A Multiple Linear Regression Analysis of Hot Corrosion Attack on a Series of Nickel Base Turbine Alloys

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SUMMARY

Multiple linear regression analysis was used to determine an equation for estimating hot corrosion attack for a series of Ni-base cast turbine alloys. The U-transform (i.e., \( \sin^{-1} \left( \frac{\%DA}{100} \right)^{1/2} \)) was shown to give the "best" estimate of the dependent variable, y. A complete second degree equation is described for the "centered" weight chemistries for the elements Cr, Al, Ti, Mo, W, Cb, Ta, and Co. In addition linear terms for the minor elements C, B, and Zr were added for a basic 47-term equation. The "best" reduced equation was determined by the stepwise selection method with essentially 13 terms. The Cr term was found to be the most important accounting for 60 percent of the explained variability for turbine alloys. At a typical Cr level of 8 and 10 percent Co with the restriction that the total refractory metal content is not to exceed 20 percent the most resistant alloy is predicted to be Ni-10Co-8Cr-4Ti-7Al-10W-5Mo-5Ta with the nominal C, B, and Zr variations not critical. If the Co level were reduced to 0 percent and even more resistant alloy is estimated to be Ni-8Cr-4Ti-4Al-10Ta-5Mo-5W. The regression analysis described herein explained roughly 79 percent of the total variability with a 13-term equation. Of the remaining 21 percent residual error, only 2 percent was shown to be experimental indicating good control of the experiment.

INTRODUCTION

In a previous study (ref. 1) multiple linear regression techniques were successfully used to analyze high temperature cyclic oxidation data as a function of sample alloy content. A series of cast nickel-base \( \gamma/\gamma' \) alloys were formulated to represent a one-quarter replicate of a \( 2^7 \)-factorial statistical design for two levels of Cr, Al, Ti, Mo, W, Ta and Cb content. Those 32 alloys were supplemented by 18 additional alloys with compositions within the statistical design. For all 50 compositions, the levels of Co, C, Zr, and B (so-called "tramp" variables) were nominally held constant. A regression model was developed which was based on the seven design variables and four tramp variables with their linear terms plus first order interactions for the design variables with an attack parameter as the dependent variable. The initial model equation involved 32 terms at a 90 percent rejection level. From this an estimating equation was evolved which had 10 terms at each of two test temperatures and explained 93 percent of the total variability.

This same approach has now been used to analyze burner rig hot corrosion data obtained for the same alloy compositions used in the cyclic oxidation study. In addition to these compositions, 36 other alloy samples were also tested in the hot corrosion investigation (ref. 2). Rationale for alloy composition selection, test procedures, and experimental results are detailed by Deadmore (ref. 2). The primary attack parameter identified by Deadmore was
the percentage change in cross-sectional area (\%\Delta A) of a wedge-shaped burner bar measured after 300 1-hr cycles of hot corrosion testing at 900 °C in the flame of a Mach 0.3 burner rig seeded with 0.5 ppm Na as NaCl.

The basic approach used here was to find the "best" transformation of the dependent variable, \%\Delta A, using compositions (here weight percent) in the statistical design field. Then an additional group of test values for modified commercial alloys were added to expand the model to include cobalt level variations. Finally, the model equation was tested by applying it to test data obtained for 16 similar commercial and experimental nickel-base turbine superalloys.

RESULTS AND DISCUSSION

The test results for the 94 burner rig bars are listed in table I along with the alloy designation, bar number, and alloy chemistries in weight percent (wt %). The alloy attack is in terms of percent cross-sectional area charge, termed \%\Delta A, as measured metallographically after the test is completed and the sample is cut and mounted. The alloy bars were exposed for 300 1-hr cycles to a jet fuel and air flame doped with 0.5 ppm Na as NaCl environment at 900 °C in Mach 0.3 burner rigs.

The data are divided into two separate groups for purpose of analysis as indicated in table I. In the first group are a series of statistically designed Ni-Base γ/γ' alloys with 10 wt %, Co-0.05 wt %, Zr-0.10 wt %, C-0.01 B and with high or low levels of Cr, Al, Ti, W, Mo, Ta, andCb. These alloy compositions are based on a one-quarter replica of a 2^7 factorial design supplemented with 18 additional design compositions. This group includes eight sets of replicates minus a bar of alloy M-33 which was lost. To this basic set of 57 samples are added an additional group of 21 samples that are variations in most cases of the alloys U-700 and MAR-M-247 with selected levels of cobalt. This grouping is termed the n = 78 data set. The second grouping defined as the n = 94 data set includes the n = 78 data set as well as 16 additional alloy samples used for testing predictability of the derived estimating equation. Test results for the data set lead to the basic model regression equation used to estimate corrosion y in terms of alloy chemistry designated in weight percent by the element abbreviations

\[ y = a_0 + b_1 Al + b_2 Cr + \ldots + b_{11} B + b_{12} Al^2 + b_{13} Cr^2 + \ldots + b_{18} Ta^2 + b_{19} Al \cdot Cr + b_{20} Al \cdot Ti + \ldots + b_{47} Ta \cdot Co \pm S.E.E. \]  \( (1) \)

This is a complete second degree estimating equation in terms of Al, Cr, Ti, Mo, W, Cb, Ta, and Co along with linear terms of Zr, C, and B for a total of 47 terms. A nominal rejection level of 0.900 (F = 2.88) was used by the stepwise method (ref. 3) and analyzed by means of MINITAB on an IBM 370 computer. Also in the analysis each elemental weight percent was adjusted or "centered" by subtracting the group mean (ref. 4). In equation (1) S.E.E. stands for the Standard Error of Estimate.

The strategy in regression analysis is to reduce the number of terms in the original estimating equation as far as possible while still explaining as much of the total variability as possible with the lowest possible S.E.E. This is termed the coefficient of multiple determination designated as \( R^2 \). In the
the case of a perfect fit $R^2$ is 100 percent. In addition the equation should be able to predict results for the same alloy variables (here alloy chemistry) for values similar to those used to derive the regression coefficients.

A major problem in using equation (1) is to determine the proper transformation of the dependent variable $y$. Three possible choices are: $\%\Delta A$ (linear), $\log_{10}(\%\Delta A)$, and a third $U = \sin^{-1}(\%\Delta A/100)^{1/2}$ (ref. 5). The U-transform is often used in corrosion studies where attack is sometimes described in terms of a percentage of a pipe wall consumed. All three of the above transformations were tried on a subset of the $n = 78$ data set involving the 57 original test samples. The U-transformation appeared to be the best of the three giving the highest $R^2$ for the same basic modeling equation. When these estimates are then transformed back to the linear values and compared to the original the U-transformation is still superior. This was determined by subtracting the original $\%\Delta A$ from the derived value transformed back to a $\%\Delta A$ estimate, taking the absolute value, and then averaging all the values. For this study this value will be defined as the average deviation. On this basis the U-transform appears satisfactory and the best available. It will be used exclusively for the remainder of the analysis.

The $n = 78$ data set contains 14 sets of replicates. These are tests preformed on identical samples from the same alloy heat. Here they were tested randomly throughout the three phases of the program to give a measure of the experimental error. They were not necessarily run together during the same run or even in the same test rig if run at different times. This "experimental" error is thought to be more realistic because it contains any rig-to-rig and/or run-to-run variability confounded with the replicate error. Table II lists the 14 replicate tests separately by the bar number along with the $\%\Delta A$ value and the U-transform that belong to the $n = 78$ data set. A 15th replicate set is also listed but it belongs to the last group of 16 tested.

The regression analysis performed on the $n = 78$ data set is summarized in table III. At the rejection level chosen based on the original 47 term model (i.e., $Z_r = 47$) a total of 12 (i.e., $Z_f = 12$) coefficients are significant including seven of the eight main effects: Al, Cr, Ti, Mo, W, Cb, Ta, but not Co. There are five two-factor terms: Al·Mo, Al·W, Al·Ta, Cr·W and Mo·Co. However, because Co is considered so important it was added back in the model and the regression was recomputed. This analysis is summarized in the second part of the table. The coefficients are slightly altered to accommodate the Co coefficient with standard error of estimate, S.E.E. slightly raised from 0.1100 to 0.1108. Both reduced equations explain close to 74 percent of the total variability (i.e., $R^2 = 73.8$ and 73.8, respectively).

The derived U-transform estimates from the reduced 13-term model equation for the $n = 78$ data set are plotted in figure 1 versus the original $\%\Delta A$ values using the U-transform. A perfect fit would fall along the diagonal straight line passing thru the origin. The two parallel lines bounding the diagonal line represent one standard deviation, $S = \pm 0.1108$. The data plotted is broken into two groups: the circles represent the original M-series factorial design alloys of 57 samples while the squares represent the additional supplement to makeup the full $n = 78$ data set. Possible outliers ($>\pm 2$ s) are shaded and their bar numbers indicated. Possible outliers are automatically "flagged" in most regression analysis programs usually at values that exceed $\pm 2$ standard deviations. It is up to the investigator to determine whether there is some physical reason to reject any of these data points.
Usually about 5 percent of the test values would be expected to fall within 2 to 3 standard deviation units. Values that exceed ±3 standard deviation units are more apt to be true outliers. Based on these criteria these five results were retained in the analysis. All U-values are listed in table IV. Also included in the table are the U-estimates for the reduced 12-term model as well without the Co term. Note they are extremely close.

A plot of the residuals (i.e., $U_{\text{obs}} - U_{\text{est}}$), (fig. 2), shows a nearly complete random nature. This indicates there is no reason to reject the validity of this 13-term equation.

If the five possible outliers shown in figure 1 are dropped and the 13 coefficients are recomputed the $R^2$ is raised to 84.2 percent from the original 73.8 percent. The S.E.E. drops to 0.0827 from 0.1108. This technique could be used to improve the model equation providing these are true outliers.

A more practical indicator of the validity of the final reduced model is how well the %AA estimate derived from the U-transform estimate agrees with the original %AA values. These were generated by MINITAB by taking the sine of the U-transform, squaring it and multiplying by 100. These are plotted against the original %AA values in figure 3 for $Z_f = 13$. Again the values should fall on the diagonal straight line for a perfect fit. The average deviation computed from the absolute difference as described above forms the upper and lower bounds with a value of ±4.78. The possible outliers from figure 1 are shown also. In general the fit appears satisfactory although the model tends to underestimate the attack in the higher consumption region of 40 percent and greater.

The ultimate test on the validity of this regression model approach is how well it predicts corrosion results for a series of mostly commercial Ni-base turbine alloys similar in composition to the $n = 78$ data set. These are bars 81 to 96 listed in table I. Table V lists the %AA and U-transforms for these 16 bars as well as the U-transforms and %AA estimates derived from the U-transform for the $n = 78$ set coefficients listed in table III. Figure 4 shows the U-transform plot for the $Z_f = 13$ (i.e., 12 plus Co) set of coefficients. The perfect fit line along with the 1 σ parallel lines are also shown. Ten of the 16 predicted values fall within the 1 σ standard deviation line with only one value, bar 85 the experimental alloy IV-E, as a possible outlier. Since 12 of the 16 are below the perfect fit line there is a tendency as before to underestimate the degree of attack. Figure 5 shows the derived %AA estimates for the 16 bars plotted against the actual values. The perfect fit and average deviation parallel lines are shown also.

The final step in this analysis is to use all 94 data values (including one additional set of replicate samples) and perform a final regression analysis using the U-transform starting with the same 47-term equation with the same approximate 0.900 rejection criterion. The equation reduces to 12 coefficients but again omits a Co main effect that is then added back in. These terms are summarized in table VI. The results are quite similar to those for the $n = 78$ data set as summarized in table III. Two minor terms the Cr · W and Mo · Co are replaced by a W · Co and Ti² terms. However, when taken together they account for under 5 percent of the total explained variability.
For the sake of brevity, table VII lists the U-transform estimates and derived %ΔA estimates for the \( n = 94 \) data set only for bar numbers 81 to 96 for the \( Z_f = 13 \) (i.e., 12 plus Co) set of coefficient. These can be compared to similar values in table V. Figure 6 cross-plots the U-estimates in tables V and VII versus U-observed values. It shows the improvement as would be expected adding an additional 16 data points to the regression analysis and then including these 16 alloy chemistries in predicting the U-transforms and their derived %ΔA estimates. Figure 6 shows both sets of U-estimates using in both cases the \( Z = 13 \) (12 plus Co) coefficients, respectively. Figure 7 is a similar plot for the %ΔA values.

This final estimating equation was also used to predict results for an additional set of statistically designed alloys (ref. 6) which are currently being tested and evaluated. It will further test the validity of this approach.

As more data becomes available it can be added to the basic data sets and equation (1) can be reanalyzed or even expanded with new variables and/or other regression criteria. One would like to drive the \( R^2 \) values to over 90 percent as was done for cyclic oxidation for a similar group of alloys (ref. 1). But this is not so critical as giving "good" predictability for similar alloys which have not as yet been incorporated in the estimating equation.

The replicate runs in each data set are useful in evaluating the model. Because there are replicates the residual error (i.e., sum of squares) can be broken into two components - the experimental error and the lack of fit. This total residual error accounts for just under 21 percent of the total variability. Of this total error the replication component is quite small, just under 2 percent. This tells us the experimental runs and test alloys were closely controlled with good reproducibility. Thus if we are to drive the \( R^2 \) to over 90 percent we have to do it by improving the model rather than refining the test.

The model might be modified enough so the critical \( F \) ratio in table VIII for example would fall below a value of 2.15. This could include altering the original 47-term model as given in equation (1), finding even a better transformation than the U-transform used here or pin-pointing any true outliers and drop them from the model. This would be the ultimate goal for this type of analysis. In the meantime the existing analysis appears satisfactory for a number of applications. Table VIII shows a summary of the ANOVA (Analysis of Variance) for the \( n = 94, Z_f = 13 \) data set.

The estimating equation(s) can be used to note the effect of the various alloy constituents and in theory to design alloys that have low attack rates. By far the most important element is Cr which in general should be as high as possible. The Al level should be as low as possible unless, W, Ta, and Mo are present. Co in general is detrimental though the effect is small. The other effects are interrelated. This analysis is summarized in table IX for the \( n = 94 \) data set for three typical Cr levels of 8, 12, and 16 percent at three typical Co levels of 0, 5, and 10 percent. The U-values can be computed for each of these 9 sets and 11 levels of Al (2.0, 2.5, 3.0, 3.5, ... 7.0), 6 levels of Tl, (0, 1, 2, 3, 4, 5) and 3 levels each (low, medium, and high) of Mo, Cb, W, and Ta. These 5346 values in each set can be scanned for the minimum of U-values.
In actual commercial turbine alloys the total refractory metal content rarely exceeds 20 percent. If this restriction is imposed on the estimating equation used above this reduced to 4422 values in each of the nine sets. Table IX shows the results when these sets are scanned for the minimum value of U-estimated. In four of the nine sets the U-estimate value is actually in the negative region which is effectively zero attack. The results are quite similar for the optimum composition levels for ranges of 0 to 10 Co, and 8 to 16 Cr, Cb is 0, and Ti is 4 in all the minimum cases. Al somewhat surprisingly is at its maximum since the interaction terms with W, Ta, and Mo override the single +Al term. The only difference is at 0 Co where the minimum is at 5 W and 10 Ta while at the 5 and 10 Co levels these compositions are reversed.

Since these optimal alloy compositions are near the extremes of the alloy sample space where the sampling error is higher than the center of the data, extrapolation should be checked by further experimentation.

**SUMMARY OF RESULTS AND CONCLUSIONS**

Multiple linear regression analysis was used to determine an estimating equation for hot corrosion attack for a series of Ni-base cast turbine alloys. The U-transform (i.e., sin⁻¹ (%%A/100)¹/²) was shown to give the "best" estimate of the dependent variable, y. A complete second degree equation is described for the "centered" weight chemistries for the elements Cr, Al, Ti, Mo, W, Cb, Ta, and Co. In addition linear terms for the minor elements C, B, and Zr were added for a basic 47 term equation. The "best" reduced equation was determined by the stepwise selection method with essentially 13 terms. The Cr term was found to be the most important accounting for 60 percent of the explained variability for turbine alloys. At a typical Cr level of 8 and 10 percent Co with the restriction that the total refractory metal content is not to exceed 20 percent the most resistant alloy is predicted to be Ni-10Co-8 Cr-4Ti-7Al-10W-5Mo-5Ta with the C, B, and Zr variations not critical. If the Co level were reduced to 0 percent and even more resistant alloy is estimated to be Ni-8Cr-4Ti-4Al-10Ta-5Mo-5W. The regression analysis described herein explained roughly 79 percent of the total variability with a 13-terms equation. Of the remaining 21 percent residual error, only 2 percent was shown to be experimental indicating good control of the experiment.

**REFERENCES**


TABLE I - HOT CORROSION RESULTS FOR 94 TEST BARS FROM REF. 1 ALONG WITH ALLOY CHEMISTRY

[Some chemistries upgraded since publication of Ref. 2. Divided into 2 sets of
n = 78; plus 16 additional samples.]

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<th>Al</th>
<th>Ti</th>
<th>W</th>
<th>Mo</th>
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<td>7.93</td>
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Other: 
- Other alloy compositions are available upon request.
TABLE I. - Continued.

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<th>Mo</th>
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<th>Cb</th>
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| W78    | 10.63  | 4.45 | 2.81 | 4.86 | 2.69 | 4.02 | 2.09 | 9.35 | .08 | .03 | .08 | ------- | ---- |
| TAZ-8A | 81     | 5.77 | 6.60 | 0.0  | 3.84 | 3.73 | 8.74 | 2.20 | 0.0  | .66 | .01 | .12 | ------- | 38.5 |
| U-710  | 82     | 17.88 | 2.52 | 5.04 | 1.34 | 3.00 | 0.0  | 13.98 | 0.0  | .01 | .11 | ------- | 1.9  |
| X1-A   | 83     | 4.45 | 4.85 | .87  | 5.17 | 4.47 | 5.18 | 4.63 | 10.14 | .10 | .01 | .12 | ------- | 43.5 |
| M-421  | 84     | 18.02 | 4.06 | 1.81 | 3.45 | 1.71 | 0.0  | 1.87 | 9.88 | .10 | .02 | .17 | ------- | 6.2  |
| LV-E   | 85     | 7.07 | 4.87 | .98  | 3.66 | 3.48 | 3.86 | 4.08 | 10.13 | .11 | .01 | .10 | ------- | 59.8 |
| M-246  | 86     | 10.70 | 5.10 | 1.80 | 0.0  | 2.00 | 0.0  | 11.30 | .01 | .08 | ------- | 47.6 |
| X0-1   | 87     | 4.11 | 5.94 | .87  | 5.16 | 4.57 | 5.21 | 4.76 | 10.34 | .11 | .09 | -------- | 52.0 |
| U-700  | 88     | 15.07 | 4.43 | 3.43 | 0.0  | 4.07 | 0.0  | 0.0  | 15.58 | .01 | .02 | .07 | ------- | 27.9 |
| U-700  | 91     | 15.07 | 4.43 | 3.43 | 0.0  | 4.07 | 0.0  | 0.0  | 15.58 | .01 | .02 | .07 | ------- | 16.7 |
| IN-738 | 89     | 15.95 | 3.50 | 3.46 | 2.97 | 1.70 | 1.65 | .88  | 8.35 | .09 | .01 | .18 | ------- | .7  |
| B-1900 | 90     | 8.98 | 6.21 | 1.12 | 4.84 | 5.83 | 4.44 | .16  | 10.35 | .08 | .01 | .11 | ------- | 46.7 |
| TAZ-8A | 92     | 5.94 | 6.20 | 0.0  | 6.20 | 3.94 | 7.97 | 2.45 | 0.0  | .96 | .02 | .14 | ------- | 41.0 |
| IN-100 | 93     | 8.95 | 5.77 | 4.81 | 0.0  | 3.03 | 0.0  | 0.0  | 15.60 | .07 | .02 | .16 | 1.02 V | 54.9 |
| M-211  | 94     | 8.90 | 5.04 | 2.00 | 4.88 | 2.42 | 0.0  | 2.82 | 10.20 | .03 | .02 | .17 | ------- | 37.0 |
| R-125  | 95     | 8.90 | 4.70 | 2.60 | 7.00 | 1.90 | 3.70 | 0.0  | 9.90 | .07 | .01 | .01 | 1.6 Hf | 26.8 |
| TRD-R  | 96     | 8.10 | 6.53 | .83  | 4.00 | 2.76 | 6.30 | .32  | 7.95 | .12 | .02 | .13 | 1.07 Hf | 30.0 |

---

| W94    | 10.57 | 4.55 | 2.67 | 2.77 | 3.81 | 2.00 | 9.52 | .09 | .03 | .09 | ------- | ---- |
TABLE II. - REPLICATE TEST BAR RESULTS
SELECTED FROM TABLE I.

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TABLE III. - REGRESSION RESULTS FOR THE U-TRANSFORM \( U = \sin^{-1}(y/100)^{1/2} \) AS A FUNCTION OF ALLOY COMPOSITION IN wt % OF Al, Ti, Cr, Mo, Cb, Ta, W and Co, THEIR TWO FACTOR INTERACTIONS AND QUADRATIC EFFECTS AS WELL AS LINEAR EFFECTS OF C, Zr and B. NUMBER OF DATA VALUES, \( n = 78 \).

\[ U = \sin^{-1}(\%A/100)^{1/2}; \quad n = 78 \text{ Data Set} \]

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<tr>
<th>Coefficient</th>
<th>T-stat</th>
<th>% explained S.S.</th>
<th>Coefficient</th>
<th>T-stat</th>
<th>% explained S.S.</th>
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<td>( Z_f = 12 )</td>
<td>( Z_f = 12 + \text{Co} )</td>
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<td></td>
<td></td>
<td></td>
</tr>
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<td>+5.12</td>
<td>12.7</td>
<td>+0.03976</td>
<td>+5.07</td>
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<tr>
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<td>-9.88</td>
<td>56.6</td>
<td>-0.04799</td>
<td>-9.81</td>
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<td>-3.45</td>
<td>8.0</td>
<td>-0.03556</td>
<td>-3.43</td>
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<tr>
<td>( 4) ) Mo</td>
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<td>-2.82</td>
<td>6.0</td>
<td>-0.02677</td>
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</tr>
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<td>-5.43</td>
<td>1.4</td>
<td>-0.02628</td>
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<td>-0.02221</td>
<td>-3.90</td>
<td>8.6</td>
<td>-0.02217</td>
<td>-3.88</td>
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<tr>
<td>( 8) ) Co</td>
<td>-------</td>
<td>----</td>
<td>----</td>
<td>-------</td>
<td>----</td>
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<tr>
<td>( 9) ) Al·Mo</td>
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<td>2.2</td>
<td>+0.00134</td>
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<td>2.8</td>
<td>-0.00647</td>
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<tr>
<td>( 11) ) Al·Ta</td>
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<td>-1.98</td>
<td>2.0</td>
<td>-0.00609</td>
<td>-1.96</td>
</tr>
<tr>
<td>( 12) ) Cr·W</td>
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<td>2.8</td>
<td>+0.00305</td>
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<tr>
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<td>1.9</td>
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<td>+1.42</td>
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</tbody>
</table>

\( R^2 = 0.39625 \), S.E.E. = \( \pm 0.1100 \), Prob. = \( 0.90, F = 2.88 \)

\( 2 \) Initial = 47 terms with \( R^2 = 89.1 \% \) and S.E.E. = 0.1045; total sum of squares (i.e. variability) = 3.00070.
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<th>Alloy</th>
<th>Bar No.</th>
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<th>U-trans. OBS</th>
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<th>U-trans-EST Z = 12 + Co(13)</th>
<th>% AA EST.</th>
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TABLE IV. - REGRESSION RESULTS FOR THE U-TRANSFORM and % AA VALUES DERIVED FROM THE REGRESSION COEFFICIENTS IN TABLE THE n = 78, DATA SET.
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<tr>
<td>M-48</td>
<td>58</td>
<td>18.1</td>
<td>.4394</td>
<td>.4920</td>
<td>.4898</td>
<td>22.13</td>
</tr>
<tr>
<td>M-49</td>
<td>45</td>
<td>21.2</td>
<td>.4785</td>
<td>.4980</td>
<td>.4972</td>
<td>22.75</td>
</tr>
<tr>
<td>M-50</td>
<td>64</td>
<td>46.9</td>
<td>.7544</td>
<td>.6282</td>
<td>.6273</td>
<td>34.45</td>
</tr>
</tbody>
</table>

TABLE V. - ESTIMATES OF THE U-TRANSFORM AND %AA VALUES FOR THE
16 EXTRA RUNS BASED ON THE 13 REGRESSION COEFFICIENTS FROM
TABLE III DERIVED FROM THE n = 78 DATA SET.

<table>
<thead>
<tr>
<th>Bar No.</th>
<th>Alloy</th>
<th>%AA</th>
<th>U-trans.</th>
<th>U-trans-EST Z = 12 + Co(13)</th>
<th>%AA EST.</th>
</tr>
</thead>
<tbody>
<tr>
<td>81</td>
<td>TAZ-8A</td>
<td>38.5</td>
<td>0.6694</td>
<td>0.6003</td>
<td>31.91</td>
</tr>
<tr>
<td>82</td>
<td>U-710</td>
<td>1.9</td>
<td>.7202</td>
<td>.6247</td>
<td>34.20</td>
</tr>
<tr>
<td>83</td>
<td>X1-A</td>
<td>43.5</td>
<td>.7202</td>
<td>.6247</td>
<td>34.20</td>
</tr>
<tr>
<td>84</td>
<td>MM-421</td>
<td>6.2</td>
<td>.2516</td>
<td>.1685</td>
<td>2.81</td>
</tr>
<tr>
<td>85</td>
<td>IV-E</td>
<td>59.8</td>
<td>.8840</td>
<td>.6324</td>
<td>34.93</td>
</tr>
<tr>
<td>86</td>
<td>MM-246</td>
<td>47.6</td>
<td>.7614</td>
<td>.7825</td>
<td>49.71</td>
</tr>
<tr>
<td>87</td>
<td>XD-1</td>
<td>52.0</td>
<td>.6054</td>
<td>.6426</td>
<td>35.93</td>
</tr>
<tr>
<td>88</td>
<td>U-700</td>
<td>27.9</td>
<td>.5565</td>
<td>.3591</td>
<td>12.35</td>
</tr>
<tr>
<td>91</td>
<td>U-700</td>
<td>16.7</td>
<td>.4210</td>
<td>.3591</td>
<td>12.35</td>
</tr>
<tr>
<td>89</td>
<td>IN-738</td>
<td>7</td>
<td>.0838</td>
<td>.1707</td>
<td>2.89</td>
</tr>
<tr>
<td>90</td>
<td>B-1900</td>
<td>46.7</td>
<td>.7524</td>
<td>.6796</td>
<td>39.50</td>
</tr>
<tr>
<td>92</td>
<td>TAZ-8A</td>
<td>41.0</td>
<td>.6949</td>
<td>.4701</td>
<td>20.52</td>
</tr>
<tr>
<td>93</td>
<td>In-100A</td>
<td>54.9</td>
<td>.8345</td>
<td>.8226</td>
<td>53.72</td>
</tr>
<tr>
<td>94</td>
<td>MM-211</td>
<td>37.0</td>
<td>.6539</td>
<td>.7997</td>
<td>51.43</td>
</tr>
<tr>
<td>95</td>
<td>R-125</td>
<td>26.8</td>
<td>.5441</td>
<td>.5046</td>
<td>23.38</td>
</tr>
<tr>
<td>96</td>
<td>TRW-R</td>
<td>30.0</td>
<td>.5796</td>
<td>.6295</td>
<td>34.66</td>
</tr>
</tbody>
</table>

aV content not factored in.
TABLE VI. - REGRESSION RESULTS FOR THE U-TRANSFORM \( U = \sin^{-1}(%A/100)^{1/2} \)
AS A FUNCTION OF ALLOY COMPOSITION IN wt % OF Al, Ti, Cr, Mo, Cb, Ta, W AND Co, THEIR TWO FACTOR INTERACTIONS AND QUADRATIC EFFECTS AS WELL AS LINEAR EFFECTS OF C, Zr and B. NUMBER OF DATA VALUES, \( n = 94 \).

\[
U = \sin^{-1}\left(\frac{\%A}{100}\right)^{1/2}; \quad n = 94 \text{ Data Set}
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>T-stat</th>
<th>% explained S.S.</th>
<th>Coefficient</th>
<th>T-stat</th>
<th>% explained S.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Al</td>
<td>+0.04470</td>
<td>+5.91</td>
<td>+0.04372</td>
<td>+5.85</td>
<td>8.5</td>
</tr>
<tr>
<td>2) Cr</td>
<td>-0.04590</td>
<td>-10.78</td>
<td>-0.04587</td>
<td>-10.93</td>
<td>60.0</td>
</tr>
<tr>
<td>3) Ti</td>
<td>-0.04409</td>
<td>-5.09</td>
<td>-0.05012</td>
<td>-5.47</td>
<td>9.7</td>
</tr>
<tr>
<td>4) Mo</td>
<td>-0.02142</td>
<td>-2.67</td>
<td>-0.02266</td>
<td>-2.82</td>
<td>2.8</td>
</tr>
<tr>
<td>5) W</td>
<td>-0.02803</td>
<td>-6.53</td>
<td>-0.02563</td>
<td>-5.79</td>
<td>1.7</td>
</tr>
<tr>
<td>6) Cb</td>
<td>-0.01333</td>
<td>-1.70</td>
<td>-0.01831</td>
<td>-4.06</td>
<td>1.1</td>
</tr>
<tr>
<td>7) Ta</td>
<td>-0.02085</td>
<td>-4.20</td>
<td>-0.02079</td>
<td>-4.25</td>
<td>7.4</td>
</tr>
<tr>
<td>8) Co</td>
<td></td>
<td></td>
<td>+0.00829</td>
<td>+1.83</td>
<td>2.2</td>
</tr>
<tr>
<td>9) Al+Mo</td>
<td>-0.01178</td>
<td>-2.34</td>
<td>-0.01220</td>
<td>-2.46</td>
<td>8.8</td>
</tr>
<tr>
<td>10) Al+W</td>
<td>-0.00739</td>
<td>-2.94</td>
<td>-0.00721</td>
<td>-2.91</td>
<td>2.9</td>
</tr>
<tr>
<td>11) Al+Ta</td>
<td>-0.00694</td>
<td>-2.53</td>
<td>-0.00649</td>
<td>-2.39</td>
<td>1.9</td>
</tr>
<tr>
<td>12) W+Co</td>
<td>-0.00164</td>
<td>-1.88</td>
<td>-0.00075</td>
<td>-1.76</td>
<td>0.8</td>
</tr>
<tr>
<td>13) Ti²</td>
<td>+0.02038</td>
<td>+2.26</td>
<td>+0.02710</td>
<td>2.81</td>
<td>1.1</td>
</tr>
<tr>
<td>0</td>
<td>.37949</td>
<td></td>
<td>.36994</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \sum \) = 4 terms with \( R^2 = 88.1 \) percent and S.E.E. = \( 0.1071 \); total sum of squares (i.e. variability) = 4.44332.

TABLE VII. - ESTIMATES OF THE U-TRANSFORM AND % ΔA VALUES FOR THE 16 EXTRA RUNS BASED ON THE 13 REGRESSION COEFFICIENTS FROM TABLE VI DERIVED FROM THE \( n = 94 \) DATA SET

<table>
<thead>
<tr>
<th>Bar No.</th>
<th>Alloy</th>
<th>% ΔA OBS</th>
<th>U-trans. OBS</th>
<th>U-trans-EST Z = 12 + Co (13)</th>
<th>% ΔA-EST Z = 12 + Co (13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>81</td>
<td>TAZ-BA</td>
<td>38.5</td>
<td>0.6694</td>
<td>0.7336</td>
<td>44.83</td>
</tr>
<tr>
<td>82</td>
<td>U-710</td>
<td>1.9</td>
<td>.1383</td>
<td>.1232</td>
<td>1.51</td>
</tr>
<tr>
<td>83</td>
<td>X1-A</td>
<td>43.5</td>
<td>.7202</td>
<td>.7102</td>
<td>42.51</td>
</tr>
<tr>
<td>84</td>
<td>MM-421</td>
<td>6.2</td>
<td>.2516</td>
<td>.1830</td>
<td>3.31</td>
</tr>
<tr>
<td>85</td>
<td>LV-E</td>
<td>59.8</td>
<td>.6804</td>
<td>.6821</td>
<td>39.74</td>
</tr>
<tr>
<td>86</td>
<td>MM-246</td>
<td>47.6</td>
<td>.7614</td>
<td>.7630</td>
<td>47.76</td>
</tr>
<tr>
<td>87</td>
<td>XD-1</td>
<td>52.0</td>
<td>.8054</td>
<td>.7308</td>
<td>44.55</td>
</tr>
<tr>
<td>88</td>
<td>U-700</td>
<td>27.9</td>
<td>.5565</td>
<td>.3995</td>
<td>15.13</td>
</tr>
<tr>
<td>91</td>
<td>U-700</td>
<td>16.7</td>
<td>.4210</td>
<td>.3995</td>
<td>15.13</td>
</tr>
<tr>
<td>89</td>
<td>IN-738</td>
<td>.7</td>
<td>.0838</td>
<td>.1294</td>
<td>1.67</td>
</tr>
<tr>
<td>90</td>
<td>B-1900</td>
<td>46.7</td>
<td>.7524</td>
<td>.6999</td>
<td>41.39</td>
</tr>
<tr>
<td>92</td>
<td>TAZ-BA</td>
<td>41.0</td>
<td>.6949</td>
<td>.6636</td>
<td>37.94</td>
</tr>
<tr>
<td>93</td>
<td>IN-100</td>
<td>54.9</td>
<td>.8345</td>
<td>.8718</td>
<td>58.60</td>
</tr>
<tr>
<td>94</td>
<td>MM-211</td>
<td>37.0</td>
<td>.6539</td>
<td>.7407</td>
<td>45.54</td>
</tr>
<tr>
<td>95</td>
<td>R-125</td>
<td>26.8</td>
<td>.5441</td>
<td>.4482</td>
<td>18.78</td>
</tr>
<tr>
<td>96</td>
<td>TRW-R</td>
<td>30.0</td>
<td>.5796</td>
<td>.6661</td>
<td>38.18</td>
</tr>
</tbody>
</table>

\( \text{aV content not factored in.} \)
TABLE VIII. - ANALYSIS OF VARIANCE (ANOVA) SUMMARY FOR
n = 94 DATA SET; Zf = 13 SHOWING SOURCES
OF VARIATION INCLUDING LACK OF FIT
OF THE ESTIMATING EQUATION

<table>
<thead>
<tr>
<th>Source</th>
<th>d.f.</th>
<th>Sum of squares</th>
<th>Mean squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>13</td>
<td>3.5254154</td>
<td>0.27118580</td>
</tr>
<tr>
<td>Residual</td>
<td>80</td>
<td>.91790306</td>
<td>.011473788</td>
</tr>
<tr>
<td>lack of fit(a)</td>
<td>(65)</td>
<td>(.83804974)</td>
<td>(.01289307)</td>
</tr>
<tr>
<td>replication</td>
<td>(15)</td>
<td>(.07985332)</td>
<td>(.00532355)</td>
</tr>
<tr>
<td>Total</td>
<td>93</td>
<td>4.4433185</td>
<td></td>
</tr>
</tbody>
</table>

\[ F\text{-ratio}^b = \frac{\text{MS(LOF)}}{\text{MS(REPS)}} = \frac{0.01289307}{0.00532355} = 2.422 \]

S.E.E = \((\text{MSResidual})^{1/2}\) = \(0.1071\);
Rep. STD. Deviation = \((\text{MS rep})^{1/2}\) = \(0.0730\)

\(a\)The lack of fit term appears to be significant so an attempt to improve the model and/or drop some outliers would be in order.

\(b\)This F-ratio > 2.15 for (1-\(\alpha\)) = 0.95 but < 3.04 for (1-\(\alpha\)) = .99

TABLE IX. - APPROXIMATE OPTIMAL COMPOSITION LEVELS FOR HOT CORROSION RESISTANCE FOR Ni-BASE TURBINE ALLOYS AT FIXED LEVELS OF Cr AND Co
[Total refractory metal level (W + Cb + Ta + Mo) not to exceed 20 wt %. Coefficients used from Table VII, \(N = 94\) SET, \(Z_f = 13\).]

<table>
<thead>
<tr>
<th>Fixed Cr level</th>
<th>Fixed Co level</th>
<th>Optimum Levels(^a) for minimum attack</th>
<th>Estimate U-trans.</th>
<th>Attack % ΔA</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>16</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>10</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>10</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>10</td>
<td>10</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

\(a\)Levels scanned: W - 0, 5, 10; Cb - 0, 2.5, 5; Ti - 0, 1, 2, 3, 4, 5; Mo - 0, 2.5, 5; Ta - 0, 5, 10; Al - 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7. (5313 combinations scanned for minimum U-trans. at each fixed Cr, Co level.)
Figure 1. - Derived U-transform from the reduced 13-term estimating equation vs. U-observed for the \( n = 78 \) set. Vector lines represent the perfect fit line ± one standard deviation.

Figure 2. - Plot of residuals from the regression fit in figure 1. Symbols vary randomly about 0 as a function of the U-transform estimated.
Figure 3. - %ΔA values obtained from the derived U-transform values shown in figure 1 plotted against the original %ΔA observed values. Vector lines represent the perfect fit line ± one average deviation.

Figure 4. - Derived U-transform from the reduced 13-term estimating equation from the n = 78 vs U-observed values for bar numbers 81 to 96 to test the predictability of the regression equation. Vector lines represent the perfect line ± one standard deviation.
Figure 5. - %ΔA values obtained from the derived U-transform values shown in figure 4 plotted against the %ΔA observed values for bar numbers 81 to 96. Vector lines represent the perfect fit line ± one average deviation.

Figure 6. - Derived U-transform from the reduced 13-term estimating equations derived from the n = 78 and n = 94 data sets, respectively, vs the U-observed values for bar numbers 81 to 96. Vector lines represent the perfect fit line ± one standard deviation from the n = 78 data set.
Figure 7. - %ΔA values obtained from the derived U-transform values comparing the estimates plotted in figure 5 (shown as circles for the n = 78 data set) and the %ΔA values from figure 6 (the n = 94 data set plotted against the %ΔA observed values for bar numbers 81 to 96). Vector lines represent the perfect fit line ± one average deviation from the n = 78 data set.
Multiple Linear regression analysis was used to determine an equation for estimating hot corrosion attack for a series of Ni-base cast turbine alloys. The U-transform (i.e., $\sin^{-1}(\%A/100)^{1/2}$) was shown to give the "best" estimate of the dependent variable, y. A complete second degree equation is described for the "centered" weight chemistries for the elements Cr, Al, Ti, Mo, W, Cb, Ta, and Co. In addition linear terms for the minor elements C, B, and Zr were added for a basic 47-term equation. The "best" reduced equation was determined by the stepwise selection method with essentially 13 terms. The Cr term was found to be the most important accounting for 60 percent of the explained variability hot corrosion attack.
End of Document