SPLINE CURVES, WIRE FRAMES AND BVALUE

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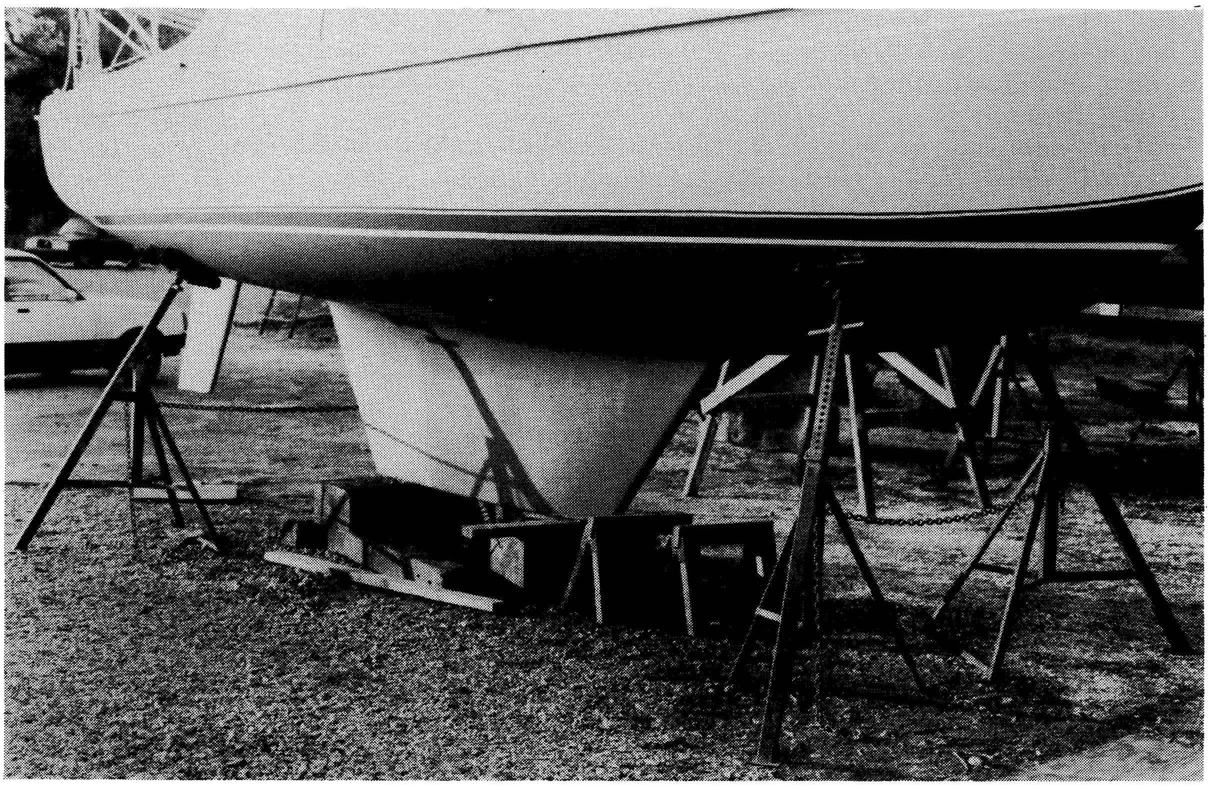

ABSTRACT

Engineers, including naval architects, begin the design of sculptured surfaces by constructing wire-frame curve nets. The boat pictured here first took shape on a drawing board. The wire frame that outlines its shape is called a lines drawing. It is a collection of plane curves drawn with a mechanical spline. Each curve is drawn to fit point, differential, and integral constraints. Curve shape is important.

Modern engineers substitute a terminal for the drawing board and polynomial splines for their mechanical predecessors. Many of them construct their curves with B-splines and find deBoor's FUNCTION BVALUE[1] useful for that purpose. A method centered on BVALUE gives the designer precise control of the fit of each curve to point, differential and integral constraints. It also gives him good control of the shape of each curve and of the whole frame.

The fundamentals of B-spline construction are well described in the literature. They were first applied to design by Gordon and Riesenfeld[2]. This article presents an extension of the Gordon-Riesenfeld method using BVALUE. The motivation is engineering design, in particular, curved surface design. We use curves constructed with B-splines to create a wire frame and then define a mathematical surface patchwise over that frame. The curves are open in the sense of having free ends; however, one or more elements of the frame may be self intersecting to create a surface that is at least partly closed.

The surface itself may be divisible into regions. Within a region, geodesics and lines of curvature may be required to be continuous which, in turn, implies that the patched surface be at least C^3. The surface continuity requirement is reflected in the continuity of the spline curves that make up the wire frame; the curves must be at least C^4. The boundaries of regions may be space curves; the remaining curves in the designer's wire frame are plane.

The engineering surfaces of concern here must meet geometric constraints and be fair. Fairness implies no unwanted humps, hollows, flats or saddles in the surface.

Some point and differential constraints are defined at regional boundaries. They may be known a priori or they may be determined during construction. For example, the locus of a point on the boundary may be defined early, but the exact distribution of surface curvature along a boundary may be decided during the design process and may be influenced by other constraints.

Data for the surface may not be known directly. They are more likely to be known indirectly as constraints and boundary conditions on a frame that supports the surface. We apply these data to construct the elements of the wire frame, one by one.

The designer first constructs the boundary curves of a region. Each boundary curve meets point, tangent vector, curvature and possibly torsion conditions at its ends. Point constraints may also be imposed along the arc of the curve. If the boundary curve is planar, then an area bounded partially by the curve may be specified. Finally, good distributions of curvature and torsion along boundary curves are important if the surface which they support is to be fair.

The plane curves of the wire frame are designed next, one by one. They define the interior of the region and are labeled s- and t-curves. We emphasize that the initial data require a constructive approach and that they are taken line by line. We have not found an automatic or wholesale method for satisfying the data that guide engineering surfaces. For example, the s-curves may be plane cross sections of an air intake duct which has a prescribed distribution of area along its flow axis. We can form each of the s-curves to satisfy the local area requirement, but we can do so only one at a time.

Some geometric constraints must be met precisely on each s- and t-curve. An elementary but sharp example is the intersection of an s-curve with a t-curve. Measurable error in each of the x-y-z coordinates is allowed at the nominal intersection. The amount allowed is affected by manufacturing tolerance and is likely to be small when compared to the dimensions of the cross section. The designer needs a curve construction method that gives him precise control of the loci of the two curves at their nominal intersection, within a tolerance, and without disturbing the positions of either curve at neighboring mesh points.

The curvature at the end of a wire-frame element contrasts with point constraints along its arc. The curvature of one s-curve at a regional boundary is affected by two wire-frame considerations. First, it must be similar to the end curvatures of its neighboring s-curves. Second, the curvature at one end of a wire-frame element influences shape over much of the arc of that element.

The demand for a fair surface poses a dilemma. We do not know of a design method that guarantees a multi-constraint surface which has no unwanted humps, hollows, flats or saddles. We do not know how to prescribe constraints and boundary conditions that are guaranteed not to conflict with each other in the sense of fairness. Lacking such guidance, we retreat to interactive design. We fair graphs of the boundary derivatives, we fair the graph of cross-section areas, and we fair each s- and t-curve. Fairness in a plane curve implies the absence of unwanted inflections. Further, it means a good shape or a good distribution of curvature; it is largely subjective. The fairness of a graph of boundary derivatives is also subjective.

The main goal of this article is to describe the methods that we have developed for wire-frame design. The principal tools for control of a curve during interactive design are mathematical ducks. The simplest of these devices is an analog of the draftsman's lead weight that he uses to control a mechanical spline. We also create ducks for controlling differential and integral properties of curves.

Other methods presented include:

- constructing the end of a Bézier polygon to gain quick and reasonably confident control of the end tangent vector, end curvature and end torsion

- keeping the magnitude of unwanted curvature oscillations within tolerance

- constructing the railroad curves that appear in many engineering design problems

- controlling the frame to minimize errors at mesh points and to optimize the shapes of the curve elements

REFERENCES

1. Carl deBoor: A Practical Guide to Splines, Springer Verlag, 1978.
2. William J. Gordon and Richard F. Riesenfeld: "B-spline Curves and Surfaces," Computer-Aided Geometric Design, Robert E. Barnhill and Richard F. Riesenfeld, eds., Academic Press, 1974, pp. 95-126.