N85-31197

SENSOR/ACTUATOR SELECTION FOR THE CONSTRAINED VARIANCE CONTROL PROBLEM

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ABSTRACT

This paper considers the problem of designing a linear controller for systems subject to inequality variance constraints. A quadratic penalty function approach is used to yield a linear controller. Both the weights in the quadratic penalty function and the locations of sensors and actuators are selected by successive approximations to obtain an optimal design which satisfies the input/output variance constraints. The method is applied to NASA's 64 meter Hoop-Column Space Antenna for satellite communications. In addition the solution for the control law, the main feature of these results is the systematic determination of actuator design *requirements* which allow the given input/output performance constraints to be satisfied.

I. INTRODUCTION

Consider the task of controlling the linear, stochastic system

(1a) $\dot{x} = Ax + B(u+w)$, $x \epsilon R^n$, $u \epsilon R^m$, $y \epsilon R^k$

(1b) $y = Cx$

(1c) $z = Mx + v$, $z \epsilon R^{\ell}$

$$E\begin{pmatrix} x(0) \\ w(t) \\ v(t) \end{pmatrix} = 0 , \quad E\begin{pmatrix} x(0) \\ w(t) \\ v(t) \end{pmatrix} (x^T(o), w^T(\tau), v^T(\tau)) = \begin{bmatrix} X_0 & 0 & 0 \\ \cdot & W\delta(t-\tau) & \\ 0 & 0 & V\delta(t-\tau) \end{bmatrix}$$

$$W = \text{diag} [\dots W_{ii} \dots], \quad V = \text{diag} [\dots V_{ii} \dots],$$

such that these four control design goals are met:

$$\text{(I)} \quad E_\infty y_i^2(t) \overset{\Delta}{=} \lim_{t\to\infty} E\, y_i^2(t) \le \sigma_i^2, \; i = 1, \dots, k$$

$$E_\infty u_i^2(t) \overset{\Delta}{=} \lim_{t\to\infty} E\, u_i^2(t) \le \mu_i^2, \; i = 1, \dots, m$$

(II) *Only* $\bar{\ell} < \ell$ *sensors are used*

$$\bar{z} = \begin{pmatrix} z_1 \\ \vdots \\ z_{\bar{\ell}} \end{pmatrix} = \begin{pmatrix} m_1^T x + v_1 \\ \vdots \\ m_{\bar{\ell}}^T x + v_{\bar{\ell}} \end{pmatrix} = \bar{M}x + \bar{v} \tag{2}$$

from the admissible set of ℓ *sensors described from* (1c)

$$z = \begin{pmatrix} z_1 \\ \vdots \\ z_{\ell} \end{pmatrix} = \begin{pmatrix} m_1^T x + v_1 \\ \vdots \\ m_{\ell}^T x + v_{\ell} \end{pmatrix} = Mx + v. \tag{1c}$$

(III) *Only* $\bar{m} < m$ *actuators are used*

$$\bar{B}(\bar{u} + \bar{w}) = \sum_{i=1}^{\bar{m}} b_i(u_i + w_i) \tag{3}$$

from the admissible set of m *actuators described from* (1a)

$$B(u + w) = \sum_{i=1}^{m} b_i(u_i + w_i) \tag{4}$$

(IV) *The control* $\bar{u}(t)$ *is a linear function of the present and past measurements* $\bar{z}(\tau)$, $\tau \leq t$.

Many engineering control design problems can be stated with performance constraints of the form (I). For example, large space telescopes are feasible only if the RMS pointing errors $(E_\infty y_i^2)^{1/2}$ are within certain bounds $((E_\infty y_i^2)^{1/2} \leq \sigma_i)$ so as to achieve diffraction-limited performance (σ_i) of the optics. The designer may also have the freedom to choose from a number of different types of sensors and actuators at a number of different locations. The locations and the types of actuators (sensors) determine the vectors b_i (m_i) in (4) and (1c).

A straight-forward approach to accommodate the bounded input/output problem (I) yields nonlinear controllers [1-2], violating goal (IV). A straight-forward approach to accommodate goals (IV) and (I) is to use a penalty function method [3-5], minimizing

(5) $$V = E_\infty \frac{1}{t}\int_0^t (||y||_Q^2 + ||u||_R^2)\, d\tau \ , \quad ||y||_Q^2 = y^T Q y$$

while adjusting Q and R until (I) is satisfied. These successive approximation schemes [3-5] presume a fixed measurement/control structure, and hence do not satisfy goals (II) and (III). It is important to unify the treatment of all four goals (I-IV) since it has been shown [6-7] they are inherently interdependent problems. In particular, for the *isolated* problems; [6] has shown the optimal sensor and actuator selection for LQG problems (5) with *fixed* (Q,R), and [3-5] have adjusted Q and R to satisfy the constrained-variance problem (I) with *fixed* sensors and actuators (i.e. fixed B, M).

Unfortunately, the optimal answer for the simultaneous solution of both problems turns out *not* to be the juxtaposition of results [6] and [3-5], due to the interdependence of the two problems.

The purpose of this paper is to present a unified treatment of the entire problem (I-IV), which we call the Constrained Variance Sensor/Actuator Selection (CVSAS) problem. Section II describes the approach. Section III gives the formulas for sensor and actuator effectiveness to deal with goals (II) and (III). Section IV presents the numerical algorithm for iteratively dealing with goal (I). Section V gives the algorithm for solving the entire problem (I-IV). Section VI illustrates the application to the Hoop-Column Antenna.

II. APPROACH

The solution of the problem with inequality constraints (I) is generally not unique. To be a bit more specific than statement (I) we define two variations of the problem. The first is called the "Constrained-Input Variance" option of the CVSAS. In this option the input constraints in (I) are *binding* and the output constraints in (I) are *relaxed*.

<u>CIVSAS: The Constrained-Input Variance, Sensor/Actuator Selection Problem</u>

Satisfy (II), (III), *and with all input-constraints binding,*

(6) $$\mu_i^{-2} E_\infty u_i^2 = 1, \quad i = 1, \ldots, \bar{m} \ ,$$

minimize (recall $y_i = c_i^T x$*),*

(7a) $$V_y = \sum_i \sigma_i^{-2} E_\infty y_i^2 \quad \forall\ i: \quad \sigma_i^{-2} E_\infty y_i^2 > 1 \ .$$

If however, there is no i *for which* $\sigma_i^{-2} E_\infty y_i^2 > 1$ *then minimize*

(7b) $$V_y = \sum_{i=1}^{k} \sigma_i^{-2} E_\infty y_i^2$$

with all input constraints binding (6).

Definition: *The phrase "minimal achievable output performance" for the* CIVSAS *will mean the minimum constraint violation in the sense of the minimum value of* V_y *in* (7) *with input constraints binding* (6).

The CIVSAS problem is useful when one wishes to determine the best performance achievable for a *given* power limitation on the input devices (actuators). That is, for a *given* set of μ_i the CIVSAS *finds* the minimum achievable output performance.

The second variation of the CVSAS problem is called the Constrained Output Variance Sensor/Actuator Selection (COVSAS).

COVSAS: The Constrained-Output Variance, Sensor/Actuator Selection Problem

Satisfy (II), (III), *and with all output constraints binding*

(8) $$\sigma_i^{-2} E_\infty y_i^2 = 1 \,, \quad i = 1, \ldots, k \,,$$

minimize

(9a) $$V_u = \sum_i \mu_i^{-2} E_\infty u_i^2 \;\forall\; i: \;\; \mu_i^{-2} E_\infty u_i^2 > 1.$$

If however, there is no i *for which* $\mu_i^{-2} E_\infty u_i^2 > 1$ *then minimize*

(9b) $$V_u = \sum_{i=1}^{\bar{m}} \mu_i^{-2} E_\infty u_i^2$$

with all output constraints binding, (8).

Definition 2: *The phrase "minimum achievable input performance" for the* COVSAS *will mean the minimum constraint violation in the sense of* (9), *with all output constraints binding* (8).

The COVSAS is useful when one wishes to determine the necessary capabilities (design requirements) of the actuators in order to achieve the *specified* output performance. That is, for a *given* set of σ_i the COVSAS *finds* the minimum achievable input performance.

III. SENSOR/ACTUATOR EFFECTIVENESS

In this Section we temporarily assume that Q and R in (5) are *specified* diagonal matrices $Q = \text{diag}\,[\ldots\ q_i\ \ldots]$, $R = \text{diag}\,[\ldots\ r_i\ \ldots]$, and we wish to determine a ranking of the effectiveness of the admissible set of sensors and actuators for the LQG problem described by (1) and (5). To help with this task a price or "cost" is assigned to each input and output by decomposing the total system cost function (5) into contributions from each input and output. This task is called "input or output cost analysis" and from [6] we have the results

$$V = \sum_{i=1}^{m} V_i^u + \sum_{i=1}^{k} V_i^y = \sum_{i=1}^{m} V_i^w + \sum_{i=1}^{\ell} V_i^v \tag{10}$$

where V_i^u, V_i^y, V_i^w, V_i^v is the contribution in V of, respectively, the i^{th} control u_i, output y_i, noise w_i, or noise v_i, and

$$V_i^u = r_i \| g_i \|^2_{\hat{X}} \qquad i = 1, \ldots, m \tag{11a}$$

$$V_i^y = q_i \| c_i \|^2_{P+\hat{X}} \qquad i = 1, \ldots, k \tag{11b}$$

$$V_i^w = W_{ii} \| b_i \|^2_{K+L} \qquad i = 1, \ldots, m \tag{11c}$$

$$V_i^v = V_{ii} \| f_i \|^2_{L} \qquad i = 1, \ldots, \ell \tag{11d}$$

where P, K, $\hat{X}$ and L satisfy

$$0 = PA^T + AP - PM^TV^{-1}MP + BWB^T, \quad [f_1, \ldots, f_\ell] \stackrel{\Delta}{=} F = PM^TV^{-1} \tag{12a}$$

$$0 = KA + A^TK - KBR^{-1}B^TK + C^TQC, \quad [g_1, \ldots, g_m] = G^T = -KBR^{-1} \tag{12b}$$

$$0 = \hat{X}(A + BG)^T + (A + BG)\hat{X} + FVF^T \tag{12c}$$

$$0 = L(A - FM) + (A - FM)^TL + G^TRG \tag{12d}$$

The effectiveness of the i^{th} sensor is measured by

(13) $$\mathcal{V}_i^{sens} = \mathcal{V}_i^{v}$$

and the effectiveness of the i^{th} actuator is measured by

(14) $$\mathcal{V}_i^{act} \stackrel{\Delta}{=} \mathcal{V}_i^{u} - \mathcal{V}_i^{w}$$

These terms $\mathcal{V}_i^{sens}$ and $\mathcal{V}_i^{act}$ represent the particular combinations of the input/output costs $\mathcal{V}_i^{u}$, $\mathcal{V}_i^{w}$, $\mathcal{V}_i^{v}$ which are involved in the performance of each sensor and actuator. (The distinction here is that the effect of the *input* w_i can be calculated by $\mathcal{V}_i^{w}$, but the effect of an *actuator* involves both $\mathcal{V}_i^{u}$ and $\mathcal{V}_i^{w}$ since the actuator is noisy, and this dependence is accounted for in (14)). To see that $\mathcal{V}_i^{sens}$ and $\mathcal{V}_i^{act}$ gives the appropriate measure of the effect of *deleting* the i^{th} sensor or the i^{th} sensor or the i^{th} sensor or the i^{th} actuator, refer to the numerical work in [7].

Two results from [6] add insight into the use of (13), (14).

Theorem 1, [6,7]:

For a specified (Q,R), *the optimal value of the* LQG *performance metric* (5) *cannot be reduced by the deletion of any of the admissible sensors* z_i, $i = 1, \ldots, \ell$.

Theorem 2, [6,7]:

For a specified (Q,R) *the optimal value of the* LQG *performance metric* (5) *can possibly be reduced by the deletion of some of the admissible actuators* u_i, $i = 1, \ldots, m$.

These theorems partially explain why the sensor effectiveness $\mathcal{V}_i^{sens}$ is a much simpler calculation than $\mathcal{V}_i^{act}$. Since the magnitude of the gain on the i^{th} sensor signal $||f_i||^2 = ||m_i||^2_{pp} V_{ii}^{-2} \to 0$ as $V_{ii} \to \infty$, an extremely noisy sensor simply will not affect the optimal LQG controller. Hence, the effectiveness of the i^{th} sensor can be calculated by the input cost $\mathcal{V}_i^{v}$. Section V will show how to use (13) and (14) in the solution of the COVSAS problem.

IV. THE COVLQG ALGORITHM

Now we cite an algorithm (COVLQG) to solve the COVSAS problem under the temporary assumption that $\bar{\ell} = \ell$ and $\bar{m} = m$. That is, all admissible sensors and actuators are used ($\bar{B} = B$ and $\bar{M} = M$). The COVLQG algorithm will first be stated and then its theoretical properties will be discussed.

<u>The COVLQG algorithm (i.e. the COVSAS with $\bar{\ell} = \ell$, $\bar{m} = m$)</u>:

Step A: *Compute* P *from* (12a). *If* $\sigma_i^{-2}||c_i||_P^2 > 1$ STOP. *No solution to the* COVLQG *problem exists. Otherwise initialize*

$$q_i(o) = \sigma_i^{-2}\ , \quad r_i(o) = \mu_i^{-2}\ .$$

Discussion of Step A: The lower bound on $E_\infty y_i^2$ in an LQG problem is $E_\infty y_i^2 \geq ||c_i||_P^2$ (from the well known lower bound tr CPC^T on V in (5)), and this result is independent of the choice of $Q \geq 0$, $R > 0$.

Step B: *Compute*

$$E_\infty y_i^2 = q_i^{-1}\, V_i^y \quad \forall\ i:\ q_i > 0$$

$$E_\infty u_i^2 = r_i^{-1}\, V_i^u$$

using (11), (12). *If* $\sigma_i^{-2}\, E_\infty y_i^2 = 1\ \forall\ i:\ q_i > 0$ *and if* $\mu_i^{-2}\, E_\infty u_i^2 \geq 1\ \ \forall\ i = 1, \ldots m$, STOP. *The* COVLQG *solution has been found.*

Discussion of Step B: In the COVLQG option all necessary control effort is applied to force the constraints $E_\infty y_i^2 \leq \sigma_i^2$ to be binding. A formal proof that the stopping criterion of Step B indicates a solution of the COVLQG problem is given by Theorem 5 of [7].

Step C: Q *and* R *update equations: Let the iteration index be* j *and set* $q_i(j+1) = [\sigma_i^{-2}\, E_\infty y_i^2]q_i(j)$, $i = 1, \ldots, k$. *If* $(\varepsilon\sigma_i^2)^{-1} < q_i(j+1) < \varepsilon\sigma_i^{-2}$, ($\varepsilon < 0$ *small specified constant) then set* $q_i(j+1) = 0$. *If* $\sigma_i^{-2}\, E_\infty y_i^2 = 1\ \forall\ i:\ q_i > 0$, *then set* $r_i(j+1) = [\mu_i^{-2}E_\infty u_i^2]^{1/2} r_i(j)$, $\forall\ i:\ \mu_i^{-2}\, E_\infty u_i^2 < 1$. *For all other* i, *set* $r_i(j+1) = r_i(j)$. *Return to* Step B.

Discussion of Step C: The $r_i(j+1)$ of Step C are clearly adjusted toward the stopping condition of Step B ($\mu_i^{-2}\, E_\infty u_i^2 \geq 1$), since a reduction in r_i causes $E_\infty u_i^2$ to increase. The justification for setting $q_i = 0$ when *either* $q_i(j+1) \to 0$

or when $q_i(j+1) \to \infty$ is as follows: The tendency of q_i toward zero indicates a lack of output controllability due to a degenerate rank of C (rank $C < k$). In this case, the algorithm ceases to attempt the impossible (i.e. to force two *dependent* outputs to arbitrary values) by removing this particular y_i (the least critical one as indicated by the smallest $q_i \to 0$) from the cost function by setting its coefficient $q_i = 0$. Now let rank $C = k$. The tendency of q_i toward ∞ can result only when a stabilizable, detectable system is not output controllable, (even though $C = k$) *and* an uncontrollable output converges to a value which violates its constraint ($E_\infty y_i^2 > \sigma_i^2$). The constraint is violated the smallest amount possible since in this case the corresponding $q_i \to \infty$ on successive iterations of the update equations. When this condition is determined, such y_i's are removed from the cost function on future iterations (by setting $q_i = 0$) since it now has been established that they *cannot* be brought within specification $E_\infty y_i^2 \leq \sigma_i^2$.

A similar algorithm exists for the Constrained Input Variance LQG problem (CIVLQG) and details are given in [7].

V. THE COVSAS ALGORITHM

The sensor/actuator effectiveness formulas (13), (14) derived in Section III and the COVLQG algorithm of Section IV are now integrated to solve the COVSAS problem posed in Section II.

<u>COVSAS Algorithm</u>:

Step 1. *Specify* $\{A, B, C, M, V, \bar{\ell}, \bar{m}, \sigma^2, \mu^2\}$. *Run* COVLQG *algorithm using* ℓ *actuators,* m *sensors.*

Step 2. *Compute* V_i^{sens}, V_i^{act} *from* (13), (14) *and rank sensors and actuators according to their effectiveness:*

$$V_1^{sens} \geq V_2^{sens} \geq \dots \geq V_\ell^{sens} \tag{15a}$$

$$V_1^{act} \geq V_2^{act} \geq \dots \geq V_m^{act} \tag{15b}$$

Delete the sensor and actuator with the lowest effectiveness values V_i^{sens}, V_i^{act}, *provided such deletion does not cause loss of* *controllability or observability.*[‡] *Unless* $\ell < \bar{\ell} + 1$, *reset* ℓ *to* $\ell-1$. *Unless* $m < \bar{m} + 1$, *reset* m to m-1. *If* $\sigma_i^{-2} \mathcal{E}_\infty y_i^2 = 1 \ \forall\ i = 1, \ldots, k$ *and* $\forall$ i: $\mu_i^{-2} \mathcal{E}_\infty u_i^2 > 1$, *if* $[\frac{1}{\ell} \sum_{i=1}^{\ell} \mu_i^{-2} \mathcal{E}_\infty u_i^2]_{(j+1)\text{iteration}} <$ $[\frac{1}{\ell} \sum_{i=1}^{\ell} \mu_i^{-2} \mathcal{E}_\infty u_i^2]_{(j+1)\text{iteration}}$ *return to* Step 1. *Otherwise STOP. A solution to the* COVSAS *has been found.*

Discussion of Step 2: Numerical experience with this algorithm suggests that more than one sensor and more than one actuator may be deleted on each iteration. In fact, for many cases the same result can be obtained by reducing ℓ to $\bar{\ell}$ and m to $\bar{m}$ on the *first* iteration. However, this *quicker* convergence can sometimes converge only to *suboptimal* answers, and the algorithm above is written in its most conservative form (deleting only one sensor and/or actuator per iteration) where convergence to optimal values is more reliable [7].

VI. CONTROL OF A SPACE ANTENNA

Fig. 1 depicts the Hoop-Column Antenna arrangement for a proposed NASA communications satellite. Stationed in a geosynchronous orbit, the objective of the antenna control system is to regulate the orientation and focus of the satellite antenna relative to its multiple feed horns (at node 10). Table 1 lists the 24 linear and angular displacements which make up the outputs y_i, $i = 1, \ldots, k$, where k = 24. Table 2 lists the 39 admissible sensors and Table 3 lists the 12 admissible actuators. Note that ARX2 stands for angular rate about the x axis at node 2. AX2 stands for angular displacement about axis x at node 2. Z10-Z2 stands for a rectilinear displacement between nodes 10 and 2 in the z direction. The specifications for the outputs are $\sigma_i = 22.8$ are seconds for $i = 1, \ldots, 6$, and $\sigma_i = .158$ mm for $i = 7, \ldots, 24$. The specifications for the inputs u_i are $\mu_i = 10$ dn-cm, $i = 1, \ldots, 12$. The actuator noise is described by $W = \text{diag}\,[\ldots W_{ii} \ldots]$, $W_{ii} = .1\ (\text{dy-cm})^2$, $\forall\ i = 1, \ldots 12$. The sensor noise is $V = \text{diag}\,[\ldots V_{ii} \ldots]$, $V_{ii} = 7.615\text{x}10^{-7}\ \text{rad}^2$, $i = 1,2,3,13,14,15$, $V_{ii} = 2.5\text{x}10^{-7}\ \text{m}^2$, $i = 4, \ldots, 12, 16, \ldots, 27$, $V_{ii} = 4.76\text{x}10^{-5}\ (\text{rad/sec})^2$, $i = 28, \ldots, 39$. It is desired to limit the number of actuators to $6 = \bar{m}$ and the number of sensors to $12 = \bar{\ell}$. The dynamics of the antenna structure were described by 10 elastic modes and 3 rigid body modes. The square of the frequencies

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[‡]Observability, controllability checks are particularly simple for flexible space structures using the tests in [8]. That is, rank tests of matrices $[B; AB, \ldots A^{n-1}B]$, $[C^T, A^TC^T, \ldots A^{Tn-1}C^T]$ can be *avoided*.

ω_i^2, i = 1, ..., 10 of the elastic modes are

$$(\omega_1^2, \omega_2^2, \ldots, \omega_{10}^2) = (.40579, 7.2090, 7.2362, 13.27/, 44.834, 132.14, 14?.66, 445.01, 448.69, 775.86)\ (\text{rad/sec})^2.$$

More complete information for the antenna model may be found in [7].

The results of the COVSAS algorithm applied to the Hoop-Column Antenna are summarized in Table 4. The 6 actuators deleted from the admissible set of Table 3 are (listed in order of deletion): u_{12}, u_9, u_6, u_{10}, u_7, u_4. The 27 sensors deleted (in order of deletion) are: z_{15}, z_3, z_6, z_{12}, z, z_{13}, z_2, z_1, z_{24}, z_{27}, z_4, z_5, z_{18}, z_{21}, z_{30}, z_{39}, z_{33}, z_7, z_8, z_{31}, z_{23}, z_{20}, z_{35}, z_{25}, z_{22}, z_{16}. Notice that even though the output constraints are still binding the total control effort is *less* using only 6 actuators, (6x5.021 = 30.12) than using 12 actuators (12x3.275 = 39.30 > 30.12). Thus, *better* performance is possible with *fewer* actuators, since for several actuators the noise effect V_i^w is greater than the signal effect V_i^u in (14) (note the negative values of V_i^{act} in Table 4).

Perhaps the most important information from the COVSAS is the determination of the minimum achievable actuator specification ... from Table 5 that all of the 24 outputs are held within their design constraints (σ_i = 22.8 arc secs. for angles and σ_i = .158 mm for rectilinear displacements) by actuators which *must be designed* for the capabilities of TABLE 5. That is, the given output specifications, σ_i^2 are possible to meet if μ_i is changed (=> actuators are redesigned) (from Table 5) to μ_1 = 73, μ_2 = 26, μ_3 = 105, μ_4 = 26, μ_5 = 32, μ_6 = 39.

VII. CONCLUSIONS

Presented is an algorithm COVSAS which integrates the following tasks:

Selects sensors and actuators from an admissible set.

Designs a linear feedback controller which satisfies output variance constraints.

Determines *actuator design requirements* which allow the output variance constraints to be satisfied.

Numerical properties of the convergence of this algorithm are given for NASA's Hoop-Column Antenna. Additional theoretical properties of convergence of this algorithm are given in [7].

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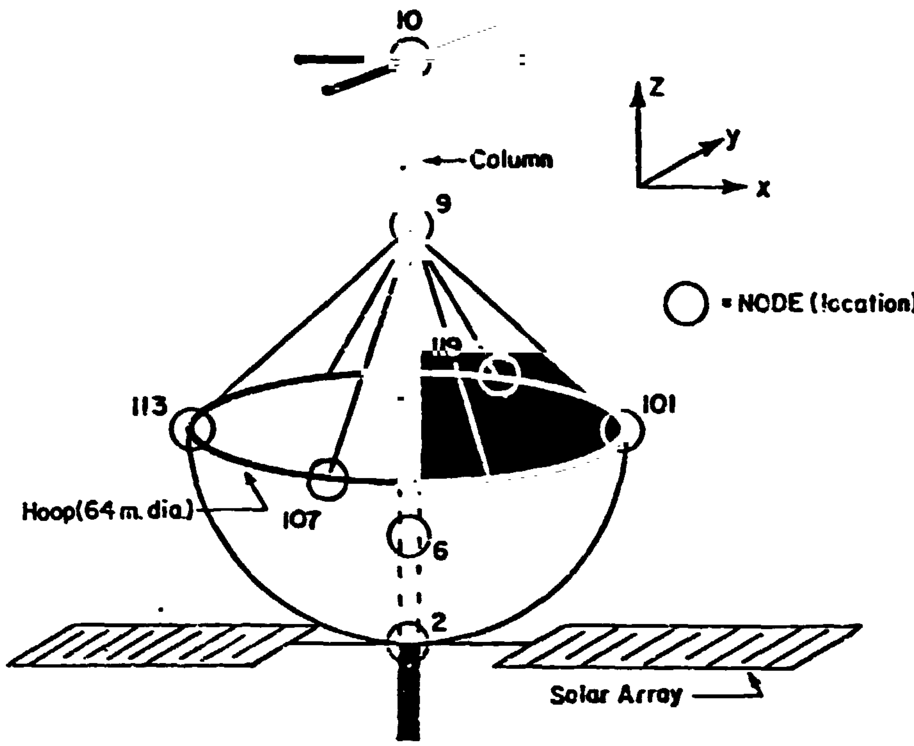

Figure 1: Hoop Column Antenna

Table 1: Hoop Column Output Description

Output #	Type	Nodal Location	Direction
1	Inertial Angle	2	X
2	"	2	Y
3	"	2	Z
4	Relative Angle Between	10 and 2	X
5	"	"	Y
6	Inertial Angle	10	Z
7	Relative Linear Disp. Between	6 and 2	X
8	"	"	Y
9	"	9 and 2	X
10	"	"	Y
11	"	10 and 2	X
12	"	"	Y
13	"	101 and 10	X
14	"	"	Y
15	"	"	Z
16	"	107 and 10	X
17	"	"	Y
18	"	"	Z
19	"	113 and 10	X
20	"	"	Y
21	"	"	Z
22	"	119 and 10	X
23	"	"	Y
24	"	"	Z

Table 2: Hoop-Column Sensor Labels

Sensor Number	Label	Sensor Number	Label	Sensor Number	Label
1	AX2	14	AY10	27	Z119-Z10
2	AY2	15	AZ10	28	ARX2
3	AZ2	16	X101-X10	29	ARY2
4	X6-X2	17	Y101-Y10	30	ARZ2
5	Y6-Y2	18	Z101-Z10	31	ARX6
6	Z6-Z2	19	X107-X10	32	ARY6
7	X9-X2	20	Y107-Y10	33	ARZ6
8	Y9-Y2	21	Z107-Z10	34	ARX9
9	Z9-Z2	22	X113-X10	35	ARY9
10	X10-X2	23	Y113-Y10	36	ARZ9
11	Y10-Y2	24	Z113-Z10	37	ARX10
12	Z10-Z2	25	X119-X10	38	ARY10
13	AX10	26	Y119-Y10	39	ARZ10

Table 3: Hoop Column Actuator Description

Actuator	torque about axis at Node location
u_1 =	T X 2
u_2 =	T Y 2
u_3 =	T Z 2
u_4 =	T X 6
u_5 =	T Y 6
u_6 =	T Z 6
u_7 =	T X 9
u_8 =	T Y 9
u_9 =	T Z 9
u_{10} =	T X 10
u_{11} =	T Y 10
u_{12} =	T Z 10

Table 4: Hoop Column Output Constrained COVSAS Results

Iteration Number	Identified Sensors (V_i^{sen})	Identified Actuators (V_i^{act})	Ave Input Value (7.6)	Number of Sensors/Actuators
1	AZ10(.0004116) AZ2(.000397) Z6-Z2(0) Z9-Z2(0) Z10-Z2(0)	TZ10(-1.362) TZ9(-1.369)	3.275	39/12
2	AY1(.003362) AX10(.003358) AY2(.00226) AX2(.00226) Z113-Z10(.001942) Z119-Z10(.001884)	TZ6(-2.1405)	3.592	34/10
3	X6-X2(.01457) Y6-Y2(.01455) Z101-Z10(.0110) Z107-Z10(.0108)	TX10(-1.2055)	3.699	28/9
4	ARZ2(.02844 ARZ10(.02232) ARZ6(.02238)	TX9(-1.2917)	3.997	24/8
5	X9-X2(.0986) Y9-Y2(.0839)	TX6(-1.4793)	4.377	21/7
6	ARX6(.07648) ARX2(.07648)	----	4.829	19/6
7	Y107-Y10(.13395) XRY9(.1098)	----	4.857	17/6
8	X119-X10(.1557) X113-X10(.1555) X101-X10(.1551)	----	4.905	15/6
9	----	----	5.021	12/6

Table 5: Output-constrained Specifications

Output #	$E_\infty y_i^2$	Actuator #	$E_\infty u_i^2$ (minimum achievable)
1(AX2)	.015 sec	1 TX2	72.91 dn-cm
2(AY2)	.015 sec	2 TY2	26.145 dn-cm
3(AZ2)	11.588 sec	3 TZ2	105.47 dn-cm
4(AX10-AX2)	.001 sec	4 TY6	26.138 dn-cm
5(AY10-AY2)	.001 sec	5 TY9	31.750 dn-cm
6(AZ10)	12.000 sec	6 TY10	38.812 dn-cm
7(X6-X2)	.010 mm		
8(Y6-Y2)	.010 mm		
9(X9-X2)	.068 mm		
10(Y9-Y2)	.068 mm		
11(X10-X2)	.158 mm		
12(Y10-Y2)	.158 mm		
13(X101-X10)	.104 mm		
14(Y101-Y10)	.158 mm		
15(Z101-Z10)	.007 mm		
16(X107-X10)	.158 mm		
17(Y107-Y10)	.156 mm		
18(Z107-Z10)	.008 mm		
19(X113-X10)	.122 mm		
20(Y113-Y10)	.158 mm		
21(Z113-Z10)	.001 mm		
22(X119-X10)	.158 mm		
23(Y119-Y10)	.091 mm		
24(Z119-Z10)	.001 mm		