ON THE COSMIC RAY DIFFUSION IN A VIOLENT INTERSTELLAR MEDIUM

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1. Introduction. A variety of the available observational data on the cosmic ray (CR) spectrum, anisotropy and composition are in good agreement with a suggestion on the diffusion propagation of CR with energy below 10^15 eV in the interstellar medium (see Ginsburg and Ptuskin, 1983). The magnitude of the CR diffusion coefficient and its energy dependence are determined by ISM magnetic field spectra.

Direct observational data on magnetic field spectra are still absent. In this work we present a theoretical model to the turbulence generation in the multiphase ISM. Our model is based on the multiple generation of secondary shocks and concomitant large-scale rarefactions due to supernova shock interactions with interstellar clouds. We derive (also see Bykov and Toptygin, 1985) the distribution function for ISM shocks with account for supernova statistics, diffuse cloud distribution and various shock wave propagation regimes. This allows us to calculate the ISM magnetic field fluctuation spectrum and CR diffusion coefficient for the hot phase of ISM.

2. The ISM turbulence model. Current observational data on the structure of ISM turbulence have been summarized in the review by Armstrong et al. (1981). They have concluded that a variety of data obtained by different methods do not contradict to the assumption on the presence of continuous spectrum of ISM inhomogeneities in a wide scale range. The energy density dependence on wavenumber k takes the form

\[ W(k) \propto k^{-\alpha}, \]

where \(1.4 \leq \alpha \leq 1.8\) for \(10^{-20} < k < 10^{-8}\) cm \(^{-1}\) (Armstrong et al., 1981).

The ISM is characterized by a wide variety of phase states, so that the ISM turbulence is highly nonuniform. The possibility of developing an inertial range of weak MHD turbulence produced by energy cascading from long wavelengths to short wavelengths under the ISM conditions has been discussed by McIvor (1977). This discussion is based on the assumption about collisional damping of MHD waves. We remark (Bykov and Toptygin, 1983) that the weak MHD turbulence spectrum can be developed by the energy cascading if only the linear Landau damping of collisionless MHD waves is effectively suppressed. The above suppression can be realized due to the distortion of the thermal plasma distribution function by finite amplitude waves (\(\delta B / B \approx 0.1\)) with scales \(1 \sim 3 \times 10^{17}\) cm. If such waves are absent, the energy cascading of weak MHD turbulence seems to be highly unlikely as well as the appearance of some inertial range in the hot ISM.

On the other hand, the fluctuations of ISM magnetic field and electron density may be connected with the presence of strong turbulence. Here we treat the strong turbulence as an ensemble of noncoherent weakly interacting nonlinear
waves. For such a system the energy redistribution between wave-scales are determined by the evolution of nonlinear waves. The strong turbulence of Langmuir solitons has been discussed by Rudakov and Tsytovich (1978).

In the present paper we propose the model for a strong ISM turbulence. The main structural elements of our model are nearly spherical collisionless shock waves with rarefaction regions in the inner parts of the spheres. Such structures may be formed due to multiple interactions of strong supernova shocks with interstellar clouds. The total energy input to the ISM turbulence in our model can be estimated as 10% from the kinetic energy of large-scale motions produced by supernovae and stellar wind of OB stars (about $10^{-27}$ erg cm$^{-3}$ s$^{-1}$). Now we obtain the distribution function for ISM shocks of different strengths (also see Bykov and Toptygin, 1985).

3. ISM shock wave distribution function. Theoretical consideration of an average distribution of ISM shocks is important to study the ISM structure (McKee and Ostriker, 1977; Heathcote and Brand, 1983) and GR propagation (Blandford and Ostriker, 1980; Axford, 1981; Ginzburg and Ptuskin, 1984). We define the shock strength distribution function $P(M)$ as an average number of passings of shocks with Mach numbers $M_i$, $M + dM$ through an arbitrary point of ISM disk per unit time (cf. Blandford and Ostriker, 1980). Supernova explosions with local rate $S$ ($\sim 10^{-13}$ pc$^{-1}$ yr$^{-1}$) produce strong primary shock waves in the ISM. The primary shock evolution is described by selfsimilar spherical solutions with radius

$$R(M) = R_0 M^{-\alpha/3}$$

where $R_0$ depends on the energy release and ambient gas parameters. The exponent $\alpha$ is equal to $\alpha = 2$ for the Sedov solution; $\alpha = 4.5$ for the McKee and Ostriker solution, and $\alpha = 1.2$ for the snow-plow model.

It is known that an average number of passings of primary shocks with Mach numbers $M > M$ through an arbitrary point of ISM per unit time is given by (see e.g. Blandford and Ostriker, 1980; Axford, 1981)

$$P(M) = \frac{S}{3} R_0^3 M^{-\alpha/3}$$

Thus, $P^{(1)}(M) = -dP/dM$. Primary shock propagation through ISM with embedded diffuse clouds will be accompanied by generation of secondary shocks (e.g. see Spitzer, 1982). The number of secondary shocks produced by a primary shock will be about $10^{(4)}$, provided the cloud number density $n_{cl} \approx 5 \times 10^{-4}$ pc$^{-3}$, and $R \approx 10^{(2,3)}$ pc. The distribution function of secondary shocks (without account for secondary shock reflection from clouds) takes the form:

$$P^{(2)}(M) = 3 n_{cl} \int_1^{\infty} \int_1^{\infty} \frac{dM_1}{M} \frac{dM_2}{M} P^{(1)}(M_1) \delta \left( M_1 - \frac{\frac{3M_2^2 + 1}{5M_2^2}}{\frac{M_1}{M - 1}} \right) \frac{(M_0 - 1)^3}{(M - 1)^4},$$

(3)
where $f_1$ is the diffuse cloud filling factor in the ISM. This formula takes account for shock reflection from dense sphere and the secondary shock propagation regime. For simplicity, we consider a perfect gas with $\gamma = 5/3$ and take into account that secondary shock will be weak (i.e. $M_s - 1 < 1$) if the primary shock has $M_s \leq 3$ (for more detailed discussion see Bykov and Toptygin, 1985).

As a result, we derive an approximate distribution function with account for the weak secondary shock contribution:

$$
P(M) = \alpha S \frac{4\pi}{3} R_0^3 \left[ M^{-\alpha - 1} + 3 f_{cl} C(3; \alpha)(M^{-1})^{-4} \right],
$$

where $C(3; 4, 5) \approx 4.1 \cdot 10^{-3}$. It should be noted the strong dependence of $P$ on $M$ determined by the secondary shock contribution. Among the waves with $M \leq 1, 2$, secondary shocks are dominant. Multiple interactions of secondary shocks with clouds can be taken into account by iterations of eq. (3), i.e. by series expansion of $P(M)$ with respect to $f_{cl}$.

4. ISM fluctuation spectra. In the discussed model nearly spherical shocks with rarefaction regions in the inner parts of spheres represent well-defined structural cells. The ISM velocity, density and magnetic field fluctuations are determined by noncoherent overlapping of a large number of such structures. For statistically homogeneous and isotropic ensemble of shocks (with strength $M$) and concomitant rarefaction the spectral energy density takes the form:

$$
W^*(k, M) = \left( \frac{4}{\pi} \right)^{1/2} \frac{1}{C_2} (M - 1)^2 K^2 R_0^3 M \left[ 1 + k^2 R_0^2 \right]^{-2}
$$

where $C(3; 4, 5) \approx 4.1 \cdot 10^{-3}$, where $\delta$ is the shock front thickness. For collisionless weak shocks in the hot ISM $\delta \approx 10(12)$ cm, Eq. (5) quite satisfactorily extrapolates the well-known relations at $kR \gg 1$ and $kR \ll 1$ (Bykov and Toptygin, 1985).

To evaluate the spectral energy density, $W(k)$, one should average Eq. (5) over an ensemble of shocks with different strengths. This yields $W(k) = W^{(1)}(k) + W^{(2)}(k)$, where

$$
W^{(1)}(k) = \int_{\Lambda} W(k, \mu) d\mu,
$$

$$
W^{(2)}(k) = \int_{\Lambda} f_{cl} \int_{\Lambda} \int_{\Lambda} d\mu_1 d\mu_2 d\mu_3 P^{(1)}(\mu_1) P^{(1)}(\mu_2) W(k, \mu_3) \left( \frac{M_3 - 1}{M_3 - 1} \right)^2 \delta(M_3 - \sqrt{\frac{3M_3^2 + 1}{5 - M_3^2}}).
$$

Let us present the obtained expression for $W(k)$ in the short - wavelength limit ($kR_o \gg 1$) for the McKee and Ostriker (1977) model ($\alpha > 3$):

$$
W(k) = \left( 4 \alpha c_s^2 / \pi R_0 \right) \left[ 1 + 3 f_{cl} C(3; \alpha) \frac{R_o}{R_{cl}} \ln \left( \frac{\sqrt{5} - 1}{M_s - 1} \right) \right] k^{-2}
$$

$R_{cl}$ being the mean radius of clouds; $(M_s - 1) \approx 10(-2)$; $C(2; 4, 5) \approx 9.4 \cdot 10^{-3}$. The long-wavelength spectral range has been considered earlier by the authors. The magnetic fluctuation
spectrum appears to be similar to the velocity fluctuation spectrum given by Eqs. (6), (7).

Thus, Eqs. (6) determine the fluctuation spectrum in the model of turbulence produced by shocks from supernova explosions. The main mechanism of energy redistribution over wavelengths is that due to the generation and subsequent self-similar evolution of secondary shocks.

5. The CR diffusion. The magnitude of the CR diffusion coefficient along the field lines \( \chi_{\parallel} \) and its dependence on the energy \( E \) of energetic particles are determined by the magnetic fluctuation spectra (e.g. Toptygin, 1985),

\[
\chi_{\parallel} = \frac{C G(\nu)}{2} \left( \frac{B_0}{\delta B} \right)^{(\nu+3)/4} (E/\varepsilon R_0 B_0)^{2-\nu} R_0,
\]

where the effect of large-scale magnetic fields on the particle scattering at \( \nu \approx \Pi/\lambda \) is taken into account. According to our result (7) for the McKee and Ostriker (1977) model, the spectral turbulence index \( \nu = 2 \). Hence, it follows from Eq. (8) that the CR diffusion coefficient along the regular magnetic field is energy-independent as long as \( E \sim e R_0 B_0 \).

For the above mentioned parameters of the model we have

\[
\chi_{\parallel} \approx 5 \times 10^{28} \text{ cm}^2 \text{s}^{-1}
\]

which is in satisfactory agreement with observational data on CR anisotropy and abundance.

References


Toptygin I.N. Cosmic Rays in Interplanetary Magnetic Fields.