NASA TM-87053

NASA Technical Memorandum 87053

NASA-TM-87053 19850020980

Vibration Control of Rotor Shaft Systems by Active Control Bearings

Kenzou Nonami
Lewis Research Center
Cleveland, Ohio

Prepared for the
Design Engineering Technical Conference
sponsored by the American Society of Mechanical Engineers
Cincinnati, Ohio, September 10–13, 1985

LIBRARY COPY

MAY 12 1986

LANGLEY RESEARCH CENTER
LIBRARY, NASA
HAMPTON, VIRGINIA

NF00140

VIBRATION CONTROL OF ROTOR SHAFT SYSTEMS BY ACTIVE CONTROL BEARINGS

Kenzou Nonami*
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

E-2616

ABSTRACT

This study describes suppression of flexural forced vibration or the self-excited vibration of a rotating shaft system not by passive elements but by active elements. Namely, the distinctive feature of this method is not to dissipate the vibration energy but to provide the force cancelling the vibration displacement and the vibration velocity through the bearing housing in rotation. Therefore the bearings of this kind are appropriately named "Active Control Bearings."

A simple rotor system having one disk at the center of the span on flexible supports is investigated in this paper. The actuators of the electrodynamic transducer are inserted in the sections of the bearing housing. First, applying the optimal regulator of optimal control theory, the flexural vibration control of the rotating shaft and the vibration control of support systems are performed by the optimal state feedback system using these actuators. Next, the quasi-modal control based on a modal analysis is applied to this rotor system. This quasi-modal control system is constructed by means of optimal velocity feedback loops. The differences between optimal control and quasi-modal control are discussed and their merits and demerits are made clear. Finally, the experiments are described concerning only the optimal regulator method.

INTRODUCTION

It is well known that a damped flexible support rotor system suppresses the vibrations of the rotating shaft system. On this point of view, the several studies deal with the optimum tuned condition of a damped flexible support system applying a flexible support (1,2). But it is very difficult for this method to supply appropriate damping on practical equipment or to change a damping coefficient according to system variation. So there are some suggestions about an active vibration control method which is different from that by a passive element. G. Schweitzer (3) tried vibration control of a rotating shaft by means of an active damper using electromagnetic forces. In the same way, J.L. Nikolajsen et al. (4) carried out lateral vibration control of a marine transmission shaft with an electromagnetic damper. Most recently J. Salm et al. (5) have discussed a problem of vibration reduction for a rotor with multiple degrees of freedom and have performed vibration control of an unbalance response using a magnetic bearing. H. Ulbrich et al. (6) have conducted an experiment which enlarges a stable region of an asymmetric rotor by applying a magnetic bearing. U. Gondhalekar et al. (7) have shown how to apply a control force to three poles of a magnetic bearing from two directional signals using a microcomputer and have indicated they are able to shift their critical speeds as a result of changing the stiffnesses.

However the above expressed methods are not available when the setting positions of actuators are restricted. It is complicated and dangerous to control the shaft directly in high speed rotation. Therefore it is considered to suppress the vibration of the rotating shaft by controlling the nonrotating bearing housings. R. Stanway et al. have suggested an active control of bearing housings and discussed both controllability and observability (8). Moreover they have studied a pole assignment problem for a three degree of freedom system (9). J.W. Moore et al. have tested the efficiency of this method by means of control systems using a transfer function with velocity feedback loops (10) and experiments using loud speakers (11).

This paper is on the basis of an idea similar to that of R. Stanway et al. Namely, a control method which moves the characteristic roots of a rotor system to the optimum positions is applied. The efficiency of this control method is demonstrated by simulations and experiments. As the bearing housings are actuators of the vibration control, the bearings of this kind are appropriately named to the "Active Control Bearings." In this paper two control methods for active control bearings are discussed in particular.

*NRC-NASA Research Associate.

N85-29292 #

One is the control method by an optimal regulator with all state variable feedback (12) and the other is by a quasi-modal control with velocity feedback based on a modal analysis. First, their design methods are explained about how to construct the control loop. Next, the two methods are compared as to their vibration control effects, their merits and demerits are made clear. The rotor system has two degree of freedom system with one disk at the center of the span on flexible supports. And the actuators were made by the use of a principle of an electrodynamic transducer and the experiments were performed by setting them. The results obtained agree with the simulations qualitatively. Therefore it is clear that unbalance vibrations can be suppressed sufficiently by active control bearings.

NOMENCLATURE

$\underset{\sim}{A}$ system matrix

$\underset{\sim}{b}$ control vector

C_r rotor damping coefficient

C_s support damping coefficient

$\underset{\sim}{F}$ modal feedback gain matrix, n x n

K stiffness ratio, $K = k_r/k_s$

$\underset{\sim}{K_n}$ stiffness matrix, n x n

k_r rotor shaft stiffness coefficient

k_s support stiffness coefficient

M mass ratio, $M = m_r/m_s$

$\underset{\sim}{M_n}$ mass matrix, n x n

m_r rotor mass

m_s bearing housing mass

$\underset{\sim}{P}$ forced external vector, n

$\underset{\sim}{Q}$ weighting matrix

$\underset{\sim}{R}$ solution matrix of Riccati equation, solution of Eq. (8)

$\underset{\sim}{T}$ modal matrix, n x n

$\underset{\sim}{U}$ control input vector, n

U_x control input in x direction

U_y control input in y direction

$\underset{\sim}{X}$ state vector

$\underset{\sim}{X_n}$ column vector, n

X_r rotor absolute displacement in x direction

X_s bearing housing absolute displacement in x direction

Y_r rotor absolute displacement in y direction

Y_s bearing housing absolute displacement in y direction

ε rotor eccentricity

ρ weighting coefficient between vibration energy and control energy

ζ_r rotor damping ratio, $\zeta_r = C_r/2\sqrt{m_r k_r}$

ζ_s support damping ratio, $\zeta_s = C_s/2\sqrt{m_s k_s}$

ω rotor angular velocity

ω_r single support critical speed, $\omega_r = \sqrt{k_r/m_r}$

VIBRATION CONTROL BY MEANS OF OPTIMAL REGULATOR METHOD

Model of shaft system

Figure 1 represents the model of a rotor shaft system. The shaft is considered as a massless elastic member and the rotor mass is concentrated in a disk mounted at the center of the span. The bearings are assumed rigid. The bearing housings are supported on damped flexible supports. If the control forces are added at both bearing housings as shown in Fig. 1, equations of motion for the rotor shaft system in complex notation reduce to the following.

$$\left.\begin{aligned} m_r\ddot{Z}_r + C_r(\dot{X}_r - \dot{Z}_s) - k_r(Z_r - Z_s) &= m_r\varepsilon\omega^2 e^{i\omega t} \\ m_s\ddot{Z}_s + C_s\dot{Z}_s + C_r(\dot{Z}_s - \dot{Z}_r) + k_s Z_s + k_r(Z_s - Z_r) &= U \end{aligned}\right\} \quad (1)$$

where

$$Z_r = X_r + iY_r,\ Z_s = X_s + iY_s,\ U = U_x + iU_y \quad (2)$$

Considering dimensionless parameters, if only the x direction is shown because there is no coupling between x and y directions, their forms can be written as

$$\left.\begin{aligned} &\ddot{X}_r + 2z_r\omega_r(\dot{X}_r - \dot{X}_s) + \omega_r^2(X_r - X_s) = \omega^2 \cos \omega t \\ &\ddot{X}_s + \left(2\zeta_r M\omega_r + 2\zeta_s\sqrt{\frac{M}{K}}\,\omega_r\right)\dot{X}_s - 2\zeta_r M\omega_r\dot{X}_r \\ &\qquad + \left(M\omega_r^2 + \frac{M}{K}\,\omega_r^2\right)X_s - M\omega_r^2 X_r = \frac{U_x}{M_s} \end{aligned}\right\} \quad (3)$$

where $X_{r/\varepsilon}$ and $X_{s/\varepsilon}$ are newly expressed as X_r and X_s. Transforming Eq. (3) into the state equation yields

$$\dot{\underset{\sim}{X}} = \underset{\sim}{A}\underset{\sim}{X} + \underset{\sim}{b}U_x + \underset{\sim}{w}f_x \quad (4)$$

where

$$\underset{\sim}{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_r^2 & -2\zeta_r\omega_r & \omega_r^2 & 2\zeta_r\omega_r \\ 0 & 0 & 0 & 1 \\ M\omega_r^2 & 2\zeta_r M\omega_r & -M\omega_r^2-\frac{M}{K}\omega_r^2 & -2\zeta_r M\omega_r-2\zeta_s\sqrt{\frac{M}{K}}\,\omega_r \end{bmatrix}$$

$$\underset{\sim}{X} = \begin{bmatrix} X_r \\ \dot{X}_r \\ X_s \\ \dot{X}_s \end{bmatrix}, \quad \underset{\sim}{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_s} \end{bmatrix}, \quad \underset{\sim}{w} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad f_x = \omega^2 \cos \omega t$$

In the same manner, the equation for the y direction may be expressed as follows:

$$\dot{\underset{\sim}{Y}} = \underset{\sim}{A}\underset{\sim}{Y} + \underset{\sim}{b}U_y + \underset{\sim}{w}f_y \tag{5}$$

where

$$\underset{\sim}{Y} = (Y_r, \dot{Y}_r, Y_s, \dot{Y}_s)^T, \; f_y = \omega^2 \sin \omega t$$

The superscript T denotes the transpose. The state Eqs. (4) and (5) mean the control systems for the case of one input and four outputs. Comparing two control inputs U_x and U_y, they are identical except for the phase between U_x and U_y. Therefore the system of only the x direction is considered after this.

Control system design with optimal regulator

In this section, the control input U_x as shown in Fig. 1 is determined based on the optimal regulator theory in the modern control fields (13). Namely for the state Eq. (4), the problem is to find the control input U_x to minimize the following cost function

$$J = \frac{1}{2}\int_0^\infty \rho\left[\underset{\sim}{X}^T \underset{\sim}{Q}\underset{\sim}{X} + U_x^2\right]dt \tag{6}$$

where Q is a symmetric positive semidefinite matrix and it is assumed Eq. (4) is controllable. The optimal control input U_x to minimize Eq. (6) is given with state variable feedback as follows:

$$U_x^0 = -\underset{\sim}{b}^T\underset{\sim}{R}\underset{\sim}{X} \tag{7}$$

where R satisfies the following Riccati type algebra equation

$$\underset{\sim}{R}\underset{\sim}{A} + \underset{\sim}{A}^T\underset{\sim}{R} - \underset{\sim}{R}\underset{\sim}{b}\underset{\sim}{b}^T\underset{\sim}{R} + \underset{\sim}{Q} = 0 \tag{8}$$

In Eq. (7), the following expression

$$\underset{\sim}{f}^0 = \underset{\sim}{b}^T\underset{\sim}{R} \tag{9}$$

is the optimal feedback vector. Substituting Eq. (7) into Eq. (4), the optimal closed loop system is expressed except for the external force as follows:

$$\dot{\underset{\sim}{X}} = (\underset{\sim}{A} - \underset{\sim}{b}\underset{\sim}{b}^T\underset{\sim}{R})\underset{\sim}{X} \tag{10}$$

Equation (10) is called an optimal regulator.

Weighting matrix and root locus

Equation (10) is asymptotic stable in any case of Q and the eigen values are automatically determined by Eq. (8) if Q is given. Therefore it is examined that the eigen values of the closed loop system show the behaviors in what manner as the variations of the weighting coefficients ρ. Hence the parameters used are $M = K = 1$, $\zeta_r = \zeta_s = 0$, $\omega_r = 100$ rad/s and $m_s = 1$ kg. Figure 2 represents the root loci: (a) is the case of Q = diag (1,1,1,1) and weighting coefficients of the rotor displacement, its velocity, the bearing housing displacement and its velocity are all the same. When $\rho = 0$, it shows the open loop system or the uncontrolled system. Consequently the first and the second natural frequencies become the undamped ones and they are 0.62 and 1.62. As ρ increases, it is found that the damping ratio of the first and the second natural frequencies increase and the damped natural frequencies decrease. In this Fig. (a), it shows from $\rho = 0$ to $\rho = 10^8$. The root loci in the case which ρ increases larger is considered by the root loci of the square of Kalman equation. From these results, in the case that ρ is infinite, three roots approach to equivalent zeros $S = 0$, $S = \pm i\omega_r$ and the remaining root goes toward to the infinite point for the left direction along the real axis as like the first Butterworth pattern.

(b) is in the case of Q = diag(1,1,0.1,0.1). It draws the root loci that the weights of the rotor displacement and its velocity are ten times as much as the ones of the bearing housing displacement and its velocity. When ρ increases, they separate from the imaginary axis and their damping ratios increase. But the second damped natural frequency is inclined to increase. In the case of $\rho = 10^5$, the second damped natural frequency increases one and half times as much as the one of (a) and both damping ratios are almost the same. When ρ increases beyond this, their loci are similar to the ones of (a). That is, three roots converge to three equivalent zeros and one root indicates the first Butterworth pattern. (a) in more sensitive than (b) concerning the sensitivity of behaviors of their roots. Their roots hardly move beyond $\rho = 10^6$ in (a) and $\rho = 10^7$ in (b).

Steady state unbalance response

In order to compute the steady state unbalance response extending whole rotational speeds, Eq. (3) is solved analytically. U_x of Eq. (3) is expressed by Eq. (7) as follows:

$$U_x = -f_1X_r - f_2\dot{X}_r - f_3X_s - f_4\dot{X}_s \tag{11}$$

where f_1, f_2, f_3 and f_4 are elements of the row vector of f^0. Substituting Eq. (11) into Eq. (3), the following equation is obtained.

$$\underset{\sim}{M_n}\ddot{\underset{\sim}{X}}_n + \underset{\sim}{C_n}\dot{\underset{\sim}{X}}_n + \underset{\sim}{K_n}\underset{\sim}{X}_n = e \cos \omega t \tag{12}$$

$$\underset{\sim}{M_n} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \underset{\sim}{C_n} = \begin{bmatrix} 2\zeta_r\omega_r & -2\zeta_r\omega_r \\ \frac{f_2}{M_s} - 2\zeta_r M\omega_r & 2\zeta_r M\omega_r + 2\zeta_s\sqrt{\frac{M}{K}}\,\omega_r + \frac{f_4}{M_s} \end{bmatrix}$$

$$\underset{\sim}{K_n} = \begin{bmatrix} \omega_r^2 & -\omega_r^2 \\ \frac{f_1}{M_s} - M\omega_r^2 & M\omega_r^2 + \frac{M}{K}\omega_r^2 + \frac{f_3}{M_s} \end{bmatrix}, \quad e = \begin{bmatrix} \omega^2 \\ 0 \end{bmatrix}, \quad \underset{\sim}{X_n} = \begin{bmatrix} X_r \\ X_s \end{bmatrix}$$

From Eq. (12), the steady state solutions $\underset{\sim}{X_n}$ are described as follows:

$$X_r = \frac{M_{11}\omega^2\sqrt{P_1^2 + P_2^2}}{a^2 + b^2}\cos(\omega t - \theta_1)$$
$$X_s = \frac{M_{11}\omega^2\sqrt{P_3^2 + P_4^2}}{a^2 + b^2}\cos(\omega t - \theta_2) \qquad (13)$$

where

$$a = (K_{11} - \omega^2 M_{11})(K_{22} - \omega^2 M_{22}) - \omega^2 C_{11}C_{22} - K_{12}K_{21} + \omega^2 C_{12}C_{21}$$

$$b = (K_{11} - \omega^2 M_{11})\omega C_{22} + (K_{22} - \omega^2 M_{22})\omega C_{11} - K_{12}\omega C_{21} - K_{21}\omega C_{12}$$

$$P_1 = a(K_{22} - \omega^2 M_{22}) + \omega C_{22} b$$

$$P_2 = \omega C_{22} a - (K_{22} - \omega^2 M_{22}) b$$

$$P_3 = -aK_{21} - \omega C_{21} b$$

$$P_4 = -a\omega C_{21} + bK_{21}$$

$$\theta_1 = \tan^{-1}(P_2/P_1)$$

$$\theta_2 = \tan^{-1}(P_4/P_3)$$

M_{ij}, C_{ij}, and K_{ij} are elements of $\underset{\sim}{M_n}$, $\underset{\sim}{C_n}$, and $\underset{\sim}{K_n}$ matrixes. Figure 3 shows the unbalance responses X_r and X_s given by Eq. (13). In this case, the parameters used are $M = K = 1$, $\zeta_r = \zeta_s = 0$, $\omega_r = 100$ rad/s $m_s = 1$ kg and $\underset{\sim}{Q_n} = \mathrm{diag}\,(1,1,1,1)$. The peak amplitudes at the first and the second critical speeds decrease in the case when ρ is large. If the same peak amplitudes in the case of $\rho = 10^4$ are to be realized by passive devices, damping ratios ζ_r and ζ_s of them must be about 0.1. It seems that it is generally difficult for rotor systems supported on rolling bearings to supply such large damping ratios. For any the second peak amplitudes are higher than ones of the first. This is caused by that the damping ratio of the second mode is smaller than one of the first mode for root loci in Fig. 2(a). Even if ρ increases 100 or 1000 times as much as $\rho = 10^4$, it can not be expected that the peak amplitudes at resonance decrease remarkably. And yet, the critical speeds can be actually ignored in the neighborhood of $\rho = 10^6$. For the case of the larger weighting coefficient ρ, it seems from the root loci in Fig. 2(a) that damping ratios of complex eigen values decrease. However, since the root of the second mode hardly moves and the unvibrational root on the real axis have an effect on the complex root, the resonance peak does not appear nearby the critical speed on simple supports. This point fundamentally differs from the design of the optimum tuned dynamic damper and indicates definitely that active control bearing systems of this kind are superior to any passive vibration control device. This reason is caused by the all state variable feedback, specially the optimal state variable feedback. If an active control bearing system is designed by an incomplete state variable feedback, for example, only the displacement feedback or only the velocity feedback, the resonance peak arises nearby the original critical speed ω_r in the occasion of large loop gains. The control system becomes unstable in the worst case. These are discussed in detail in next Chapter.

VIBRATION CONTROL BY MEANS OF QUASI-MODAL CONTROL METHOD

Control system design method with quasi-modal control

First, a control system design method is described in general. The following equation is considered.

$$\underset{\sim}{M_n}\ddot{\underset{\sim}{X_n}} + \underset{\sim}{K_n}\underset{\sim}{X_n} = \underset{\sim}{P}(t) + \underset{\sim}{U}(t) \qquad (14)$$

Equation (14) is generally written and however a damping term is neglected for the simplicity. For the $\underset{\sim}{X_n}$ vector of Eq. (14), the following transformation is performed.

$$\underset{\sim}{X_n} = \underset{\sim}{Ta} \quad \text{or} \quad a = \underset{\sim}{T}^{-1}\underset{\sim}{X_n} \qquad (15)$$

where $\underset{\sim}{T}$ is the modal matrix and it is assumed to be normalized. From Eq. (15), Eq. (14) is transformed as follows:

$$\underset{\sim}{T}^T\underset{\sim}{M_n}\underset{\sim}{T}\ddot{a} + \underset{\sim}{T}^T\underset{\sim}{K_n}\underset{\sim}{T}a = \underset{\sim}{T}^T\underset{\sim}{P}(t) + \underset{\sim}{T}^T\underset{\sim}{U}(t) \qquad (16)$$

Hence $\underset{\sim}{T}^T\underset{\sim}{M_n}\underset{\sim}{T}$ is the unit matrix and $\underset{\sim}{T}^T\underset{\sim}{K_n}\underset{\sim}{T}$ is the frequency matrix. Therefore Eq. (16) is reduced to the following equation.

$$\ddot{\underset{\sim}{a}} + \underset{\sim}{\Omega}^2\underset{\sim}{a} = \underset{\sim}{T}^T\underset{\sim}{P}(t) + \underset{\sim}{T}^T\underset{\sim}{U}(t) \qquad (17)$$

where Ω^2 symbolizes the frequency matrix. After all, Eq. (14) is reduced to Eq. (17) separated the each mode and uncoupled. By the way, it is very easy to determine the optimal control inputs to each mode in the above expressed modal domain. That is, it is assumed to determined as follows:

$$\ddot{\underset{\sim}{a}} + \underset{\sim}{\Omega}^2\underset{\sim}{a} = \underset{\sim}{T}^T\underset{\sim}{P}(t) + \underset{\sim}{U}^0(t) \qquad (18)$$

where $\underset{\sim}{U}^0$ is the new term determined as the desirable optimal control force. As Eq. (17) and Eq. (18) should be equivalent, the following relation is introduced.

$$\underset{\sim}{U}^0 = \underset{\sim}{T}^T\underset{\sim}{U} \qquad (19)$$

From this, the control force supplied to the actual system is

$$\underset{\sim}{U} = (\underset{\sim}{T}^T)^{-1}\underset{\sim}{U}^0 = \underset{\sim}{H}^{-1}\underset{\sim}{U}^0 \qquad (20)$$

where $\underset{\sim}{H} = \underset{\sim}{T}^T$. Now for the undamped vibration system it is considered that optimal control inputs are velocity feedbacks assigned the critical damping on each mode. In this case, the modal control system is expressed by the block diagram as shown in Fig. 4.

where

$$\underset{\sim}{F} = \begin{bmatrix} -2\zeta_1\omega_1 & & & 0 \\ & -2\zeta_2\omega_2 & & \\ & & \ddots & \\ 0 & & & -2\zeta_n\omega_n \end{bmatrix} \tag{21}$$

Finally the modal control input is given from Fig. 4 by

$$\underset{\sim}{U} = \underset{\sim}{H}^{-1}\underset{\sim}{F}\underset{\sim}{T}^{-1}\dot{\underset{\sim}{X}}_n \tag{22}$$

In n degree of freedom system, if the number of control inputs are less than n or the number of vibration velocities measured are less than n, this control system can not be called a modal control system. This paper names the such control system to a quasi-modal control system. For example, for two inputs and three outputs, the quasi-modal control system is expressed by

$$\underset{\sim}{U} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ U_a \\ \vdots \\ U_b \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \cdots h_{1n} \\ h_{21} & h_{22} \cdots h_{2n} \\ \vdots & \vdots \\ h_{n1} & h_{n2} \cdots h_{nn} \end{bmatrix} \begin{bmatrix} -2\zeta_1\omega_1 & & & 0 \\ & -2\zeta_2\omega_2 & & \\ & & \ddots & \\ 0 & & & -2\zeta_n\omega_n \end{bmatrix}$$

$$\times \begin{bmatrix} t_{11} & t_{12} \cdots t_{1n} \\ t_{21} & t_{22} \cdots t_{2n} \\ \vdots & \vdots \\ t_{n1} & t_{n2} \cdots t_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{X}_p \\ \dot{X}_q \\ \vdots \\ \dot{X}_r \\ 0 \end{bmatrix}$$

If it is desired that modes from the first to the sth are controllable, matrix $\underset{\sim}{F}$ has to be regarded till the sth mode. And higher order modes above this can be disregarded. That is,

$$\underset{\sim}{F} = \begin{bmatrix} -2\xi_1\omega_1 & & & & & & 0 \\ & -2\xi_2\omega_2 & & & & & \\ & & \ddots & & & & \\ & & & -2\xi_s\omega_s & & & \\ & & & & 0 & & \\ & & & & & 0 & \\ 0 & & & & & & \ddots \\ & & & & & & & 0 \end{bmatrix} \tag{24}$$

Anyway the feedback gain of the quasi-modal control is given by in Eq. (22) and a linear relation exists between $\underset{\sim}{U}$ and $\underset{\sim}{X}$.

Steady state unbalance response

Figure 5 represents the unbalance response for the model in Fig. 1. The rotor vibration velocity and the housing velocity are measured and control forces are supplied the bearing housing as well as Chapter 3. Accordingly, this model becomes a quasi-modal control system with one input and two outputs. If this system has two inputs and two outputs, it is an ideal modal control system. Figure 5 shows the results for the several modal damping ratios ζ of Eq. (24). In this figure, displacements X_r and X_s show the minimization nearby 0.35 or 0.7. For the case that ζ is larger than it, new peak amplitudes appear nearby $\omega/\omega_r = 0.85$. The above expression leads to the result that the optimal damping ratio is the vicinity from 0.35 to 0.7 in this quasi-modal control system. On the contrary, for the ideal modal control system, the more the damping ratio ζ increases, the more the amplitudes X_r and X_s decrease. Namely it has the most desirable characteristics of the vibration control.

Comparison optimal regulator system with quasi-modal control system

Figure 5 illustrates the amplitude characteristics in the case of the optimal regulator with $\rho = 10^4$ and $\rho = 10^6$. From Fig. 5, it is obvious that the peak amplitude by means of the optimal regulator is equivalent to the minimum amplitude by means of the quasi-modal control method. This means that the efficiency of the vibration control by using the optimal regulator method approaches to a saturation. Because, even if the control system is constructed by the optimal control theory, it is theoretically impossible to reduce the amplitude beyond the minimum amplitude of the quasi-modal control system.

From two standpoints, a modern control theory and a classical control theory, the design methods of active control bearings have been discussed. The merits and demerits between the optimal regulator system and the quasi-modal control system are summarized as follows:

The optimal regulator method:

merits:
(1) If a control object and a cost function are given, the feedback coefficients are automatically determined.
(2) A closed loop system is always stable and optimum regions that peak amplitudes become minimum don't exist.
(3) A closed loop system is reasonable because the feedback control system uses all state variable.

demerits:
(1) It is difficult for a multidegree of freedom system to give a weighting coefficient matrix.
(2) It is complicated for a multidegree of freedom system to compute a feedbackgain.
(3) All state variables are required.

The quasi-modal control method:

merits:
(1) The physical prospects are good.
(2) It is easy for a multidegree of freedom system to determine the feedback coefficients.
(3) Since this method consists of only the velocity feedback, the number of measure ments are a few.

demerits:
(1) If a damping term measurements are regarded, it is complicated to make the modal transformation.
(2) Optimum regions that peak amplitudes become minimum always exist. And for a large feedback gain the closed loop system becomes unstable.
(3) Since it consists of only the velocity feedback, the control system apt to be unreasonable.

As damping forces always exist in the actual rotating machinery, these amounts have to be correctly estimated if possible. For the quasi-modal control system in particular, estimated damping amounts have sensitively an influence to the efficiency of the vibration control on account of the existence of the tuned condition. Occasionally the rotor system becomes unstable because of the miss matching of this kind. Judging totally from the above mentioned results, it is able to conclude that the optimal regulator method is superior to the quasi-model control method for the control design of an active control bearing system.

EXPERIMENTS

Test rig

Figure 6 shows the overview of the test rig. The rotor shaft is 10 mm in the diameter, the drill rod with the span length 800 mm and has one disk of the mass 1 kg at the center of the span. The mass of the bearing housing is 0.5 kg including the ball bearing. Therefore the mass ratio between the rotor and the bearing housing is unit. There are four springs suspending the bearing housing and its spring constant is 0.49×10^4 N/m (0.5 kgf/mm). The actuators are trial manufactured using a principle of a electrodynamic transducer. The active control bearing set up by means of four actuators is indicated in Fig. 7. The generating force of one actuator is approximately 18 N versus the coil current 1 A and the force is in proportion to the current. Figure 8 shows the sectional view of one actuator.

Figure 9 illustrates the experimental block diagram. In this Chapter, it is described in only the case that the active control bearing system is designed by an optimal regulator. First the rotor displacement and the housing displacement are measured by the gap sensor.

The first order lag filters are used in order to remove some noise. $\dot{X}_r$ and $\dot{X}_s$ are obtained by differentiating these displacements. Next multiplying feedback gains f_1, f_2, f_3, f_4 and adding them, the actuators are controlled through the servo amplifier supplied this state feedback signal. The control system in y direction is also constructed in the same manner.

Discussion of results

For $M = 1$, $K = 0.5$, $\omega_r = 100$ rad/s, $M_s = 1$ kg, $\rho = 10^4$ and $\underset{\sim}{Q} = \mathrm{diag}\,(1,1,1,1)$, the optimal feedback gains are given as $f^0 = (-3100, 50, 3100, 1000)$. Figure 10(b) shows the impulse response obtained by the experiments with this feedback gain. The efficiency of the active vibration control is well recognized in both the rotor and the bearing housing. Figure 11 shows the experimental data of the unbalance response with the parameter as same as in Fig. 10(b). In the uncontrolled case, there are two critical speeds at about 750 and 1600 rpm. The maximum amplitude at the first critical speed attains about 2.3 mm in X_r. However, in the controlled case, the amplitude at the first critical speed reduces to about 0.4 mm and the second critical speed can not be confirmed. Figure 10(c) represents the results in the case of the larger weighting coefficient ρ, namely $\rho = 3.5 \times 10^4$ and $f^0 = (-9000, 100, 9000, 200)$. Under this case, the impulse response of the housing is made better in comparison with Fig. 10(b) and the unbalance response is shown in Fig. 12. In spite of the response of the rotor in Fig. 12(a) is as same as one in Fig. 11(a) except for the drop of the first critical speed, the efficiency of the vibration control of the housing is improved better than Fig. 11(b). Figure 13 indicates the numerical solutions concerning the experimental conditions in Fig. 11 and Fig. 12. The used parameters are $M = 1$, $K = 0.5$, $\omega_r = 100$ rad/s and $M_s = 1$ kg. Besides the damping ratios are estimated as $\zeta_r = 0.02$, $\zeta_s = 0.03$ by the analysis of the uncontrolled experimental data. From these results, it can be admitted that both the experiments and the simulations have good agreements.

CONCLUSIONS

This paper proposes two control design techniques to the active control bearings. The superiority or inferiority of these design technquies has been described in detail by the simulations. If a rotor shaft system is supported by active control bearings, it is possible to change damping ratios of the system because the arbitrary state feedback system or the velocity feedback system can be easily constructed. From this, the rotor system supported by the active control bearings system can be designed as satisfying a design specification for resonance amplitudes. Considering the difficulty of estimation of damping forces in a practical rotating machinery, it is expected that the optimal regulator method is superior to the quasi-modal control method for the control design of the active control bearing system. The active control bearing system is effective not only for unbalance forces but for unstable forces or external forces transmitted from a foundation.

REFERENCES

1. Saito, S. and Kobayashi, M., Transactions of the Japan Society of Mechanical Engineers, (in Japanese) Vol. 48, No. 436, Series C, 1982, p. 1883.
2. Kirk, R.G. and Gunter, E.J., "Effect of Support Flexibility and Damping on the Dynamic Response of a Single Mass Flexible Rotor in Elastic Bearings," NASA CR-2083, 1972.
3. Schweitzer, G., Dynamics of Rotors, Springer, New York, 1975, p. 472.
4. Nikolajsen, J.L., Holmes, R. and Gondhalekar, V., "Investigation of an Electromagnetic Damper for Vibration Control of a Transmission Shaft," Proceedings of the Institute of Mechanical Engineers, Vol. 193, 1979, pp. 331-336.
5. Salm, J. and Schweitzer, G., I. Mech. E. Conf. "Vibration in Rotating Machinery", C277/84 (1984) p. 553.
6. Ulbrich, H. and Anton, E., I. Mech. E. Conf. "Vibration in Rotating Machinery," C299/84 (1984) p. 543.
7. Gondhalekar, V. and Holmes, R., "Design of an Electromagnetic Bearing for Vibration Control of Flexible Transmission Shaft," Rotordynamic Instability Problems in High-Performance Turbomachinery, NASA CP-2338, 1984.
8. Stanway, R. and Burrows, C.R., "Active Vibration Control of a Flexible Rotor on Flexibility-Mounted Journal Bearings," Journal Dynamic Systems, Meaurements, and Control, Vol. 103, No. 4, Dec. 1981, pp. 383-388.
9. Stanway, R. and O'Reilly, J., I. Mech. E. Conf. "Vibration in Rotating Machinery", C274/84 (1984) p. 515.
10. Moore, J.W., Lewis, D.W. and Heizman, J.D., "Rotordynamic Instability Problems in High-Performance Turbomachinery," NASA CP-2133, 1980.

11. Lewis, D.W., Moore, J.W., Bradley, P.L. and Allaire, P.E., "Rotordynamic Instability Problems in High-Performance Turbomachinery," NASA CP-2250, 1982.

12. Nonami, K., Preprint of Japan Society of Mechanical Engineers, (in Japanese) No. 840-11, 1984 pp. 76-83.

13. Athans, M. and Falb, P.L., Optimal Control, Chapter 9, McGraw Hill, New York, 1966.

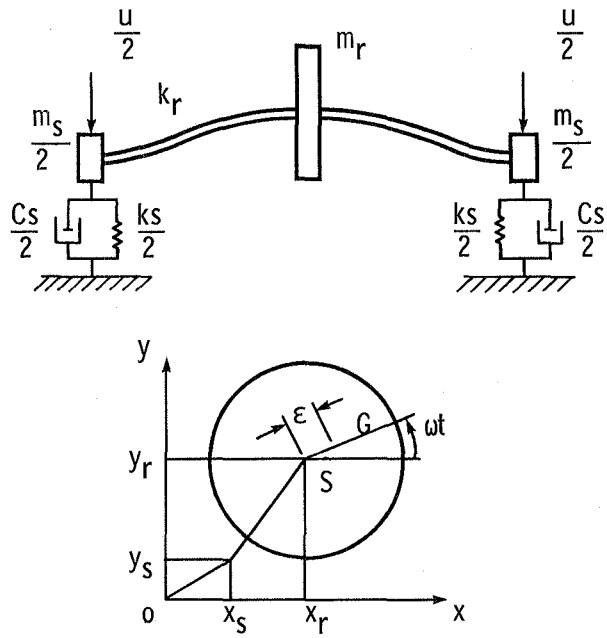
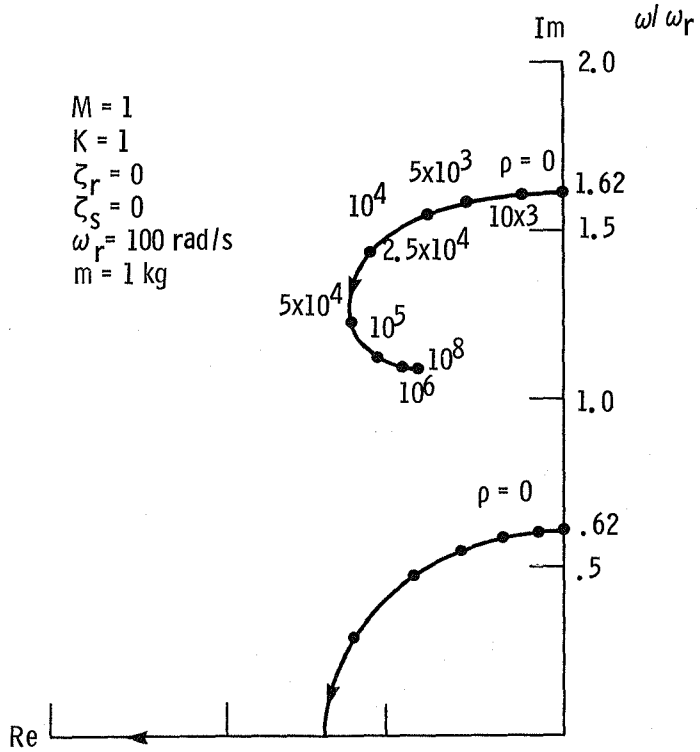
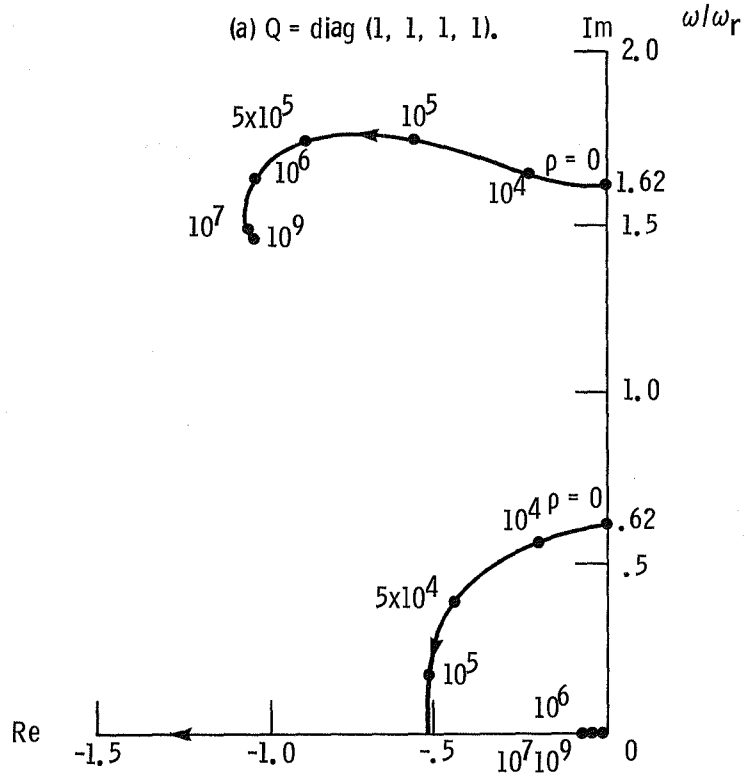


Fig. 1. - Rotor shaft model and coordinates.

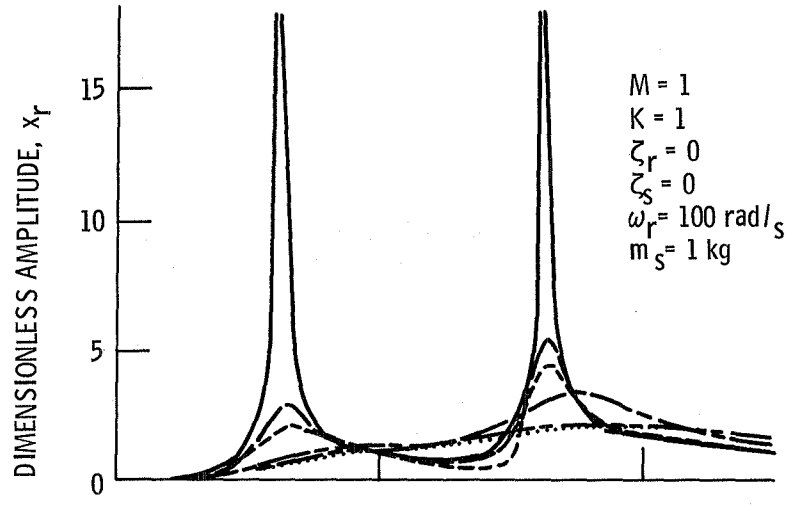


(a) Q = diag (1, 1, 1, 1).

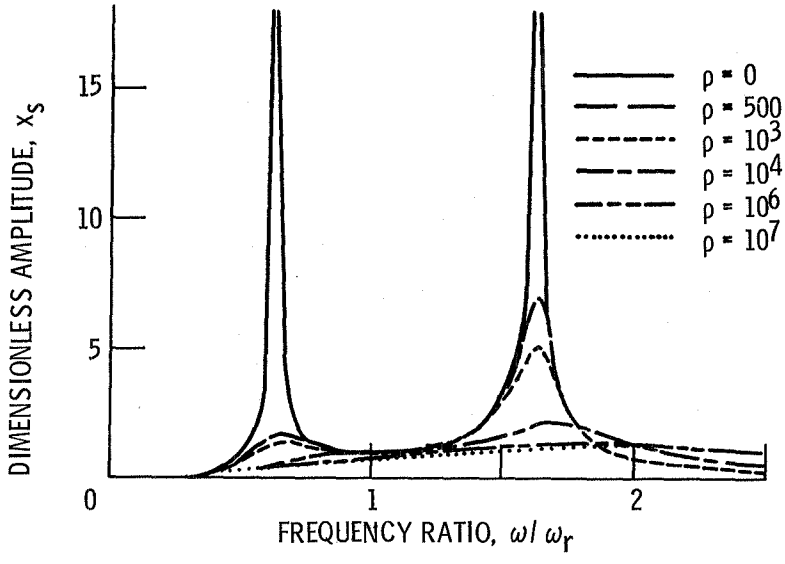


(b) Q = diag (1, 1, 0.1, 0.1).

Fig. 2. - Weighting matrix and root locus.



(a) Rotor displacement.



(b) Housing displacement.

Fig. 3. - Steady state unbalance response using optimal regulator method.

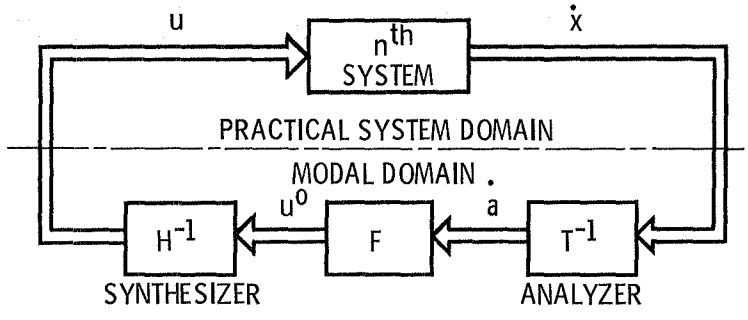
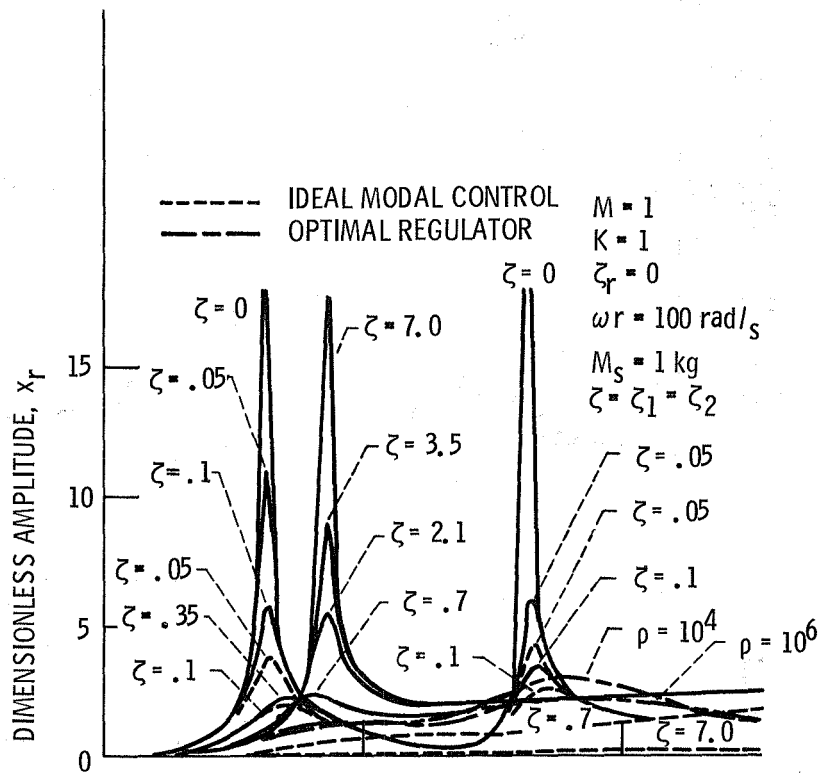
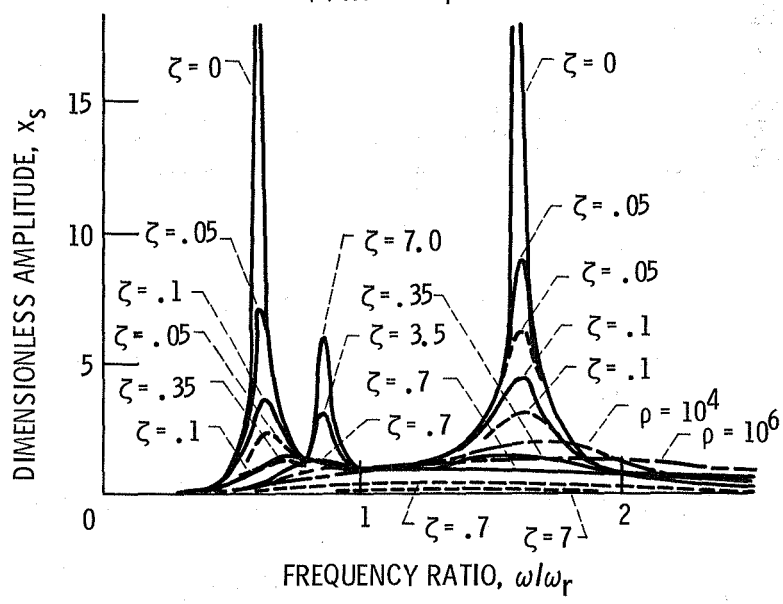


Fig. 4. - Modal control system with velocity feedback.



(a) Rotor displacement.



(b) Housing displacement.

Fig. 5. - Steady state unbalance response using quasi modal control method.

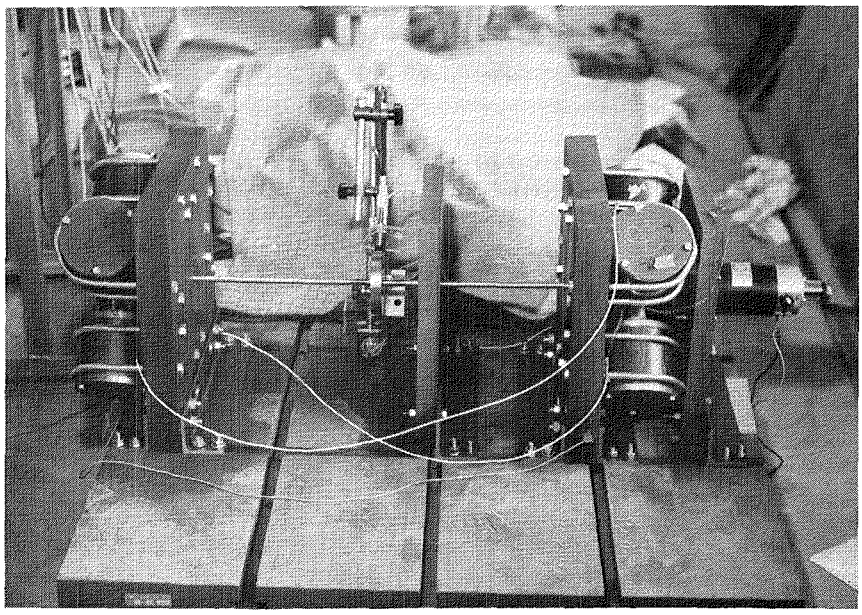


Fig. 6. - Test rig overview.

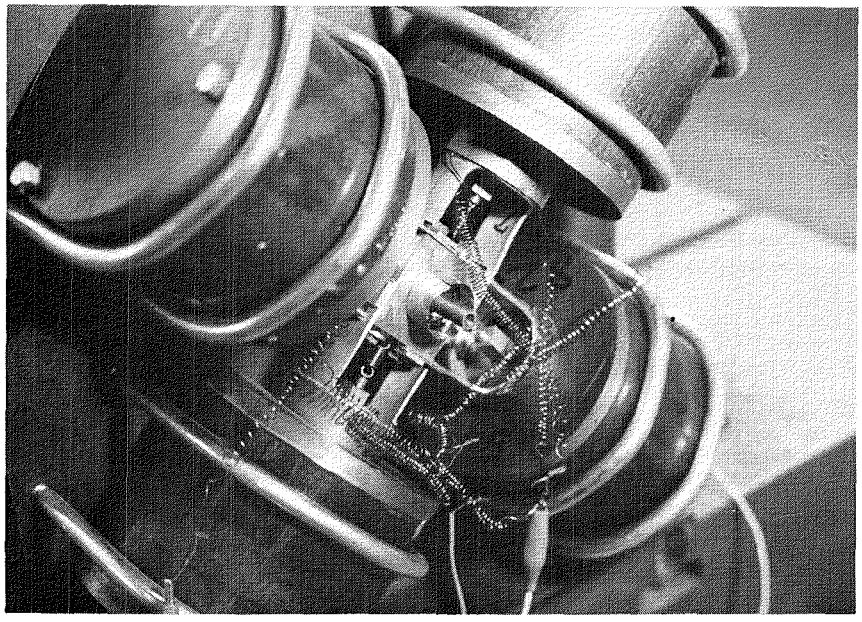
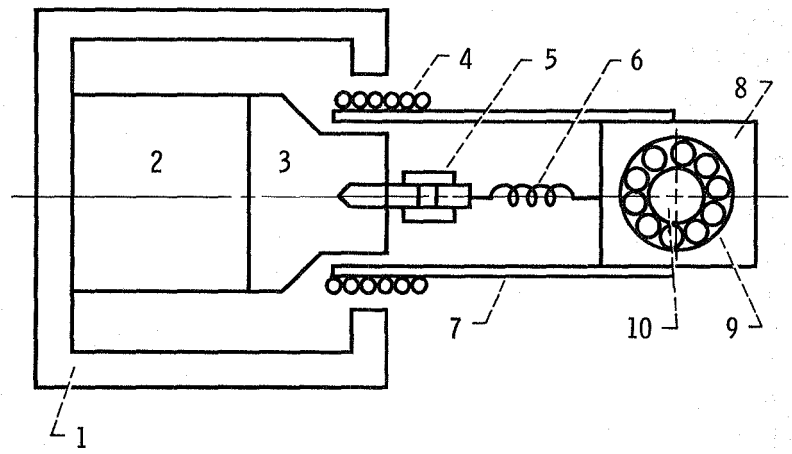


Fig. 7. - Active control bearing supported by four actuators.



1. YOKE
2. MK MAGNET
3. POLE PIECE
4. MOVING COIL
5. ADJUSTER
6. TENSION COIL SPRING
7. HOLDER
8. BEARING HOUSING
9. BALL BEARING
10. SHAFT

Fig. 8. - Sectional view of one actuator.

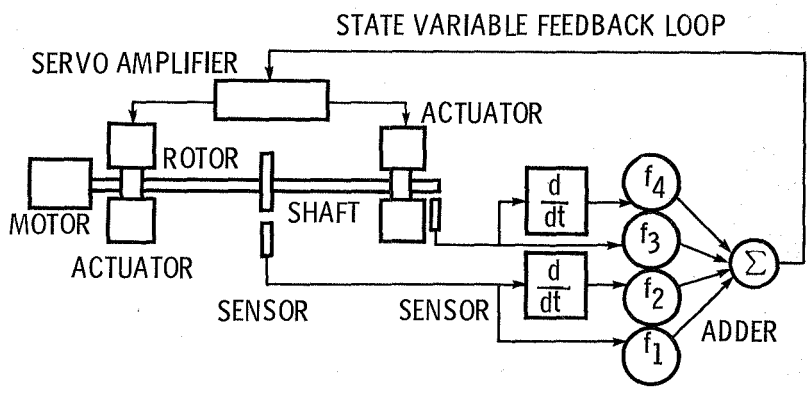
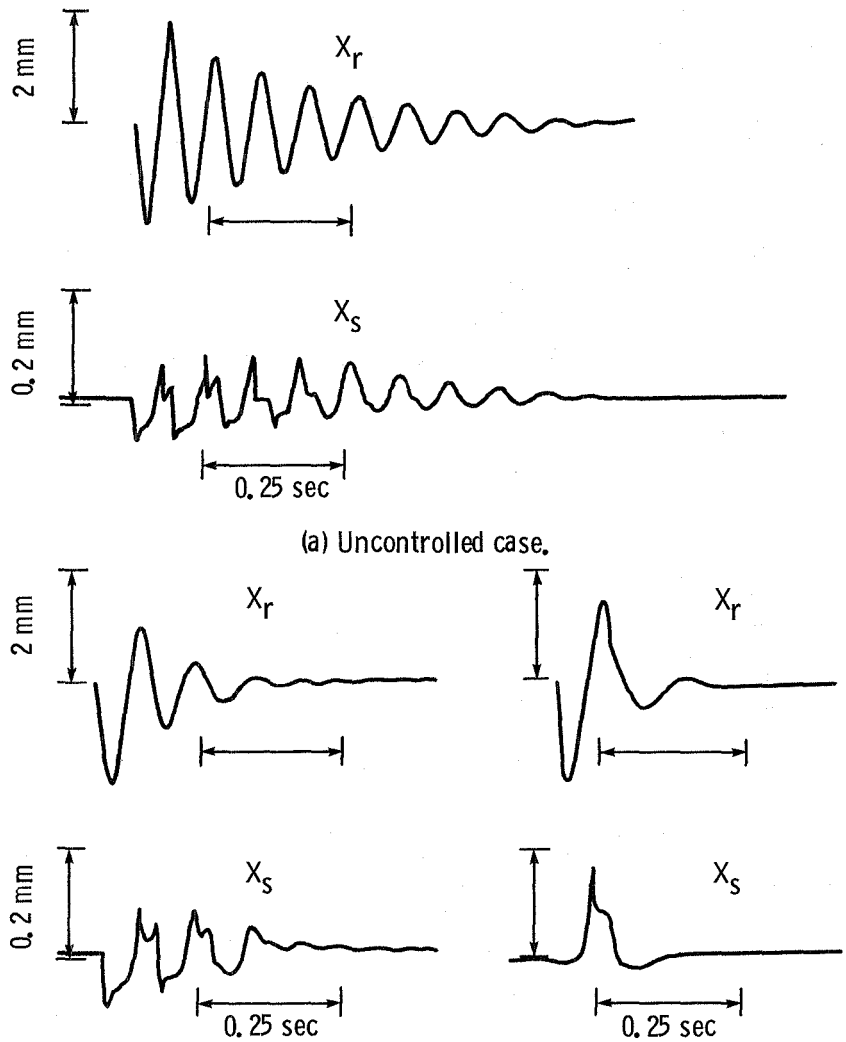


Fig. 9. - Experimental block diagram.

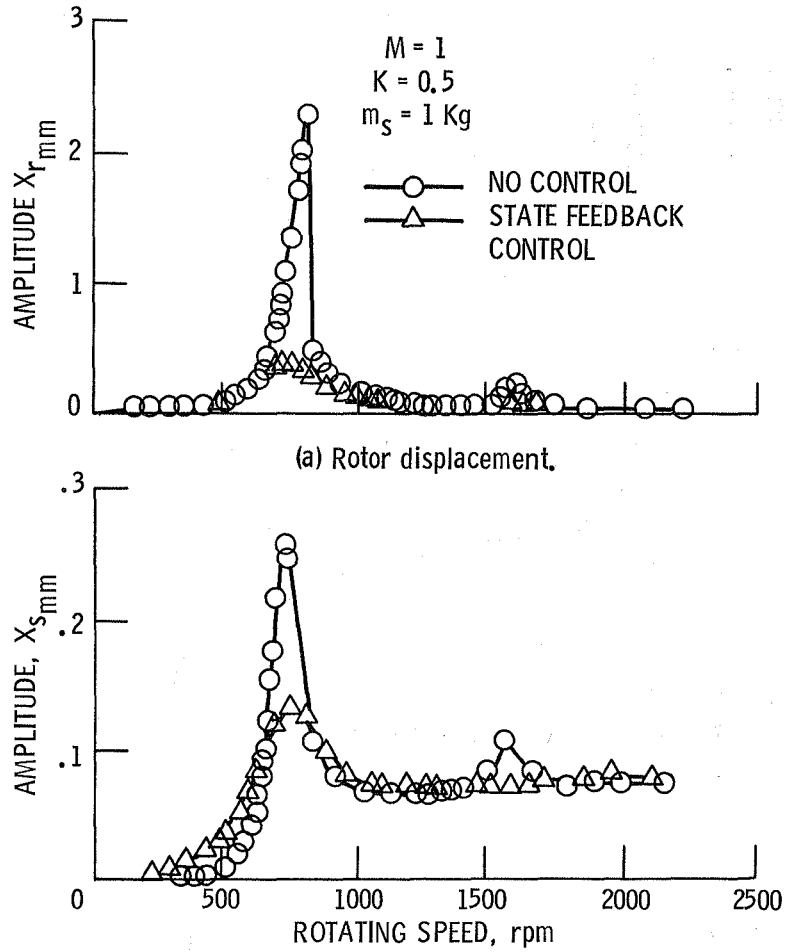


(a) Uncontrolled case.

(b) Controlled case, $\rho = 10^4$.

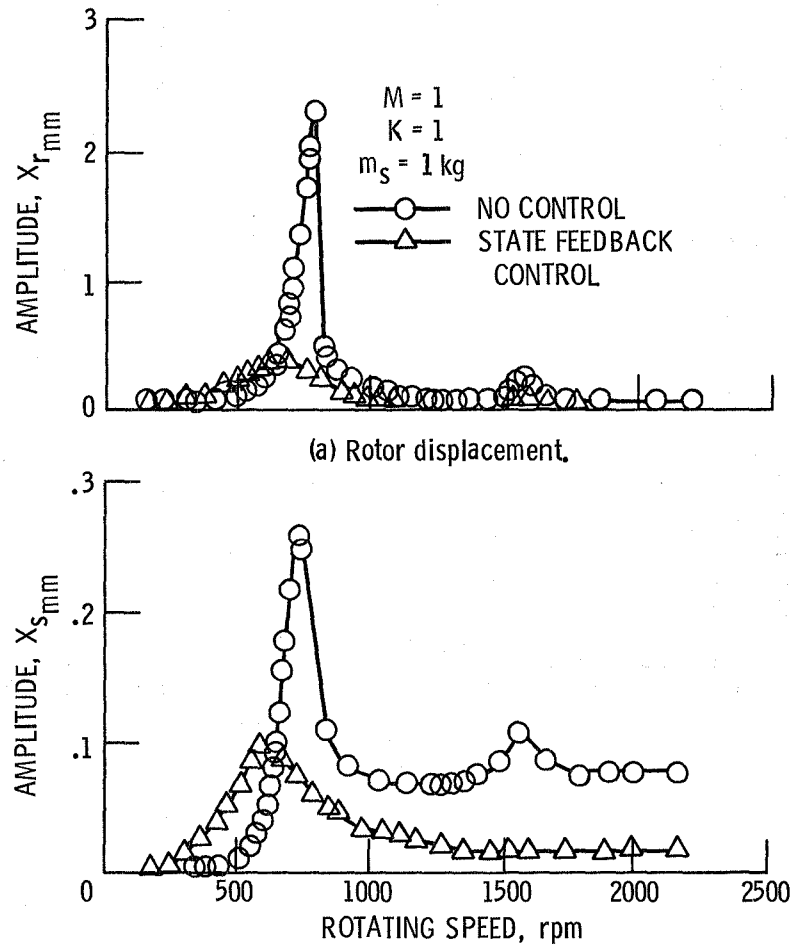
(c) Controlled case, $\rho = 3.5 \times 10^4$.

Fig. 10. - Impulse response.



(b) Housing displacement.

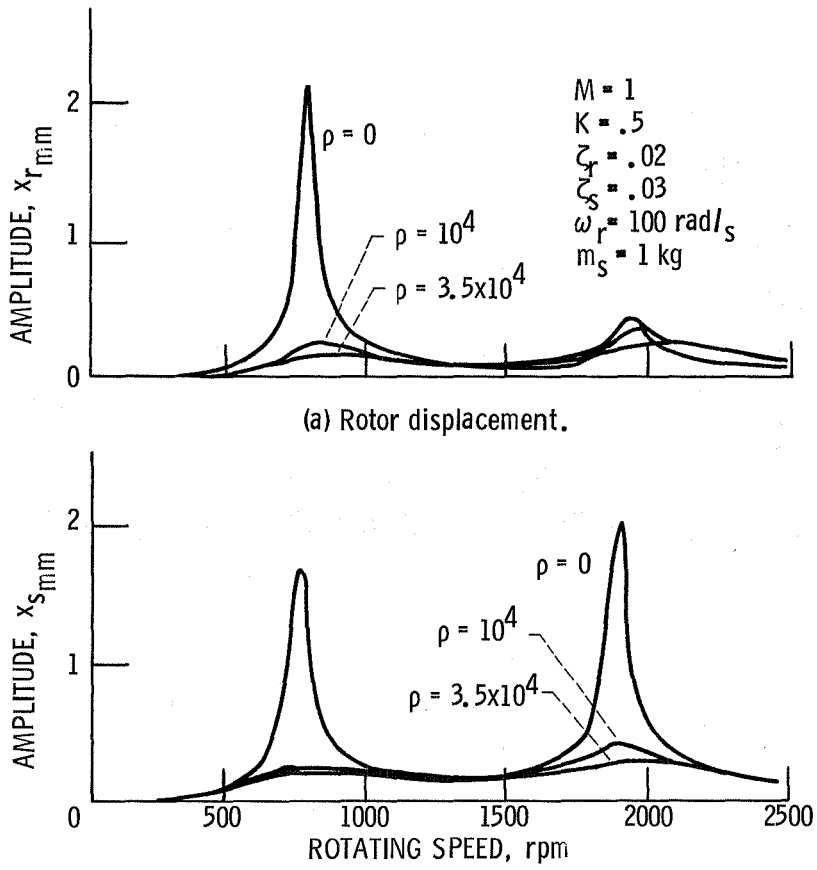
Fig. 11. - Unbalance response (experimental), $\rho = 10^4$.



(a) Rotor displacement.

(b) Housing displacement.

Fig. 12. - Unbalance response (experimental), $\rho = 3.5 \times 10^4$.



(a) Rotor displacement.

(b) Housing displacement.

Fig. 13. - Unbalance response (computed).

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-87053		

4. Title and Subtitle	5. Report Date
Vibration Control of Rotor Shaft Systems by Active Control Bearings	6. Performing Organization Code 505-33-7B
7. Author(s) Kenzou Nonami*	8. Performing Organization Report No. E-2616
	10. Work Unit No.
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135	11. Contract or Grant No.
	13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Prepared for the Design Engineering Technical Conference, sponsored by the American Society of Mechanical Engineers, Cincinnati, Ohio, September 10-13, 1985.
*NRC-NASA Research Associate.

16. Abstract

This study describes suppression of flexural forced vibration or the self-excited vibration of a rotating shaft system not by passive elements but by active elements. Namely, the distinctive feature of this method is not to dissipate the vibration energy but to provide the force cancelling the vibration displacement and the vibration velocity through the bearing housing in rotation. Therefore the bearings of this kind are appropriately named "Active Control Bearings." A simple rotor system having one disk at the center of the span on flexible supports is investigated in this paper. The actuators of the electrodynamic transducer are inserted in the sections of the bearing housing. First, applying the optimal regulator of optimal control theory, the flexural vibration control of the rotating shaft and the vibration control of support systems are performed by the optimal state feedback system using these actuators. Next, the quasi-modal control based on a modal analysis is applied to this rotor system. This quasi-modal control system is constructed by means of optimal velocity feedback loops. The differences between optimal control and quasi-modal control are discussed and their merits and demerits are made clear. Finally, the experiments are described concerning only the optimal regulator method.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Vibration control Rotor dynamics	Unclassified - unlimited STAR Category 37

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified		

*For sale by the National Technical Information Service, Springfield, Virginia 22161

End of Document