n\theta_2 \cos \phi J_1(n\phi_1) \right] \sin \omega_{sc} t
\]

\[
\text{cross modulation on subcarrier}
\]

\[
- \frac{8}{\pi} \sqrt{P_2} d_1(t) d_2(t) \left[ \sum_{n=1}^{\infty} \frac{1}{n} \sin n\theta_2 \sin \phi J_1(n\phi_1) \right] \sin \omega_{sc} t
\]

\[
+ \frac{4}{\pi} \sqrt{P_2} d_2(t) \left[ \sum_{n=1}^{\infty} \frac{1}{n} \sin n\theta_2 \cos \phi J_0(n\phi_1) \right]
\]

\[
\text{32-kbps data at baseband}
\]

\[
+ \frac{4}{\pi} \sqrt{P_2} \left[ \sum_{n=1}^{\infty} \frac{1}{n} \cos n\theta_2 \sin \phi J_0(n\phi_1) \right]
\]

\[
\text{DC term}
\]

Hence, we see that the 16-kbps data modulated on the 1.024-MHz subcarrier is present, as is the 32-kbps baseband data and they are separable by filtering. A cross-modulation term is also present on the subcarrier, but its amplitude goes to zero as the PI carrier-loop phase error, \( \phi \), goes to zero.
6.C POWER LEVELS OUT OF THE LIMITER

The first three terms of (33) were programmed on a digital computer with parameters $\theta_1 = 0.8$ radian, $\theta_2 = 1.0$ radian and $P_2 = 1$, with the result that all the power out of the limiter was in the 32-kbps data stream. However, by computer evaluation using 5000 terms, it was observed that, when $\theta_1 = 0.8$ radian and $\theta_2 = 0.6$ radian, the subcarrier and baseband relative-power levels were given by

$$P_{sc} = -4.5 \text{ dB} \quad ; \quad P_{bb} = -5.6 \text{ dB}$$

Thus, varying the modulation indices causes the relative-power levels to vary drastically. As seen at the output of channel 2 of the 8.5-MHz subcarrier demodulator, the baseband and subcarrier signal levels are plotted in Figure 3 as a function of $\theta_1$ for $\theta_2 = 1.0$ radian. When the specification values of $\theta_1 = 0.8$ and $\theta_2 = 1.0$ are used, it is seen that all the power resides in the baseband signal; i.e., complete suppression of the subcarrier signal occurs.

Figure 4 illustrates the same signal powers for the condition that $\theta_1 = 0.8$ when $\theta_2$ is variable.

In order to check the sensitivity of the results to the modulation indices, the indices were allowed to vary $\pm5\%$ and $\pm10\%$ from the nominal values of $\theta_1 = 0.8$ and $\theta_2 = 1.0$. In addition, phase errors of $\phi = 0$ and 0.1 radian were used in the program. When tolerances of $15\%$ were used with $\phi = 0.1$, all the power appeared in the baseband (32-kbps) signal out of the QPSK demodulator. When 10% tolerances were considered, it was also found that, when $\phi = 0.1$, $\theta_1 = 0.8 + 10\% = 0.88$ and $\theta_2 = 1 - 10\% = 0.9$, the subcarrier is again fully suppressed. If $\pm15\%$ variations could occur so that $\theta_1 > \theta_2$, then some 1.024-MHz subcarrier signal would appear along with the baseband signal at the output of the subcarrier demodulator.

When the baseband signal is turned off (assuming $\theta_2 = 0$), the normalized power output in channel 2 of the QPSK demodulator is 0.81 (total power = 1), or -0.9 dB, which is the power in the fundamental of the 1.024-MHz subcarrier. Clearly, when $\theta_2 = 0$, no power will appear at the 32-kbps data rate.
Figure 3. 32-kbps (BB) and 16-kbps (SC) Power Out of Channel 2 QPSK Demodulator.

Conditions:
\[ \phi = 0, \theta_1 \text{ variable}, \theta_2 = 1.0 \]

Total power = 1.0 = \( P_2 \)

Subcarrier 16-kbps Power

32-kbps Baseband Power
Figure 4. 32-kbps (BB) and 16-kbps (SC) Power Out of Channel 2 QPSK Demodulator.

Conditions:
\[ \phi = 0, \theta_2 \text{ variable, } \theta_1 = 0.8 \]
\[ N = 5000 \]
\[ \text{Total power} = 1.0 = P_2 \]
7.0 CONCLUSION

We conclude (perhaps, by good fortune) that the existing specification values of $\theta_1 = 0.8$ and $\theta_2 = 1.0$ radian fully suppress the subcarrier signal and leave the desired baseband 32-kbps at full power at the output of the QPSK subcarrier TDRSS earth terminal. Since the 5-MHz low-pass filter was not included in the analysis, the measured values of the power levels of the subcarrier and baseband signals might vary slightly from the predicted values given here.

REFERENCES


3. S. Kindorf, private communication with TRW TDRSS engineer.
TECHNICAL MEMORANDUM NO. M8401-1

TO: Peter Nilsen  Date: January 10, 1984
FROM: Jack Holmes  cc: T. Costello, JSC  J. Ratliff, JSC
SUBJECT: A Possible Explanation of Solar Max Retrieval
I-Q Swapping due to Noise Filtering in the
8.5-MHz Subcarrier Demodulator

1.0 SUMMARY

In this memo, it is shown that the power ratio of channel 2 to
channel 1, out of the lowpass arm filters in the TDRSS ground station subcar-
rier demodulator, can be as low as 1:1 when the input lower is -113 dBm.

Although approximations were necessary, it seems reasonable to assume
that the noise power present in the 5-MHz Shuttle Payload Interrogator shares
the total power in channel 2 with the 32-kbps signal but, depending on the SNR
in the 5-MHz bandwidth, a good portion of the noise is filtered out in the TDRS
QPSK subcarrier demodulator arm filters. Thus, the QPSK demodulator has great
difficulty in deciding whether the strong or weak channel is phased in the
strong channel position. This could account for the tendency to lock in the
wrong I-Q relationship and not be detected.

2.0 APPROACH

The purpose of the calculations given in this memo is to show that,
at low SNR's into the PI, the noise power contained in the 5-MHz lowpass PI
filter is partly filtered out in the arm filters of the QPSK tracker (subcarrier
demodulator) and therefore, the power ratio is condensed close to 1:1 out of
the QPSK demodulator. To show this, we will model the system in the next section.
A model of the complete Solar Max signal path is indicated in Figure 1. The signal transmitted from the Solar Max transmitter is of the form

\[ y(t) = \sqrt{2P} \sin(\omega_c t + \theta_1 d_1(t) \sin \omega_s c t + \theta_2 d_2(t)) \]  

(1)

where

- \( P \) = signal power
- \( \omega_c = 2\pi f_c \), angular carrier frequency
- \( \theta_1 \) = 16-kbps phase-modulation angle (0.8±5% rad)
- \( \theta_2 \) = 32-kbps phase-modulation angle (1.0±5% rad)
- \( d_1(t) \) = 16-kbps biphase-M data
- \( d_2(t) \) = 32-kbps biphase-M data

As discussed in [1], the carrier-loop demodulated signal into the 5-MHz lowpass filter is given by

\[ e_{WB}(t) = \sqrt{P} \cos(\theta_2) \sin\left( J_0(\theta_1) + \sum_{n=2}^{\infty} 2J_n(\theta_1) \cos n\omega_s c t \right) \]

\[ + \sqrt{P} d_2(t) \sin \theta_2 \cos\left( J_0(\theta_1) + \sum_{n=2}^{\infty} 2J_n(\theta_1) \cos n\omega_s c t \right) \]

\[ + \sqrt{P} d_1(t) \cos \theta_2 \cos\left( \sum_{n=1}^{\infty} 2J_n(\theta_1) \sin n\omega_s c t \right) \]

\[ - \sqrt{P} d_1(t) d_2(t) \sin \theta_2 \sin\left( \sum_{n=1}^{\infty} 2J_n(\theta_1) \sin n\omega_s c t \right) + n(t) \]  

(2)

where \( \sum \) and \( \sum_0 \) denote the summation over odd and even indices, respectively. The term \( n(t) \) is a sample function of white Gaussian noise. Expanding some of the key terms, we have
Figure 1. Mode: for the Solar Max Retrieval Data Dump
\[ e_{\text{MB}}(t) = \sqrt{P} J_0(\theta_1) \cos \theta_1 \sin \phi + \sqrt{P} d_2(t) \sin \theta_2 \cos \phi J_0(\theta_1) + \sqrt{P} J_1(\theta_1) d_1(t) \cos \theta_1 \cos \phi \sin \omega_{\text{sc}} t + 2 \sqrt{P} J_2(\theta_1) \sin \theta_2 \sin \phi \sin \omega_{\text{sc}} t \]

16-kbps data on 1.024-MHz subcarrier

\[ + 2 \sqrt{P} J_1(\theta_1) d_1(t) \cos \theta_1 \sin \phi \sin \omega_{\text{sc}} t \]

cross modulation on subcarrier

\[ - 2 \sqrt{P} d_2(t) \sin \theta_2 d_1(t) \cos \theta_1 \cos \phi \sin \omega_{\text{sc}} t \]

second harmonic of subcarrier

\[ + 2 \sqrt{P} J_2(\theta_1) \sin \theta_2 d_2(t) \cos \omega_{\text{sc}} t \cos \phi + n(t) \]

+ terms at 3\omega_{\text{sc}}, 4\omega_{\text{sc}}, etc. (3)

When the phase error, \( \phi \), is assumed to be small, we have that the dominant term out of the carrier demodulator is

\[ e_{\text{MB}}(t) = \sqrt{P} d_2(t) \sin \theta_2 J_0(\theta_1) + n(t) + 2 \sqrt{P} J_1(\theta_1) d_1(t) \cos \theta_1 \sin \omega_{\text{sc}} t \] (4)

When the modulation indices are \( \theta_1 = 0.8 \) and \( \theta_2 = 1.0 \), the relative power levels out of the 5-MHz lowpass filter, following the carrier demodulator, is given by

\[ P_{\text{MB}} = 0.507 P + 0.0794 P + 0.00814 P + \cdots + P_n \] (5)

with \( P_n \) being the power out of the 5-MHz lowpass filter. In order to size the thermal noise power, we note that, with the 3.5-dB noise figure of the PI receiver, the \( P/N_0 \) ratio is given by

\[ \frac{P}{N_0} = P \text{ dBm} + 170.5 \text{ dBm/Hz} \] (6)
So that, for a -98 dBm signal level in a noise bandwidth of $5 \times 10^6 \times \pi/2$ Hz (from the 5-MHz LPF), we have

$$\frac{p}{N_0} = 170.5 - 10 \log\left(5 \times \frac{5}{2} \times 10^6\right) - 98 \quad (7)$$
or

$$\frac{p}{N_0} = 3.5 \text{ dB} \quad (8)$$

Hence, the three significant components plus noise from (5) are given by

$$P_{MB} = 0.507P + 0.0794P + 0.00814P + 0.44P \quad (9)$$

Note that the third and remaining signal-type components are negligible since the second signal term is 8 dB down from the 32-kbps baseband data term. Since the analysis with both a baseband Manchester signal and a Manchester-modulated subcarrier signal into a limiter is not available in the literature, we will modify the model slightly so that known results can be used. Hence, we will model the input to the limiter terminal of the QPSK subcarrier as a baseband signal (32 kbps) plus thermal noise. Our model is illustrated in Figure 2.

In Appendix A, it is shown that the signal plus noise out of the (conceptual limiter is given by

$$y(t) = d(t) \text{ erf}\left(\frac{5}{\sqrt{2N}}\right) + N(t) \quad (10)$$

where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (11)$$

*The limiter is inherent in the QPSK modulator.*
Figure 2. Model of Signal Plus Noise into the Multiplexer (Modulator)
with \( d(t) \) representing the 32-kbps baseband data stream and \( N(t) \) is the resulting noise. The limiter is modeled as outputting a value of \( \pm 1 \) for convenience. In Appendix B, the autocorrelation function of \( y \) is given by

\[
R_y(\tau) = R_d(\tau) \left\{ \text{erf} \left[ \frac{S}{\sqrt{2N}} \right] + 4 \sum_{m=0}^{\infty} z(2m+1) \left( \frac{S}{\sqrt{N}} \right)^{2m+2} \right\} / (2m+2)!
\]

where

\[
z(n)(x) = \frac{d^n}{dx^n} \left( \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \right)
\]

The first term is the \( S^2S \) term, second is the \( S^2N \) terms, and the last term is the \( N^2N \) terms. Notice that, at high SNR,

\[
R_y(\tau) \approx R_d(\tau)
\]

where \( R_d(\tau) \) is the autocorrelation function of the 32-kbps data stream. At low SNR, it is shown in Appendix B that the autocorrelation function converges to

\[
R_y(\tau) = \frac{2}{\pi} \text{ arc sin } \left( \frac{R_n(\tau)}{\sigma^2} \right)
\]

where \( R_n(\tau) \) is the input autocorrelation function of the noise and \( \sigma^2 = R_n(0) \), the variance of the input noise process.

Expanding (12) produces
So that the first term is signal x signal and the rest are either signal x noise or noise x noise terms. As can be seen the terms tend to diminish in size assuming that \( z'(\sqrt{SN}) \) does not increase too fast with \( i \). For each SNR evaluation we will determine the significant terms. Once the spectral density of the noise is known, the power loss at the receiver 1.8 MHz demod arm filters can be computed.

4.0 DEMODULATED POWER RATIO AT \( P = -98 \text{ dBm} \)

From (8) and (9) we see that the signal to noise ratio \( S/N \) is given by

\[
\frac{S}{N} = 3.5 - 10 \log(0.507) = 0.56 \text{ dB} = 1.14 \quad (18)
\]

Hence by evaluating the \( z \) parameters we have

\[
\begin{align*}
    z(0)(1.07) &= 0.225 & z(3)(1.07) &= 0.446 \\
    z(1)(1.07) &= -0.241 & z(4)(1.07) &= -0.57 \\
    z(2)(1.07) &= 0.032 & z(5)(1.07) &= -1.18
\end{align*}
\]

From (17) we can write

\[
R_y(\tau) = R_d(\tau) \operatorname{erf}(\sqrt{\frac{S}{2N}}) + R_N(\tau) \quad (20)
\]
with $R_N(\tau)$ defined to be equal to all the terms in $R_y(\tau)$ except

$$R_d(\tau) = \text{erf}\left(\sqrt{\frac{S}{2N}}\right)$$

Combining only the non-negligible terms from (17) we have

$$R_N(\tau) \equiv 0.1 R_d(\tau) \left[ \frac{R_N(\tau)}{\sigma^2} \right]^2 + 0.033 R_d(\tau) \left[ \frac{R_N(\tau)}{\sigma^2} \right]^4 + 0.2 \left[ \frac{R_N(\tau)}{\sigma^2} \right]$$

We see that the spectral density (with the help of Appendix B) is given by (neglecting $R_d(\tau)$ since $d(t)$ is much narrower banded than $n(t)$)

$$\mathcal{S}_N(f) = \frac{0.05}{B} \left(1 - \frac{|f|}{2B}\right) U_{2B}(f) + \frac{0.0083}{B^2} \left(\frac{4}{3} B - \frac{f^2}{2B} + \frac{|f|^3}{8B^2}\right) + \frac{0.1}{B} U_B(f)$$

$$0 \leq |f| \leq 2B$$

or

$$\mathcal{S}_N(f) = \frac{0.0083}{B^2} \left(\frac{8}{3} B - 2|f| + \frac{f^2}{2B} - \frac{|f|^3}{24B^2}\right) 2B \leq |f| \leq 4B$$

with

$$U_B(f) = 1 \quad |f| \leq B$$
$$= 0 \quad |f| > B$$

The spectral density is sketched in Figure 3.

From (10), when $S/N = 1.14$ (eq. (18)), we have, out of our apparent limiter, the signal structure

$$y_L(t) = 0.711 \, d_2(t) + N(t)$$

with the spectral density of $N(t)$ being given by (21) and (22) and is plotted in Figure 3. Now, as shown in [1], we have that at the subcarrier demodulator the signal is filtered to 1.8 MHz which reduces the power in the strong channel
Figure 3. Sketch of Spectral Density of the Noise at $P = -98$ dBm
from the original ratio of 4:1 to something less. Since channel 1 is a 192 kbps data stream without additive thermal noise, the weak channel is not affected by the 1.8 MHz filter. The power ratio after filtering is approximately

$$PR = \frac{0.51 + (0.145)(1.8)^2}{0.25} = 2.46$$

Hence in the subcarrier loop the power ratio is approximately 2.46:1.

4.0 DEMODULATED POWER RATIO AT $P = -103$ dBm

Now the $S/N$ is given by

$$S/N = -4.44 \text{ dB}$$

The signal suppression factor

$$\alpha = \text{erf}(\sqrt{\frac{S}{2N}})$$

is then given by

$$\alpha = \text{erf}(0.42) = 0.45$$

so that the voltage out of the (apparent) limiter is given by

$$y_L(t) = 0.45 d_2(t) + N(t)$$

We can evaluate the $z$ parameters by using the fact that $\sqrt{S/N} = 0.6$ so that

$$z^{(0)}(0.6) = 0.33 \quad z^{(3)}(0.6) = 0.53$$
$$z^{(1)}(0.6) = -0.2 \quad z^{(4)}(0.6) = 0.32$$
$$z^{(2)}(0.6) = -0.21 \quad z^{(5)}(0.6) = -2.31$$
Combining all the non-negligible terms we obtain

$$R_N(\tau) \approx 0.08 R_d(\tau) \left[ \frac{R_n(\tau)}{2} \right]^2 + 0.047 R_d(\tau) \left[ \frac{R_n(\tau)}{2} \right]^4 + 0.44 R_n(\tau)$$

(31)

Hence, the noise power spectral density is given by (using Appendix B)

$$\mathcal{S}_N(f) = \frac{0.04}{B} \left( 1 - \frac{|f|}{2B} \right) U_{2B}(f) + \frac{0.012}{B^2} \left( \frac{4}{3} B - \frac{f^2}{2B} + \frac{|f|^3}{8B^2} \right) + \frac{0.22}{B} U_B(f)$$

$$0 < |f| < 2B$$

(32)

and

$$\mathcal{S}_N(f) = \frac{0.012}{B^2} \left( \frac{8}{3} B - 2 |f| + \frac{f^2}{2B} - \frac{|f|^3}{24B^2} \right), \quad 0 < |f| < 2B$$

(33)

Hence, we have

$$y_L(t) = 0.45 d_2(t) + N(t),$$

(34)

with $N(t)$ having the spectral density of equations (32) and (33). The spectral density is plotted in Figure 4. Again filtering to 1.8 MHz yields a power ratio of

$$PR = \frac{0.203 + 2(0.265)(\frac{1.8}{5})}{0.25} = 1.58$$

(35)

Therefore, in the subcarrier loop, the power ratio is approximately 2.15:1.

Now consider the $P = -108$ dBm case.
Figure 4. Power Spectral Density of the Noise When $P = -103$ dBm.
5.0 DEMODULATED POWER RATIO AT P = -108 dBm

In this case, the S/N is 5 dB lower than in the previous case, hence

\[ S/N = -9.44 \text{ dB} \]  \quad (36)

The signal suppression factor is given by

\[ \alpha = \text{erf}(0.238) = 0.265 \]  \quad (37)

Hence, the voltage out of the limiter can be written as

\[ y_L(t) = 0.265 d_2(t) + N(t) \]  \quad (38)

In order to estimate the noise spectral density we must compute the \( z \) parameters again with argument \( \sqrt{S/N} = \sqrt{0.114} = 0.337 \)

\[ z(0)(0.337) = 0.376 \quad z(3)(0.337) = 0.3693 \]
\[ z(1)(0.337) = -0.128 \quad z(4)(0.337) = 0.873 \]  \quad (39)
\[ z(2)(0.337) = -0.333 \quad z(5)(0.337) = -1.774 \]

It follows that the noise autocorrelation function is given by

\[ R_N(\tau) = 0.033 R_d(\tau) \left[ \frac{R_n(\tau)}{2} \right]^2 + 0.566 \left[ \frac{R_n(\tau)}{2} \right] + 0.074 \left[ \frac{R_n(\tau)}{2} \right]^3 \]  \quad (40)

Hence, from Appendix B, we have that the noise power spectral density is given by

\[ S_N(f) = \frac{0.0165}{B} \left( 1 - \frac{|f|}{2B} \right) U_{28}(f) + 0.283 U_B(f) + \frac{0.0093}{B} \left( 3 - \left( \frac{f}{B} \right)^2 \right), \quad 0 < |f| < B \]  \quad (41)
\[ = \frac{0.0165}{B} \left( 1 - \frac{|f|}{2B} \right) U_{28}(f) + 0.0047 \left( 3 - \left( \frac{|f|}{B} \right)^2 \right), \quad B < |f| < 3B \]  \quad (42)
By adding up the total noise power and the signal power, assuming statistical independence, we see that the power totals much less than unity (~ 0.7 instead of 1.0), indicating that more terms should be taken to represent the noise waveform. However, since this analysis is only meant to imply feasibility of the I-Q switching problem, we continue on with our three term approximation. The power spectral density is plotted in Figure 5.

After filtering to 1.8 MHz we obtain a power ratio of

$$\text{PR} = \frac{0.07 + (0.32)^2 \frac{1.8}{5}}{0.25} = 1.2$$

Hence we see that the power ratios are near unity at $P = -108 \text{ dBm}$ within the limits of our approximation.

6.0 DEMODULATED POWER RATIO AT $P = -113 \text{ dBm}$

From (36), we have

$$\text{S/N} = -14.45 \text{ dB}$$

(44)

The signal suppression factor is given by

$$\alpha = \text{erf}(0.134) = 0.145$$

(45)

It follows that the voltage out of the limiter can be written as

$$y_1(t) = 0.145 d_2(t) + N(t)$$

(46)

The $z$ parameters for $\sqrt{S/N} = 0.19$ are

$$z^{(0)}(0.19) = 0.3915 \quad z^{(4)}(0.19) = 1.09$$

$$z^{(1)}(0.19) = -0.074 \quad z^{(3)}(0.19) = 0.22$$

$$z^{(2)}(0.19) = -0.377 \quad z^{(5)}(0.19) = -1.09$$

(47)

$$z^{(6)}(0.19) = -5.25$$

(47)
Figure 5. Sketch of Spectral Density of the Noise at $P = -106$ dBm
The noise autocorrelation function is given by

\[ R_N(\tau) = 0.613 \left( \frac{R_n(\tau)}{\sigma^2} \right) + 0.09 \left( \frac{R_n(\tau)}{\sigma^2} \right)^3 \]  \hspace{1cm} (48)

Using just the dominant term we obtain

\[ \mathcal{A}_N(f) = \frac{0.307}{B} U_B(f) \]  \hspace{1cm} (49)

Hence, the power ratio after the 1.8 MHz arm filtering is

\[ PR = \frac{0.021 + 0.307 (2) \left( \frac{18}{5} \right)}{0.25} \]  \hspace{1cm} (50)

which is unity! Hence, we conclude that filtering at the receiver could cause I-Q interchange in the vicinity of \( P = -113 \) dBm since the ratio of powers is about equal.

The channel power detector is illustrated in Figure 6. Basically it measures the voltage in the I and Q channels and forms the difference which is then filtered.

7.0 CONCLUSION

Based on some calculations which required certain approximations, it appears that the filtering of the channel signal pulse noise in the arm filters of the QPSK subcarrier demodulator can cause the power ratio out of the arm filters (what the QPSK loop "sees") is near 1:1 rather than 4:1 at an input power of about -113 dBm. Since the calculations involved important approximations, the results can only be described in approximate.
Figure 6. Channel Power Ratio Detector-Decision Circuit
REFERENCES


4. Operation and Maintenance Instructions, KSAR Medium-Range Demodulator For TDRSS Tracking, Return, and Modulation Equipment (TRAM)
APPENDIX A

BASEBAND DIGITAL SIGNAL-PLUS-NOISE THROUGH A HARD LIMITER

In this appendix, we determine the output spectral density of a baseband digital signal plus bandlimited Gaussian noise. This calculation allows one to determine the output $C/N_0$ value as a function of the input $C/N_0$ value. The derivation of the autocorrelation function follows that of Painter [1] with minor corrections in the final results. The model is shown in Figure A.1 below.

\[ x(t) = A d(t) + n(t) \]
\[ y(t) = \text{sgn}[x(t)] \]

The first- and second-order density functions are given by

\[ p_x(x_t, t) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2\sigma^2} (x_t - \mu_t)^2 \right] \quad (A-3) \]
\[ p_x(x_t, x_{t-}) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \exp \left[ -\frac{1}{2(1-\rho^2)} \left\{ \frac{(x_t - \mu_t)^2}{\sigma^2} - \frac{2\rho(x_t - \mu_t)(x_{t-} - \mu_{t-})}{\sigma^2} + \frac{(x_{t-} - \mu_{t-})^2}{\sigma^2} \right\} \right] \quad (A-4) \]
where
\[
x_t = x(t) \\
x_{t+r} = x(t+r) \\
\mu_t = E[x(t)] = Ad(t) s(t) \\
\mu_{t+r} = E[x(t+r)] = Ad(t+r) s(t+r) \\
\sigma^2 = E[(x_t-\mu_t)^2] = E[(x_{t+r}-\mu_{t+r})^2] \\
\rho = \frac{1}{\sigma^2} E[(x_t-\mu_t)(x_{t+r}-\mu_{t+r})] \\
or
\rho = \frac{1}{\sigma^2} E[n(t+r)n(t)] = \frac{R_n(\tau)}{\sigma^2} \tag{A-5}
\]

First let us compute the mean value of our process. We will temporarily drop the time subscript.

\[
\mu = \int_{-\infty}^{\infty} \text{sgn}(x) p_x(x) \, dx \tag{A-6}
\]

which can be expressed as
\[
\mu = \int_{0}^{\infty} \frac{1}{\sqrt{2\pi} \sigma} \exp \left[ -\frac{1}{2\sigma^2} (x-s)^2 \right] \, dx - \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi} \sigma} \exp \left[ -\frac{1}{2\sigma^2} (x-s)^2 \right] \, dx \tag{A-7}
\]

where \( s = s(t) = Ad(t) \). Letting \( \frac{x-s}{\sqrt{2\sigma}} = v \) produces

\[
\mu = \frac{2}{\sqrt{\pi}} \int_{0}^{2/\sqrt{2\sigma}} e^{-v^2} \, dv = \text{erf} \left[ \frac{s(t)}{\sqrt{2\sigma}} \right] \tag{A-8}
\]

Now let us determine the output autocorrelation function defined by
\[
R_y(t+r,t) = E[y(t+r)y(t)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \text{sgn}[x_t] \text{sgn}[x_{t+r}] p_x(x_t,x_{t+r}) \, dx_t \, dx_{t+r} \tag{A-9}
\]
Equation (A-9) can be written

\[ R_y(t+\tau,\tau) = \int_0^\infty \int_0^\infty p_x(x_t, x_{\tau}) \, dx_t \, dx_{\tau} + \int_0^\infty \int_0^0 p_x(x_t, x_{\tau}) \, dx_t \, dx_{\tau} \]

\[ -\int_0^\infty \int_0^0 p_x(x_t, x_{\tau}) \, dx_t \, dx_{\tau} - \int_0^\infty \int_0^\infty p_x(x_t, x_{\tau}) \, dx_t \, dx_{\tau} \]  \hspace{1cm} (A-10)

Letting \( x_t - \mu_t = u \sigma, \ x_{\tau} - \mu_{\tau} = v \sigma \) produces

\[ R_y(t+\tau,\tau) = \int_{\frac{\mu_t}{\sigma}}^{\infty} \int_{\frac{\mu_{\tau}}{\sigma}}^{\infty} g(u,v,\rho) \, dudv + \int_{-\infty}^{\frac{\mu_t}{\sigma}} \int_{-\infty}^{\frac{\mu_{\tau}}{\sigma}} g(u,v,\rho) \, dudv \]

\[ -\int_{\frac{\mu_t}{\sigma}}^{\infty} \int_{-\infty}^{\frac{\mu_{\tau}}{\sigma}} g(u,v,\rho) \, dudv - \int_{-\infty}^{\frac{\mu_t}{\sigma}} \int_{\frac{\mu_{\tau}}{\sigma}}^{\infty} g(u,v,\rho) \, dudv \]  \hspace{1cm} (A-11)

where \( g(u,v,\rho) \) is the density function of the random variables \( u \) and \( v \) induced from the variables \( x_t \) and \( x_{\tau} \).

Let

\[ h = \frac{\mu_t}{\sigma} \quad k = \frac{\mu_{\tau}}{\sigma} \]

then

\[ R_{yy}(\tau,\tau) = L(-h,-k,\rho) + L(h,k,\rho) - L(-h,k,-\rho) - L(h,-k,\rho) \]  \hspace{1cm} (A-12)

so using [2]

\[ L(h,k,\rho) = Q(h)Q(k) + \sum_{n=0}^\infty \frac{z^{(n)}(h)z^{(n)}(k)}{(n+1)!} \rho^{n+1} \]  \hspace{1cm} (A-13)
and

\[ Q(h) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2} \, dt \quad (A-14) \]

we arrive at

\[ R_{yy}(t, \tau) = Q(-n)Q(-k) + \sum_{n=0}^{\infty} \frac{z^n(-h)z^n(-k)}{(n+1)!} \rho^{n+1} \]

\[ + Q(h)Q(k) + \sum_{n=0}^{\infty} \frac{z^n(h)z^n(k)}{(n+1)!} \rho^{n+1} \]

\[ - Q(-h)Q(k) - \sum_{n=0}^{\infty} \frac{z^n(-h)z^n(k)(-\rho)^{n+1}}{(n+1)!} \]

\[ - Q(h)Q(-k) - \sum_{n=0}^{\infty} \frac{z^n(h)z^n(-k)(-\rho)^{n+1}}{(n+1)!} \quad (A-15) \]

where

\[ z^{(n)}(x) = \frac{d^n}{dx^n} \left\{ \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \right\} \quad (A-16) \]

Now

\[ R_y(\tau, t) = [Q(h) - Q(-h)][Q(k) - Q(-k)] \]

\[ + \sum_{n=0}^{\infty} \frac{\rho^{n+1}}{(n+1)!} \left\{ z^{(n)}(-h)z^n(-k) + z^{(n)}(h)z^n(k) \right\} \]

\[ - (-1)^{n+1} \left[ z^{(n)}(-h)z^{(n)}(k) + z^{(n)}(h)z^{(n)}(-k) \right] \quad (A-17) \]
Now note that

\[ Q(-x) - Q(x) = \text{erf} \left( \frac{x}{\sqrt{2}} \right) \quad (A-18) \]

Also note that

\[ z^{(n)}(x) = z^n(-x) \quad , \quad n \text{ even} \]
\[ z^{(n)}(x) = -z^n(-x) \quad , \quad n \text{ odd} \quad (A-19) \]

So

\[ R_y(t, \tau) = \text{erf} \left( \frac{h}{\sqrt{2}} \right) \text{erf} \left( \frac{k}{\sqrt{2}} \right) + \sum_{n=0}^{\infty} 4 \frac{z^{(n)}(h)z^{(n)}(k)p^{n+1}}{(n+1)!} \]
\[ + 4 \sum_{n=1}^{\infty} \frac{z^{(n)}(h)z^{(n)}(k)p^{n+1}}{(n+1)!} \quad (A-20) \]

where \( \sum \) denotes sums over \( n \) odd and \( \Sigma \) denotes sums over \( n \) even. Changing indices yields

\[ R_y(t, \tau) = \text{erf} \left( \frac{\mu t}{\sqrt{2} \sigma} \right) \text{erf} \left( \frac{\mu \tau}{\sqrt{2} \sigma} \right) + 4 \sum_{m=0}^{\infty} z^{(2m+1)} \left( \frac{\mu t}{\sigma} \right) z^{(2m+1)} \left( \frac{\mu \tau}{\sigma} \right) \frac{\rho^{2m+2}}{(2m+2)!} \]
\[ + 4 \sum_{m=0}^{\infty} \frac{\rho^{2m+1}}{(2m+1)!} z^{(2m)} \left( \frac{\mu t}{\sigma} \right) z^{(2m)} \left( \frac{\mu \tau}{\sigma} \right) \quad (A-21) \]

We have, since \( s(t) = \text{Ad}(t) \),
\( R_y(t, \tau) = d(t)d(t+\tau) \text{erf}^2 \left[ \sqrt{\frac{S}{2N}} \right] \)

\[ = \frac{4}{2m+1} \sum_{m=0}^{\infty} \left( \frac{2m+1}{\sqrt{\frac{S}{N}}} \right)^2 \left( \frac{R_n(\tau)}{\sigma^2} \right)^{2m+2} / (2m+1)! \]

\[ + 4 \sum_{m=0}^{\infty} \left( \frac{2m}{\sqrt{\frac{S}{N}}} \right)^2 \left( \frac{R_n(\tau)}{\sigma^2} \right)^{2m+1} / (2m+1)! \]

where we have let

\[ \frac{A^2}{\sigma^2} = \frac{S}{N} \]  

(A-23)

Now consider \( d(t) \),

\[ d(t) = \sum_{i=-\infty}^{\infty} d_i p(t-iT) \]

where \( p(t) = 1, t \in (0,T) \) and \( d_i = \pm 1 \) with equal likelihood. Hence, the time and ensemble average of \( d(t)d(t+\tau) \) yields

\[ R_d(\tau) = \left\langle d(t)d(t+\tau) \right\rangle = \left\langle \sum_{i=-\infty}^{\infty} p(t-iT) p(t+iT) \right\rangle \]

or

\[ R_d(\tau) = \left[ 1 - \frac{1}{T} \right], \quad \text{NRZ waveforms,} \]

so that we have
The first term is the $S_xS$ term, second is the $S_xN$ terms, and the last term is the $N_xN$ term. Notice at high SNR

\[ R_y(\tau) = R_d(\tau) \]  

At low SNR

\[ \lim_{S/N \to 0} \text{erf} \left( \frac{S}{\sqrt{2N}} \right) = 0 \]

\[ \lim_{S/N \to 0} \left[ z \left( \frac{(2m)}{\sqrt{2N}} \right) \right]^2 = \frac{1}{2\pi} \left( \frac{1}{2} \right)^{2m} \left( \frac{(2m)!}{m!} \right)^2 \]

\[ \lim_{S/N \to 0} \left[ z \left( 2m+1 \right) \left( \frac{S}{\sqrt{2N}} \right) \right]^2 \to 0 \]  

Therefore, (A-24) converges at $S/N = 0$ to

\[ \lim_{S/N \to 0} R_y(\tau) = 4 \sum_{m=0}^{\infty} \left( \frac{1}{2\pi} \right) \left( \frac{1}{2} \right)^{2m} \left( \frac{2m!}{m!} \right)^2 \frac{1}{(2m+1)!} \left( \frac{R_{nn}(\tau)}{2} \right)^{2m+1} \]  

or

\[ R_y(\tau) = \frac{2}{\pi} \arcsin \left( \frac{R_n(\tau)}{\sigma^2} \right) \]

which is a well-known result. It is easy to show that $R_d(\tau)$ is given by
\[ R_d(\tau) = \begin{cases} 1 - \frac{|\tau|}{T} & |\tau| \leq T \\ 0 & |\tau| > T \end{cases} \]  
\text{NRZ} (A-29)

\[ R_d(\tau) = \begin{cases} 1 - \frac{3|\tau|}{T} & |\tau| \leq \frac{T}{2} \\ \frac{1}{T} & \frac{T}{2} < |\tau| < T \end{cases} \]  
\text{Manchester} (A-30)

Therefore, at the input to the FM summer due to channels 1 and 2, we have

\[ s(t) = A \frac{4}{\pi} d_2(t) \cos \theta \sin(\omega_{sc} t) + A \frac{4}{\pi} d_1(t) \sin \theta \cos(\omega_{sc} t) \]  
(A-31)

where \( d_2(t) \) has the spectral density given by

\[ \mathcal{S}_d(f) = \mathcal{S}_d(f) \left[ \frac{S_2}{N_2} \right]^2 + 4 \mathcal{S}_d(f) \sum_{m=0}^{\infty} \left[ \frac{z}{2} \frac{S_2^2}{N_2^2} \frac{R_n(\tau)^{2m+2}}{\sigma_2^{2m+2}} \right] \left[ \frac{R_n(\tau)^{2m+2}}{(2m+2)!} \right] \]  
(A-32)

with \( \mathcal{S}(\cdot) \) the Fourier transform and where \( d_1(t) \) has the spectral density given by

\[ \mathcal{S}_d(f) = \mathcal{S}_d(f) \left[ \frac{S_1}{N_1} \right]^2 + 4 \mathcal{S}_d(f) \sum_{m=0}^{\infty} \left[ \frac{z}{2} \frac{S_1^2}{N_1^2} \frac{R_n(\tau)^{2m+2}}{\sigma_1^{2m+2}} \right] \left[ \frac{R_n(\tau)^{2m+2}}{(2m+2)!} \right] \]  
(A-33)
and where $S_d(t)$ denotes the data spectrum of that respective channel.

Evaluation of the $S_{xN}$ and $N_{xN}$ spectral densities is made in Appendix B.

References--Appendix A


APPENDIX B

SPECTRAL DENSITY ASSOCIATED WITH

\[
\left[ \frac{R_n(\tau)}{\sigma^2} \right]^j, \quad j=1,2,3,4
\]

In this appendix, we summarize the power spectral density associated with the jth power of

\[
\left[ \frac{R_n(\tau)}{\sigma^2} \right]
\]

for \( j=1,2,3,4 \).

First consider the Fourier transform of \( R_n(\tau)/\sigma^2 \):

\[
\mathcal{S}_n^{(1)}(\tau) = \int_{-\infty}^{\infty} \frac{R_n(\tau)}{\sigma^2} e^{-i2\pi f \tau} \, dt
\]

For convenience, we assume that the lowpass filter of Figure 2 is an ideal low-pass filter. Hence, \( R_n(\tau) \) is given by

\[
R_n(\tau) = \int_{-B}^{B} \frac{N_0}{2} e^{i2\pi f \tau} \, df = N_0 B \frac{\sin(2\pi B \tau)}{(2\pi B \tau)}
\]

The corresponding spectral density is given by

\[
\mathcal{S}_n^{(1)}(f) = \int_{-\infty}^{\infty} \frac{R_n(\tau)}{\sigma^2} e^{-i2\pi f \tau} \, d\tau = \int_{-\infty}^{\infty} \frac{\sin(2\pi B \tau)}{2\pi B \tau} e^{-i2\pi f \tau} \, d\tau
\]

or

\[
\mathcal{S}_n^{(1)}(f) = 2 \int_{0}^{\infty} \frac{\sin x \cos(fB \ x)}{x} \, \frac{dx}{2\pi B}
\]

(B-4)
\[ \mathcal{S}_n^{(1)}(f) = \frac{1}{2B} U_B, \quad (B-5) \]

where

\[ U_B = \begin{cases} 1, & |f| < B \\ 0, & |f| > B \end{cases} \quad (B-6) \]

Now consider the second power of \( R_n(\tau)/\sigma^2 \). We have that the spectral density is given by

\[ \mathcal{S}_n^{(2)}(f) = \int_{-\infty}^{\infty} \frac{\sin^2 2\pi B \tau}{(2\pi B \tau)^2} \cos(2\pi f \tau) \, d\tau \quad (B-7) \]

Changing variables, we have

\[ \mathcal{S}_n^{(2)}(f) = 2 \int_{0}^{\infty} \frac{\sin^2 x}{x^2} \cos \left( \frac{f}{B} x \right) \frac{dx}{2\pi B} \quad (B-8) \]

Using [1, p. 450, §3.82, #5], we have

\[ \mathcal{S}_n^{(2)}(f) = \begin{cases} \frac{1}{2B} \left( 1 - \frac{|f|}{2B} \right) & |f| < 2B \\ 0 & |f| > 2B \end{cases} \quad (B-9) \]

Now consider the cube term which is given by

\[ \mathcal{S}_n^{(3)}(f) = \int_{-\infty}^{\infty} \frac{\sin^3(2\pi B \tau)}{(2\pi B \tau)^3} e^{-2\pi f \tau} \, d\tau \quad (B-10) \]

Letting \( x = 2\pi B \tau \) yields

\[ \mathcal{S}_n^{(3)}(f) = 2 \int_{0}^{\infty} \frac{\sin^3 x}{x^3} \cos \left( \frac{f}{B} x \right) \frac{dx}{2\pi B} \quad (11) \]
\[ \mathcal{J}_n^{(3)}(f) = \frac{1}{8B} \left( 3 - \frac{|f|}{B} \right)^2 \quad 0 < |f| < B \]
\[ \mathcal{J}_n^{(3)}(f) = \frac{1}{16B} \left( 3 - \frac{|f|}{B} \right)^2 \quad B < |f| < 3B \] (B-12)

Now consider the fourth power term:

\[ \varkappa_n^{(4)}(f) = \int_{-\infty}^{\infty} \frac{\sin^4 (2\pi B \tau)}{(2\pi B \tau)^4} e^{-i2\pi ft} \, dt \] (B-13)

\[ = \mathcal{F}\left\{ \frac{\sin^2 (2\pi B \tau)}{(2\pi B \tau)^2} \right\} \star \mathcal{F}\left\{ \frac{\sin^2 (2\pi B \tau)}{(2\pi B \tau)^2} \right\} \] (B-14)

\[ \varkappa_n^{(4)}(f) = \left\{ \frac{1}{2B} \left( 1 - \frac{|f|}{2B} \right) U_B(f) \right\} \star \left\{ \frac{1}{2B} \left( 1 - \frac{|f|}{2B} \right) U_B(f) \right\} \] (B-15)

Hence,

\[ \varkappa_n^{(4)}(f) = \frac{1}{4B^2} \int_{-2B+|f|}^{2B} \left( 1 - \frac{|f'|}{2B} \right) \left( 1 - \frac{|f''|}{2B} \right) U_f(f') U_f(f-f') \, df' \] (B-16)

\[ \varkappa_n^{(4)}(f) = \frac{1}{4B^2} \left[ \frac{4}{3} B - \frac{f^2}{2B} + \frac{|f|^3}{24B^2} \right] \quad 2B < |f| < 4B \] (B-17)

\[ \varkappa_n^{(4)}(f) = \frac{1}{4B^2} \left[ \frac{8}{3} B - 2|f| + \frac{f^2}{2B} - \frac{|f|^3}{24B^2} \right] \quad 2B < |f| < 4B \] (B-18)

References--Appendix B

ANALYSIS OF THE KU-BAND SHUTTLE/MRD

I-Q REVERSAL PHENOMENA

Interim Report
Contract NAS9-16893

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Axiomatix Report No. R8406-1
June 1, 1984
TABLE OF CONTENTS

| LIST OF FIGURES | 11 |
| 1.0 | INTRODUCTION TO THE PROBLEM 1 |
| 2.0 | SYSTEM MODEL 1 |
| 3.0 | MRD TRACKING-LOOP DESCRIPTION 1 |
| 4.0 | CONDITIONS REQUIRED TO INDUCE I-Q SWAPPING 22 |
| 5.0 | TAPE-RECORDER-SIGNAL MODEL 30 |
| 6.0 | CONCEPTUAL BASIS FOR THE TAPE-RECORDER-INDUCED I-Q FLIPPING 30 |
| 7.0 | SIMULATION OF THE I-Q FLIPS 34 |
| 8.0 | SOLAR MAX WEAK-SIGNAL-CASE I-Q FLIPPING 37 |
| 9.0 | SUMMARY OF THE NASA LABORATORY TESTS AND THE AXIOMATIX MODULE SIMULATION 41 |
| 10.0 | CONCLUSIONS 41 |
| REFERENCES | 43 |

APPENDICES

A  Model of the HAC Subcarrier Modulator
B  Lock-Detector Analysis
C  S-Curve of a Modified Four-Phase Costas Loop for a Particular Quadrature Signal Input
D  Impulse Response of an Nth-Order Butterworth Filter
E  Simulation of the MRD Modified Four-Phase Costas Loop
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Model for the Solar Max Retrieval Data Dump and the OPS DataDump</td>
<td>2</td>
</tr>
<tr>
<td>3.1</td>
<td>Modified Four-Phase Costas Loop</td>
<td>3</td>
</tr>
<tr>
<td>3.2</td>
<td>S-Curve $P_2: P_1 = 4:1; A_v = 2; SNR = $</td>
<td>5</td>
</tr>
<tr>
<td>3.3</td>
<td>S-Curve $P_2: P_1 = 1:1; A_v = 2; SNR = $</td>
<td>6</td>
</tr>
<tr>
<td>3.4</td>
<td>S-Curve $P_2: P_1 = 1/2:1; A_v = 2; SNR = $</td>
<td>7</td>
</tr>
<tr>
<td>3.5</td>
<td>S-Curve $P_2: P_1 = 1/4:1; A_v = 2; SNR = $</td>
<td>8</td>
</tr>
<tr>
<td>3.6</td>
<td>S-Curve when $P_2: P_1 = 1/8:1; $A_v = 2, SNR = $</td>
<td>9</td>
</tr>
<tr>
<td>3.7</td>
<td>S-Curve when $P_2: P_1 = 1/16:1; $SNR = $</td>
<td>10</td>
</tr>
<tr>
<td>3.8</td>
<td>S-Curve $A_v = 2$, Channel 2 Drops Out (Strong Channel); $P_2: P_1 = 0:1; $SNR = $</td>
<td>11</td>
</tr>
<tr>
<td>3.9</td>
<td>S-Curve $A_v = 2$, Channel 1 Drops Out (Weak Channel Drops Out); $P_2: P_1 = 1:0; $SNR = $</td>
<td>12</td>
</tr>
<tr>
<td>3.10</td>
<td>S-Curve; Sine Wave on Channel 2, Biphase-L Random on Channel 1; One-Pole 1.8-MHz Filter Bandwidth; $f_{SW} = 1$ MHz, $P_2: P_1 = 2.48:1$</td>
<td>15</td>
</tr>
<tr>
<td>3.11</td>
<td>S-Curve; Sine Wave on Channel 2, Biphase-L Random on Channel 1; One-Pole 1.8-MHz Bandwidth; $f_{SW} = 2$ MHz, $P_2: P_1 = 1.45:1$</td>
<td>16</td>
</tr>
<tr>
<td>3.12</td>
<td>S-Curve; Sine Wave on Channel 2, Biphase-L Random on Channel 1; One-Pole 1.8-MHz Bandwidth; $f_{SW} = 3$ MHz, $P_2: P_1 = 0.86:1$</td>
<td>17</td>
</tr>
<tr>
<td>3.13</td>
<td>S-Curve with One-Pole 1.8-MHz Bandwidth; Sine Wave on Channel 2; Biphase-L Random on Channel 1; $f_{SW} = 4$ MHz, $P_2: P_1 = 0.55:1$</td>
<td>18</td>
</tr>
<tr>
<td>3.14</td>
<td>S-Curve with One-Pole 1.8-MHz Bandwidth; $f_{SW} = 5$ MHz, $P_2: P_1 = 0.37:1$; Sine Wave on Channel 2; Biphase-L Random on Channel 1</td>
<td>19</td>
</tr>
<tr>
<td>3.15</td>
<td>S-Curve with One-Pole 1.8-MHz Bandwidth; $f_{SW} = 6$ MHz, $P_2: P_1 = 0.27:1$; Sine Wave on Channel 2; Biphase-L Random on Channel 1</td>
<td>20</td>
</tr>
<tr>
<td>3.16</td>
<td>S-Curve with One-Pole 1.8-MHz Bandwidth; $f_{SW} = 7$ MHz, $P_2: P_1 = 0.2:1$, Sine Wave on Channel 2; Biphase-L Random on Channel 1</td>
<td>21</td>
</tr>
</tbody>
</table>
3.17 S-Curve; Random Data on Channel 1; Sine Wave on Channel 2; 
f = 2.5 MHz ........................................... 23

3.18 S-Curve; Random Data on Channel 1; Sine Wave on Channel 2; 
f = 3.0 MHz ........................................... 24

3.19 S-Curve; Random Data on Channel 1; Sine Wave on Channel 2; 
f = 3.2 MHz ........................................... 25

3.20 S-Curve; Random Data on Channel 1; Sine Wave on Channel 2; 
f = 3.4 MHz ........................................... 26

3.21 S-Curve; Random Data on Channel 1; Sine Wave on Channel 2; 
f = 3.5 MHz ........................................... 27

3.22 S-Curve; Random Data on Channel 1; Sine Wave on Channel 2; 
f = 3.6 MHz ........................................... 28

3.23 S-Curve; Random Data on Channel 1; Sine Wave on Channel 2; 
f = 4.0 MHz ........................................... 29

3.24 (a) Normal Playback Spectra; (b) Transient Start-Up Spectra ... 31

3.25 Simulation of I-Q Inversion with the 5-MHz Prelimiter LPF Plotted 
Against Time and the Lowest Frequency Value ................. 33

3.26 Simulation of Unsuccessful I-Q Inversion Using a 2.5-MHz LPF 
Plotted Against Time and the Lowest Frequency Value ........ 35

7.1 Plot of Phase Errors Versus Time and Lowest Tone Frequency at 
Channel $E_b/N_0 = 2$ dB .................................. 38
SUMMARY

In this report, the I-Q flipping problem of the MRD is studied by analysis and simulation to discover why the flipping occurred. In the case of the tape-recorder dump during start-up or wraparound, it was found that the composite tape-recorder signal contained two tones increasing with frequency, plus a spectrally flat noise term. According to the prelimiter-filter bandwidth into the subcarrier modulator, this mixture determined the conditions for I-Q flipping. The narrower filter bandwidth (2.5 MHz, six-pole) stopped the I-Q flipping completely.

In the case of the Solar Max I-Q flipping, the small SNR at the Shuttle PI caused mainly noise to be transmitted on channel 2. At the receiver, most of the noise is filtered out, greatly reducing the effective I-Q ratio at the MRD subcarrier loop.

Table 9.1 summarizes the NASA laboratory measurements and Axiomatix predictions for a variety of cases with excellent agreement. It should be noted that observing the existence and modeling of two tones, along with the noise output from the tape recorder start-up or wraparound, allowed Axiomatix to make very accurate performance predictions. Specifically, the comparisons between the NASA measurements and Axiomatix predictions based on simulations or calculations are very good for a variety of cases, as can be depicted in Table 9.1.
1.0 INTRODUCTION TO THE PROBLEM

Figure 1.1 illustrates the Shuttle Ku-band transmitter, TDRSS terminal, Shuttle Payload Interrogator (PI) and Solar Max transmitter. The problem we address is the I-Q interchange which occurs in the medium-rate demodulator (MRD) under the following conditions: during bent-pipe acquisition of the 8.5-MHz subcarrier at low SNR, and during playback operation when the recorder either starts up or changes tracks (wrap-around). It should be noted that, in addition to the above sources of I-Q inversion, a square-wave data sequence on channel 2 will induce I-Q inversion if the frequency of the square wave is high enough. In the following sections, we will elaborate on the different mechanisms of this I-Q flipping.

2.0 SYSTEM MODEL

Figure 1.1 illustrates the system block diagram under study. First, we consider the Solar Max signal which is sent from the transmitter to the Shuttle receiver (PI), where it is demodulated to baseband, filtered and QPSK modulated onto an 8.5-MHz square-wave subcarrier. The modulator intrinsically limits both of the channel inputs in the modulation process. However, the bandpass filter following the QPSK modulator restores the QPSK signal to the standard QPSK sine wave and cosine wave format, being the fundamental of the square-wave rate. This process is discussed in Appendix A. This subcarrier is added to the channel 3 data, frequency modulated, and transmitted through the TDRS to the TDRSS terminal. At this ground terminal, the FM demodulates the carrier and produces the QPSK 8.5-MHz subcarrier which contains both channel 1 and channel 2 data. The medium-rate demodulator (MRD) is a QPSK tracking loop which produces, at its output, channel 1 and channel 2 baseband data. The problem we are addressing in this report is the flipping, or interchanging of channel 1/channel 2 data in the MRD.

3.0 MRD TRACKING-LOOP DESCRIPTION

Figure 3.1 illustrates the MRD tracking loop which is capable of tracking QPSK signals having an I-Q power ratio of either 4:1 or 1:1 in the two phases of the transmitted signal. The gain (or attenuation in the opposite arm) is set at the ratio of $A_V = \sqrt{P_2/P_1}$, where $P_1$ and $P_2$ are the respective channel powers. Basically, the loop forms an orthogonal set of sinusoids
Figure 1.1. Model for the Solar Max Retrieval Data Dump and the OPS Data Dump
Figure 3.1. Modified Four-Phase Costas Loop

$A_v = \sqrt{P_2/P_1}$

$L_1$ and $L_2$

$\sqrt{2} \sin (\omega_0 t + \phi)$

$\sqrt{2} \cos (\omega_0 t + \delta)$

$L_{HF}$

$\text{Sign}$

$\text{VCXO}$

$\text{Sign}$

$\text{LPF}$

$s(t) + n(t)$
that attempt to demodulate the respective data streams which are filtered and processed through data-removal processing to yield an error-control signal. This error-control signal drives the loop filter and, in turn, controls the VCO. Normally, there are four stable lock points: 0°, 90°, 180° and 270°; however, the angles can change according to SNR and the amount of distortion present, as well as the power ratio. Figures 3.2 through 3.9 depict the S-curves (average control signals, given φ) for various channel 1/channel 2 power ratios, assuming no distortion in the data signals and no additive thermal noises in the channel.

The S-curves are based on the average value of the error-control signal given a particular value of φ. Here the quadriphase signal is given by

\[ s(t) = \sqrt{2} \sqrt{(1-\alpha)^2} m_2(t) \sin(\omega_0 t + \theta_0) + \sqrt{2} \sqrt{\alpha S} m_1(t) \cos(\omega_0 t + \theta_0) \]  

(1)

where \( m_1(t) \) is the data signal in channel 1, \( m_2(t) \) is the data signal in channel 2, \( \omega_0 \) is the radian carrier frequency, and \( \theta_0 \) is the carrier phase. From Figure 3.1, we can write

\[ \epsilon_2(t) = \sqrt{(1-\alpha)^2 \alpha S} \bar{m}_2(t) \cos \phi - \sqrt{\alpha S} \bar{m}_1(t) \sin \phi \]

\[ \epsilon_1(t) = \sqrt{(1-\alpha)^2 \alpha S} \bar{m}_2(t) \sin \phi + \sqrt{\alpha S} \bar{m}_1(t) \cos \phi \]  

(2)

where \( \alpha S \) is the power in channel 1, \( (1-\alpha)^2 \alpha \) is the power in channel 2, \( \bar{m}_1(t) \) is the arm-filtered channel 1 data signal, and \( \bar{m}_2(t) \) is the arm-filtered channel 2 data signal. Combining the signal as in Figure 3.1 produces the control

\[ C(\phi) = A_v \left[ \sqrt{(1-\alpha)^2 \alpha S} \bar{m}_2(t) \sin \phi + \sqrt{\alpha S} \bar{m}_1(t) \sin \phi \right] \]

\[ \times \text{sgn} \left[ \sqrt{(1-\alpha)^2 \alpha S} \bar{m}_2(t) \cos \phi - \sqrt{\alpha S} \bar{m}_1(t) \sin \phi \right] \]

\[ - \left[ \sqrt{(1-\alpha)^2 \alpha S} \bar{m}_2(t) \cos \phi - \sqrt{\alpha S} \bar{m}_1(t) \sin \phi \right] \]

\[ \times \left[ \sqrt{(1-\alpha)^2 \alpha S} \bar{m}_2(t) \sin \phi - \sqrt{\alpha S} \bar{m}_1(t) \cos \phi \right] \]  

(3)

Since the channel 1 data rate is low compared to the arm-filter bandwidth (1.8 MHz), we may assume that \( \bar{m}_1(t) \) takes on the values ±1. For convenience, we also initially assume that \( \bar{m}_2(t) = ±1 \) independent of \( \bar{m}_1(t) \). Hence, assuming that both \( \bar{m}_1(t) \) and \( \bar{m}_2(t) \) are random 1's or -1's, the average value of \( C(\phi) \) becomes
Figure 3.3. S-Curve $P_2:P_1 = 1:1; A_v = 2; = SNR
Figure 3.4. S-Curve $P_2:P_1 = 1/2:1; A_v = 2$; = SNR

$\bullet =$ STABLE LOCK POINT
Figure 3.6. S-Curve when $P_2:P_1 = 1/8:1$
$A_v = 2; SNR = \infty$
Figure 3.7. S-Curve when $P_2/P_1 = 1/16$: $SNR = \infty$
Figure 3.8. S-Curve When $A_v = 2$, Channel 2 Drops Out (Strong Channel); $P_2:P_1 = 0:1$; $SNR = \infty$

NOTE: $f(\phi) = f(\phi + \pi)$
Figure 3.9. S-Curve when $A_v = 2$, Channel 1 Drops Out (Weak Channel Drops Out); $P_2/P_1 = 1:0$;

$\text{SNR} = \infty$
\( \bar{C}(\phi) = \frac{1}{2} A_\nu [\sqrt{(1-\alpha)S} \sin \phi + \sqrt{\alpha S} \cos \phi] \text{sgn}[\sqrt{(1-\alpha)S} \cos \phi - \sqrt{\alpha S} \sin \phi] \\
- \frac{1}{2} [\sqrt{(1-\alpha)S} \cos \phi - \sqrt{\alpha S} \sin \phi] \text{sgn}[\sqrt{(1-\alpha)S} \sin \phi + \sqrt{\alpha S} \cos \phi] \\
+ \frac{1}{2} A_\nu [\sqrt{(1-\alpha)S} \sin \phi - \sqrt{\alpha S} \cos \phi] \text{sgn}[\sqrt{(1-\alpha)S} \cos \phi + \sqrt{\alpha S} \sin \phi] \\
- \frac{1}{2} [\sqrt{(1-\alpha)S} \cos \phi + \sqrt{\alpha S} \sin \phi] \text{sgn}[\sqrt{(1-\alpha)S} \sin \phi - \sqrt{\alpha S} \cos \phi] \\
+ \frac{1}{2} A_\nu [-\sqrt{(1-\alpha)S} \sin \phi - \sqrt{\alpha S} \cos \phi] \text{sgn}[-\sqrt{(1-\alpha)S} \cos \phi + \sqrt{\alpha S} \sin \phi] \\
- \frac{1}{2} [-\sqrt{(1-\alpha)S} \cos \phi + \sqrt{\alpha S} \sin \phi] \text{sgn}[-\sqrt{(1-\alpha)S} \sin \phi - \sqrt{\alpha S} \cos \phi] \\
+ \frac{1}{2} A_\nu [-\sqrt{(1-\alpha)S} \sin \phi + \sqrt{\alpha S} \cos \phi] \text{sgn}[-\sqrt{(1-\alpha)S} \cos \phi - \sqrt{\alpha S} \sin \phi] \\
- \frac{1}{2} [-\sqrt{(1-\alpha)S} \cos \phi - \sqrt{\alpha S} \sin \phi] \text{sgn}[-\sqrt{(1-\alpha)S} \sin \phi + \sqrt{\alpha S} \cos \phi] \) \hspace{1cm} (4)

Equation (4) as the basis of Figures 3.2 through 3.9. The lock points are shown as large dots. In all of these curves, notice that the lock points are stable at 0°, 90°, 180° and 270°. When the power ratio is 1/8:1 or less, additional lock points occur, yielding eight admissible lock points.

We now consider the case when filtering of channel 2 is severe and is based on a one-pole LPF and when channel 1 has negligible filtering. We also assume a square-wave signal on channel 2 for mathematical convenience rather than a random-data sequence. The results are developed in Appendix C.

From this appendix, the S-curve is given by

\( S(\phi) = -A_\nu G(\phi) - G(\phi + \pi/2) \) \hspace{1cm} (5)

where
G(\phi) = \text{sgn}([\sin \phi] \cos \phi)

\[ G(\phi) = \frac{2}{\pi} \left( \cos \phi \text{sgn}([\sin \phi] \cos^2 \left( \frac{\tan(\phi)}{A} \right) \right) \sin(\phi) \text{sgn}(\cos \phi) \sqrt{1 - \left( \frac{\tan(\phi)}{A} \right)^2} \]

with

\[ K(\phi) = \begin{cases} 1, & |\sin \phi| \leq A |\cos \phi| \\ 0, & \text{otherwise} \end{cases} \]

and

\[ \text{sgn} \ x = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \]

and

\[ A = \sqrt{\frac{1 - \gamma}{\alpha}} \gamma(f_0) \]

with

\[ \gamma(f_0) = \frac{(4/\pi)}{\sqrt{1 + \left( \frac{f_0}{1.8} \right)^2}}, \quad f_0 \text{ in MHz}, \]

the sine-wave attenuation to the tone at \( f_0 \). It should be noted that the hard limiter illustrated in Figure 1.1 at the \( 8.5 \) MHz subcarrier converts the sine wave into a square wave but, at the TDRSS terminal, all the harmonics of the square wave are filtered out when the tone is greater than about \( 1.2 \) MHz. Thus, the peak fundamental is \( 4/\pi \) of the peak square wave. Figures 3.10 through 3.16 illustrate the S-curve for various sine-wave (or square-wave) frequencies and the channel-2-to-channel-1 power ratio. Note that, when the sine wave is \( 6 \) MHz, the S-curve is almost flat at \( \phi=0 \) and \( 180^\circ \). At \( 7 \) MHz, the curve has negative slope at \( 0^\circ \) and \( 180^\circ \). Thus, if the loop is at \( 0^\circ \) or \( 180^\circ \), it would move to either \( 90^\circ \) or \( 270^\circ \) if the channel 2 data changed from random Manchester to sine wave at \( 7 \) MHz. These graphs illustrate the fact that severe distortion has a marked influence on the S-curve. The previous case of negligible distortion yielded no unstable S-curves at \( \phi=0^\circ \) or \( 180^\circ \).
Figure 3.10. S-Curve; Sine Wave on Channel 2, Biphase-L Random on Channel 1; One-Pole 1.8-MHz Filter Bandwidth; $f_{SW} = 1$ MHz; $P_2:P_1 = 2.48:1$
Figure 3.11. S-Curve; Sine Wave on Channel 2, Biphase-L Random on Channel 1; One-Pole 1.8-MHz Bandwidth; $f_{SW} = 2$ MHz; $P_2:P_1 = 1.45:1$
Figure 3.12. S-Curve: Sine Wave on Channel 2, Biphasé-L Random on Channel 1; One-Pole 1.8-MHz Bandwidth; $f_{SW} = 3$ MHz; $P_2:P_1 = 0.86:1$
Figure 3.13. S-Curve with One-Pole 1.8-MHz Bandwidth; Sine Wave on Channel 2; Biphasic Random on Channel 1; \( f_{SW} = 4 \text{ MHz}, P_2:P_1 = 0.55:1 \)
Figure 3.14. S-Curve with One-Pole 1.8-MHz Bandwidth; $f_{sw} = 5$ MHz, $P_2:P_1 = 0.37:1$; Sine Wave on Channel 2, 3iphae-L Random on Channel 1
Figure 3.15. S-Curve with One-Pole 1.8-MHz Bandwidth; $f_{SW} = 6$ MHz, $P_2:P_1 = 0.27:1$; Sine Wave on Channel 2; Biphasel Random on Channel 1
Figure 3.16. S-Curve with One-Pole 1.8-MHz Bandwidth; $f_{SW} = 7$ MHz; $P_2:P_1 = 0.2:1$, Sine Wave on Channel 2, Biplase Random on Channel 1
The final S-curve case is based on an attempt to accurately model the filters in the system. Again, a tone was used on channel 2 and random data on channel 1. A six-pole LPF of 5 MHz was employed to model the filter prior to the channel 2 limiter; a six-pole 6-MHz bandpass filter was used to model the bandpass filter prior to the QPSK tracking loop. Details of this filter are discussed in Appendix D. Figures 3.17 through 3.23 depict the resulting S-curves. These differ somewhat from the case of one-pole filtering (Figures 3.10 through 3.16). In particular, when the tone is at 3.2 MHz (Figure 3.23), the S-curve possesses a slightly positive slope at 0° or 180°. While at 3.5 MHz, a stable lock point near $\phi = 63.4^\circ$ appears with a weak lock point at 90° and 270°. At $f = 3.6$ MHz and greater, the only lock point is at $\phi = 63.4^\circ$ and $116.6^\circ$. Also, at 3.6 MHz, the power ratio of channel 2 to channel 1 at the QPSK tracking loop is about 0.4:1 while, at 4 MHz, it is about 0.12:1.

4.0 CONDITIONS FOUND TO INDUCE I-Q SWAPPING

There were basically three methods of testing which caused I-Q swapping. The first was noticed during bent-pipe operation with the Solar Max during the medium-rate demodulator (MRD) acquisition of the 8.5-MHz subcarrier. As the received signal power at the MRD is reduced, the probability of locking with the correct I-Q relationship decreases. This problem is inherently an acquisition problem; hence, it is due to a power loss of channel 2 in the MRD and the error induced in the power-ratio detector (see Appendix B).

The second means by which the MRD loop interchanges channels 1 and 2 occurs when the tape recorder driving channel 2 starts up or changes tracks (wraps around). During this start-up or wrap-around time, the tape recorder puts out a noise-like burst rather than data, which causes channels 1 and 2 to interchange.

The third way the channel interchange was observed was when a sine wave was used as the modulation input of channel 2. When the sine-wave frequency reached 2.1 MHz by stepping from 2 MHz, the MRD would interchange channels 1 and 2. This was not a random phenomena but, rather, a precise deterministic process.
Figure 3.17: S-Curve; Random Data on Channel 1; Sine Wave on Channel 2; f = 2.5 MHz

Six-pole LPF @ 5 MHz
Six-pole BPF @ 6 MHz
Figure 3.18. S-Curve; Random Data on Channel 1: Sine Wave on Channel 2; f = 3.0 MHz

Six-pole LPF @ 5 MHz
Six-pole BPF @ 6 MHz
Figure 3.19. S-Curve: Random Data on Channel 1; Sine Wave on Channel 2, f = 3.2 MHz

Six-pole LPF @ 5 MHz
Six-pole BPF @ 6 MHz
Six-pole LPF @ 5 MHz
Six-pole BPF @ 6 MHz

Figure 3.22. S-Curve; Random Data on Channel 1; Sine Wave on Channel 2; 
f = 3.6 MHz
Six-pole LPF @ 5 MHz
Six-pole BPF @ 6 MHz

Figure 3.23. S-Curve; Random Data on Channel 1; Sine Wave on Channel 2; $f = 4.0$ MHz
5.0 TAPE-RECORDERSIGNAL MODEL

The tape recorder was observed to produce a "noise-like" burst during the tape recorder startup or turnaround which was viewed by NASA/JSC personnel to be the source of the I-Q flipping. In Figure 3.24(a)/(b) [2, page 145] are the spectral densities of both the steady-state and start-up transient conditions. In the steady-state case, the spectral density appears to be the usual Manchester-encoded data, whereas the start-up spectral density appears to be the sum of flat noise plus two sine-wave tones that start at about 2.5 MHz and 3.8 MHz. However, since the spectral plot occurs only at a given time, as time progresses, the lines move farther out in frequency, then essentially disappear (when the data spectral density shows itself at the steady state). Observations by NASA/JSC engineers have noted that the lines do move out as time progresses.

Thus, our model for the output of the tape recorder startup or turnaround is given by (for up to one second or more)

\[ V(t) = n(t) + A_1 \cos \left( \omega_1 \left( t + \frac{a}{2} t^2 \right) \right) + A_2 \cos \left( \omega_2 \left( t + \frac{a}{2} t^2 \right) \right) \]  

where \( A_1 \) and \( A_2 \) are the tone peak amplitudes and \( \omega_1 \) and \( \omega_2 \) are the radian frequencies corresponding to 2.5 MHz and 4 MHz when a 5-MHz prefilter bandwidth was used and 1.25 MHz and 2 MHz when a 2.5-MHz prefilter bandwidth was used. The rate of change of frequency coefficient, \( a \), was modeled as 1.2. The first term \( n(t) \) is modeled as bandlimited white Gaussian noise. The power ratios of tone power to noise power is made a variable in the computer simulation program. It will shortly be shown that the existence of these tones accounts for the I-Q flipping phenomena in a well determined way.

6.0 CONCEPTUAL BASIS FOR THE TAPE-RECORDER-INDUCED I-Q FLIPPING

In this section, we discuss the reason why the tape recorder being turned on or wrapping around caused the I-Q inversion. During the tape-recorder startup, both a noise burst and the two sweeping tones are generated. Assume that a 5-MHz LPF (prelimiting) is used first. We can view Figures 3.17 through 3.23 to see what happens. Since the second tone (the higher frequency) will be filtered first, we neglect it and concern ourselves with the lower frequency tone only. From Figures 3.17 through 3.23, we see that, as
Photo of Normal Tape Playback
Center Connector of Breakout Box Used
- 960-kbps playback
- No filter

Figure 3.24(a). Normal Playback Spectra

Photo at Tape Start-Up
Center Connector of Breakout Box Used
- 960 kbps playback
- No filter

Figure 3.24(b). Transient Start-Up Spectra
the tone frequency (the value of $f$ in these figures) increases, the S-curve lock point at $\phi=C$ starts to diminish. By the time $f=3.2$ MHz, the S-curve has a very weak lock at $\phi=0$ while, at $f=3.4$ MHz, the lock point disappears (the slope at $\phi=0$ is negative). Hence, at this point, the loop attempts to push the phase error towards either $+90^\circ$ or $-90^\circ$. As the tape recorder continues to increase its tape speed, the two tones continue to increase in frequency, leading to Figure 3.22, which shows a lock point at $63.4^\circ$. The loop will remain locked at this phase error until the tone frequency goes beyond the 5-MHz cutoff of the channel 2 lowpass prelimiter. The noise in the tape system will then be the dominant power term and the S-curves will revert to those of Figures 3.2 through 3.9. However, since the power in channel 2 is based essentially on thermal noise and enters a limiter (at the transmitter), the bandwidth is greater than 5 MHz but the 6-MHz BPF of Figure 1.1 removes some noise power, as does the 1.8-MHz arm filters of the loop. So, instead of a 4:1 power ratio, a smaller ratio (say, 1:1) occurs initially in the loop. But, from Figure 3.3, we see that a phase error of $63.4^\circ$ will be carried into $90^\circ$ (or $-63.4^\circ$ into $-90^\circ$). Thus, I-Q swapping has occurred. Figure 3.25 shows a simulation of this phenomena, while Appendix E discusses the simulation model. In Figure 3.25, we see that, within a few tenths of a millisecond, the phase reaches about $63.4^\circ$ and hangs there as discussed above, then slowly reaches $90^\circ$.

Now let us consider what happens when the 2.5-MHz LPF is employed prior to the channel 2 limiter. Recall that the BPF located prior to the sub-carrier demodulator (MRD) has a baseband bandwidth of 3 MHz. Now, as the tape recorder is turned on, the recorder noise starts and the two previously discussed tones start increasing their frequencies. Again, we will concern ourselves with only the lower frequency tone since it is the dominant one in terms of I-Q flipping. As the lower frequency tone reaches 2.5 MHz, it starts getting filtered by the 2.5-MHz LPF in front of the channel 2 limiter. When it gets to 3 MHz, it is down about 10 dB; hence, the dominant term is the tape-recorder noise—not the tone. But, when only noise is present, it passes through the channel 2 limiter and looks like a data stream with heavy phase or time modulation. Hence, the S-curves for data modulation are applicable. Since the prefilter is set to 2.5 MHz, the limiter-output bandwidth is increased somewhat to, say, 4 MHz. But the arm filter "captures" most of this energy so that Figure 3.3 applies (or maybe Figure 3.4—it doesn't matter).
Figure 3.25. Simulation of I-Q Inversion with the 5-MHz Prelimiter LPF Plotted Against Time and the Lowest Frequency Value
From these S-curves, we see that there is no force to push the loop away from the lock point. From this discussion, it is clear that both the bandwidth and the number of poles are important since they determine how much attenuation is present to the lowest frequency tone.

Figure 3.26 illustrates the results of a simulation of the system when a 2.5-MHz prelimiter LPF was used. We see that, when the tone (which is now attenuated by about 6.5 dB at 2.9 MHz) reaches the point of the skirt of the 6-MHz bandpass filter, it is much weaker than the case when the 5-MHz LPF was used; hence, the mixture of noise-like energy to tone energy is not great enough to push the loop 90° away from the correct lock point. From Figure 3.26, the peak deviation from the lock point is about 5°.

We can now see why the square-wave test caused the loop to lose lock. Since this test was composed of only one square-wave signal, we see from Figures 3.19 and 3.20 that, at about 3.2 MHz, the S-curve has a negative slope at $\phi = 0$; hence, the loop is pushed towards the lock point at 90°. Note that, once at the 90° point, the loop will not slip again if the square wave is applied when $\phi = 90^\circ$ since $90^\circ$ is a stable lock point. Hence, once the loop I-Q flipped, it would not flip again on turnaround.

The measured value of square-wave-induced I-Q interchange was 2.1 MHz, whereas our theory predicts 3.2 MHz. Since we did not make a precise estimate of the tone-to-tape-noise power, we should not expect an exact agreement between the experimental measurement and theory.

7.0 SIMULATION OF THE I-Q FLIPS

In this section, we compare the NASA measurements with the simulation results performed at Axiomatix. In Table 7.1, simulation results are given for the case when no channel noise was present.

In the first two rows of Table 7.1, the individual tone-to-recorder noise spectral density was set at 87 dB-Hz which, for the two prelimiter LPF bandwidths considered, yielded SNR's of 20 and 23 dB (tone-to-noise-power ratio). The initial frequencies of the recorder tones are set so that the loop has time to respond to them before they are "outside" the LPF bandwidth. The loop bandwidth of the simulation was set to 18 kHz in order to reduce computer computation time and cost. The result shown in the last column of Table 7.1 was that, with a 5-MHz LPF prior to the limiter, it I-Q flipped whereas, with the 2.5-MHz LPF, it did not flip. This result is compatible with the observed measurements at NASA.
\( \alpha = 0.01 = -20 \text{ dB} \)
Test Filter = 2.5 MHz

Figure 3.26. Simulation of Unsuccessful I-Q Inversion Using a 2.5-MHz LPF Plotted Against Time and the Lowest Frequency Value
### Table 7.1. No Channel Noise

<table>
<thead>
<tr>
<th>Tone-to-Noise PSD (Recorder Model) (dB-Hz)</th>
<th>Test Filter Cut-Off Frequency (MHz)</th>
<th>In-Band Tone-to-Noise Ratio (dB)</th>
<th>Initial Frequency of the Tones (MHz)</th>
<th>Loop BW (kHz)</th>
<th>Description of Simulation Result</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>β = 87</strong></td>
<td>(6-pole) 5</td>
<td>20.0</td>
<td>2.5 &amp; 4.0</td>
<td>18</td>
<td>FLIPPED Moved to 63.4°, then to 90° (noise takes over)</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>23.0</td>
<td>1.25 &amp; 2.0</td>
<td>18</td>
<td>DID NOT FLIP</td>
</tr>
<tr>
<td><strong>β = 77</strong></td>
<td>5</td>
<td>10.0</td>
<td>2.5 &amp; 4.0</td>
<td>18</td>
<td>FLIPPED Less tendency to stay at 63.4° (noise takes over sooner than previously)</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>13.0</td>
<td>1.0 &amp; 2.0</td>
<td>18</td>
<td>DID NOT FLIP</td>
</tr>
<tr>
<td><strong>β = 72</strong></td>
<td>5</td>
<td>4.8</td>
<td>2.5 &amp; 4.0</td>
<td>18</td>
<td>FLIPPED Faster than previously</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>β = 67</strong></td>
<td>5</td>
<td>0.0</td>
<td>2.5 &amp; 4.0</td>
<td>18</td>
<td>FLIPPED Takes more time to take off, then flips faster</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>3.0</td>
<td>1.0 &amp; 2.0</td>
<td>18</td>
<td>DID NOT FLIP</td>
</tr>
<tr>
<td><strong>β = -</strong> (Only white noise as recorder output)</td>
<td>5</td>
<td></td>
<td>18</td>
<td></td>
<td>DID NOT FLIP</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td></td>
<td>18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The same comparison was made in the second, third and fourth rows of Table 7.1, except that the tone-to-recorder noise spectral density was reduced by 10 dB. Again, the results agreed with NASA's observations. This test was repeated twice more, each time with a 10-dB lower tone-to-noise power spectral density with the same flipping results. Finally, recorder noise only without a tone was tried and the I-Q loop did not flip. Later, we will see that, when channel noise was added to the receiver, the loop would not I-Q flip except at very low $E_b/N_0$ values.

Next, channel noise (white Gaussian thermal noise) was added to the receiver in order to simulate receiver front-end noise in the usual high $E_b/N_0$ level that this link displays (see Table 7.2). Again, it is seen that the loop flips only with the 5-MHz LPF—not with the 2.5-MHz filter.

In the data of Table 7.3, we considered the case when the channel $E_b/N_0$ was low. The purpose was to try to simulate the observed I-Q flip and return at the low SNR noted by NASA/JSC. Using the 5-MHz filter, the first test was simulated with a $B_L = 18$ kHz. It appeared that the loop SNR was too low, so this bandwidth was reduced to 4.5 kHz. At 20-dB tone-to-tape-recorder noise power ratio and $E_b/N_0$ of 2 dB, a flip occurred, then returned to $\phi = 0$, similar to what NASA had described. Removing the tone in the recorder-output model prevented this I-Q flip and return from happening. A plot of this phenomena is depicted in Figure 7.1. Note initially that the lowest frequency tone causes the loop to start to I-Q flip ($\phi = \pm 90^\circ$), then the error gradually returns to zero phase error, that is, the I-Q flip corrects itself.

8.0 SOLAR MAX WEAK-SIGNAL-CASE I-Q FLIPPING

Even though the Solar Max case was not analyzed extensively, it is believed that the basic reason why the I-Q flipping occurred in the Solar Max case was due to the weak signal conditions near 10-mi ranges, along with the fact that the SNR in the 5-MHz filter at these ranges was low; hence, most of the power transmitted in channel 2 was due to thermal noise. This noise was bandwidth expanded in the limiter of channel 2; hence, when this signal was captured by the four-phase loop, only a small fraction of the original amount of power remained in the arm filters. Therefore, as the SNR in channel 2 decreased, the I-Q power-ratio detector would cause the probability of I-Q flipping to increase because the captured channel 2 power decreases as the Solar Max SNR decreases (range increases). In other words, it is believed that this problem is basically one of the effective I-Q power ratio diminishing as the range of the Solar Max from the Shuttle increases.
Figure 7.1. Plot of Phase Error Versus Time and Lowest Tone Frequency at Channel $E_b/N_0 = 2$ dB
Table 7.2 Weak Noise Added to the Link

<table>
<thead>
<tr>
<th>Tone-to Noise PSD (dB-Hz)</th>
<th>Test Filter (MHz)</th>
<th>In-Band Tone-to Recorder Noise Ratio (dB)</th>
<th>Initial Frequency of the Tones</th>
<th>Loop BW (kHz)</th>
<th>Eb/N0 (dB)</th>
<th>Description of Simulation Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10.0</td>
<td>2.5 &amp; 4.0</td>
<td>18</td>
<td>16</td>
<td>FLIPPED</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>13.0</td>
<td>1.25 &amp; 2.0</td>
<td>18</td>
<td>16</td>
<td>DID NOT FLIP</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>~</td>
<td>~</td>
<td>18</td>
<td>16</td>
<td>DID NOT FLIP</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>~</td>
<td>~</td>
<td>18</td>
<td>16</td>
<td>~</td>
<td></td>
</tr>
</tbody>
</table>

\( \beta = 77 \)

\( \beta = \infty \) (Only white noise as recorder output)
Table 7.3. Strong Noise in the Link

\[ \frac{E_b}{N_0} \leq 6 \text{ dB} \]

<table>
<thead>
<tr>
<th>Test Filter (MHz)</th>
<th>n-Band Tone-to-Recorder Noise (dB)</th>
<th>Initial Frequency of the Tones (MHz)</th>
<th>Loop BW (kHz)</th>
<th>( E_b/N_0 )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10.0</td>
<td>2.5 &amp; 4.0</td>
<td>18.0</td>
<td>6.0</td>
<td>FLIPPED*: did not return to 0° (it might have needed more time)</td>
</tr>
<tr>
<td>5</td>
<td>10.0</td>
<td>2.5 &amp; 4.0</td>
<td>18.0</td>
<td>4.0</td>
<td>FLIPPED*: returned to 0°, but did not stay in lock</td>
</tr>
<tr>
<td>5</td>
<td>10.0</td>
<td>2.5 &amp; 4.0</td>
<td>4.5</td>
<td>4.0</td>
<td>FLIPPED*: did not return to 0° (more running time?)</td>
</tr>
<tr>
<td>5</td>
<td>10.0</td>
<td>2.5 &amp; 4.0</td>
<td>4.5</td>
<td>2.0</td>
<td>DID NOT FLIP (?) (More running time)</td>
</tr>
<tr>
<td>5</td>
<td>20.0</td>
<td>2.5 &amp; 4.0</td>
<td>4.5</td>
<td>2.0</td>
<td>FLIPPED; returned to 0° and stayed in lock</td>
</tr>
<tr>
<td>Only noise at recorder output</td>
<td>-</td>
<td>---</td>
<td>4.5</td>
<td>4.0</td>
<td>DID NOT FLIP</td>
</tr>
</tbody>
</table>

**NOTES:**

1. Reduced sampling rate
2. Loop bandwidth reduced by a factor of 4
3. Rate of change of tone instantaneous frequency also reduced by a factor of 4 \( (a = 3/10 \text{ kHz/s}) \)
9.0 SUMMARY OF THE NASA LABORATORY TEST AND THE AXIOMATIX MODEL SIMULATION

In this section, we summarize some of the pertinent tests made by NASA and compare them to Axiomatix's model simulation described in Table 9.1.

First, in the NASA test, the tape recorder was turned on in channel 2 and the MRD loop flipped with a 15-MHz prelimiter LPF, but did not flip with a 2.5-MHz filter. Then, as a comparison, the Axiomatix model was utilized with a simulated tape-recorder input with a 5-MHz prelimiter LPF and the loop I-Q flipped. With a 2.5-MHz LPF, the MRD loop would not flip, agreeing with the NASA measurement.

Next, noise was introduced by NASA into channel 2 as a possible model of the tape-recorder start-up transient; it was found that, with both 15 MHz and 2.5 MHz, the MRD loop would not I-Q flip. This test was simulated on the Axiomatix simulation and found to agree.

At NASA, it was found that a 2.1-MHz square-wave signal injected into the channel 2 input would cause I-Q inversion when a 15-MHz LPF was used prior to the limiter, but it would not flip at a lower frequency. Axiomatix's model would I-Q flip with a 5-MHz filter with a 3.6-MHz square wave, but not at a lower frequency. Finally, NASA reported that, when the SNR was reduced, the I-Q flipping could be prevented. (Actually, the flip would occur, 90° change, then return back to 0°.) This apparently occurred at about $E_b/N_0 = 5 \text{ dB}$ at NASA whereas, for the Axiomatix model, it occurred at $E_b/N_0 = 2 \text{ dB}$.

10.0 CONCLUSIONS

It is believed that the tape-recorder model developed herein allows us to predict or replicate the actual MRD hardware performance of I-Q flipping quite well, as noted in Table 9.1. Furthermore, we believe that it is the only theory presented which can explain all the known tests made by NASA at Johnson Space Center.

We wish to emphasize that it is our view that the tape-recorder start-up transient I-Q flipping problem is not just a noise-burst problem but, in fact, tones are also generated along with the noise in the tape recorder during start-up or wraparound. The Axiomatix theory also predicts the square-wave I-Q flipping problem which a noise-only theory does not.
Table 9.1. NASA Measurements Compared to the Axiomatix Model/Simulation

<table>
<thead>
<tr>
<th>NASA Test or Axiomatix Model</th>
<th>Test Conditions</th>
<th>$E_b/N_0$ (dB)</th>
<th>LPF Bandwidth (MHz)</th>
<th>Data Width (kbps)</th>
<th>I-Q Flip?</th>
<th>Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASA</td>
<td>Tape recorder turned on in Channel 2</td>
<td>16</td>
<td>15</td>
<td>192</td>
<td>N/I</td>
<td>Yes</td>
</tr>
<tr>
<td>Axiomatix</td>
<td>Tape recorder turned on in Channel 2</td>
<td>16</td>
<td>5</td>
<td>192</td>
<td>N/I</td>
<td>Yes</td>
</tr>
<tr>
<td>NASA</td>
<td>Tape recorder turned on in Channel 2</td>
<td>16</td>
<td>2.5</td>
<td>192</td>
<td>N/I</td>
<td>No</td>
</tr>
<tr>
<td>Axiomatix</td>
<td>Tape recorder turned on in Channel 2</td>
<td>16</td>
<td>2.5</td>
<td>192</td>
<td>N/I</td>
<td>No</td>
</tr>
<tr>
<td>NASA</td>
<td>Lab noise injected into Channel 2</td>
<td>16</td>
<td>15</td>
<td>192</td>
<td>N/I</td>
<td>No</td>
</tr>
<tr>
<td>Axiomatix</td>
<td>Noise injected into Channel 2</td>
<td>16</td>
<td>5</td>
<td>192</td>
<td>N/I</td>
<td>Yes</td>
</tr>
<tr>
<td>NASA</td>
<td>Lab noise injected into Channel 2</td>
<td>16</td>
<td>2.5</td>
<td>192</td>
<td>N/I</td>
<td>No</td>
</tr>
<tr>
<td>Axiomatix</td>
<td>Noise injected into Channel 2</td>
<td>16</td>
<td>2.5</td>
<td>192</td>
<td>N/I</td>
<td>No</td>
</tr>
<tr>
<td>NASA</td>
<td>2.1-MHz square wave injected into Ch 2*</td>
<td>*</td>
<td>15</td>
<td>192</td>
<td>N/I</td>
<td>Yes</td>
</tr>
<tr>
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<td>3.6-MHz square wave injected in Ch 2*</td>
<td>*</td>
<td>5</td>
<td>192</td>
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<td>Yes</td>
</tr>
<tr>
<td>NASA</td>
<td>Lower channel SNR with recorder</td>
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<td>15</td>
<td>192</td>
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<td>192</td>
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<tr>
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<td>15</td>
<td>192</td>
<td>992</td>
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</tr>
<tr>
<td>Axiomatix</td>
<td>Lower channel SNR with recorder</td>
<td>2.0</td>
<td>5</td>
<td>192</td>
<td>960</td>
<td>No**</td>
</tr>
</tbody>
</table>

NOTE: N/I = not important.

*Minimum frequency that causes I-Q flips.

**Flips, then returns to 0°.
REFERENCES


ADDENDUM 2

ANALYSIS OF THE DEPLOYED ELECTRONIC ASSEMBLY
EXCITER LOOP DROP LOCK PROBLEM
DEPLOYED ELECTRONIC ASSEMBLY
EXCITER LOOP DROP LOCK PROBLEM

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Axiomatix Report No. R8410-2
May 15, 1984
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>i</td>
</tr>
<tr>
<td>SUMMARY</td>
<td>ii</td>
</tr>
<tr>
<td>1.0 PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>2.0 POSSIBLE SOURCES OF THE PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>3.0 SOLUTION TO THE LOSS OF LOCK PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>4.0 MEETINGS ATTENDED</td>
<td>4</td>
</tr>
<tr>
<td>5.0 ZERO BEAT DETECTOR FREQUENCIES</td>
<td>4</td>
</tr>
<tr>
<td>6.0 WHAT FREQUENCIES ARE PRESENT AT THE PHASE DETECTOR</td>
<td>8</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1 Exciter Functional Block Diagram .................. 2
Figure 2 Exciter System Model ............................... 3
Figure 3 ZBD Comparator Hysteresis Design Changes ...... 5
Figure 4 ZBD Gain/BW Design Changes ..................... 6
SUMMARY

The Deployed Electronic Assembly phase locked loop exciter drop lock problem was investigated, meetings with HAC were held, and some preliminary analysis was completed. By changing gains and adding hysteresis to the zero beat detector, the drop out and resulting resweep were eliminated. More details are contained within this report.
1.0 PROBLEM

The Deployed Electronic Assembly (DEA) provides clean signals for the transmitter TWTA in each of the five Radar frequency channels, and for the single channel in the COMM mode of operation. In the TWTA bypass mode of Radar operation, the exciter output is used as the transmitter signal. In addition, the exciter provides the first and second LO signals for use in the Receiver mixer downconverters.

The problem that was addressed was the intermittent loss of phase lock in the exciter with the relock requiring an additional 20 µs (which is the time for a single sweep). The occurrence of this loss of phase lock appeared to be random with a mean time between losses of about one hour.

A block diagram model of the exciter is shown in Figure 1. When the exciter loop is locked up, the zero beat detector (ZBD) output occasionally would drop below the threshold initiating the sweep circuit. Figure 2 illustrates a system loop model for the exciter.

2.0 POSSIBLE SOURCES OF THE PROBLEM

The following reasons were suggested as a source of the problem. First, it was suggested that the noise or transients coupled onto the 156 MHz signal could cause the drop out. Secondly, the power supply noise or the transients generated external to the DEA could cause the problem. Phase noise generated internal to the exciter could be the problem source. Also, the power supply noise or transients generated within the DEA, but external to the exciter, could be the cause. Finally, too low of a threshold in the ZBD could be a basis for the problem.

3.0 SOLUTION TO THE LOSS OF LOCK PROBLEM

The proposed solution to prevent the intermittent re-sweeping is as follows:

1. Reduce the zero beat detector gain.
FIGURE 1:

EXCITER FUNCTIONAL BLOCK DIAGRAM
\[ \sum_{k=1}^{N} \alpha_k \sin \left( k \omega_0 t + \frac{\pi}{3} \right) \]

\[ \sqrt{2} \sin \left( \omega_0 t + \frac{\pi}{3} \right) \]

\[ \sqrt{2} \sin \left( 3 \omega_0 t + \frac{\pi}{3} \right) \]

\[ \sqrt{2} \sin \left( 3 \omega_0 t + \frac{4\pi}{3} \right) \]

\[ 2 \sin \left( \omega_1 t + \pi \right) \]

\[ f_0 \text{= 66kHz} \]

\[ \text{LPF} \]

\[ \text{Thres.} \]

\[ \text{Sweep} \]

\[ \text{10kHz Ampl} \]

\[ \text{LPF} \]

\[ \text{DET} \]

\[ \text{Loop Filter} \]

\[ \text{VCO} \]

\[ \text{Harmonic Gen.} \]

\[ \text{BPF} \]

\[ \text{X83} \]

\[ \text{156.285 MHz} \]

\[ \text{4.8MHz BPF} \]

\[ \text{Figure 2: Exciter System Model} \]
2. Increase the zero beat detector post amplifier high pass filter cutoff to 3 MHz.

3. Add hysteresis to the re-sweep comparator.

The proposed changes are illustrated in Figures 3 and 4. These combined changes stopped the occurrence of the occasional drop out and the resulting resweep.

4.0 MEETINGS ATTENDED

Axiomatix attended two meetings at Hughes Aircraft Company in building S-13, on March 16th and March 29th, 1984, to familiarize ourselves with the problem. Lance Mohler discussed the nature of the problem, and some planned tests.

We, also, attended the Ku Band MPR held at the Hacienda Hotel on April 3rd and 4th, 1984. In addition, we were in attendance at the Exciter Design Review held April 24, 1984.

Sections 5.0 and 6.0 are preliminary analyses that were completed.

5.0 ZERO BEAT DETECTOR FREQUENCIES

Consider Figure 2 for the following analysis:

We have,

\[ e_s = \sqrt{2} \sqrt{2} \sin (\omega_1 t + \hat{\theta} + \theta_v) \cdot \sqrt{2} \sin (83\omega_0 t + 83 \theta_0 (t)) \]  

(1)

or neglecting the sum frequency we have,

\[ e_s = \sqrt{2} \cos (83\omega_0 t - \omega_1 t) + \theta^2 \theta_0 - \hat{\theta} - \theta_v \]  

(2)

so that the error signal and the signal to the ZBD is,

\[ \epsilon = \sqrt{2} \cos [(83\omega_0 t - \omega_1 t) + 83 \theta_0 - \hat{\theta} - \theta_v] \cdot \sum_{k=3}^{N} \sqrt{2} \sin \left| \frac{k\omega_0}{3} t + \frac{k\theta_0}{3} \right| \]  

(3)

which becomes,
FIGURE 3:
ZBD COMPARATOR HYSTERESIS DESIGN CHANGES
(THE RESISTOR ADDS HYSTERESIS TO THE THRESHOLD (3 CHANGES))
\[
\epsilon = \sum_{k=1}^{N} \sin \left[ \frac{k \omega_0 t}{3} + 83 \omega_0 t - \omega_1 t + 83 \theta_0 - \hat{\theta} - \theta_v + \frac{k \theta_0}{3} \right]
\]

\[
(A + B) = \sum_{k=1}^{N} \sin \left[ \frac{k \omega_0 t}{3} + \frac{k \theta_0}{3} - 83 \omega_0 t + \omega_1 t - 83 \theta_0 + \hat{\theta} + \theta_v \right]
\]

\[
(A - B) = \sum_{k=1}^{N} \sin \left[ \frac{k \omega_0 t}{3} - 83 \omega_0 + \omega_1 \right] \quad \text{(neglecting } \hat{\theta})
\]

Hence, we can neglect the \((A - B)\), since \(4 \frac{\omega_0}{3} \gg 100 \text{ MHz filter cutoff}\).

We, then have as the error signal,

\[
\epsilon = \sum_{k=1}^{N} \sin \left[ \frac{k \omega_0 t}{3} + 83 \omega_0 t - \omega_1 t + 83 \theta_0 - \theta_v + \frac{k \theta_0}{3} \right]
\]

when lock occurs \((k_1 = 3, \text{ or greater})\) so

\[
\frac{k_1 \omega_0}{3} + 83 \omega_0 - \omega_1 = 0
\]

or

\[
\omega_1 = 83 \omega_0 + \frac{k_1 \omega_0}{3}
\]

Now the signal to the filter detector bank has frequency,

\[
\omega_{\text{DET}} = \omega_1 - 83 \omega_0
\]
hence,

$$\omega_{\text{DET}} = \frac{k_1 \omega_0}{3}$$  \hspace{1cm} (11)

when \( k = 3 \)

$$\omega_{\text{DET}} = \omega_0 = 2\pi \cdot 156 \text{ MHz}$$

\( k = 4 \) \quad $$\omega_{\text{DET}} = \frac{4}{3} \omega_0 = 2\pi \cdot 208 \text{ MHz}$$

\( k = 5 \) \quad $$\omega_{\text{DET}} = \frac{5}{3} \omega_0 = 2\pi \cdot 260 \text{ MHz}$$

\( k = 6 \) \quad $$\omega_{\text{DET}} = \frac{6}{3} \omega_0 = 2\omega_0 = 2\pi \cdot 312 \text{ MHz}$$

\( k = 7 \) \quad $$\omega_{\text{DET}} = \frac{7}{3} \omega_0 = 2\pi \cdot 364 \text{ MHz}$$  \hspace{1cm} (12)

so \( k = 3 \) through 7 are important.

6.0 WHAT FREQUENCIES ARE PRESENT AT THE PHASE DETECTOR?

From 7 we write,

$$\varepsilon = \sum_{k=1}^{N} a_k \sin \left[ \frac{k \omega_0}{3} t + 33 \omega_0 t - \omega_1 t + 33 \theta_0 - \theta - \theta_1 + \frac{k \omega_0}{3} \right]$$  \hspace{1cm} (13)

when \( k_1 = 3 \) (CH 1 lock), we have \( \omega_1 = 84 \omega_0 \)  \hspace{1cm} (14)

so,

$$\varepsilon_1 = \sum_{k=1}^{N} a_k \sin \left[ (\frac{3 \omega_0}{3} + 84 \omega_0) t + 33 \theta_0 - \hat{\theta} - \theta_1 + \frac{k \omega_0}{3} \right]$$  \hspace{1cm} (15)

or,

$$\varepsilon_1 = \sum_{k=1}^{N} a_k \sin \left[ (\frac{k}{3} - 1) \omega_0 t + 33 \theta_0 - \hat{\theta} - \theta_1 + \frac{k \theta_0}{3} \right]$$  \hspace{1cm} (16)
At lock
\[ 8 \theta_0 - \dot{\theta} - \dot{\theta}_0 = 0 \] (17)
so that
\[ \varepsilon_1 = \sum_{k=1}^{N} \frac{a_k}{2} \sin \left[ \left( \frac{k}{3} - 1 \right) \omega_0 t - \theta_0 + \frac{k \theta_0}{3} \right] \] (18)
or
\[ \varepsilon_1 = a_1 \sin \left( - \frac{2}{3} \omega_0 t - \frac{2}{3} \theta_0 \right) + a_2 \sin \left( - \frac{1}{3} \omega_0 t - \frac{1}{3} \theta_0 \right) \] (k=1)
\[ + a_3 \sin \left[ \omega_0 t + \theta_0 \right] + a_5 \sin \left[ \frac{2}{3} \omega_0 t + \frac{2}{3} \theta_0 \right] \] (k=2)
\[ + a_6 \sin \left[ \omega_0 t + \theta_0 \right] + a_7 \sin \left[ \frac{4}{3} \omega_0 t + \theta_0 \right] \] (k=3)

Hence, the following frequencies are present at the output of the phase detector in the channel 1 mode:

\[ f_1 = \frac{2}{3} 156 \text{ MHz} = 104 \text{ MHz} \text{ (two components combine)} \]

\[ f_2 = \frac{1}{3} 156 \text{ MHz} = 52 \text{ MHz} \text{ (two components combine)} \]

\[ f_3 = 156 \text{ MHz} \]

\[ f_4 = \frac{4}{3} 156 \text{ MHz} = 208 \text{ MHz} \]

\[ f_5 = \frac{5}{3} 156 \text{ MHz} = 212 \text{ MHz} \]

\[ f_6 = \frac{6}{3} 156 \text{ MHz} = 312 \text{ MHz} \]

\[ f_7 = \frac{7}{3} 156 \text{ MHz} = 364 \text{ MHz} \] (20)
APPENDIX A

MODEL OF THE HAC SUBCARRIER MODULATOR

By

Jack K. Holmes

May 1984
In this appendix, we discuss the Hughes Aircraft Company (HAC) implementation of the quadriphase modulator. A digital subcarrier oscillator (SCO) running at 8.5 MHz supplies a clock and its complement. Both clocks are passed through a fixed-time delay \( T \) corresponding to a fixed-phase delay \( 2\theta_0 \) in the square-wave fundamental. When not filtered, the result is a constant-envelope four-phase signal, as shown on the I-Q diagram of Figure A.1, which has an I-Q ratio of 4:1. The actual signal wave is filtered so that only the first harmonics are applied to the FM modulator.

In order to understand the power relationships of the I and Q channels, we establish the form of the filtered quadriphase signal following the work of Cager [1] in the case of no noise. The digital subcarrier phase modulator is described by

\[
\begin{align*}
s(t) &= \left\lfloor \sin \left[ \omega_c t + \phi(t) \right] \right\rfloor \\
\phi(t) &= \phi_0 (d_1(t) - d_2(t)) - \pi/2 (d_2(t) - 1)
\end{align*}
\]

where

\[
\phi_0 = \text{value of } \phi \text{ such that the I and Q relationship yields a power ratio of 4:1 in the first harmonic. Expanding (A-1) in a sine-Fourier series yields}
\]

<table>
<thead>
<tr>
<th>( \phi(t) )</th>
<th>( d_1(t) )</th>
<th>( d_2(t) )</th>
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<tbody>
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<td>( \theta )</td>
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<td>1</td>
</tr>
<tr>
<td>( -\theta + \pi )</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>( \theta + \pi )</td>
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<td>-1</td>
</tr>
<tr>
<td>( -\theta )</td>
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Table A.1. Square-Wave Oscillator Phase Relationships
Figure A.1. QPSK Subcarrier Oscillator Modulator for Mode 2 Ku-Band Return Link
where $\sum$ denotes summation on the odd terms. Expanding (A-3) yields
\[ s(t) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin[n(\omega_0 t + \phi(t))] \quad (A-3) \]

Using (A-2) for $g(t)$ yields
\[ \cos[n\phi(t)] = \cos[n(d_1d_2\theta)] \cos[n(d_2-1)\pi/2] \quad (A-5) \]
\[ \sin[n\phi(t)] = \sin[n(d_1d_2\theta)] \sin[n(d_2-1)\pi/2] \quad (A-6) \]

since $\sin[n(d_2-1)\pi/2]$ equals zero. Simplifying (A-5) and (A-6) yields
\[ \cos[n\phi(t)] = d_2 \cos n\theta \quad n \text{ odd} \quad (A-7) \]
\[ \sin[n\phi(t)] = d_1 \sin n\theta \quad n \text{ odd} \quad (A-8) \]

Therefore, the quadriphase signal is of the form
\[ s(t) = d_2(t) \sum_{n=1}^{\infty} \frac{4}{n\pi} \cos(n\theta) \sin(n\omega_0 t) + d_1(t) \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin(n\theta) \cos(n\omega_0 t) \quad (A-9) \]

Let
\[ U(t) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \cos n\theta \sin(n\omega_0 t) \quad (A-10) \]

and
\[ V(t) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin n\theta \cos(n\omega_0 t) \quad (A-11) \]

then
\[ s(t) = d_2(t)U(t) + d_1(t)V(t) \quad (A-12) \]

To identify the waveforms expressed by (A-9), note that
\[ U(t) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \cos(n\theta) \sin(n\omega_0 t) = \frac{1}{2} \int \sin(\omega_0 t-\theta) + \int \sin(\omega_0 t+\theta) \quad (A-13) \]
where \( \sum_{n=1}^{\infty} \sin(\omega_0 t + \phi) \) is a hard-limited sine wave of the argument \( \omega_0 t + \phi \).

Since

\[
\sum_{n=1}^{\infty} \sin(\omega_0 t) + \sum_{n=1}^{\infty} \sin(\omega_0 t + \phi) = \frac{4}{\pi n} \cos(-n\phi) \sin(n\omega_0 t) + \sin(-n\phi) \cos(n\omega_0 t)
+ \frac{4}{\pi n} \cos(n\phi) \sin(n\omega_0 t) + \sin(n\phi) \cos(n\omega_0 t)
\]

In the same manner, it can also be shown that

\[
V(t) = \frac{4}{\pi} \sin \omega_0 \cos(\omega_0 t) = \frac{\sum_{n=1}^{\infty} \sin(\omega_0 t + \phi) - \sum_{n=1}^{\infty} \sin(\omega_0 t - \phi)}{2}
\]

Now the right-hand sides of (A-13) and (A-15) can easily be plotted as shown in Figure A.2. Thus, we see that the digital quadriphase signal can be broken down into two orthogonal basis functions that span the space of periodic functions. Note that the sum of \( U(t) \) and \( V(t) \) yields a square-wave signal that is early (from \( t=0 \)) by \( 2\phi/\omega_0 \) seconds, whereas \( U(t) - V(t) \) generates a square wave with a delay of \( \phi/\omega_0 \) seconds, etc.

Note that the total power in the waveform of (A-9) is given by

\[
P_5 = \frac{1}{2} \sum_{n=1}^{\infty} \frac{16}{n^2 \pi^2} \left( \cos^2(n\phi) + \sin^2(n\phi) \right) = \frac{8}{\phi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}
\]

Furthermore, the power in channel 1 is given by

\[
P_V = P_1 = \frac{1}{2} \sum_{n=1}^{\infty} \frac{16}{n^2 \pi^2} \cos^2(n\phi) = \frac{1}{(2\pi)} \pi/2 \geq \phi \geq 0
\]

and the power in channel 2 is given by

\[
P_U = P_2 = \frac{1}{2} \sum_{n=1}^{\infty} \frac{16}{n^2 \pi^2} \cos^2(n\phi) = 1 - \frac{1}{(2\pi)} \pi/2 \geq \phi \geq 0
\]

Notice that, when \( \phi = 0.464 \) radians (26.58°), the power split is

\[
P_1 = 0.295
\]

and

\[
P_2 = 0.705
\]
Figure A.2. Two Basis Functions for QPSK Channels 1 and 2. Signal...
and $P_2/P_1$ is 2.38:1. However, since the BPF at the output of the MUX (quadriphase modulator) passes only the fundamental, the expressions for the respective channel powers become

$$P_1 = \frac{8}{\pi^2} \sin^2 \theta, \quad (A-21)$$

and

$$P_2 = \frac{8}{\pi^2} \cos^2 \theta \quad (A-22)$$

and, for $\theta = 0.464$, we have that $P_2/P_1$ is 4:1, as desired.

We note from (A-9), using only the first harmonics, that the signal can be expressed as

$$s_f(t) = \frac{4}{\pi}d_1(t) \sin \cos(\omega_0 t) + \frac{4}{\pi}d_2(t) \cos \sin(\omega_0 t) \quad (A-23)$$

where $d_1(t)$ is the output of the channel 1 limiter and $d_2(t)$ is the output of channel 2. Thus, we see that we can model the bandpass process in terms of its baseband processes.

**Reference**

APPENDIX B

LOCK-DETECTOR ANALYSIS

By

Andreas Polydoros

May, 1984
Consider the detector for the unbalanced-QPSK carrier-tracking loop, as shown in Figure B.1. Here $P_i, R_i; i=1,2$ represent the power and the data rate in the weak (1) and strong (2) channels, respectively.

Let the received waveform be

$$r(t) = \sqrt{2P_1} m_1(t) \cos(\omega_0 t + \phi) + \sqrt{2P_2} m_2(t) \sin(\omega_0 t + \phi) + n(t) \quad (B-1)$$

where $m_i(t); i=1,2$ is the baseband, ±1-valued data stream for each channel, $\omega_0 = 2\pi f_0$ is the carrier radian frequency, $\phi$ is the phase to be tracked and $n(t)$ is additive white Gaussian noise with the usual bandpass representation

$$n(t) = \sqrt{2} \left[n_c(t) \cos(\omega_0 t + \phi) + n_s(t) \sin(\omega_0 t + \phi)\right] \quad (B-2)$$

In (B-2), $n_c(t), n_s(t)$ is a pair of baseband, independent, identically distributed Gaussian processes, with zero mean and flat (one-sided) power spectral density (PSD) of $N_0$ W/Hz. Note that the expansion of (B-2) is conveniently done around the loop (estimator) phase $\hat{\phi}$.

Upon baseband conversion (see Figure B.1), the lowpass portions (LP) of the prefiltered waveforms $e_j(t); i=1,2$ (i.e., neglecting double-frequency terms) become

$$e_1(t)_{\text{LP}} = \sqrt{P_1} m_1(t) \cos\phi + \sqrt{P_2} m_2(t) \sin\phi + n_c(t) \quad (B-3a)$$

and

$$e_2(t)_{\text{LP}} = \sqrt{P_2} m_2(t) \sin\phi - \sqrt{P_1} m(t) \cos\phi + n_s(t) \quad (B-3b)$$

where

$$\phi = \hat{\phi} - \theta \quad (B-4)$$

is the loop phase error, assumed to be slowly varying with respect to the data.

After lowpass filtering (LPF) by the 1.8-MHz arm filters, we get

$$e_1(t) = \sqrt{P_1} a_1 m_1(t) \cos\phi + \sqrt{P_2} a_2 m_2(t) \sin\phi + \tilde{n}_c(t) \quad (B-5a)$$

and

$$e_2(t) = \sqrt{P_2} a_2 m_2(t) \cos\phi - \sqrt{P_1} a_1 m_1(t) \sin\phi + \tilde{n}_s(t) \quad (B-5b)$$
Figure B.1. Unbalanced DPSK Loop Lock Detector
where $n_i(t); i=1,2$ represents the filtered noise in each arm and

$$
\alpha_i = \int \left| H_{ARM}(f) \right|^2 S_{m_i}(f) \, df
$$

is the power-reduction coefficient for the $i$th data stream due to filtering (here, $S_{m_i}(f)$ represents the $i$th data stream PSD). Note that the effect of filtering is accounted for by a reduction in power, but not pulse-shape distortion. This is a reasonable assumption since the arm filter bandwidth (1.8 MHz) is approximately twice the data rate of the strong channel ($R_2 = 0.96$ Mb/s), which is Manchester encoded. Thus, the pulse distortion which results from filtering is insignificant.

The waveforms $\xi_i(t)$ then go through an envelope detector, whose output is $|\xi_i(t)|$, prior to subtraction. Since the signal-to-noise ratio (SNR) in each waveform is sufficiently large, the envelope detectors can be approximated by square-law detectors, as shown in Figure 6.2, whose analysis is much more tractable. Such a substitution should have a negligible effect on the overall conclusions.

Within this approximation, the inputs to the subtractor are

$$
\xi_1(t) = \alpha_1 P_1 \cos \phi + \alpha_2 P_2 \sin \phi + \tilde{n}_c(t)
$$

$$
+ 2 \sqrt{P_1} \alpha_1 m_1(t) \tilde{n}_c(t) \cos \phi + 2 \sqrt{P_2} \alpha_2 m_2(t) \tilde{n}_c(t) \sin \phi
$$

$$
+ (1/2) \sqrt{P_1 P_2} \alpha_1 \alpha_2 m_1(t) m_2(t) \sin 2\phi
$$

and

$$
\xi_2(t) = \alpha_2 P_2 \cos \phi + \alpha_1 P_1 \sin \phi + \tilde{n}_s(t)
$$

$$
+ 2 \sqrt{P_2} \alpha_2 m_2(t) \tilde{n}_s(t) \cos \phi - 2 \sqrt{P_1} \alpha_1 m_1(t) \tilde{n}_s(t) \sin \phi
$$

$$
- (1/2) \sqrt{P_1 P_2} \alpha_1 \alpha_2 m_1(t) m_2(t) \sin 2\phi
$$

Each of the above quantities can be divided into a mean value $\mu_i$ and a remainder noise $\eta_i(t)$, i.e.,

$$
\varepsilon_i = \mu_i + \eta_i(t)
$$
Figure B.2. Substituting an Envelope Detector with a Square-Law Detector in the Analysis
where
\[
\mu_1 = a_1 p_1 \cos \phi + a_2 p_2 \sin \phi + \sigma \{ \rho_{c}(t) \}^2 \tag{B-9a}
\]
\[
\mu_2 = a_2 p_2 \cos \phi + a_1 p_1 \sin \phi + \sigma \{ \rho_{a}(t) \}^2 \tag{B-9b}
\]
and
\[
\eta_1(t) = \left[ \rho_{c}(t) - \sigma \{ \rho_{c}(t) \}^2 \right] + 2 \sqrt{f_1} a_1 m_1(t) \eta_{o}(t) \cos \phi \\
+ 2 \sqrt{f_2} a_2 m_2(t) \eta_{o}(t) \sin \phi \\
+ \left( \frac{1}{2} \right) \sqrt{f_1 f_2} a_1 a_2 m_1(t) m_2(t) \sin 2\phi \tag{B-10a}
\]
\[
\eta_2(t) = \left[ \rho_{a}(t) - \sigma \{ \rho_{a}(t) \}^2 \right] + 2 \sqrt{f_2} a_2 m_2(t) \eta_{o}(t) \cos \phi \\
- 2 \sqrt{f_1} a_1 m_1(t) \eta_{o}(t) \sin \phi \\
- \left( \frac{1}{2} \right) \sqrt{f_1 f_2} a_1 a_2 m_1(t) m_2(t) \sin 2\phi \tag{B-10b}
\]

Although the exact statistical nature of \(\eta_1(t)\) and \(\eta_2(t)\) is complicated, certain facts can be assessed, particularly in connection with the bandwidth of the output filter \(H_0(f)\) (see Figure B.1). This is a one-pole RC filter with cut-off frequency \(f_0 = 15\) Hz and transfer function
\[
H_0(f) = \frac{1}{1 + j(f/f_0)} \tag{B-11}
\]

Since the 3-dB bandwidth of 75 Hz is much smaller than that of each noise term in (B-10), \(\eta_1(t)\) is essentially "white" with respect to \(H_0(f)\). Furthermore, by a central-limit argument on the integration effects of \(H_0(f)\), its output \(z(t)\) is approximately Gaussian. Thus, we may model the input noises \(\eta_1(t)\) as zero-mean Gaussian processes, each with a constant PSD equal to \(N_1\) W/Hz, to be evaluated below.

Further, it can easily be shown that each term constituting the noise \(\eta_1(t)\) (and similarly for \(\eta_2(t)\)) is statistically uncorrelated from the others. Except for the very last term in (B-10a) and (B-10b), which is identical, the same is true between any one component of \(\eta_1(t)\) and any other of \(\eta_2(t)\). Since the contribution of the last term is small compared to the rest, we conclude that \(\eta_1(t)\) and \(\eta_2(t)\) are approximately uncorrelated. Furthermore, the total PSD \(N_1\) equals the sum of the PSD's of the individual components.
The final output $z(t)$ is the difference

$$z(t) = z_2(t) - z_1(t)$$  \hspace{1cm} (B-12)

where $z_i(t); i=1,2$ is the output of $H_0$ due to $\epsilon_i^2(t)$. Within the Gaussian assumption, performance is uniquely determined by the mean and variance of $z(t)$.

- **Mean of $z(t)$**

  We have that

  $$\mathcal{E}\{z(t)\} = \mathcal{E}\{z_2(t)\} - \mathcal{E}\{z_1(t)\} = H_0(0)\left[\mathcal{E}\{\epsilon_2^2(t)\} - \mathcal{E}\{\epsilon_1^2(t)\}\right]$$

  or

  $$\mathcal{E}\{z(t)\} = \mu_z = \mu_2 - \mu_1$$  \hspace{1cm} (B-13)

  since $H_0(0) = 1$ (see (B-11)). But

  $$\mathcal{E}\{n_0(t)\} = \mathcal{E}\{n_3(t)\} = \left(\frac{N_0}{2}\right) \int_{-\infty}^{\infty} |H_{ARM}(f)|^2 \, df$$  \hspace{1cm} (B-14)

  Therefore, from (B-9), (B-13) and (B-14), we have that

  $$\mu_z = \left(2a_2^2 - 2a_1^2\right) \cos^2 \phi + \left(2a_1^2 - 2a_2^2\right) \sin^2 \phi$$  \hspace{1cm} (B-15)

- **Variance of $z(t)$**

  From (B-8) and (B-12), it follows that

  $$\text{var}\{z(t)\} \triangleq \sigma_z^2 = \int_{\eta_2-\eta_1}^{\infty} |H_0(f)|^2 S(f) \, df$$  \hspace{1cm} (B-16)

  where $S(f)$ is the PSD of the difference $n_2(t) - n_1(t)$. However, due to the wideband nature of this difference (see previous discussion), it follows that

  $$\sigma_z^2 = S(0) \int_{\eta_2-\eta_1}^{\infty} |H_0(f)|^2 \, df$$  \hspace{1cm} (B-17)
From the functional expression of $H_0(f)$ in (B-11), it is easily shown that (B-17) reduces to

$$
\sigma_z^2 = S(0) \cdot \left( \pi f_0 \right) \quad \text{(B-18)}
$$

In order to derive the value of $S(f)$ at the original $f = 0$, we write the difference $n_2(t) - n_1(t)$ as

$$
n_2(t) - n_1(t) = n_1(t) \cdot 2 \cos \phi \cdot n_2(t) - 2 \sin \phi \cdot n_3(t) - \sin 2\phi \cdot n_4(t) \quad \text{(B-19)}
$$

where the (uncorrelated) noises $n_i(t); i=1,2,3,4$ are given by

$$
\begin{align*}
n_1(t) &= n_0^2(t) - \mathcal{E}\left\{\tilde{n}_0(t)\right\} + \mathcal{E}\left\{\tilde{n}_0(t)\right\} \\
n_2(t) &= \alpha_2 \sqrt{P_2} \cdot m_2(t) \cdot \tilde{n}_3(t) - \alpha_1 \sqrt{P_1} \cdot m_1(t) \cdot \tilde{n}_3(t) \\
n_3(t) &= \alpha_1 \sqrt{P_1} \cdot m_1(t) \cdot \tilde{n}_3(t) + \alpha_2 \sqrt{P_2} \cdot m_2(t) \cdot \tilde{n}_3(t) \\
n_4(t) &= \alpha_1 \alpha_2 \sqrt{P_1 P_2} \cdot m_1(t) \cdot m_2(t)
\end{align*}
$$

Then, if we denote by $S_1(0)$ the PSD at $f=0$ of $n_1(t)$, we conclude from (B-18) and (B-19) that

$$
\sigma_z^2 = \left( \pi f_0 \right) \left[ S_1(0) + 4 \cos^2 \phi \cdot S_2(0) + 4 \sin^2 \phi \cdot S_3(0) + \sin^2 2\phi \cdot S_4(0) \right] \quad \text{(B-21)}
$$

However, by inspecting the noises $n_2(t)$ and $n_3(t)$ in (B-20a) and (B-20c) and realizing the existing symmetry (substitute $n_0$ for $n_3$ and disregard the sign change, as it does not affect the PSD), we immediately conclude that

$$
S_2(f) = S_3(f) \quad \text{(B-22)}
$$

in which case, (B-21) simplifies to

$$
\sigma_z^2 = \left( \pi f_0 \right) \left[ S_1(0) + 4 \cdot S_2(0) + \sin^2 2\phi \cdot S_4(0) \right] \quad \text{(B-23)}
$$

since $\cos^2 \phi + \sin^2 \phi = 1$. The individual forms in (B-23) are now evaluated.

Let $R_2(\tau)$ and $R_3(\tau)$ indicate the autocorrelation functions of the squared filtered noises $\tilde{n}_3^2(t)$ and $\tilde{n}_0^2(t)$, while $R_m(\tau)$ and $R_s(\tau)$ correspond to $\tilde{n}_3(t)$ and $\tilde{n}_0(t)$ (not squared). Because of the statistical symmetry between $n_3(t)$ and $n_0(t)$

$$
R_3(\tau) = R_0(\tau); R_s(\tau) = R_s(\tau) \quad \text{(B-24)}
$$
But, from (B-20a),

\[
S_1(0) = \int_{-\infty}^{\infty} \left( R_2(t) + R_2(t) \right) dt = 2 \int_{-\infty}^{\infty} R_2(t) dt \quad \text{(B-25)}
\]

where

\[
R_2^2(t) = \mathcal{E}\left\{ \bar{\mathcal{S}}_2(t) - \mathcal{E}\left\{ \bar{\mathcal{S}}_2(t+\tau) \right\} \right\} \mathcal{E}\left\{ \bar{\mathcal{S}}_2(t) - \mathcal{E}\left\{ \bar{\mathcal{S}}_2(t) \right\} \right\}
\]

\[
= \mathcal{E}\left\{ \bar{\mathcal{S}}_2(t+\tau) \bar{\mathcal{S}}_2(t) \right\} - \mathcal{E}\left\{ \bar{\mathcal{S}}_2^2(t) \right\}
\]

\[
= \mathcal{E}\left\{ \bar{\mathcal{S}}_2(t) \right\} + 2 \mathcal{E}\left\{ \bar{\mathcal{S}}_2(t+\tau) \bar{\mathcal{S}}_2(t) \right\} - \mathcal{E}\left\{ \bar{\mathcal{S}}_2^2(t) \right\}
\]

\[
= 2 R_2^2(t) \quad \text{(B-26)}
\]

In deriving (B-26), the familiar identity

\[
\mathcal{E}\left\{ \prod_{i=1}^{4} X_i \right\} = \mathcal{E}\{X_1X_2\} \mathcal{E}\{X_3X_4\} + \mathcal{E}\{X_1X_3\} \mathcal{E}\{X_2X_4\} + \mathcal{E}\{X_1X_4\} \mathcal{E}\{X_2X_3\}
\]

for jointly Gaussian random variables has been used. Thus, from (B-25) and (B-26) and by using Parseval's theorem,

\[
S_1(0) = 4 \int_{-\infty}^{\infty} R_2^2(t) dt = 4 \int_{-\infty}^{\infty} \left| \mathcal{S}_{\mathcal{S}}(f) \right|^2 df = 4 \int_{-\infty}^{\infty} \left( \frac{N_0}{2} \right)^2 \left| \mathcal{H}_{\text{ARM}}(f) \right|^4 df
\]

or

\[
S_1(0) = N_0^2 \int_{-\infty}^{\infty} \left| \mathcal{H}_{\text{ARM}}(f) \right|^4 df \quad \text{(B-27)}
\]

Proceeding in a similar manner, we get

\[
S_2(0) = a_2 P_2 \int_{-\infty}^{\infty} R_{m_2}(f) R_{c}(f) df + a_1 P_1 \int_{-\infty}^{\infty} R_{m_1}(f) R_{c}(f) df
\]

\[
= a_2 P_2 \left( \frac{N_0}{2} \right) \int_{-\infty}^{\infty} S_{m_2}(f) \left| \mathcal{H}_{\text{ARM}}(f) \right|^2 df
\]

\[
+ a_1 P_1 \left( \frac{N_0}{2} \right) \int_{-\infty}^{\infty} S_{m_1}(f) \left| \mathcal{H}_{\text{ARM}}(f) \right|^2 df
\]

or

\[
S_2(0) = \left( \frac{N_0}{2} \right) \left[ a_2 P_2 + a_1 P_1 \right] \quad \text{(B-28)}
\]
where the definition for $a_j^2$ in (B-6) has been used. Finally,

$$S_q(0) = a_1 a_2 P_1 P_2 \int_{-\infty}^{\infty} S_{m_1}(f) S_{m_2}(f) \, df$$  \hspace{1cm} (B-29)

To recapitulate our findings, let us define

$$C_1 \triangleq a_1^2 = \int_{-\infty}^{\infty} S_{m_1}(f) |H_{\text{ARM}}(f)|^2 \, df$$ \hspace{1cm} (B-30a)

$$C_2 \triangleq c_c^2 = \int_{-\infty}^{\infty} S_{m_2}(f) |H_{\text{ARM}}(f)|^2 \, df$$ \hspace{1cm} (P-30b)

$$C_3 \triangleq \int_{-\infty}^{\infty} |H_{\text{ARM}}(f)|^4 \, df$$ \hspace{1cm} (B-30c)

$$C_4 \triangleq \int_{-\infty}^{\infty} S_{m_1}(f) S_{m_2}(f) \, df$$ \hspace{1cm} (B-30d)

Then, from (B-15) and (B-30),

$$\mu_2 = (C_2 P_2 - C_1 P_1) \left( \cos^2 \phi - \sin^2 \phi \right)$$

or

$$\mu_2 = (C_2 P_2 - C_1 P_1) \cos 2\phi$$ \hspace{1cm} (B-31)

and, from (B-23) and (B-27) through (B-30),

$$\text{var}(z) = (nf_0) \cdot N_0 \left[ C_3 + 2C_1 \left( \frac{P_1}{N_0} \right) + 2C_2 \left( \frac{P_2}{N_0} \right) + \sin^2 2\phi \cdot C_1 C_2 C_H \left( \frac{P_1}{N_0} \frac{P_2}{N_0} \right) \right]$$ \hspace{1cm} (B-32)

Note from (B-32) that the last term vanishes for $\phi = 0$ (locked-on condition) or $\phi = \pi/2$ (flip condition).

The output signal-to-noise ratio, $\text{SNR}_0$, is defined as

$$\text{SNR}_0 = \frac{\mu_2^2}{\text{var}(z)}$$ \hspace{1cm} (B-33)
From (B-31) through (B-33), we obtain

$$SNR_0 = \left( \frac{1}{\pi f_0} \right) \cos^2 2\phi \cdot \frac{\left[ C_2 \left( \frac{P_2}{N_0} \right) - C_1 \left( \frac{P_1}{N_0} \right) \right]^2}{C_3 + 2C_1 \left( \frac{P_1}{N_0} \right) + 2C_2 \left( \frac{P_2}{N_0} \right) + \sin^2 2\phi \ C_1 C_2 C_4 \left( \frac{P_1}{N_0} \right) \left( \frac{P_2}{N_0} \right)}$$

which, for $\phi = 0$ (correct lock) or $\phi = \pi/2$ (flip), simplifies to

$$SNR_0 = \left( \frac{1}{\pi f_0} \right) \frac{\left[ C_2 \left( \frac{P_2}{N_0} \right) - C_1 \left( \frac{P_1}{N_0} \right) \right]^2}{C_3 + 2C_1 \left( \frac{P_1}{N_0} \right) + 2C_2 \left( \frac{P_2}{N_0} \right)} \text{ (B-34)}$$

It should be pointed out that $\mu_2$ changes sign (see (B-31)) when $\phi$ goes from 0 to $\pi/2$ rad, provided that

$$\frac{P_2}{P_1} > \frac{C_1}{C_2}$$

This latter condition is satisfied in the UQPSK design since

$$\frac{P_2}{P_1} = 4 \quad \text{and} \quad \frac{C_1}{C_2} = 1$$

Therefore, the mean of the Gaussian distribution for $z(t)$ is positive in the correct lock condition and negative in the flip condition, as it should be (see threshold comparison in Figure B.1).

Let $(1 - P_e)$ indicate the probability that a sample of $z(t)$ at any $t = t_0$ will be positive, assuming the correct lock condition. In other words, $(1 - P_e)$ is the probability of correct indication conditioned on the correct lock. Due to the symmetry of expressions (B-31) and (B-32), $(1 - P_e)$ is also equal to the probability of a correct (negative) indication, given a flip. Thus, in both circumstances, $P_e$ stands for the probability of an erroneous indication. Because of the Gaussian assumption,
where
\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-z^2/2} dz \tag{B-36}
\]

Now suppose that \( N \) samples are taken and each is \( T_s \) seconds apart. We assume that \( T_s \) is sufficiently large (few time constants of \( k_0(f) \), i.e., 2 - 3 multiples of 1/15 s) to render the successive samples statistically independent. Then the probability that all \( N \) samples indicate correctly is

\[
Pr[\text{correct indication in } NT_s \text{ seconds}] = 1 - Q\left[\sqrt{\text{SNR}_0}\right]^N = 1 - NQ\left[\sqrt{\text{SNR}_0}\right] \tag{B-37}
\]

where it is assumed that \( NPe \ll 1 \) (justified numerically below). Thus,

\[
Pr[\text{false indication in } NT_s \text{ seconds}] = NQ\left[\sqrt{\text{SNR}_0}\right] \tag{B-38}
\]

For this latter probability to be considered appreciable, we set

\[
NQ\left[\sqrt{\text{SNR}_0}\right] = 0.1 \tag{B-39}
\]

Therefore, the number of samples \( N \) (or, equivalently, the length of time \( NT_s \)) required to reach such a level of probability can be specified from (B-39) once \( \text{SNR}_0 \) is known for a particular system. Equation (B-24) can be rewritten in terms of the following parameters:

\[
\frac{E_b}{N_0} = \text{bit SNR in channel } 2 = \frac{P_2 T_2}{N_0} = \frac{P_2}{N_0 R_2} \tag{B-40a}
\]

\[
\rho = \frac{P_2}{P_1} \quad \text{power ratio} \tag{B-40b}
\]

\[
\frac{f_0}{R_2} \quad \text{c} \tag{B-40c}
\]

\[
\frac{f_{\text{ARM,C}}}{R_2} \quad \text{c} \tag{B-40d}
\]
where we assume that the arm filters are identical, one-pole, with a 3-dB cut-off frequency of $f_{\text{ARM},C}$ Hz. Then,

$$SNR_0 = \frac{\left( \frac{E_{b_2}}{N_0} \right)^2 \left( \frac{C_2}{C_1} - \frac{1}{\rho} \right)^2}{\left( \frac{E_{b_2}}{N_0} \right)^2 \left( \frac{C_2}{C_1} + \frac{1}{\rho} \right)^2 + \left( \frac{\pi}{2} \right) \delta_{\text{ARM}}^2}$$  \quad (B-41)

For the particular system at hand, $f_c = 15$ Hz, $f_{\text{ARM},C} = 1.8$ MHz, $R_1 = 192$ Mb/s and $R_2 = 0.96$ Mb/s so that $\rho = 4$, $C_1 = 1$, $C_2 = 0.75$ (from tables), $\delta_c = 15.625 \times 10^{-6}$ and $\delta_{\text{ARM}} = 1.88$.

Table B.1 provides the values of SNR$_0$ from (A-41), where the above numbers have been used. The parameter $\left( \frac{E_{b_2}}{N_0} \right)$ was ranged from 2 dB to 10 dB for both $\rho = 4$ and $\rho = 2$. It is clear that, in all cases, SNR$_0$ is extremely high, thus assuring correct operation of the lock detector for practically an indefinite interval (hundreds of years!).

In conclusion, this analysis has shown the lock detector herein to be a most reliable component in both the correct-lock and the phase-flip conditions.

**Table B.1. Values of SNR$_0$ as a Function of $E_{b_2}/N_0$, For Two Values of the Power Ratio $\rho$**

<table>
<thead>
<tr>
<th>$E_{b_2}/N_0$ (dB)</th>
<th>10</th>
<th>8</th>
<th>6</th>
<th>4</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{SNR}_0$ (dB)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho = 4$</td>
<td>44.2</td>
<td>4.9</td>
<td>39.3</td>
<td>36.6</td>
<td>33.6</td>
</tr>
<tr>
<td>$\rho = 2$</td>
<td>37.2</td>
<td>34.9</td>
<td>32.4</td>
<td>29.8</td>
<td>27.0</td>
</tr>
</tbody>
</table>
APPENDIX C

S-CURVE OF A MODIFIED FOUR-PHASE COSTAS LOOP
FOR A PARTICULAR QUADRATURE SIGNAL INPUT

By
Raimundo Sampaio
In this appendix, we derive the tracking characteristic (S-curve) for a modified four-phase Costas loop when the input signal is of the following form: the inphase component is biphase modulated and the quadrature component is sine-wave modulated. The presence of thermal noise is not considered in this analysis.

The received signal \( s(t) \) is of the form
\[
s(t) = \sqrt{2} A_1 \, m_1(t) \cos(\omega_0 t + \phi) + \sqrt{2} A_2 \, m_2(t) \sin(\omega_0 t + \phi) \tag{C-1}
\]
where \( m_1(t) \) is a (\( \pm \)) binary data modulation,
\[
m_2(t) = \sin(\omega_a t + \zeta)
\]
is a sinusoidal modulation with radian frequency \( \omega_a \) and arbitrary phase parameter \( \zeta \), and \( \omega_0 \) and \( \phi \) are the radian frequency and phase of the inphase and quadrature components of \( s(t) \).

According to Figure C.1 herein, the received signal is cross correlated simultaneously with the inphase and quadrature versions of the VCO output reference signal
\[
r(t) = \sqrt{2} \cos(\omega_0 t + \phi)
\]
where \( \hat{\phi}(t) \) is the instantaneous loop estimate of the unknown phase \( \phi \).

Assuming that \( \theta \) and \( \phi \) are slow-varying quantities and that the lowpass arm filters reject double-frequency components, their outputs \( \epsilon_1(t) \) and \( \epsilon_2(t) \) can be written as
\[
\epsilon_1(t) = A_1 \, m_{1F}(t) \cos(\phi - \hat{\phi}) + A_2 \, m_{2F}(t) \sin(\phi - \hat{\phi}) \tag{C-2a}
\]
and
\[
\epsilon_2(t) = A_2 \, m_{2F}(t) \cos(\phi - \hat{\phi}) - A_1 \, m_{1F}(t) \sin(\phi - \hat{\phi}) \tag{C-2b}
\]
where \( m_{1F} \) and \( m_{2F} \) are the filtered versions of \( m_1(t) \) and \( m_2(t) \), respectively. If we consider as negligible the data distortion introduced by arm filtering of \( m_1(t) \), we have
\[
m_{1F}(t) = \sqrt{a_1} \, m_1(t) \quad ; \quad m_{2F}(t) = \sqrt{a_2} \, \sin(\omega_a t + \phi_a)
\]
where
\[
a_1 = \int_{-\infty}^{+\infty} \left| L(f) \right|^2 S_m(f) \, df \quad \text{(assume } \left| L(0) \right| = 1) \tag{C-3}
\]
and
\[
a_2 = \left| L(f_0) \right|^2 \tag{C-4}
\]
Figure C.1: Block Diagram of a Modified Four-Phase Costas Loop
are the power losses introduced by an arm filter with frequency characteristic \( L(f) \).

If we let \( \phi = \theta - \hat{\theta} \) represent the loop phase-tracking error, then (C-2) can be rewritten as

\[
\epsilon_1(t) = A_1 \sqrt{\alpha_1} m_1(t) \left[ c_\alpha + \beta \frac{m_2(t)}{m_1(t)} \sin \phi \right]
\]

(C-5a)

and

\[
\epsilon_2(t) = A_1 \sqrt{\alpha_1} m_1(t) \left[ \beta \frac{m_2(t)}{m_1(t)} \cos \phi - \sin \phi \right]
\]

(C-5b)

where

\[
\beta = \frac{A_2 \sqrt{\alpha_2}}{A_1 \sqrt{\alpha_1}}
\]

(C-6)

With \( \epsilon_1(t) \) and \( \epsilon_2(t) \) as in (C-5a) and (C-5b), the control signal

\[
z(t) = A_v \epsilon_1(t) \text{sign}[\epsilon_2(t)] - \epsilon_2(t) \text{sign}[\epsilon_1(t)]
\]

assumes the form

\[
z(t, \phi) = A_1 \alpha_1 \left\{ A_v \left( \cos \phi + \beta \frac{m_2(t)}{m_1(t)} \sin \phi \right) \text{sign} \left[ \beta \frac{m_2(t)}{m_1(t)} \cos \phi - \sin \phi \right] 

- \left( \beta \frac{m_2(t)}{m_1(t)} \cos \phi - \sin \phi \right) \text{sign} \left[ \cos \phi + \beta \frac{m_2(t)}{m_1(t)} \sin \phi \right] \right\}
\]

(C-7)

The feedback signal \( z(t, \phi) \) has random components and periodic oscillations. The decomposition

\[
z(t, \phi) = \langle z(t, x) \rangle + \langle z(t, x) - \langle z(t, x) \rangle \rangle \bigg|_{x=\phi}
\]

(\( \text{where the overbar denotes statistical average and } \langle \rangle \text{ denotes time average} \)) allows us to write \( z(t, \phi) \) as a nonrandom, nontime-varying function of the tracking error \( \phi \) (control voltage) plus a random component that acts as "modulation self noise." The S-curve of the system is therefore defined as the function \( s(\cdot) \) given by

\[
s(x) = \langle z(t, x) \rangle
\]
Averaging $z(t,x)$ with respect to the random $(x)$ data signal $m_1(t)$, we obtain

$$\frac{z(t,x)}{A_1^2 a_1} = \frac{A_y}{2} \cdot (\cos x \cdot \beta m_2(t) \sin x) \operatorname{sign} [\beta m_2(t) \cos x - \sin x]$$

$$+ \frac{A_y}{2} \cdot (\cos x \cdot \beta m_2(t) \sin x) \operatorname{sign} [-\beta m_2(t) \cos x - \sin x]$$

$$- \frac{1}{2} \cdot (\beta m_2(t) \cos x - \sin x) \operatorname{sign} [\cos x + \beta m_2(t) \sin x]$$

$$- \frac{1}{2} \cdot (-\beta m_2(t) \cos x - \sin x) \operatorname{sign} [\cos x - \beta m_2(t) \sin x] \quad (C-8)$$

The above function is periodic in $t$ with the same period of $m_2(t)$. We also note that the particular value of the phase parameter $\theta_8$ of $m_2(t)$ is irrelevant for the computation of time average and can be taken as $\theta_8 = 0$; that is,

$$m_2(t) = \sin(\omega_3 t)$$

If we now define the periodic functions

$$f(t,x) = (\cos x \cdot \beta \sin(\omega_3 t) \sin x) \operatorname{sign} [\beta \sin(\omega_3 t) \cos x - \sin x] \quad (C-9a)$$

and

$$g(t,x) = (\beta \sin(\omega_3 t) \cos x - \sin x) \operatorname{sign} [\cos x + \beta \sin(\omega_3 t) \sin x] \quad (C-9b)$$

then (C-8) can be expressed as

$$\frac{z(t,x)}{A_1^2 a_1} = \frac{A_y}{2} [f(t,x) + f(-t,x)] - \frac{1}{2} [g(t,x) + g(-t,x)] \quad (C-10)$$

The time average of (C-10) therefore reduces to

$$\frac{s(x)}{A_1^2 a_1} = A_y \langle f(t,x) \rangle - \langle g(t,x) \rangle$$

or, by noting that $f(t,x + \pi/2) = -g(t,x)$, to

$$\frac{s(x)}{A_1^2 a_1} = A_y G(x) + G(x + \pi/2) \quad (C-11)$$

where

$$G(x) \triangleq \langle f(t,x) \rangle = \frac{1}{T_S} \int_0^{T_S} f(t,x) \, dt \quad (C-12)$$

with $T_S$ being the period of the sinusoid $m_2(t)$. 
Evaluation of $G(x)$

For a given $x$, $f(t,x)$ can be written from (C-9a) as

$$f(t,x) = \begin{cases} 
K(t,x) = \cos x + \beta \sin \left( \frac{2\pi}{T_3} t \right) \sin x; t \in \left( \frac{2\pi}{T_3} x \right) \cos x \geq \sin x \\
- K(t,x); \quad \text{otherwise}
\end{cases} \quad (C-13)$$

In evaluating $G(x)$, it is convenient to consider the following cases:

1. $0 \leq x < \pi/2$; $G(x) \cap G_1(x)$ ($\cos x > 0$, $\sin x \geq 0$)

$$f(t,x) = \begin{cases} 
K(t,x); \quad \text{teI} = \left\{ t; \sin \left( \frac{2\pi}{T_3} t \right) \geq \frac{\tan x}{\beta} \right\} \\
- K(t,x); \quad \text{otherwise}
\end{cases}$$

Figure C.2 illustrates set I. Now,

$$G_1(x) = \frac{1}{T_3} \int_0^{T_2} f(t,x) dt = \frac{1}{T_3} \left[ \int_0^{T_3} (-K(t,x)) dt - \int_I (-K(t,x)) dt + \int_I K(t,x) dt \right]$$

$$= \frac{1}{T_3} \left[ 2 \int_{T_1}^{T_2} K(t,x) dt - \int_0^{T_3} K(t,x) dt \right] \quad (C-15)$$

Since

$$\int K(t,x) dt = t \cos x + \frac{T_3}{2\pi} \beta \cos \left( \frac{2\pi}{T_3} t \right) \sin x$$

then (C-15) reduces to

$$G_1(x) = \begin{cases} 
\frac{2(T_2-T_1)}{T_3} \cos x - \sin x \left[ \cos \left( \frac{2\pi}{T_3} T_2 \right) - \cos \left( \frac{2\pi}{T_3} T_1 \right) \right] \cos x; 0 \leq \frac{\tan x}{\beta} \leq 1 \\
- \cos x; \quad 1 \leq \frac{\tan x}{\beta}
\end{cases} \quad (C-16)$$
\[ \sin \left( \frac{2\pi}{T_S} T_1 \right) = \sin \left( \frac{2\pi}{T_S} T_2 \right) = \frac{\tan \beta}{\beta} \]

\[ \cos \left( \frac{2\pi}{T_S} T_2 \right) = -\cos \left( \frac{2\pi}{T_S} T_1 \right) = -\sqrt{1 - \left( \frac{\tan \beta}{\beta} \right)^2} \]

Figure C.2. Interval I and the Relations Between $T_1$, $T_2$ and $\frac{\tan \beta}{\beta}$
The quantity between brackets in (C-16) is given by (see (C-14) in Figure C.2)

\[
\cos\left(\frac{2\pi}{T_s} T_2\right) - \cos\left(\frac{2\pi}{T_s} T_1\right) = -2 \cos\left(\frac{2\pi}{T_s} T_1\right) = -2 \sqrt{1 - \left(\frac{\tan \beta}{\beta}\right)^2} \quad (C-17)
\]

Now, since \(\frac{T_2 + T_1}{2} = T_s/4\), then

\[
\frac{T_2 - T_1}{2} = T_2 - \frac{T_s}{4}
\]

Therefore,

\[
\cos\left(\frac{2\pi}{T_s} \left(\frac{T_2 - T_1}{2}\right)\right) = \cos\left(\frac{2\pi}{T_s} \frac{T_2 - \pi}{2}\right) = \sin\left(\frac{2\pi}{T_s} T_2\right) = \frac{\tan \beta}{\beta} = \frac{|\tan(x)|}{\beta} \quad (\tan \beta \geq 0)
\]

which yields

\[
\frac{T_2 - T_1}{T_s} = \frac{1}{\pi} \cos^{-1} \left(\frac{|\tan(x)|}{\beta}\right) \quad (C-18)
\]

Inserting (C-18) and (C-17) into (C-16) we obtain

\[
G_1(x) = \begin{cases} 
-\cos(x) + \frac{2}{\pi} \left[ \cos(x) \cos^{-1} \left(\frac{|\tan(x)|}{\beta}\right) + \beta \sqrt{1 - \left(\frac{\tan(x)}{\beta}\right)^2} \sin x \right] ; & 0 \leq \frac{\tan(x)}{\beta} \leq 1 \\
-\cos(x) ; & \frac{\tan(x)}{\beta} \geq 1
\end{cases} \quad (C-19)
\]

\(2\) \quad \frac{\pi}{2} \leq x < \pi \; ; \; G(x) \geq G_2(x) \quad (\tan x \leq 0, \sin x > 0)

\[
f(t,x) = \begin{cases} 
K(t,x) ; \; t \in I = \left\{ t : \sin \left(\frac{2\pi}{T_s} t\right) \leq \frac{\tan(x)}{\beta} \right\} \\
-K(t,x) ; \text{ otherwise}
\end{cases}
\]

The expression for \(G_2(x)\) can be written in a form similar to (C-15) as*

\[
G_2(x) = \frac{1}{T_s} \left[ \int_{T_3}^{T_4} K(t,x) dt - \int_{0}^{T_3} K(t,x) dt \right]
\]

*Also refer to Figure C.3.
\[
\sin\left(\frac{2\pi}{T_s} T_4\right) = \sin\left(\frac{2\pi}{T_s} T_3\right) = \frac{\tan x}{\beta}.
\]

\[
\cos\left(\frac{2\pi}{T_s} T_4\right) = -\cos\left(\frac{2\pi}{T_s} T_3\right) = \sqrt{1 - \left(\frac{\tan x}{\beta}\right)^2}
\]

Figure C.3. Interval I and the Relation Between $T_3$, $T_4$ and $\frac{\tan x}{\beta}$
yielding
\[
G_2(x) = \begin{cases} 
\frac{2}{\beta} (T_h-T_3) \cos x - \frac{\beta}{\pi} \sin x & \cos \left(\frac{2\pi}{T_h} \right) - \cos \left(\frac{2\pi}{T_3} \right) - \cos x; -1 \leq \frac{\tau x}{\beta} \leq 0 \\
-\cos x ; \frac{\tau x}{\beta} \leq -1 
\end{cases}
\]
(C-21)

Using the fact that \((T_h+T_4)/2 = 3T_s/4\) and working in a way similar to that for \(G_1(x)\), we obtain

\[
\cos \left(\frac{2\pi}{T_s} \left(\frac{T_h-T_4}{2}\right) \right) = -\frac{\tau x}{\beta} \left| \frac{\tau x}{\beta} \right| \text{ (tg} x, 0) \]

which yields

\[
\frac{T_h-T_3}{T_s} = -\cos^{-1} \left(\frac{\tau x}{\beta} \right) \]
(C-22)

Combining (C-20), (C-21) and (C-22) results in

\[
G_2(x) = \begin{cases} 
-\cos x + \frac{2}{\beta} \left[ \cos x \cos^{-1} \left(\frac{\tau x}{\beta} \right) - \beta \sqrt{1 - \left(\frac{\tau x}{\beta} \right)^2} \sin x \right] ; -1 \leq \frac{\tau x}{\beta} \leq 0 \\
-\cos x ; \frac{\tau x}{\beta} \leq -1 
\end{cases}
\]
(C-23)

By developments similar to those performed for \(G_1(x)\) and \(G_2(x)\), we find that

(3) \( \pi < x < 3\pi/2 \) \( ; \ G(x) = G_3(x) = -G_1(x) \)
(C-24)

and that

(4) \( 3\pi/2 < x < 2\pi \) \( ; \ G(x) = G_4(x) = -G_2(x) \)
(C-25)

By using signs of \(\cos(x)\) and \(\sin(x)\) in different quadrants, expressions (C-19) and (C-23)-(C-25) can be expressed in a unified form by
\[
G(x) = - \text{sign} [\sin(x)] \cos(x) + c(x) \frac{2}{\pi} \left\{ \text{sign} [\sin(x)] \cos^{-1} \left( \frac{\text{tg}(x)}{\beta} \right) \cos(x) + \text{sign} [\cos(x)] \sqrt{1 - \left( \frac{\text{tg}(x)}{\beta} \right)^2} \sin(x) \right\}
\]

where

\[
c(x) = \begin{cases} 
1 & ; \quad |\sin(x)| \leq \beta |\cos(x)| \\
0 & ; \quad \text{otherwise}
\end{cases}
\]

The loop's S-curve, given by (C-11), is therefore

\[
s(x) = A_1 \alpha_1 \left[ A_v G(x) + G \left( x + \frac{\pi}{2} \right) \right]
\]

with \(G(x)\) given by (C-26), \(\beta\) defined through

\[
\beta = \frac{A_2}{A_1} \sqrt{\frac{\alpha_2}{\alpha_1}}
\]

and where \(\alpha_1\) and \(\alpha_2[f_3]\) are given by (see (C-3) and (C-4))

\[
\alpha_1 = \int_{-\infty}^{+\infty} |L(f)|^2 S_{m_1}(f) \, df
\]

and

\[
\alpha_2 = |L[f_3]|^2, \quad (f_3 = \text{sinusoid frequency in Hz})
\]

Note that the ratio \(A_2/A_1\) in (C-27b) denotes the ratio of the amplitude of the sinusoidal modulation in channel 2 to that of the data modulation in channel 1 at the input of the tracking device.
Slope of the S-Curve at the Origin (x=0)

Let \( s'(0) \) denote the derivative of \( s(x) \) at \( x=0 \); then, from (C-27a), we have

\[
\frac{s'(0)}{A_1^2 a_1} = A_V \cdot \frac{G'(0) + G'(\frac{\pi}{2})}{2}
\]

The quantities \( G'(0) \) and \( G'(\pi/2) \) can be shown to be given by

\[
G'(0) = \frac{2}{\pi}(1/\beta)(\beta^2 - 1)
\]

and

\[
G'(\pi/2) = 1,
\]

which, when inserted into (C-28), yield

\[
\frac{s'(0)}{A_1^2 a_1} = \frac{2 A_V}{\pi \beta} \left( \beta^2 - 1 \right) + 1
\]

Figure C.4 illustrates the dependence of \( s'(0) \) on the parameter \( \beta \). Note that \( s'(0) \) becomes negative if \( \beta \) is less than a threshold value \( \beta^* \). In this range, the loop will no longer have a stable point at \( \phi=0^\circ \). By equating (C-29) to zero and solving for \( \beta \), we find that

\[
\beta^* = \frac{1}{2} \left[ \sqrt{\left( \frac{\pi}{2 A_V} \right)^2 + 4} - \frac{\pi}{2 A_V} \right]
\]
Figure C.4. Dependence of $s'(0)$ on the ratio $\beta = \frac{A_2}{A_1} \sqrt{\frac{\alpha_2}{\alpha_1}}$
APPENDIX D

IMPULSE RESPONSE OF AN $n$th-ORDER BUTTERWORTH FILTER

By

Raimundo Sampao
Consider the squared-magnitude system function of an $N$th-order analog Butterworth filter (BF) given by

$$|H(j\omega)|^2 = \frac{1}{1 + (j\omega/j\omega_c)^{2N}} \quad (D-1)$$

In (D-1), $\omega_c$ is the cut-off radian frequency. It follows from (D-1) that, in the Laplace domain,

$$H(s)H(-s) = \frac{1}{1 + (s/j\omega_c)^{2N}} \quad (D-2)$$

The poles of $H(s)H(-s)$ are

$$s_p = (-1)^{1/2N} (j\omega_c) \quad (D-3)$$

i.e., they are equally spaced in angle on a circle of radius $\omega_c$ in the s plane. For a causal and stable design, $H(s)$ is chosen to correspond to the poles in the left-half s plane, as shown in Figure D.1. Note that the poles have been assigned in complex conjugate pairs $(b_j, b_j^*)$, $j=1, ..., (N-1)/2$ for $N$ odd and $j=1, ..., n/2$ for $n = even$. Furthermore, for $N$ odd, there is an additional negative pole $b_j = -\omega_c$.

It follows from the above that $H(s)$ can be expressed as

$$H(s) = \begin{cases} 
(N-1) & (N-1) \\
\frac{1}{K} \prod_{i=1}^{\frac{N}{2}} \frac{1}{(s-b_0)} \prod_{i=1}^{\frac{N}{2}} \frac{1}{(s-b_1)} \prod_{i=1}^{\frac{N}{2}} \frac{1}{(s-b_1^*)} \quad N \text{ odd} \\
\frac{1}{N/2} \prod_{i=1}^{N/2} \frac{1}{(s-b_1)} \prod_{i=1}^{N/2} \frac{1}{(s-b_1^*)} \quad N \text{ even} 
\end{cases} \quad (D-4)$$

where, if we require the normalization $H(0)=1$, we must have

$$K = (\omega_c)^N \quad (D-5)$$

Now let $H_U(s)$ be the transfer function corresponding to $\omega_c=1$, with poles $\{a_i\}$ on the unit circle. Since

$$b_1 = \omega_c a_1 \quad (D-6)$$

it is easy to verify that the transfer function $H(s)$ associated with a general $\omega_c$ is related to $H_U(s)$ by
*Complex conjugate

Figure D.1. Causal and Stable Butterworth Filter Designs in the Complex s Plane
(Pole identification is shown for N odd or even)
In the time domain, let \( h_u(t), h(t) \) be the impulse responses corresponding to \( H_u(s) \) and \( H(s) \), respectively. In other words,

\[
\begin{align*}
H_u(s) &= \int_0^\infty h_u(t) e^{-st} dt \quad \text{(D-8a)}
\end{align*}
\]

and

\[
\begin{align*}
H(s) &= \int_0^\infty h(t) e^{-st} dt \quad \text{(D-8b)}
\end{align*}
\]

Combining (D-7) and (D-8a), we conclude that

\[
\begin{align*}
H(s) &= H_u \left( \frac{s}{\omega_c} \right) = \int_0^\infty h_u(t) e^{-st} dt = \int_0^\infty \omega_c h_u(\omega_c t') e^{-st'} dt' \quad \text{(D-9)}
\end{align*}
\]

which, by mere comparison with (D-8b), results in

\[
\begin{align*}
h(t) &= \omega_c h_u(\omega_c t) \quad \text{(D-10)}
\end{align*}
\]

Therefore, the normalized and unnormalized impulse responses are related by an amplitude and time scaling by \( \omega_c \).

With the above in mind, let us now proceed to calculate \( h_u(t) \); conversion to \( h(t) \) is then immediate.

From (D-4), setting \( \omega_c = 1 \); hence, \( K = 1 \), we get

\[
\begin{align*}
H_u(s) &= \left\{ \begin{array}{l}
\frac{1}{(s-a_0)} \sum_{i=1}^{\frac{N-1}{2}} \frac{1}{(s-a_i)(s-a_i^*)} = \frac{1}{(s-a_0)} \sum_{i=1}^{\frac{N-1}{2}} \frac{1}{(s-a_i)} \sum_{i=1}^{\frac{N-1}{2}} \frac{1}{(s-a_i^*)} \quad ; N=\text{odd} \\
\prod_{i=1}^{\frac{N}{2}} \frac{1}{(s-a_i)(s-a_i^*)} = \prod_{i=1}^{\frac{N}{2}} \frac{1}{(s-a_i)} \prod_{i=1}^{\frac{N}{2}} \frac{1}{(s-a_i^*)} \quad ; N=\text{even}
\end{array} \right. \\
\end{align*}
\]

Recall that the poles of \( H_u(s) \) are called \( a_i \), as per (D-6).
Performing a partial-fraction expansion yields

\[
H_u(s) = \begin{cases} 
\frac{A_0}{s-a_0} + \sum_{i=1}^{\frac{N-1}{2}} \frac{B_i}{s-a_i} + \frac{C_i}{s-a_i^*} & \text{N=odd} \\
\sum_{i=1}^{N/2} \frac{B_i}{s-a_i} + \frac{C_i}{s-a_i^*} & \text{N=even}
\end{cases}
\] (D-12)

where the coefficients \(A_0, B_i, C_i\) are found by standard formulae, as shown below. Assuming for a minute that \(N=\text{odd}\), we have

\[
A_0 = H_u(s) |_{s=a_0}
\]

\[
B_i = H_u(s)|_{s=a_i} \quad i=1,2,\ldots,\frac{(N-1)}{2} \quad (D-13)
\]

\[
C_i = H_u(s)|_{s=a_i^*}
\]

Thus, using (D-13) into (D-11) yields

\[
B_i = \frac{1}{2} \prod_{k=1}^{\frac{N-1}{2}} \frac{1}{\left(\frac{a_i-a_k}{a_i-a_0}\right)} = \prod_{k=1}^{\frac{N-1}{2}} \frac{1}{\left(\frac{a_i-a_k^*}{a_i-a_0^*}\right)} \quad \text{N=odd} \quad (D-14)
\]

\[
C_i = \frac{1}{2} \prod_{k=1}^{\frac{N-1}{2}} \frac{1}{\left(\frac{a_i^*-a_k}{a_i^*-a_0}\right)} = \prod_{k=1}^{\frac{N-1}{2}} \frac{1}{\left(\frac{a_i^*-a_k^*}{a_i^*-a_0^*}\right)} \quad (D-15)
\]

Since \(a_0\) is real (it corresponds to a real pole), we conclude from (D-14) that

\[
C_i = B_i^* \quad i=1,2,\ldots,\frac{(N-1)}{2} \quad (D-15)
\]

so that only the \(B_i\)'s of (D-14) need to be evaluated. Furthermore,

\[
A_0 = \prod_{i=1}^{\frac{N-1}{2}} \frac{1}{a_0-a_i^*} = \prod_{i=1}^{\frac{N-1}{2}} \frac{1}{a_0-a_i} \quad (D-16)
\]

It follows from (D-12) and (D-15) that \(H_u(s)\) is given by

\[
H_u(s) = \frac{A_0}{s-a_0} + \sum_{i=1}^{\frac{N-1}{2}} \frac{B_i}{s-a_i} + \frac{B_i^*}{s-a_i^*} \quad (D-17)
\]
Let
\[ \phi_i = \text{Re}[a_i] \]  
and
\[ \omega_i = \text{Im}[a_i] \]  
so that
\[ a_i = \phi_i + j\omega_i \]  

The inverse Laplace transform of (9-17) is then
\[
h_u(t) = A_0 e^{-\alpha t} \sum_{i=1}^{(N-1)/2} B_i e^{\phi_i t} + B_i e^{\phi_i t} = A_0 e^{-\alpha t} \left( 2 \sum_{i=1}^{(N-1)/2} \text{Re}(B_i e^{\phi_i t}) \right) \]
or
\[
h_u(t) = A_0 e^{-\alpha t} \sum_{i=1}^{(N-1)/2} \phi_i t e^{\phi_i t} \left( \sum_{i=1}^{(N-1)/2} \text{Re}(B_i e^{\phi_i t}) \right) \]

which is a real function, as expected. In (D-19), \( A_0 \) and \( B_i \) are given in (D-16) and (D-14).

In an entirely similar way, when \( N=\text{even} \),
\[
h_u(t) = 2 \sum_{i=1}^{N/2} \phi_i t e^{\phi_i t} \left( \sum_{i=1}^{N/2} \text{Re}(B_i e^{\phi_i t}) \right) \]

with
\[
B_i = \prod_{k=1}^{N/2} \frac{1}{(a_i - a_k)} \prod_{k=1}^{N/2} \frac{1}{(a_i - a_k^*)} \]  

The poles \( a_i \) can also be expressed in trigonometric form as

**N=odd**
\[
a_i = -\cos \left( \frac{i\pi}{N} \right) + j \sin \left( \frac{i\pi}{N} \right) ; i=0,1,\ldots,(N-1)/2
\]

\[ \begin{array}{cc}
\phi_i & \omega_i \\
\hline
\hline
\end{array} \]

**N=even**
\[
a_i = -\cos \left( \frac{(2i-1)\pi}{2N} \right) + j \sin \left( \frac{(2i-1)\pi}{2N} \right) ; i=1,\ldots,N/2
\]

\[ \begin{array}{cc}
\phi_i & \omega_i \\
\hline
\hline
\end{array} \]
APPENDIX E

SIMULATION OF THE MRD MODIFIED FOUR-PHASE COSTAS LOOP

By

Jack K. Holmes
The analog version of the modified four-phase Costas loop is illustrated in Figure E.1. Let the received signal be described by

\[ r(t) = \sqrt{2P_1} m_1(t) \cos(\omega_{sc} t + \theta) + \sqrt{2P_2} m_2(t) \sin(\omega_{sc} t + \theta) \]

then the detected signals will be

\[ e_1(t) = \sqrt{P_1} m_1(t) \cos(\theta - \hat{\theta}) + \sqrt{P_2} m_2(t) \sin(\theta - \hat{\theta}) \] (1)

and

\[ e_2(t) = -\sqrt{P_1} m_1(t) \sin(\theta - \hat{\theta}) + \sqrt{P_2} m_2(t) \cos(\theta - \hat{\theta}) \] (2)

In order to utilize the results on a digital computer, we digitize the above equations to yield

\[ e_1(k) = \sqrt{A} m_1(k) \cos(\theta(k) - \hat{\theta}(k)) + 2\sqrt{A} m_2(k) \sin(\theta(k) - \hat{\theta}(k)) \] (3)

and

\[ e_2(k) = -\sqrt{A} m_1(k) \sin(\theta(k) - \hat{\theta}(k)) + 2\sqrt{A} m_2(k) \cos(\theta(k) - \hat{\theta}(k)) \] (4)

where we have let \( P_2 = 4P_1 \) because of the 4:1 power ratio, and \( A = P_1 \).

Out of the 1.8-MHz lowpass filters, we can write

\[ e_1(k) = \sum_{n=0}^{k} e_1(n) h_L(k-n) \] (5)

and

\[ e_2(k) = \sum_{n=0}^{k} e_2(n) h_L(k-n) \] (6)

where

\[ h_L(n) = e^{-2\pi n f_0 T_3} \] (7)

with \( T_3 \) representing the symbol duration and \( f_0 \) being the 3-\( \omega \) cutoff of the arm lowpass filters.

Now let the output of the limiters be denoted \( L_1(k) \) and \( L_2(k) \). Then

\[ L_1(k) = \text{sgn}(e_1(k)) \] (8)

and

\[ L_2(k) = \text{sgn}(e_2(k)) \] (9)
The analog version of the modified four-phase Costas loop is illustrated in Figure E.1. Let the received signal be described by

\[ r(t) = \sqrt{2P_1} m_1(t) \cos(\omega_{\text{lo}} t + \phi) + \sqrt{2P_2} m_2(t) \sin(\omega_{\text{lo}} t + \phi) \]

then the detected signals will be

\[ e_1(t) = \sqrt{P_1} m_1(t) \cos(\phi) + \sqrt{P_2} m_2(t) \sin(\phi) \]

(1)

and

\[ e_2(t) = -\sqrt{P_1} m_1(t) \sin(\phi) + \sqrt{P_2} m_2(t) \cos(\phi) \]

(2)

In order to utilize the results on a digital computer, we digitize the above equations to yield

\[ e_1(k) = \sqrt{A} m_1(k) \cos(\phi(k) - \delta(k)) + 2\sqrt{A} m_2(k) \sin(\phi(k) - \delta(k)) \]

(3)

and

\[ e_2(k) = -\sqrt{A} m_1(k) \sin(\phi(k) - \delta(k)) + 2\sqrt{A} m_2(k) \cos(\phi(k) - \delta(k)) \]

(4)

where we have let \( P_2 = 4P_1 \) because of the 4:1 power ratio, and \( A = P_1 \).

Out of the 1.8-MHz lowpass filters, we can write

\[ e_1(k) = \sum_{n=0}^{k} e_1(n) h_L(k-n) \]

(5)

and

\[ e_2(k) = \sum_{n=0}^{k} e_2(n) h_L(k-n) \]

(6)

where

\[ h_L(n) = e^{-2\pi n f_0 T_s} \]

(7)

with \( T_s \) representing the symbol duration and \( f_0 \) being the 3-dB cutoff of the arm lowpass filters.

Now let the output of the limiters be denoted \( L_1(k) \) and \( L_2(k) \). Then

\[ L_1(k) = \text{sgn}(e_1(k)) \]

(8)

and

\[ L_2(k) = \text{sgn}(e_2(k)) \]

(9)
Figure E.1. Modified Four-Phase Costas Loop

\[ A_v = \sqrt{P_2/P_1} \]

\[ s(t), p(t) \]

\[ \frac{1}{2} \sin(\omega_0 t + \phi) \]

\[ \frac{1}{2} \cos(\omega_0 t + \phi) \]

\[ \text{LPF} \]

\[ \text{VCXO} \]

\[ \text{Sign} \]

\[ \text{Sign} \]
The output of the second multipliers is given by

$$z_1(k) = \epsilon_1(k) L_2(k)$$  \hspace{1cm} (10)

and

$$z_2(k) = \epsilon_2(k) L_1(k)$$  \hspace{1cm} (11)

Out of the difference, we have

$$z(k) = A_y z_1(k) - z_2(k)$$  \hspace{1cm} (12)

where $A_y = 2$ for our case. Now the output-phase estimate is given by

$$\hat{\theta}(k+1) = \hat{\theta}(k) + K_L T_s z(k)$$  \hspace{1cm} (13)

for the first-order loop which is utilized in this simulation. Now,

$$K_L = \frac{2 \pi B_L T_s f_0}{A_y^2 \gamma_2^{1/2} + \gamma_1^{1/2}}$$  \hspace{1cm} (14)

relates the closed-loop one-sided noise bandwidth, $B_L$, to the recursion gain $K_L$. The $\gamma$ parameters are the filtering loss of the respective data bit streams, i.e.,

$$\gamma_1' = \int_{-\infty}^{\infty} \frac{M_1(f)}{1 + \left(\frac{f}{f_0}\right)^2} df \leq 1$$  \hspace{1cm} (15)

and

$$\gamma_2' = \int_{-\infty}^{\infty} \frac{M_2(f)}{1 + \left(\frac{f}{f_0}\right)^2} df \leq 1$$  \hspace{1cm} (16)

Before the loop, the test filter (prelimiter) and limiter, plus a bandpass filter, are located and modeled in the simulation. The resulting baseband-equivalent model is illustrated in Figure E.2. The recorder signal model composed of a noise burst plus two sweeping tones were coded in the program.
Figure E.2. Baseband Equivalent MRD Loop Model Simulation
ADDENDUM 3

RF COVERAGE ANALYSIS
TECHNICAL MEMORANDUM, AXIOMATIX

TO: JSC/EE8/J. Ratliff
FROM: Axiomatix/J. Porter
SUBJECT: STS-9 Ascent RF Coverage; Comm Support Formulation

The enclosure provides the recommended STS-9 ascent-phase STTN support plan and the corresponding predicted Orbiter S-band PM automatic antenna switching up to and including TDRS ascent support, as derived from analysis of the STS-9 Cycle 1 reference trajectory. This information has been provided informally to FS4 GC's (Ground Controllers) per request and is currently baselined for STS-9.

The use of MIL and PDL embodied in items (1) through (7) on the enclosure is the standard scenario utilized for past missions and is unchanged for STS-9 (but the Orbiter automatic antenna switching called out is different from the past as a result of the $57^\circ$ inc.). The ground-station and TDRS scenario from item (8) through item (11) is all new and is the result of two new considerations for the STS-9 mission: (1) the first TDRS support of ascent, and (2) the first $57^\circ$ inc. mission, all previous missions being $28^\circ$Q1 or approximately due east.

Because of the $57^\circ$ inc, unlike previous missions BDA will be a low-EL ($<5^\circ$) and hence marginal pass. This led to the recommendation of the new MIL/BDA H/O time in item (8) of 6.5 min versus the previous 5.5 min. BDA 0°-EL horizon break will occur later for STS-9 (about 5:40 MiS MET) while MIL is still above 5° EL until 6:30 and also offers a significantly stronger link than BDA.

The recommended BDA/TDRS H/O of 9 to 9.5 min in item (10) was predicated on BDA showing line-of-sight terrain masking just after 9.5 min. Based on this recommendation, 9.5 min is currently baselined.

James A. Porter

Enclosure

cc: JSC, EE8/W. Teasdale
    B. Batson
    JSC, EE/R. Sawyer
    J. Johnson
    Axiomatix, G. Ruth
Recommended STTN Support and Corresponding Predicted Orbiter Antenna Switching for STS-9 Ascent

1. 0:00 (Liftoff) Orbiter S-land PM antenna: ULA, i.e., Upper-Left/Aft-Beam

2. ~0:09 (M/S MET) Switches to LLA at PDL horizon break. PDL is now switching target. (Link is still thru MIL and remains strong because slant range is small.)

3. ~0:12 Line-of-sight changing rapidly (see attached figure, PDL part); switch to LRA indicated. Switching target still PDL, link still MIL.

4. ~0:21 To URA

5. ~0:23 To URF (Note: may not occur; line-of-sight barely goes into URF region)

6. ~0:46 Back to (or still on) URA. PDL is still target; link is also normally handed over to PDL at about 0:45.

7. ~2:30 Link and back to MIL, and MIL becomes target for ant. switching. Probably stays on URA but also close to ULA (see attached figure, MIL plot).

8. ~6½ min* Handover to BDA; goes to ULF.

9. ~8:05 Still BDA; goes to ULA and remains thru BDA LOS or until BDA/TDRS H/O.

10. 9-9½ min* BDA/TDRS H/O; goes to LLA

11. 27½ min Still TDRS; goes to ULA

* New; recommended
Shuttle Line-of-Sight Look Angles to MIL, PDL, BDA, and TDRS During STS-9 Nominal Ascent (Ref. Traj: Cycle 1)

AOS = Acq. of Sight
LOS = Loss of Sight
TO: EE8/J. Ratliff
FROM: Axiomatix/J. Porter
SUBJECT: Shuttle STS-8 Mission DTO 702 Report

To: EE8/J. Ratliff
From: Axiomatix/J. Porter
Subject: Shuttle STS-8 Mission DTO 702 Report

STS-8 was the first Shuttle mission operation of communications via a TDRS satellite for both S-band and K-band comm. DTO 702 was an STS-8 comm flight test to exercise different operational configurations of the Orbiter's S-band comm system towards establishing flight readiness of TDRS-mode operation. K-band had separate DTO's, 710, 712, and 724.

DTO 702 required exercising S-band comm thru TDRS using HDR and LDR, with and without spreading, with and without encryption, Hi-Freq and Lo-Freq operation, acquisition without doppler compensation, plus a test part to verify that antenna and beam switching occurs at the projected boundaries.

The DTO assessment report is enclosed. DTO 702 is considered successful and completed. Assessment was based on the EE8 standard postmission data product CCOOlF, with data printouts extracted per EE8 direction by Lockheed-EMSCO; and postmission reports distributed by the different mission operation groups; plus discussions with mission personnel. The report is co-authored by D. O'Herron/Lockheed-EMSCO, being a cooperative effort, and thus will also be provided under Lockheed-EMSCO cover.

Enclosure

CC:
JSC, EE8/W. Teasdale
S. Novosad
B. Batson
EE/R. Sawyer
J. Johnson
EE3/J. Seyl, W. Seibert
EE4/M. Luse
Axiomatix, G. Huth
Lockheed-EMSCO, D. O'Herren
REPORT ON STS-8 DTO 702

January 26, 1984

Prepared for James E. Ratliff, Manager NASA/JSC Task 530, RF Coverage Analysis, EE8/Systems Analysis Office, Tracking and Communications Division, Johnson Space Center, National Aeronautics and Space Administration.

by

Dave O’Herren
Lockheed Engineering and Management Services Company

James A. Porter
Axiomatix

Approved:

J.E. Ratliff
Manager, Task 530
Report on STS-8 DTO 702

STS-8 was the first Shuttle mission operation of communications via a TDRS satellite for both S-band and K-band comm. DTO 702 was an STS-8 comm flight test to exercise different operational configurations of the Orbiter's S-band comm system, towards establishing flight readiness for TDRSS-mode operations. K-band comm had separate DTO's, 710, 712, and 724, and these are being addressed in other reports.

Specifically, DTO 702 requires exercising S-band comm thru TDRSS using HDR and LDR, with and without spreading, with and without encryption, Hi-Freq and Lo-Freq operation, acquisition without doppler compensation, plus a test part for assessing antenna and beam switching. Table I identifies the comm system configuration elements addressed by each of the parts of 702. The criteria for success for each test part is simply proper end-to-end operation of the S-band link when exercising the configuration elements of that test part. More specifically, success is promptly establishing and continuing data lock. For purposes of the DTO, timely lock plus 3-min sustained operation is defined as adequate demonstration of success.

For this report, postflight data was available only from the Orbiter, and hence the assessment is based largely on forward (FWD) link performance; return (RTN) link observations from mission operation reports are included in the assessment when and where applicable.

Assessment

All tests required by DTO 702 were performed. Each designated configuration element was exercised. Since the purpose of the DTO was to simply establish that each Table I configuration is operational, timely lock plus a nominal 3-min sustained operation is defined as successful demonstration. To minimize the operational inconvenience of frequent configuration changes, the mission plan allowed that each configuration might be exercised the entire TDRS pass time, roughly an hour, of the orbit scheduled for that configuration. The scheduled orbits are listed in Table II. With these orbits as a guide, the mission time periods shown in Table II were selected for ordering post-mission data for analysis and assessment.

Data tabulations for these periods were prepared and first reviewed to ascertain if the proper configurations were employed and for what portions of these periods. The configurations selected by mission personnel for DTO 702 purposes are listed with corresponding DTO parts in Table II. These are total configurations and thus include selection elements which are incidental to the DTO, as can be seen by comparison with Table I. For example, per Table I, 702-1 requires a period of
Orbiter operation with the low set of operating frequencies, 2041.9-MHz receive/2217.5-MHz transmit, as opposed to 2106.4-MHz receive/2287.5-MHz transmit. Per Table II, the planned or designated configuration for 702-1 correctly includes Lo-Freq selection, whereas the other configuration elements are optional with 702-1.

The processed data shows that designated configurations were in some cases employed throughout the Table II periods, some roughly half, and some others much less. Since these configurations per se were not required, attention was directed to any configurations meeting Table I requirements. The data was then reviewed for segments meeting this criteria and also of at least 3-min duration, per the assessment guideline.

Such segments were found in the data set for the periods in Table II in all cases except 702-2 and of course 702-9 since, as noted in Table II, the source-data tape for the 702-9 period was faulty and a re-order failed to clear the problem. The 702-2 data shows HDR with spreading (SPD) beginning at the very end of the data period and providing only a few seconds of data lock. However, mission reports indicate this was successfully demonstrated shortly thereafter in a 5-min interval early in the next orbit, orbit 77 (247:00:27 - 00:32).

The assessment of the individual parts of 702 is as follows:

- **702-1. Successful.** Extensively demonstrated throughout an entire TDRS pass, ~50 min.

- **702-2. Not adequately demonstrated by the data but mission reports indicate successful demonstration not in orbit 76 but shortly thereafter early in 77, in a 5-min segment 247:00:27 - 00:32. A supplemental data request is considered unmerited. Rated: (qualified) Successful.**

- **702-3. Successful.** Much like 702-1 above. (702-4 replaced by 702-10 and 11 as explained in Table I)

- **702-5. Successful.** Data indicates approximately 6 min of good operation during orbit 25. Non-related procedural problems and shortened TDRS support reported.

- **702-6. Successful.** Acquisition without doppler compensation was demonstrated during normal mission operation. There is one exception, high positive doppler such as at the beginning of a TDRS pass where the doppler is +50 kHz or more. With doppler comp inhibited at AOS on orbit 88, the Orbiter transponder did not lock to the FWD link, confirming a previous ground test result.

- **702-7. Successful.** Data shows an interval approximately of 20 min of successful 702-7 operation during orbit 54.
• 702-8. Orbit 37 data shows only a short 702-8 test interval. Non-related procedural problems were reported for much of this orbit but the data appears to indicate a minimal period of successful 702-8 operation. Mission reports also state successfully demonstrated. Rated: Successful.


• 702-10, 11. Successful. Assessment of roll plane and pitch plane antenna and beam switching, comm mode configuration optional. Data shows switching occurs at each antenna and beam switching boundary. Some residual minor discrepancies as to precise boundaries remain after allowing for 2° hysteresis, and also allowing for relatively fast 10/2-sec rate-of-change versus a 2-sec staleness factor. Discrepancies typically less than 5°, with one on the order of 10°.

Conclusion

DTO 702 is considered successful and completed. Adequate tests are conducted for all ten DTO parts.
<table>
<thead>
<tr>
<th>Part</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>702-1</td>
<td>Lo-Freq Operation</td>
</tr>
<tr>
<td>702-2</td>
<td>HDR; SPD</td>
</tr>
<tr>
<td>702-3</td>
<td>HDR</td>
</tr>
<tr>
<td>702-4</td>
<td>(Deleted*)</td>
</tr>
<tr>
<td>702-5</td>
<td>LDR</td>
</tr>
<tr>
<td>702-6</td>
<td>Acq. without Doppler comp</td>
</tr>
<tr>
<td>702-7</td>
<td>Encrypted; D/L LDR, U/L LDR, SPD</td>
</tr>
<tr>
<td>702-8</td>
<td>Encrypted; D/L HDR, U/L HDR, SPD</td>
</tr>
<tr>
<td>702-9</td>
<td>Encrypted; D/L LDR, U/L LDR</td>
</tr>
<tr>
<td>702-10, 11</td>
<td>Antenna and beam switching assessment (roll and pitch planes, respectively)</td>
</tr>
</tbody>
</table>

*Part 702-4, originally the part requiring antenna and beam switching assessment, was deleted and replaced by 702-10 and 11 to parallel 712-1 and 2, a K-band DTO which fortuitously already provides roll and pitch plane rotations.*
### TABLE II. - STS-8 DTO 702 Postmission Data Set

<table>
<thead>
<tr>
<th>DTO Part</th>
<th>STS-8 Designated Configuration for DTO Part</th>
<th>Reported Orb. No.</th>
<th>Period of Postmission Data Obtained (GMT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>702-1</td>
<td>LDR/Coding/SPD/Lo-Freq</td>
<td>Orb. 9</td>
<td>242:19:05 - 20:01 (55 min)</td>
</tr>
<tr>
<td>702-2</td>
<td>HDR/Coding/SPD/Hi-Freq</td>
<td>Orb. 76</td>
<td>246:22:06 - 23:00, plus 23:43 - 23:59 (70 min)</td>
</tr>
<tr>
<td>702-3</td>
<td>HDR/Coding/Hi-Freq</td>
<td>Orb. 13</td>
<td>243:01:31 - 02:25 (54 min)</td>
</tr>
<tr>
<td>702-5</td>
<td>LDR/Coding/Hi-Freq</td>
<td>Orb. 25</td>
<td>243:19:40 - 20:30 (50 min)</td>
</tr>
<tr>
<td>702-6</td>
<td>LDR/Coding/SPD/Hi-Freq, No Doppler Comp</td>
<td>Orb. 88</td>
<td>247:17:04 - 17:57 (53 min)</td>
</tr>
<tr>
<td>702-7</td>
<td>LDR/Coding/SPD/ENC/Hi-Freq</td>
<td>Orb. 54</td>
<td>245:14:30 - 15:25 (55 min)</td>
</tr>
<tr>
<td>702-8</td>
<td>HDR/Coding/SPD/ENC/Hi-Freq</td>
<td>Orb. 37</td>
<td>244:12:50 - 13:50 (60 min)</td>
</tr>
<tr>
<td>702-9</td>
<td>LDR/Coding/ENC/Hi-Freq</td>
<td>Orb. 69</td>
<td>(Not available; bad data tape)</td>
</tr>
<tr>
<td>702-10</td>
<td>LDR/Coding/SPD/Hi-Freq</td>
<td>Orb. 49</td>
<td>245:06:38 - 07:30 (52 min)</td>
</tr>
<tr>
<td>702-11</td>
<td>LDR/Coding/SPD/Hi-Freq</td>
<td>Orb. 65</td>
<td>246:06:20 - 07:31 (71 min)</td>
</tr>
</tbody>
</table>

* 702-4 deleted (achieved in 702-10, 11)
TO: EE8/J. E. Ratliff
FROM: Axiomatix/J. A Porter
SUBJECT: Shuttle STS-8 Mission DTO 710 Report

March 26, 1984

STS-8 was the first Shuttle mission operation of communications via a TDRS satellite for both S-band and K-band comm. DTO 710 was an STS-8 comm flight test to exercise different operational configurations of the Orbiter's K-band comm system towards establishing flight readiness of K-band operations. S-band had an analogous separate DTO (702) which was previously reported.

The intent of DTO 710 was that K-band comm be exercised and verified in each of the various combinations of FWD and RTH modes, with and without spreading, and with and without encryption. Additionally, that K-band acq be demonstrated in both the automatic and the manual modes, that K-band failover to S-band be demonstrated and finally, a demonstration of maintaining tracking and comm during an OMS burn.

The DTO assessment report is enclosed. DTO 710 is considered successful overall. One item is assigned TBD, as data products are being further analyzed for completeness. Assessment was based on the EE8 standard post-mission data product CC001F as extracted and reported on by LEMSCO/D. O’Herren per EE8 direction, and various mission reports and data from direct mission support personnel or organizations. The report is co-authored with D. O’Herren and hence will also be forwarded under Lockheed-EMSCO cover.

James A. Porter

Enclosure

cc: JSC, EE8/W. Teasdale
    S. Novosad
    EE/R. Sawyer
    J. Johnson
    EE3/J. Seyl
    W. Seibert
    EE4/M. Luse
    Axiomatix, G. Huth
    Lockheed-EMSCO, D. O’Herren
REPORT ON STS-8 DTO 710

March 14, 1984

Prepared for James E. Ratliff, Manager NASA/JSC Task 530, RF Coverage Analysis, EEB/Systems Analysis Office, Tracking and Communications Division, Johnson Space Center, National Aeronautics and Space Administration.

by

Dave O’Herren
Lockheed Engineering and Management Services Company

James A. Porter
Axiomatix
AXIO-3/84-HR1

Approved:

James E. Ratliff
Manager, Task 530
Report on STS-8 DTO 710
3/14/84

STS-8 was the first Shuttle mission operation of communications via a TDRS satellite for both S-band and K-band comm. DTO 710 was an STS-8 comm flight test to exercise different operational configurations of the Orbiter's K-band comm system, towards establishing flight readiness for K-band comm via TDRSS. S-band comm had an analogous separate DTO (702) which was previously reported.

The intent of DTO 710 was that K-band comm be exercised for each of the various combinations of FWD and RTN modes, with and without spreading, and with and without encryption. Additionally, that K-band acquisition be demonstrated both in the automatic mode and the manual mode, that K failover to S-band be demonstrated and finally, a demonstration of maintaining antenna tracking and the comm link during an OMS burn. Table I identifies the comm system configuration elements addressed by each of the parts of 710. The criteria for success for each test part is simply proper end-to-end operation of the K-band link when exercising the configuration elements of that test part. More specifically, success is promptly establishing and continuing data lock. For purposes of the DTO, timely lock plus 3-min sustained operation is defined as adequate demonstration of success.

For this report, postflight data was available only from the Orbiter, and hence the assessment is based largely on forward (FWD) link performance; return (RTN) link observations from mission operation reports are included in the assessment when and where applicable.

Assessment

All tests required by DTO 710 were performed. Each designated configuration element was exercised. Since the purpose of the DTO was to simply establish that each Table I configuration item is operational, timely lock plus a nominal 3-min sustained operation is defined as successful demonstration. For each item in Table I, mission personnel defined a system configuration to include that item, and a mission period (orbit) in which that configuration would be exercised. The complete, hight-designated configurations corresponding to the Table I items are listed in Table II. The orbit during which each configuration was exercised, as reported by mission personnel, is included. With these orbits as a guide, the mission time periods shown in Table II were selected for ordering postmission data for analysis and assessment. Data tabulations for these periods were prepared and first reviewed to ascertain if proper configurations were employed and for what portions of these periods. Segments of proper configuration were then addressed for an assessment of DTO test.
results. Such segments were found in the data set for the periods in Table II in all cases except 710-1, -3, and -7. For the latter two, data either was not available and/or was faulty. For 710-1, the data for the orbit identified (21) shows the Orbiter uplink data source as being S-band rather than K-band during all of the data period except for 3 min., and for these 3-min. shows the K-band FWD mode to be mode 1 (216 KB) instead of the designated mode 2 (72 KB). This is at odds with mission support personnel reports that 72 KB was utilized and that 710-1 was successfully demonstrated in orbit 21. Whether the data is erroneous, or a processing error incurred, or the wrong orbit reported, etc., is being looked into. In the interim, DTO part 710-1 is rated as "Successful (qualified)" since it was directly reported accomplished by the mission comm officer (INCO).

Table III provides an assessment of DTO 710 by parts.
### TABLE I. - Parts of STS-8 DTO 710 (K-hand)

<table>
<thead>
<tr>
<th>710-1</th>
<th>72KB Subsystem, K U/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>710-2</td>
<td>72KB UNSPD, K U/L</td>
</tr>
<tr>
<td>710-3</td>
<td>D/L PM Mode; U/L 216KB Mode, UNSPD</td>
</tr>
<tr>
<td>710-4</td>
<td>Failover to S</td>
</tr>
<tr>
<td>710-5</td>
<td>(1) D/L PM ENCRP; (2) D/L FM ENCRP</td>
</tr>
<tr>
<td>710-6</td>
<td>Acquire while in D/L PM ENCRP</td>
</tr>
<tr>
<td>710-7</td>
<td>72KB ENCRP/UNSPD, U/L</td>
</tr>
<tr>
<td>710-8</td>
<td>(1) Manual acq; (2) Track thru OMS burn</td>
</tr>
</tbody>
</table>
## TABLE II. - STS-8 DTO 710 Postmission Data Set

<table>
<thead>
<tr>
<th>DTO Part</th>
<th>STS-8 Designated K-band Configuration</th>
<th>Reported Orbit No.</th>
<th>Period of Postmiss. Data Examined (GMT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>710-1</td>
<td>FWD Mode 2 (72KB)/RTN Mode 2 (FM). SPD On, ENCRP OFF/ENCRP OFF</td>
<td>orb. 21</td>
<td>243:12:45 - 13:40</td>
</tr>
<tr>
<td>710-2</td>
<td>FWD Mode 2 (72KB)/RTN Mode 2 (FM). SPD OFF, ENCRP OFF/ENCRP OFF</td>
<td>orb. 82</td>
<td>247:07:22 - 08:22</td>
</tr>
<tr>
<td>710-3</td>
<td>FWD Mode 1 (216KB)/RTN Mode 1 (FM). SPD OFF, ENCRP OFF/ENCRP OFF</td>
<td>orb. 87</td>
<td>247:15:30 - 16:21*</td>
</tr>
<tr>
<td>710-4</td>
<td>FWD Mode 1 (216KB)/RTN Mode 2 (FM). SPD ON, ENCRP OFF/ENCRP OFF</td>
<td>Various</td>
<td>Various</td>
</tr>
<tr>
<td>710-5</td>
<td>FWD Mode 1 (216KB)/RTN Modes 1 &amp; 2. SPD ON, ENCRP ON/ENCRP ON</td>
<td>orb. 54</td>
<td>245:14:30 - 15:23</td>
</tr>
<tr>
<td>710-6</td>
<td>FWD Mode 1 (216KB)/RTN Mode 1 (FM). SPD ON, ENCRP ON/ENCRP ON</td>
<td>orb. 37</td>
<td>244:12:50 - 13:55</td>
</tr>
<tr>
<td>710-7</td>
<td>FWD Mode 2 (72KB)/RTN Mode 2 (FM). SPD OFF, ENCRP ON/ENCRP ON</td>
<td>orb. 69</td>
<td>246:12:32 - 13:30*</td>
</tr>
<tr>
<td>710-8</td>
<td>FWD Mode 1 (216KB)/RTN Mode 2 (FM). SPD ON, ENCRP OFF/ENCRP OFF</td>
<td>orb. 48</td>
<td>245:05:03 - 06:18</td>
</tr>
</tbody>
</table>

*Data could not be obtained or was faulty.
All parts of DTO 711 were previously verified in STS-8 DTO 710 except for exercising and verifying K-band RTN link PM mode channel 3 HDR in the highest HDR, 48 MB. This report addresses this latter.

The 48-MB HDR was exercised numerous times in STS-9, per mission reports. On many occasions performance was reported as very satisfactory. Occasions when it was not can generally be traced to circumstances such as K-band link outages due to antenna blockage or obscuration and/or outages unrelated to 48-MB performance. For purposes of this DTO, a sample period of sustained proper data flow is sufficient verification of the 48-MB capability; hence, timely lock plus 3-min sustained operation with good data is defined as adequate demonstration of success.

Assessment

Discussions with mission personnel identified several periods of 48-MB operation which were free of any blockage, etc outages not germane to whether or not the 48-MB channel operates properly. One of these periods, a 13-min interval from 336:21:42 to 336:21:55 UT, was one for which EPB standard postmission data products were also available. This period was selected for DTO 711 assessment. RTN link performance, such as signal strength and data lock, is not an available postmission data product. Hence, the assessment of RTN link 48-MB performance must be evaluated by a combination of Orbiter onboard configuration status data and RTN link performance reports from mission personnel.

Orbiter data confirms that the Orbiter K-band system was configured, as required, for RTN Mode 1 (PM) with Channel 3 HDR at 48-PB throughout the period selected. Mission support personnel report 100% data quality for the received 48-PB data also throughout the period. DTO 711 is therefore rated successful and completed.
PHASES AND STAGES OF A DTO*

I. COMPREHENSIVE REVIEW OF SYSTEM TO IDENTIFY CANDIDATE FTR'S (FLT TEST REQ'S)

II. PRELIMINARY FORMULATION OF FLIGHT TESTS

III. ATTAINING FTR STATUS

IV. ATTAINING DTO STATUS

V. MISSION INCORPORATION

--------------------------------------------------------------- (FLIGHT ACHIEVED) ---------------------------------------------------------------

VI. POSTMISSION ANALYSIS AND ASSESSMENT

VII. DTO FORMAL REPORT

* A FEW DTO'S, SPONTANEOUSLY BORN IN ANSWER TO A MISSION MANIFESTED NEED, BEGAN ESSENTIALLY AT STEP IV, BYPASSING THE EXTENSIVE TIME AND EFFORTS INVOLVED IN STEPS I, II, III.
(C&T DTO'S)

PART I.: COMPREHENSIVE C&T SYSTEM REVIEW TO IDENTIFY CANDIDATE FTR'S

- OVERVIEW OF SYSTEM COMPONENTS, CONFIGURATIONS, MODES
- DELINEATION OF REQUIREMENTS, CONSTRAINTS, LIMITS, OPERATING ENVIRONMENTS
- ASSESS VEHICLE-INTEGRATED OPERATION; VARIABLE-MISSION CONSIDERATIONS
- ASSESS EACH C&T RF SYSTEM'S ROLL IN ITS OWN RF LINK, AS A TRANSMITTER AND AS A RECEIVER, IN COMBINATION WITH THE SYSTEM(S) AT THE OTHER END OF THE LINK; COMBINED-OPERATION PROCEDURES, CHARACTERISTICS, CONSTRAINTS, LIMITS
- ASSESS WHAT EACH RF SYSTEM MUST PROVIDE TO AND OBTAIN FROM OTHER ONBOARD NON-C&T SYSTEMS, PLUS FROM OTHER ONBOARD RF SYSTEMS

PART II.: PRELIMINARY FORMULATION OF FLIGHT TESTS

- TO EXERCISE COMPONENTS AND CONFIGURATIONS; IN GSTDN-MODE, TDRSS-MODE, SGLS-MODE, ETC.
- USING THE VARIOUS ANTENNAS AND THEIR LOOK-ANGLE REGIONS OF OPERATION
- IN REAL FLIGHT, OVER A RANGE OF OPERATING ENVIRONMENTS, DISTANCES, ETC.

PART III.: ATTAINING FTR STATUS

- PROGRAM REVIEWS, TECHNICAL REVIEWS. CANDIDATE-FTR STATUS.
- RE-REVIEWS, COMPROMISES, RE-FORMULATIONS. SCRUBS, FINAL REVIEWS. FTR STATUS.

PART IV.: ATTAINING DTO STATUS

- REVIEW OF MISSION MANIFESTS FOR MISSION ASSIGNMENT, MISSION SELECTION.
- DTO INCORPORATION INTO MISSION AND PROGRAM DOCUMENTATION AND CONTROL. DTO STATUS.
PART V: MISSION INCORPORATION

- Planning sessions with assigned mission personnel to formulate test scenario and specific procedures, and incorporate into mission timeline.
- Formulation of specific measurements required; correlation with list of available measurements and available forms and formats.
- Coordination with postmission-data world, who provides what, when, where, and how.
- Re-iterations required by mission re-scheduling or re-formulations.

PART VI: POSTMISSION ANALYSIS AND ASSESSMENT

- Pre-mission formulation and submittal of preliminary data request listing planned test periods and data products required.
- Post-mission identification of actual test periods and re-submittal of data request.
- Delays due to higher-priority orders, facility/manpower constraints, etc.
- Receipt of raw data products; pre-screening and re-orders for faulty data or shortages.
- Data processing and workarounds of unavailable data.
- Data reduction and analysis; incorporation of other source information.

PART VII: DTO FORMAL REPORT

- Preparation and submittal of DTO formal report.
How to Analyze the STS-11 Delayed-Acq. Problem

PREREQUISITES

1. Need management to instruct mission operation personnel to identify the passes where they experienced the delayed S-band acq. Identify by GMT start time of TDLS horizon break.

2. Need management to instruct postmission-data personnel to provide postmission data. Must be expedited with priority; otherwise can run into months. Re-orders due to bad data, etc., must be specifically spelled out in the expediting instructions or these likewise result in long delays.

METHOD

With the log from (1), and the full support per (2), could proceed as follows. First, order the Orbiter Roll/Pitch look angles to TDLS, from TDLS AOS horizon break to +15 min, and see if the bad passes show a pattern such as occurring when the line-of-sight is in the tail region, or in the nose region, or the upper left quadrant, etc. Second, obtain data and see if any coincidence between bad passes and system configuration. Third, obtain data that would show if any comm system parameters had off-nominal values at these times. This also requires ordering some good passes for reference.

This part could be completed in a few weeks if (1) and (2) were implemented with full expeditious support (i.e., a 1-day turnover, same for all re-orders).

If this investigation of the Orbiter side failed to provide any explanation, a second part would be to seek analogous status data for the TDLS to see if the problem can be explained on that end. (Efforts in past missions to obtain TDLS RTN link performance data have been without success; management assistance would be essential.)

jp
ADDENDUM 4

A TOROIDAL ANTENNA PATTERN FOR CENTAUR ISPM
September 12, 1984

TO: Heinz Wimmer
FROM: Richard S. Iwasaki
SUBJECT: A Toroidal Antenna Pattern for the Centaur ISM

CONTRACT: NAS 9-16893

The present General Dynamics Centaur antenna design is essentially omnidirectional and therefore is not suitable for the Solar Polar mission which requires a moderate gain toroidal pattern along the Centaur vehicle axis. A modified biconical array is proposed as a candidate configuration for achieving this toroidal pattern since it is readily adapted to the existing Centaur deployable antenna concept.

The biconical radiating element provides an inherent toroidal antenna pattern. Circular polarization is readily achieved by using orthogonal dipole elements spatially separated to provide the quadrature phase relationship. In this particular case, the circular symmetry of the biconical element is utilized to minimize the axial ratio considerations at the pattern extremes fore-and-aft of the Centaur vehicle since the biconical array axis will be perpendicular to the Centaur vehicle axis. The gain of the biconical array can be increased to the desired value by simply increasing the number of biconical elements in the array, as illustrated in Figure 1.

A simplified biconical dipole design has been developed based on the standing-waves of the radiating dipole element. The basic concept is to appropriately bend the dipole element to accommodate circular polarization. The dipole length is adjusted to permit three standing half wavelengths, sketched in Figure 2b. If the radiating current direction is superimposed on the shaped biconical radiating element, as shown in Figure 2c it is apparent that circular polarization is obtained. Use of an appropriate dielectric such as a ceramic can decrease the dimensions of the biconical element to optimize the antenna pattern.
Figure 2. Radiating Currents on a Standing Wave Dipole
ADDENDUM 5

BER DEGRADATION DUE TO CENTAUR TRANSMITTED
BIT TIMING JITTER NOISE THROUGH THE SHUTTLE
PAYLOAD COMMUNICATION SYSTEM
BER DEGRADATION DUE TO TRANSMITTED
BIT TIMING JITTER NOISE

NAS 9-16893

Prepared for

NASA Lyndon B. Johnson Space Center
Houston, Texas 77058

Prepared by
Dr. Jack Holmes

Axiomatix
9841 Airport Blvd., Suite 912
Los Angeles, California 90045

Axiomatix Report No. R8409-1
September 28, 1984
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>ii</td>
</tr>
<tr>
<td>SUMMARY</td>
<td>1</td>
</tr>
<tr>
<td>1.0 CENTAUR TRANSMISSION TO THE SHUTTLE</td>
<td>2</td>
</tr>
<tr>
<td>2.0 CENTAUR DCU MODEL</td>
<td>2</td>
</tr>
<tr>
<td>3.0 SOURCE OF BIT ERROR RATE DEGRADATION</td>
<td>7</td>
</tr>
<tr>
<td>4.0 BIT ERROR RATE PROBABILITY FORMULATION</td>
<td>11</td>
</tr>
<tr>
<td>5.0 NUMERICAL EVALUATION OF $\phi$</td>
<td>13</td>
</tr>
<tr>
<td>6.0 EVALUATION OF THE BER DEGRADATION</td>
<td>13</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>14</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>15</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Functional Flow Diagram - Centaur Transmission to Orbiter</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Shuttle Payload Communications Receiver</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>NASA PSP Configuration</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>Single Sided Bit Timing Phase Noise Spectral Density (From LinCom Report TR-0394-8214-200)</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>NRZ Asymmetry in a Three Bit Pattern</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>NRZ Example of Duration Jitter</td>
<td>10</td>
</tr>
</tbody>
</table>
SUMMARY

It was determined that the Bit Error Rate (BER) degradation, due to just the DCU bit timing jitter, is approximately 0.04dB. It is concluded that this is a negligible amount of degradation.
The basic concern in this report is the bit error rate caused by the transmitted bit timing jitter, including continuous or discrete spectral components. The source of the bit timing jitter originates in the Digital Computer Unit (DCU), which modulates the Centaur subcarrier and in turn, phase modulates the carrier, see Figure 1. After S-Band transmission to the Shuttle Orbiter, the Payload Interrogator (PI) demodulates the carrier, and the data bits in the PSP bit synchronizer. Frame synchronizing is also done in the PSP. Figure 1 illustrates the system in block diagram form. Figure 2 shows the PI and Figure 3 illustrates the PSP.

The transmitted signal is described by

\[ s(t) = \sqrt{2} A \sin (\omega_0 t + \theta(t)) \sin (\omega_{sc} t + \theta_{sc}) \]  \hspace{1cm} (1)

where \( d(t) \) is the data modulation on the subcarrier. Then the detected signal out of the subcarrier demodulator is given by

\[ y(t) = 2A \cos (\theta(t)) \cos (\omega_{sc}(t)) \]  \hspace{1cm} (2)

where \( A \) is the r.m.s. carrier amplitude, \( \phi(t) \) is the RF phase error, \( \phi_{sc}(t) \) is the subcarrier phase error, \( \theta \) is the subcarrier phase modulation index, and \( d(t) \) is the data bit stream. The data process \( d(t) \) has timing jitter imbedded in the transition times.

For the general case that the timing jitter has both a continuous spectral density, plus discrete line spectral components, the bit timing error expressed in radians\(^2\) is given by

\[ \sigma^2_{\phi} = 2 \int_{-\infty}^{\infty} |1-H(f)|^2 \Phi_{\phi}(f) df + \sum_{k=1}^{N} |1-H(f_k)|^2 \lambda(f_k) \]  \hspace{1cm} (3)

where the first term is due to the continuous timing bit jitter, and the second term is due to the discrete line components in the bit jitter process with weighting \( \lambda(f_k) \).

Conversion to timing (5) jitter in seconds\(^2\) is given by

\[ \sigma^2_{\epsilon} = \left( \frac{T}{2\pi} \right) \sigma^2_{\phi}, \text{ seconds}^2 \]  \hspace{1cm} (4)

where \( T \) is the bit duration in seconds. In equation (3)\( S_{\phi}(f) \) is the single sideband timing bit jitter expressed in the rad\(^2\)/Hz. Consequently, doubling the integral, weighted by the bit synchronizer loop error response [1], yields the variance of the bit timing error jitter. LinCom Corporation [2] has done the measurements for the curve of 16kBPS Manchester data and it is shown in Figure 4. We see that there are no discrete line components in the resulting spectral density.
Figure 1. Functional Flow Diagram - Centaur Transmission to Orbiter
Figure 4. Single Sided Bit Timing Phase Noise Spectral Density
(From LinCom Report TR-0394-214-200)

\[ \int_{0.01}^{10\text{kHz}} \phi(f) \, df = 2.7^\circ \]
The second term of equation (3) is the contribution of the
discrete line spectral components. Now let us determine how to compute the
value of $\lambda(f_c)$. For clarity, assume that two spurs are phase modulated on a
carrier. We model this signal as

$$x(t) = \sqrt{2}A \sin (\omega_0 t + \beta_1 \sin \omega_1 t + \beta_2 \sin \omega_2 t)$$

or

$$x(t) = \sqrt{2}A \sin (\omega_0 t \cos (\beta_1 \sin \omega_1 t + \beta_2 \sin \omega_2 t))$$

$$+ \sqrt{2}A \cos (\omega_0 t \sin (\beta_1 \sin \omega_1 t + \beta_2 \sin \omega_2 t))$$

Assuming that $J_0(\beta_1) \approx 1 \ (|\beta_1|<<1)$ allows us to obtain

$$x(t) = \sqrt{2}A \sin \omega_0 t + \sqrt{2}A J_1(\beta_1) [\sin (\omega_0 + \omega_1) t - \sin (\omega_0 - \omega_1) t]$$

$$+ \sqrt{2}A J_1(\beta_2) [\sin (\omega_0 + \omega_2) t - \sin (\omega_0 - \omega_2) t]$$

Hence, it can be deduced in the $N$th phase modulation spur case that we have

$$x(t) = \sqrt{2}A \sin \omega_0 t + \sqrt{2}A \sum_{i=1}^{N} J_1(\beta_i) [\sin (\omega_0 + \omega_i) t - \sin (\omega_0 - \omega_i) t]$$

Since

$$J_n(\beta) = \frac{\beta^n}{2^n n!}, \quad |\beta|<<1$$

We have

$$x(t) = \sqrt{2}A \sin \omega_0 t + \sqrt{2}A \sum_{i=1}^{N} \frac{\beta_i^2}{2} [\sin (\omega_0 + \omega_i) t - \sin (\omega_0 - \omega_i) t]$$

Hence the line (spur) components above $\omega_0$ have (rms) dBc values of

$$\text{dBc}_i = \frac{\beta_i^2}{4} \text{ rad}^2 \ (\text{SSB})$$

and

$$\beta_i = 2.10 \ \text{dBc}_i / 20$$

The line components below $\omega_0$, also, have the same dBc values. Hence, the
total mean squared timing jitter expressed in rad$^2$ (using single sided notation)
is given by

$$\sigma^2_\phi = \int_{-\infty}^{\infty} [1 - H(f)] \phi_{\phi_c}^2 (f) \ df + \sum_{k=1}^{N} |H(f_k)|^2 \beta_k^2 / 2$$

### 3.0 SOURCE OF BIT ERROR RATE DEGRADATION

In this section we will discuss various sources of possible bit
error rate degradation. First consider asymmetry with NRZ symbols, which are
illustrated in Figure 5.
Figure 5. NRZ Asymmetry in a Three Bit Pattern
As can be seen in the figure, "zero" bits are wider than "one" bits so that in six cases out of the eight cases there is no degradation; one case (110) has a relative energy loss of \((1 - 2^6/T)\), and the last case (010) has a relative energy loss of \((1 - 4^6/T)\), with \(\delta < T/4\). Hence, we see that the asymmetry will degrade two of the 3 bit sequences, and hence the overall Bit Error Rate will be degraded. Further details can be found in reference [3]. Since, to the first order, asymmetry degradation is not affected by bit jitter, we will neglect it in what follows.

Next we consider bit duration jitter and see how it is related to the bit tracking jitter. Let the transmitted time epochs have a nominal bit duration of \(T\) seconds, then transmit time epochs are given by

\[
T(t_j) = T_0 + jT + \epsilon_j
\]

where

\[
\epsilon_j = (T_0 + jT)
\]

and

\[
t_j = T_0 + jT
\]

with \(\epsilon_j\) being the transmitted bit jitter process. As we noted above, the time base jitter is assumed to be the linear sum of a random component, plus sinusoidal terms representing the spurs.

It follows that the duration of the \(j\)th bit has a duration \(D(t)\) given by

\[
D(t_j) = T(t_{j+1}) - T(t_j) = T + (\epsilon_{j+1} - \epsilon_j)
\]

Hence, the duration is nominally \(T\) seconds but is perturbed according to \((\epsilon_{j+1} - \epsilon_j)\), which is a random sequence. In other words, the duration of the bit integration time (bit synchronizer estimate of the bit duration) is a random sequence varying around the mean value of \(T\) seconds, with \(1/T = R\) the bit rate.

Hence, we see that the effect of duration jitter is to expand and contract the bit integration time. Consider Figure 6, which illustrates (a) a zero timing error at both edges, and (b) consecutive timing errors \(\epsilon_1\) and \(\epsilon_2\).

Note first that if \(|\epsilon_1| > |\epsilon_2|\), then duration jitter has no effect. Secondly, if the timing error process has most of the bit jitter noise spectral density located below the data rate, then \(\epsilon_1 = \epsilon_2\) and again bit duration does not contribute to BER. We conclude that bit duration jitter has a much smaller component of BER degradation than bit jitter itself.
Figure 6. NRZ Example of Duration Jitter
(a) No Error
(b) Errors $\epsilon_1$ and $\epsilon_2$
The above discussion is for NRZ, but the same argument applies to Bi-$\phi$-L data too, that is, bit duration is small compared to the bit timing error.

4.0 BIT ERROR RATE PROBABILITY FORMULATION

To estimate the bit error rate, we consider the bit timing jitter which can be broken up into (a) a component slow compared to the data rate and (b) a component fast compared to the data rate. For convenience, we use the frequency $f=R_b$ (the data rate) as the dividing point between region (a) and (b). Then we model the bit timing error $\epsilon(t)$ as the sum of two statistically independent processes in the form

$$\epsilon(t) = \epsilon_L(t) + \epsilon_H(t)$$

with $\epsilon_L(t)$ the below $R_b$ timing error component and $\epsilon_H(t)$ is above $R_b$ timing error component. We, then, assume that $\epsilon_L(t)$ is constant over the duration of one bit time and $\epsilon_H(t)$ varies very fast over one bit time. Let the respective variances be given by $\sigma_L^2$ and $\sigma_H^2$. Now the bit error rate, given that a transition occurs, is given by

$$PE(\epsilon | Tr) = Q(\sqrt{2R^2(1 - \frac{2|\epsilon_L|}{T})})$$

where

$$Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

or

$$R^2 = E[1 - \frac{2|\epsilon_L|}{T}]$$

(19)

(20)

(21)

(22)

Also, the bit error rate, given that no transition occurs, is given by

$$PE(\epsilon | \overline{Tr}) = Q(\sqrt{2R^2(1 - \frac{4|\epsilon_H|}{T})}) \quad |\epsilon| \leq \frac{T}{4}$$

(23)

where

$$R^2 = E[1 - \frac{4|\epsilon_H|}{T}]$$

or

$$R^2 = \frac{1}{\sqrt{2\pi} \sigma_H} \int_{-\infty}^{\infty} [1 - \frac{4|\epsilon_H|}{T}] e^{-\epsilon_H^2/2\sigma^2} d\epsilon_H$$

(24)

(25)

and, therefore, the resulting probability of error, for equally likely data, is given by
\[
PE = \frac{1}{4\pi \sigma_L} \int_{-\infty}^{\infty} e^{-\frac{t^2}{2\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{2\sigma^2}} \int_{-\infty}^{T} e^{-\frac{t^2}{2\sigma^2}} \frac{T}{\sqrt{2\pi}} (1 - \frac{2e_L}{T}) dt \, dt \, dt 
\]

where

\[
N_L \geq f_{NL} \leq R_b \\
\sigma_L^2 = 2 \int_{0}^{R_b} |1 - H(f)|^2 \mathcal{P}(f) \, df + \sum_{k=1}^{N_L} |1 - H(f_k)|^2 \frac{8^2}{k} 
\]

\[
\sigma_H^2 = 2 \int |1 - H(f)|^2 \mathcal{P}(f) \, df + \sum_{k=N_L}^{N_U} \frac{k \, \mathcal{P}(f_k)}{2} 
\]

where

\[
f_{NL} \leq R_b \\
f_{NL} + 1 \geq R_b 
\]

and also,

\[
f_{NU} = BW_i 
\]

with \( BW_i \) being the bandwidth into the bit synchronizer.
5.0 NUMERICAL EVALUATION OF $\sigma_\phi^2$

We need to evaluate the value of

$$\sigma_\phi^2 = 2\int_0^{f_u} S_\phi(f) \left|1-H(f)\right|^2 df, \text{ rad}^2$$ (32)

with $f_u$ being the upper limit of the timing jitter noise, as determined by the bandwidth into the bit synchronizer (250 kHz). R. Helgeson, has indicated to the author that the bit synchronizer loop bandwidth is between 2 and 10 Hz. Assuming a damping factor of 0.707, we have that

$$\left|1-H(f)\right|^2 = \frac{f^n}{f^n + f_n^4}$$ (33)

and $f_n = 0.3B_L = 0.6$ to 3 Hz. To obtain worst case, we let $f_n = 0.6$ Hz. Then (32) becomes

$$\sigma_\phi^2 = 2\int_0^{f_u} S_\phi(f) \frac{f^n}{f^n + 0.1296} df$$ (34)

where $f_u$ is the upper frequency limit and is 250 kHz, as is clear from the low-pass filter feeding the 3 bit A/D converter in Figure 3.

Evaluating the integral yields the bit jitter

$$\sigma_\phi = 0.64^\circ$$ (35)

or

$$\frac{\epsilon_{L}}{T} = 0.00178, \text{ fractions of a bit}$$ (36)

Note that we have included all the bit timing jitter (phase noise) as being below the data rate; whereas, a major portion exists above the data rate. However, this approach bounds the performance, since any other apportionment yields lower PE.

6.0 EVALUATION OF THE BER DEGRADATION

Using the worst case assumption for the phase (assumes that all the non zero density exists below $R_B$) yields a worst case estimate for the BER degradation. Using the value of 0.00178, we find using equation (26) or equivalently Figure 12.21 of Holmes [1], that 0.4 dB degradation occurs at $\frac{\epsilon_{L}}{T} = 0.015$.

Using linear extrapolation, again, a pessimistic assumption yields an estimated degradation of 0.04dB!
We have not included thermal noise, nor the fact that the loop is actually a discrete loop in our BER degradation estimate. It was understood that, the intent of the analysis was to ascertain the degradation based on bit timing jitter only, and not thermal effects or imperfections in the loop.

CONCLUSION

It was determined that the bit error rate (BER) degradation, due to DCU bit timing jitter, is approximately 0.04 dB. It is concluded that this is a negligible amount of degradation. Neither thermal noise, nor the discrete nature of the loop has been included in this analysis. It is felt that a full bore simulation would be needed to estimate the effects of the discrete loop. However, it is doubtful whether the discrete nature of the loop would add more than 0.1 dB. It was concluded that, due to the very involved nature of the loop, it would not be reasonable to construct a simulation.
REFERENCES


ADDENDUM 6

PERFORMANCE ANALYSES OF USING CONCATENATED-CODING
FOR CENTAUR SIGNALING THROUGH TDRSS
The diagram illustrates a concatenated coding scheme with Reed-Solomon outer code and convolutional inner code. The process begins with data input into a Reed-Solomon encoder (outer code), followed by a symbol interleaving buffer. The output then enters the convolutional encoder (inner code) and is transmitted through the channel. Upon reception, the data passes through a Reed-Solomon decoder (outer code) and symbol deinterleaving buffer, before being decoded by a Viterbi inner decoder. The supercoder and superdecoder organize the coding and decoding processes, respectively.
INNER AND OUTER CODING PARAMETERS

- INNER CODE PARAMETERS
  - CONVOLUTIONALLY ENCODED
  - VITERBI DECODING
  - \( R = \frac{1}{n} (1/2) \)
  - CONSTRAINT LENGTH = \( K (7) \)

- OUTER CODE PARAMETERS
  - REED-SOLOMON \((n,k)\) CODE
  - \( J \) = NUMBER OF BITS PER RS SYMBOL
  - \( n = 2^J - 1 \) NUMBER OF SYMBOLS PER RS CODEWORD \((255)\)
  - \( E \) = NUMBER OF ERRORS THAT CAN BE CORRECTED \((16)\)
  - \( k = n - 2E \) NUMBER OF RS SYMBOLS (OUT OF \( n \)) REPRESENTING INFORMATION \((223)\)
PERIODIC BLOCK INTERLEAVER STRUC TURE

\[ \{ \begin{align*} 2^J \text{ RS CODE SYMBOLS} &= J2^J \text{ BITS} \\ 2^J - 1 \text{ CODE SYMBOLS} + 1 \text{ SYNCHRONIZATION SYMBOL} \end{align*} \] 

\[ 2^J - (1 + 2E) \text{ INFORMATION SYMBOLS} \]

\[ 2E \text{ PARITY SYMBOLS} \]

I ROWS

(I RS WORDS)

ONE SYMBOL

ONE SYMBOL

IJ BIT SYNCHRONIZATION SEQUENCE

(I RS SYMBOL)

NOTES

(Delay = I words = I2^J RS symbols = IJ2^J bits)

(I \geq 16 \text{ typically for } K = 7)

(J \text{ bits per RS code symbol})
### Coding Gain Summary (dB)

<table>
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<th>CODES</th>
<th>$PE_b = 10^{-3}$</th>
<th>$PE = 10^{-5}$</th>
<th>$PE = 10^{-7}$</th>
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<tr>
<td>(a) CONVOL-VITERBI</td>
<td>4.0</td>
<td>5.35</td>
<td>6.0</td>
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<td>$K = 7, R = 1/2$</td>
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<td></td>
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<tr>
<td>(b) CONVOL-VITERBI</td>
<td>4.6</td>
<td>5.75</td>
<td>6.4</td>
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<tr>
<td>$K = 7, R = 1/3$</td>
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<td></td>
<td></td>
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<tr>
<td>(c) CONCAT RS: CONVOL</td>
<td>4.7</td>
<td>7.3</td>
<td>8.85</td>
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<td>$E = 16 (255,223), (7, 1/2)$</td>
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<td></td>
<td></td>
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<tr>
<td>(d) CONCAT RS: CONVOL</td>
<td>5.1</td>
<td>7.7</td>
<td>9.2</td>
</tr>
<tr>
<td>$E = 16 (255,223), (7, 1/3)$</td>
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<tr>
<td><strong>COMPARISON</strong></td>
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<tr>
<td>$\Delta((a)-(c))$</td>
<td>0.7</td>
<td>1.95</td>
<td>2.85</td>
</tr>
<tr>
<td>$\Delta((a)-(d))$</td>
<td>1.1</td>
<td>2.35</td>
<td>3.2</td>
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A COMPARISON OF SEVERAL CODING SCHEMES
(SIMULATION)

[Graph showing bit error rate vs. Eb/N0 for various coding schemes, including Shannon's limits and convolutional codes concatenated with Reed-Solomon codes.]

\[ E_b/N_0, \text{ dB} \]
TECHNICAL MEMORANDUM NO. M8404-1

TO: S. Novasad
FROM: U. Cheng

DATE: April 6, 1984
FILE: NAS9-16893

SUBJECT: Bit-Error Rate of the Concatenated-Coding System
With Multiple Encryptors

1.0 SUMMARY

The bit-error rate (BER) performance of a single-channel concatenated-coding system with multiple encryptors is discussed in this memorandum. The general block diagram of this system is shown in Figure 1.
The BER's of this system at the output of the RS decoder and each decryptor are shown in Figure 2. At BER = 10^{-3}, decryptor #1 causes a degradation of 0.14 dB, decryptor #2 causes an additional degradation of 0.023 dB and decryptor #3 causes an additional degradation of 0.01 dB. It is also worth noting that, due to the steepness of the curves, a small amount of degradation in SNR still implies a large difference in BER. All results presented in this memo are for the ideal concatenated-coding system. The effect of antenna dropout is not included.

2.0 INTRODUCTION

The BER performance of a single-channel concatenated-coding system with multiple encryptors is discussed in this memorandum. The general block diagram of this system is depicted in Figure 1.
The decryptor output-error model described in [1] can be summarized as follows:

• Each decryptor input error bit will result in an output error burst of a fixed length of 100 bits and a fixed bit-error density of 0.5

• If two input error bits are separated by less than 100 correct bits, the corresponding two output 100-bit error bursts abut or overlap to form an extended-length error burst with the bit-error density remaining 0.5.
Figure 1. The Concatenated-Coding System with Multiple Encoders
Figure 2. Performance of the Concatenated-Coding System with Multiple Encryptors. The Inner Code is the (7,1/2) Convolutional Code; the Outer Code is the (255,223) RS Code.
Let us call a string of correct bits of length $N$ which is sandwiched by two error bits as a wait of length $N$. Based on the above model, it is clear that a wait of length $N \geq 100$ in the input of the decryptor will result in a wait of length $N-99$ in the output of the decryptor. If all three decryptors shown in Figure 1 are identical, we have the result shown below.

**Lemma**

Let

$$N_j = \text{the length of a wait in the output of decryptor } \#j,$$

$$1 \leq j \leq 3$$

and

$$N_0 = \text{the length of a wait in the output of the RS decoder.}$$

A wait of length $N_0$ in the output of the RS decoder will then result in a wait of length $N_j$ in the output of decryptor $\#j$ such that

$$N_j = \begin{cases} 
0 & \text{if } N_0 \leq 99j \\
N_0 - 99j & \text{if } N_0 \geq 99j + 1
\end{cases} \quad (1)$$

The above result can be understood by a simple example. A wait of length 300 in the output of the RS decoder will result in a wait of length 201 in the output of decryptor $\#1$. This wait of length 201 will then result in a wait of length 102 in the output of decryptor $\#2$. Finally, a wait of length 3 will result in the output of decryptor $\#3$.

3.0 DECRYPTOR OUTPUT BER

Since the output of the RS decoder has very complicated error statistics, some assumptions must be made. We assume that the eight-bit RS symbols in the output of the RS decoder are mutually independent. Let $P_{SE}$ denote the RS symbol-error probability and $P_{BE}$ denote the bit-error probability. It is well known that

$$P_{BE} = \frac{2^{k-1}}{2^k - 1} P_{SE}$$
where we assume that each RS symbol contains \( l \) bits. If the eight-bit RS symbol is considered, one has

\[ P_{SE} = 0.502 P_{SE} \]

Letting \( L_s \) = the length of a run of incorrect RS symbols and \( N_s \) = the length of a run of correct RS symbols, clearly

\[ \text{Prob}[N_s = n] = (1 - P_{SE})^{n-1} P_{SE} \quad (3) \]

and

\[ \text{Prob}[L_s = l] = P_{SE} (1 - P_{SE}) \quad (4) \]

Therefore,

\[ L_s = E[L_s] = \frac{1}{1 - P_{SE}} \quad (5) \]

and

\[ N_s = E[N_s] = \frac{1}{P_{SE}} \quad (6) \]

Now, a wait of length \( N_s \) RS symbols in the output of the RS decoder will result in a wait of length \( N_j \) bits in the output of decryptor \#j such that

\[ N_j = \begin{cases} 
0 & \text{if } N_s \leq C_{Sj} - 1 \\
8N_s - 99j & \text{if } N_s > C_{Sj}
\end{cases} \quad (7) \]

where

\[ C_{Sj} = \begin{cases} 
13 & \text{if } j=1 \\
25 & \text{if } j=2 \\
38 & \text{if } j=3
\end{cases} \]

It is easy to see that
\[ \text{Prob}\{ N_S \geq C_{Sj} \} = (1 - P_{SE})^{C_{Sj} - 1} \]

and

\[ E\{ N_S \mid N_S \geq C_{Sj} \} = \frac{\sum_{n=C_{Sj}}^{\infty} n(1 - P_{SE})^{n-1} P_{SE}}{(1 - P_{SE})^{C_{Sj} - 1}} = \frac{1 + (C_{Sj} - 1)P_{SE}}{P_{SE}} \]

Therefore, the BER in the output of decryptor \#j can be expressed as

\[ P_{BEj} = \frac{1}{2} \left[ 1 - \frac{\text{Prob}\{ N_S \geq C_{Sj} \} \cdot E\{ N_j \}}{8(N_S + N_S)} \right] = \frac{1}{2} \left( 1 - (1 - P_{SE})^{C_{Sj}} \left[ 1 + C_{Sj} - \left( 1 - \frac{99}{8} \right)P_{SE} \right] \right) \]

Hence, one ends up with

\[ P_{BE1} = \frac{1}{2} \left( 1 - (1 - P_{SE})^{13} \left( 1 - \frac{3}{8} P_{SE} \right) \right) \quad (8) \]

\[ P_{BE2} = \frac{1}{2} \left( 1 - (1 - P_{SE})^{25} \left( 1 - \frac{6}{8} P_{SE} \right) \right) \quad (9) \]

\[ P_{BE3} = \frac{1}{2} \left( 1 - (1 - P_{SE})^{38} \left( 1 - \frac{1}{8} P_{SE} \right) \right) \quad (10) \]
Given $P_{BE}$ at the output of the RS decoder, $P_{BE1}$, $P_{BE2}$ and $P_{BE3}$ can be computed by (2), (8), (9) and (10). These results are plotted in Figure 2. At $BER = 10^{-3}$, decryptor #1 causes a degradation of 0.14 dB, decryptor #2 causes an additional degradation of 0.023 dB, and decryptor #3 causes an additional degradation of 0.01 dB. It is also worth noting that, due to the steepness of the curves, a small amount of degradation in SNR still implies a large difference in BER.

Reference

In this memorandum, the bit-error rate (BER) of the concatenated-coding system with differential encoding is studied. The differential encoder should be implemented before the convolutional encoder and after the RS code encoder and RS symbol interleaver. This scheme is shown in Figure 2. The BER's of the concatenated-coding system with and without differential encoding are shown in Figure 3, which illustrates that the use of differential encoding will cause a degradation of 0.02 dB in a wide range of BER \(10^{-3} - 10^{-7}\). The result presented in this technical memorandum is for the ideal concatenated-coding system. The effect of antenna dropout is not included.

The BER of the concatenated-coding system with differential encoding is studied. The two possible ways in which to implement the differential encoder are listed below:

1. The differential encoder is implemented before the RS encoder

2. The differential encoder is implemented after the RS encoder and RS symbol interleaver, but before the convolutional encoder.

The general block diagrams of the concatenated-coding system with differential encoding are depicted in Figures 1 and 2 for the first and second methods, respectively. The first method is worse than the second for the following two reasons:

1. The RS coder used in the first method must satisfy the property that its generator polynomial does not contain \(x-1\) as a factor. Due to this requirement, the RS code with the reversible generator polynomial introduced by Berlekamp [1] cannot be used.

2. Due to the symbol deinterleaver before the RS decoder, the differential decoder in the first method may cause more degradation than the second method.
Figure 1. The Concatenated-Coding System with Differential Encoding; The First Method
Figure 2. The Concatenated-Coding System with Differential Encoding; The Second Method
Figure 3. The BER of the Concatenated-Coding System With and Without Differential Encoding; The (7,1/2) Convolutional and the (255,223) RS Codes Are Used
Let us consider the second method in the following. We first note that the Viterbi decoder will generate error bursts at its output. The differential decoder will then process the data with these error bursts. After passing the differential decoder, the number of error bursts will not be changed and the length of each error burst will be increased by one. Assume that there are M error bursts in a string of N output bits at the output of the Viterbi decoder. Let $B_1, B_2, \ldots, B_M$ denote the length of these M error bursts. For the case without differential encoding, the number of $l$-bit RS symbols which can be in error is upper bounded by [2]

$$
\sum_{i=1}^{M} \frac{B_i + l - 1}{2} \leq 2^l
$$

(1)

Therefore, the RS symbol error probability at the input of the RS decoder is

$$
P_{CSE} = \frac{\sum_{i=1}^{M} \frac{B_i + l - 1}{2}}{N} = \frac{(l-1)M}{N} + \frac{M}{N}
$$

(2)

For the case with differential encoding, the number of $l$-bit RS symbols which can be in error is upper bounded by

$$
\sum_{i=1}^{M} \frac{(B_i+1) + l - 1}{2} \leq 2^l
$$

(3)

Therefore, the RS symbol error probability at the input of the RS decoder is, in this case, given by

$$
P_{CSE} = \frac{\sum_{i=1}^{M} \frac{B_i}{2}}{N} + \frac{M}{N}
$$

(4)

Let $P_{BE}$ = bit-error probability in the output of the Viterbi decoder, $\theta$ = bit-error rate in the error bursts at the output of the Viterbi decoder, and $P_{Burst}$ = average number of error bursts in the output of the Viterbi decoder. Then one has
The simulation results on \( P_{\text{Burst}} \), \( P_{\text{BE1}} \), and \( \theta \) for \((7,1/2),(7,1/3),(10,1/2)\) and \((10,1/3)\) convolutional codes are available in [2]. The bit-error probability following the RS decoder can be computed by the following equation:

\[
P_{\text{BE}} \leq \left( \frac{2}{2^e+1} \right) \sum_{i=e+1}^{\frac{l}{2}} \frac{1}{2} \binom{\frac{l}{2}}{i} 1 - P_{\text{CSE}} \left( 1 - P_{\text{CSE}} \right)^{2^i-1} \]

where \( P_{\text{CSE}} \) is the RS symbol error probability at the input of the RS decoder, each RS symbol contains \( l \) bits and \( 2e+1 \) is the designed minimum distance of the RS code.

In Figure 3, we show the BER of the concatenated-coding system with and without differential encoding. It shows that the use of differential encoding will cause a degradation of 0.02 dB in a wide range of BER \((10^{-3}-10^{-7})\).

References


ADDENDUM 7

INVESTIGATION OF POTENTIAL INTERFACE
BETWEEN THE SHUTTLE AND THE SPACE STATION
SPACE STATION RETURN-LINK

MULTIPLE-ACCESS COMMUNICATION STUDY

Interim Report

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Axiomatix Report No. R8310-3
October 14, 1983
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>iii</td>
</tr>
<tr>
<td>1.0 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.0 EXECUTIVE SUMMARY</td>
<td>2</td>
</tr>
<tr>
<td>3.0 BASELINE COMMUNICATION REQUIREMENTS</td>
<td>4</td>
</tr>
<tr>
<td>4.0 MULTIPLE-ACCESS LINK-DESIGN TRADE-OFFS</td>
<td>7</td>
</tr>
<tr>
<td>4.1 Frequency-Division Multiple-Access</td>
<td>7</td>
</tr>
<tr>
<td>4.1.1 Introduction</td>
<td>7</td>
</tr>
<tr>
<td>4.1.2 Discussion</td>
<td>7</td>
</tr>
<tr>
<td>4.2 Frequency-Hop FDMA for Antijam Performance</td>
<td>19</td>
</tr>
<tr>
<td>4.2.1 Introduction</td>
<td>19</td>
</tr>
<tr>
<td>4.2.2 Discussion</td>
<td>19</td>
</tr>
<tr>
<td>4.3 Time-Division Multiple-Access Return Links</td>
<td>27</td>
</tr>
<tr>
<td>4.3.1 Introduction</td>
<td>27</td>
</tr>
<tr>
<td>4.3.2 Baseline TDMA System Example</td>
<td>27</td>
</tr>
<tr>
<td>4.4 TDMA and Antijam Modulation</td>
<td>34</td>
</tr>
<tr>
<td>5.0 MULTIPLE-ACCESS ANTENNA-DESIGN CONSIDERATIONS</td>
<td>48</td>
</tr>
<tr>
<td>APPENDIX A - VIDEO DATA-COMPRESSION TECHNIQUES STUDY</td>
<td></td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>The FDMA Concept as Applied to the Communication Satellite Repeater</td>
</tr>
<tr>
<td>4.2</td>
<td>Typical MOTV of EMU FDMA User Return-Link Communication System</td>
</tr>
<tr>
<td>4.3</td>
<td>FDMA Channel Modulation with Ranging Option</td>
</tr>
<tr>
<td>4.4</td>
<td>Functional Block Diagram of the User FDMA Transmitter/Receiver with a PN Ranging Provision</td>
</tr>
<tr>
<td>4.5</td>
<td>FDMA Channel Allocation Allowing Maximum Simultaneous Return-Link Services</td>
</tr>
<tr>
<td>4.6</td>
<td>Comparison of Power Spectral Densities for Various Digital Modulation Formats</td>
</tr>
<tr>
<td>4.7</td>
<td>FDMA Channel Allocation That Meets User Return-Link Requirements Utilizing the Minimum RF Bandwidth</td>
</tr>
<tr>
<td>4.8</td>
<td>Space Station FDMA Receiver</td>
</tr>
<tr>
<td>4.9</td>
<td>Potential AGC Scheme to Accommodate 110-dB Dynamic-Range Requirement</td>
</tr>
<tr>
<td>4.10</td>
<td>Worst-Case $P_D$ versus $E_b/N_j$; FH-QPSK (Tone Jamming)</td>
</tr>
<tr>
<td>4.11</td>
<td>Basic Phase Estimator for $m$-FSK Carriers</td>
</tr>
<tr>
<td>4.12</td>
<td>Functional Block Diagram of the User FH-FDMA Transmitter/Receiver</td>
</tr>
<tr>
<td>4.13</td>
<td>Space Station FH-FDMA Receiver</td>
</tr>
<tr>
<td>4.14</td>
<td>TDMA Channelization with $M$-ary MFSK Modulation</td>
</tr>
<tr>
<td>4.15</td>
<td>TDMA Receiver</td>
</tr>
<tr>
<td>4.16</td>
<td>$P_T(\text{min})$ versus Range: Return-Link TDMA (15.84 Mbps)</td>
</tr>
<tr>
<td>4.17</td>
<td>$P_T(\text{min})$ versus Range: Return-Link TDMA (5.6 Mbps)</td>
</tr>
<tr>
<td>4.18</td>
<td>$P_T(\text{min})$ versus Range: Return-Link TDMA (484 kbps)</td>
</tr>
<tr>
<td>4.19</td>
<td>$P_T(\text{min})$ versus Range: TDMA 15.84-Mbps Burst Rate</td>
</tr>
<tr>
<td>4.20</td>
<td>$P_T(\text{min})$ versus Range: TDMA 5.6-Mbps Burst Rate</td>
</tr>
<tr>
<td>4.21</td>
<td>$P_T(\text{min})$ versus Range: TDMA 464-kbps Burst Rate</td>
</tr>
<tr>
<td>4.22</td>
<td>Baseline TDMA/FH System</td>
</tr>
<tr>
<td>4.23</td>
<td>The 4-ary MFSK with J/S Jamming Tunes</td>
</tr>
<tr>
<td>4.24</td>
<td>Time/Frequency Utilization with Interleaved TDMA/FM, Multiple Symbols per Hop</td>
</tr>
<tr>
<td>4.25</td>
<td>$m$-Diversity Frequency Hopping</td>
</tr>
<tr>
<td>5.1</td>
<td>Gains and Beamwidths of Commercially Available Parabolic-Dish Antennas</td>
</tr>
<tr>
<td>5.2</td>
<td>CCD Video-Camera Candidate for Visual-Acquisition Aid</td>
</tr>
<tr>
<td>5.3</td>
<td>Hemispheric Array of Parabolic Dishes</td>
</tr>
<tr>
<td>5.4</td>
<td>RF Switching Network Using Ferrite-Latching Circulators</td>
</tr>
<tr>
<td>5.5</td>
<td>Five-Horn Cross Feed with Dielectric Polarizers and Orthomode Transducer for Transmit/Receive</td>
</tr>
</tbody>
</table>
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Baseline User Group, MA Return Link</td>
<td>5</td>
</tr>
<tr>
<td>3.2</td>
<td>Expanded User Group, MA Return Link</td>
<td>5</td>
</tr>
<tr>
<td>4.1</td>
<td>Hard-Decision Coded-FH Performance</td>
<td>23</td>
</tr>
<tr>
<td>4.2</td>
<td>Hard-Decision Coded-FH/PN Performance</td>
<td>23</td>
</tr>
<tr>
<td>4.3</td>
<td>Summary of AJ Performance for SS FH-FDMA System</td>
<td>24</td>
</tr>
<tr>
<td>4.4</td>
<td>Energy Per Bit* and Bandwidth (Unhopped) Required as a Function of Signal Alphabet with Noncoherent Demodulation, $P_B &lt; 10^{-5}$</td>
<td>39</td>
</tr>
<tr>
<td>4.5</td>
<td>Energy Per Bit Required, $P_B &lt; 10^{-2}$</td>
<td>39</td>
</tr>
<tr>
<td>4.6</td>
<td>Maximum $J/S$ with $E_c/N_j = 20$ dB, $R = 15.84$ Mbps</td>
<td>44</td>
</tr>
<tr>
<td>4.7</td>
<td>Maximum $J/S$ with $E_c/N_j = 10.4$ dB, $R = 15.34$ Mbps</td>
<td>46</td>
</tr>
<tr>
<td>5.1</td>
<td>Idealized Planar-Phased-Array Parameters for Various Gain Levels</td>
<td>49</td>
</tr>
<tr>
<td>5.2</td>
<td>Parabolic-Dish Dimensions for Higher Gains</td>
<td>52</td>
</tr>
<tr>
<td>5.3</td>
<td>Parabolic-Dish Spherical Coverage Using Idealized Gain Values</td>
<td>52</td>
</tr>
</tbody>
</table>
1.0 INTRODUCTION

This report discusses several key issues relating to Space Station (SS) communications. The type of communications discussed are between the SS and a group of 18 users consisting of a combination of free fliers, unmanned orbital-transfer vehicles (OTV's), manned orbital-transfer vehicles (MOTV's), teleoperator maneuvering systems (TV's), and extravehicular mobility units (EMU's). The return-link communications from the aggregate of the users is referred to as the user multiple-access (MA) return link. Because the MA return links must simultaneously support TV links, voice links and telemetry links from a multitude of users, the network comprising these links poses a more difficult design than the forward link (SS to user). Thus, this report concentrates on trade-offs and performance issues for the return MA links.

Since the highest data rate which the MA return link must handle is for digital TV, it is appropriate to include a discussion of digital TV herein. Thus, a comprehensive survey and analysis of digital TV techniques and implementations comprise a vital part of this report. Motivated by a desire to provide AJ protection to the TV links, several digital TV systems capable of bit rates on the order of 2 Mbps are discussed.

The SS MA return-link antenna-design requirements are particularly challenging due to the MA return-link requirements to simultaneously communicate with up to 18 users (to a range of 2000 km). Consequently, Section 5 of this report discusses several antenna approaches that can be considered in meeting these requirements. Also included in Section 6 is the performance of solid-state, low-noise amplifiers (LNA's) and high-power amplifiers (HPA's) as a function of frequency. The performance obtainable from these devices will have a strong influence on the choice of MA link frequency. At present, it is felt that, because of the military-satellite work at 44 GHz and the NASA work at 30 GHz, a good choice of high-performance devices will be available at these frequencies.
2.0 EXECUTIVE SUMMARY

Axiom has examined alternative modes of return-link (user to Space Station) multiple-access (MA) communications. Basic requirements of the Space Station (SS) return-link system consist of the following:

1. Service 18 users simultaneously, seven with television (TV) and four with voice.
2. User vehicle range can vary from 100 m to 2000 km.

Data rates can vary from 2.048 Mbps (compressed) to 64 kbps slow scan. If required, 22 Mbps TV service can be provided by reconfiguring the receiver to a single TV channel and expanding the bandwidth slightly. Telemetry channels run at 64 kbps, which includes user synchronization, and voice (delta modulation) channels run at 16 kbps.

Two methods of MA channelization in the presence of jamming are examined: frequency-division MA (FDMA) and time-division MA (TDMA). A third, code-division MA (CDMA), is not considered practical with this number of users in a jammed environment. Principal trade-offs between FDMA and TDMA are complexity versus timing requirements. FDMA has the following advantages relative to TDMA:

1. Lower burst rate and lower user EIRP
2. Greater MA network capacity in the antijam (AJ) mode.

TDMA has the following advantages relative to FDMA:

1. Less complex receiver hardware
2. Less complex antenna: single-beam versus multiple simultaneous beams
3. Does not require user power control for simultaneous near/far operations.

The SS is assumed to be the master node in either system.

The performance of phase-shift and frequency-shift-keying (FSK) modulation is examined. Quadrature-phase-shift-keying (QPSK) modulation is practical for the longer symbol times used in FDMA. Methods of coherent and noncoherent demodulation are discussed, including a new technique for acquiring rapid phase reference. Multiple-frequency-shift-keying (MFSK) is examined; it is shown that 4-ary MFSK is superior to higher order modulations for conserving the data bandwidth.
For systems-comparison purposes, it is assumed that error-correcting coding is employed, with a channel bit-error rate (BER), $P_c$, less than $10^{-2}$, and a decoder BER, $P_B$, less than $10^{-5}$.

The FDMA system with QPSK and coherent demodulation has the performance edge over TDMA; however, the complexity of the FDMA system, particularly the antenna requirements for 18 simultaneous high-gain beams, may be the deciding factor. At this point, a clear-cut choice of multiple-access technique (i.e., TDMA or FDMA) cannot be made. Further analyses and hardware trade-offs are required.
3.0 BASELINE COMMUNICATION REQUIREMENTS

The communication systems described in subsequent sections address the multiple-access (MA) return-link requirements, which are a subset of the Space Station (SS) communication requirements described in "Space Station Program Description Document, System Requirements and Characteristics, Book 3," published in November 1982. Users in the MA network, derived from Table 2.7-2 of the requirements document, includes six free fliers (FF's), two orbital-transfer vehicles (OTV's), two manned orbital-transfer vehicles (MOTV's), two teleoperation-maneuvering systems (TMS's) and six extravehicular-mobility units (EMU's), for a total of 18 network members. Users range is from 100 m to 2000 km. Each user is given one 64-kbps telemetry channel. In addition, any seven of the 18 users can be given a 2-Mbps TV channel upon request. The manned units require voice on the return link. Any two of the six EMU's and two OTV's can be given a voice channel.

The above requirements are considered a baseline for the MA system. Enhancements will increase the capacity of the system and additional requirements for MA at extreme ranges can reduce system accessibility. The ensemble of users is outlined in Tables 3.1 and 3.2. Table 3.1 shows a baseline group of 18 users, with TV limited to a 2-MHz bandwidth. Table 3.2 depicts an expanded group of users, with one user having a 22-MHz TV channel. The MA user community consists of unmanned OTV's, FF's and TMS's, and manned OTV's and EMU's. The difference in communication requirements between the unmanned and manned units is the voice requirement for the manned units. The exact capacity of the expanded system in mode 2 is dependent on the specific overhead requirements of the various MA schemes considered.

We also discuss the acquisition requirements of the MA schemes considered. Acquisition is a cooperative procedure between the SS and a user requesting access to the network. In particular, accurate time synchronization is needed to accommodate MA waveforms with minimum multiuser interference. The requirement for time synchronization implies that relative ranges be established between the user and the SS, with either a navigation or an acquisition protocol. More detailed discussions of acquisition are covered in later sections describing specific MA schemes, but an underlying assumption is that the SS is the master node.
Table 3.1. Baseline User Group, MA Return Link

<table>
<thead>
<tr>
<th>Number of Simultaneous Users</th>
<th>Data Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2 Mbps TV or digital data</td>
</tr>
<tr>
<td>plus</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>64 kbps telemetry, including synchronization bits</td>
</tr>
<tr>
<td>plus</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>16 kbps digital voice</td>
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Table 3.2. Expanded User Group, MA Return Link

<table>
<thead>
<tr>
<th>Mode</th>
<th>Number of Simultaneous Users</th>
<th>Data Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 plus 18 plus 4</td>
<td>22 Mbps TV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>64 kbps telemetry</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16 kbps voice</td>
</tr>
<tr>
<td>2</td>
<td>Any combination of 2 Mbps TV, 64 kbps telemetry and 16 kbps voice requiring less than 25 Mbps total capacity</td>
<td></td>
</tr>
</tbody>
</table>
In addition to range knowledge requirements for time synchronization, MA schemes employing simultaneous reception of energy from multiple users (non-TDMA systems) may require transmit-power control by each user, based on range, in order to ensure that the SS receiver is needed to prevent close-in users from suppressing far-range users. All system designs considered incorporate AJ modulation.
4.0 MULTIPLE-ACCESS LINK-DESIGN TRADE-OFFS

4.1 Frequency-Division Multiple Access

4.1.1 Introduction

This section discusses the use of frequency-division multiple access (FDMA) for meeting the return-link requirements of the users given in Table 3.1. Although the discussion given here addresses a clear mode, i.e., a non-antijam communication network, it will be shown in section 4.2 that the FDMA concepts outlined here can be extended to provide antijam (AJ) protection.

One of the most common uses of FDMA is seen in communication-satellite repeaters, wherein a major communication resource--the satellite repeater--is shared by a large number of users who access the repeater by employing different carrier frequencies. This concept is illustrated in Figure 4.1. Each of the several transponders can be viewed as a separate resource that can be shared by utilizing FDMA. A typical 36-MHz repeater can have an FDMA scheme wherein approximately 1600 users, each having a bandwidth of about 20 kHz, can access the transponder simultaneously. In this discussion, this concept will be extended to the Space Station (SS).

4.1.2 Discussion

The FDMA user return links must be capable of supporting television, voice, and telemetry. For each user, these services could be FDM'd together before transmission. However, since each service would already be digitally encoded, TDMA makes much more sense when combining. This concept is shown in Figure 4.2, which illustrates a typical FDMA user employing all three services, such as MOTV or EMU. Prior to error-correction encoding, the required data rate (referring to Appendix A) would be 2 Mbps for TV, plus 16 kbps for voice, plus 64 kbps for telemetry, or 2.08 Mbps. When rate-1/2 error-correction encoding is used, the symbol rate is 4.16 Msps. QPSK or SQPSK modulation could be used, so that the required RF channel bandwidth would be on the order of $1.5 \times 2.08 = 3\mathrm{MHz}$. However, it is desirable to include a ranging capability. This capability is readily provided by means of a PN range code and can be conveniently added via quadrature phase modulation of the carrier, as shown in Figure 4.3. The PN range code could also be added by exclusive OR'ing with the symbol stream that is split in two, which is also shown in Figure 4.3. A top-level functional block diagram of a user transmitter/receiver with ranging provisions is shown in Figure 4.4.
Figure 4.1. The FDMA Concept as Applied to the Communication Satellite Repeater
Figure 4.2. Typical MOTV of EMU FDMA User Return-Link-Communication System
(a) Channel Modulation, No Ranging

(b) Channel Modulation with Ranging, Option 1

(c) Channel Modulation with Ranging, Option 2

Figure 4.3. FDMA Channel Modulation with Ranging Option
Figure 4.4. Functional Block Diagram of the User FDMA Transmitter/Receiver with a PN Ranging Provision
A candidate FDMA channel allocation is depicted in Figure 4.5. This allocation allows all seven TV users to transmit a return-video signal simultaneously. Each of the remaining 11 users are allocated a 160-kbps channel which will simultaneously provide both voice and telemetry services. This FDMA allocation plan requires approximately 55 MHz of RF bandwidth, assuming a frequency guard band of 0.5 MHz between the full-service (TV, voice and telemetry) channels and 40 kHz between the voice and telemetry channels. The choice of frequency guard bandwidth is dependent on user transmitter-oscillator stability and adjacent channel interference from modulation spectral spillover. The modulation spectral spillover is dependent on the specific modulation format, the linearity of the user transmitter, and/or the SS receiver. Since the user transmitter power amplifier would probably be operated in a saturated mode, a modulation that minimizes spectral spillover with a hard limiter will be chosen. Some of the modulation candidates are staggered quadrature-phase-shift-keying (SQPSK) or tamed frequency modulation (TFM). A plot of the relative modulation rolloff of these and several other spectrally efficient modulations is shown in Figure 4.6. An additional driver on guard bandwidth is intermodulation products. The exact choice of guard bandwidth must be investigated further.

Another possible FDMA channel allocation is shown in Figure 4.7. While it does not provide the full flexibility of the previously discussed allocation, this allocation does meet the minimum requirements for simultaneous user MA return-link service. Furthermore, it requires only 22 MHz of RF bandwidth compared to 55 MHz for the former allocation. The best choice of FDMA allocation requires further investigation but, for the purpose of this report, the former allocation (i.e., 55 MHz RF bandwidth) is chosen as the FDMA baseline.

Figure 4.8 shows the conceptual top-level block diagram of the SS return-link FDMA receiver. The common resource shared among MA users is the receiver front end, including the first two downconverters and IF's. Beyond that, a DMOD and a DMUX are required for each user channel, which leads to as many as 18 DMOD's and 18 DMUX's for the SS MA return-link service for FDMA.

This proliferation of receiver equipment is the result of any FDMA scheme wherein the stringent timing requirements and high burst rates of TDMA are traded off for an FDMA receiver entailing a much greater amount of hardware.

As indicated in the block diagram of Figure 4.8, AGC is provided at the front end of the receiver. However, the approximately 110-dB dynamic range required to accommodate the distance range of the MA user return links poses a
7 Full-Service Channels

11 Voice and TLM Channels

6.5 MHz $G_1$

$G_1 = \text{Guardband for Full-Service Channels (\sim 0.5 MHz)}$

49 MHz

$G_2 = \text{Guardband for Voice and TLM Channels (\sim 40 kHz)}$

5.7 MHz

Total FDMA RF Bandwidth = 55 MHz

Figure 4.5. FDMA Channel Allocation Allowing Maximum Simultaneous Return-Link Services
Figure 4.6. Comparison of Power Spectral Densities for Various Digital Modulation Formats
Figure 2.7. FDMA Channel Allocation That Meets User Return-Link Requirements Utilizing the Minimum RF Bandwidth
Figure 4.8. Space Station FMA Receiver
a difficult problem for the AGC design as well as maintaining the required linear-amplitude range. Two approaches can be considered in solving this problem. The first approach is to group the users' frequency allocations into two 55-dB-amplitude groups. As an example, the first group would be the "close-in" group and the distance range of this group would correspond to a signal-power range of 55 dB. Likewise, the second, or "far-away," group would also correspond to a 55-dB signal-power range. The total RF band could then be divided into two, with one half of the band dedicated to each user group. Then, the IF amplifier and AGC control scheme shown in Figure 4.9 could be used to provide AGC independently as well as 55 dB of dynamic range to each group.

The second approach in accommodating the 110-dB signal-power range is to use active power control on the return link. This would effectively narrow the signal-power range to something much more tractable, say, on the order or 50 to 60 dB, or even less if required. Users of the more distant ranges (up to 2000 km) would utilize their full EIRP, whereas users at the closer ranges would lower their EIRP. The user could determine its appropriate EIRP setting by either forward-link command or sensing the AGC level of the user forward-link receiver.
Figure 4.9. Potential AGC Scheme to Accommodate 110-dB Dynamic-Range Requirement
4.2 Frequency-Hop FDMA For Antijam Performance

4.2.1 Introduction

The concepts of FDMA that were discussed in section 4.1 can be extended to provide antijam (AJ) protection in which AJ can be added by frequency hopping the carrier frequencies. This section discusses the approaches and performance of FH FDMA for the space section (SS) multiple-access (MA) links.

4.2.2 Discussion

AJ protection can be obtained by either direct-sequence (DS) spreading or frequency hopping (FH) the data. In the US case, the 2.160-Mbps data rate necessitates an inordinately high and impractical PN chip rate, e.g., in order to obtain 30 dB of processing gain, a PN code rate of $2 \times 10^9$ chips per second would be required. This high a rate is unworkable for many reasons.

If frequency hopping is utilized, the full-service user (2.160 Mbps) would be spread over approximately 2 GHz so as to obtain 30 dB of processing gain. The amount of AJ protection is determined from the relationship:

$$J/S_{dB} = P_G_{dB} - E_b/N_j_{dB}$$

where $E_b/N_j$ is the energy-per-bit-to-noise-density ratio necessary to meet the performance requirements; usually, a final BER of $10^{-5}$. The $E_b/N_j$ also depends on the choice of data modulation and type of jammer. In keeping with the concept of extending the FDMA concepts to AJ, we shall first consider QPSK data modulation. The $E_b/N_j$ required for QPSK is determined from the curve given in Figure 4.10. We shall consider the performance requirement to be BER = $10^{-2}$ (uncoded which, assuming a rate-1/2 convolutional code with Viterbi decoding, is equivalent to BER = $10^{-5}$ coded). Then, from Figure 4.10, $E_b/N_j$ is approximately 11 dB, an amount to which we must add 3 dB in order to account for the rate-1/2 coding. Thus, the $E_b/N_j$ required for FH-QPSK against a tone jammer is approximately 14 dB. This means that, for the 2-GHz hopping bandwidth, i.e., $P_G = 30$ dB, the $J/S$ is only 16 dB. By increasing the hopping bandwidth to 4 GHz and using soft-decision decoding, the $J/S$ is approximately 20 dB.
Figure 4.10. Worst-Case $P_b$ versus $E_b/N_J$; FH-QPSK (Tone Jamming)
Use of QPSK modulation requires that a coherent carrier reference be obtained. This can be accomplished by either closed-loop techniques, such as a Costas carrier-tracking loop, or open-loop phase-estimation techniques. The open-loop phase-estimation technique is preferred since it requires far less time than a Costas loop to generate a coherent phase reference. This is particularly true since the frequency hopper would be noncoherent; thus, requiring that a new phase acquisition be accomplished at the start or each hop. If we postulate a hopping rate of \( f_h = 4 \times 10^3 \) hops/second, there are approximately \( \frac{2 \times 10^6}{4 \times 10^3} = 500 \) bits per hop. By using an open-loop phase-estimation technique, as described by Viterbi and Viterbi [2], it should be possible to lose no more than 25 of these bits to the phase-acquisition process at the start of each hop, i.e., approximately 0.2 dB loss. A block diagram of the Viterbi phase-estimation process is shown in Figure 4.11. Since this concept is relatively new, it should be investigated further for its applicability to SS MA communication.

An alternate data modulation to coherent QPSK that may be suitable to SS AJ communications is M-ary FSK. This modulation offers the advantage of not requiring a coherent reference. If straight FH MFSK is used, the AJ performance will be approximately 6 dB worse than in the QPSK case. This result is based on the 19.8 dB \( E_b/N_0 \) required for 8-ary FSK, as shown in Table 4.1 [3]. However, if a hybrid FH scheme is used in which DS PN is added, the AJ performance is found to be almost the same as that for QPSK. This is based on the \( E_b/N_0 \) required of 13.6 dB, as shown in Table 4.2 [3]. The disadvantage of this FH/PN hybrid AJ modulation is the increased system complexity. The best choice of data modulation and AJ modulation requires further investigation, but, at this point, FH-QPSK is recommended as the baseline for the SS FDMA AJ network.

The J/S of 20 dB that was derived in the preceding paragraphs may not meet the requirements of a specified jamming threat. If this is the case, two options are open. These are: (1) reduce the data rate, or (2) utilize a null-steering antenna. It is important to note that the 20-dB J/S is for the full-service user, i.e., TV plus voice plus telemetry. Thus, a user requiring voice and telemetry only, i.e., 160 kbps, represents a data-rate reduction factor of 12.5, or a J/S increase of approximately 11 dB to J/S = 31 dB. Likewise, a user requiring only slow-scan TV transmission and utilizing a 64-kbps slow-scan TV system with 160 kbps of voice and telemetry would have a J/S of approximately 30 dB. These J/S ratios are summarized in Table 4.3. Therefore, it is necessary to further investigate the user-community data requirements in a jamming scenario as well as the jamming threat to specify the SS AJ communication system.
Figure 4.11. Basic Phase Estimator for m-PSK Carriers
Table 4.1. Hard-Decision Coded-FH Performance

<table>
<thead>
<tr>
<th>Error-Correcting-Code Technique</th>
<th>Required $\left(\frac{E_b}{N_0}\right)_{\text{eff}}$</th>
<th>Worst-Case Jamming</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FH/DPSK (dB)</td>
<td>FH/DQPSK (dB)</td>
</tr>
<tr>
<td>No Coding:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_b = 10^{-3}$</td>
<td>27.0</td>
<td>24.1</td>
</tr>
<tr>
<td>$P_b = 10^{-5}$</td>
<td>47.0</td>
<td>44.1</td>
</tr>
<tr>
<td>$R = 1/2$ Convolutional Viterbi Decoding:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_b = 10^{-3}$</td>
<td>13.8</td>
<td>10.9</td>
</tr>
<tr>
<td>$P_b = 10^{-5}$</td>
<td>17.7</td>
<td>14.8</td>
</tr>
</tbody>
</table>

Table 4.2. Hard-Decision Coded-FH/PN Performance

<table>
<thead>
<tr>
<th>Error-Correcting-Code Technique</th>
<th>Required $\left(\frac{E_b}{N_0}\right)_{\text{eff}}$</th>
<th>Worst-Case Jamming</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FH-PN/DPSK (dB)</td>
<td>FH-PN/DQPSK (dB)</td>
</tr>
<tr>
<td>No Coding:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_b = 10^{-3}$</td>
<td>22.6</td>
<td>23.3</td>
</tr>
<tr>
<td>$P_b = 10^{-5}$</td>
<td>42.6</td>
<td>43.3</td>
</tr>
<tr>
<td>$R = 1/2$ Convolutional Viterbi Decoding:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_b = 10^{-3}$</td>
<td>9.5</td>
<td>10.0</td>
</tr>
<tr>
<td>$P_b = 10^{-5}$</td>
<td>13.4</td>
<td>14.0</td>
</tr>
</tbody>
</table>
Table 4.3. Summary of AJ Performance for SS FH-FDMA System

<table>
<thead>
<tr>
<th>User Service</th>
<th>Data Rate</th>
<th>J/S (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full (TV, TLM, Voice)</td>
<td>2.08 Mbps</td>
<td>20</td>
</tr>
<tr>
<td>Partial, No TV (TLM, Voice)</td>
<td>80 kbps</td>
<td>34</td>
</tr>
<tr>
<td>Partial, Slow-Scan TV (SCTV, TLM, Voice)</td>
<td>144 kbps (64 kbps for SCTV)</td>
<td>32</td>
</tr>
</tbody>
</table>

A block diagram of the FH-FDMA user transmitter/receiver and Space Station FH-FDMA receiver is illustrated in Figures 4.12 and 4.13, respectively. The most significant aspect of this frequency-hop scheme is that all users hop synchronously (adjusted for individual range to SS) so that the Space Station requires only one dehopper and synthesizer, as shown in Figure 4.13. However, this requires each user to adjust its hop clock phase so that its frequency hopping is in exact synchronization with the SS dehopper frequency at the SS. This is accomplished via the user tracking the forward link and exchanging its time-of-arrival information with the SS via a synchronization channel. This approach is commonly used in military AJ communication systems. Provisions for the synchronization channel are depicted in Figure 4.12.

The link calculation for the FH-FDMA return link at 30 GHz results in the following system parameters:

- User transmitter power: 10 W
- User transmitter antenna gain: 0 dB
- SS receive antenna gain: 50 dB
- Effective receiver noise temperature: 825°K
- Maximum range: 2000 km
- BER (coder output): 10^-5

These parameters support a full-service return link (TV + voice + TLM) with a jammer-to-signal ratio of approximately 20 dB. The 50 dB of antenna gain may represent the most difficult aspect of the FH-FDMA concept. This is certainly true if an electronically steered phase array is used which must create 18 simultaneous independent beams. On the other hand, if multiple parabolic-reflector antennas are used, 60 dB does not represent a significant problem.
Figure 4.12. Functional Block Diagram of the User FH-FDMA Transmitter/Receiver
Figure 4.13. Space Station FH-FDMA Receiver
4.3 Time-Division Multiple-Access Return Links

4.3.1 Introduction

The basic time-division multiple-access (TDMA) scheme assigns specific time slots to each user, with high-data-rate users accorded proportionally more slots than low-data-rate users. User slots can be assigned prior to a mission, which simplifies the MA assignment strategy, or assigned on demand, which results in a more flexible system. A typical TDMA channelization is shown in Figure 4.14.

A common requirement for all TDMA schemes is the need for a guard time to separate users and prevent overlays. Guard time is overhead, and reduces the effective data rate of the system; however, the duration of the guard band can be minimized if accurate relative timing is maintained between the SS and the users. Another common consideration is the high burst rate of TDMA systems. Since the time allotted to a given user is a fraction of the total time available, the burst rate is correspondingly higher. From the examples, we can see that the narrow symbol times, coupled with anticipated relative velocities, will require the SS to time track each user and broadcast updated timing and range information to each user.

4.3.2 Baseline TDMA System Example

A TDMA system based on the requirements shown in the Table 3.1 user group will consist of a single receiver, as shown in Figure 4.15, wherein time samples of received data are routed to their respective destinations. A current history of each user's parameters, e.g., time offset, range, range rate and bearing, is maintained in order to rapidly acquire the user during the next dwell time.

A total of 18 users can be accommodated, with each user given one telemetry channel (64 kbps) and one guard time. In addition, seven users are given a TV channel (2.048 Mbps) and four users are given a voice channel (16 kbps). For system-definition purposes, we assume that a guard time is equal to a least-common multiple of the channel time which, in this case, is one voice channel time, which we can call a time unit. One user frame consists of a guard time (one time unit), a telemetry time (four time units), a possible voice time (one time unit) and a possible TV time (128 time units). The total system time requirement is derived as shown below.
NOTE: Each user has $k$ accesses per second; each slot has $\frac{16 \times 10^3}{k}$ information bits.
Figure 4.15. TDM4 Receiver
18 guard times \(18 \times 1 = 18\) time units

18 telemetry channels \(18 \times 4 = 72\) time units

7 TV channels \(7 \times 128 = 896\) time units

4 voice channels \(4 \times 1 = 4\) time units

990 time units

A single time unit, e.g., one voice channel, can accommodate one 32-kbps channel, which gives a system burst rate of 990 x 16 kbps, or 15.840 Mbps. Thus, a single bit time is 57.7 ns and, with rate-1/2 coding, symbol time is 28.8 ns, assuming binary modulation. In order to achieve low-symbol error rates, timing uncertainty should be a small fraction of a symbol time, but range uncertainty must still be resolved to within several meters. Clearly, it is impossible to range track a user accurately over the extremes of range and range rate considered. Thus, data-tracking loops will be required. It is assumed that a portion of the telemetry channel for each user will be devoted to carrier, data, and frame acquisition/synchronization.

Figure 4.16 presents the results of a sample link-budget calculation in the presence of noise, with required transmitter power given as a function of range; this is a K\(_a\)-band link with a transmitter-plus-receiver antenna gain of 40 dB and a burst rate of 15.84 Mbps. We see that a peak user power of -5.6 dBW is required at 100 m, but a peak power of 30 dBW is required at 2000 km! No margins are included in this calculation, but a reasonable margin implies that a peak power in excess of 1 kW would be needed. The average power of a single user, with only telemetry, would be 4/999 \\times 1 \text{ kW} = 4.0 \text{ W}. A realistic upper limit of 10 dBW user peak power implies an antenna gain of 60 dB. The link budget calculation is based on the following:

\[ P_T = 10 \log \text{Rate} + 9.5 - 228.6 + T_{\text{dB}} - (G_R + G_T) + 92.5 + 20 \log f_{\text{CHz}} + 20 \log R_{\text{km}} \]

with \(T_{\text{dB}} = 29.2\) at 30 GHz and 31.2 at 44 GHz. No margin or implementation loss is included.

Figure 4.17 gives the required power with reduced service; 18 telemetry channels, two TV channels and four voice channels. At the extreme range, an antenna gain of 56 dB is required with a 10-dBW transmitter. If the SS receiver is reconfigured to provide dedicated service to one user with telemetry, slow-scan TV (364 kbps) and voice at 2000 km, an antenna gain of 45 dB would be required. The performance of this configuration is given in Figure 4.18.
Figure 4.17. $P_T$ (min) versus Range: Return-Link TDMA
Figure 4.18. $P_T$ (min) Versus Range: Return-Link TDMA
Figures 4.19, 4.20 and 4.21 represent the same user configuration as Figures 4.16, 4.17 and 4.18, but with a 44-GHz RF link. The antenna technology required to support these links is discussed in Section 5.

The preceding results show that we cannot expect full-user service-ability at extreme ranges unless the user has high EIRP available. The same methodology is used to show the performance with the expanded user group of Table 3.2. Figure 4.10 represents the expanded user group with a 30-GHz link.

4.4 TDMA and Antijam Modulation

The previous section discussed TDMA in general, without regard to modulation and synchronization. In this section, we discuss modulation techniques to provide jam resistance. Additional protection against a jammer can be provided by antenna-null steering, which is discussed in a separate section.

There are two candidate AJ-modulation schemes to be considered with TDMA: direct-sequence (DS) spreading and frequency hopping (FH). Direct spreading is not considered a viable alternative for SS TDMA; a chip time, which is a small fraction of a symbol time, implies sub-nanosecond chip times at the data rates envisioned for the SS KA return link.

FH AJ can employ either frequency-shift-keying (FSK) or phase-shift-keying (PSK). Generally, PSK requires as much as 3 dB less energy per bit to achieve a given bit-error rate (BER), but requires a derived phase reference that is more complex than FSK demodulation. Additionally, with the bandwidth available, a higher order alphabet can be employed (more bits per symbol) and timing constraints can be relaxed due to longer symbol times. To compare TDMA with FDMA, we will examine a system with 4-ary MFSK modulation, i.e., two bits per symbol. Assuming that all channels have rate-1/2 convolutional codes, the net result is two code-check bits per symbol, or one information bit per symbol. Symbol error rate is given as:

$$P_{E,S} = \frac{3}{2} e^{-\frac{E_s}{2}} - \frac{2E_s}{3} + \frac{3E_s}{4}$$

for a noncoherent 4-ary receiver. In this case, with rate-1/2 coding, $E_s/N_0 = E_b/N_0$. The check BER is given as:

$$P_{E,C} = \frac{2^{2-1}}{2^{2-1}} P_{E,S} = \frac{2}{3} P_{E,S}.$$
Figure 4.19: $P_T$ (min) versus Range: TDMA 15.84-Mbps Burst Rate

44 GHz 4-ary MFSK Rate-1/2 Code $P_E, CHECK = 10^{-2}$

Range, km

0.1 1.0 10.0 100.0 1000.0 2000.0

$G_R + G_T$, dB

18 Telemetry
7 TV
4 Voice

$MPD, r_d$
Figure 4.21. $P_T$ (min) versus Range: TDMA 484-kbps Burst Rate
If we desire a check BER of $10^{-5}$ or better, then $P_{E,S} < 1.5 \times 10^{-5}$ and $F/N_0 > 13.6 \text{ dB}$. For purposes of comparison, a coherent receiver would require $E_s/N_0 > 12.9 \text{ dB}$ since $P_{E,S} < 3Q\left(\sqrt{E_s/N_0}\right)$ for 4-ary coherent demodulation. $Q(x)$ is the $Q$ function defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} \, dy < \frac{1}{\sqrt{2\pi}} \frac{e^{-x^2/2}}{x}$$

Binary FSK would require $E_c/N_0$ of $13.3 \text{ dB}$, or an $E_b/N_0$ of $16.4 \text{ dB}$ since

$$P_{E,C} = \frac{1}{2} e^{-\frac{E_c/N_0}{2}}$$

and two check bits are required for one information bit. The results are summarized in Table 4.4 for various alphabet sizes, and we see that the choice of a 4-ary alphabet minimizes the signal bandwidth required as well as providing for a somewhat simpler demodulator. However, if a large enough hop bandwidth is available, larger alphabet sizes will be practical due to the improved error rate performance. In the presence of a tone jammer, however, we shall see that a 4-ary alphabet provides the best performance. Table 4.5 summarizes the performance for a check BER of $10^{-2}$, which will give a decoded BER less than $10^{-5}$.

The performance estimates given so far represent the system performance in thermal noise only. In order to communicate against a jamming threat, we propose to introduce several levels of AJ modulation. The AJ modulation is progressive; against a low-threat jammer, pure FH is employed. The adaptive nature of the TDMA system can be exploited to provide increased resistance to a more serious jammer at the expense of reduced data rate. First, we will discuss the basic system performance with wideband FH. As discussed earlier, other forms of spectrum spreading are impractical at these burst rates. In fact, at the rate of 17.344 MHz, the system must dwell on one hop for several symbols, which requires that symbols be interleaved over several decoder constraint lengths in order to prevent a jammer hit from causing errors in contiguous check bits. Figure 4.22 illustrates the block diagram of a baseline TDMA receiver which corresponds to the baseline user's group of Table 3.1. Figure 4.23 depicts a simplified example of the time/frequency occupancy of the signal tones. After interleaving, symbols $S_1$, $S_4$ and $S_7$ are sent during hop 1, with
Table 4.4. Energy per Bit and Bandwidth (Unhopped) Required as a Function of Signal Alphabet with Noncoherent Demodulation, $P_B \leq 10^{-5}$

<table>
<thead>
<tr>
<th>$M = 2^n$</th>
<th>$P_{E,S} = \frac{2^{n-1} \times 10^{-5}}{2^n - 1}$</th>
<th>$E_s^* \frac{E_b}{N_0} = \frac{E_s}{N_0}$</th>
<th>$E_b = \frac{2}{n} \frac{E_s}{N_0}$</th>
<th>$T_s = \frac{N}{T} 17.344$ Mbps</th>
<th>$BW = \frac{M}{T_s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$10^{-5}$</td>
<td>13.34 dB</td>
<td>16.35 dB</td>
<td>31.6 ns</td>
<td>63.4 MHz</td>
</tr>
<tr>
<td>4</td>
<td>$3/2 \times 10^{-5}$</td>
<td>13.62 dB</td>
<td>13.62 dB</td>
<td>63.1 ns</td>
<td>63.4 MHz</td>
</tr>
<tr>
<td>8</td>
<td>$7/4 \times 10^{-5}$</td>
<td>13.86 dB</td>
<td>12.10 dB</td>
<td>94.7 ns</td>
<td>84.5 MHz</td>
</tr>
<tr>
<td>16</td>
<td>$15/8 \times 10^{-5}$</td>
<td>14.10 dB</td>
<td>11.09 dB</td>
<td>126.3 ns</td>
<td>17.0 MHz</td>
</tr>
</tbody>
</table>

*NOTE: $P_{E,S} = \sum_{j=1}^{M-1} \frac{(-1)^{j+1}}{j+1} \left(\frac{M-1}{j}\right) e\left(-\frac{j E_s/N_0}{j+1}\right)$*

Table 4.5. Energy Per Bit Required, $P_B \leq 10^{-2}$

<table>
<thead>
<tr>
<th>$M = 2^n$</th>
<th>$P_{E,S}$</th>
<th>$E_s/N_0$, dB</th>
<th>$E_b/N_0$, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$10^{-2}$</td>
<td>8.9</td>
<td>11.9</td>
</tr>
<tr>
<td>4</td>
<td>$3/2 \times 10^{-2}$</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>8</td>
<td>$7/4 \times 10^{-2}$</td>
<td>10.0</td>
<td>8.2</td>
</tr>
<tr>
<td>16</td>
<td>$15/8 \times 10^{-2}$</td>
<td>16.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>
Figure 4.22. Baseline TDMA/FH System
Figure 4.23. The 4-ary MFSK with J/S Jamming Tones
symbol \( S_1 \) (shaded) at frequency \( f_1 \), symbol \( S_4 \) at \( f_2 \), etc. We can consider \( f_0 \) the hop frequency, \( f_1 = f_0 + 1/T_s \), \( f_2 = f_1 + 2/T_s \), and \( f_3 = f_2 + 3/T_s \), with \( T_s \) representing the symbol duration. In the example presented in Figure 4.22,

\[
T_s = \frac{1}{15.84 \times 10^6} = 63.13 \text{ ns.}
\]

In the baseline receiver, the signal is dehopped, downconverted and demodulated. During a user's time slot, the demodulated check bits are rate buffered down to 4.256 Mcps and deinterleaved. The minimum \( E_b/N_0 \) specified in the performance curves of the prior section gives a check BER of \( 10^{-2} \) at this point. This is reduced to \( 10^{-5} \) error rate or better by the Viterbi decoder.

The worst-case (WC) jammer for MFSK is a partial-band tone jammer, as depicted in Figure 4.24. The jammer's tones have just enough power \( (S+\epsilon) \) to exceed the communicator's signal \( S \). Thus, if a jamming tone hits a data band, it has a probability of \( (M-1)/M \) of causing an error. One time out of \( M^{k-1} \), it will reinforce the desired tone. If the jammer has power \( J \), then he can transmit \( J/S \) tones, each with power \( S \). In a hop band \( W \), there are

\[
\frac{W}{W_s} = \frac{W}{M} \times \frac{T_s}{M}
\]

possible data bands for the communicator; thus, the probability of symbol error due to jamming is \( (M-1)/M \) times the probability that one jammer tone hits one data band, or

\[
P_E = \left( \frac{M-1}{M} \right) \left( \frac{J}{S} \right) \left( \frac{M}{W} \right)
\]

From a previous section, we know that

\[
P_B = \frac{2^{k-1}}{2^{k-1}} \times P_E = \frac{1}{2} \frac{M}{M-1} P_E
\]

Therefore,
Figure 4.24. Time/Frequency Utilization with Interleaved TDMA/FM, Multiple Symbols per Hop
For a given energy-to-jammer power ratio, the minimum probability of error is given with \( k = 1 \) or 2. Higher order alphabets have reduced AJ performance by this measure. With \( k = 2 \),

\[
P_c = \frac{1}{E_c/N_j}
\]

and we need 20 dB of \( E_c/N_j \) to achieve a \( 10^{-2} \) BER into the decoder. In Table 4.6, we give the maximum AJ protection afforded (J/S) with this scheme, assuming that the bandwidth \( W \) is 10% of the RF bandwidth. We derive this from the relation

\[
\frac{E_c}{N_j} = \frac{W T_S}{J} \quad \text{or} \quad \frac{J}{S} = W - \frac{E_c}{N_j} \quad \text{in dB.}
\]

### Table 4.6. Maximum J/S with \( E_c/N_j = 20 \text{dB} \), \( R = 15.84 \text{ Mbps} \)

<table>
<thead>
<tr>
<th>Carrier Frequency</th>
<th>( W, \text{ GHz} )</th>
<th>( J/S, \text{ dB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>3.0</td>
<td>2.8</td>
</tr>
<tr>
<td>44</td>
<td>4.4</td>
<td>4.4</td>
</tr>
<tr>
<td>60</td>
<td>6.0</td>
<td>5.8</td>
</tr>
</tbody>
</table>
For purposes of estimating system performance in a jammed environment, we make the simplifying assumption that jammer power is the dominant factor in symbol-decision errors. Note from Table 4.6 that, even at 60 GHz, a maximum J/S of 5.8 dB is available at a data rate of 15.84 MHz and bandwidth of 6 MHz while maintaining a check BER of $10^{-2}$. The frequency-hopped TDMA system performance can be improved by employing frequency diversity, e.g., multiple hops per symbol at the expense of increased system complexity and reduced throughput.

An example of m-diversity AJ modulation is given in Figure 4.25. Symbols $S_1$ through $S_N$ are transmitted on hops 1 through $m$. The receiver has storage for $N$ symbol decisions, and is capable of determining whether or not a given symbol has been jammed. It is assumed that the jammer is asynchronous with the communicator and portions of a hop can be tone jammed. The receiver makes symbol-by-symbol decisions on hop 1, erasing symbols that are ambiguous due to jamming. This process continues until all $m$ hops have been examined by the receiver. Thus, the probability of symbol error is equal to the probability that all $m$ hops have been jammed. In the example of Figure 4.25, symbols $S_1$ and $S_2$ of hop 1 would be ignored, while $S_{N-1}$ and $S_N$ would be stored in memory. The probability of a check bit error can be shown to be:

$$P_b = \frac{1}{2} e^{-\left(\frac{k}{c} e^{-1}\right)^{E_c/N_j}}$$

with $M = c^k$ and the optimum diversity to be

$$m = e^{-1\left(\frac{E_c}{N_j}\right)^{\frac{k}{2}}}$$

For $P_b < 10^{-2}$ and $M = 4$, $m = 4$ and $E_c/N_j = 10.4$ dB. The resultant AJ margin is given in Table 4.7.
Table 4.7. Maximum J/S with $E_c/\sqrt{N_j} = 10.4$ dB, $R = 15.84$ Mbps

<table>
<thead>
<tr>
<th>Carrier Frequency</th>
<th>W, GHz</th>
<th>J/S, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>3.0</td>
<td>12.4</td>
</tr>
<tr>
<td>44</td>
<td>4.4</td>
<td>14.0</td>
</tr>
<tr>
<td>60</td>
<td>6.0</td>
<td>15.4</td>
</tr>
</tbody>
</table>
5.0 MULTIPLE-ACCESS ANTENNA-DESIGN CONSIDERATIONS

The omnidirectional antenna-coverage requirement for the Space Station (SS) for 18 simultaneous users out to 2,000 km is a difficult problem. A number of possible antenna configurations have been studied in order to develop a viable system. The outcome of the preliminary investigation is that planar phased-array antennas are the best candidates to fulfill system requirements, but the implementation is difficult to perform. An alternate scheme which shows some promise has also been developed which may overcome some of the problems associated with phased arrays uses a combination of mechanically and electronically scanned parabolic dishes.

In order to simplify the antenna-coverage problem, two range regions are delineated. For near coverage out to 50 km, antenna gains of 20 dB are adequate while, for the extreme ranges out to 2,000 km, 40 dB of gain will be considered reasonable, at least initially, with 60 dB of gain desirable. The baseline frequency will be 30 GHz, with 44 GHz also being studied since many new generations of military satellites will be using this frequency band.

The electronic planar-phased array is capable of being scanned rapidly by phase shifters for TDMA applications and forming multiple beams for FDMA systems. Nulling for A1 performance is also possible with the new beam-forming/nulling processors which adjust amplitude and phase shifts at each radiating element in order to optimize multiple-beam pointing and jammer nulling.

When considering planar-phased arrays, some important features are that very large numbers of phase shifters are required, the array-feed network is lossy, the phased array is frequency sensitive for frequency-hopped (FH) systems, and multiple beams/nulling and beam-steering degrade the array performance. Table 5.1 shows an attempt to outline the basic requirements of an idealized array at various gain levels.

At close ranges where 20 dB of gain is required, the planar-phased array is only a square-inch wide, with 36 phase shifters for one beam (i.e., TDMA). However, if 16 simultaneous multiple beams (FDMA) are required, the gain-design criteria must be increased by 12 dB, so that a 36 in² array with nearly a thousand phase shifters (filled array) are necessary. The selection of TDMA and/or FDMA techniques is therefore extremely important in determining the required size of the antenna. This choice is even more apparent when the 40-dB gain array is designed, since a TDMA planar array is only one square foot in area with 3,000 phase shifters versus an FDMA planar array 9 ft² in area with 30,000 phase shifters.
Table 5.1. Idealized Planar-Phased-Array Parameters for Various Gain Levels

<table>
<thead>
<tr>
<th>Gain</th>
<th>20 dB</th>
<th>25 dB</th>
<th>30 dB</th>
<th>35 dB</th>
<th>40 dB</th>
<th>50 dB</th>
<th>60 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>((n\lambda)^2 = \frac{G}{\lambda^2})</td>
<td>3</td>
<td>5</td>
<td>9</td>
<td>16</td>
<td>28</td>
<td>89</td>
<td>282</td>
</tr>
<tr>
<td>Dimension of Square Array ((\lambda))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Filled-Array Phase Shifters</td>
<td>36</td>
<td>100</td>
<td>324</td>
<td>1,024</td>
<td>3,100</td>
<td>31,700</td>
<td>318,000</td>
</tr>
<tr>
<td>(\theta_{3dB} = \sqrt{\frac{27,000}{G}})</td>
<td>16.4°</td>
<td>9.2°</td>
<td>5.2°</td>
<td>2.9°</td>
<td>1.6°</td>
<td>0.5°</td>
<td>0.16°</td>
</tr>
<tr>
<td>States in 90°</td>
<td>6</td>
<td>10</td>
<td>18</td>
<td>32</td>
<td>56</td>
<td>180</td>
<td>562</td>
</tr>
<tr>
<td>Dimensions:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 GHz</td>
<td>1.2 in</td>
<td>2 in</td>
<td>3.5 in</td>
<td>6.3 in</td>
<td>11 in/0.9 ft</td>
<td>35 in/2.9 ft</td>
<td>111 in/9.3 ft</td>
</tr>
<tr>
<td>44 GHz</td>
<td>0.8 in</td>
<td>1.34 in</td>
<td>2.4 in</td>
<td>4.3 in</td>
<td>7.5 in/0.6 ft</td>
<td>24 in/2.0 ft</td>
<td>75.5 in/6.3 ft</td>
</tr>
</tbody>
</table>
Due to the spherical-coverage requirement, five or six of these phased arrays will surround the SS. A multiple-gain capability may be developed where the aperture area is electronically adjusted but, if not, both a 20-dB array and a separate 40-dB array will have to be used, thus doubling the number of planar arrays. In addition, the receive and transmit functions may or may not be integrated into the same system. Practically speaking, since the transmitter uses a different frequency set in the same band and therefore requires another set of phase shifters, a separate transmit planar array is feasible because of the additional isolation obtainable. Thus, as many as 24 planar arrays may be needed to fulfill the spherical-coverage, near/far requirements.

The alternate antenna configuration considered is the parabolic dish, which has extremely high gains (up to 60 dB, i.e., desired). If mechanical beam steering is allowed, 18 separate dishes can accommodate 18 users simultaneously without TDMA and FDMA using individual transmitters/receivers. Orthogonal circular polarizations for transmit and receive provide additional isolation by using quarter-wavelength diagonal-dielectric slabs as polarizers in a square waveguide. Figure 5.1 shows the gains achievable with commercially available dishes and Table 5.2 outlines idealized parabolic dish diameters for various gain levels.

As an aside, an acquisition aid that might be considered is a sensitive CCD video camera which can visually locate a blinking stroboscopic light beacon on the user, as described in Figure 5.2. Monopulse autotracking is readily available once acquisition occurs. An obvious advantage of this concept is the inherent redundancy and the incremental growth capability.

The use of parabolic dishes at near ranges is not as practical, but a modification of the feed into an electronically scanned array offers a means of substantially reducing the number of phase shifters required. A five-horn cross feed, with the central horn fed directly and the surrounding horns with phase shifters, provides an electronic-scanning capability of ±5 beamwidths, for a total of 100 beam positions. Table 5.3 outlines the number of parabolic dishes necessary to attain spherical coverage for a given gain. For example, 25 dB of gain implies a 3-dB beamwidth of 9.2°, and 10 beam positions gives 90° coverage. Theoretically, five paraboloids would provide spherical coverage. At 30 GHz, since the aperture diameter of $6\lambda$ is 6 cm or 2.5 inches, the total antenna set of five paraboloids is quite small. Even at 35 dB gain, 46 paraboloids, eight inches in diameter, scattered about the large SS, is not unreasonable since only 184 phase shifters are required. Figure 5.3 attempts to show the appearance of one-half of a 40-sided polygon parabolic dish array. Since it uses electronic
Figure 5.1. Gains and Beamwidths of Commercially Available Parabolic-Dish Antennas
### Table 5.2. Parabolic-Dish Dimensions for Higher Gains

<table>
<thead>
<tr>
<th>Gain (dB)</th>
<th>$\theta_{3dB}$</th>
<th>Aperture Diameter (n(\lambda))</th>
<th>Aperture Diameter at 30 GHz</th>
<th>Aperture Diameter at 44 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.16°</td>
<td>318</td>
<td>10.5 ft</td>
<td>7.2 ft</td>
</tr>
<tr>
<td>55</td>
<td>0.29°</td>
<td>180</td>
<td>5.9 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>50</td>
<td>0.5°</td>
<td>100</td>
<td>3.3 ft</td>
<td>2.2 ft</td>
</tr>
<tr>
<td>45</td>
<td>0.9°</td>
<td>57</td>
<td>1.9 ft</td>
<td>1.3 ft</td>
</tr>
</tbody>
</table>

### Table 5.3. Parabolic-Dish Spherical Coverage Using Idealized Gain Values

<table>
<thead>
<tr>
<th>$\theta$ (°)</th>
<th>Gain (dB)</th>
<th>Circumference (Paraboloid Diameter)</th>
<th>Radius (Paraboloid Diameter)</th>
<th>Surface Area (Paraboloids)</th>
<th>Aperture Diameter (n(\lambda))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>40</td>
<td>22</td>
<td>3.6</td>
<td>161</td>
<td>32</td>
</tr>
<tr>
<td>2.0</td>
<td>38</td>
<td>18</td>
<td>2.9</td>
<td>103</td>
<td>28</td>
</tr>
<tr>
<td>2.5</td>
<td>36</td>
<td>14.4</td>
<td>2.3</td>
<td>66</td>
<td>20</td>
</tr>
<tr>
<td>3.0</td>
<td>35</td>
<td>12</td>
<td>1.9</td>
<td>46</td>
<td>18</td>
</tr>
<tr>
<td>3.5</td>
<td>33</td>
<td>10.3</td>
<td>1.6</td>
<td>34</td>
<td>14</td>
</tr>
<tr>
<td>4.0</td>
<td>32</td>
<td>9</td>
<td>1.4</td>
<td>26</td>
<td>13</td>
</tr>
<tr>
<td>4.5</td>
<td>31</td>
<td>8</td>
<td>1.3</td>
<td>21</td>
<td>11</td>
</tr>
<tr>
<td>5.0</td>
<td>30</td>
<td>7.2</td>
<td>1.2</td>
<td>17</td>
<td>10</td>
</tr>
<tr>
<td>5.5</td>
<td>29.5</td>
<td>6.6</td>
<td>1.1</td>
<td>14</td>
<td>9.5</td>
</tr>
<tr>
<td>6.0</td>
<td>29</td>
<td>6.0</td>
<td>1.0</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>9.2</td>
<td>25</td>
<td>4.0</td>
<td>0.6</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>16.4</td>
<td>20</td>
<td>2.2</td>
<td>0.35</td>
<td>1.5</td>
<td>3.2</td>
</tr>
</tbody>
</table>
SONY Miniature CCD Video Camera Module

FEATURES
- CCD (Charge Coupled Device) solid state image sensor replaces tube type pickup and provides extended life and improved stability
- Miniaturized design is only 1 ¼" high by 1 ¾" wide by 2 ½" deep and weighs just 4.1 ounces
- Ultra stable, extended life operation due to completely solid state componentry with virtually no maintenance
- Low lag and high image burn resistance
- Precise image geometry
- Shock and vibration resistance exceeds all tube cameras
- Instant shooting with a start-up time of ½ second due to elimination of electron gun/pick up tube
- Anti-magnetic properties of CCD allow camera to be used in areas with strong magnetic fields
- High sensitivity enables camera to shoot in 3 lux illumination
- Low power consumption of less than 2.3 Watts
- Special mini bayonet lens mount plus C lens mounting with an optional adaptor

SPECIFICATIONS (TENTATIVE)
- Pick up device: Interline Transfer Type CCD
- Optical system: Special mini bayonet/"C" mount with adaptor
- Scanning system: 525 lines, 50 fields/sec. 2:1 interlace
- Video output: 1.0 Vp-p sync negative, 75 Ohms
- Resolution: Horizontal: 280 lines
- Vertical: 350 lines
- Sensitivity: 400 lux, F 4, typical
- (3200°K, S/N 40dB with Infrared cut filter)
- Minimum illumination: 3 lux F 1.4
- (without infrared cut filter)
- Signal-to-noise ratio: more than 46 dB
- Power requirement: 12 V DC with power unit
- Power consumption: 2.3 W
- Weight: 4 ½ oz. 115 grams (without lens & power unit)
- Dimensions: 1 ½" × 1 ¾" × 2 ½" (HWD)
- 28.5mm × 43mm × 72.5mm (HWD)

Specifications subject to change without notice.

Figure 5.2. CCD Video-Camera Candidate for Visual-Acquisition Aid
Figure 5.3. Hemispheric Array of Parabolic Dishes
scanning for the limited coverage of one paraboloid, a parabolic-dish array is suitable for TDMA techniques within the coverage area of one dish and multiple beams from individual parabolic dishes are attained by selective IF signal combining, so that FDMA is feasible.

The beam-selection process uses latching-reciprocal ferrite phase shifters and latching-ferrite SPDT switches. Figure 5.4 shows a possible switching network connecting \(2^N\) parabolo'lds, where \(N\) is the number of switching levels. This complexity can be avoided if individual transmitters/receivers are provided at each paraboloid, so that switching is done at the IF level.

The five-horn feed configuration with phase shifters in the outer horns would allow electronic acquisition even for the high-gain, mechanically steered parabolic dish. If the monopulse-tracking magic tee is located after the phase shifters, the sum signal indicates acquisition, and the difference signal will allow autotracking, even off-axis, or indicate the correct phase adjustments needed to mechanically align the beam. Since latching-ferrite phase shifters with 1 μs switching times are used, TDMA within the 100-beam electronic-scan sector is possible if more than one user is in that sector.

Since the orthogonal modes of circular polarization for the transmitter require separate sets of phase shifters, the required phase shifts for the FH signal can be individually tailored to point at the user, independent of the receive FH signals. Figure 5.5 sketches the transmit/receive shifter configuration. Note that this approach doubles the number of phase shifters required but, since the parabolic-dish-array concept uses fewer phase shifters, the total number of phase shifters is not unreasonable.
Figure 5.4. RF Switching Network Using Ferrite-Latching Circulators
Figure 5.5. Five-Horn Cross Feed with Dielectric Polarizers and Orthomode Transducer for Transmit/Receive
When evaluating antenna configurations, component availability is an important consideration because the feasibility of an approach hinges on realistic design practices. Two frequency ranges studied for the SS communication links are as follows: The 30-GHz range is desirable due to NASA/DOD technology developments and 44 GHz is used for military satellites such as MILSTAR. The baseline frequency of 30 GHz is attractive due to the FET low-noise amplifiers (LNA's) and the higher power transmitters available. The 44-GHz region allows wider frequency-hopping (FH) bandwidths and therefore greater antijam (AJ) capability, but the device technology is not as well developed and thus is deemed a higher risk, although the SS program will not be fully implemented until the 1990's time frame.

Both antenna configurations discussed--planar-phased array and phased-array-fed parabolic dish--use many common components. Latching ferrite phase shifters having 1 μs switching speeds are now commercially available at both 30 and 44 GHz. Similarly, low-noise-figure, double-balanced mixers are available off the shelf. The primary difference between the choice of these two frequencies is that low-noise FET preamplifiers with 3.5-dB noise figures at 30 GHz are being developed, which greatly enhances receiver sensitivity and therefore increases link margin. In addition, the available transmitter powers are inherently higher at 30 GHz, approximately double that at 44 GHz--an especially critical factor since the free fliers are transmitter-power limited.

The Cassegrain configuration parabolic dish with five horn feeds and orthomode transducers is a standard antenna system. The proposed high-gain, mechanically steered dish is closely related to existing ground station dishes. The major advantages of this system are: (1) extremely high gains are readily achievable and, (2) adding electronic beam-scanning requires only eight phase shifters per dish (a set of four each for the transmitter and receiver). Each dish has its own LNA and transmitter amplifier and, therefore, does not have to be shared. Microwave circuit losses are lower since no power-dividing, beam-switching networks are needed, which also greatly adds to system complexity. The independence of each dish provides redundancy and incremental growth capability, valuable features for space operation. Full-spherical antenna coverage is available with high gains. Since each parabolic dish in the set of fixed low-gain parabolas points in a different direction, the off-axis axial ratio and beam-spreading performance is greatly improved--a definite limitation of planar-phased arrays. Finally, if each dish has its own low-noise receiver and transmitter, the antenna gain and transmitter powers do not have to be shared, as in the case of planar-phased arrays with multiple beams, which sacrifices gain to produce multiple beams.
On the other hand, the planar-phased array is definitely a viable antenna-configuration candidate due to its flexibility. For TDMA operation, it can rapidly scan a spot beam. Conversely, multiple-beam forming and nulling capabilities are realizable for FDMA and A3 applications. However, there are some definite design constraints which must be considered when evaluating the antenna configurations. First, the multiple-beam forming and nulling capabilities are complex and require much larger apertures in order to compensate for sharing the achievable gains since the antenna pattern is simply being redistributed. For example, when two main beams are formed instead of one, there is at least a 3-dB loss in gain in each beam when the pattern is shared. Therefore, the phased-array aperture area must be correspondingly doubled in order to maintain a constant value of main-beam gain, also doubling the number of phase shifters required. Since each radiating element is spaced approximately a half-wavelength apart and large aperture areas are needed for high gain, very large numbers of phase shifters are required, even for thinned arrays. The power-dividing/combining networks for these large arrays are complex and quite lossy, thus degrading the receiver sensitivity and increasing the transmitter power requirements. Simultaneous receiver and transmitter operation demands a separate array for each function, especially since both forward and return links are independently frequency hopped. Another limitation is that the off-axis performance of a planar array is severely degraded, both in axial ratio and beam broadening, which reduces the gain. Under these conditions, it is not unreasonable to consider a larger number of smaller planar arrays distributed about the SS to overcome some of these limitations, as proposed for similar reasons for the parabolic dishes. The increased number of planar-phased arrays also increases the available transmit power since it does not have to be shared, at the expense of additional high-power amplifiers and low-noise receivers, but which provides redundancy.

In conclusion, the high-gain antenna requirement for spherical coverage about the SS can be satisfied by either large planar arrays or mechanically steered large parabolic dishes with approximately the same size apertures. If there are 12 simultaneous users for the far ranges (50-2000 km), then 12 mechanically steered parabolic dishes or six equivalent-sized planar-phased arrays faced in the appropriate directions are required. If one planar-phased array services one quadrant for the 12 users employing FDMA, the aperture size must increase by a factor of 16 in order to compensate for the gain loss due to redistributing the pattern for multiple beams with the same gain, so the TDMA approach is appealing for high-gain applications using the planar-phased arrays.
The low-gain spherical coverage can be handled readily by either the planar-phased array or multiple parabolic dishes. The lower gain permits the use of smaller apertures and, therefore, fewer phase shifters. The dimensions of the apertures are reasonable and acceptable. The problems of separate transmit and receive arrays as well as acquisition and tracking still exist for the planar-phased array, but they are not insurmountable. Again, the driving consideration is the type of multiple access used, either TDMA or FDMA or combinations thereof.

One aspect of the communication link which is not fully discussed is the antenna requirement of the free flier, which generally is omnidirectional in order to avoid pointing problems. If directional antennas with higher gain can be used, the SS gain problem can be correspondingly reduced.

It is obvious that no apparent solution to the SS antenna-gain-coverage requirement exists. Much of the technology being developed for geosynchronous satellites cannot be readily adapted to SS use since the antenna coverage is limited to earth coverage from a distance—a very small sector compared to the spherical coverage requirements of the SS. The SS antenna design is a challenging area worthy of extensive investigation in the future.
APPENDIX A

VIDEO DATA-COMPRESSION TECHNIQUES STUDY
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>LIST OF FIGURES</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>iv</td>
</tr>
<tr>
<td>1.0 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.0 VIDEO DATA-COMPRESSSION TECHNOLOGY STATUS AND TRENDS</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Factors Contributing to Digital Video Data Compression</td>
<td>3</td>
</tr>
<tr>
<td>2.1.1 Advantages of Digital Video Data Transmission</td>
<td>3</td>
</tr>
<tr>
<td>2.1.2 Current Digital Video Uses Which Provide a Data Base for SS Video Bandwidth-Compression Techniques</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Video-Digitizing and Bandwidth-Compression Techniques Identification</td>
<td>5</td>
</tr>
<tr>
<td>2.2.1 Technique Identification</td>
<td>5</td>
</tr>
<tr>
<td>2.2.2 Encoding-Scheme Evaluation Criteria</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Video Bandwidth-Compression-Techniques Description</td>
<td>11</td>
</tr>
<tr>
<td>2.3.1 Motion-Compensation Technique</td>
<td>12</td>
</tr>
<tr>
<td>2.3.2 Hybrid Coding for Bandwidth Compression of Video Signals</td>
<td>17</td>
</tr>
<tr>
<td>2.3.2.1 General Considerations</td>
<td>17</td>
</tr>
<tr>
<td>2.3.2.2 Harris Corp. hybrid-coding system for RPV use</td>
<td>21</td>
</tr>
<tr>
<td>2.3.2.3 Other efforts in hybrid coding</td>
<td>26</td>
</tr>
<tr>
<td>2.3.3 Delta-Modulation Techniques for 15-t0-30-Mbps Digital TV Links</td>
<td>28</td>
</tr>
<tr>
<td>2.3.3.1 Overview</td>
<td>28</td>
</tr>
<tr>
<td>2.3.3.2 Bistate adaptive delta modulation</td>
<td>28</td>
</tr>
<tr>
<td>2.3.3.3 Tri-state delta modulation for run-length encoding</td>
<td>33</td>
</tr>
</tbody>
</table>
2.4 VLSI Utilization Potential .......................... 43

2.4.1 Capabilities and Limitations of VLSI Applications to Bandwidth-Reduction Systems ............ 43

2.4.2 Specific Systems to be Potentially Reduced Through VLSI 46

2.4.2.1 VTS 1.5E System made by Compression Labs, Inc. ........................................ 46
2.4.2.2 NETES-XITV Codec, made by Nippon Electric Co., Ltd. ..................................... 47

3.0 RELATED VIDEO-TECHNOLOGY DEVELOPMENTS ................. 48

3.1 CCD Cameras ............................................. 48
3.2 Digital TV Devices Applicability ................................. 48

4.0 SUMMARY AND CONCLUSIONS .................................. 50

REFERENCES ....................................................... 51
LIST OF FIGURES

2.1 Comparison of Analog FM and Digital Transmission in Terms of RF Bandwidth and Transmitter [1] ........................................ 4
2.2 Digitized Video Signal Lends Itself to a Relatively Simple Encryption/Decryption Method ........................................... 4
2.3 Television Source-Encoding Categorization ............................ 7
2.4 Alternative Methods of Digitized TV Signal Coding ............... 8
2.5 Coding Bit Rate Versus Picture Quality for Typical Coding Methods [1] ................................................................. 9
2.6 Interframe-Coding Performance Features [1] ........................ 9
2.7 Relative Performance of Coding Techniques in Terms of Picture Quality Versus Transmission Rate [2] .............................. 10
2.8 Principle of Basic Interframe Coding ................................... 13
2.9 Generic Configuration of an Interframe Encoder .................... 13
2.10 The Principle of Motion-Compensated Interframe Coding ....... 13
2.11 Motion-Compensated Coder and Decoder Block Diagram ........ 15
2.12 Mean-Square Error Performance of Different Transforms for the Two-Dimensional Markov Image Source with Horizontal Correlation Factor of $\rho_c$ and Vertical Correlation Factor of $\rho_r$ [3] .................. 18
2.13 Block Diagram of a Hybrid Transform/DPCM Encoder .......... 20
2.14 Functional Block Diagram of the Video-Bandwidth Compression and Reconstruction Units ............................................. 23
2.15 Hardware Systems Design of Compression and Reconstruction Units ................................................................. 24
2.16 Conventional (Bistate) Delta Modulator Functional Block Diagram ................................................................. 30
2.17 Field-Sequential Color System [10] ................................... 32
2.18 Tri-State Delta Modulator (TSDM) Functional Block Diagram .. 34
2.19 Tri-State Video Digitizer (and Reconstructor) Implementation .. 37
2.20 Run-Length Encoder (Transmitter Side) ............................ 39
2.21 Run-Length Decoder (Receiver Side) ................................... 41
2.22 NTSC Color TV Component Digitization ............................ 42
2.23 Speed-Power-Density Diagram [14] .................................... 45
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3-1</td>
<td>Salient Characteristics of the Harris Corp. Hybrid-Coding System [4]</td>
<td>22</td>
</tr>
<tr>
<td>2.3-2</td>
<td>RPV Compression Hardware Evaluation [5]</td>
<td>27</td>
</tr>
<tr>
<td>2.3-3</td>
<td>Encoding Table for TSDM</td>
<td>40</td>
</tr>
</tbody>
</table>
EXECUTIVE SUMMARY

Video data-compression techniques applicable to space station (SS) communication requirements are considered from the standpoint of their realizability with existing hardware. Projections toward future implementations with respect to size reduction are made. Specifically, recent developments in the area of frame-to-frame (interframe) coding are examined in terms of commercially available existing equipment. Also, examples of existing hybrid-coding equipment developed for remotely piloted vehicles (RPV'S) are presented and discussed in terms of SS applicability. Furthermore, relatively simple encoding systems, such as delta modulation, are considered for utilization in links which do not permit complex-terminal encoding equipment.

The study addresses two general types of bandwidth-compression schemes. One scheme is characterized by the rather complex equipment yielding good-quality pictures at rates less than 3 Mbps. One such scheme is motion compensation combined with interframe encoding. Hybrid-transform coding with reducible frame rates is also described in this category. The second scheme considered is that of delta modulation with relatively simple intraframe encoding, such as run-length encoding. The channel rates for these types of systems are in the 15-to-30-Mbps range, with about 20 Mbps being a baseline number.

Also in this study, the applicability of VLSI techniques for potential size reduction of existing schemes is discussed, and projections based on our engineering judgment are made. Furthermore, recent developments in CCD technology are considered, particularly with respect to miniaturized components such as the CCD-based TV camera.

Our findings can thus be summarized as follows:

- Good-quality, moderate-motion TV color pictures can be transmitted at a rate of 2.048 Mbps using interframe encoding with motion compensation. The video processor for this transmission can be packaged to within 1 ft$^3$ using VLSI.

- For channel rates much lower than 2.048 Mbps, low-frame rate, black-and-white pictures can be transmitted using hybrid coding. The processor volume for this type of transmission is about 25 in$^3$ using LSI.

- For requirements where a lot of picture motion has to be accommodated, delta modulation can be used provided that a channel capacity of 15-30 Mbps is available. Depending on the requirements (i.e., color versus black and white, resolution, etc.), the processor volume may range from 25 in$^3$ to 0.5 ft$^3$. 
1.0 INTRODUCTION

This report describes recent developments in video data-compression techniques from the standpoint of applicability to Space Station (SS) requirements. The particular slant of this report is the hardware aspect of the latest video data compression. Thus, the techniques themselves are described only in general terms and, wherever applicable, the hardware implementation aspects of such techniques are mentioned.

The methodology used to obtain the material for this report consisted of the following activities:

1. Survey of recent technical literature pertaining to video data compression
2. Study of vendor/manufacturer/supplier brochures
3. Direct vendor/representative contact (by telephone)
4. Our own interpretation and projection of the gathered information, i.e., analysis/synthesis method.

Due to the formative nature of the SS requirements, it is difficult to make a specific and final recommendation at this time with respect to the "optimum" technique for digital video compression. Requirements will have to be defined by various users and these requirements, in turn, will be mission dependent.

Consequently, the best we can do at this point is to describe what is available today in terms of the video data-compression techniques in order to identify the trends and make some reasonable prediction of what will be available in the near future.

In making such assessments and predictions, we chose to be guided by the following assumptions:

1. The video-compression techniques must be compatible with an all-digital communication channel
2. The baseline goal for the maximum data-transmission rate for reasonable quality TV picture transmission is 2.048 Mbps, with only special situations requiring up to 22 Mbps
(3) No video data-compression technique can be considered "superior" to any other technique in that the customer requirements dictate the trade-offs in selecting a particular data-compression method.

Thus, the contents of this report should be judged in terms of the aforementioned assumptions. Furthermore, it must be emphasized that, because of the rapidly evolving developments in TV video data-compression techniques, any opinion expressed at this time must be reexamined at some later date when: (1) the SS detailed requirements will come into better focus and, (2) the video data-compression techniques will undergo a considerable evaluation in their respective fields of application.
2.0 VIDEO DATA-COMPRESSION TECHNOLOGY Status AND TRENDS

2.1 Factors Contributing to Digital Video Data Compression

2.1.1 Advantages of Digital Video Data Transmission

The advantages of video data transmission via digital, rather than analog, methods have been demonstrated in the past by both theory and practical implementations. However, it is only recently that the digital signal-processing technology has reached the point where operational use of the digital techniques is possible. In general, for an equivalent picture quality, digital technology offers a marked advantage over analog methods in that it has the following features: (1) lower RF power requirements, (2) lower RF bandwidth requirements, (3) data-encryption capability, (4) picture quality is solely determined by terminal equipment—not by the communication link, and (5) error-control techniques are applicable.

Figure 2.1 demonstrates the transmitter-power-requirements advantage of the digital-picture-transmission method as compared with the FM analog techniques. Although Figure 2.1 does not contain any reference to the absolute quality of analog and digital transmissions, it clearly demonstrates the power and RF bandwidth advantages of the digital-transmission techniques.

The relative ease with which a digital picture transmission can be encrypted is demonstrated in Figure 2.2. As shown there, encryption/decryption can be implemented, at least in principle, by means of EXCLUSIVE-OR digital-logic gates. In comparison, an equivalent process for analog video signal encryption would require a considerable amount of signal-processing equipment.

Since the quality of a digital transmission is determined by the processing capability of the terminal equipment—not by the transmission media—the digitized video signal becomes particularly attractive for digital RF links utilizing multiplexing for multiple access. This is particularly true for the case of time multiplexing, which is one of the candidates for the SS communication link design.

Furthermore, the error-control techniques to which the digital signal lends itself provide a means for improving the tolerance of the digital video-transmission systems to the system bit-error rate (BER). This feature is of great advantage of the space-borne links, where the RF transmitter power is at a premium and, thus, a significant savings in power, size and weight can be realized by using digital error-correction codes implemented with a relatively
Figure 2.1. Comparison of Analog FM and Digital Transmission in Terms of RF Bandwidth and Transmitter [1]

Figure 2.2. Digitized Video Signal Lends Itself to a Relatively Simple Encryption/Decryption Method
low-power technology such as CMOS. Utilization of the latter technology for encoding and decoding is particularly attractive for the relatively low video digital-data rates such as those baselined at 2.048 Mbps for SS operation.

2.1.2 Current Digital Video Uses Which Provide a Data Base for SS Video Bandwidth-Compression Techniques

As the result of the aforementioned advantages of digital video transmission, there recently emerged several areas of active development and utilization of such transmissions. Of particular interest for SS use are the existing techniques for bandwidth compression of the digital video signal. At least three areas can presently be identified, where efficient use of advanced techniques for bandwidth compression are successfully utilized. These techniques are used for transmission of video pictures, either still or time variant, consistent with the desired picture quality. The three areas are: (1) network high-quality digital TV, (2) video conferencing and, (3) remotely piloted vehicle (RPV) video transmission.

The first two techniques are driven by commercial, nonmilitary users. Thus, a considerable amount of competitive development activity has been exhibited in these areas in recent years. Not bound by military-component specifications, these two civilian application areas of digital video techniques have benefited greatly from the recent development of computer-oriented digital LSI technologies. On the other hand, the military-oriented RPV video-transmission technologies make use of custom developments that are tailored to specific video digitizing and processing techniques. Consequently, a considerable data base exists for SS digital video requirements from which applicable techniques can be synthesized.

2.2 Video-Digitizing and Bandwidth-Compression Techniques Identification

2.2.1 Technique Identification

Depending on the coding-bit rate, the existing video-digitizing and signal-processing techniques can be divided into three general categories, as follows: (1) conventional PCM, (2) intraframe coding, (3) interframe coding.

The conventional PCM does not fall into the bandwidth-compression techniques area. It is characterized by reasonably high quality and requires digital channel-bit rates in the 50-80 Mbps range. Typically, it consists of straightforward analog-to-digital conversion of video information. The
levels of conversion may range from six to eight bits, and no attempt is made
to perform any coding of the digitized-video information. Because of its rea-
sonably high quality, the conventional PCM is often used as a quality reference
for video-digitizing processes.

To achieve bandwidth compression, encoding techniques must be applied
to the digitized video stream. Because the encoding is applied to the digitized
signal per se and its purpose is bandwidth compression rather than the protection
of the digitized data within the communication channel, it is referred to as
"source encoding." Figure 2.3 shows the categorization of source-encoding
techniques.

Source coding can be applied either within a single frame (intraframe
coding) or over a series of several frames (interframe coding). For the case
of intraframe coding, the signal processing can be applied either in one dimen-
sion, such as on a single-line basis, or over two dimensions, which include
coding across a space extending over several lines within a single frame. Fig-
ure 2.4 provides a qualitative illustration of the intraframe and interframe
coding. Motion compensation is one of the examples where efficient interframe-
encoding algorithms are applied to achieve significant bandwidth compression.

2.2.2 Encoding-Scheme Evaluation Criteria

Unfortunately, no single scheme has yet been developed for evaluating
picture quality versus bandwidth compression of various encoding schemes. This
is because each case is not only applications oriented, but is also a function
of picture content. Figure 2.5 shows a general relationship between the coding
bit rate, coding technique, and picture quality. From Figure 2.5, it is evident
that some sort of interframe coding, such as that used for teleconferencing, must
be utilized for the SS to achieve channel rates below 10 Mbps, such as a baseline
rate of 2.048 Mbps.

On the other hand, once one resorts to interframe coding, the picture
quality becomes highly content dependent, as illustrated in Figure 2.6. From
this figure, it is evident that, in order to accommodate picture-content varia-
tions such as those caused by motion, rather sophisticated interframe-coding al-
gorithms have to be employed to preserve an acceptable degree of picture quality.
Figure 2.7 demonstrates the importance of coding algorithms for interframe (i.e.,
frame-to-frame) signal processing for bandwidth compression. As indicated in this
figure, neither simple frame-to-frame coding nor even complex interframe coding,
Figure 2.3. Television Source-Encoding Categorization
INTRAFRAME (Spatial)

One Dimensional

Two Dimensional

INTERFRAME (Temporal)

Three Dimensional

Figure 2.4. Alternative Methods of Digitized TV Signal Coding
Figure 2.5. Coding Bit Rate Versus Picture Quality for Typical Coding Methods [2]

Figure 2.6. Interframe-Coding Performance Features [1]
Figure 2.7. Relative Performance of Coding Techniques in Terms of Picture Quality versus Transmission Rate [2]
such as conditional replenishments, can provide good quality with transmission rates in the range of 1.5 to 3.0 Mbps. A special motion-compensation algorithm is required that achieves the most efficient utilization of interframe-comparison information. Such an algorithm is described in section 2.3.1.

Before concluding this rather general discussion of the relationship between the coding techniques for bandwidth compression and picture quality, the following criteria for judging picture quality [2] should be listed. These are: (1) edge sharpness, (2) edge accuracy, (3) flat-area busyness, (4) image continuity and, (4) distortion.

Edge sharpness defines the degree of edge blurring that is typical of bandwidth-limited TV pictures. Edge accuracy deals with the degree of waviness occasionally observed with some of the coding schemes. Flat-area busyness is a result of quantization noise-limited numbers of bits in a sample. This phenomenon is manifested as a grainy noise or false contouring in the flat areas. Image continuity manifests itself as breaks in the image resulting from frame-rate (interframe) processing techniques. Distortion defines the displacement of objects from their correct relative positions within the image.

For each given mission, the relative weight of these criteria can be only roughly assessed. Precise weighting will be impossible until there is more agreement between the users on the subjective criteria used for evaluation.

2.3 Video Bandwidth-Compression-Techniques Description

In this section, we consider some of the video bandwidth-compression techniques applicable to SS, TV and picture transmission at rates comparable to the baseline maximum channel rate of 2.048 Mbps. In this case, the emphasis is on those techniques whose feasibility has been demonstrated either in the field or in the laboratory. Not all of these techniques have been reduced to the size required for SS-type use but, at least in principle, such reduction appears feasible in the future, particularly in view of the continued progress in LSI and VLSI technology developments. In section 2.4, we make our first-cut predictions on the possible size reduction due to LSI/VLSI utilization.
2.3.1 Motion-Compensation Technique

Motion compensation is an algorithm which is added to interframe (i.e., frame-to-frame) encoding in order to improve picture quality at lower transmission rates (1.5 to 6 Mbps). Relative improvements due to this algorithm were shown in Figure 2.7. Prior to discussing the details of the motion-compensation principle, a brief description of interframe-coding principles is in order.

The interframe coding achieves the reduction in transmission bit rate by transmitting the difference signal between two successive frames. As shown in Figure 2.8, an object moving across the TV screen is positioned differently at each successive frame. The position difference, when digitized, represents a relatively small amount of information compared to that contained in the entire frame. Thus, if there is no movement, no new information needs to be transmitted. On the other hand, large displacements require a considerable amount of new information to be transmitted from frame to frame.

Figure 2.9 shows the generic configuration of an interframe coder. The analog video signal is first quantized by an A/D converter. The digital samples are then applied to the interframe coder which stores at least one entire frame in a digital format. When the subsequent frame is received and digitized, the encoder compares it to the previous frame. The difference information developed as a result of this comparison is then processed by a variable-length coder that removes the redundancy contained in the frame-difference signal. The buffer memory then performs smoothing of the difference signal. This smoothing ensures that the transmission rate is constant, independent of the movement of picture objects. In order to prevent buffer-memory overflow, a feedback-control is used to suppress excessive generation of information as a result of the frame-differencing process. Because the control of information rate is implemented by changing the quantization-step size, the signal-to-noise ratio (SNR) degrades as the amount of motion increases. Consequently, when the information rate exceeds the permissible transmission channel rate, the quality of the encoded picture starts to deteriorate. Conversely, if the picture quality is not allowed to deteriorate below a certain limit, the channel rate must increase, as indicated in Figure 2.7.

The motion-compensation algorithm provides picture-quality improvement for a given channel bit rate. Figure 2.10 illustrates the principle of motion compensation. As shown there, the picture element \( S_n(i,j) \) is the sample to be transmitted, where the subscript \( n \) designates the frame number and \( i \) and \( j \) give the \( x \) and \( y \) coordinates of the element within the frame.
Interframe Coding, Converter - Present Previous Frame Difference

Figure 2.8. Principle of Basic Interframe Coding

A/D Converter
Interframe Coding
Variable-Length Coding
Buffer Memory
Feedback Control

Figure 2.9. Generic Configuration of an Interframe Encoder

Motion-compensated Interframe Prediction

Figure 2.10. The Principle of Motion-Compensated Interframe Coding
If the object displacement in one time frame is given by the motion vector \( V(x,y) \), one can then predict the position of the element \( S_n(i,j) \) for the time of arrival of the \( n \)th frame using an estimate of this vector. Such a prediction thus minimizes the interframe difference between the element of the previous frame, i.e., \( S_{n-1}(i-x,j-y) \) and the same element \( S_n(i,j) \) of the current frame. Thus, it is the function of the motion-compensation interframe-coding algorithm to determine the estimate of the motion vector \( V(x,y) \) from the picture.

Figure 2.11 shows a functional block diagram of the motion-compensated interframe coder and decoder such as Nippon Electric Company's (NEC) models no. NETEC-6/3 and NETEC-X1. The input video in Figure 2.11 is the NTSC color television signal. This signal is first digitized, then it is applied to a color signal processor (CSP-5) which converts the NTSC formatted signal into a time-division-multiplex (TDM) signal. The TDM signal is a baseband video signal with a time-compressed chrominance signal inserted into the horizontal-blanking interval. The TDM signal is then applied to the movement detector and an interframe coder.

In the NEC system, the picture field is subdivided into subblocks of seven picture elements (pel's) times seven lines. Calculation of the motion vector \( V(x,y) \) is then performed for each subblock using high-speed parallel processors. Interframe coding is then performed on a frame-to-frame basis, with the variable delay controlled by the motion vector. As in sample interframe coding, special techniques are used to prevent buffer overflow in the coder. The techniques used are: (1) adaptive quantization, (2) field repeating and, (3) subsampling. These techniques correspond, respectively, to resolution control in the amplitude, time and space domains. Huffman codes and run-length codes are then used to compress video data corresponding to interframe differences and the motion vectors.

The decoding process is the inverse of coding. But, because the motion vector is transmitted as a separate signal, motion detection is not required at the decoder. The interframe decoder unit converts the predetection error signals to TDM signals. This is accomplished by adding the error signals to the motion-compensated prediction signals. Subsequently, the TDM signal is converted back to the NTSC format and the latter applied to the D/A converter. In all, the receiver (i.e., decoder) configuration is much simpler than the transmitter.

Figure 2.7 shows that good-to-excellent picture quality can be obtained at channel bit rates in the range from 1.5 Mbps to 3 Mbps, with the motion-compensation algorithm added to the simple and complex frame-to-frame
Figure 2.11. Motion-Compensated Coder and Decoder Block Diagram
encoding. Realization of this algorithm is provided in operational video-conferencing systems such as NEC's NETEC-6/3 and NETEC-X1 equipment as well as the VTS 1.5E system of Compression Labs, Inc. Specifications for these systems are provided in Appendix A.
2.3.2 Hybrid Coding for Bandwidth Compression of Video Signals

2.3.2.1 General considerations

Hybrid coding usually refers to an encoding process whereby the video information is first subjected to a transform process, then the transform is digitized according to some quantizing process, such as differential pulse-code modulation (DPCM).

With a "full-blown" transform coding, either the entire picture or a frame may be transformed using either of the techniques listed in Figure 2.3 (see analysis-synthesis methods), then the transform coefficients are transmitted by means of DPCM. Such "pure" transform coding, however, is rather complex to be implemented in real time because: (1) it requires excessive storage of data and, (2) a large number of operations per pel are required. On the other hand, hybrid coding offers a partial solution to this problem. Specifically, hybrid coding is characterized by: (a) use of relatively small blocks for obtaining transform coefficients and (b) implementing DPCM of the transform coefficients for these blocks by making use of the coefficients of the previously transmitted block, i.e., using the past coefficients as predictors.

The use of smaller blocks for encoding is justified since, as shown in Figure 2.12, little is gained in picture-error reduction as the number of pels per block is increased. With respect to selecting the method, or an "algorithm" for encoding the transform coefficients via DPCM, the following three types have been considered for the hybrid coding:

1. A one-dimensional block along the horizontal scan line with DPCM in the vertical direction

2. A small, two-dimensional block and DPCM using the coefficients of a horizontal previous block for predictions

3. A two-dimensional block with the DPCM implemented in the temporal direction.

It must be noted that methods (1) and (2) above are intraframe (or intrafield), whereas method (3) is interframe. It is also interesting to point out that considerable savings in data storage is realized with the third hybrid-coding scheme as compared to the "pure" transform-coding method which uses three-dimensional blocks.
Figure 2.12. Mean-Square Error Performance of Different Transforms for the Two-Dimensional Markov Image Source with Horizontal Correlation Factor of $\rho_C$ and Vertical Correlation Factor of $\rho_r$ [3]
Figure 2.13 shows a generic block diagram of a hybrid-transform/DPCM encoder. As shown therein, the video is first transformed, then applied to DPCM encoders which, in turn, are followed by the code-assignment units. We show more than one DPCM encoder in order to indicate that, ideally, each transform coefficient should be encoded by its own DPCM unit that is specifically adapted to the particular coefficient. However, such implementation is not efficient from the standpoint of hardware realization. Thus, as described in following sections, in a practical system, an attempt is made to design a single DPCM encoder which can handle all the coefficients. Also, as indicated in Figure 2.13, certain transform coefficients are dropped so as to minimize the bits/rel ratio.

The code-assignment unit and a quantizer that precedes it (not shown in Figure 2.13) are very important subblocks of a hybrid coder. These units determine the manner in which the quantization levels are established for the purpose of digitizing and transmitting the transform coefficients for the various blocks.
Figure 2.13. Block Diagram of a Hybrid Transform/DPCM Encoder

- Video In
- DPCM Encoder
- Code Assignment
- Multiplex
- DPCM Encoder
- Dropped Coefficients
- Demultiplex
- DPCM Decoder
- Inverse Transform
- DPCM Decoder
- Display
- To Channel

From Channel
2.3.2.2 Harris Corp. hybrid coding system for RPV use

A system developed by Harris Corporation for RPV applications [4] is an example of the practical implementation of a hybrid-coding method. We have chosen to describe this system because it has been implemented with LSI components to dimensions suitable for space-borne applications in addition to having been tested in the laboratory. This system is not in the same class as the previously described motion-compensated, full-color, video-conferencing equipment in that it is black and white only and has a maximum frame rate of 7-1/2 frames per second. However, because of its already demonstrated size-reduction and variable-bandwidth-compression capabilities, it is of definite interest to potential applications in SS communications. Table 2.3-1 lists the salient characteristics of the Harris Corporation hybrid-coding system.

Figure 2.14 shows the functional block diagram of the video-bandwidth-compression system made by Harris. This system utilizes the discrete-cosine transform (DCT). As shown there, the image is first cosine transformed, then applied to the scaler, which then feeds the DPCM unit. The transform size is 16 elements along the scan line. From this ensemble, 16 transform coefficients are obtained. Harris claims [4] that their system implements an optimum-bit assignment with respect to the block-quantization process. This claim is based on their assumption that each transform coefficient at the output of the DCT is an independent Markov process with different correlation coefficients for each frequency term.

Figure 2.15 explains some details of the Harris system hardware implementation. One of the unique features of this implementation is the use of a table look-up technique instead of either a hardware or software multiplication which is required for the DCT process. Thus, the hardware complexity is simplified, and the high-speed-transform process is realized. Another feature of this system is the use of a scaler to equalize the values of the transform coefficients prior to the DPCM. Specifically, the scale factors are selected in such a manner that the scaled values of the non-DC component variances are as close as possible to the DC component's variance. Thus, although a limiter is shown in the block diagram after the scaler, proper design of the latter minimizes the possibility of coefficient clipping prior to DPCM. The use of the scaler prior to DPCM allows a single one-step predictor to be employed for all 15 coefficients. This, too, simplifies the complexity of the hardware implementation.
Table 2.3-1. Salient Characteristics of the Harris Corp. Hybrid-Coding System [4]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/Description</th>
</tr>
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<tbody>
<tr>
<td>Transform type</td>
<td>Cosine</td>
</tr>
<tr>
<td>Maximum resolution</td>
<td></td>
</tr>
<tr>
<td>Horizontal: 640 pixels</td>
<td></td>
</tr>
<tr>
<td>Vertical: 420 lines</td>
<td></td>
</tr>
<tr>
<td>Frame rate</td>
<td>Maximum: 7-1/2 fps*</td>
</tr>
<tr>
<td>Minimum: 15/128 fps**</td>
<td></td>
</tr>
<tr>
<td>Channel-rate requirement</td>
<td>Maximum: 4.608 Mbps (at 7-1/2 fps)</td>
</tr>
<tr>
<td>Minimum: 72 kbps (at 15/128 fps)</td>
<td></td>
</tr>
<tr>
<td>Bits/pel range (selectable)</td>
<td>Two, one, and one-half</td>
</tr>
<tr>
<td>Dimensions</td>
<td>Two modules</td>
</tr>
<tr>
<td>(LSI implementation)</td>
<td>4.5&quot; x 4.5&quot; x 0.5&quot; each</td>
</tr>
</tbody>
</table>

*Frame refreshed every 0.133 seconds.
**New frame every 8.53 seconds.
Figure 2.14. Functional Block Diagram of the Video-Bandwidth Compression and Reconstruction Units

*DCT = Discrete Cosine Transform
Figure 2.15. Hardware System Design of Compression and Reconstruction Units.
The system operates in the following manner. The A/D converter digitizes the input video to eight bits. An entire frame is stored in the frame-grab memory at full resolution and at eight bits per pixel. Loading of the frame is done in real time. At the rate of 7-1/2 frames per second, there are four frame periods of the standard 30-fps rate. During the first frame period, the memory is loaded with data. The processing commences immediately, continues during the loading period, then extends for the duration of four frame periods. Thus, at the input rate of 30 fps, three of the four frames are not processed. The resulting average output data rate, \( D \), in megabits per second (Mbps), is

\[
D = F \times H \times V \times b \times T \times 10^{-6}
\]

where:

- \( F \) = transmitted frame rate in frames per second
- \( H \) = number of pixels in the horizontal line
- \( V \) = number of lines (vertical resolution)
- \( b \) = bits per pixel
- \( T \) = truncation area ratio

For example, if \( F = \frac{7}{2} \text{ fps} \), \( H = 640 \text{ pixels} \), \( V = 480 \text{ lines} \), \( b = 2 \) bits per pixel, and \( T = 1 \), the required channel rate is 4.6080 Mbps. On the other hand, if \( F = \frac{15}{128} \text{ fps} \), \( H = 640 \), \( V = 480 \), \( b = 2 \), and \( T = 1 \), then \( D \) is 0.072 Mbps or 72 kbps. For comparison purposes, with uncompressed, eight-bit quantized PCM video, \( F = 30 \), \( H = 640 \), \( V = 480 \), \( b = 8 \) and \( T = 1 \), and the corresponding required channel rate is 73.728 Mbps. Thus, an approximate compression ratio of 1000:1 can be realized with this hybrid-coding system.

According to Harris Corporation, the entire DCT hybrid-compression system has been implemented in LSI and packaged onto two 4.5"x4.5"x0.5" modules. One module contains the bandwidth-compression circuitry and the second contains the frame memory.
2.3.2.3 Other efforts in hybrid coding

As mentioned earlier, the requirement for transmission of bandwidth-compressed video from RPV's resulted in the development of compact, low-power-consuming equipment. Several of these developments are listed in [5] and are presented in Table 2.3-2. Although the information in this table is somewhat out of date (1979), it is indicative of the trends in this area of development. Because of the military application of the RPV's and the potentially classified nature of the associated hardware, no effort was made on our part to obtain an update to Table 2.3-2.
<table>
<thead>
<tr>
<th>Dimensionality</th>
<th>RCA</th>
<th>Northrop</th>
<th>Motorola</th>
<th>TRW</th>
<th>Lockheed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transform operator</td>
<td>Cosine; DPCM between lines</td>
<td>Haar</td>
<td>Hadamard</td>
<td>Adaptive</td>
<td>Constant-area quantization</td>
</tr>
<tr>
<td>Frame memory type</td>
<td>Sample</td>
<td>Full Grabber</td>
<td>Full Grabber</td>
<td>Full Grabber</td>
<td>Full Grabber</td>
</tr>
<tr>
<td>Relative MIPS required</td>
<td>Moderate</td>
<td>Low</td>
<td>Moderate</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Quantizer complexity</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>Moderate</td>
</tr>
<tr>
<td>Frame-rate-reduction complexity</td>
<td>Low</td>
<td>Moderate</td>
<td>High</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>Effective compression range bits/pixel</td>
<td>0.75 - 2.0</td>
<td>0.75 - 2.0</td>
<td>75 - 2.0</td>
<td>1.0 - 2.0</td>
<td>1.2 - 2.0</td>
</tr>
<tr>
<td>Edge quality</td>
<td>Moderate</td>
<td>Good</td>
<td>Good</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>Distortion</td>
<td>Poor for fast-moving scenes</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>Flat-region noise</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>Noise immunity</td>
<td>Good</td>
<td>Moderate</td>
<td>Good</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>Size of physical hardware</td>
<td>Small</td>
<td>Moderate</td>
<td>Moderate</td>
<td>Small</td>
<td>Small</td>
</tr>
</tbody>
</table>
2.3.3 Delta-Modulation Techniques for 15-to-30-Mbps Digital TV Links

2.3.3.1 Overview

Several video-digitizing techniques exist, or are in the development stage, which can provide transmission of either field-sequential or NTSC-format color signals. These techniques utilize adaptive delta-modulation systems that can yield reasonably good-quality pictures, with full-color capability, at channel rates in the 15-to-30 Mbps range.

The primary advantage of these techniques is that they can be implemented with relatively few components. Thus, they can be used where space and power limitations are of importance. The disadvantage of using these techniques is a relatively high-channel-data rate. In a power-constrained link, this limits the operating range of the system.

Over the past several years, three approaches for employing delta modulation for TV purposes have evolved. One is to use a single adaptive delta modulator for transmitting either black-and-white (B/W) or field-sequential color television. Another approach is to use the same delta modulation for the transmission of digitized composite NTSC color signals. A third approach is to first decode the composite NTSC signal into the Y (luminescence) component and the I&Q (chrominance) components, then transmit these components by means of separate delta modulators. With the latter approach, intraframe techniques such as run-length encoding can be used to reduce the overall channel bandwidth rate. Some of the technical aspects of these approaches are described herein.

2.3.3.2 Bistate adaptive delta modulation

Successful applications of delta modulation to TV signal digitization has been reported in the literature \(^ {6,7}\) and experimental work continues with respect to different algorithms used for digitization and encoding. The commonly used delta-modulation encoder produces a sequential output that indicates whether the final output voltage should increase or decrease. It acts like a DPCM system with only one bit per sample. Hence, it is referred to as a "bistate" encoder. Acceptable performance with this encoding scheme is realized by an increased sampling rate as compared to a PCM coding scheme. Because the bistate delta modulator requires relatively low data rates compared to PCM, considerable effort has been spent trying to perfect a delta modulation system.
The goal of every design approach is to trade off fidelity of reproduction versus the allowable data transmission rate and the amount of hardware used. The best solution also depends on the qualities of the signal being sampled. Video information tends to be a series of amplitude steps from one luminance level of another. These level steps are connected by voltage changes with large $dV/dt$ values. When handling a video signal, a delta modulator must handle very sharp rise and fall time signals, then settle rapidly to a new luminance value, with no overshoot, ringing or other added noise. The relative delay for different frequency components in the picture is also important.

Figure 2.16 is a block diagram of a bistate delta modulator which illustrates its operation. $D_k$ is the difference between the input signal $S_k$ and the output of the estimator $X_k$. This difference signal is passed through a hard limiter whose output is a bit $B_k$ which causes the estimator to change its output. This bit is the signal which is transmitted over a communication link to the receiver. The receiver consists of only the estimator portion of the transmitter block diagram. The sampling rate of the delta modulator determines the amount of delay inherent in the digitizing process. The faster the sampling rate, the smaller the inherent delay.

The hard limiter shown in Figure 2.16 guarantees that the estimator will increase or decrease its output every time the sample clock cycles. This characteristic of indicating only changes results in the inherent "granularity" in the reproduction of a constant-luminance signal since the modulation scheme must always output a sample corresponding to an increase or decrease in voltage. The reconstructed signal will average to the correct level by varying up and down around the desired output level. Any attempt to smooth the delta modulation output will slow the response to step changes and, therefore, reduce picture "sharpness."

Conversely, if the bistate delta-modulation scheme is designed to provide good rise and fall times, the amplitude of the level jitter around the desired output will increase and more granularity will appear. Furthermore, the amount of overshoot and ringing after an abrupt edge will usually increase as the delta-modulation system is designed to reduce slew time. Various attempts to alleviate these conflicting requirements and reduce the overshoot and ringing have been proposed [8,9].
where:

\[ S_k = \text{signal to be transmitted (sampled and digital)} \]

\[ X_k = \text{estimate of signal to be transmitted} \]

\[ D_k = \text{quantized difference} \quad (D_k = S_k - X_k) \]

\[ B_k = \text{activity vector (signal rising or falling)} \]

Figure 2.16. Conventional (Bistate) Delta Modulator Functional Block Diagram
Figure 2.17 shows how a bistate adaptive delta modulator (ADM) is used to provide field-sequential encoding of color signals. As illustrated in this figure, the ADM sequentially encodes the red, green and blue signals generated by the TV camera [10]. Thus, a typical sequence of color encoding and transmission would be: field 1--red, field 2--green, field 3--blue. The advantages of this scheme of color-picture transmission are twofold: (1) extreme simplicity of the camera and, (2) the channel-bit rate is the same as that required for transmission of black-and-white pictures. Another sideline advantage of this scheme is that the edge busyness, which is typical of delta modulators and is a random process within each frame, tends to be masked when the three color fields are recombined in the scan converter prior to being displayed on the screen.

The disadvantage of this field-sequential system is that only one color is transmitted per field. Thus, the color components tend to separate at the edges of moving objects. This is because the color rate frame is only 10 frames per second, i.e., only one-third of the normal rate. The disadvantage associated with the color misregistration when using the ADM for transmitting field-sequential color video can be overcome if a camera providing an NTSC composite signal is used as a source of video. With this approach, the ADM samples and encodes the composite signal, which includes both luminance and color modulated subcarrier signals at 3.58 MHz. Although reasonably good color picture quality can be obtained with this scheme, some interaction between the ADM sampling rate and the color subcarrier signal is inevitable. Thus, efforts to optimize the ADM algorithm with respect to sampling the color subcarrier signal have been reported in recent literature [11].

---

*A simple color-wheel camera can be used for this application.*
2.3.3.3 Tri-state delta modulation for run-length encoding

2.2.2.3.1 General principles

The effort to take advantage of video signal statistics has been applied to delta-modulation encoding. Specifically, an algorithm has been developed that allows the delta modulator to "sense" or "detect" those regions of the video picture where virtually no change in intensity is present. By providing a special code that indicates a "no-change" condition, only the number of samples of such a no-change condition has to be transmitted instead of the samples themselves. Thus, a potential channel data-rate reduction can be realized by using such a run-length (R/L) encoding technique.

The tri-state delta modulator (TSDM) [12,13] has characteristics that make it especially suitable for R/L encoding of video information. By indicating the "no-change condition" with a special code, the TSDM breaks the connection between rise-time performance and steady-state level jitter inherent in a bistate delta modulator.

In comparison with the bistate delta modulation, the TSDM approach shown in Figure 2.18 quantizes $D_k$ into three values instead of two. The third value indicates that $D_k$ is between $-e_v$ and $+e_v$. This condition causes the output of the estimator to remain at its previous value. Just as in the case of bistate delta modulation, the receiver consists of only the estimator. This third state eliminates the granularity inherent in bistate delta modulation; it permits the TSDM to be optimized for transient performance without considering granularity problems. With the exception of the third state, operation of the TSDM is identical to the bistate delta modulator shown in Figure 2.17.

As indicated in Figure 2.18, the TSDM state is determined by comparing the estimate $X_k$ with the input signal $S_k$ to produce a voltage difference signal $D_k$. This voltage $D_k$ is tested for sign and magnitude to produce $B_k$. The rule is:

$$B_k = \begin{cases} \text{Sgn}[D_k], & \text{whenever } |D_k| > e_v \\ 0, & \text{whenever } |D_k| \leq e_v. \end{cases}$$

Since $D_k = S_k - X_k$, a positive voltage for $D_k$ signifies that $X_k$ must be made more positive in order to match $S_k$; therefore, $B_k = +1$ must cause $X_k$ to become more positive. The equation used to calculate $X_{k+1}$ from $X_k$ is simply
*With TSDM, the $B_k$ vector can also represent a "signal-steady" condition.

Figure 2.18. Tri-State Delta Modulator (TSDM) Functional Block Diagram
\[ X_{k+1} = X_k + B_k \lfloor \Delta_{k+1} \rfloor \]

where \( B_k \) may be +1, 0 or -1, and \( \lfloor \Delta_{k+1} \rfloor \) is given by

\[
\lfloor \Delta_{k+1} \rfloor = \begin{cases} 
1.5 \Delta_k & \text{if } B_k = B_{k-1} \neq 0 \text{ and } |\Delta_k| > 2\Delta_0 \\
3\Delta_0 & \text{if } B_k = B_{k-1} \neq 0 \text{ and } |\Delta_k| \leq 2\Delta_0 \\
0.5 \Delta_k & \text{if } B_k \neq B_{k-1}, B_k \neq 0, B_{k-1} \neq 0 \\
0 & \text{if } B_k = 0 \\
3\Delta_0 & \text{if } B_{k-1} = 0, B_k \neq 0
\end{cases}
\]

When \( |D_k| \leq e \), the feedback signal \( X_k \) is close enough in value to the input signal \( S_k \) so that the system should maintain its state. It does this by making \( X_{k+1} = X_k \) and setting \( \Delta_{k+1} = 0 \) whenever \( B_k = 0 \). That process is unique to tri-state delta modulation.

The adaptive characteristic of the system is realized by the \( |\Delta_{k+1}| = 1.5 \Delta_k \) term. When \( B_k \) has the same sign \( m \) times in sequence, the value of \( \Delta_k \) is multiplied by \( (1.5)^{m-1} \), so the slope of the feedback signal increases without limit until \( X_k \geq S_k \). In this case, the \( B_k \) vector changes sign, and the value of \( |\Delta_{k+1}| = 0.5 \Delta_k \) reduces the slope as the \( X_k \) signal changes direction. This provides for the eventual settling of \( X_k \) to a value near \( S_k \) by reducing the value of \( \Delta_k \) by a factor of 2 every time \( X_k \) crosses \( S_k \). If \( S_k \) remains constant long enough, \( X_k \) will eventually settle to within \( +e \) of \( S_k \) if \( e \) is at least as large as one-half the resolution of the DAC that produces \( X_k \). Once this happens, \( B_k \) will be equal to 0 and the systems will stay at rest until \( S_k \) changes again.

When \( B_k \) changes from \( B_{k-1} = 0 \) to either \( B_k = +1 \) or \( B_k = -1 \), the system must inject a starting value for \( |\Delta_{k+1}| \) since \( |\Delta_k| \) was 0. That is handled by the \( 2\Delta_0 \) term which is simply a constant value selected to provide a large enough initial slope to the \( S_k \) feedback signal when it must adjust to a step input in \( S_k \), so that the slew time is minimized.
2.3.3.3.2 TSDM digitizer/reconstructor implementation*

Figure 2.19 is a block diagram of a tri-state video digitizer with the receiver input and output indicated by dotted lines. Because the receiver is only a portion of the transmitter, it will not be discussed separately. As shown in this diagram, the analog video signal $S_k$ is compared with the reconstructed analog estimate of the previous sample $X_k$. The analog difference $D_k$ is applied to the A/D converter. The hysteresis bias, or "dead zone," $\pm \epsilon$, is also applied to the analog bias input of the A/D converter. The analog error is then sampled by the A/D unit and transformed into $B_k$ values according to the previously described algorithm.

The hardware block diagram illustrated in Figure 2.19 implements the required tri-state algorithm. To provide the fastest sampling rate possible, the various alternative computations are performed in parallel and the appropriate result is selected by a 4-to-1 MUX. There are two 4-to-1 multiplexers: one for the next value of $x$ and one for the next value of $\Delta x$. The decision concerning which input to select is made by the output of the A/D converter $B_k$ and the comparator which operates on both $2\Delta_0$ and the output of the $\Delta x$ register. The implementation shown in Figure 2.19 provides the maximum amount of time for the DAC, analog subtractor and three-state A/D converter to settle by performing all possible calculations while they are settling, then selecting the actual output needed. The $x$ register input, for example, may be any one of the following numbers: $x, x + 3\Delta_0, x - 3\Delta_0, x + 1.5 \Delta x, x - 1.5 \Delta x, x + 0.5 \Delta x,$ and $x - 0.5 \Delta x$.

Depending on the present vector $B_k$ and the preceding vector $B_{k-1}$, the logic and decision unit (LDU) sends the appropriate commands to the 4-to-1 multiplexers. It also sends the appropriate add/subtract commands to the $x$ register arithmetic logic units (ALU's) A, B and C. To ensure that the delta modulator does not "hang up" with a dead zone, the output of the $\Delta x$ register is always compared with the $2\Delta_0$ value. If the value of the $\Delta x$ register is less than $2\Delta_0$, the comparator controls the 4-to-1 multiplexers to select $3\Delta_0$ as the next increment. The value of $3\Delta_0$ is also selected when the LDU indicates that a transition from a no-change state (i.e., constant intensity) to a change stage, i.e., an increase or decrease in intensity has occurred on the latest sample.

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*This implementation was carried out under a NASA contract by Axiomatix Corporation of Los Angeles, California.
Figure 2.19. Tri-State Video Digitizer (and Reconstructor) Implementation
For the receive node, the logic implementation is the same as that shown in Figure 2.19, but without the analog subtractor or A/D converter. As indicated by the dotted lines, the received data stream is applied directly to the $B_k$ and $B_{k-1}$ storage registers and the reconstructed analog video is taken from the output of the D/A converter.

2.3.3.3.3 Run-length encoder/decoder implementation

The block diagram of the TDSM R/L encoder is depicted in Figure 2.20. The high-speed encoding circuitry accepts the $B_k$ values at the sample clock rate and outputs a sequence from that sample, as shown in Table 2.3-3. The sample clock rate is fixed. Since the encoding process produces a variable number of bits out, depending on the sample inputs, the data output rate of the encoder varies.

The rotating buffer provides the necessary elastic storage between the varying input data rate and the fixed output rate. The output of the FIFO, under normal circumstances, is clocked by the transmission channel clock that is supplied to the TSDM from an external source. If the picture being transmitted happens to encode very efficiently so that insufficient data is entering the FIFO to replace the data being clocked out, the R/L encoder circuitry is apprised of this by the "potential underflow" line shown in Figure 2.20. When a potential underflow problem exists, the R/L encoder circuitry waits until a new horizontal synchronization pulse occurs. At this time, the R/L encoder generates a codeword indicating the end of valid data. The rotating buffer has another output that indicates potential overflow. This output feeds back to the encoding circuitry, which changes its operating mode when the possibility of overflow occurs.

Figure 2.21 is a block diagram of the receiver R/L decoder. It is essentially the encoder turned around. The FIFO provides data to the decoding circuitry upon demand, so the output of the decoder can be a constant sample rate despite the variable-length decoding process. The elastic storage in the receiver rotating buffer forms the other half of the elastic storage in the transmitter. The total information stored in the two memories should vary, roughly together, depending on how the picture data is modified by the encoding algorithm. The sample rate into the encoder is constant, as is the output at the decoding end of the link. The link between the encoder and decoder also has a fixed rate which is somewhat greater than the sample rate in order to obtain the improved picture quality of tri-state delta modulation. The actual link data rate and the type of picture being scanned determine how often the system degrades to bistate delta modulation.
Figure 2.20. Run-Length Encoder (Transmitter Side)
Table 2.3-3. Encoding Table for TSDM

<table>
<thead>
<tr>
<th>$B_k$ Vector</th>
<th>Transmitted Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>10</td>
</tr>
<tr>
<td>-1</td>
<td>11</td>
</tr>
<tr>
<td>0</td>
<td>01110</td>
</tr>
<tr>
<td>Two 0's in a row</td>
<td>01111</td>
</tr>
<tr>
<td>Three 0's in a row</td>
<td>01100</td>
</tr>
<tr>
<td>Four 0's in a row</td>
<td>01011</td>
</tr>
<tr>
<td>Five 0's in a row</td>
<td>01010</td>
</tr>
<tr>
<td>Six 0's in a row</td>
<td>01001</td>
</tr>
<tr>
<td>Seven 0's in a row</td>
<td>01000</td>
</tr>
<tr>
<td>Eight 0's in a row</td>
<td>00</td>
</tr>
</tbody>
</table>

2.3.3.3.4 Extension of TSDM to NTSC color TV

In the previous discussion, TSDM was considered as a technique for digitizing a single video signal that is presumed to be the NTSC luminance component used for black-and-white TV transmission. Because field-sequential color transmission utilizes a sequence of such components, as provided at the output of a rotating color-wheel filter, the approach described is applicable without modification for systems such as those in use today by the Space Shuttle. For standard NTSC color TV transmission, however, the TSDM technique is applied to each of the three color components, Y, I and Q, where Y is the luminance component and I and Q are the chrominance components.

Figure 2.22 shows the method of handling those components, each with its own R/L encoder. For maximum data comparison, each component should have its own R/L encoder before multiplexing. Otherwise, since the color and luminance components do not correlate well, there is little data-rate reduction for the chrominance components when multiplexed with the luminance information.

The current breadboard for the NTSC color TSDM system requires a volume of about 5 ft$^3$ each for the transmitter and receiver. With LSI packaging, this system may be reduced to a volume of about 0.5 ft$^3$. 
Figure 2.21. Run-Length Decoder (Receiver Side)
Figure 2.22. NTSC Color TV Component Digitization

<table>
<thead>
<tr>
<th>Component</th>
<th>Maximum Bandwidth (MHz)</th>
<th>Maximum Bit Rate (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>4.2</td>
<td>15 - 20</td>
</tr>
<tr>
<td>I</td>
<td>1.5</td>
<td>4 - 8</td>
</tr>
<tr>
<td>Q</td>
<td>0.5</td>
<td>1 - 2</td>
</tr>
</tbody>
</table>

Total ≈ 20 - 30

*After coding.
2.4 VLSI Utilization Potential

2.4.1 Capabilities and Limitations of VLSI Applications to Bandwidth-Reduction Systems

VLSI (very large-scale integration) techniques are being applied to more electronic systems every day. The computer field, in particular, is amenable to their application, and many devices have already been developed. For example, 64k RAM chips are now on the market, with a number of alternative manufacturers. Other examples of products already developed in VLSI include a two-chip, 32-bit generalized data processor by Intel and a 32-bit CPU chip by Hewlett Packard which is capable of operating with an 18-MHz clock. These chips were developed several years ago, so VLSI is clearly a practical technology whose "time has come."

Packaging of more circuitry on one chip has more than one advantage. In addition to reducing the volume required to perform a given system function, the overall system reliability is improved. Specifically, because there are fewer chips in the system, reduced interconnections between chips increase reliability. However, in terms of the ultimate size reduction possible through VLSI techniques, no absolute value exists for the ratio of volume between VLSI and previously developed systems employing SSI and MSI.

The actual size reduction depends on many details of the system being redesigned and how well the functions to be implemented will mesh in a few VLSI chips. If the number of gates on a chip is used as the criteria, ratios of 10 or more are obtainable. The dividing line between LSI and VLSI is generally taken to be somewhere around 15,000 gates per chip, and some VLSI layouts have realized 150,000 gates on one chip. A new system under development, however, would not be amenable to the regularity of patterns, such as those in a CPU and memory chips. Therefore, 80,000 gates per chip should be a good average, and the ratio of chip-count reduction for motion compensation or interframe encoding should be about 10:1. Also, since the VLSI chip will generally require a larger package due to the greater number of inputs and outputs to each chip, the total volume reduction will be significantly less than the 10:1 ratio.

Since the package for a VLSI chip could easily be twice as large as the presently used SSI/SMI/LSI chip, a rough estimate of the overall volume reduction would be 4:1. This estimate may seem quite conservative considering the potential gate-density increase, but a number of limiting factors combine to lessen the potential volume reduction.
These limiting factors include thermal and geometrical considerations as well as propagation times. Reference [14] addresses these problems; Figure 2.23 is from that reference. The idea represented in Figure 2.23 is that several factors converge at approximately the same practical limit to the realizable density of the VLSI implementation. The process of "breaking through" this density limit will therefore require solving several problems at once. However, it must be noted that [14] does not claim that VLSI gate densities will not improve in the future but, rather, that a natural limit exists which, as it is approached, will reduce the rate of progress.
Figure 2.23. Speed-Power-Density Diagram
2.4.2 Specific Systems to be Potentially Reduced Through VLSI

In this section, we make estimates of the potential size reduction to be realized if some of the existing systems were reduced by means of VLSI for SS application. The two systems we consider here are motion-compensated, video-conferencing types of systems which are compatible with our baseline data rate of 2.048 Mbps or less. We picked these systems because they typify a rather complex real-time video-processing system, yet processing has been reduced to operational levels in the field.

We cannot say that the processing algorithms are final for these systems. In fact, Nippon Electric Co. (NEC) feels that the potential limit of the motion-compensation interframe encoding has not yet been reached and, therefore, it is too early (at least for them) to go to custom-made VLSI. Such thinking is understandable because, once an investment is made in custom-made VLSI, the design is essentially "frozen", and no further improvements can be realized.

Both systems to be discussed with respect to size reduction via VLSI were developed for commercial use. They were packaged for low-cost fabrication, ease of maintenance, and with relatively low-density packaging in order to ease field modifications. A thorough review of each system would be required to produce a very accurate estimate of the final package size. The following estimates were made with a rough description of the systems involved; they do not represent the most accurate size estimate possible, given enough time and complete data on the present system.

2.4.2.1 VTS 1.5E System made by Compression Labs, Inc.*

The present system weighs 60 pounds, is 25" wide by 24" deep by 31" high and contains approximately 3,000 integrated circuits. Based on a picture of the unit, a rough estimate is that one-third of the volume is used up by the power supply, base and top of the cabinet, which leaves 25" x 24" x 34" for the actual circuitry. Again judging by the picture, not all of that volume has cards plugged in so that, after repackaging, the system ought to fit in a space 2.4 ft³. It is very conceivable that the volume could drop further if all the blank spaces in the picture are not, in fact, used. Since the system is a two-way system, a one-way system for the SS would fit, after LSI'ing, in a space of about 1.0 ft³.

*See Appendix A for data sheets.
2.4.2.2 NTEC-XITV Codec, made by Nippon Electric Co., Ltd.*

The Nippon system is 42" high by 22" wide by 24" deep and weighs about 350 pounds. It has approximately 10,000 chips. Once again, it is a two-way system, so only about one-half of it needs to be flown into space for SS use. Using the same ground rules as the previous system, it should require about 1.5 ft$^3$ once it has been repackaged and LSI'ed.

*See Appendix A for data sheets.
3.0 RELATED VIDEO-TECHNOLOGY DEVELOPMENTS

In addition to the application of VLSI to SS TV bandwidth compression, several technological developments have taken place. Although perhaps not directly identifiable with bandwidth compression, these developments can be of great value in assembling an efficient system. Some of these innovations are described below.

3.1 CCD Cameras

The charge-coupled devices (CCD's) have progressed sufficiently to yield extremely small and lightweight CCD cameras. However, information available to us so far indicates that only black-and-white cameras are available in CCD implementations. In addition to being small and lightweight, CCD cameras are compatible, at least in principle, with discrete-sample types of signal processing. In essence, the CCD camera sensor consists of an array of multiple light-sensitive elements arranged in rows and columns in the focal plane of the lens. Each CCD element consists of an analog storage device which develops and stores a voltage proportional to the light impinging on the element. Thus, the video-image readout is performed on a dot by-dot basis and is controlled by "clocking" the analog information along the horizontal and vertical axes. This clocking determines the sampled nature of the output video signal.

The grid structure of the CCD camera mosaic appears to be ideally suited for implementing video-image transforms directly onto the image sensor. Information available to us, however, indicates that such a transform implementation is rather difficult to manufacture, particularly when compared to CMOS technology. Consequently, it appears that an approach which uses A/D conversion of the discrete readout of a CCD camera, with subsequent digital signal processing and storage using CMOS devices, is more feasible despite the initial attractiveness of using CCD signals directly for video bandwidth-compression processing.

3.2 Digital TV Devices Applicability

The application of VLSI to commercial television may provide a technology which can be of potential use for SS bandwidth-compression techniques. For example, International Telephone and Telegraph Corp. (IT&T) has introduced several VLSI chips which will replace most of the circuits in modern-day color TV sets [15]. These chips are designed for implementing "digital television." In this case, the word "digital" is not to be confused with digital transmission
such as that considered in this report for SS link applications. Specifically, in a digital TV set, the received video signal is A/D converted at the output of the analog IF, and all of the functions presently performed in an analog fashion are then performed in a digital format. This provides for superior color and luminescence signal processing. Signal averaging on an interframe basis is also realizable with the appropriate memory capability. Although not immediately applicable to our requirements, the development of these chips will synergistically affect the bandwidth-compression technology.
4.0 SUMMARY AND CONCLUSIONS

Various existing video data-compression techniques have been considered from the standpoint of their applicability to space station (SS) digital data links. The emphasis was made on these techniques which are either operational or have been tested and verified in real-time. As the result of this study, the following findings can be summarized:

- Good-quality, moderately-motion TV color pictures can be transmitted at a rate of 2.048 Mbps using interframe encoding with motion compensation. The video processor for this transmission can be packaged to within 1 ft$^3$ using VLSI.

- For channel rates much lower than 2.048 Mbps, low-frame rate, black-and-white pictures can be transmitted using hybrid coding. The processor value for this type of transmission is about 25 in$^3$ using LSI.

- For requirements where a lot of picture motion has to be accommodated, delta modulation can be used provided that a channel capacity of 15-30 Mbps is available. Depending on the requirements (i.e., color versus black and white, resolution, etc.), the processor volume may range from 25 in$^3$ to 0.5 ft$^3$.

One can conclude, therefore, that a number of verified techniques are available for potential SS application. Implementation of these techniques for space application will be greatly facilitated by utilization of LSI/VLSI technologies.
REFERENCES


