DM N85-32483

2.3A A MODIFIED FRESNEL SCATTERING MODEL FOR THE PARAMETERIZATION OF FRESNEL RETURNS

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ABSTRACT

This paper presents a modified Fresnel scatter model and compares the revised model with observations from the Poker Flat, Alaska, radar, the SOUSY radar and the Jicamarca radar. The modifications to the original model have been made to better account for the pulse-width dependence and height dependence of backscattered power observed at vertical incidence at lower VHF. Vertical profiles of backscattered power calculated using the revised model and routine radiosonde data show good agreement with observed backscattered power profiles. Relative comparisons of backscattered power using climatological data for the model agree fairly well with observed backscattered power profiles from Poker Flat, Jicamarca and SOUSY.

INTRODUCTION

Since the first observations at vertical incidence of enhanced specular-like echoes at lower VHF (GAGE and GREEN, 1978; ROTTGER and LIU, 1978), there has been considerable debate and controversy concerning the nature of the scattering/reflection mechanism responsible for these echoes. As discussed in GAGE and BALSLEY (1980), the enhanced echoes obtained at vertical incidence by VHF radars might be attributed to some combination of anisotropic turbulence scatter, Fresnel reflection from regions of strong refractive index gradients, and from Fresnel scattering from a volume filled with stable laminae of radio refractive index coherent transverse to the probing wave. Subsequently, GAGE et al. (1981) presented a model for Fresnel scattering which enabled the parameterization of radar echoes using ordinary rawinsonde data as input. While this model seemed to simulate the dependence of echo magnitude with stability, it did not properly account for the pulse-width dependence of the observed echoes (HOCKING and ROTTGER, 1983; GREEN et al., 1983). Also, it now appears that the height-dependence (BALSLEY and GAGE, 1981) was not properly accounted for in the original model. This paper presents a modified Fresnel scattering model which takes into account the contributions of FARLEY (1983), LIU (1983) and DOVIAK and ZRNIC (1984) as well as HOCKING and ROTTGER (1983) and hopefully corrects these deficiencies. The revised model is compared to observations of the Poker Flat, Alaska MST radar as well as the SOUSY radar located in the Federal Republic of Germany and the Jicamarca radar located in Peru.

A MODIFIED FRESNEL SCATTER MODEL

As originally presented in GAGE et al. (1981) the Fresnel scatter model assumed a coherent scattering from the half-wavelength Fourier component of the refractivity structure directed along the beam. This procedure was justified by GAGE et al. (1981) in order to account for a pulse length squared $(\Delta r)^2$ dependence of backscattered power which was observed in early VHF radar studies of specular echoes (GREEN and GAGE, 1980). Subsequent studies, however, have shown that a $(\Delta r)^1$ dependence is more typical (GREEN et al., 1983). Theoretical arguments in favor of a linear dependence on Δr have been advanced by HOCKING and ROTTGER (1983), LIU (1983) and FARLEY (1983). Clearly, the Fresnel scattering model must be modified to account for this.

Under conditions of stable stratification gradients of radio refractivity tend to be concentrated in thin layers that are horizontally coherent. Under these circumstances it is appropriate to consider the backscattered power P_r that arises from a partial reflection process. This is given by the radar equation

$$P_r = \frac{\alpha^2 P_T A_e^2}{4 r^2 \lambda^2} |\rho|^2 \tag{1}$$

where P_T is transmitted power (per pulse), A_e is effective antenna area, α is an efficiency factor, r is the range to the target, λ is radar wavelength and $|\rho|^2$ is a power reflection coefficient which depends upon the refractivity structure in the volume of the atmosphere being observed. The conventional approach to partial or Fresnel reflection is to determine the reflection coefficient deterministically based on a particular gradient structure. Since the detailed structure of individual layers is unknown, this approach is of limited value for the real atmosphere. More importantly, if the backscattered power were due to a single thin layer, there would be no Δr-dependence. The implication of the observed increase in backscattered power with Δr is clearly that the volume is filled with many thin layers. Furthermore, the reflectivity of this layered structure is greatly enhanced in hydrostatically very stable regions of the atmosphere.

The reflection coefficient pertinent to a medium filled with horizontally layered structure can be determined in the context of a one-dimensional scattering problem following LIU (1983) and FARLEY (1983). The voltage reflection coefficient ρ is given by

$$\rho = \frac{1}{2}\int_{-\ell/2}^{\ell/2} \frac{dn}{dz}\, e^{-2kiz}\, dz = ik \int_{-\ell/2}^{\ell/2} n(z) e^{-2ikz}\, dz \tag{2}$$

where ℓ is the thickness of the region of layered structure and $k(= 2\pi/\lambda)$ is radar wavelength.

When the layered structure which gives rise to the specular echoes is the result of vertical displacement due to either turbulence or waves, it is appropriate to replace n in Equation (2) by the generalized potential refractive index $\mathcal{n}$ (OTTERSTEN, 1969). Then, Equation (2) can be re-expressed as

$$\rho = \frac{1}{2}\int_{-\ell/2}^{\ell/2} Me^{-2kiz}\, dz = ik \int_{-\ell/2}^{\ell/2} \mathcal{n} e^{-2ikz}\, dz = ik \int_{-\ell/2}^{\ell/2} \Delta\mathcal{n}\, e^{-2ikz}\, dz \tag{3}$$

where $M = d\mathcal{n}/dz$ is the gradient of generalized refractive index and Δn is the fluctuating component of refractivity. The power reflection coefficient pertinent to a radar observing at vertical incidence with a probing pulse of length Δr is

$$|\rho|^2 = k^2 \int_{-\Delta r/2}^{\Delta r/2} dz_1 \int_{-\Delta r}^{\Delta r} d(z_1 - z_2)\ \langle \Delta\mathcal{n}(z_1)\Delta\mathcal{n}(Z_2)\rangle\ e^{-2ik(z_1 - z_2)} \tag{4}$$

which leads to

$$|\rho|^2 = k^2 \cdot \Delta r \cdot \Phi_{\mathcal{n}}(2k)$$

where $\Phi_{\mathcal{n}}(k)$ is the vertical wave number spectrum of generalized potential radio refractive index

$$\Phi_n(k) = \int_{-\infty}^{+\infty} d(z_1 - z_2) \langle \Delta n(z_1)\ \Delta n(z_2)\rangle\ e^{-2ik(z_1 - z_2)} \tag{6}$$

For purposes of modeling the backscattered power in a way that can be evaluated from meteorological data it is convenient to introduce the mean gradient of generalized potential index of radio refractive index

$$M \equiv -77.6 \times 10^{-6}\ \frac{P}{T}\ \left(\frac{\partial\ \ln\ \theta}{\partial z}\right)$$

$$\times \left[1 + \frac{15{,}500q}{T}\ \left(1 - \frac{1}{2}\ \frac{\partial\ \ln\ q/\partial z}{\partial\ \ln\ \theta/\partial z}\right)\right] \tag{7}$$

where P is atmospheric pressure in millibars, T is absolute temperature, θ is potential temperature [$\theta \equiv T(1000/P)^{2/7}$], and q is specific humidity. M(z) and $\Phi_n(k)$ are related through the spectrum of vertical displacements $E_\zeta(k)$ (VANZANDT and VINCENT, 1983).

$$\Phi_n(k) = E_\zeta(k)\ \overline{M}^2\ . \tag{8}$$

Combining Equation (8) with Equation (5) we find

$$|\rho|^2 = K^2 \cdot \Delta r \cdot E_\zeta(2k)\ \overline{M}^{\,2} \tag{9}$$

and defining

$$F(2k;z)^2 \equiv 4k^2 \cdot E_\zeta(2k,z) \tag{10}$$

we obtain

$$|\rho|^2 = \frac{1}{4} \cdot \Delta r \cdot F(2k,z)\ \overline{M}^2\ . \tag{11}$$

For generality in Equation (10) and Equation (11) we have indicated an altitude dependence for F and E_ζ. Substitution of Equation (11) in Equation (1) leads to the backscattered power

$$P_r = \frac{\alpha^2\ P_t\ A_e^{\ 2}\ \Delta r}{16\ r^2\ \lambda^2}\quad [\overline{M}\ F(2k,z)]^2 \tag{12}$$

The height dependence of P_r can tell us much about the height dependence of F and E_ζ. To investigate the height dependence of P_r, it is necessary to consider the height dependence of each factor in Equation (12). Since

$$P_r \propto \frac{\overline{M}^2\ E_\zeta}{r^2} \propto \frac{[\rho_{air}(z)]^2 E_\zeta}{r^2} \tag{13}$$

it is clear that the height dependence of P_r is dominated by the $\rho_{air}{}^2(z)$ factor implicit in $\overline{M}^2$.

If $E_\zeta(k)$ were due to a spectrum of vertically propagating waves that conserved their energy, it would follow that $E_\zeta \propto \rho_{air}^{-1}$ and $P_r \propto \rho_{air}(z)/r^2$. If however, the growth with altitude of E_ζ were limited by some saturation process, E_ζ would become independent of altitude and $P_r \propto \rho_{air}{}^2(z)/r^2$.

BALSLEY and GAGE (1981) explored the height dependence of P_r and found that it decreased more rapidly than $\overline{M}^2/z^2$. Indeed, they reported the observed $P_r \propto \overline{M}^2/z^4$ and suggested that this might be the result of reflection from structures limited in their horizontal dimensions (by their horizontal coherency length) to a fraction of Fresnel zone. As pointed out by DOVIAK and ZRNIC (1984) there is no reason for the horizontal dimensions of the reflecting region to be limited by the horizontal coherency length. If, indeed, the height dependence observed by BALSLEY and GAGE (1981) is real, it must be explained in some other way. One way to explain this height dependence would be to account for it in the height dependence of $E_\zeta(k,z)$. In what follows we have supposed that

$$E_\zeta(k,z) \propto e^{-z/z_*} \tag{14}$$

where z_* is a scale-height. Up to about 20 km the exponential fall-off approximates z^{-2}, but at higher altitudes the exponential fall-off is increasingly more rapid. The exponential dependence has been adopted here because it fits the data very well, especially at the higher heights, as will be shown below.

An exponential fall-off of $E_\zeta(k)$ is not easily explained, however at the meter scales pertinent to VHF radar studies it would not be surprising if viscous damping and turbulence processes played an important role. In the remainder of this paper we adopt the exponential dependence of $E_\zeta(k,z)$ and set

$$F(2k,z)^2 = F_1{}^2(\lambda)e^{-z/z_*} \tag{15}$$

where $F_1(\lambda)$ corresponds to the value of "F" used in GAGE et al. (1981). With this change we obtain the backscattered power

$$P_r = \frac{\alpha^2 P_t A_e{}^2 \Delta r}{16 r^2 \lambda^2} \quad [\overline{M} F_1(\lambda)]^2 e^{-z/z_*} \tag{16}$$

for the modified Fresnel scattering model.

Following GAGE and GREEN (1978) it is convenient to define a normalized received power S_v by

$$S_v \equiv 10^{17} P_r(\text{watts}) \left\{\frac{r(\text{m})}{10^3\text{m}}\right\}^2 \tag{17}$$

which when combined with Equation (16) leads to

$$S_v = 10^{17} \frac{\alpha^2 P_T A_e{}^2}{16 \lambda^2 10^3\text{m}} \left(\frac{\Delta r}{10^3\text{m}}\right) \{M F(\lambda)\}^2 e^{-z/z_*} \tag{18}$$

In the following section we compare S_v calculated according to Equation (15) with observed values of S_v.

COMPARISON OF SOME POKER FLAT MST RADAR OBSERVATIONS WITH THE MODIFIED FRESNEL SCATTERING MODEL

The calculated values of S_v presented here have been made under the assumption that the generalized potential refractive index M is determined in the upper troposphere and stratosphere by its dry part:

$$M_d = -77.6 \times 10^{-6} \frac{P}{T} \frac{\partial \ln \theta}{\partial z} \tag{19}$$

where P is atmospheric pressure in mb, T is absolute temperature in Kelvins, θ is potential temperature in Kelvins and z is altitude in meters. M_d is easily calculated for each significant level of the radiosonde soundings. Typically, there are several significant levels for each radar range gate. The individual values of M_d are then weighted according to the thickness of each significant level and averaged together to give $\overline{M_d}$ corresponding to each range gate. Of course, the larger the range gate the more effective will be the smoothing of the $\overline{M_d}(z)$ profile. The data reported in this section were taken at Poker Flat, Alaska in the fall of 1979 when one-quarter section (100 m x 100 m) of the final antenna was phased to look vertically. During this period a single transmitter was used with peak power of about 55 kW. Most of the data were obtained with 15 μs pulses (Δr = 2.25 km) but a limited sample of data was also taken with 5 μs pulses (Δr = .75 km).

A comparison of modeled and observed S_v for the Poker Flat MST radar is contained in Figures 1 through 4. The model calculations make use of the routine radiosonde soundings at Fairbanks which are launched about 40 km from the MST radar site at Poker Flat. The model profiles were calculated using $F_1(\lambda) = 8 \times 10^{-3}$ and $\alpha = 0.5$ in Equation (15).

Figure 1 contains a comparison of the modified Fresnel scattering model with Poker Flat radar observations taken on 13 September 1979 with .75 km height resolution. The radar observations are an average of two hours of data which bracket the time of the balloon launch in Fairbanks. The overall agreement in the magnitude and shape of the two profiles is excellent. We do not attach too much significance to the agreement in the upper troposphere (in this case below about 10 km) since we expect that, in the less stable environment of the troposphere, turbulent processes are dominant.

A second example of the agreement between modeled and observed profiles of S_v for the Poker Flat radar using .75 km height resolution is contained in Figure 2. In this case the overall agreement in magnitude is not quite so good but the agreement in shape of the two profiles is excellent. Even some of the detailed structure evident in the observed profile is reproduced in the model profile.

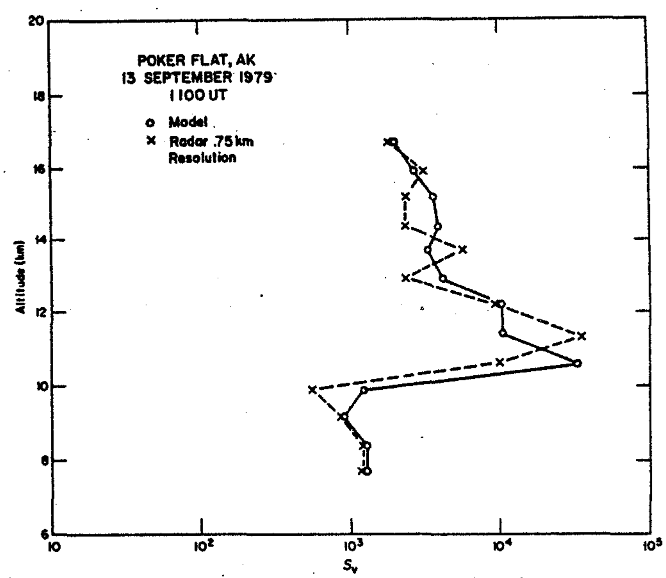

Figure 1. Comparison of calculated and observed fine-resolution profiles of normalized backscattered power for 13 September 1979.

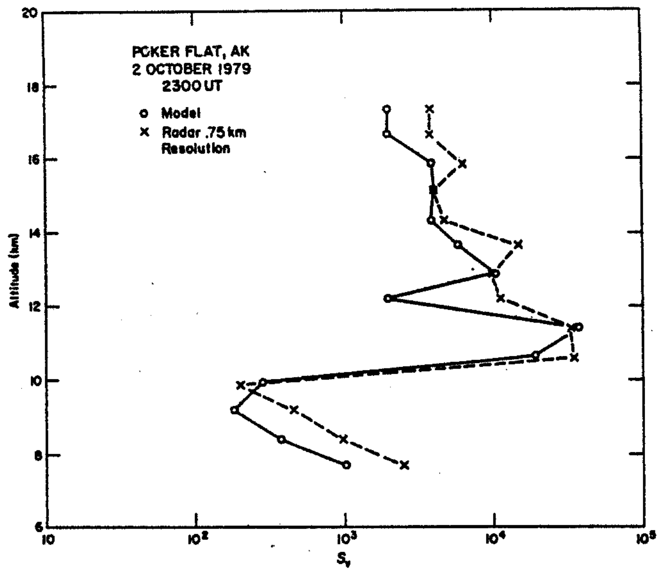

Figure 2. Comparison of calculated and observed fine-resolution profiles of normalized backscattered power for 2 October 1979.

Only a very limited sample of data was available at .75 km resolution from the Poker Flat MST radar during the Fall of 1979. However, nearly continuous coarse-resolution data were available during this period (2.25 km resolution). Two examples of comparisons obtained during this period are contained in Figure 3 and Figure 4. The most obvious change in comparison to the fine resolution profiles is the increase in backscattered power. The change in Δr in the modified version of the model accounts very well for this observed change in backscattered power. The coarse resolution profiles show low values of backscattered power in the troposphere with a significant increase in the lower stratosphere followed by a systematic decrease with altitude due mainly to decreasing atmospheric density. Both comparisons show very good agreement with the modified Fresnel scattering model. Note that the tropopause on 6 November 1979 was unusually high. Also note that both model and observed coarse-resolution profiles contain much less structure than is found in the fine-resolution profiles.

CLIMATOLOGICAL MODEL COMPARISONS WITH OBSERVATIONS

The modified Fresnel scatter model can be used with climatological data to examine the altitude dependence of this process. This procedure is similar to that followed in BALSLEY and GAGE (1981) but here we use a different approach to the height dependence as explained above. Figure 5 contains the results of these model calculations.

Figure 5a compares the relative backscattered power calculated using our modified ("exponential") Fresnel scatter model with a set of observations made in October-November 1979 using the Poker Flat, Alaska, MST radar (BALSLEY et al., 1980) (note that all comparisons with observtions shown in this section are relative). The model profile is calculated using 60°N climatological data (U.S. Standard Atmosphere Supplement, 1980). The agreement is comparable to that found by BALSLEY and GAGE (1981) using an assumed z^{-4} dependence.

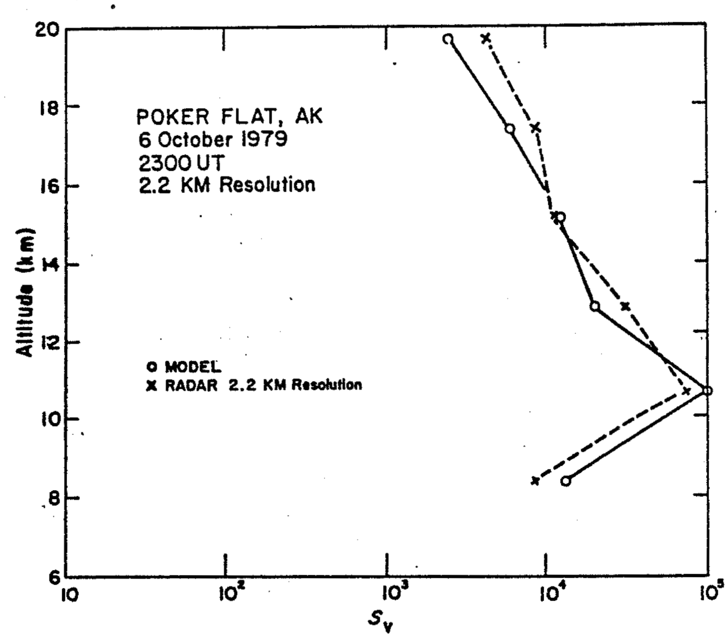

Figure 3. Comparison of calculated and observed coarse-resolution profiles of normalized backscattered power for 6 October 1979.

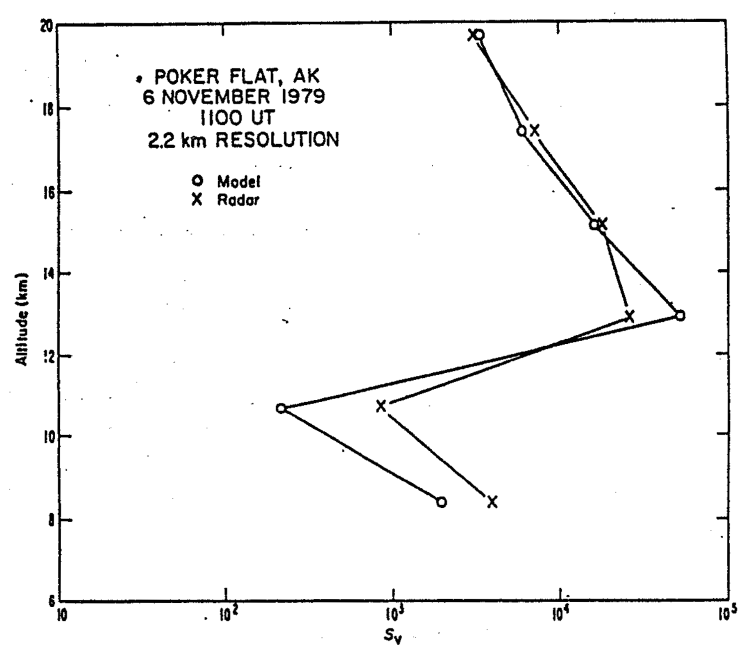

Figure 4. Comparison of calculated and observed coarse-resolution profiles of normalized backscattered power for 6 November 1979.

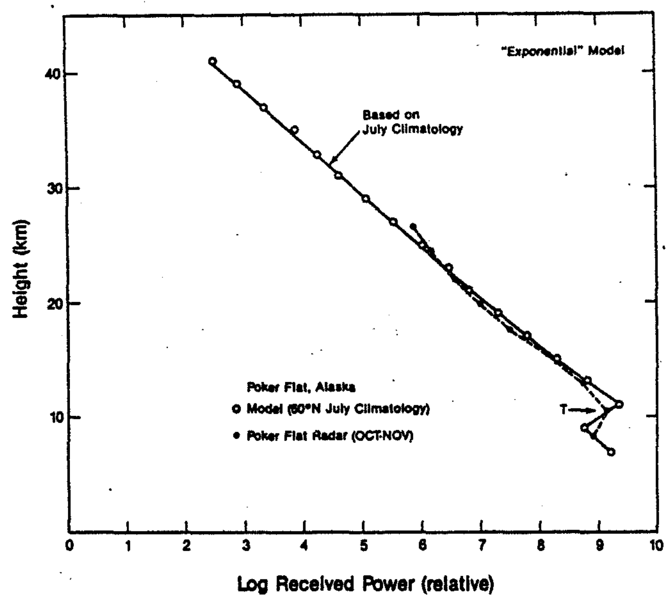

Figure 5(a).

Figure 5. Comparisons of observed profiles of backscattered power from three radars with theoretical profiles calculated from the U. S. Standard Atmosphere Supplement (1966): (a) Poker Flat, Alaska MST radar (b) Jicamarca radar (BALSLEY, 1978), and (c) SOUSY radar, Federal Republic of Germany (RUSTER et al., 1980).

Shown in Figure 5b is a comparison of the modeled backscattered power profile calculated using climatological data for 15°N with observations of the Jicamarca radar located at 12°S near Lima, Peru. The overall fit of the new model profile to the observed backscattered power profile is a considerable improvement over the old model (BALSLEY and GAGE, 1981). This result is especially significant since the observations in this case cover most of the stratosphere. As a third example we compare in Figure 5c a climatological calculation for 45°N with observations from the SOUSY radar (RUSTER et al., 1980). Again, the comparison is significantly improved over the earlier results.

CONCLUSIONS

In this paper we have formulated a modified Fresnel scattering model which permits the parameterization of VHF radar echoes obtained at vertical incidence in terms of routinely measured atmospheric parameters. It is assumed the radar observing volume is filled by a collection of transversely coherent, horizontally stratified, stable laminae of radio-index of refraction. The revised model assumes that the layered structure is randomly distributed as discussed in HOCKING and ROTTGER (1983). It also assumes an exponential decrease with altitude in the magnitude of the vertical displacement spectrum. While the exponential decrease in the vertical displacement spectrum is somewhat surprising, it is not too unreasonable at the meter scales pertinent to VHF radar probing since at these scales turbulence and viscous effects are expected to be important. With these changes the Fresnel scattering model simulates very well the observed profiles of backscattered power as seen by several MST radars.

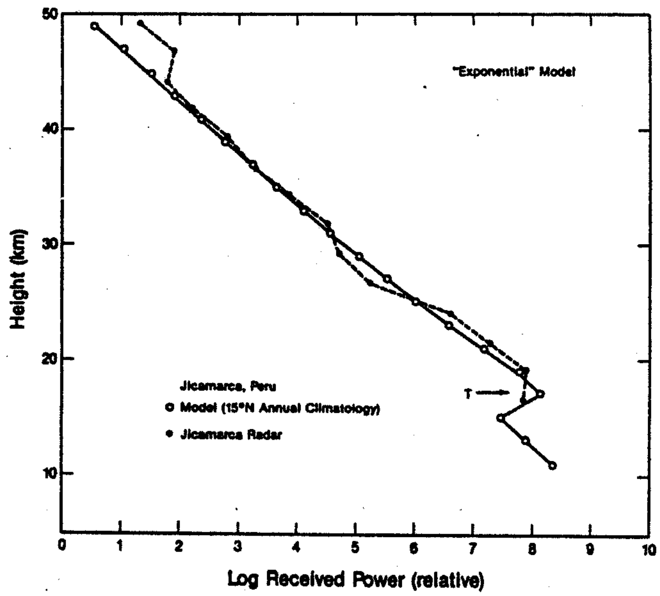

Figure 5(b).

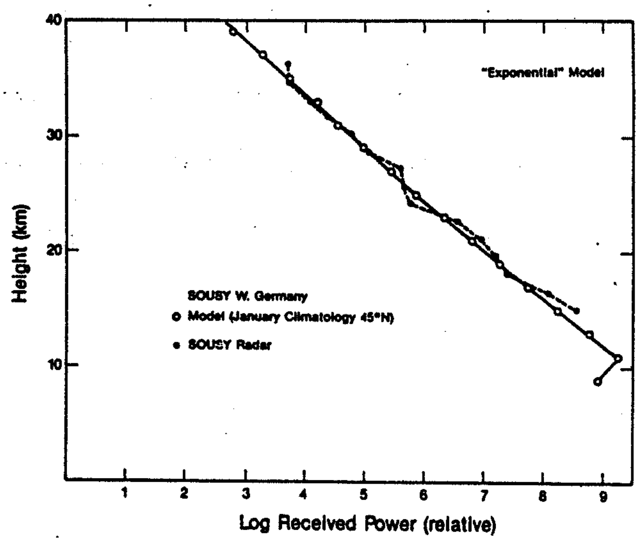

Figure 5(c).

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