DETERMINATION OF THE PITCH-ANGLE DISTRIBUTION AND TRANSVERSE ANISOTROPY OF INTERPLANETARY PARTICLES

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1. INTRODUCTION
We present a method to determine the directional differential intensity (d.d.i.), expressed in terms of spherical harmonics, from sectored particle data, concurrent interplanetary magnetic field (IMF) and solar-wind velocity. In Section 2, we show the relation between the d.d.i. and the mean sector count rates \( X_1 \). In Section 3, we show how to estimate the d.d.i. from the measured \( X_1 \) and the associated errors due to Poisson statistics. In Section 4, using the above method, we determine the pitch-angle distribution and the transverse anisotropy of the d.d.i. of low energy protons for the Day 118, 1978 solar particle event. In Section 5, we discuss an interesting correlation between the transverse anisotropy and the IMF direction.

2. RELATION BETWEEN DIRECTIONAL INTENSITY AND SECTOR RATES
We express the particle directional differential intensity as

\[
J(p, \theta, \phi) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} P_m^{(n)}(\cos \theta) \left\{ A_{nm}(p) \cos m\phi + B_{nm}(p) \sin m\phi \right\},
\]

where \( P_m^{(n)} \) are the associated Legendre functions, \( p \) is the particle momentum, \( \theta \) is pitch-angle and \( \phi \) is gyrophase in the standard coordinate system (Ng, 1985). When the telescope points in the direction \((\gamma, \eta)\), it measures the differential count rates

\[
C(p, \gamma, \eta) = \int_{\Omega} J(p, \theta, \phi) \sin \theta \, d\theta \, d\phi,
\]

where, following Sentman and Baker (unpublished manuscript), we express the angular response function of the particle telescope as

\[
S(\theta') = \sum_{k=0}^{\infty} S_k P_k(\cos \theta') = \sum_{k=0}^{\infty} S_k \left\{ P_k(\cos \gamma)P_k(\cos \theta) + \right. \\
+ 2 \sum_{m=1}^{\infty} \left[ \frac{(k-m)!}{(k+m)!} \right] P_m^{(k)}(\cos \gamma)P_m(\cos \theta) \cos m{}'(\gamma-\phi) \right\}.
\]

It follows from the orthogonality of the spherical harmonics that

\[
C(p, \gamma, \eta) = 4\pi \sum_{n=0}^{\infty} \left[ S_n/(2n+1) \right] \sum_{m=0}^{\infty} \left\{ A_{nm}(p) \cos m\eta + B_{nm}(p) \sin m\eta \right\} P_m(\cos \gamma).
\]

As the telescope sweeps over sector 1, we average (4) to obtain \( X_1(p) \) the mean differential count rate over sector 1,

\[
X_1(p) = 4\pi \sum_{n=0}^{\infty} \left[ S_n/(2n+1) \right] \sum_{m=0}^{\infty} \left\{ A_{nm}(p) \right\} \left\{ P_m(\cos \gamma) \cos m\eta \right\} + \right.
\]

\[
+ B_{nm}(p) \left\{ P_m(\cos \gamma) \sin m\eta \right\} \right\},
\]

where \( \gamma' = (1/\Delta \gamma) \int_{\gamma'_{i+1}}^{\gamma'_{i+1}} d\gamma \), \( \gamma' \) is the spacecraft (s/c) spin-angle measured from the \( \gamma' \) projection of the IMF onto the spin plane, and \((\gamma'_{i}, \gamma'_{i+1})\) defines sector 1 (see the 2nd coordinate system in Fig. 1). For multiple-telescope systems (Sanderson & Hynds, 1977), eqn (5) should
be repeated for each telescope.

We now illustrate by specialising (1) and (5) to 8-sectored data for a telescope sweeping in the spacecraft's spin plane:

\[ j(p, \theta, \phi) = \sum_{n=0}^{4} A_{n}P_{n}(\cos \theta) + \sum_{n=1}^{3} A_{n+1}P_{n}(\cos \theta) \cos \phi, \]  
\[ X_{i} = 4\pi \sum_{n=0}^{4} S_{n}A_{n}X_{n}/(2n+1) + 4\pi \sum_{n=1}^{3} S_{n}A_{n+1}X_{n}/(2n+1), \]

where

\[ Q_{n} = \sum_{j=0}^{n} a_{j} \sin^{j} \theta_{B} \cos^{j} \psi_{i}, \]  
\[ Q_{n} = \sum_{j=0}^{n} b_{j} \sin^{j} \theta_{B} \cos^{j} \psi_{i}, \]

\[ a_{j} = \text{coefficient of } x^{j} \text{ in } P_{n}(x), \]
\[ b_{j} = \text{coefficient of } x^{j} \text{ in } P_{n}(x)/(1-x^2)^{1/2}, \]
\[ \theta_{B} = \text{angle between IMF and s/c spin-axis}, \]
\[ (\hat{W}_{1x}, \hat{W}_{1y}, \hat{W}_{1z}) \text{ denotes a unit vector in the direction of } \mathbb{E}_{x}\mathbb{B}, \]
\[ (\text{Fig. 1}). \]
Note that the integrations in (8) and (9) may be performed readily in closed form.

3. Determination of \( j \) from Measured \( X_{i} \)

To simplify notation in the following, let

\[ D_{n} = A_{n+1} \quad (n=0,4), \quad D_{n} = A_{n+4,1} \quad (n=5,7), \]  
\[ R_{n} = Q_{n} \quad (n=0,4), \quad R_{n} = Q_{n+4} \quad (n=5,7). \]

We least-square fit

\[ X_{i} = \sum_{n=0}^{7} C_{n}R_{n}, \quad (i=0,7), \]

to the 8 measured sector rates \( \bar{X}_{i} \). This yields

\[ \sum_{j=0}^{7} H_{n,j}C_{j} = \sum_{i=0}^{7} R_{i}X_{i}, \quad (n=0,7), \]

where

\[ H_{n,j} = \sum_{i} R_{n}R_{j}, \quad (n=0,7; j=0,7). \]

Inverse (13), we have

\[ C_{n} = \sum_{n=0}^{7} H_{n,j}C_{j} = \sum_{i=0}^{7} R_{i}X_{i}, \quad (n=0,7), \]

and hence

\[ D_{n} = \sum_{i=0}^{7} M_{n,i}X_{i}, \quad (n=0,7), \]

where \( M_{n,i} \) is ultimately expressed in terms of \( Q_{n}, Q_{n+1} \), and \( S_{n} \). Using (16), we may determine the coefficients \( A_{n,1} \) in (6) by a matrix multiplication into the measured sector rates. When the IMF projection lies on a sector boundary, the symmetric matrix \( H_{n,j} \) becomes singular. So we drop the \( A_{n,0} \) term in (6) whenever the IMF projection comes within 2° of a sector boundary.

Suppose that \( K_{i} \) counts are registered over the time interval \( t_{c} \) in sector \( i \). Assuming Poisson statistics, we estimate

\[ X_{i} = K_{i}/t_{c}, \quad \text{var}(X_{i}) = K_{i}/t_{c}^{2}, \quad \text{var}(D_{n}) = \sum_{i} M_{n,i}^{2} \text{var}(X_{i}). \]  

If we define the anisotropy \( \xi_{n} = D_{n}/D_{0} \), then, providing the counts are not too low,

\[ \text{var}(\xi_{n}) \approx (1/\bar{D}) \sum_{i} (M_{n,i} - \xi_{n} M_{0,i})^{2} \text{var}(X_{i}). \]  

Systematic errors are, of course, much harder to estimate.
4. APPLICATION  As an example, we show in Fig. 2 the 15-min averages of $A_{00}$, $\xi_{10}$, $\xi_{20}$, $\xi_{30}$, and $-\xi_{11}$ for 1.4 - 2.5 MeV protons, determined
with the above method using 8-sectored particle data (P.I.: R.E. Vogt, CalTech), concurrent IMF (P.I.: N.F. Ness, GSFC), and hourly solar wind speed (P.I.: H. Bridge, MIT), measured aboard IMP-8 and accessed through NSSDC. Some typical standard errors due to Poisson statistics only are indicated by vertical bars.

Estimating the spectral slopes by using the corresponding results for 4-13 MeV protons, we have found the Compton-Getting corrections (Ng, 1985) for transformation to the ExB drift frame to be small, <.002 A00, <.01, <.01, <.02 for A00, E10, E20, and E30 respectively. (For transformation to the solar wind frame, the corrections are of the order of 0.04 A00, 0.1, 0.3, and 0.5 respectively). Thus A00, E10, E20, E30 essentially characterise the pitch-angle distribution in the ExB drift frame.

5. DISCUSSION The Compton-Getting correction, \( \xi(H) (3 - pA00/A00 + pA00/5A00) \) to the transverse anisotropy \( -\xi_{11} \) is indicated by the dots in Fig. 2. The observed anisotropy varies in phase with this correction but is much larger in magnitude. The same feature, even more marked, is seen for 4-13 MeV protons. What is the cause of this large discrepancy?

For Fig. 2, IMP-8's GSE coordinates in R \( \pm \) varies from (21.6, 21.5, 5.5) to (5.7, 29.5, 18.1). The times when the IMF is connected to the nominal bow shock (BS) are indicated by horizontal bars in Fig. 2 (Ng & Roelof, 1977). At \( \sim 1537 \) UT Day 119, some solar particles with guiding centres below the IMF through IMP-8 are probably shadowed by the nose of the BS, resulting in the observed peak value of \( -\xi_{11} = 0.53 \). However BS connection does not account for the general variation of \( -\xi_{11} \) in Fig. 2.

Close inspection reveals that the sector plot of \( \xi_1 \) lags behind the IMF in directional changes. Hence a tentative explanation is that some observed 15-min averages contain a substantial fraction of non-gyrotropic distributions which reside a short distance (\( \sim 1 \) gyroradius) beyond a 'kink' in the IMF. An alternative explanation is as follows. When \( \Theta_0 \) swings rapidly in an averaging interval such that its average value is close to one end of the range of values, then a field-aligned anisotropy \( \xi_{10} \) would "induce" a nonzero value of \( -\xi_{11} \), i.e., the apparent value of \( -\xi_{11} \) is not real. Further studies with smaller averaging intervals would clarify this matter.

6. CONCLUSION We have shown how to obtain the directional differential intensity referred to the standard coordinate system (Ng, 1985) from sectored particle data and concurrent IMF and solar wind data. The corrections for transformation to the ExB drift frame are explicitly calculated and found to be small for these \( \sim 1.5 \) MeV protons. However, the new correction formulae would be important for \( \lesssim 500 \) keV protons. It is tentatively suggested that the 'observed' transverse anisotropy may in large part be induced by a rapidly changing IMF in the presence of a field-aligned anisotropy.

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References