

Selfsimilar Time dependent Shock Structures

R. BECK & L. O'C. DRURY
Max-Planck-Institut für Kernphysik,
Postfach 103980, D-6900 Heidelberg.

1 Introduction

Diffusive shock acceleration as an astrophysical mechanism for accelerating charged particles has the advantage of being highly efficient. This means however that the theory is of necessity nonlinear; the reaction of the accelerated particles on the shock structure and the acceleration process must be self-consistently included in any attempt to develop a complete theory of diffusive shock acceleration. Considerable effort has been invested in attempting, at least partially, to do this and it has become clear that in general either the maximum particle energy must be restricted by introducing additional loss processes into the problem or the acceleration must be treated as a time dependent problem (Drury, 1984).

In stationary modified shock structures where a significant part of the downstream pressure is carried by the energetic particles, because these are in part relativistic and have a softer equation of state than a perfect monatomic gas, the compression ratio of the shock will exceed four and tend towards the limiting value of seven appropriate to a strong shock in a relativistic gas. If, as is almost always the case, the mean free path of an energetic particle and hence the diffusion coefficient increases with particle energy, the diffusion length scale of the particles will be much greater than the thickness of the shock structure above a certain energy. The acceleration of these particles can then be described by test particle theory which applied to a shock of compression greater than four implies a spectrum of accelerated particles flatter than $f(p) \propto p^{-4}$ where p is the particle momentum. Thus without an upper cutoff the energetic particle pressure and energy density will diverge.

We conclude that stationary modified shock structures can only exist for strong shocks if additional loss processes limit the maximum energy a particle can attain. This is certainly possible and if it occurs the energy loss from the shock will lead to much greater shock compressions. It is however equally possible that no such processes exist and we must then ask what sort of nonstationary shock structure develops. The same argument which excludes stationary structures also rules out periodic solutions and indeed any solution where the width of the shock remains bounded. It follows that the width of the shock must increase secularly with time and it is natural to examine the possibility of selfsimilar time dependent solutions.

2 Equations

Our basic idea is that the upper cutoff in the particle energy spectrum is determined through the finite acceleration time scale by the age of the shock. From test-particle theory we know that the cutoff momentum p after time t is given implicitly by (Axford, 1981).

$$t \approx \frac{3}{U_- - U_+} \int_0^p \kappa(p') \left(\frac{1}{U_-} + \frac{1}{U_+} \right) \frac{dp'}{p'}$$

where U_-, U_+ are the upstream and downstream velocities of the plasma relative to the shock and κ is the diffusion coefficient. Thus if the diffusion coefficient has a powerlaw dependence on momentum, $\kappa \propto p^\alpha$ with $\alpha > 0$, the diffusion coefficient of the maximum energy particles is of order $\alpha t U^2$ and increases linearly with time.

If these particles carry significant amounts of energy, as strongly suggested by the high energy divergence of steady solutions, the associated modification of the shock structure will also have a length scale which increases linearly with time. This suggests the possibility of selfsimilar solutions depending on the similarity variable $\xi = x/t$. Indeed on making this ansatz and writing for the effective diffusion coefficient of the high energy particles $\kappa(x, t) = \tilde{\kappa}(\xi)t$ the equations of ideal energetic particle hydrodynamics (see paper OG 8.1-5) reduce to the system of ordinary differential equations,

$$\begin{aligned} (U - \xi) \frac{\partial \rho}{\partial \xi} + \rho \frac{\partial U}{\partial \xi} &= 0, \\ (U - \xi) \frac{\partial U}{\partial \xi} + \frac{1}{\rho} \frac{\partial}{\partial \xi} (P_g + P_u) &= 0, \\ (U - \xi) \frac{\partial P_g}{\partial \xi} + \gamma_g P_g \frac{\partial U}{\partial \xi} &= 0, \\ (U - \xi) \frac{\partial P_u}{\partial \xi} + \gamma_u P_u \frac{\partial U}{\partial \xi} &= \frac{\partial}{\partial \xi} \tilde{\kappa} \frac{\partial P_u}{\partial \xi}, \end{aligned}$$

where ρ denotes the density, P_g the pressure, U the velocity of the background plasma and P_u the pressure of the ultrarelativistic particles near the upper cutoff. In the related problem for stationary solutions the diffusion coefficient can be eliminated by a change of independent variable but the adiabatic exponent of the energetic particles is uncertain; here $\tilde{\kappa}$ cannot be eliminated, but for the ultrarelativistic particles the adiabatic exponent $\gamma_u = 4/3$.

This system is easily found to have singularities at the points where

$$(U - \xi) \left[(U - \xi)^2 - \frac{\gamma_g P_g}{\rho} \right] = 0,$$

i.e. where the Doppler shifted velocity (note that we have transformed to an expanding coordinate system) is zero or equal to the local sound speed. A major distinction between such time dependent selfsimilar solutions and stationary solutions is that these singularities cannot be avoided; if we start with some asymptotic positive value of U far upstream, $U \rightarrow U_-$ as $\xi \rightarrow -\infty$, and integrate into the shock U decreases monotonically and ξ increases monotonically so that at some point $U - \xi$ will fall to the local sound speed. Thus selfsimilar shock structures always contain a subshock (whereas in the stationary case the high Mach number solutions are usually smooth).

The matching conditions at this subshock are essentially the same as those in the stationary case, namely that the energetic particle pressure be continuous and that the jump in the energy flux (if any) be positive and result from injection at the subshock,

$$[P_u] = 0, \quad \left[\frac{\gamma_u}{\gamma_u - 1} P_u (U - \xi) - \frac{\tilde{\kappa}}{\gamma_u - 1} \frac{\partial P_u}{\partial \xi} \right] = Q,$$

where Q is the injection energy flux. The solution downstream from the subshock must be spatially homogeneous, at least as far as the point where $U = \xi$ where a contact discontinuity may occur.

In the stationary case the subshock jump conditions for the plasma properties are simply the ordinary Rankine–Hugoniot conditions applied to a gas with $\gamma_g = 5/3$ and the injection energy flux, though hard to estimate, is probably sufficiently small to be ignored. However in the selfsimilar case the subshock is itself probably modified by the reaction of mildly relativistic particles and the injection flux may be significant.

3 Results

The above system of equations can be integrated in close analogy to the stationary system. In an earlier report on this work (Beck & Drury, 1984) we considered the problem of constructing stationary modified structures for the subshock, but neglected injection. This was found to be possible for moderate Mach numbers, but in general could not be done for shocks with high Mach numbers. The problem is that for strong shocks the subshock, though weaker than the main shock, is still so strong that we return to the problem from which we started; the difficulty of constructing consistent modified structures for stationary shocks. The additional freedom allowed by including the injection flux resolves this problem, but at the expense of an extra unknown parameter (which one can however attempt to estimate using physical arguments based on the concept of selfregulating injection, *cf* Eichler 1981, 1984).

An interesting feature of the timedependent solutions is that the total shock compression is usually large, of order 10 or 20, so that both methods of resolving the divergence problem in strong shocks, the introduction of additional loss mechanisms or the explicit inclusion of secular broadening, lead to the same qualitative conclusion; that strong modified shocks are expected to have large compression ratios. This of course implies that the acceleration is efficient at accelerating particles so that, at least qualitatively, we have a consistent picture.

References

- Axford, W. I. 1981 *Proc 17th ICRC, Paris*, **12** 155.
 Beck, R. and Drury, L. O'C. 1984 . . *Plasma Astrophysics, Varenna, (edited by T. D. Guyenne and J. J. Hunt)*, ESA SP-207 181.
 Drury, L. O'C 1984 *Adv. Space Res.*, **4** 185.
 Eichler, D. 1981 *Ap. J.*, **244** 711.
 Eichler, D. 1984 *Ap. J.*, **277** 429.