PARAMETRIZED ENERGY SPECTRUM OF COSMIC-RAY PROTONS WITH KINETIC ENERGIES DOWN TO 1 GeV

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A new estimation of the interstellar proton spectrum is made in which the source term of primary protons is taken from shock acceleration theory and the cosmic-ray propagation calculation is based on our proposed nonuniform galactic disk model.

It appears that above 10 GeV the interstellar proton spectrum can be determined with an absolute accuracy of ±20%. However, at lower energies the divergence among various spectral estimations is much more serious; this is due to the existence of solar modulation. Thus one needs to deduce the interstellar proton spectrum through his own demodulation calculation. The deduced proton spectrum is obviously dependent upon the model of solar modulation used.

Recently, an interesting attempt for deducing the interstellar proton spectrum is reported(1), in which a source term of primary protons suggested by shock acceleration theory is adopted and the cosmic-ray propagation calculation is based on the leaky box model. Since from our recent analysis of the high-energy electron spectrum(2) we are aware that the dominant part of observed protons comes from the dense H\textsubscript{2} cloud region, it is argued that the corresponding propagation calculation should be based on the nonuniform galactic disk (NUGD) model(3).

Here we will present a brief introduction to the NUGD model. According to it the observed cosmic rays contain two components: the distant component and the local component. As shown in Fig. 1 by the double-line arrows, the distant (left) component of cosmic rays, starting from the cosmic-ray confinement volume above the H\textsubscript{2} cloud region (Box 2) can reach the solar neighbourhood by escaping into the H\textsubscript{2} cloud region (Box II), then by propagating along the magnetic tube (Box I) inside which the solar system is located. On the other hand, the local (right) component of cosmic rays originates from their confinement volume in the solar vicinity (Box I). The fraction, $f$, of locally produced protons in the observed proton flux is found to be only $5\pm1\%$(2).

The main problem in extending the application range of the NUGD model to the low energy range is caused by a lack of knowledge of the convection velocity $(V)$ of cosmic rays. Thus at low energies one is unable to determine a precise shape of the cosmic-ray escape pathlength $(\lambda)$ in Box II or to estimate the cosmic-ray intensity variation along the magnetic tube (Box I). Nevertheless, it is noted that if we limit ourselves to interstellar protons with kinetic energy, $T \geq 1$ GeV, to a great extent we can avoid the difficulties shown above. Actually, the proton intensity in the H\textsubscript{2} cloud region (Box II) should be the same as in...
Box 1 or Box 2, because it is found experimentally that there is a lack of any cosmic-ray gradient, at least in the inner Galaxy (see Ref. 4). Note that at any given rigidity (R) protons are the dominant component of cosmic rays. Hence we do not need to use the unknown parameter $\lambda_{\text{eII}}$ to estimate the proton intensity in Box II (hereafter the subscripts 1, 2, I and II represent the quantities referred to Boxes 1, 2, I and II respectively).

Regarding the $\epsilon$ value in Box 1 or Box 2 we will use the empirical relationship (1) deduced from measured data on heavy nuclei. Furthermore, in Box I the path length ($x_1$) of cosmic rays to reach the solar neighbourhood has been estimated (2). It is noticeable that even if cosmic-ray particles lack any convection motion in Box I, protons with $T \geq 1$ GeV should have a value of $x_1$ less than $0.3 \lambda^1$, where $\lambda^1$ is the mean inelastic interaction length of interstellar protons, to reach the solar neighbourhood. As a result, the proton intensity variation along the magnetic tube should be insignificant, and we will only consider two extreme cases ($V = 0$ and $V = 300$ km/s) to estimate the range of variation of the proton spectrum.

In order to deduce the proton intensity $N_{p12}$ in Box 1 or Box 2 we need to solve the continuity equation of primary protons,

$$ N_{p12} \left( \frac{1}{\lambda_{e12}} + \frac{1}{\lambda_p^1} \right) = q_{p12} + \int_{T_p^1}^{\infty} \frac{dN_{p12}(T', T')}{dT'} P_{p12} \lambda_p^1 dT', \quad (1) $$

where $q_p$ is the source term of primary protons and $dN_p/dT = 1/T'$ is the energy distribution of protons after their inelastic interactions. From shock acceleration theory we have

$$ q_{p12} = k_{0p} p^{-(2+\eta)/\theta_p c}, \quad (2) $$

where $p$ is the proton momentum, $k_{0p}$ is a constant, $\eta = 0.05$ and $\theta_p c$ is the proton velocity. Since at low energies the power law approximation of $N_{p12}$ and the constant approximation of $\lambda_p^1$ cannot be used, an iteration procedure is used to obtain the numerical solution of $N_{p12}$. 
Further, the propagation of interstellar protons along the magnetic tube (Box I) can be described by using a slab model,

$$
\frac{\mathrm{d}N_I(T, x_I)}{\mathrm{d}x_I} = \frac{1}{\lambda_p^i T_p} \int \frac{\mathrm{d}N (T', x_I')}{\mathrm{d}T'} \frac{\mathrm{d}T'}{\lambda_p^i T_p} ,
$$

where the initial value of $N_I$ should be $N_{II}$ after any possibly adiabatic deceleration in the assumed boundary layer $s_1$ (see Fig. 1)(3). At lower energies we also have

$$
x_{I} = 4x_{10} R^{-0.7} (1 + (1 + 2q_s) 1/2)^{-2} ,
$$

where $x_{10}$ is a constant, $q_s = \frac{V_1}{K_0}$ and $K_0$ is the diffusion coefficient of cosmic rays. Similarly, a numerical method is developed to obtain the solution of Eq. (3) in the solar neighbourhood, $N$. Thus the proton intensity predicted for the NUGD model should be

$$
N_{pp} = (1 - \epsilon) N_{ps} + \epsilon N_{pl2} ,
$$

where $\epsilon = 5 \pm 1$ % as deduced from Ref. (2).

It is found that the effect of taking various values of $V$ is negligible, and the resultant cosmic-ray proton spectrum in the solar neighbourhood can be parametrized as

$$
j_{pp} (T_p (\text{GeV})) = \frac{2 \times 10^4}{1 + C_1 T_p} ,
$$

where $C_1 = 15.0 - 6.05 \ln T_p + 2.84 \ln^2 T_p + 0.169 \ln^3 T_p$ .

The predicted $j_{pp}$ spectrum is not only in accordance with the existing proton data at $T_p \geq 10$ GeV, but also is consistent with the low-energy proton data measured in the period of solar minimum(5) (see Fig.2).
The modulated spectra in Fig. 2 are obtained from our predicted \( j \) spectrum by using the force field approximation. It appears that the measured data are consistent with our modulated spectrum with \( \phi = 200 \pm 200 \) MV, where \( \phi \) is the mean energy loss per nucleon in the heliosphere for cosmic-ray nuclei\(^6\).

Finally, in Fig. 3 our deduced interstellar proton spectrum (TA) is compared with the results of other authors (OP\(^1\), MO\(^7\), ZU\(^8\) and JK\(^9\)). It is noticeable that our proton spectrum is close to the demodulated spectra recently suggested in Refs. \(^8\) and \(^9\), indicating the general reasonableness of the demodulation processes used in these works. In conclusion, the parametrized expression of our deduced interstellar proton spectrum should be useful in various cosmic-ray calculations, in particular in the estimation of the production rates of cosmic-ray secondary particles in interstellar space.

References: