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A NONLINEAR DUAL-ADAPTIVE CONTROL STRATEGY FOR IDENTIFICATION AND CONTROL OF FLEXIBLE STRUCTURES*

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ABSTRACT

A technique is presented for obtaining a control law to regulate the modal dynamics and identify the modal parameters of a flexible structure. The method is based on using a min-max performance index to derive a control law which may be considered to be a best compromise between optimum one-step control and identification inputs. Features of the approach are demonstrated by a computer simulation of the controlled modal response of a flexible beam.

I. INTRODUCTION

A class of indirect adaptive control systems proposed for the control of large space structures [1] is based on a modal decomposition of the system dynamics and may incorporate one or more on-line testing schemes [2] to determine when successful parameter identification has been achieve. The control strategy used in calculating the actuator inputs must achieve adequate regulation or tracking performance and, at the same time, provide inputs to allow adequate parameter identification. A control system designer is thus faced with the problem of devising a control strategy to ensure acceptable system performance even when on-line parameter identifiability tests have failed because the system

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configuration has changed or the environment in which the system operates has changed.

In this paper we formulate and examine the performance of a nonlinear dual-adaptive control scheme in which a sampled-data controller is designed to select a best compromise between an input signal that is optimum for mean-square system regulation and an input signal that is optimum for parameter identification. Dual control theory, originally formulated by Feldbaum [3,4], has been studied in [5-7] and in the references cited therein. A key concept introduced by Feldbaum is the dual control strategy based on a performance index that takes into account the fact that future observations on the process will be made. A controller may be able to "probe" the system for state and parameter estimation improvement, which then may improve future regulation and tracking performance. In many situations where the dual nature of stochastic control is not taken into account the controller becomes "cautious" [5,6] and tends to "turn-off". This undesirable phenomenon is avoided by the approach described below.

II. FORMULATION OF AN ADAPTIVE PERFORMANCE INDEX

The discrete-time dynamics for each mode is assumed to be described by the ARMA model

$$y(t)+a_1y(t-1)+a_2y(t-2) = b_1u(t-1)+b_2u(t-2)+e(t) \tag{1}$$

where $y(t)$ denotes modal displacement, $u(t)$ denotes modal force, and $e(t)$ is a sequence of independent, equally-distributed, normal $(0,\sigma^2)$ random variables. It is assumed that $e(t)$ is independent of $y(t-1),y(t-2),\ldots,$ $u(t-1),u(t-2),\ldots$ and that the parameters a_1,a_2,b_1,b_2 are unknown constants. If we let Y_t denote the information available to the controller at time t,

$$Y_t = \{y(t),y(t-1),\ldots,\ u(t-1),u(t-2),\ldots\} \tag{2}$$

$x(t)$ denote the modal parameter vector and $\theta(t)$ denote a modal measurement vector,

$$\begin{aligned} x^T(t) &= (a_1,a_2,b_1,b_2); \\ \theta^T(t) &= (-y(t-1),-y(t-2),u(t-1),u(t-2)) \end{aligned} \tag{3}$$

where $(\ .\ )^T$ denotes vector or matrix transpose, then (1) may be rewritten as

$$y(t) = \theta^T(t)x(t)+e(t) \tag{4}$$

where the constant parameter "dynamics" satisfies

$$x(t+1) = x(t) \tag{5}$$

It can than be shown, following the analysis of [8], that the conditional distribution of $x(t+2)$ given Y_{t+1} is normal with mean $x(t+2)$ and covariance matrix $P(t+2)$ where $x(t)$ and $P(t)$ satisfies the difference equations

$$\hat{x}(t+1) = \hat{x}(t)+K(t)(y(t)-\theta^T(t)x(t)) \tag{6}$$

$$K(t) = P(t)\theta(t)/(\sigma^2+\theta^T(t)P(t)\theta(t)) \tag{7}$$

$$P(t+1) = P(t)-(P(t)\theta(t)\theta^T(t)P(t))/ (\sigma^2+\theta^T(t)P(t)\theta(t)) \tag{8}$$

Furthermore, the control law that minimizes the regulation criterion

$$V_c(u(t)) = E\left\{y^2(t+1)\,|\,Y_t\right\} \tag{9}$$

is given by

$$u(t) = - \frac{\sum x_i(t+1)x_3(t+1)+P_{3i}(t+1))\theta_i(t+1)}{x_3^2(t+1)+P_{33}(t+1)} \tag{10}$$

where $\sum$ denotes the sum over i = 1 to 4 with the value 3 excluded.

To provide bounded modal inputs that improve parameter identification accuracy while guaranteeing that the modal amplitude will not become excessively large, the controller is designed to optimize, at each sampling instant t, the following performance criterion:

$$\min_{u(t)} \max_{\lambda} [V(\lambda, u(t))] \tag{11}$$

subject to the constraints

$$u(t) \le M, \; 0 \le \lambda \le 1 \tag{12}$$

where

$$V(\lambda, u(t)) = \lambda \frac{V_c(u(t))}{V_c^0} + (1-\lambda)\frac{V_I(u(t))}{V_I^0} \tag{13}$$

V_C° denotes an acceptable or desired level of regulation cost. $V_I(u(t))$ denotes and identification cost function of u(t),

$$V_I(u(t)) = \text{trace}\left[P(t+2)\right] \tag{14}$$

V_I° denotes and acceptable or desired level of identification cost. The maximization indicated in (11) yields a function V(u(t)) which, although not convex, is interpreted as specifying, for each admissible u(t), the most costly linear combination of relative regulation and relative identification cost. Minimization of V(u) thus yields the modal input that minimizes this most costly combination of relative identification and regulation performance.

III. SIMULATION RESULTS

Since $V_C(u(t))$ and trace P(t+2) are relatively simple functions of u(t) the numerical solution of the one-step optimization problem (11)-(13) at each sampling time is quite feasible. Results of simulation studies described below illustrate an interesting feature of this approach: since the parameters involved in the evaluation of $V_C(u(t))$ and $V_I(u(t))$ depend on system measurements, the optimum distribution of relative cost, $\lambda^*(u)$ depends on on-line measurement data and hence, at each sampling instant, the weighting between identification and regulation will change depending on the on-line system performance. This is in contrast to[9] in which a fixed weighting between absolute control and identification cost is used at each sample time.

In the simulation study we compare the performance of three control systems:

a) A constrained adaptive controller that minimizes (9) subject to the control magnitude constraint.
b) An optimum identification controller that minimizes (14) subject to the control magnitude constraint.
c) The one-step dual-adaptive controller based on (11)-(13).

In Figures 1-3 we present simulated modal response data for the first flexible mode of the Langley beam experiment described in [10] where we assume here that a single actuator is used. The accumulated on-line regulation cost, VT, shown in Figure 1 is defined as

$$VT(N) = \sum_{k=1}^{N} y^2(k) \tag{15}$$

and the on-line identification cost, PT, is defined as

$$PT(N) = \text{trace}\ [P(N)] \tag{16}$$

where P(N) is calculated on-line using (8). Note that for the first 10 to 15 sampling times the regulation cost of the dual-adaptive controller is close to that of the constrained minimum-variance controller and the identification cost of the dual-adaptive control system is close to that of the constrained one-step optimum identification controller. Figure 2 indicates that the dual-adaptive controller's actuator signals switch between its limits, ± 0.5, more frequently than do the actuator signals of the other controllers. This may be due to the lack of any energy constraint in the above problem formulation.

A future study will examine the performance of the energy-constrained dual-adaptive controller in comparison with energy-constrained minimum-variance and one-step optimum identification controllers. The relative regulation cost and relative identification cost defined in (13) are plotted in Figure 3 where

$$V_C^{\circ}(N) = \sigma^2 N \tag{17}$$

is the accumulated control cost that would be achieved if the parameters of the system where known precisely and if an unconstrained control law were used; $\sigma^2 = 10^{-4}$ was used in the simulation runs. A constant value $V_I^{\circ} = 10^{-4}$ was chosen as indicating the acceptable level of parameter identification. Figure 3 indicates that, depending on on-line measurements, the one-step identification and regulation cost at one sampling instant can have widely differing shapes from their respective distributions at other sampling times. This leads to the on-line variations in the dual-adaptive control strategy mentioned earlier.

The simulation results indication that the one-step, constrained dual-adaptive controller has the feature of providing, based on measured data, system inputs that result in parameter identification while maintaining bounded modal amplitude response.

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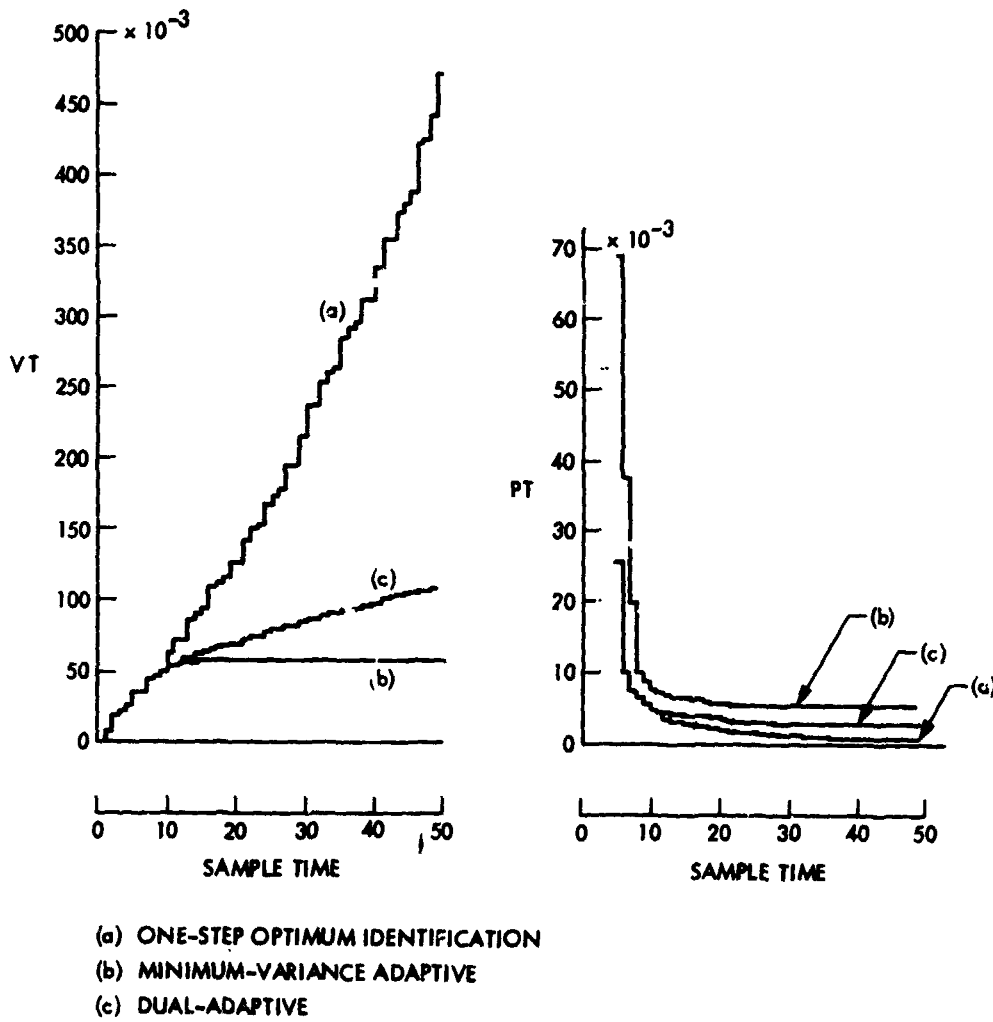
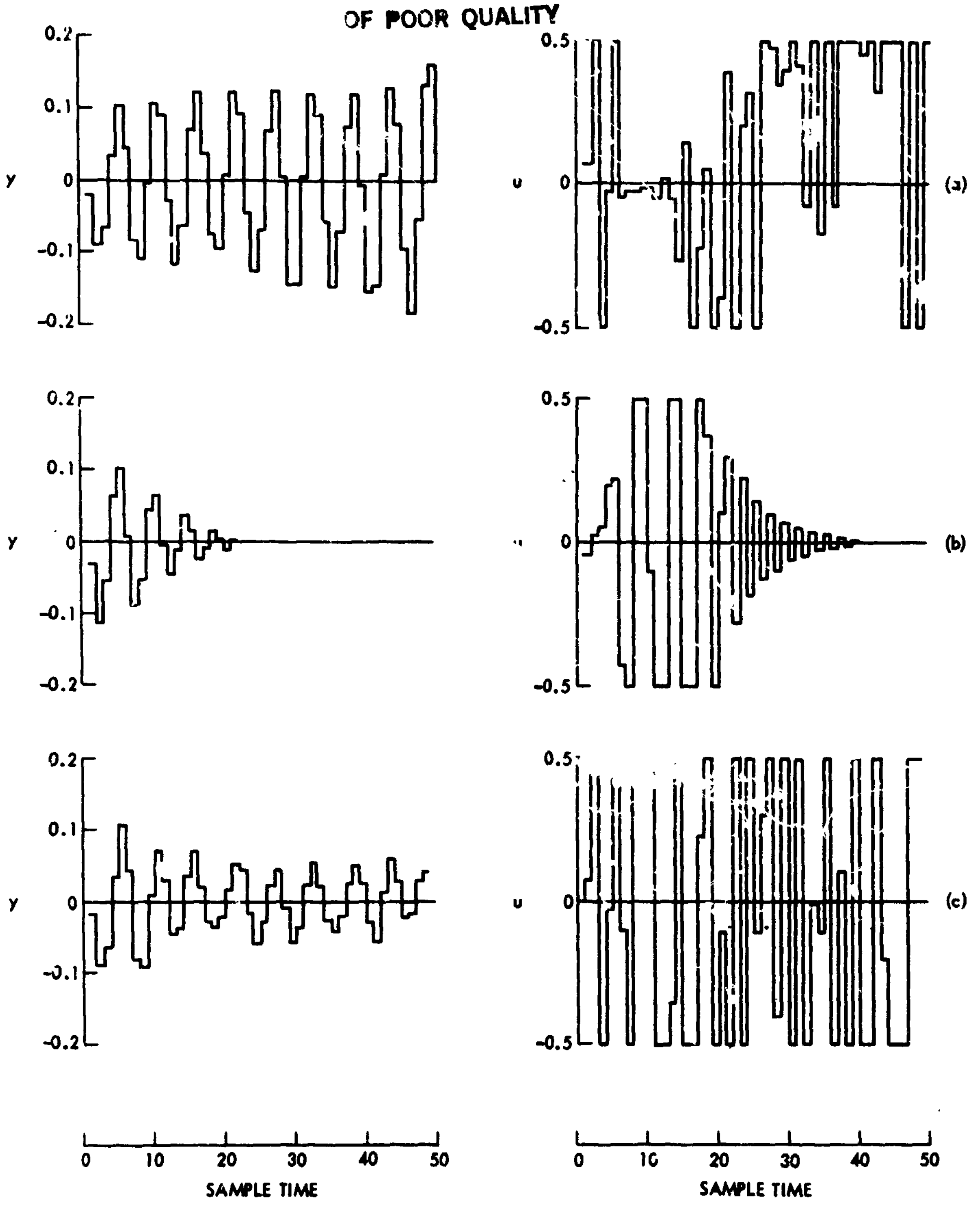


Fig. 1. On-Line Regulation and Identification Cost for Three Feedback Controllers



(a) ONE-STEP OPTIMUM IDENTIFICATION
(b) MINIMUM-VARIANCE ADAPTIVE
(c) DUAL-ADAPTIVE

Fig. 2. Modal Displacement and Modal Force for First Flexible Mode

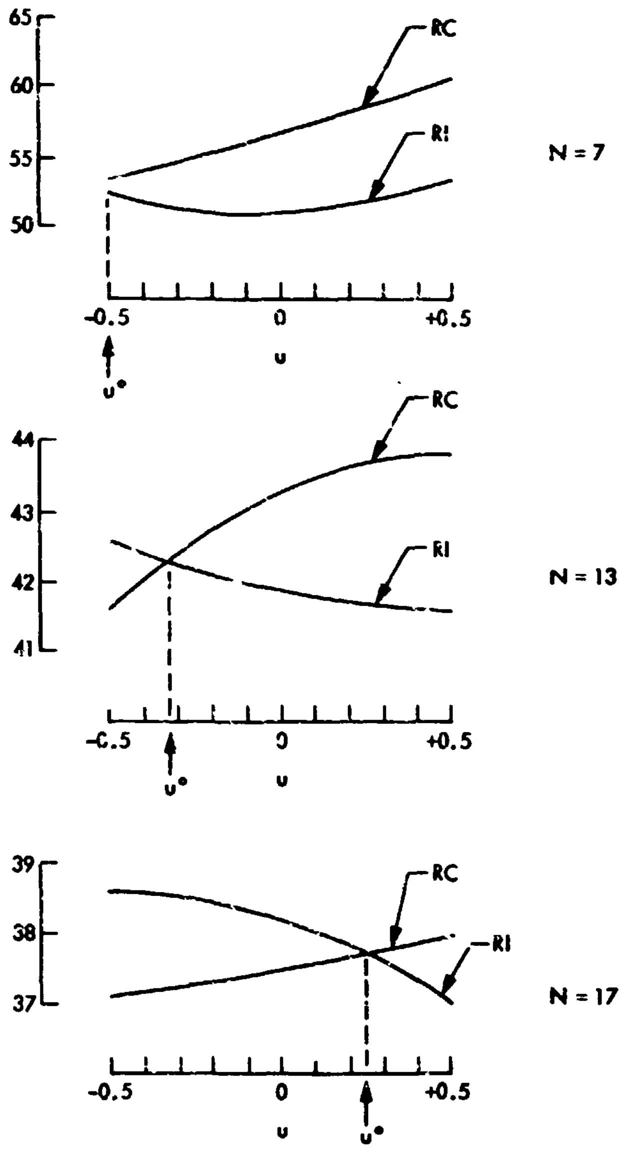


Fig 3. Relative Control Cost RC = $V_C(u)/V_C^\circ$ and Relative Identification Cost RI = $V_I(u)/V_I^\circ$ for 3 Sample Times