ABSTRACT. We compare the simultaneous first-order Fermi shock acceleration of electrons, protons, and alpha particles to solar energetic particle events. For each event, a unique shock compression ratio in the range, $r \approx 1.5$ to 3 produces spectra in good agreement with observation. The range in $r$ predicts that the wide spread in $e/p$ ratios observed at MeV energies is considerably reduced at an assumed injection energy of 100 keV. The model predicts that the acceleration time to a given energy will be approximately equal for electrons and protons and can be on the order of 1 second to $\sim 100$ MeV.

1. Introduction. We model the simultaneous acceleration of protons, alpha particles, and relativistic electrons by first-order Fermi shock acceleration (e.g. Axford et al. 1977). Shock acceleration in solar flares has been considered previously (e.g. Achterberg and Norman 1980; Lee and Fisk 1982; Bai et al. 1983; Lee and Ryan 1984). Here, we compare the predictions of Fermi shock acceleration to solar energetic particle events where interplanetary electron, proton and, in some cases, alpha-particle spectra are available (Lin et al. 1982; Evenson et al. 1984; McDonald and Van Hollebeke 1985; R. McGuire private communication) (see also Ellison and Ramaty 1985). In all cases examined, we find that for any given event, a single shock compression ratio in the range $\sim 1.5$ to 3 simultaneously produces reasonably good fits to the electron, proton, and alpha-particle spectra.

We also determine the effects of the shock strength and injection conditions on the electron to proton ($e/p$) intensity ratio. This ratio is sensitive to the injection conditions and large variations in the $e/p$ ratio at MeV energies result from variations in shock strength. We find that these results are in good agreement with the observations.

The acceleration time for shock acceleration is estimated for typical solar conditions. We find that for scattering mean free paths that scale as the gyroradius, shocks can produce rapid acceleration with equal acceleration times for electrons and protons. This result is consistent with the gamma-ray observations (Chupp 1984).

2. Model. In the test particle limit of shock acceleration with no losses, wherein particles gain energy by scattering freely between the converging upstream and downstream plasmas, the accelerated distribution function (at the shock) is a power law in momentum (Blandford and Ostriker 1978). The spectral index depends only on the compression ratio, $r = u_1/u_2$, where $u_1$ ($u_2$) is the upstream (downstream) bulk plasma flow velocity. The corresponding differential particle intensity is given by

$$\frac{dJ}{dE} \propto n_{\text{inj}} (E/E_1)^{3/2} \left( E + 2E_0c^2 \right)^{-1/2} \left( E + 2E_0c^2 \right)^{-1/2},$$

where $n_{\text{inj}}$ is the density of seed particles injected far upstream and
\( m_0c^2 \) is the rest mass energy. Equation (1) assumes that the particles are injected at kinetic energy, \( E_1 \), where \( E_1 \) is much greater than thermal energy. This expression holds, however, for any injected distribution which is steeper, at a given energy, than the resultant power law. For flatter injected distributions, the shock will boost the intensity of the injected spectrum while maintaining its slope. The limiting non-relativistic and ultra-relativistic spectral indexes are \( \Gamma_{NR} = \frac{1}{2} (r+2)/(r-1) \) and \( \Gamma_{UR} = (r+2)/(r-1) \), respectively. The differential intensity steepens in going from the non-relativistic to the ultra-relativistic regime and the spectral index doubles. If, as is normally the case for solar flare events, non-relativistic proton and relativistic electron spectra are observed, acceleration by a single shock of a given \( r \) produces an electron spectrum steeper than the proton spectrum (Achterberg and Norman 1980).

The compression ratio can be determined from the Rankine-Hugoniot conservation relations. We find that for typical solar conditions, \( r \) lies in the range 1.5 to 3. For example, a coronal shock with \( u_1 = 1000 \) km \( s^{-1} \), temperature \( T_1 = 10^6 \) K, density \( n_1 = 10^9 \) cm\(^{-3}\), and magnetic field perpendicular to the shock normal \( B_1 = 10 \) Gauss, yields \( r \approx 1.54 \). If \( u_1 \) is increased to 2000 km \( s^{-1} \), \( r \approx 2.7 \).

To model effects such as adiabatic deceleration (Lee and Fisk 1982; Lee and Ryan 1984), finite shock lifetimes (Forman 1981), and finite shock sizes (e.g. Ellison 1984), we assume the spectra will turnover approximately exponentially such that, \( dJ/dE = (dJ/dE)_0 \exp(-E/E_0) \), where \( E \) and \( E_0 \) are energy for electrons and protons and energy per nucleon for ions.

For concreteness, we assume the diffusion coefficient of all energetic particles to be, \( \kappa = \kappa_0 (v/v_0)(R/R) \), where \( R = pc/(Ze) \) is rigidity, \( v \) is velocity measured in the local plasma frame, and \( \kappa_0, v_0, \) and \( R_0 \) are constant.

We also assume that the high energy turnover is produced by a shock of finite spatial extent. This implies that the electron turnover energy will be twice that of the protons and that the alpha particle turnover energy will be \( \frac{1}{2} \) that of the protons.

If spectra that are power laws in momentum are plotted as a function of energy, the ratio \( e/p \) will decrease with increasing energy as the electrons become relativistic. The decrease in the \( e/p \) ratio will be greater for smaller \( r \). If shocks of different strengths produce a range of particle spectra, as observed for solar flare accelerated particles, a wide range in \( e/p \) will result at MeV energies even if the injection conditions are similar (see Fig. 3).

Estimates of the acceleration time (Ellison and Ramaty 1985) suggest that the solar environment should allow acceleration of electrons and ions to \( 10^5 \) s of MeV in times on the order of 1 sec. These estimates assume, of course, that the background turbulence has sufficient power to scatter both electrons and protons from injection energies to \( 10^5 \) s of MeV.

3. Comparison with Observations. We have compared the model, using a single set of parameters to simultaneously describe electron, proton, and alpha-particle spectra, to spectra observed in space (see Figs. 1 and 2).

In all cases we choose, for each event, a single compression ratio and \( E_0 \) that produces the best combined fits to the electron, proton, and alpha-particle spectra. We find that: (1) In all cases, the model

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provides an excellent fit to the electron spectra above the assumed injection energy of ~100 keV. (2) In all events where multiple species are observed, a single set of parameters (apart from the normalization) provides a fair description of all spectra. (3) The model provides a natural explanation for events with very flat proton spectra such as 3 June 1982 and can be applied over a wide energy range (see Ellison and Ramaty 1985).

![Electron intensity vs. energy](image1)

**Fig. 1.** Electron intensity vs. energy. The line is the spectrum expected from shock acceleration and is calculated for the r and E shown. E is the injection energy of 100 keV. Data is from Lin et al. (1982).

![Proton intensity](image2)

**Fig. 2.** Proton (crosses) and alpha-particle (diamonds) intensities vs. energy. The data is from IMP 7 and 8 (R. McGuire private communication).

The essential point of the spectral fits is that a single model with a single set of shock parameters produces a reasonable fit to all species. In some cases, in fact, all spectra are fit by a single parameter, r.

A wide range in e/p ratios is observed at MeV energies (Lin et al. 1982; Evenson et al. 1984). Shock acceleration can produce a spread in e/p in two ways. First, steep spectra produce a smaller e/p ratio at high energies than flat spectra assuming the same injection conditions. Second, the injection conditions may vary for electrons and protons.
Using the data of Evenson et al. (1984), we have plotted in Fig. 3 the e/p ratio measured in the energy interval 25-45 MeV (crosses) versus the relativistic electron spectral index, \( \Gamma_e \). There is a very clear correlation showing that flatter spectra generally produce higher e/p ratios. We also show with the lower solid lines, the predicted correlation between e/p and spectra index or, equivalently, \( r \). The two lines indicate a factor of 10 spread in injection conditions.

The solid dots in Fig. 3 show the e/p ratios measured at ~ 33 MeV extrapolated to the assumed injection energy of 100 keV. A strong reduction in the spread of e/p results. Gamma-ray events are identified in Fig. 3 with a "G" and it is clear that they tend to have the largest injection e/p ratios and the largest compression ratios.

4. Conclusions. We find that shock acceleration can model many solar flare energetic particle spectra with a single set of parameters for both electrons and protons. The production of power laws in momentum by shocks provides a natural explanation for much of the large spread in the observed e/p ratios at MeV energies. Events that produce gamma rays can be accommodated in this model and do not need to be treated as a distinct class of events. We also find that shocks should allow the acceleration of both electrons and protons to 100's of MeV in seconds.

5. Acknowledgements. We are grateful to Bob McGuire for allowing us to use unpublished data supported, in part, by a NASA STTP grant and NSF Grant AST-83-17755.

References